Status of 704 MHz cavity design for the Muon Cooling Demonstrator

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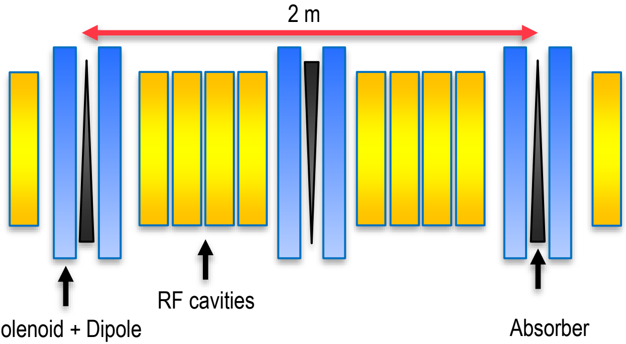
Outline

- Introduction
  - Motivation of the study
- Eigenmode RF simulations in CST Studio Suite®
  - 704 MHz pillbox cavity
  - 704 MHz pillbox-type cavity for the Muon Cooling
- RF-thermo-mechanical simulations in COMSOL Multiphysics®
  - Coupled RF-thermo-mechanical studies on the cavity
  - Lorentz Force Detuning (LFD) analysis
- Conclusions and Perspectives

Motivation of the study

- Build a conceptual RF design and Multiphysics numerical model of a 704 MHz pillbox-type cavity for the Muon Cooling Demonstrator

Preliminary Cooling Cell Concept

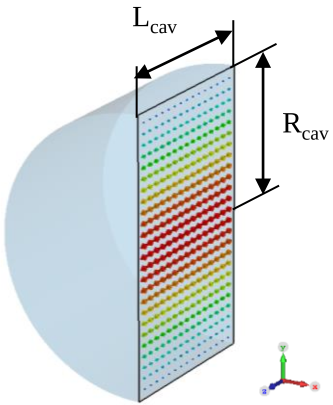


Chris Rogers

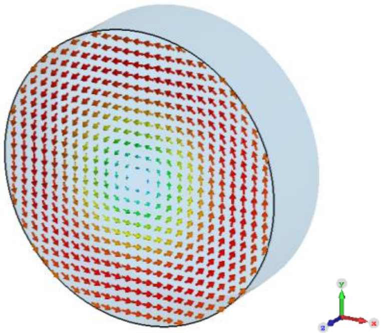
- Normal conducting cavities
- $f_0 = 704$ MHz
- Short RF pulses ($\sim\mu s$)
- High peak accelerating gradient ($E_{\mathrm{pk}} = 44\ \mathrm{MV/m}$)
- High peak solenoid field (up to 14 T)

704 MHz Pillbox cavity

- Vacuum model simulated in CST according to the beam dynamics specification given by Chris Rogers.



E-field



H-field

Beam dynamics specifications

Parameter	Unit	Pillbox - Cu	Description
f_0	[MHz]	704	Operating frequency
L_{cav}	[mm]	125	Cavity length
R_{cav}	[mm]	163.10	Cavity radius
E_{nom}	[MV/m]	44	Average Nominal gradient
V_{nom}	[MV]	5.50	Nominal voltage
β	[-]	0.884	Relativistic beta
TTF	[-]	0.83	Transit-time factor
V_{acc}	[MV]	4.56	Accelerating voltage
E_{acc}	[MV/m]	36.44	Accelerating gradient

Transit-Time factor

$$TTF = \frac{\int_{z_{\min}}^{z_{\max}} \underline{E_z}(0,0,z)e^{jkz}dz}{\int_{z_{\min}}^{z_{\max}} \underline{E_z}(0,0,z)dz} \quad \text{with} \quad k = \frac{\omega}{\beta c}$$

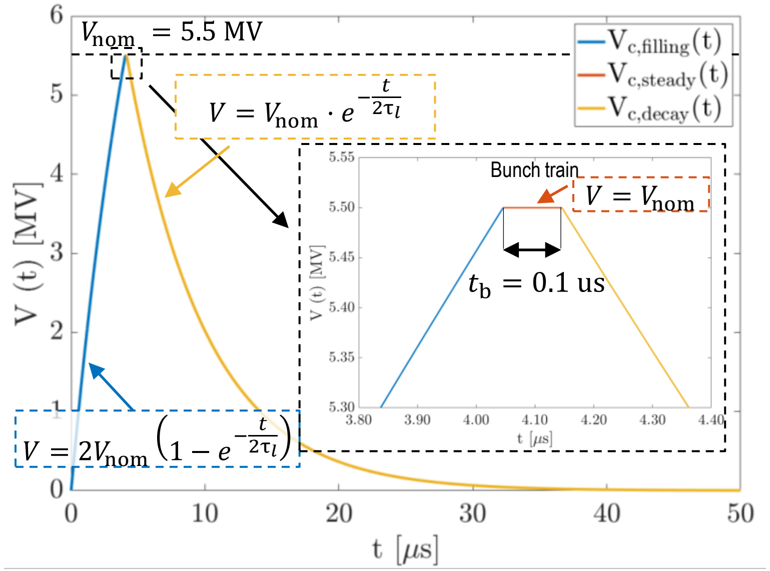
Nominal and accelerating voltage

$$V_{\text{nom}} = E_{\text{nom}} L_{\text{cav}}$$

$$V_{\text{acc}} = TTF V_{\text{nom}}$$

704 MHz Pillbox cavity – Pulse shape and power dissipation

- The voltage profile was calculated for a short beam pulse, considering the coupling factor $\beta_{coupling}$=1.2
- The average power dissipation was computed from the obtained duty factor (DF)



Parameter	Unit	Pillbox - Cu	Description
f_0	[MHz]	704	Operating frequency
Q_0	[-]	2.84e+04	Intrinsic quality factor
*R/Q	[Ω]	194.73	Geometric shunt impedance
$R/Q \cdot Q_0$	[MΩ]	5.53	Shunt impedance (σ_{Cu} = 5.8e+07 S/m)
P_{diss}	[MW]	3.75	Peak power dissipated on the cavity walls
$\beta_{coupling}$	[-]	1.2	Coupling factor
τ_l	[us]	2.92	Filling time constant
$t_{filling}$	[us]	4.05	Total filling time (f_{rep} = 5 Hz)
DF	[-]	7.00e-05	Beam duty factor
P_{ave}	[W]	262.6	Average power

Filling time

$$t_{filling} = \tau_l \ln(4) = \frac{Q_0}{\omega_0(1+\beta_{coupling})}\ln(4)$$

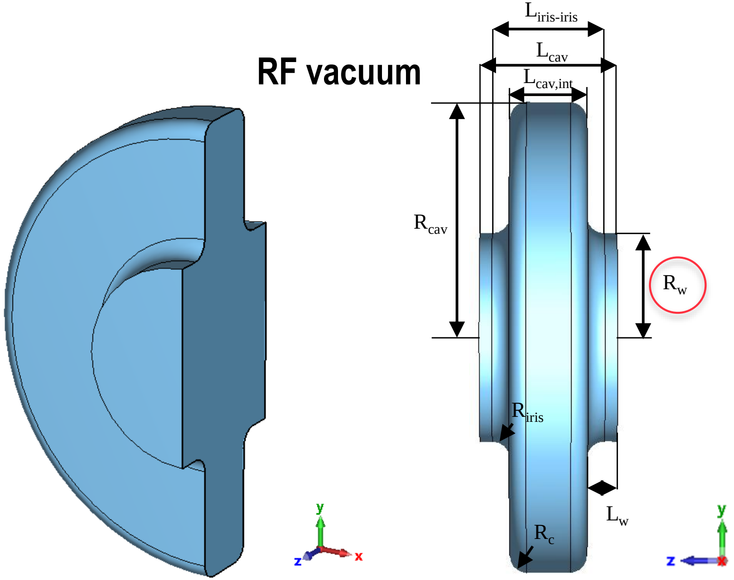
Duty factor

$$\mathrm{DF} = \frac{P_{ave}}{P_{diss}} = \frac{\int V(t)dt}{V_{acc}^2/(R/Q \cdot Q_0)}$$

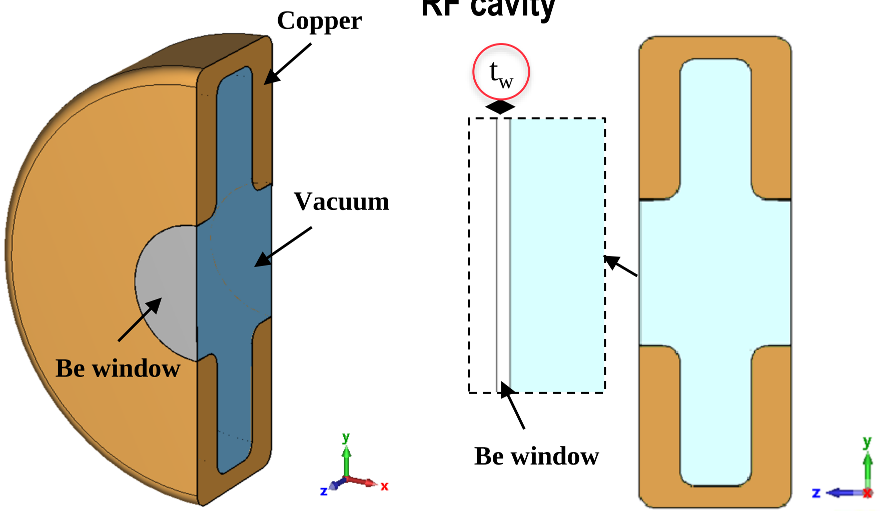
704 MHz Pillbox-type cavity for the Muon Cooling

- The 704 MHz pillbox-type cavity was built based on a design similar to the LBNL 805 MHz pillbox cavity.
- Beryllium (Be) window: 60 mm ($3\sigma_{beam}$) or 120 mm radius (**R_w**); 60 um or 120 um thickness (**t_w**)

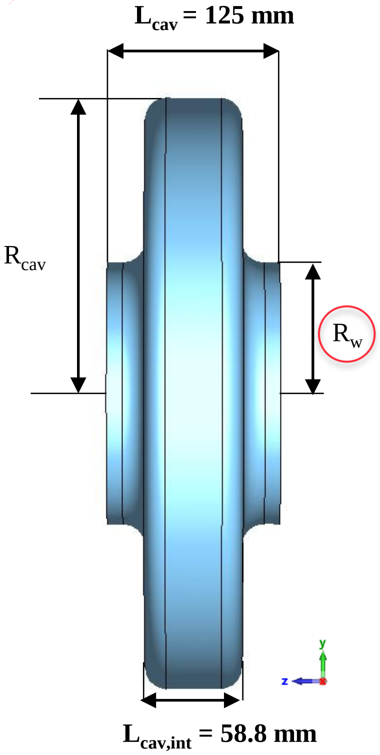
RF vacuum



RF cavity



704 MHz Pillbox-type cavity for the Muon Cooling

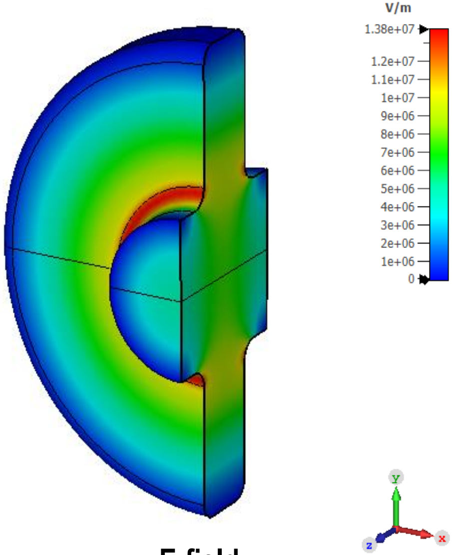


Parameter	Unit	MC Cu Cavity (R_w = 60 mm)	MC Cu Cavity (R_w = 120 mm)	Description
f_0	[MHz]	704	704	Operating frequency
R_{cav}	[mm]	175.66	191.00	Cavity radius
t_w	[um]	60-120	60-120	Be window thickness
Q_0	[-]	1.82E+04	2.02E+04	Intrinsic Quality Factor
R/Q	[Ω]	98.14	159.32	Geometric shunt impedance
$R/Q \cdot Q_0$	[MΩ]	1.79	3.22	Shunt impedance
P_{diss}	[MW]	12.18	6.45	Peak P_{diss} on walls
DF	[-]	1.19e-05	2.49e-05	Beam duty factor
P_{ave}	[W]	144.99	128.80	Peak P_{diss} on walls
E_{peak}	[MV/m]	97.29	44.22	Peak surface E-field
B_{peak}	[mT]	173.53	110.63	Peak surface B-field

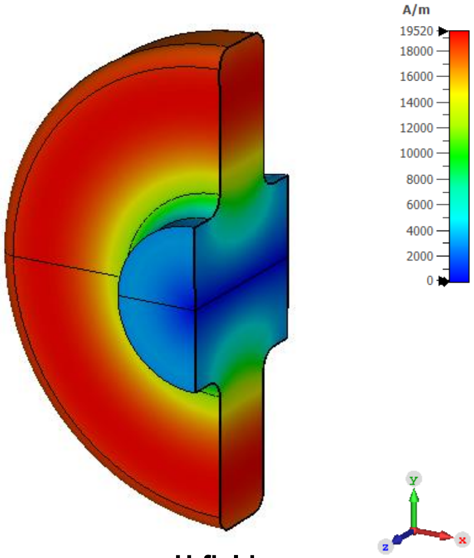
- Peak dissipated power P_{diss} on walls (~factor 2 less for larger window because of higher shunt impedance)
- Average power P_{ave} on walls (11% less for larger window)
- E_{peak} (~2 less for larger window)

Peak E and H fields on the cavity

- Maximum E-field is localized on the cavity iris, while maximum H-field on the upper cavity walls
- Peak E-field needs to be minimized to reduce the risk of RF-breakdown and mitigate Lorentz Forces
- Peak H-field needs to be reduced for the RF-heating



E-field

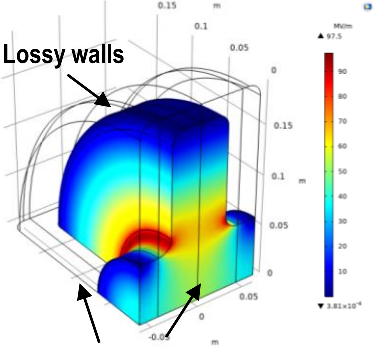


H-field

RF-thermo-mechanical simulations

RF eigenmode

$$\boldsymbol{E} = (E_x, E_y, E_z) \rightarrow f_0$$



Lossy walls

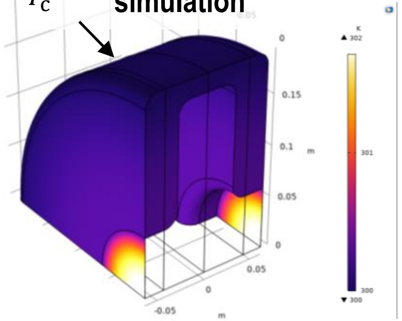
PMC (symmetry planes)

Power losses
(Heat source on lossy walls)

$$\frac{\partial P_{\mathrm{diss}}}{\partial A} \approx \frac{1}{2} DF \cdot R_s(T, f_0)\mathbf{H}\cdot\mathbf{H}^*$$

Stationary thermal simulation → T

T_c



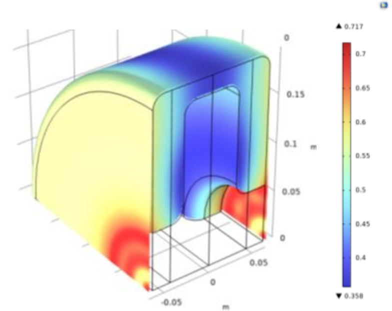
Thermal expansion

$$\epsilon_{\mathrm{th}} = \alpha(T)(T - T_\mathrm{c}) \quad \text{with} \quad T_\mathrm{c} = 300\mathrm{K}$$

Non-linear thermo-mechanical simulation

$\boldsymbol{u} = (u_x, u_y, u_z)$ *Displacement field*

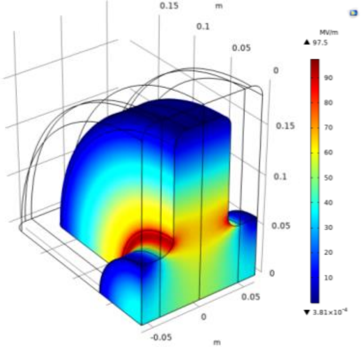
$\boldsymbol{s} = (s_x, s_y, s_z)$ *Spatial mesh displacement*



$$\textit{On symmetry planes}\begin{cases}\mathbf{u}\cdot\mathbf{n}_{\mathbf{ZX}} = 0\\ \mathbf{u}\cdot\mathbf{n}_{\mathbf{XY}} = 0\\ \mathbf{u}\cdot\mathbf{n}_{\mathbf{ZY}} = 0\\ \mathbf{u}\cdot\mathbf{n}_{\mathbf{r}} = 0 & \textit{Roller condition}\end{cases}$$

2nd RF Eigenmode

$$\boldsymbol{E} = (E_x, E_y, E_z) \rightarrow f_{0,\mathrm{th}} \rightarrow \Delta f_{\mathrm{th}} = f_{0,\mathrm{th}} - f_0$$



Simulations are carried out for the MC Cu Cavity
(R_w = 60 mm, t_w = 60 mm) – Higher power and peak fields

Power loss and Temperature Gradient

Average power losses

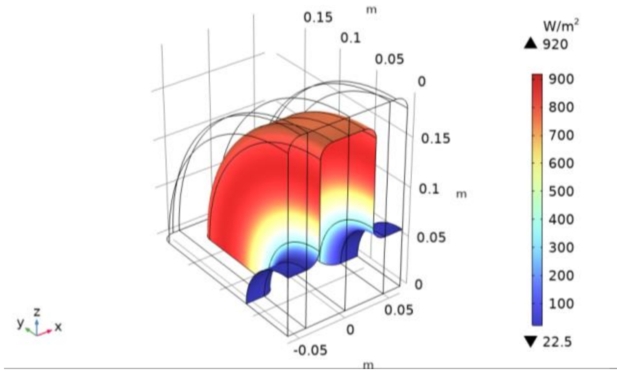
$$P_{\text{diss,ave}} = \int_A \frac{1}{2} DF \cdot R_s(T, f_0)\,\mathbf{H} \cdot \mathbf{H}^* \, dA$$

$P_{\text{diss,ave,walls}}$=169.2 W

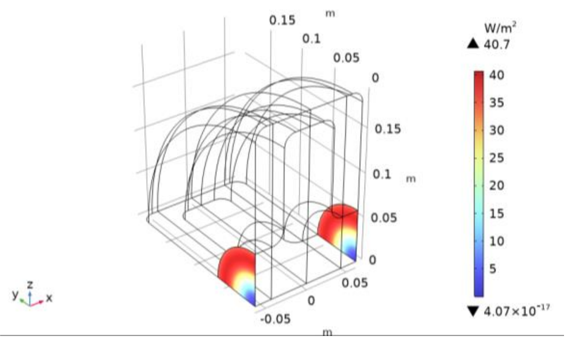
$P_{\text{diss,ave,Windows}}$=0.7 W

at $E_{\text{nom}} = 44\ \text{MV/m}$

Power loss on the cavity walls [W/m^2]



Power loss on the windows [W/m^2]



Temperature gradient

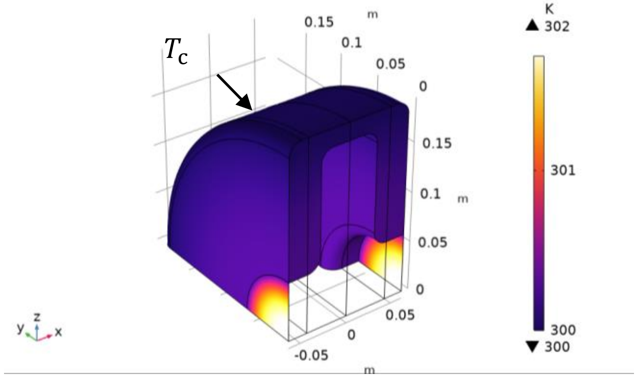
T_{max}=302 K

T_c=300 K

ΔT = 2K

at $E_{\text{nom}} = 44\ \text{MV/m}$

Temperature [K]



Thermal stress, displacement and frequency shift

von Mises Stress vs Yield strength

$$\sigma_{Mises,Cu} = 373\ \text{kPa} < 250 - 350\ \text{MPa} = \sigma_{Yield,Cu}$$

$$\sigma_{Mises,Be} = 5.49\ \text{kPa} < 240\ \text{MPa} = \sigma_{Yield,Be}$$

No plastic deformations, good safety margin to failure.

Thermally induced total displacement

$$\boxed{u_{\text{th,max}} = 0.717\ \mu\text{m}}$$

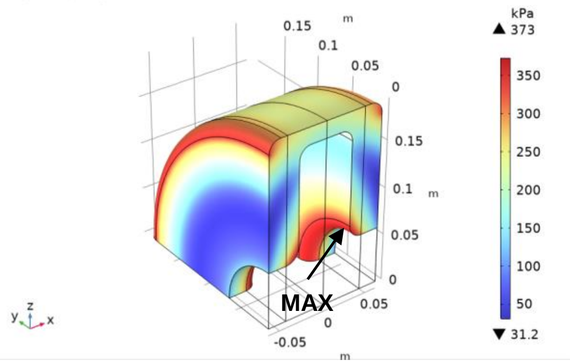
Thermally-induced frequency shift

$$\boxed{\boldsymbol{\Delta f_{\text{th}} = -0.195\ \text{kHz}}}$$

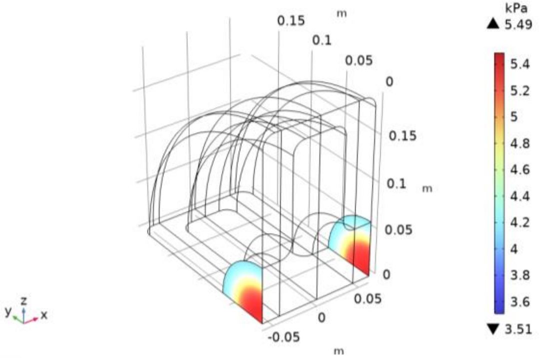
$$\Delta f_{3\text{dB}} = 38.7\ \text{kHz}$$

The frequency shift is lower than the 3 dB bandwidth.

von Mises Stress on the cavity walls [MPa]

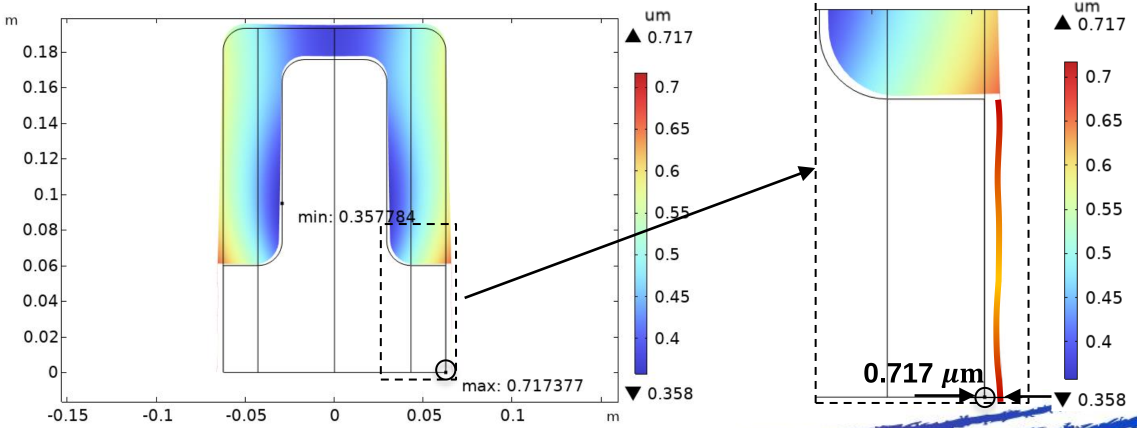


von Mises Stress on the windows [MPa]



Total displacement [um]

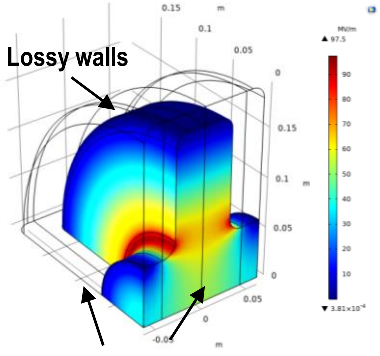
Deformed geometry (x 5000)



Lorentz Force Detuning (LFD) - RF-mechanical simulations

RF eigenmode

$$\boldsymbol{E} = (E_x, E_y, E_z) \longrightarrow f_0$$



Lossy walls

PMC (symmetry planes)

RF field pressure (Boundary load walls)

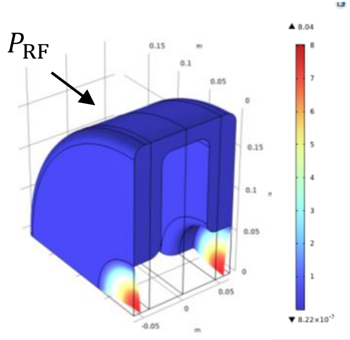
$$P_{\mathrm{RF}} = \frac{1}{4}\left(\mu_0 \mathbf{H}^2 - \varepsilon_0 \mathbf{E}^2\right)$$

Non-linear mechanical simulation

$\boldsymbol{u} = (u_x, u_y, u_z)$ *Displacement field*

$\boldsymbol{s} = (s_x, s_y, s_z)$ *Spatial mesh displacement*

P_{RF}

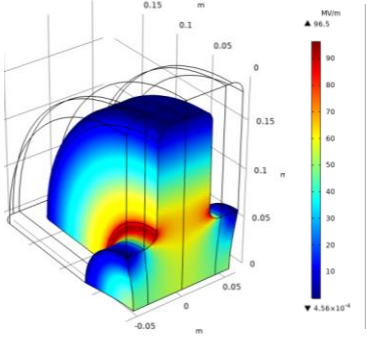


On symmetry planes

$$\begin{cases} \mathbf{u}\cdot\mathbf{n_{ZX}} = 0 \\ \mathbf{u}\cdot\mathbf{n_{XY}} = 0 \\ \mathbf{u}\cdot\mathbf{n_{ZY}} = 0 \\ \mathbf{u}\cdot\mathbf{n_{r}} = 0 \quad \textit{Roller condition} \end{cases}$$

2nd RF Eigenmode

$$\boldsymbol{E} = (E_x, E_y, E_z) \longrightarrow f_{0,\mathrm{LFD}} \longrightarrow \Delta f_{\mathrm{LFD}} = f_{0,\mathrm{LFD}} - f_0$$



Simulations are carried out for the MC Cu Cavity

(R_w = 60 mm, t_w = 60 mm) – Higher power and peak fields

Lorentz Force Detuning: stress and RF field pressure

von Mises Stress vs Yield strength

$$\sigma_{\mathrm{Mises,Cu}} = 54.9\ \mathrm{MPa} < 250 - 350\ \mathrm{MPa} = \sigma_{\mathrm{Yiel}}$$

$$\sigma_{\mathrm{Mises,Be}} = 116\ \mathrm{MPa} < 240\ \mathrm{MPa} = \sigma_{Yield,Be}$$

No plastic deformations, good safety margin to failure.

RF field pressure

$P_{\mathrm{max,walls}} = 54.2\ \mathrm{mbar}$ (Compression)

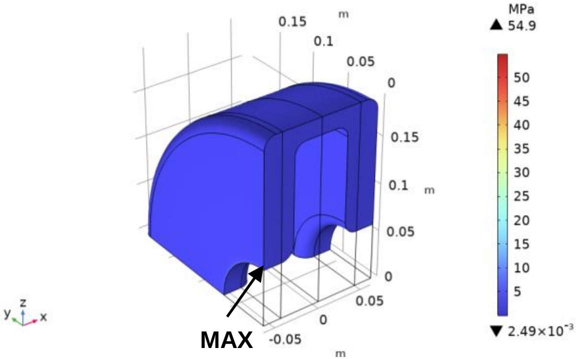
$P_{\mathrm{min,walls}} = -198\ \mathrm{mbar}$ (Traction)

$P_{\mathrm{max,window}} = 1.49\ \mathrm{mbar}$ (Compression)

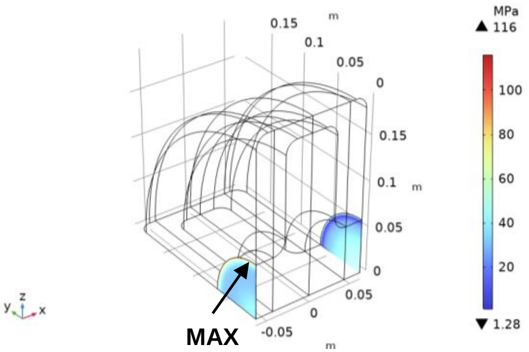
$P_{\mathrm{min\ window}} = -27.5\ \mathrm{mbar}$ (Traction)

Significant pressure on the 60 mm thick window

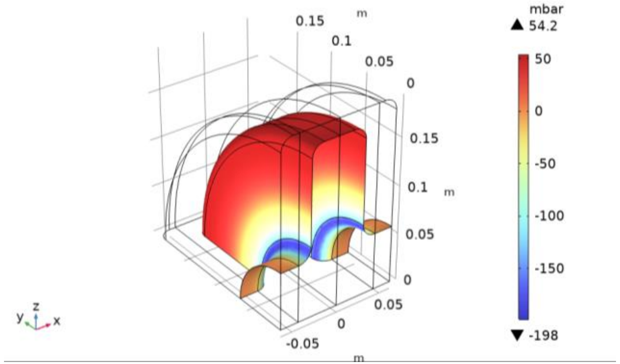
von Mises Stress on the cavity walls [MPa]



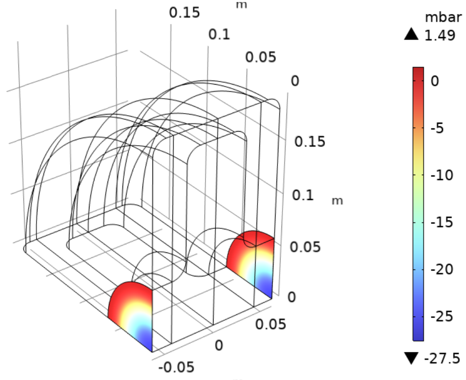
von Mises Stress on the windows [MPa]



Radiation pressure on the cavity walls [mbar]



Radiation pressure on the windows [mbar]



Lorentz Force Detuning – Frequency shift on the cavity walls

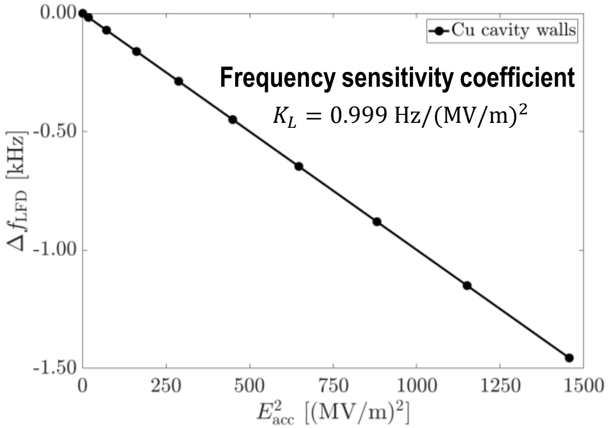
- The frequency shift due the Lorentz Force was initially investigated on the Cu walls only.

Total displacement

$$\boxed{u_{\max} = 0.41\ \text{um}}$$

Frequency shift $\Delta f_{LFD} \propto K_L E_{acc}^2$

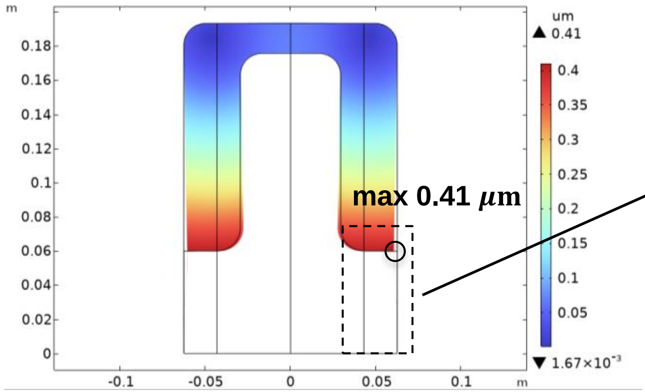
$\boxed{\Delta f_{LFD} = -1.392\ \text{kHz}}$ at $E_{\text{nom}} = 44\ \text{MV/m}$



Frequency sensitivity coefficient

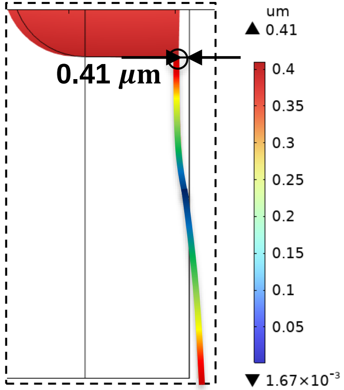
$K_L = 0.999\ \text{Hz}/(\text{MV/m})^2$

Lorentz Force-induced total displacement [um]



max 0.41 μm

Deformed geometry (x 5000)



0.41 μm

- The calculated frequency shift is lower than the 3 dB bandwidth ($\Delta f_{LFD} = 38.7\ \text{kHz}$) if the RF field pressure is applied on the Cu cavity walls only.

Preliminary results: LFD effect on both cavity walls and windows

Total displacement

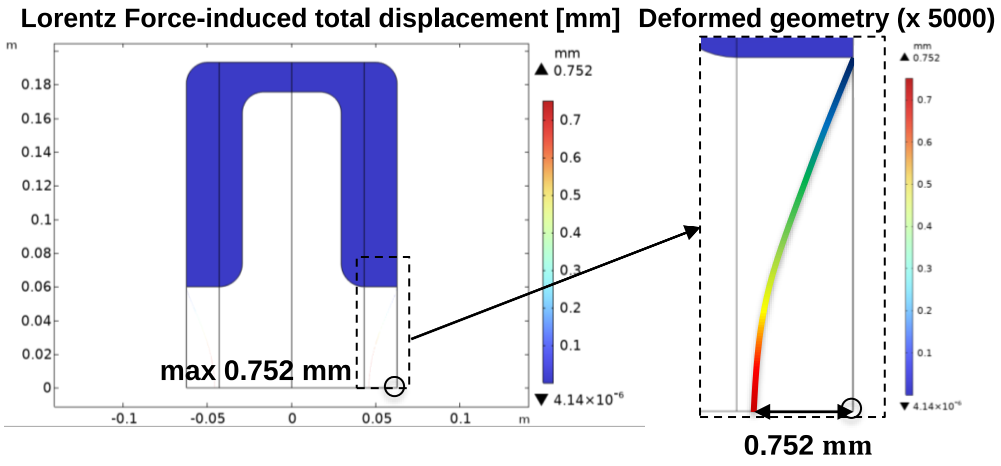
$$\boxed{u_{\mathrm{max}} = 0.752\ \mathrm{mm}}$$

Frequency shift

$$\Delta f_{LFD} \propto K_L E_{acc}^2$$

$$\boxed{\Delta f_{LFD} = -157.26\ \mathrm{kHz}} \quad \text{at} \quad E_{\mathrm{nom}} = 44\ \mathrm{MV/m}$$

$$\Delta f_{3\mathrm{dB}} = 38.7\ \mathrm{kHz}$$



Lorentz Force-induced total displacement [mm] **Deformed geometry (x 5000)**

- The frequency shift is **higher** than the 3 dB bandwidth when RF field pressure is applied on both cavity walls and windows. This is due to the large deformation and displacement detected at the Be windows.
- The implemented LFD model is based on the Slater theorem, which is valid only for small deformation.
- The used linear elastic assumption is typically valid within the **2-3% of deformations**. Even the model includes geometric non-linearity, the assumption that a purely elastic material can undergo arbitrary large deformations may not be entirely valid.

Conclusions and Perspectives

- A conceptual RF design and Multiphysics model of a Copper 704 MHz pillbox-type operating at 44 MV/m for the Muon Cooling Demonstrator was built based on beam dynamics specification.
- RF figures of merit were computed for two different Be window sizes (60 mm and 120 mm radius). The peak dissipated power and surface electric field are a factor 2 less for the cavity featuring 120 mm radius window. RF parameters still need to be optimized to reduce peak power and surface electric field.
- RF thermo-mechanical and Lorentz Force Detuning simulations on the 60 mm window radius cavity show that:
  - Most of the power is dissipated on the Cu cavity walls (169.2 W), only 0.7 W are dissipated on the Be windows. The maximum temperature of 302 K is reached on the window under ideal cooling condition (T_c = 300 K).
  - The maximum thermally-induced total displacement is 0.717 µm on the Be window. The von Mises stress is lower than the yield stress for both materials (no risk of rupture). The thermally-induced frequency shift is -0.195 kHz, which is lower than the cavity bandwidth (38.7 kHz).
  - The von Mises stress is lower than the yield stress for both materials. A significant RF field pressure of ranging from -27.5 mbar to 1.49 mbar was detected on the Be window.
  - The maximum total displacement induced by Lorentz forces is 752 µm on the Be window, which induces a frequency shift of –157 kHz. The computed frequency shift is 4 times higher than the bandwidth.
- Larger Be window and mechanical deformations need to be addressed. The implementation of a more accurate numerical model for simulating Lorentz Forces in the large deformation regime might be necessary.

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Thank you
for your attention