PRECISION PREDICTIONS FOR THE LHC

Rencontres de Blois 2024

Blois - October 22th 2024

Lorenzo Tancredi - Technical University Munich

LMU

ORIGINS Excellence Cluster

Technische Universität München

HUGE COMMUNITY WORKING AT LHC PRECISION PHYSICS

TOO MANY INTERESTING RESULTS TO SUMMARISE IN FEW MINUTES

SO I AM NOT EVEN GOING TO TRY…

Quantum Chromodynamics

Resummation, parton showers, non perturbative corrections, physics of jets…

Production of electroweak vector bosons

DY, di-boson, three-boson production
QCD, EW corrections

Heavy quarks

top, bottom, charm quarks
QCD and EW effects, fragmentation, hadronization….

Higgs Physics

Higgs couplings, Higgs potential,
Electroweak symmetry breaking

WHAT CAN I DO IN 25 MINUTES…

1. MOTIVATE WHY WE ARE DOING THIS AND MOST IMPORTANT PHYSICS

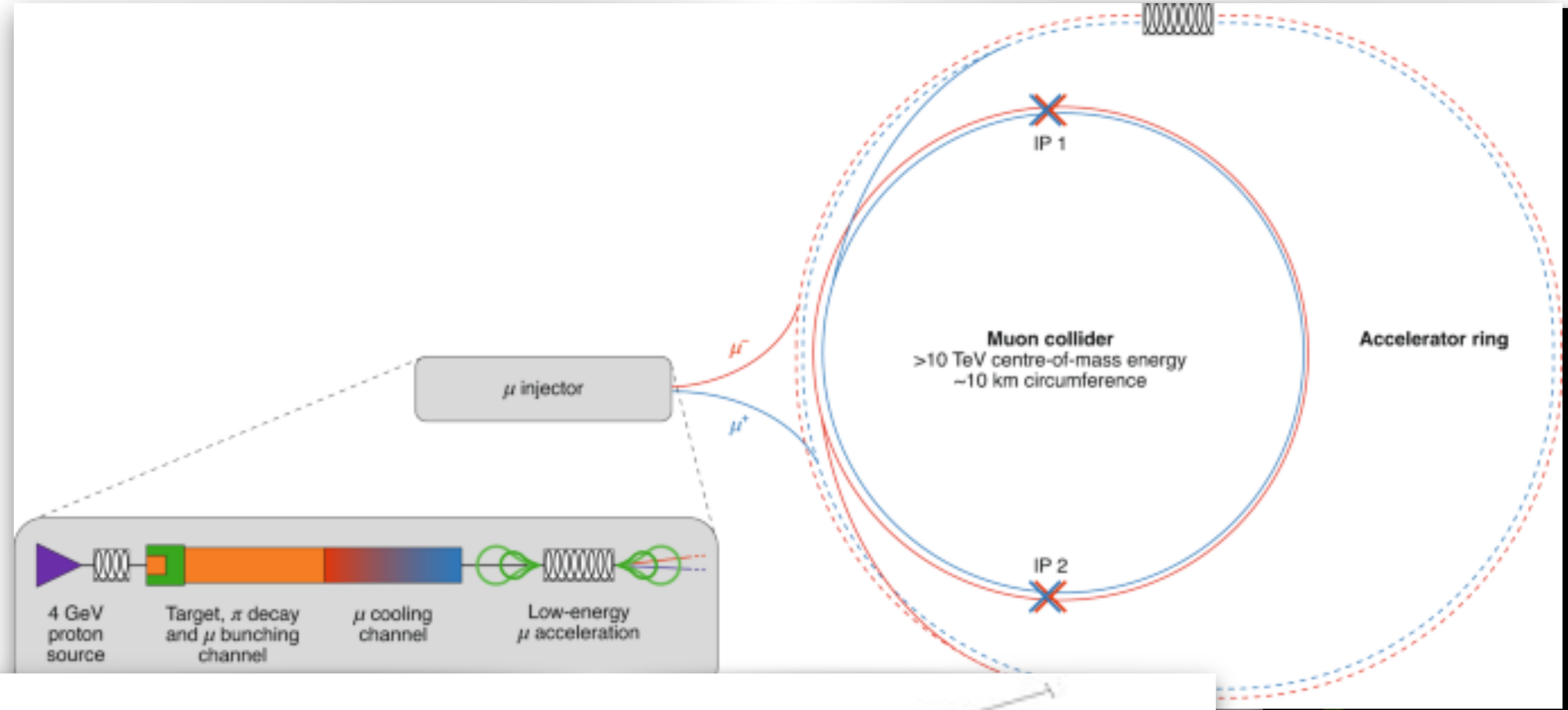
2. INTRODUCE THE FRAMEWORK ON WHICH THESE RESULTS ARE BASED

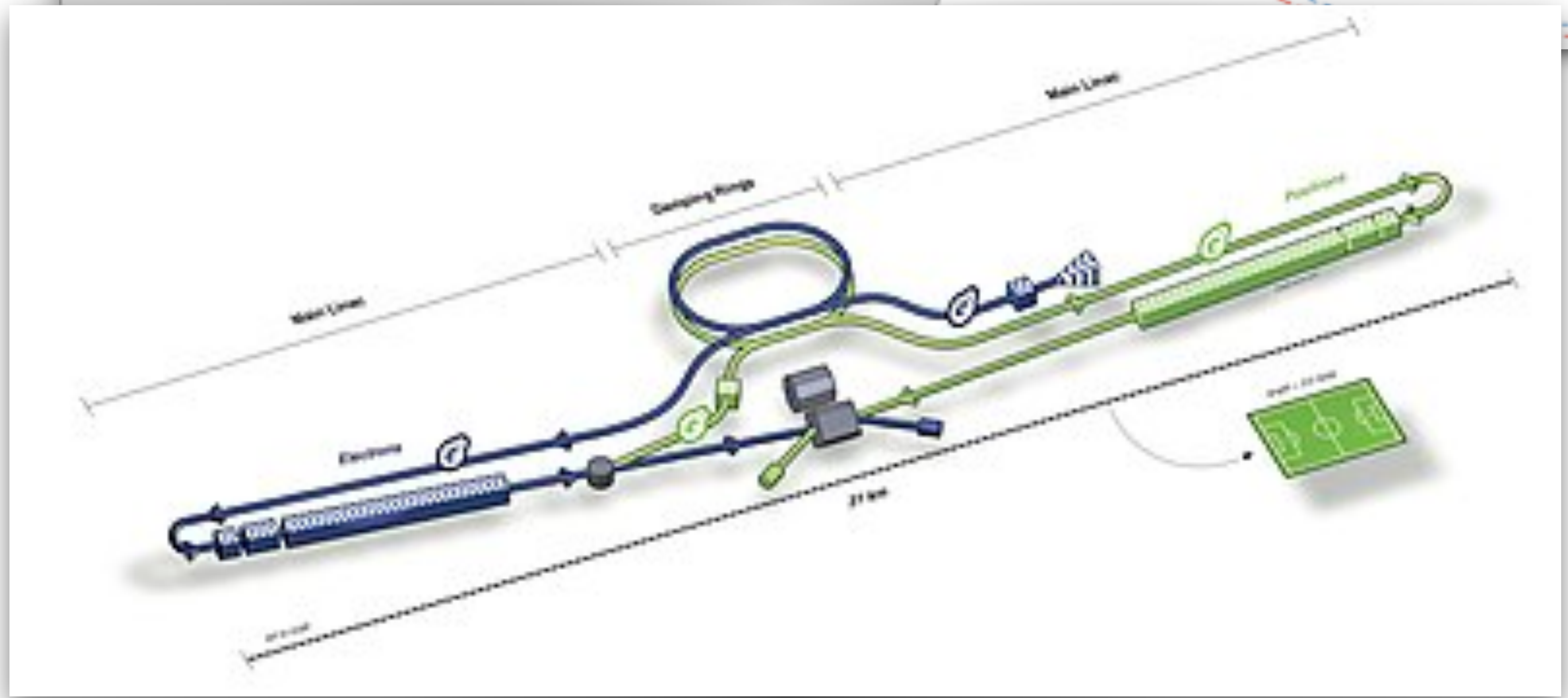
3. GIVE YOU AN IDEA OF THE STATE-OF-THE-ART

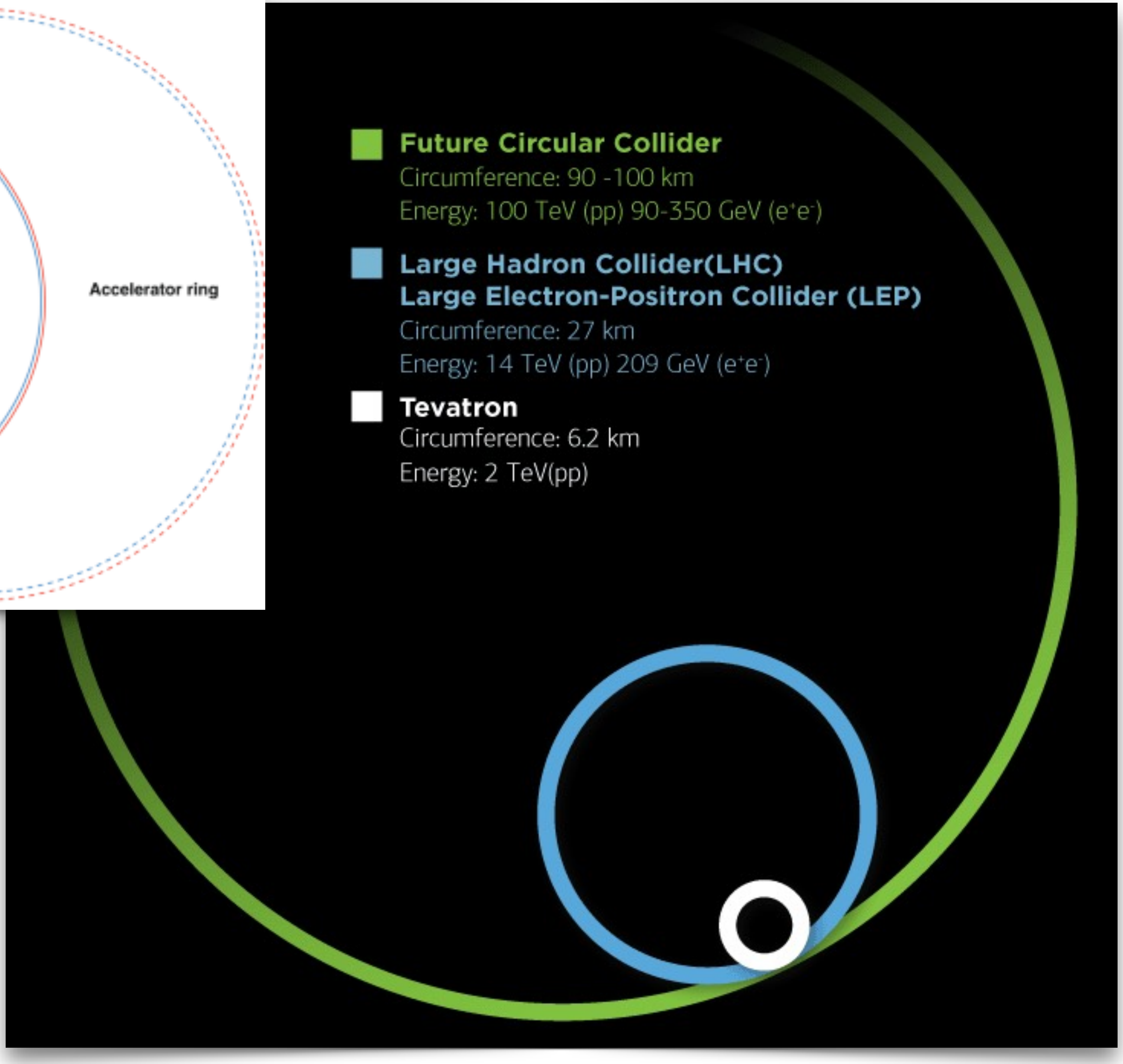
4. STRESS THE LIMITATIONS WE STILL NEED TO OVERCOME

5. … AND CONVEY SOME EXCITEMENT FOR THIS FIELD… :)

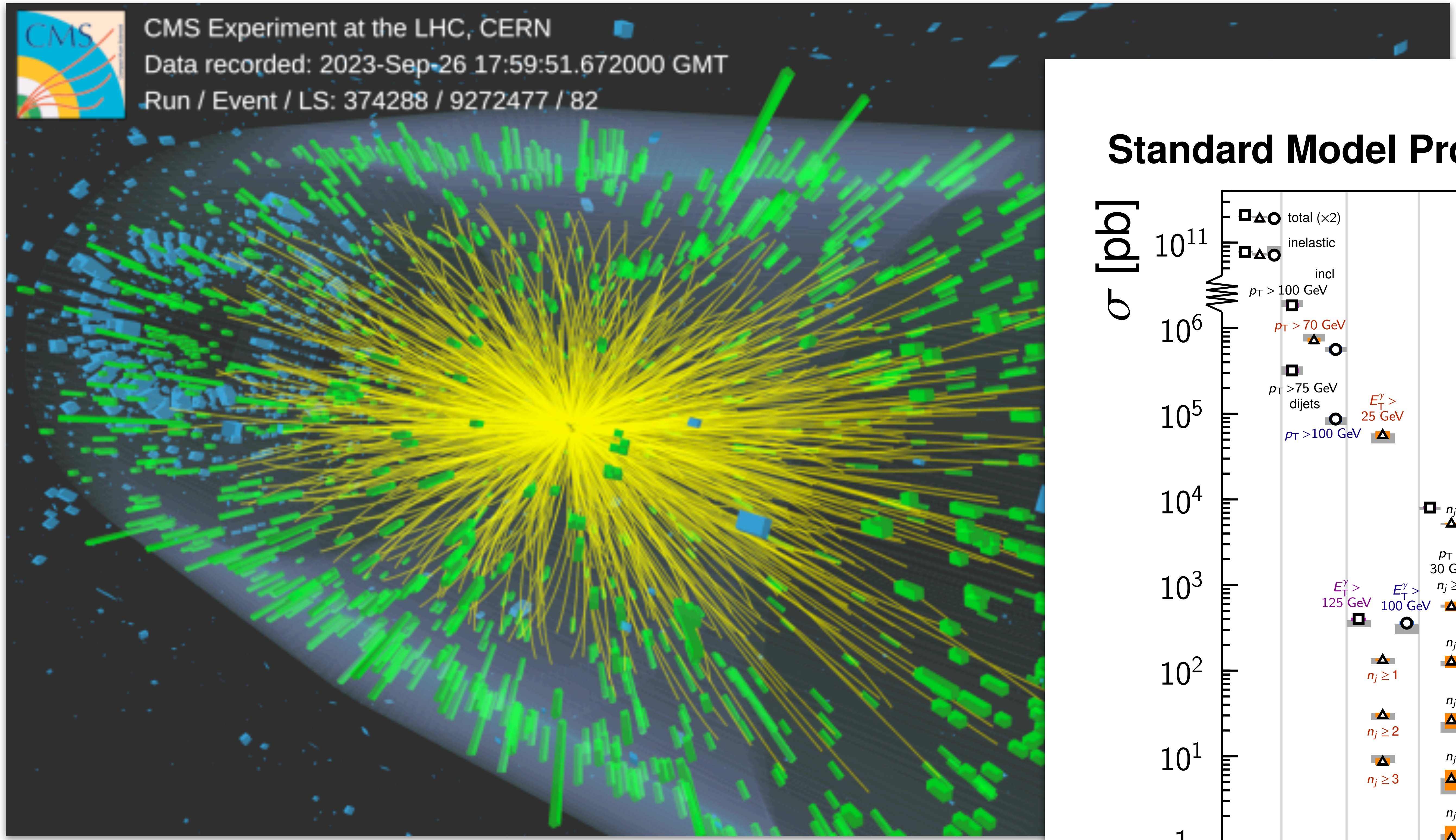
HIGHER AND HIGHER PRECISION AT THE LHC (AND BEYOND)…

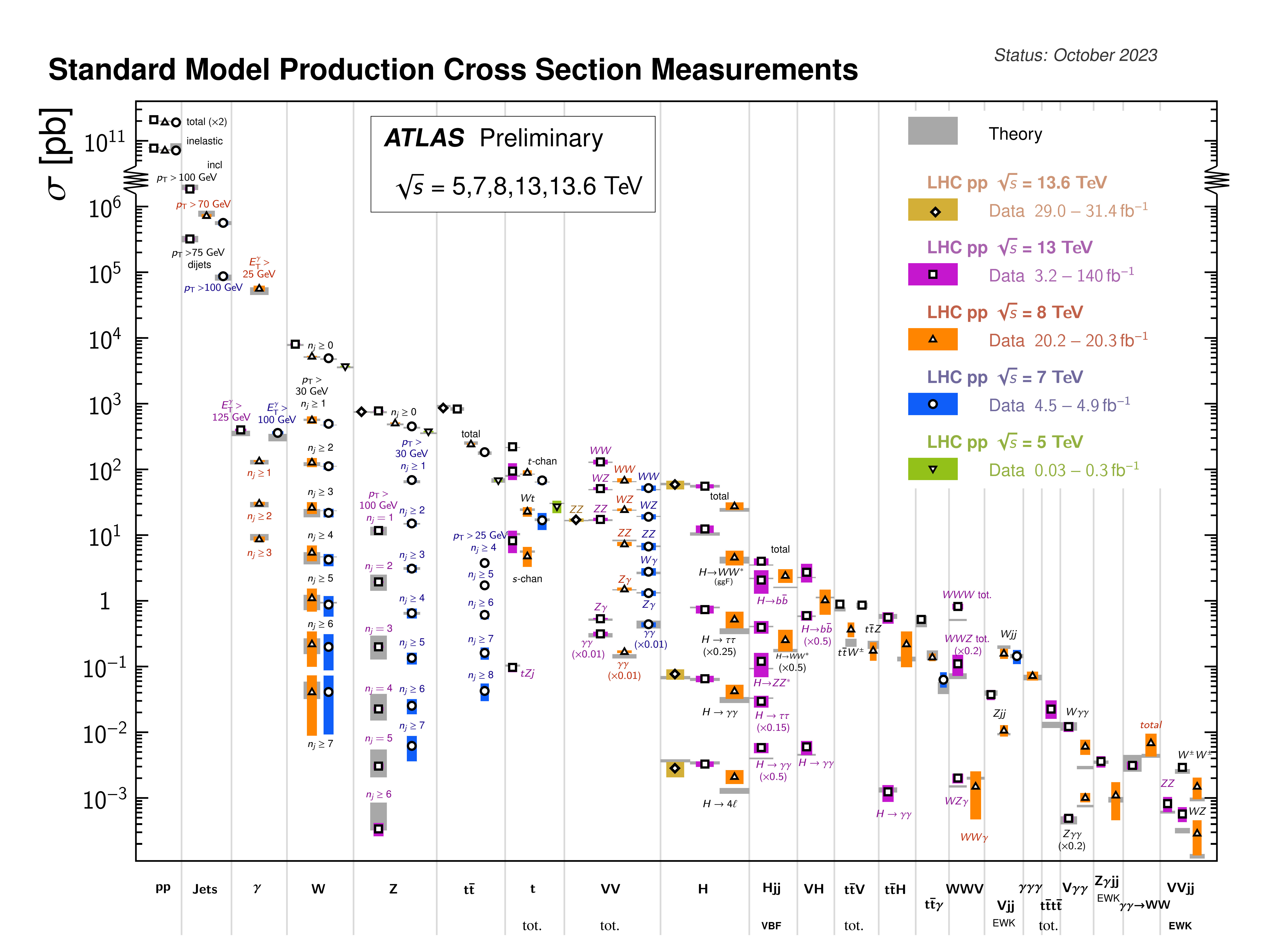






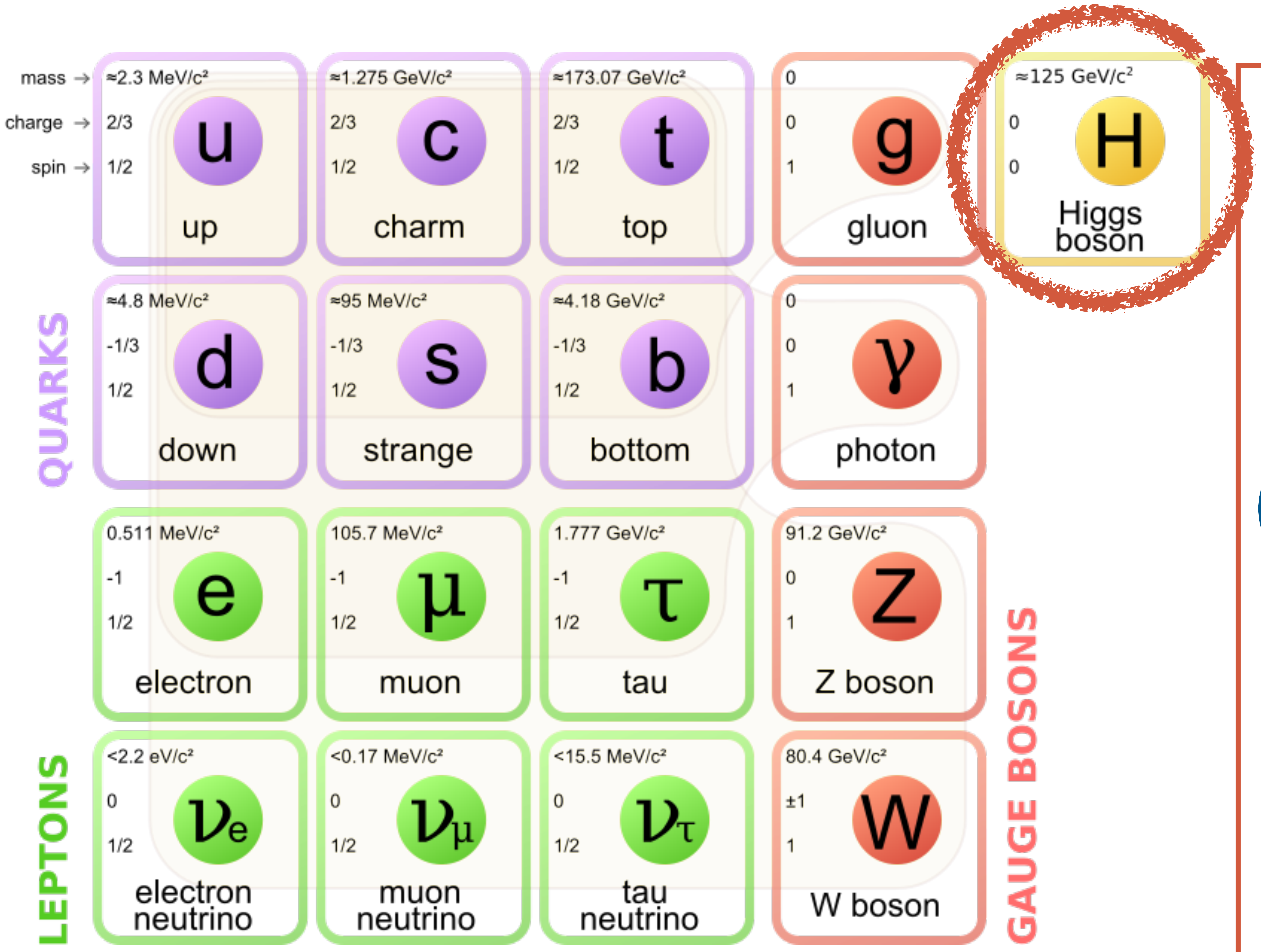
THE LHC HAS BECOME A PRECISION MACHINE

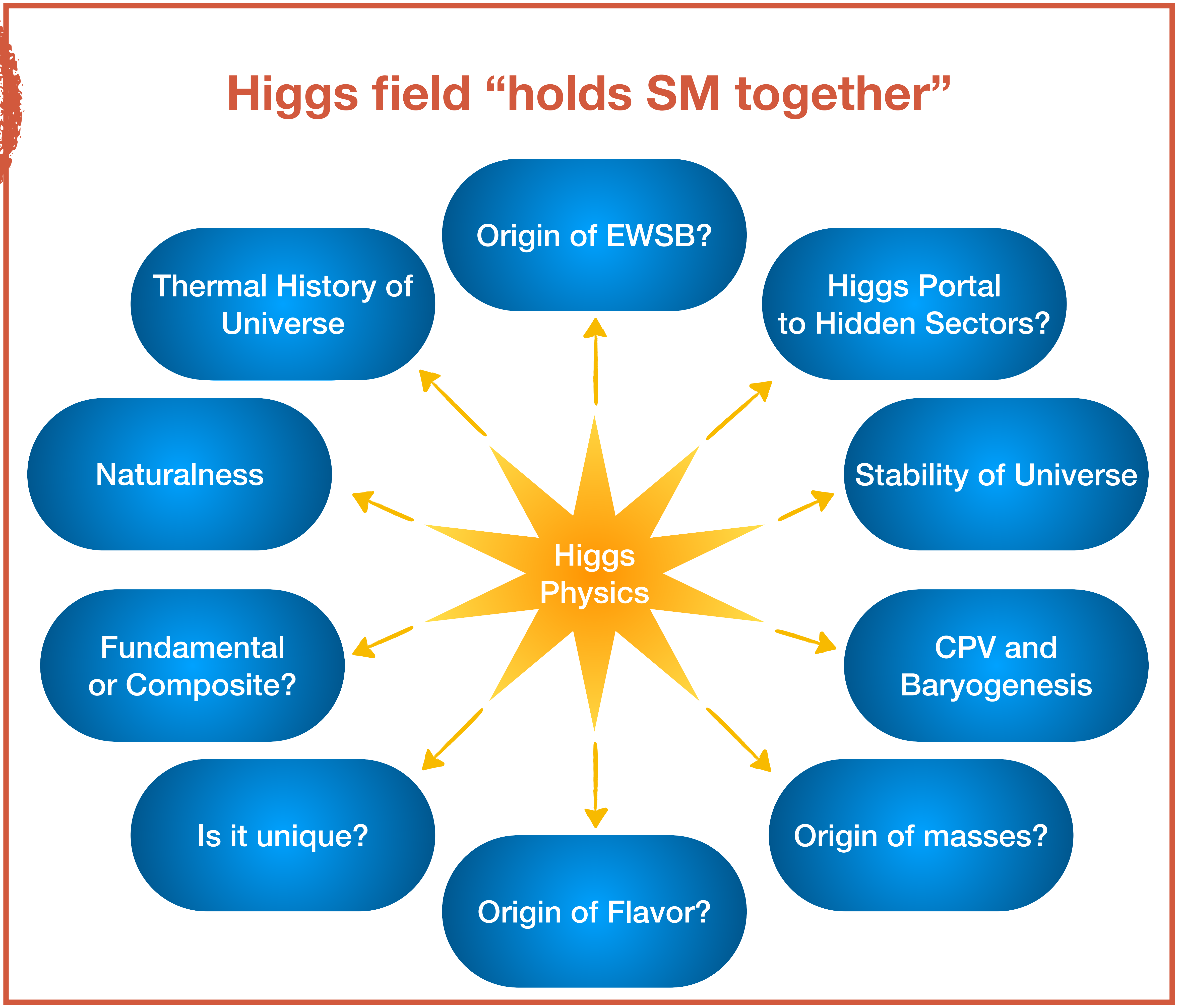




After its discovery in 2012, a lot (but not only) revolving around **Higgs boson's properties**

THE HIGGS BOSON: THE LAST MISSING PIECE

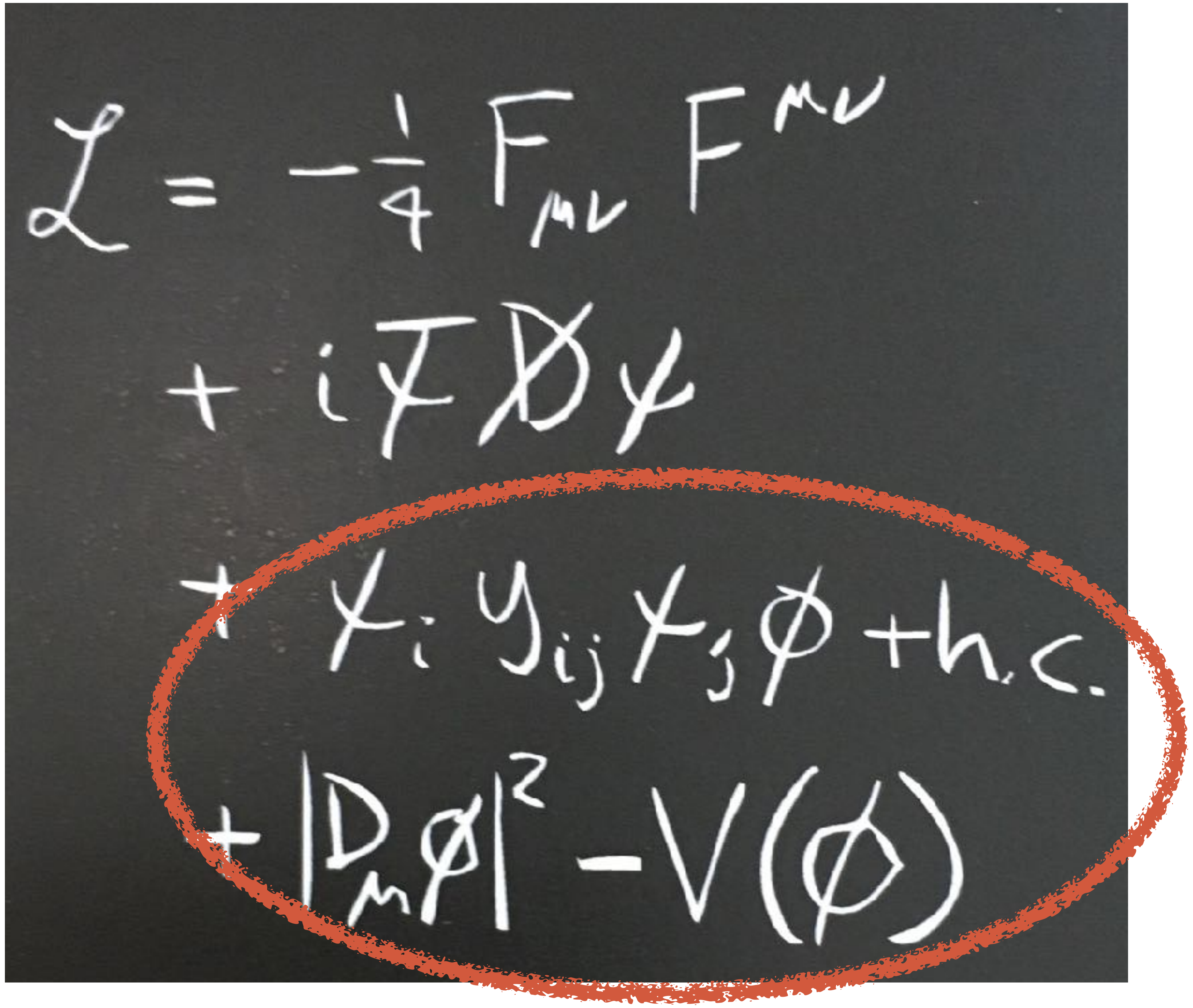




[Snowmass 2022 arXiv:2209.0751]

HIGGS INTERACTIONS AT THE LHC

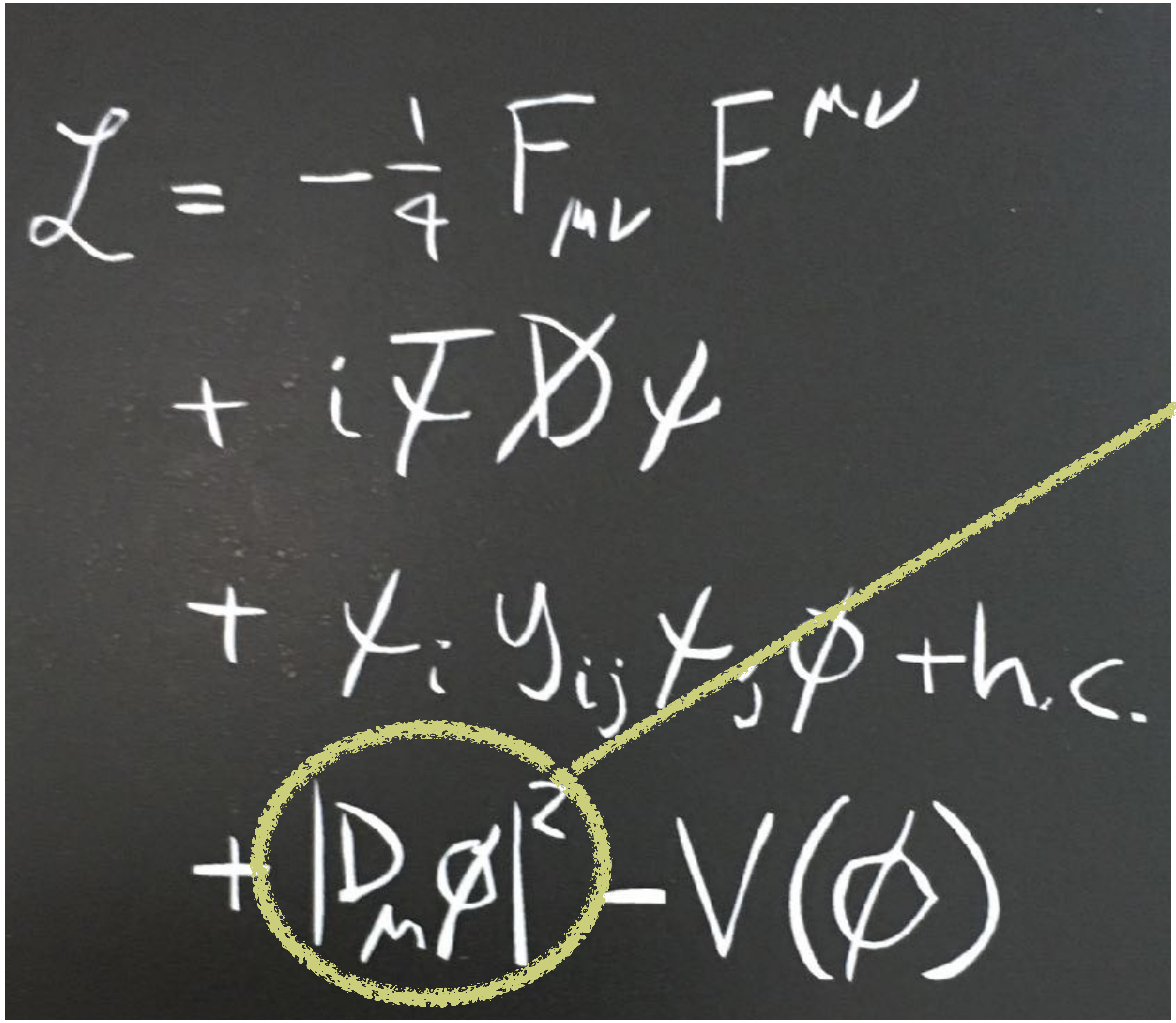
Hints to answer these questions hidden in the **details of Higgs interactions to SM particles**

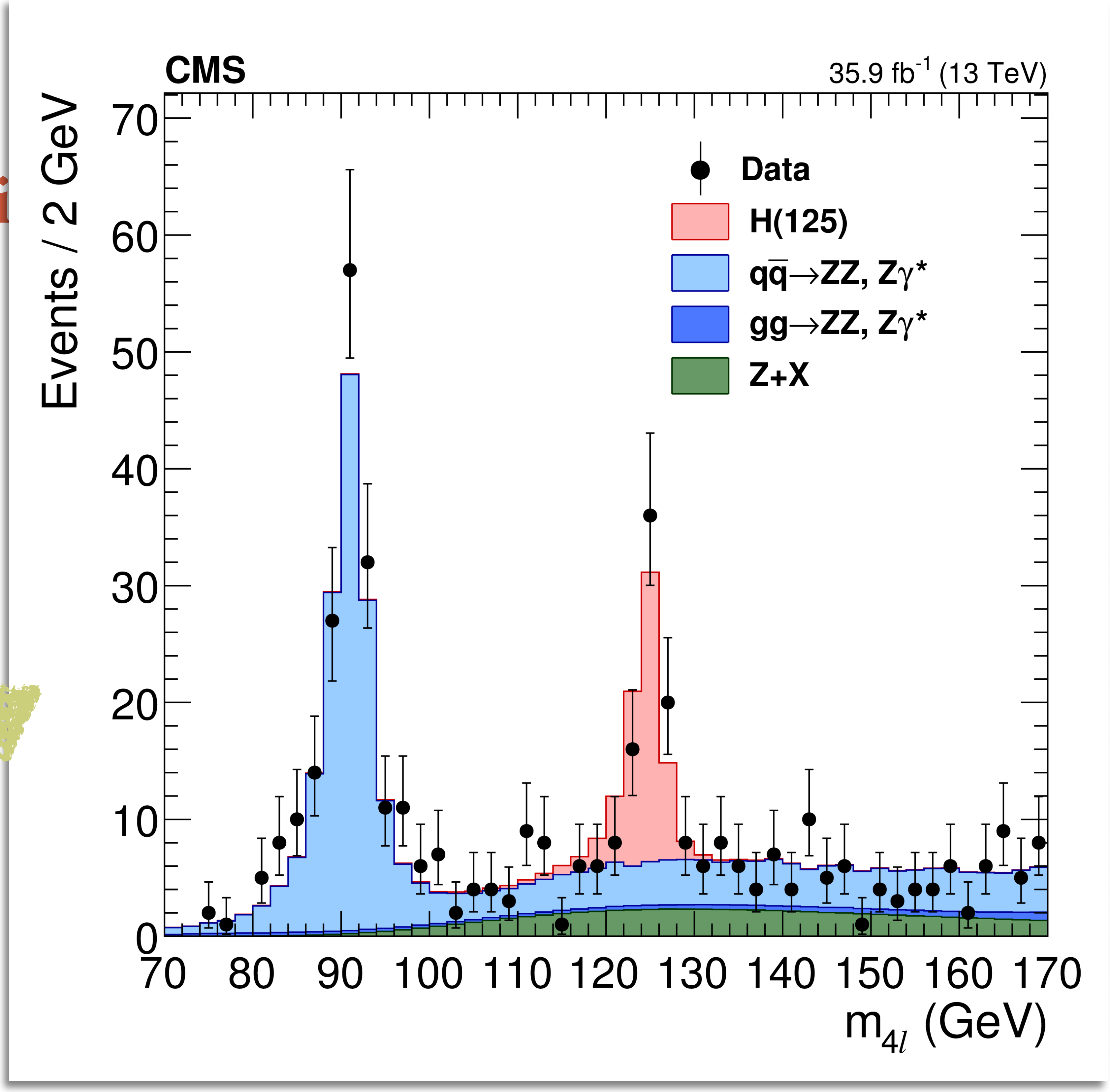


LHC has opened a window for us to peak at Higgs' interactions for the first time !

HIGGS INTERACTIONS AT THE LHC

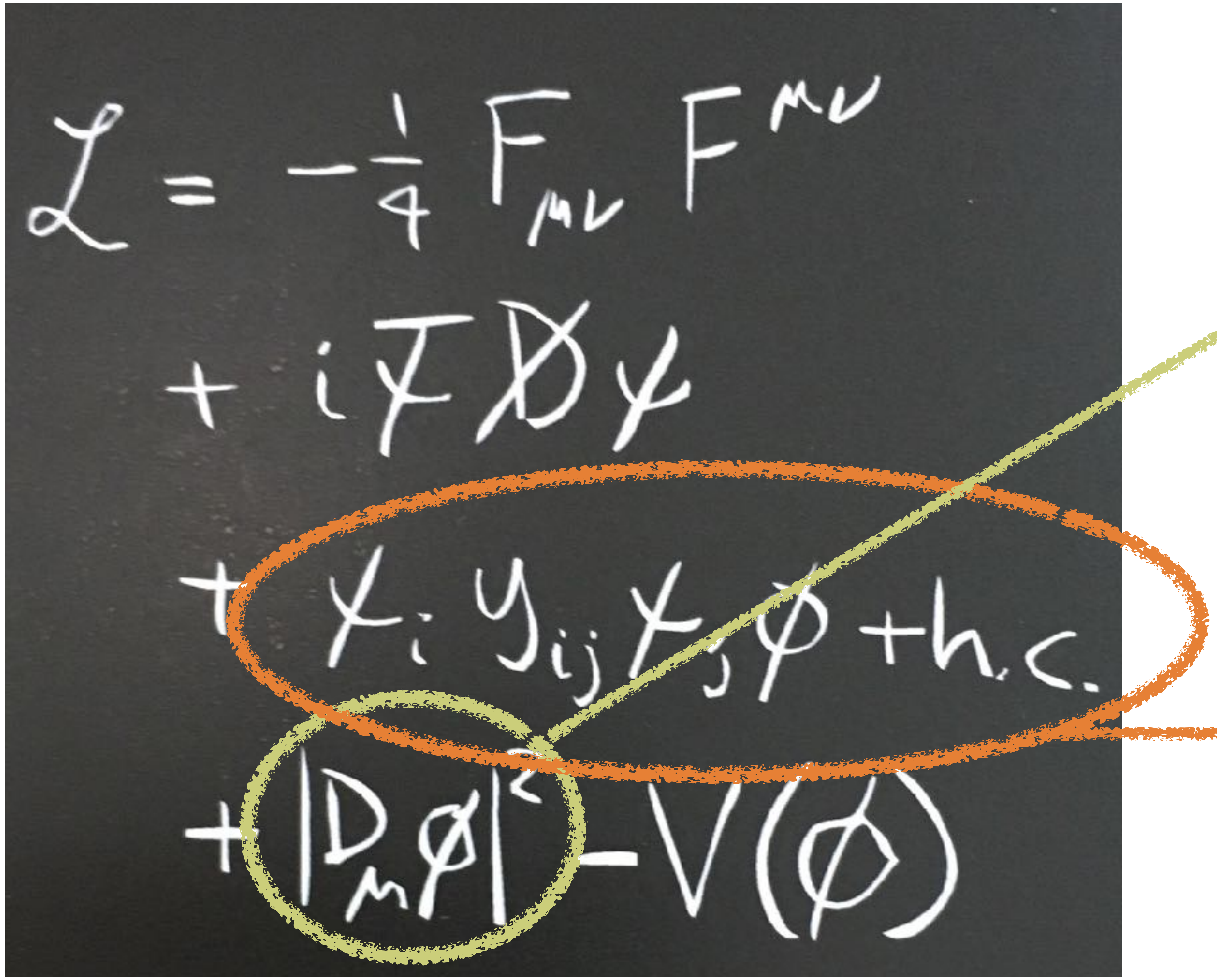
Hints to answer these questions hidden in the **details of Hi**

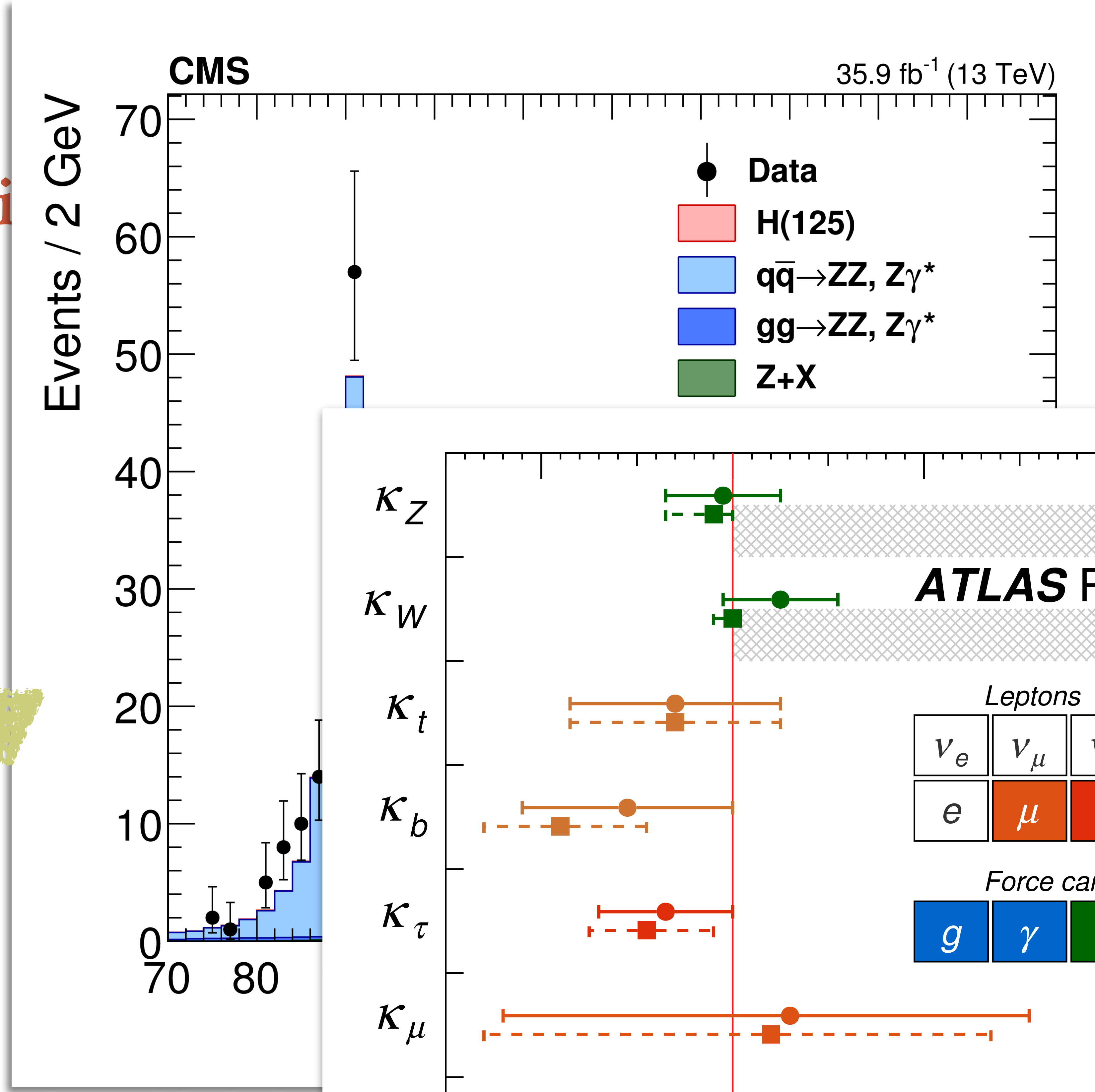


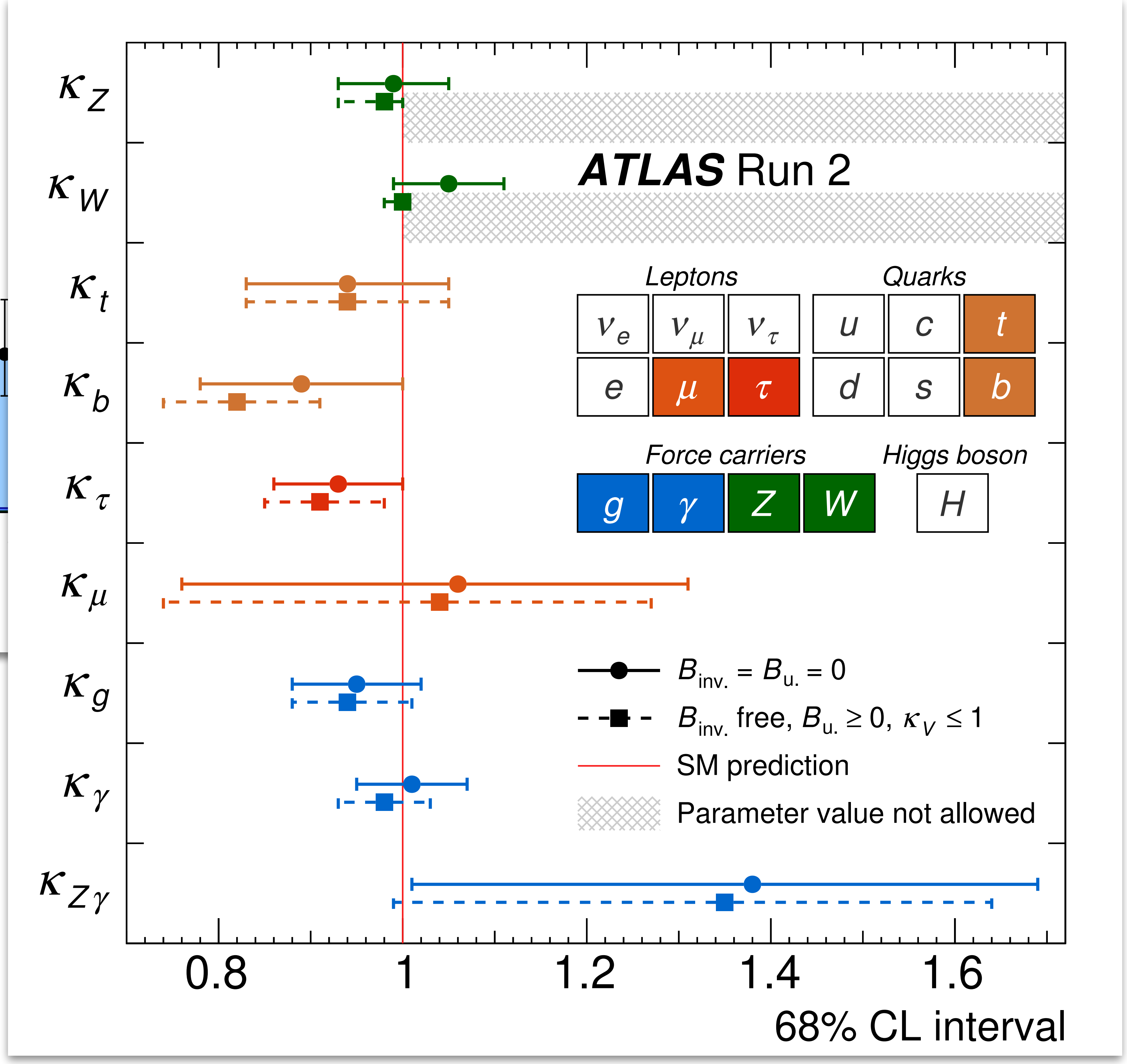


HIGGS INTERACTIONS AT THE LHC

Hints to answer these questions hidden in the **details of Hi**

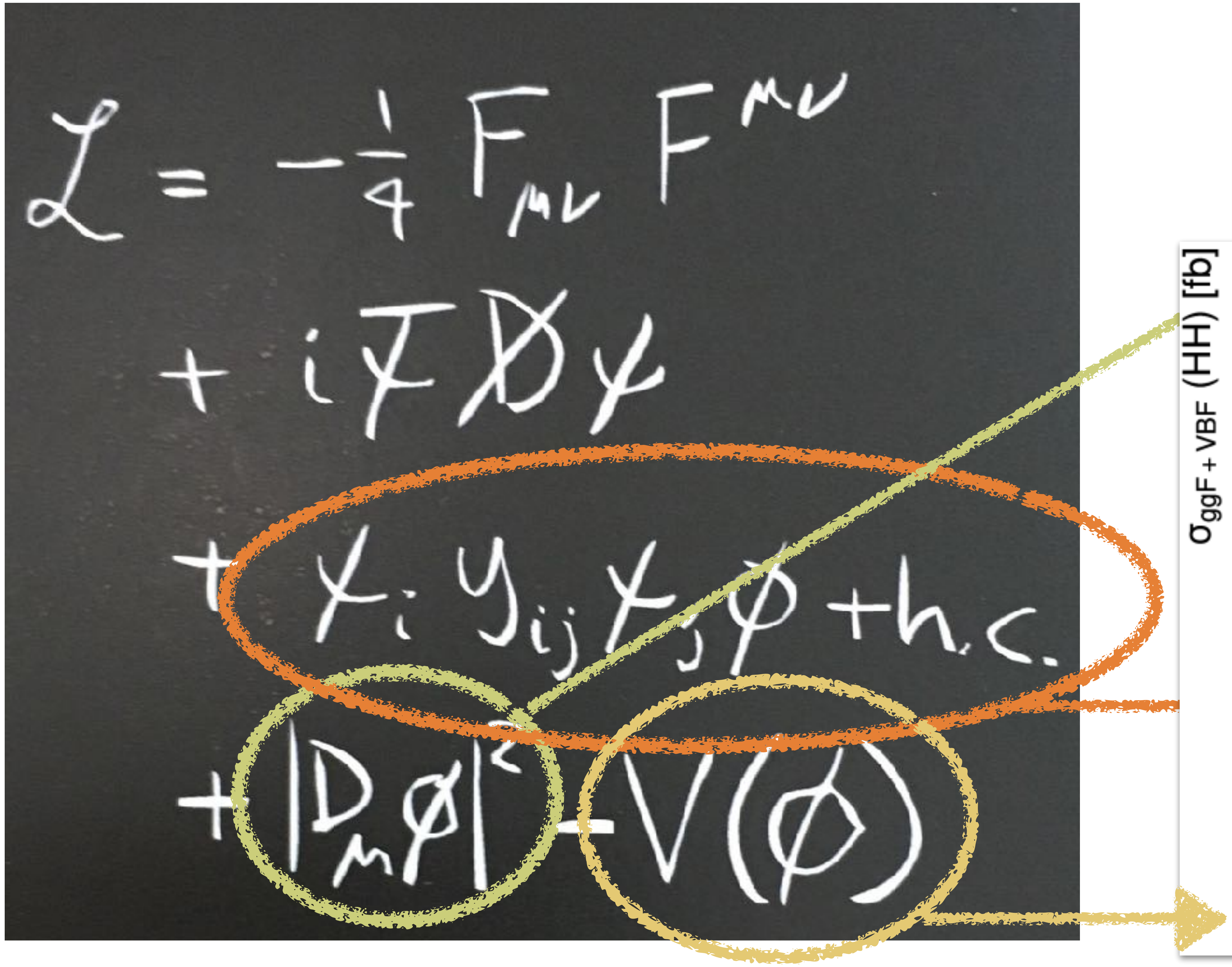


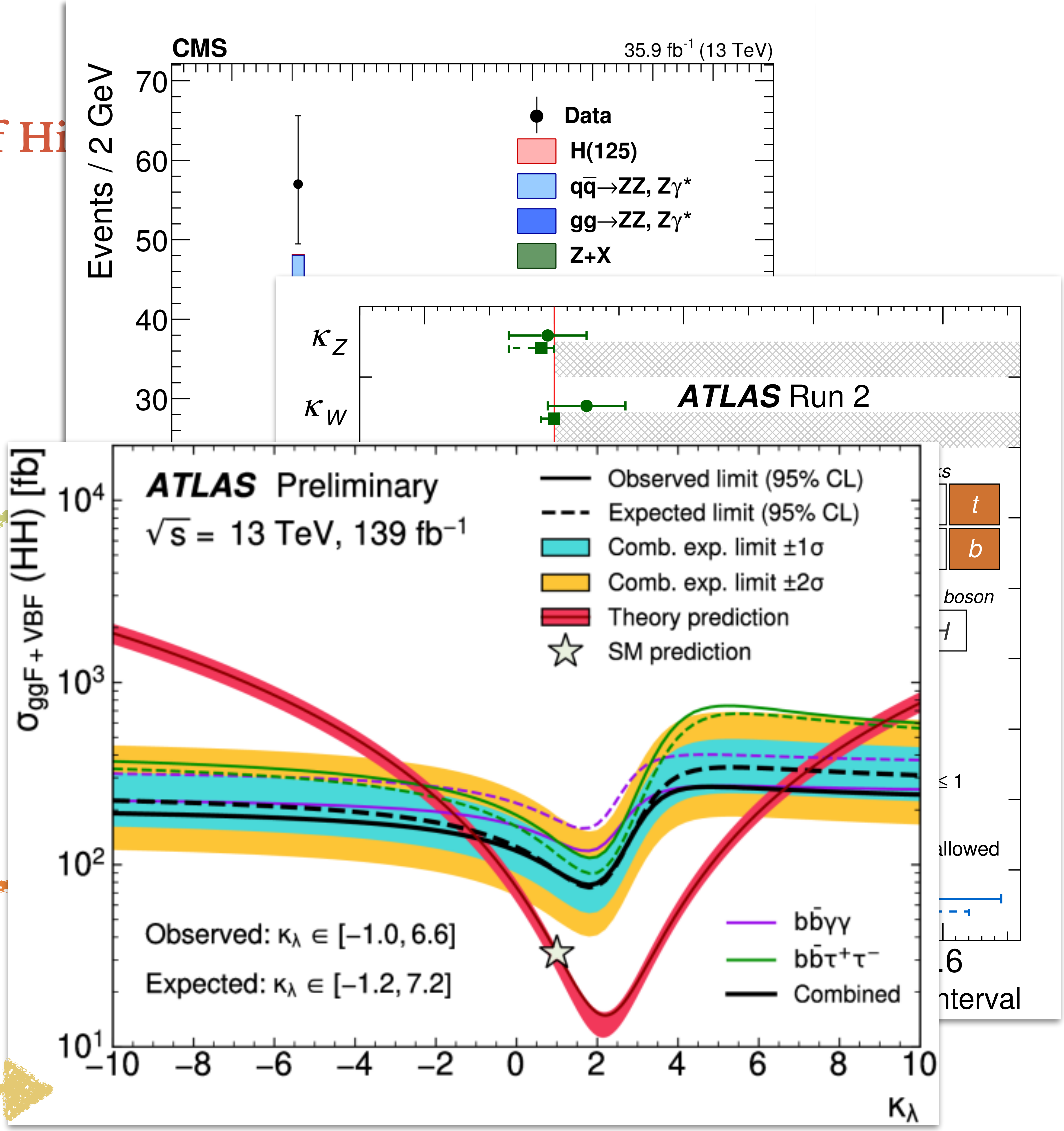




HIGGS INTERACTIONS AT THE LHC

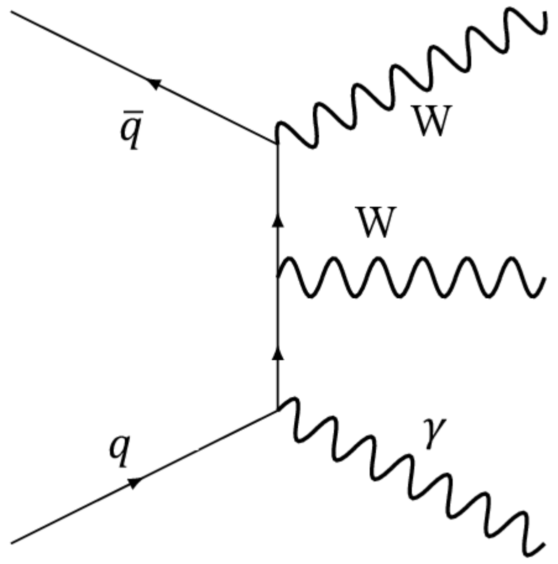
Hints to answer these questions hidden in the details of Hi



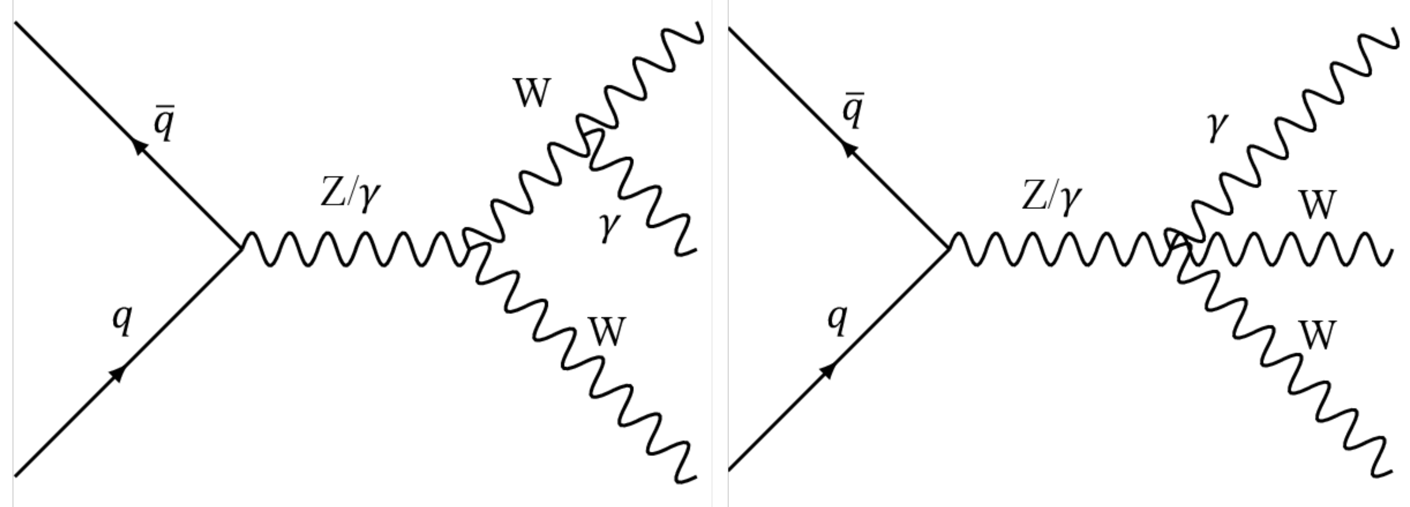


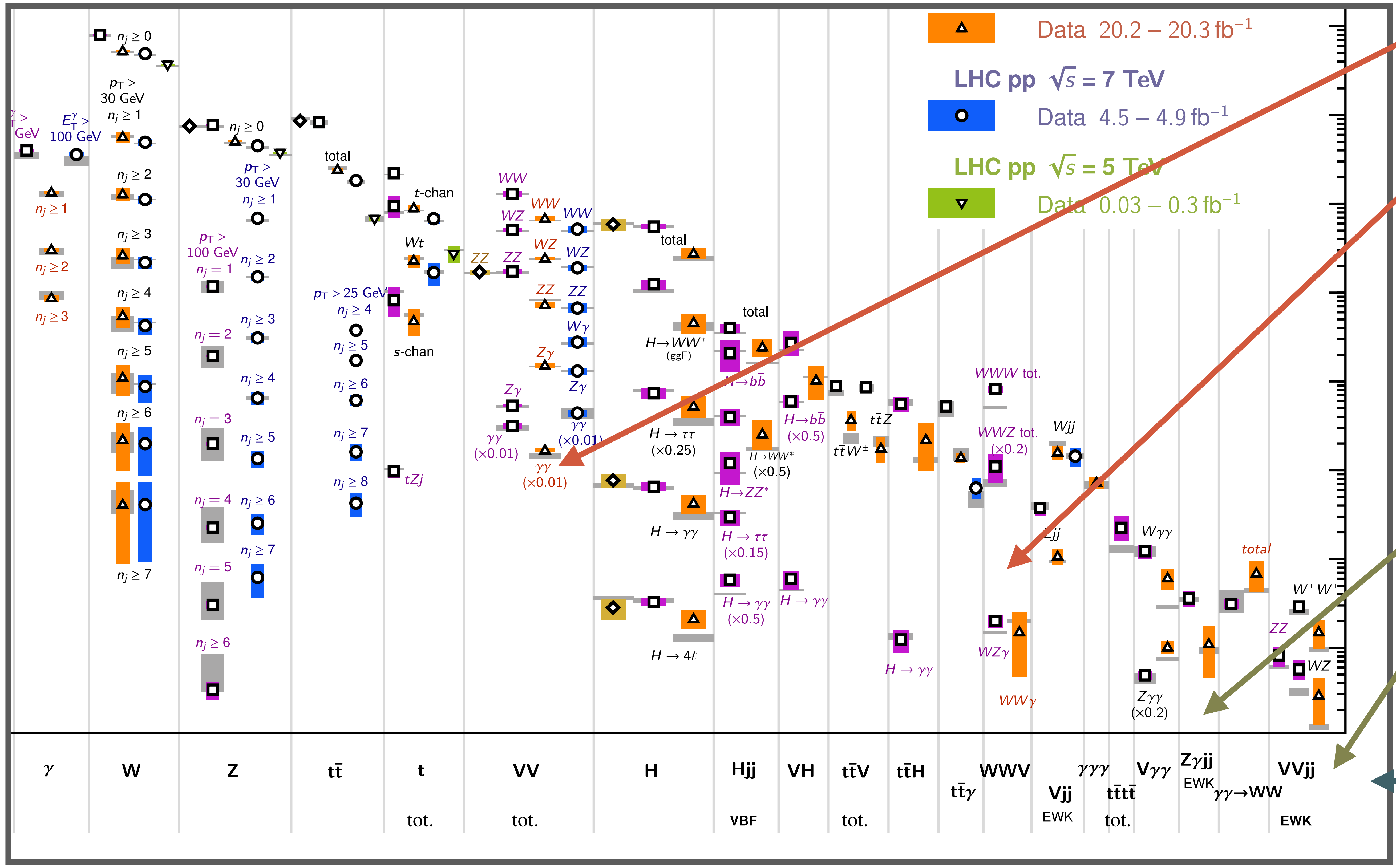
PROBING THE GAUGE SECTOR

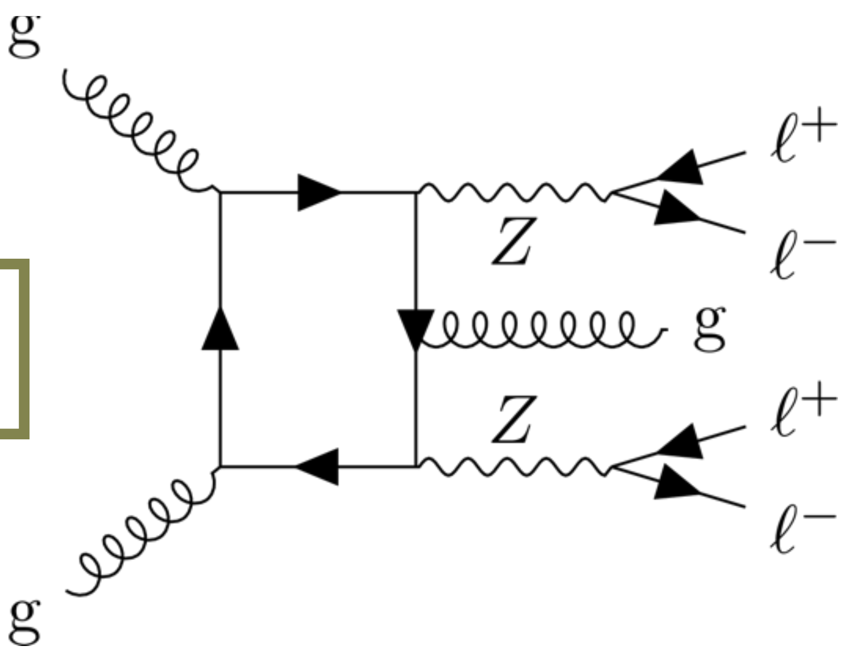
Multiboson final states as probe of electroweak sector of SM



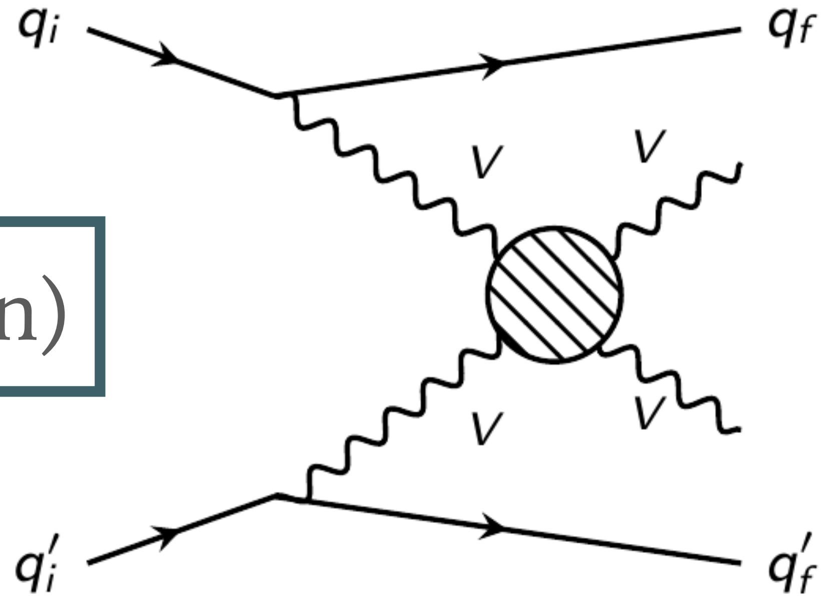
VV & VVV production





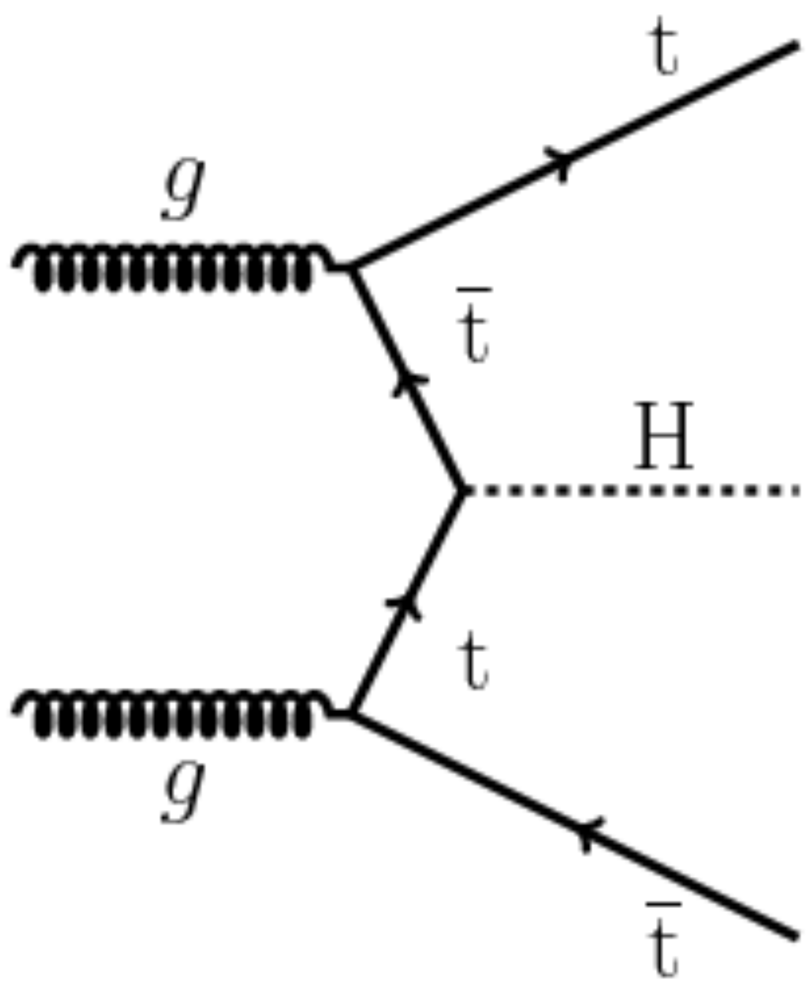


V & VV + jets production

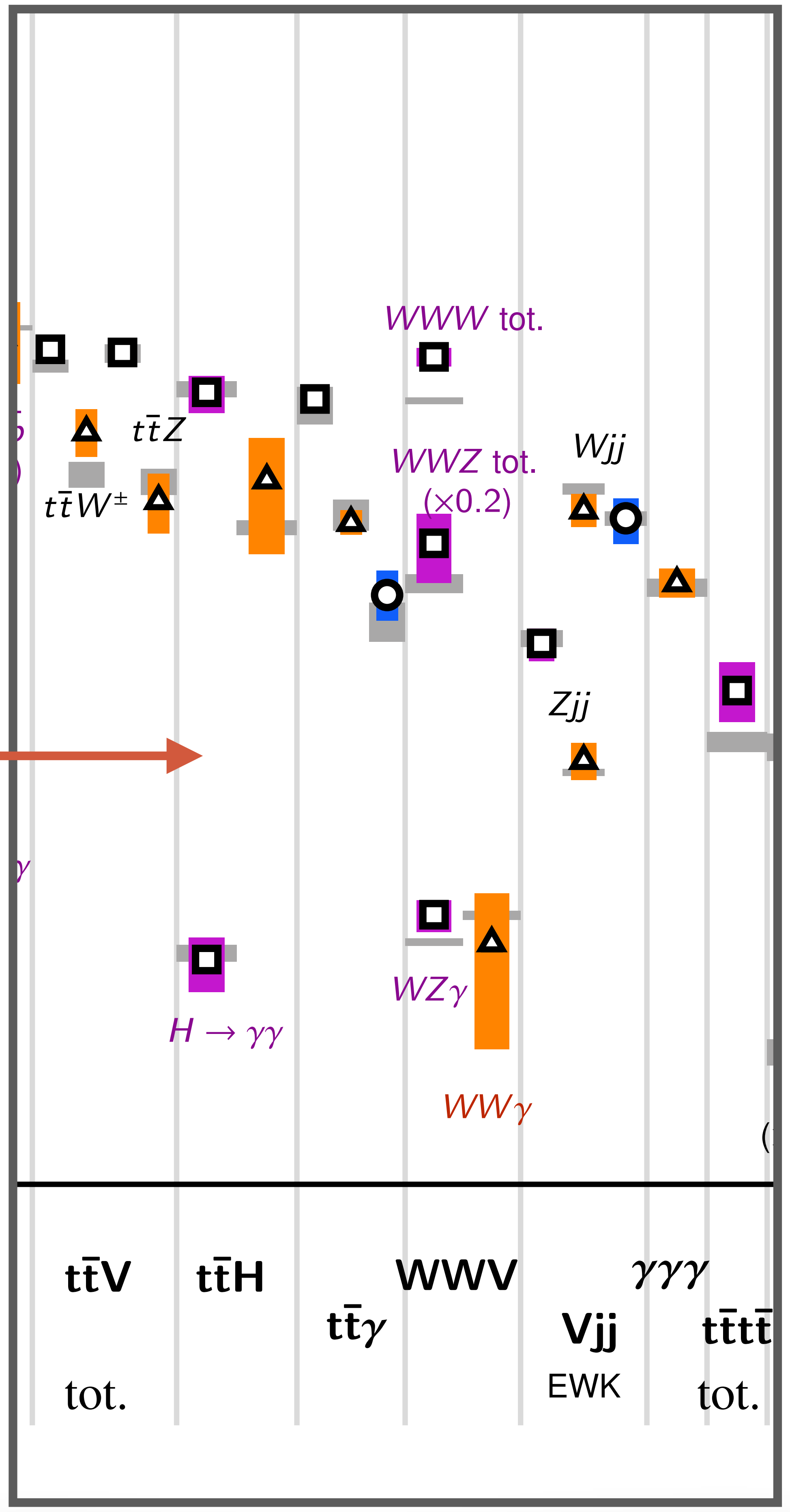


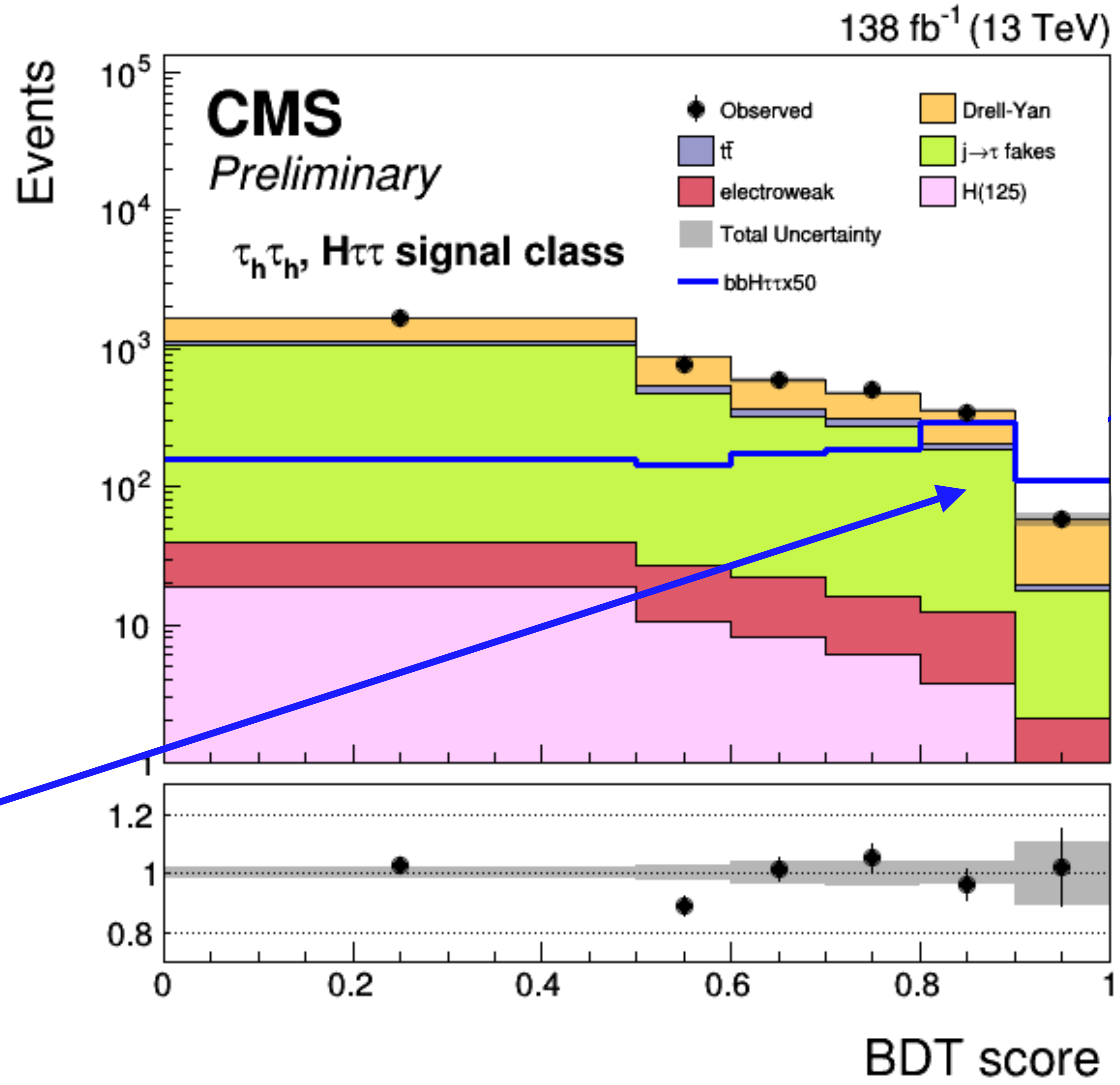
VBF (vector boson fusion)

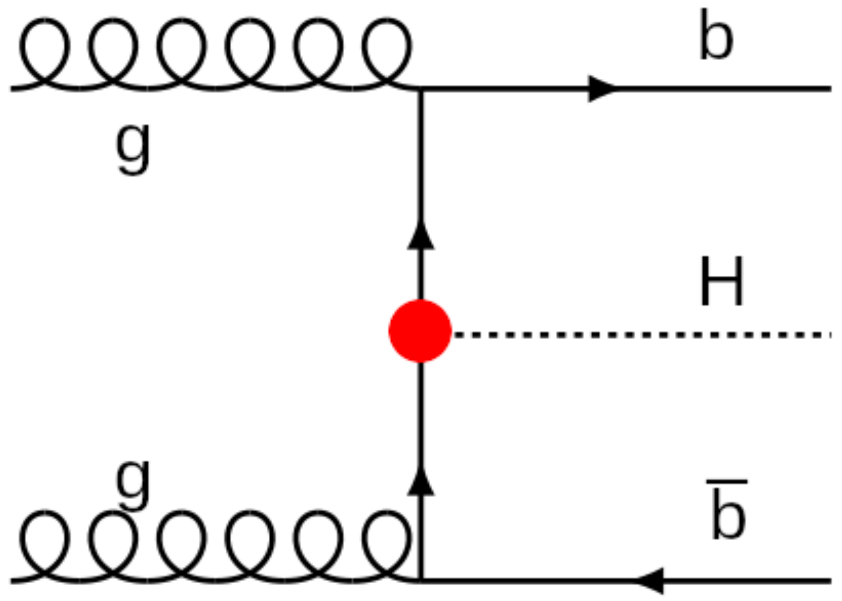
PROBING THE YUKAWA SECTOR

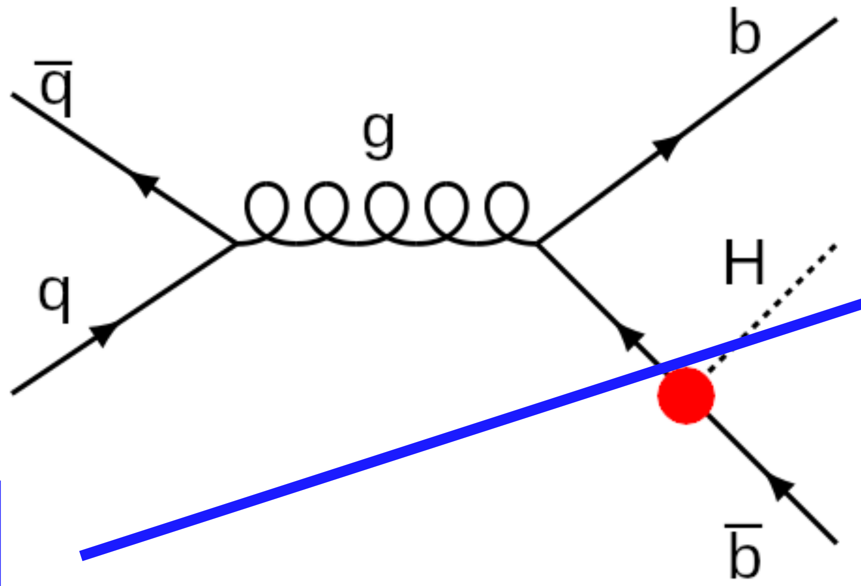


$t\bar{t}H$ production





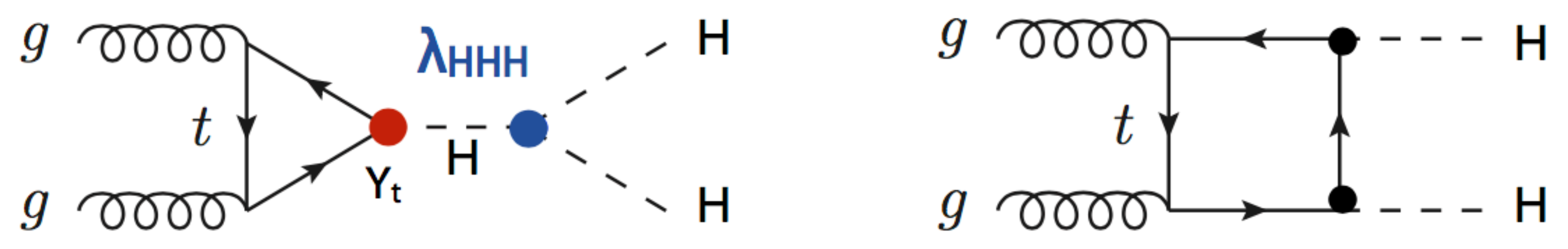




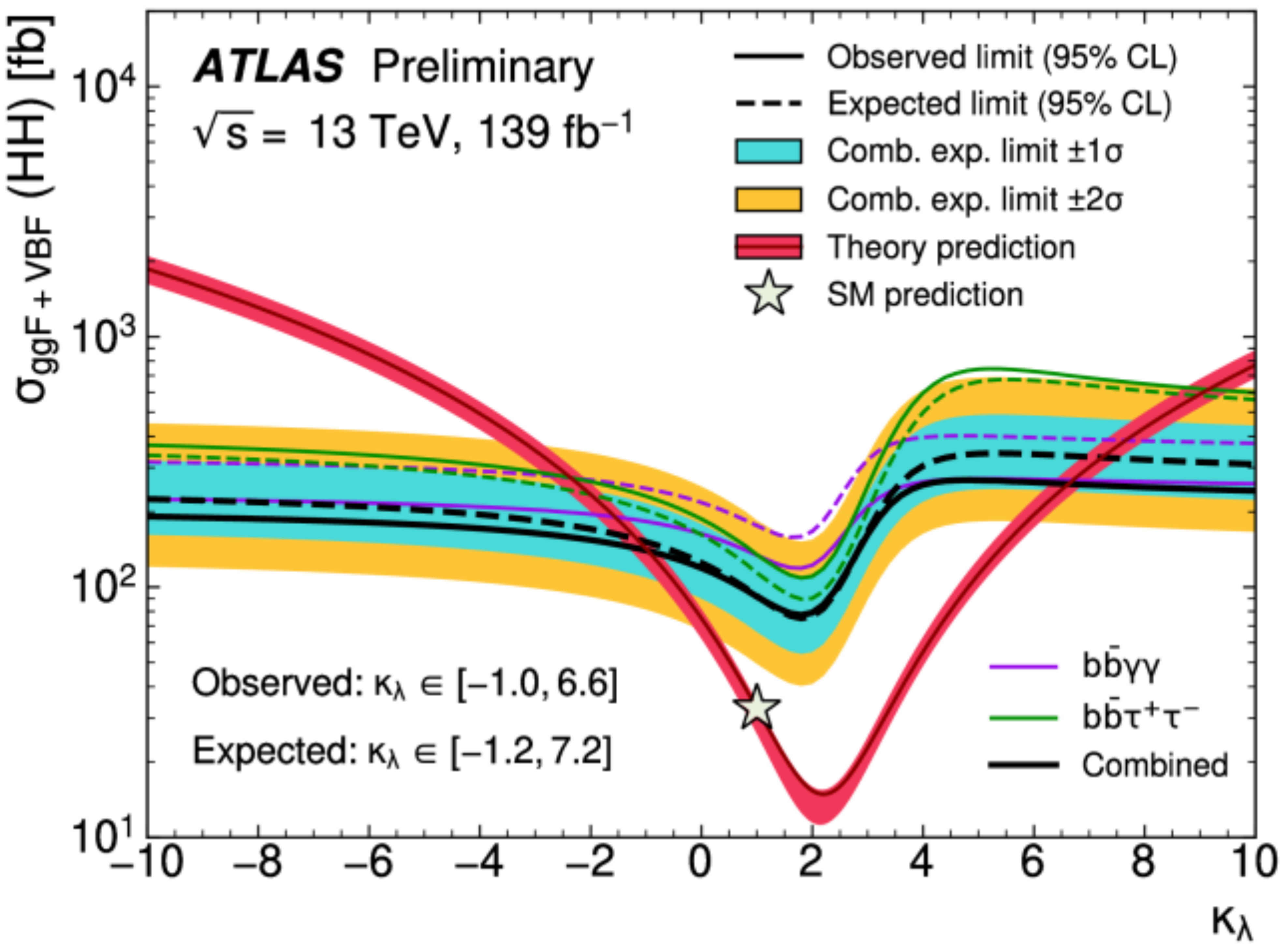
$b\bar{b}H$ production

PROBING H SELF INTERACTION THE CHALLENGES AHEAD

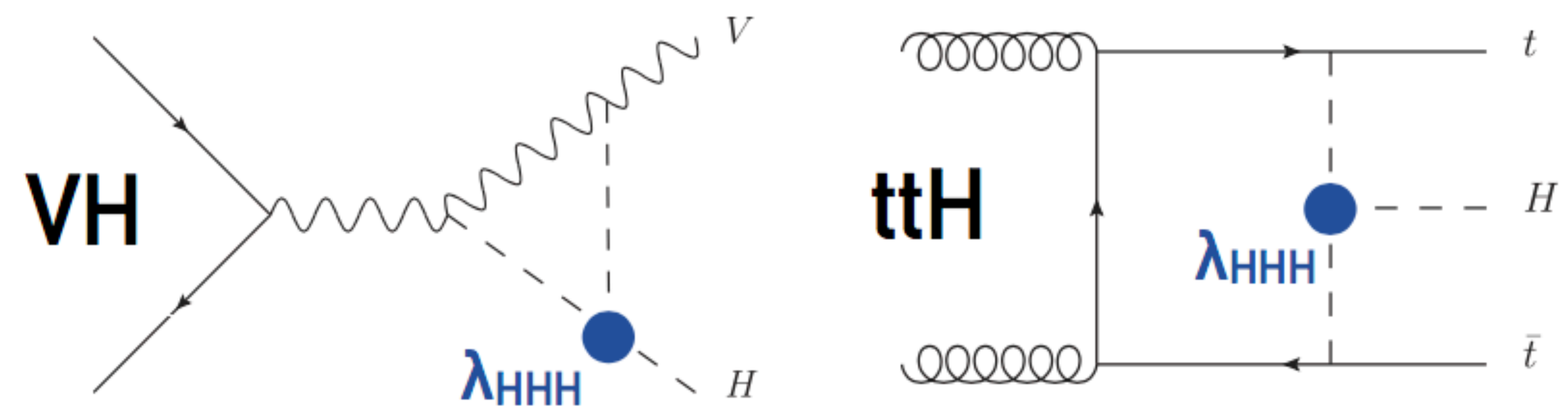
Direct sensitivity in HH production: Progress, but extremely hard to measure even at (HL-)LHC

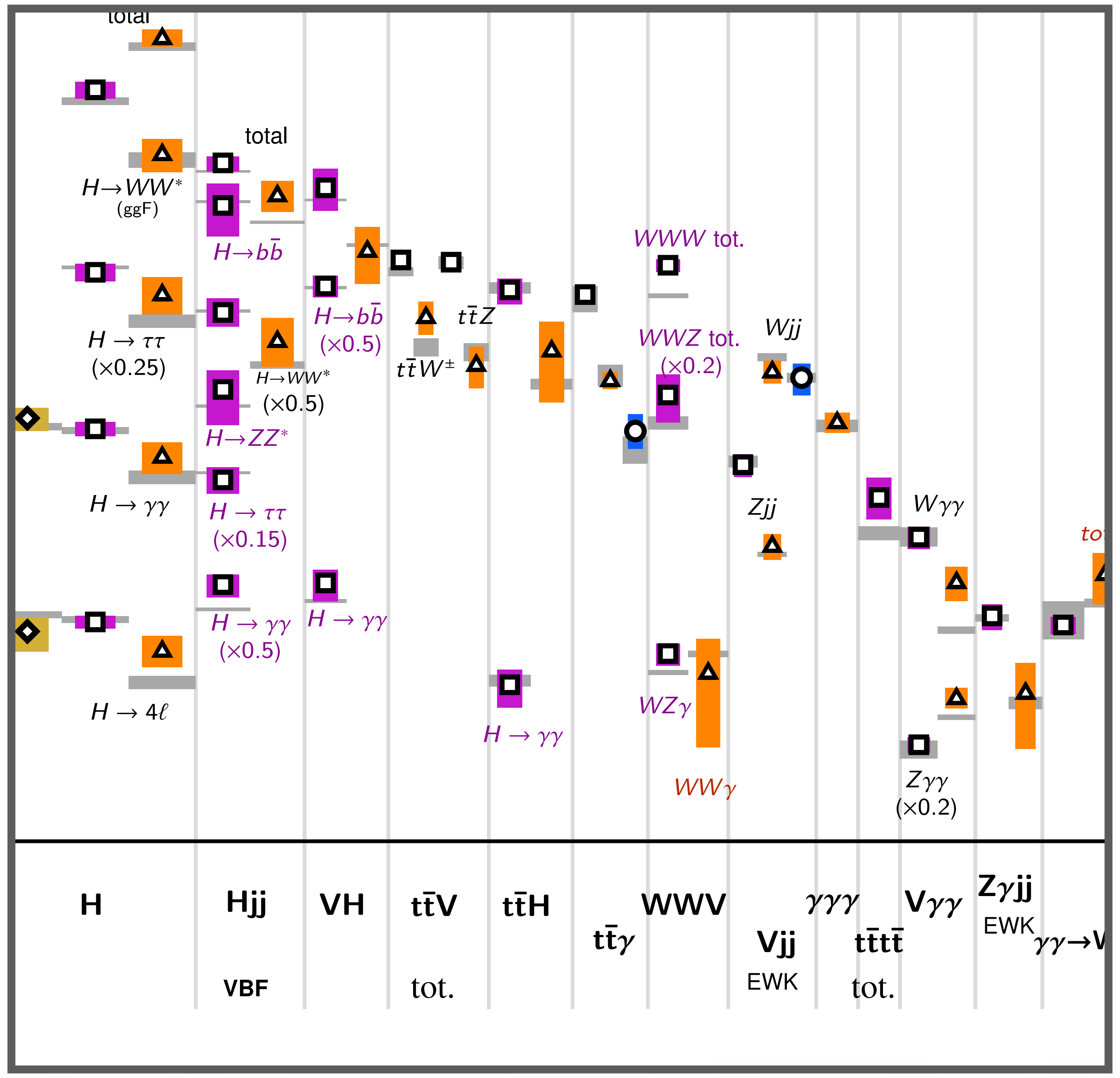


$b\bar{b}\tau\tau + b\bar{b}\gamma\gamma + b\bar{b}b\bar{b}$

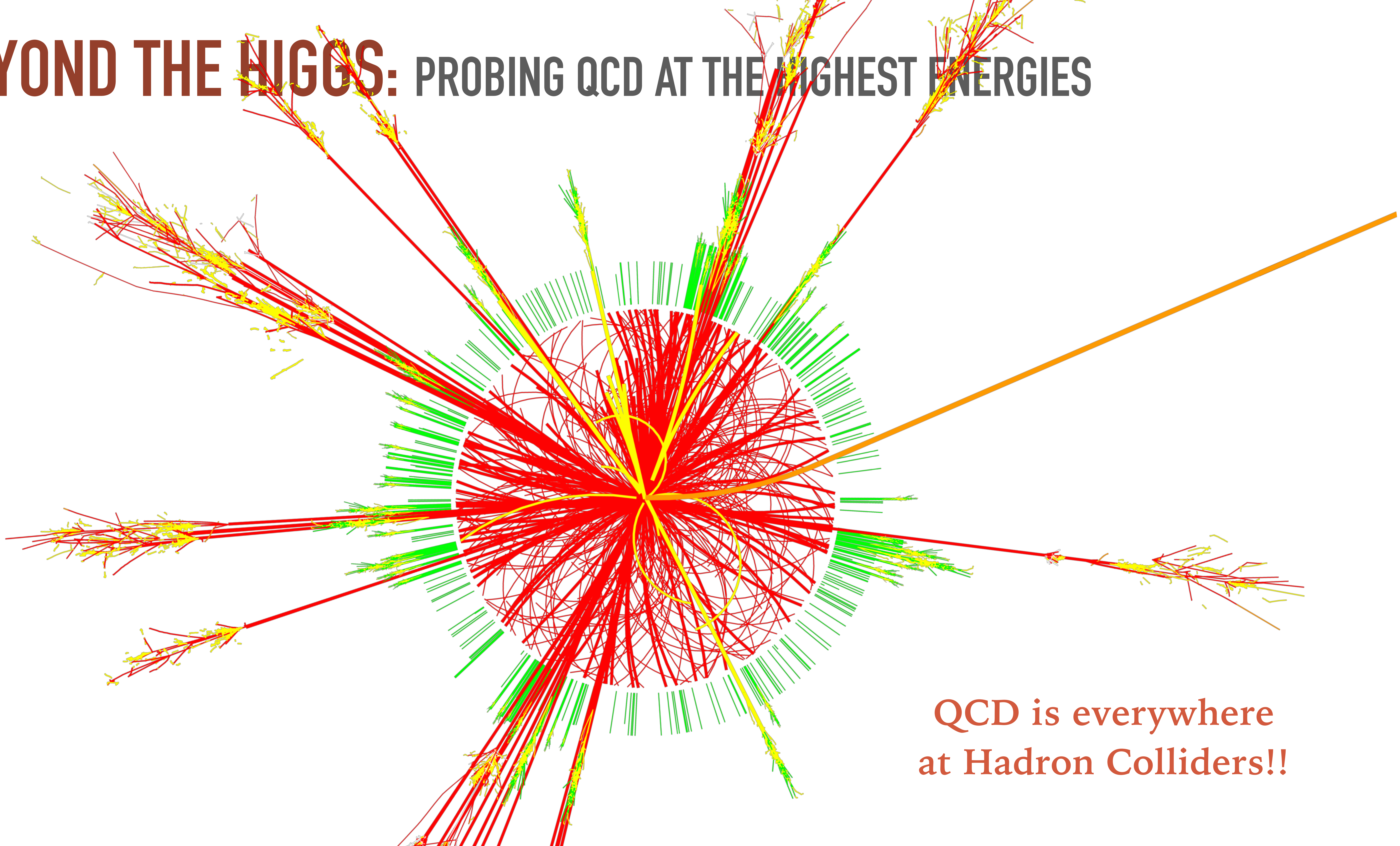


Indirect sensitivity through precision studies!



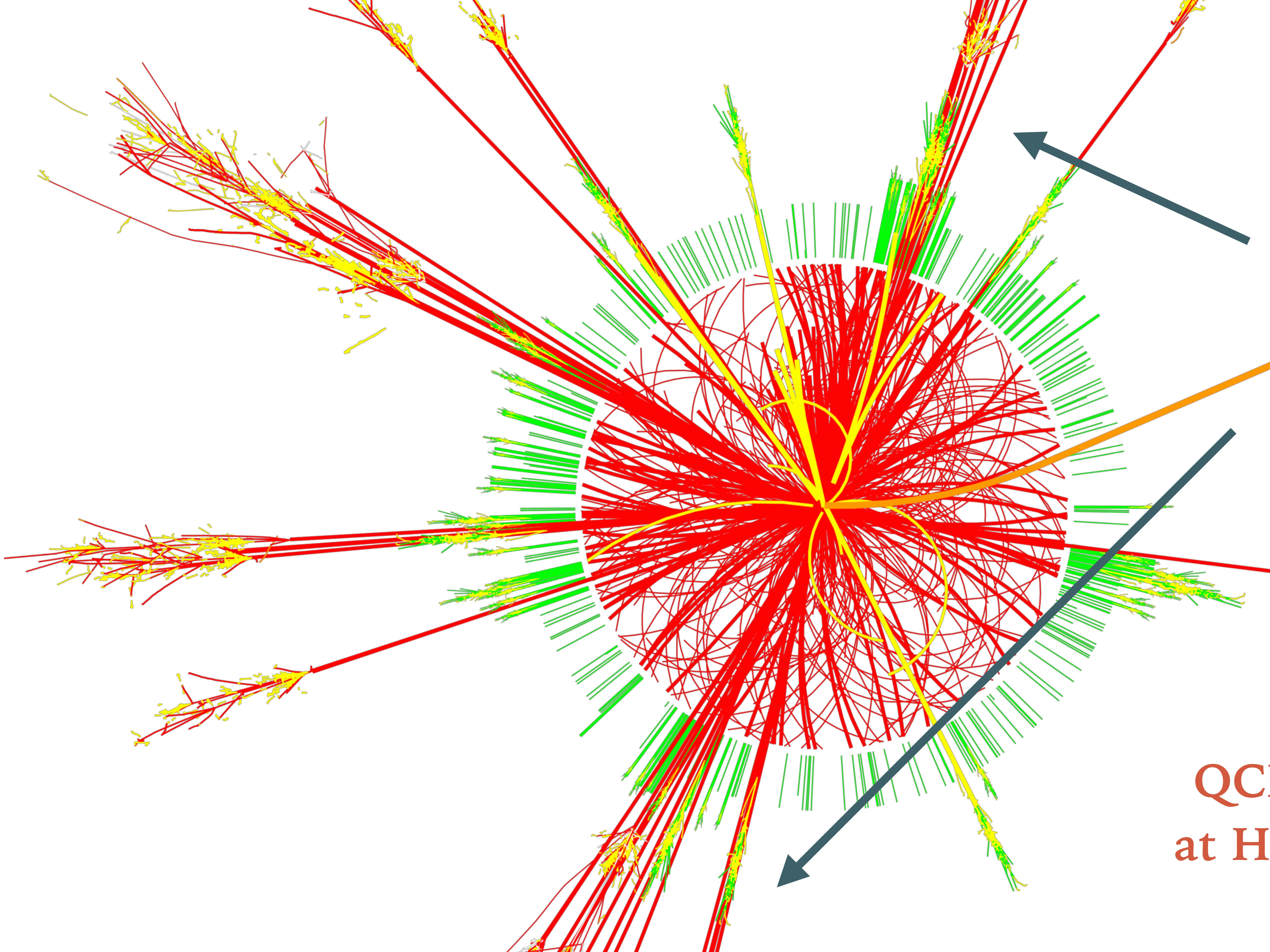


BEYOND THE HIGGS: PROBING QCD AT THE HIGHEST ENERGIES

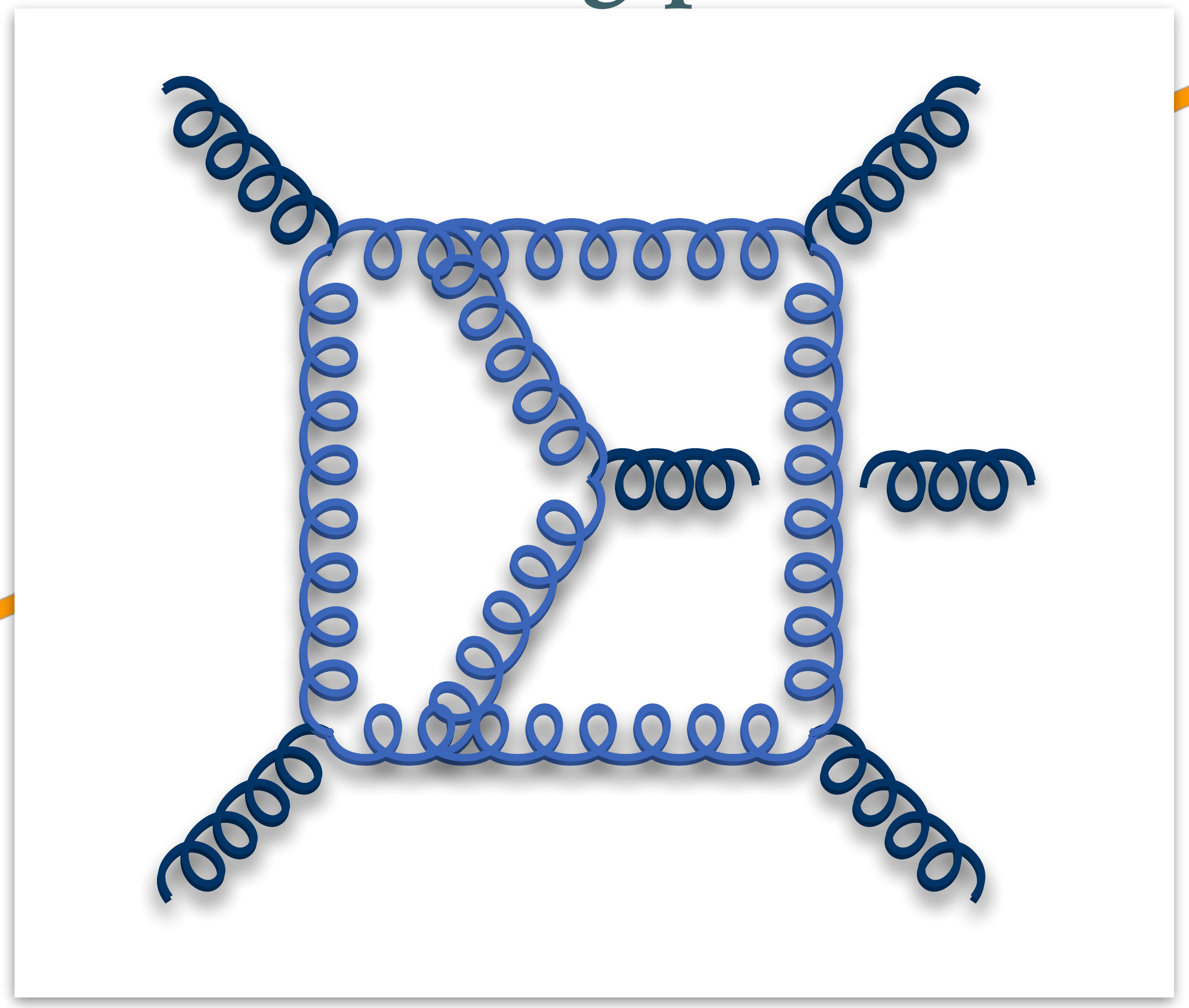


QCD is everywhere
at Hadron Colliders!!

BEYOND THE HIGGS: PROBING QCD AT THE HIGHEST ENERGIES

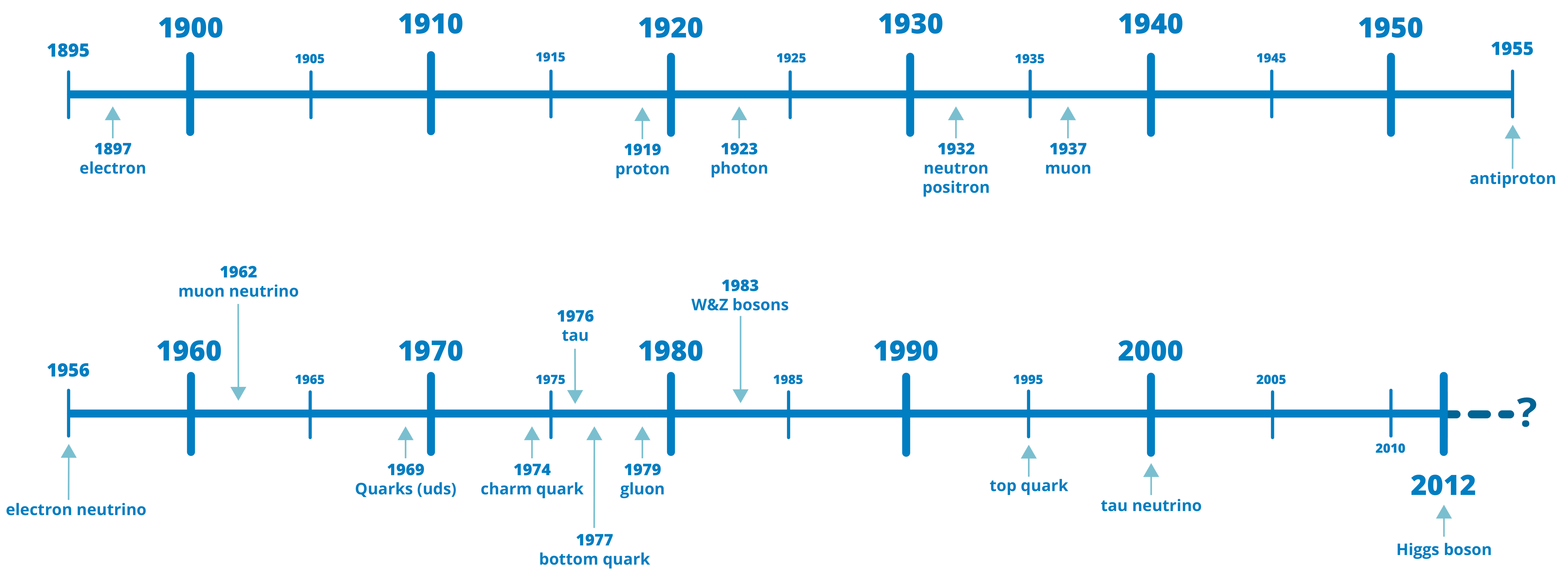


jets of strongly interacting particles



QCD is everywhere at Hadron Colliders!!

PLENTY OF POTENTIAL FOR "DISCOVERY" AHEAD

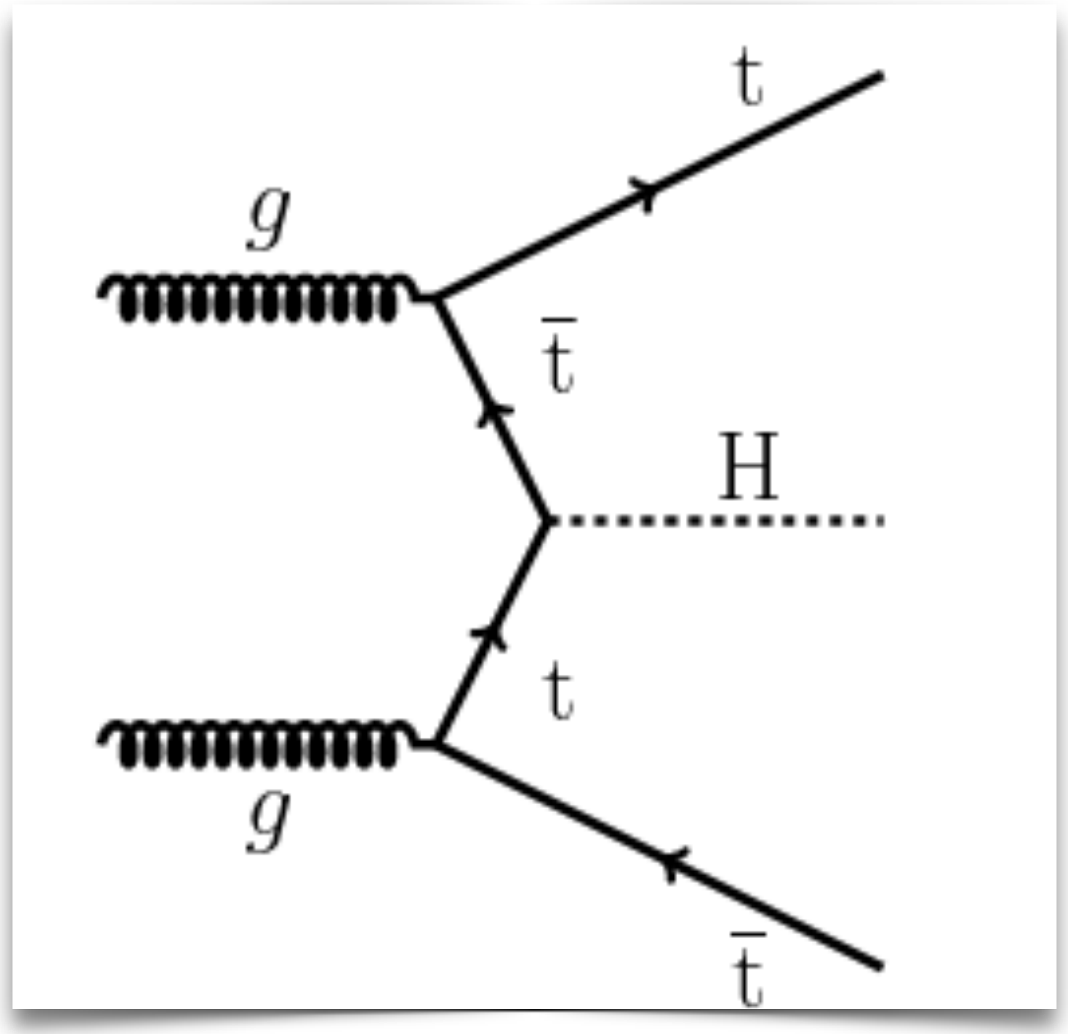


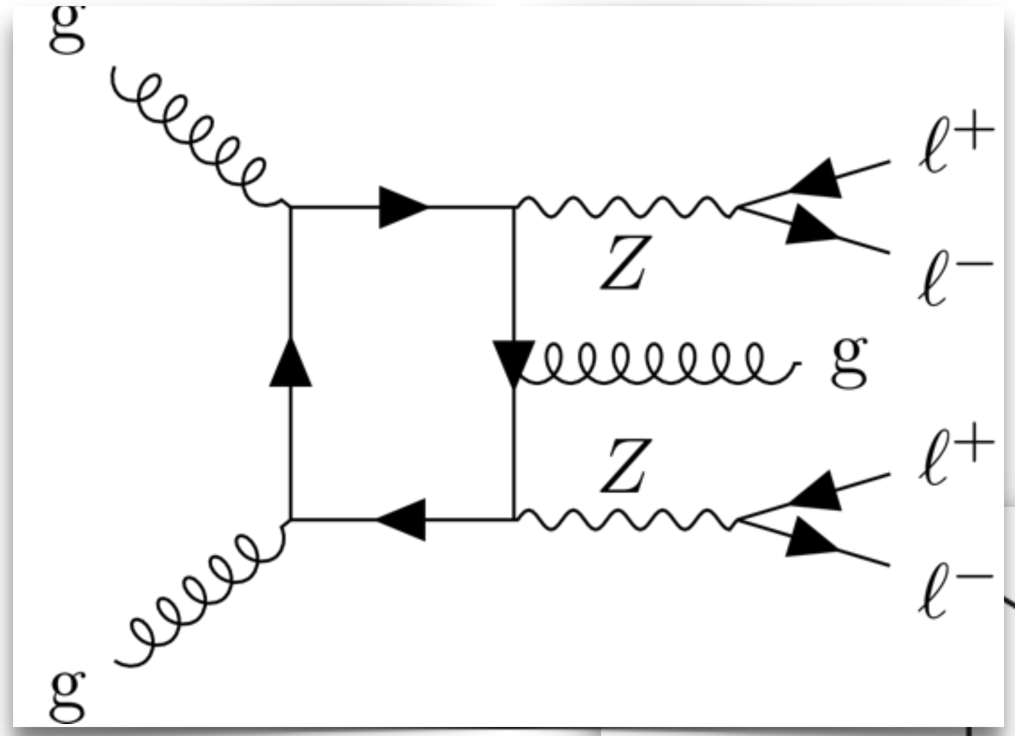
For the first time in decades, we might not expect new particles ahead…

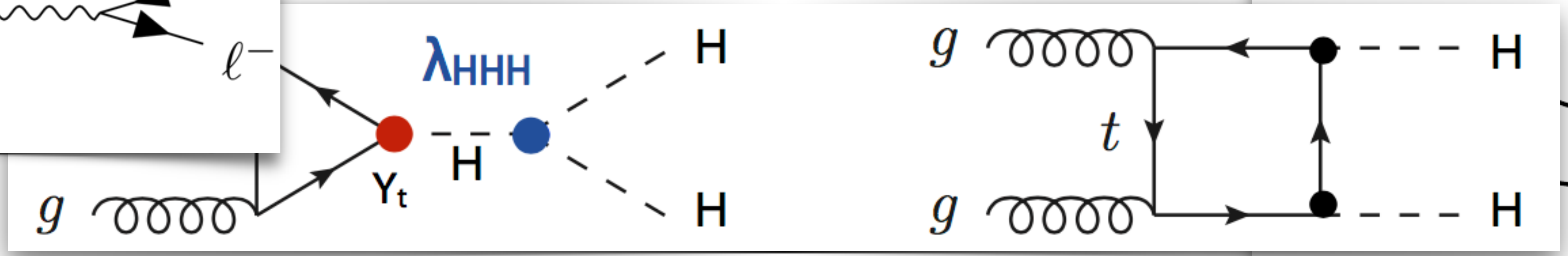
Still, thanks to % precision physics program at colliders, we have the chance to investigate these "new interactions", and **scrutinize quantum field theory to the highest precisions**

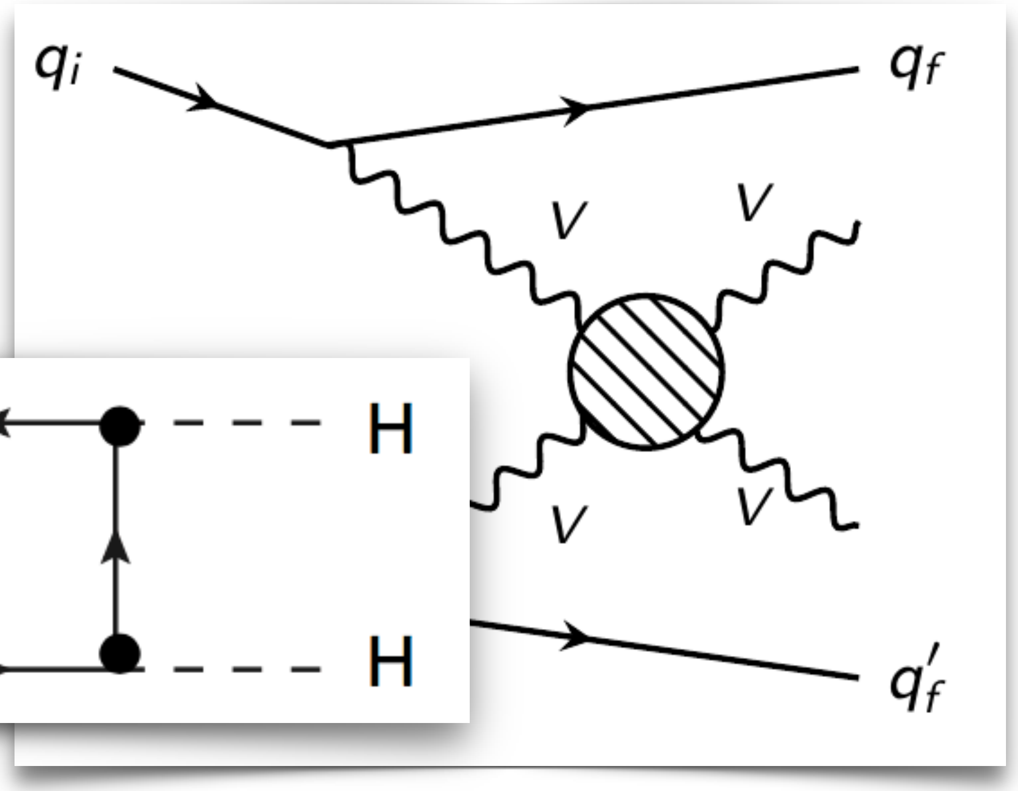
PRECISION STUDIES “OPPORTUNITIES” ALL OVER

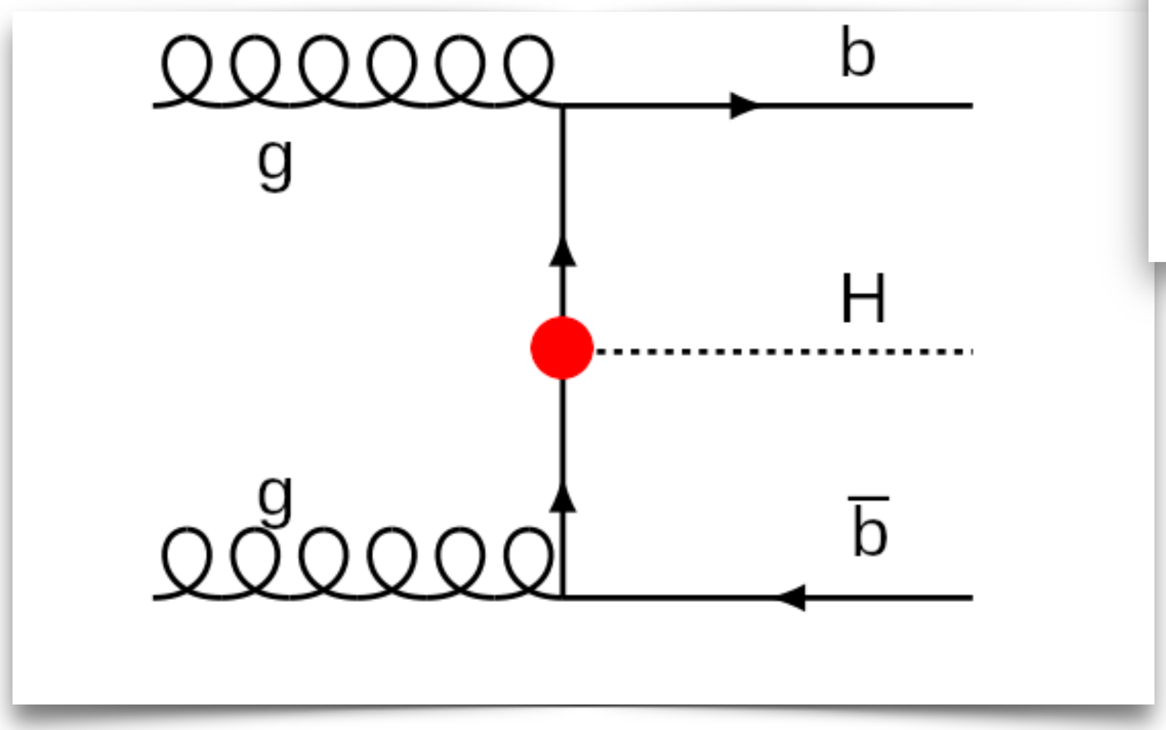
Rule of thumb: 1 more loop ~ 1 more leg

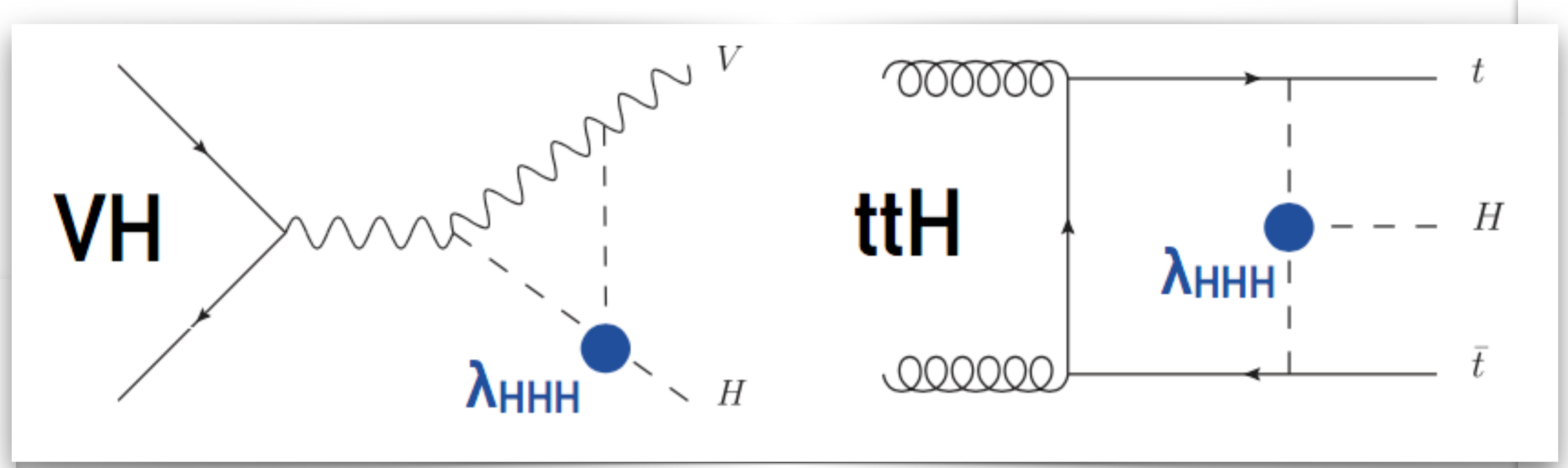


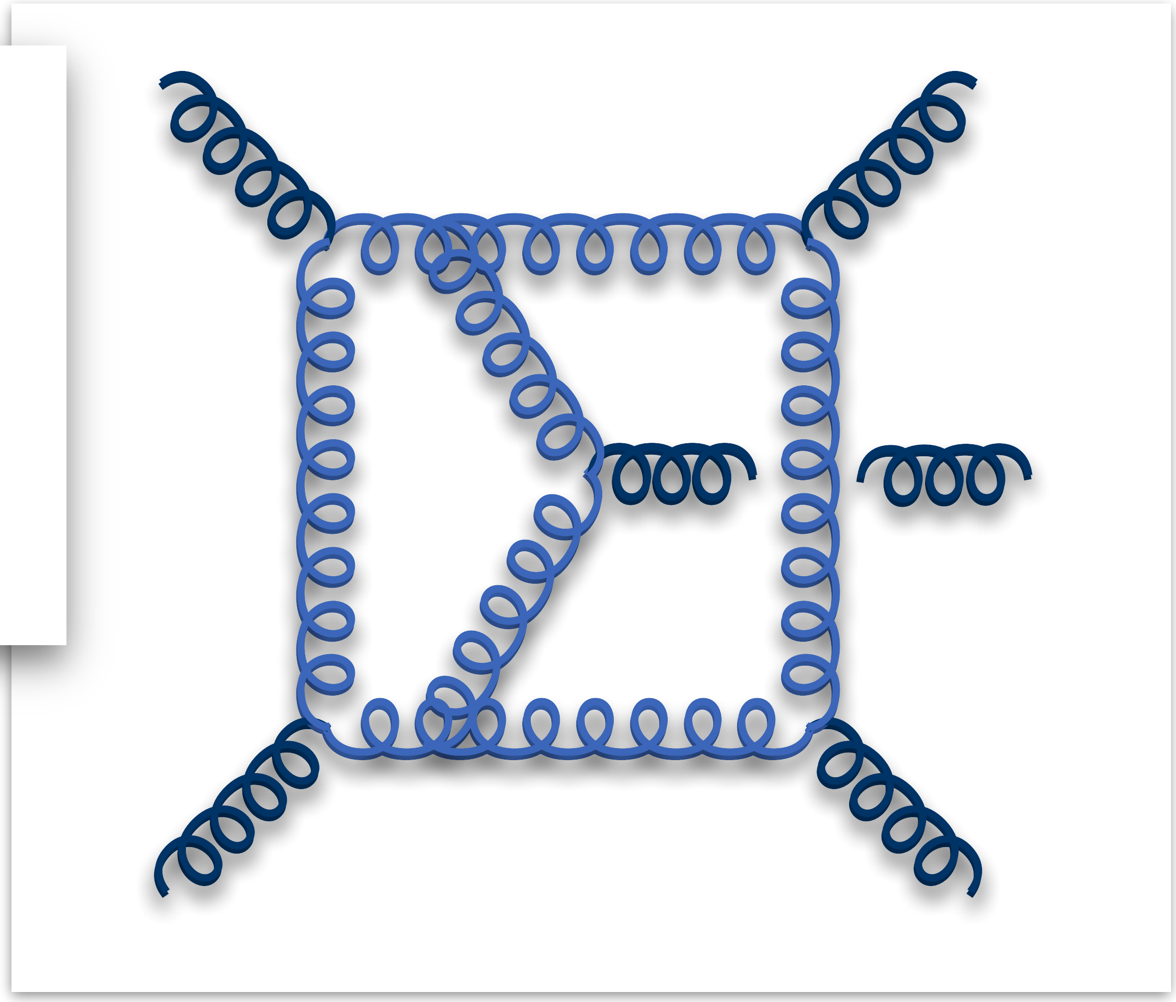






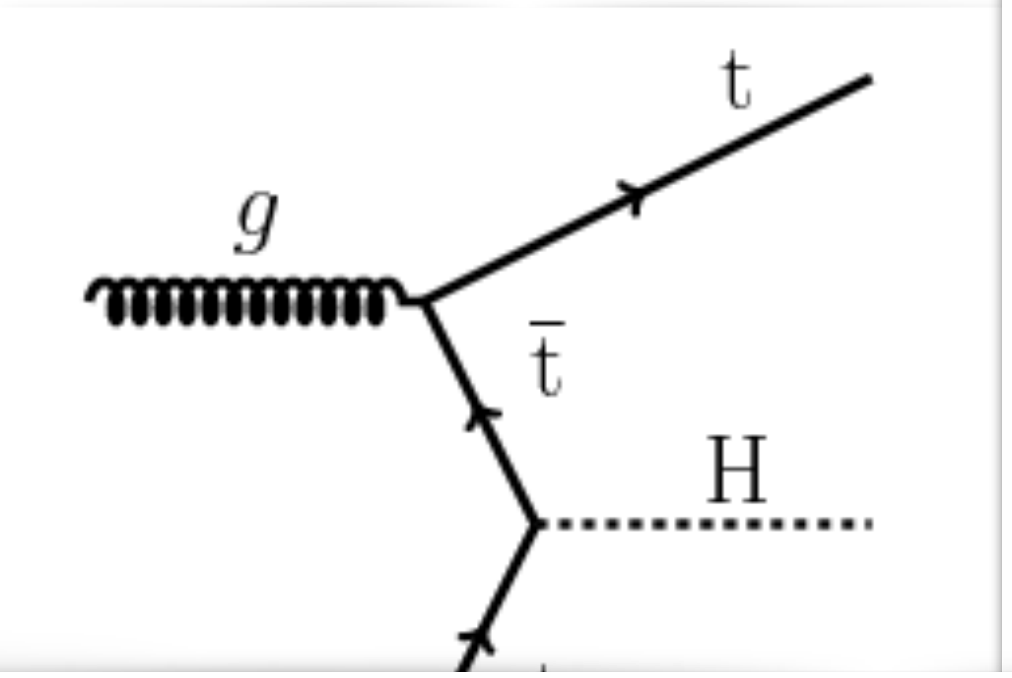


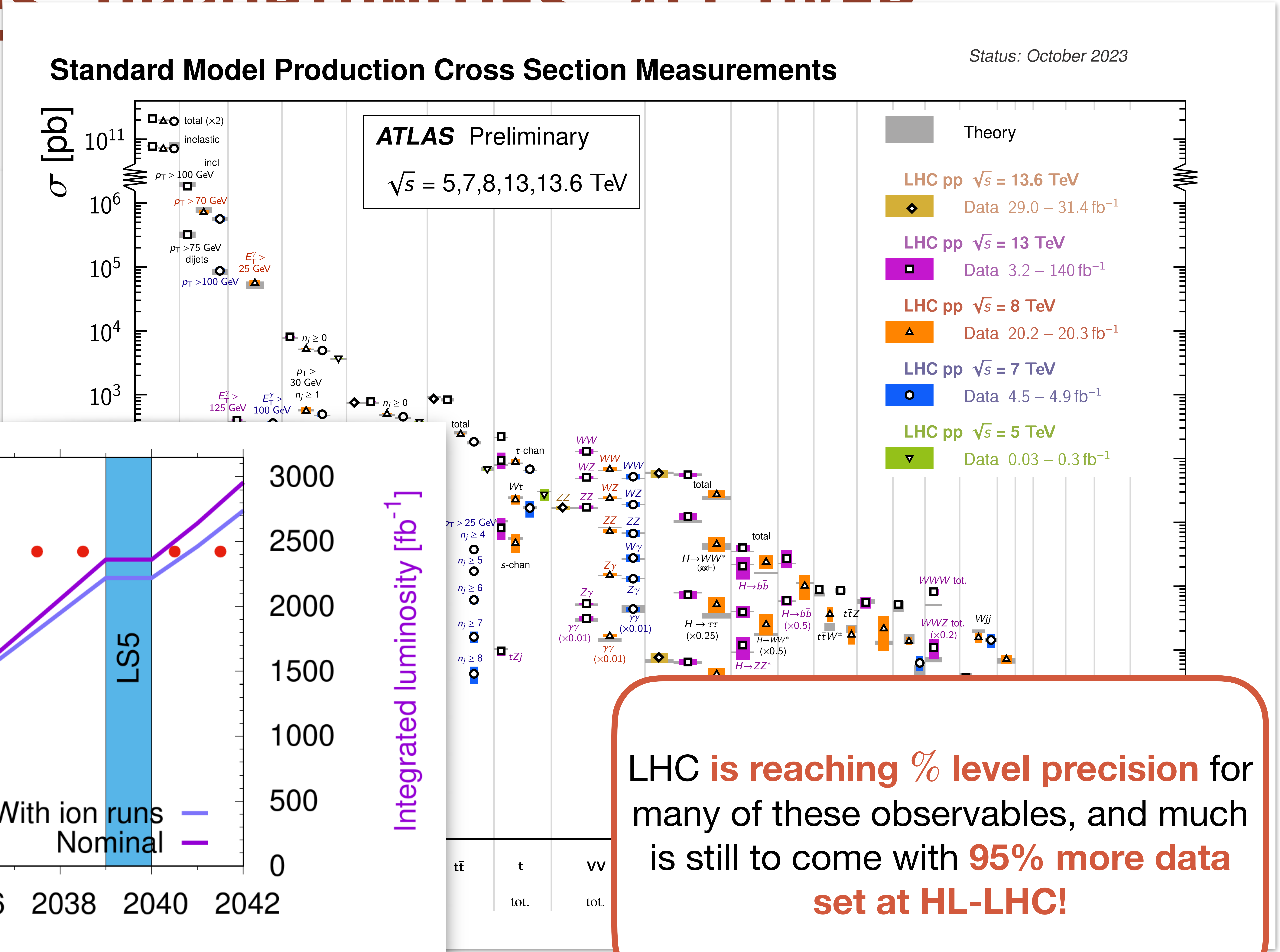


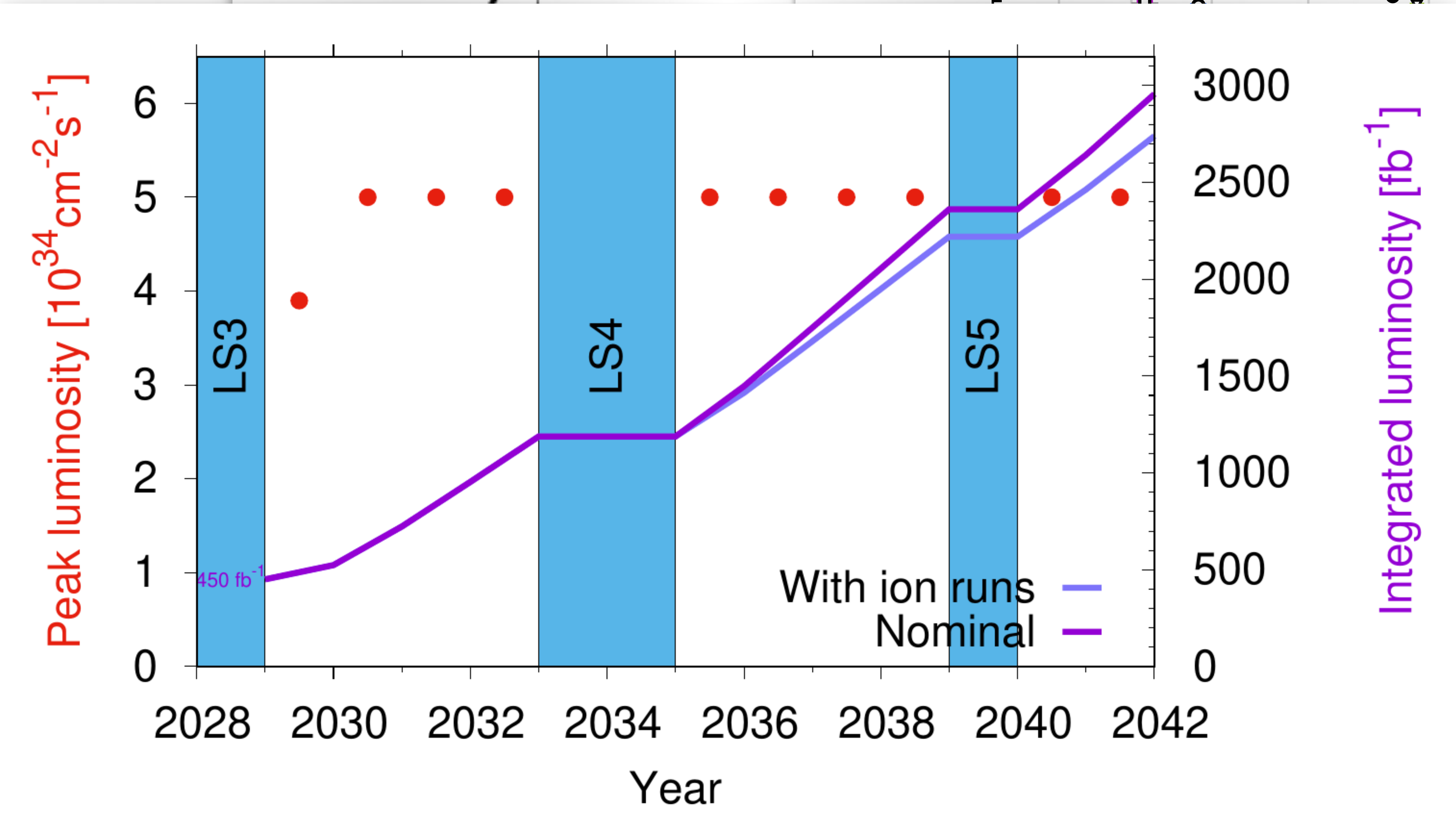


complexity scales very badly with
number of masses and external legs

PRECISION STUDIES “OPPORTUNITIES” ALL OVER



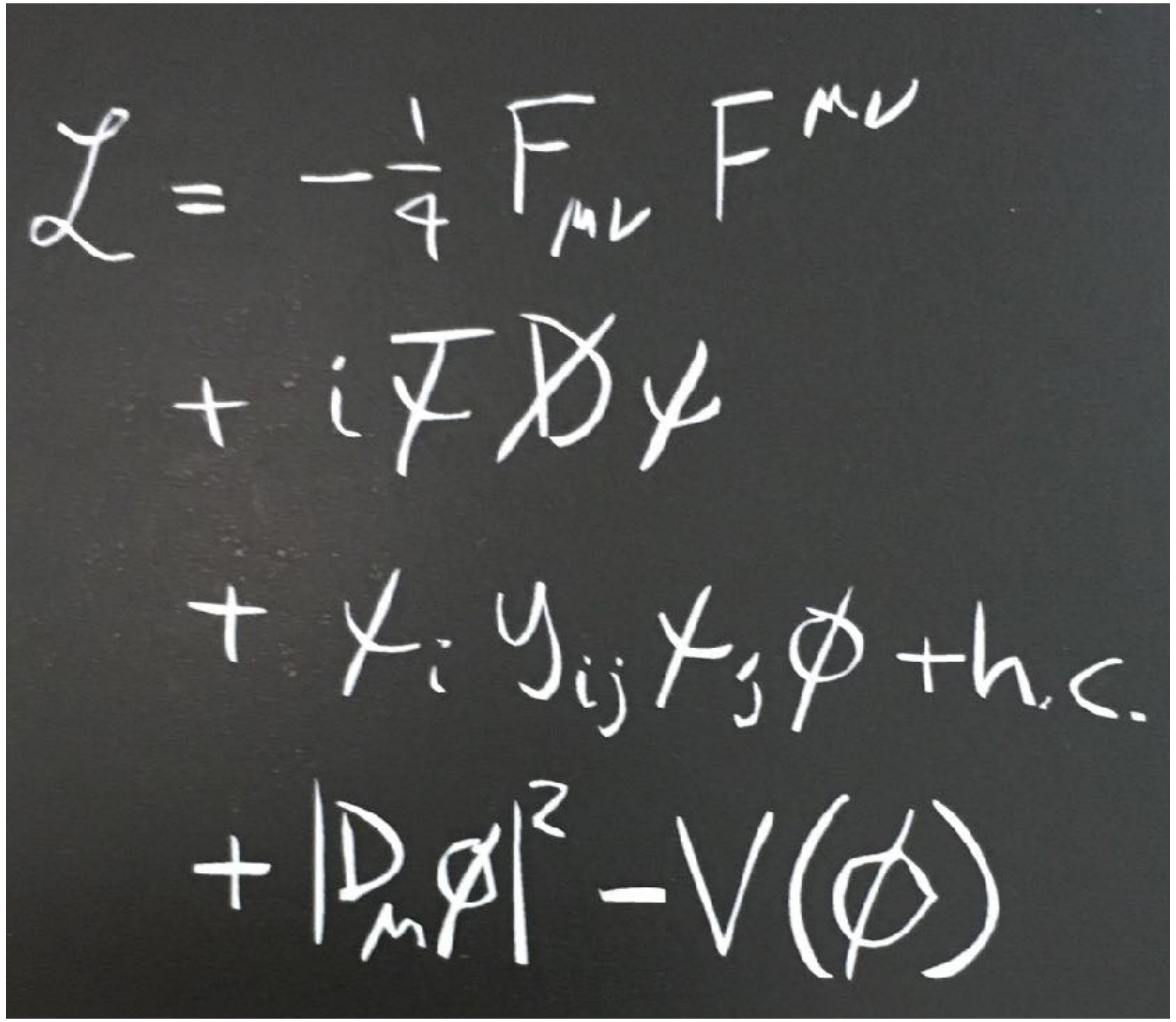




LHC **is reaching % level precision** for many of these observables, and much is still to come with **95% more data set at HL-LHC!**

% PRECISION, HOW DO WE GET THERE?

FROM THEORY TO THEORY PREDICTIONS IT'S A LONG WAY!



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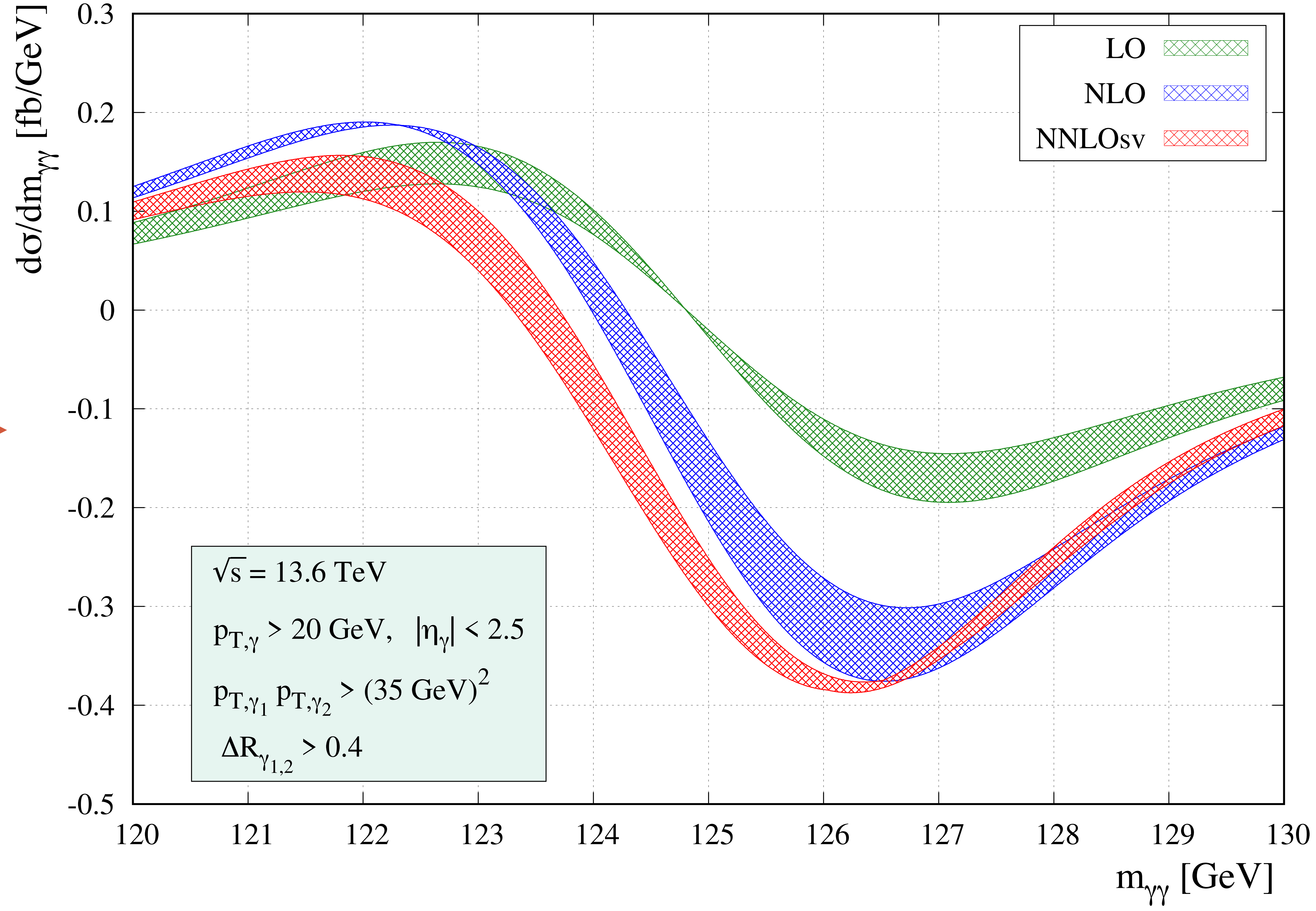
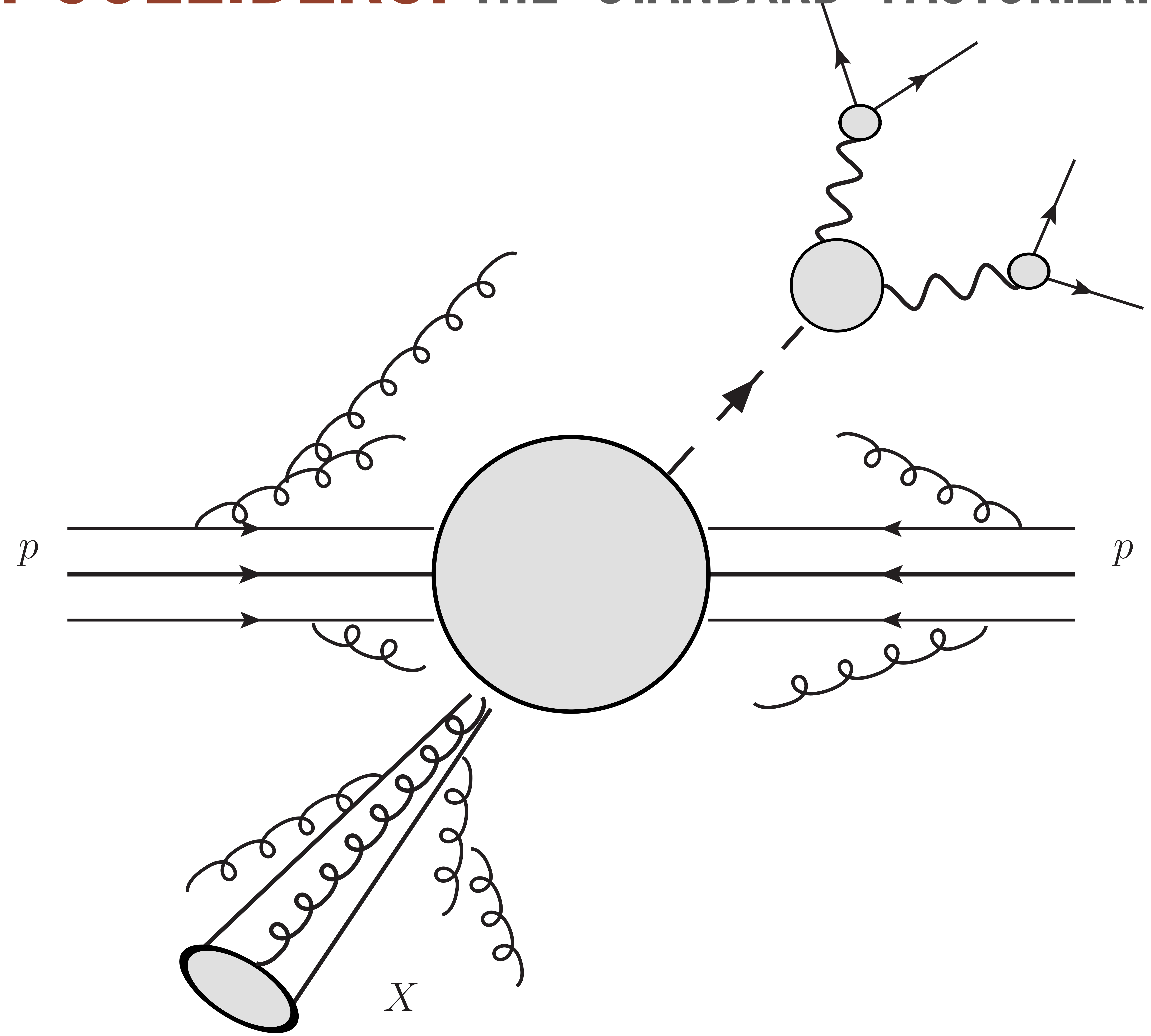


Signal to BKG interference for $gg \to H \to \gamma\gamma$

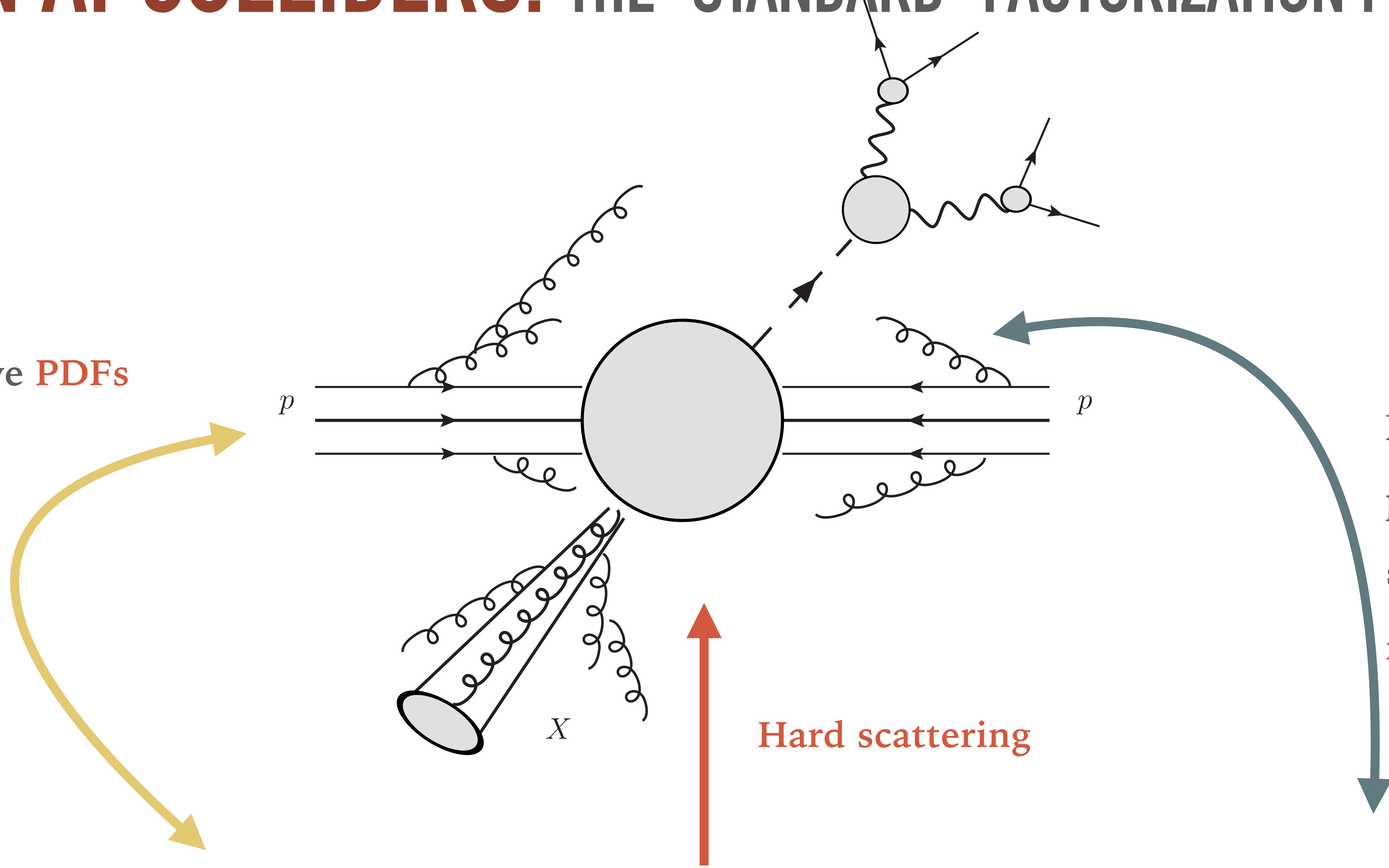
[Bargiela, Buccioni, Caola, Devoto, Manteuffel, Tancredi '22]

PRECISION AT COLLIDERS: THE "STANDARD" FACTORIZATION PICTURE



PRECISION AT COLLIDERS: THE "STANDARD" FACTORIZATION PICTURE

Non-perturbative PDFs



Hard scattering

Non-perturbative

power corrections

soft/collinear physics

minimal value of n?

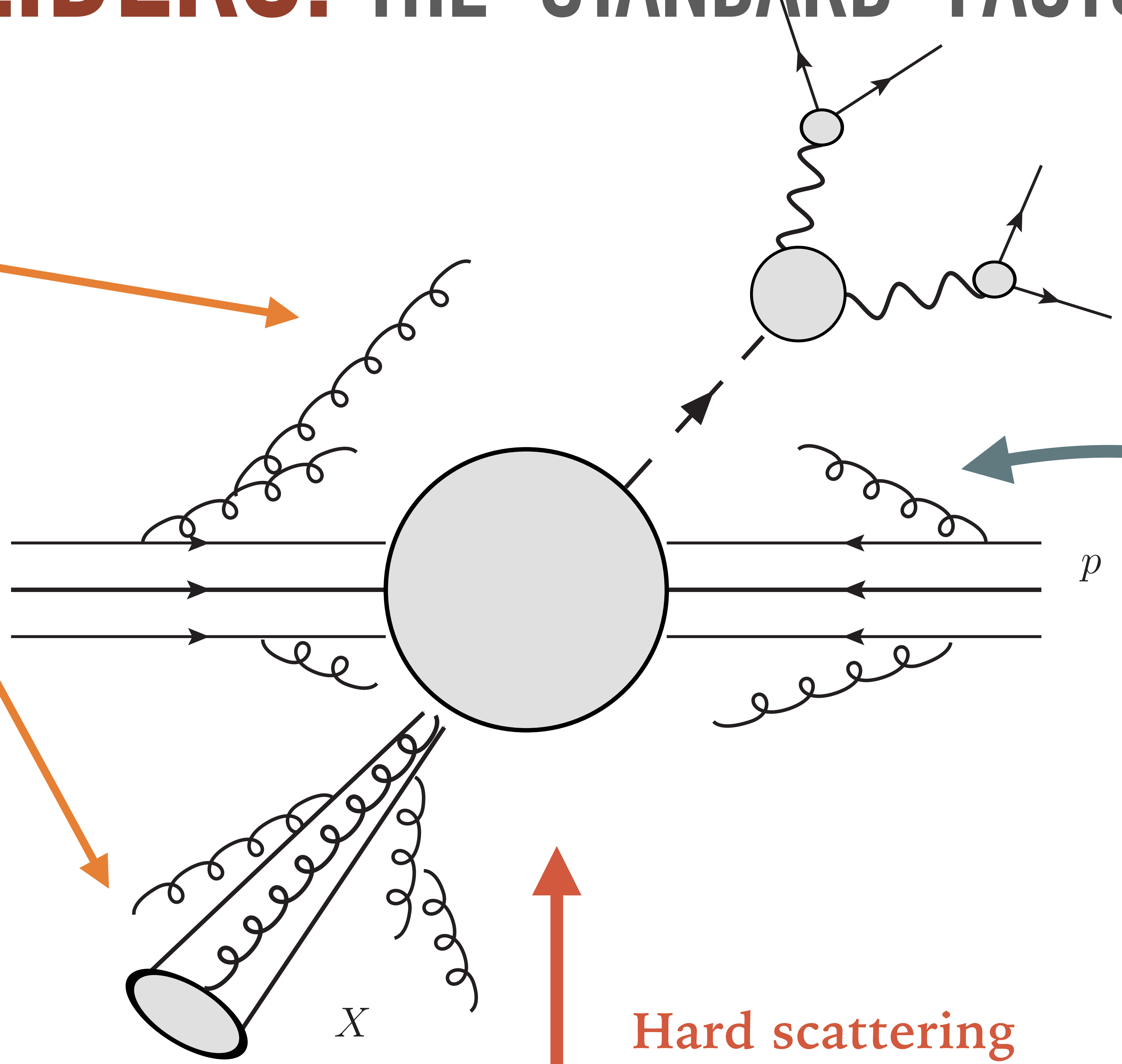
$$d\sigma = \int dx_1 dx_2\, f(x_1) f(x_2)\, d\sigma_{part}(x_1, x_2)\left(1 + \mathcal{O}(\Lambda^n_{\mathrm{QCD}}/Q^n)\right)$$

PRECISION AT COLLIDERS: THE “STANDARD” FACTORIZATION PICTURE

Resummation, parton showers, matching…

See C. Biello’s talk this afternoon

Non-perturbative PDFs



Non-perturbative power corrections soft/collinear physics minimal value of n?

Hard scattering

$$\mathrm{d}\sigma = \int dx_1 dx_2\, f(x_1) f(x_2)\, \mathrm{d}\sigma_{part}(x_1, x_2)(1 + \mathcal{O}(\Lambda^n_{\mathrm{QCD}}/Q^n))$$

PRECISION AT COLLIDERS

THE "STANDARD" FACTORIZATION PICTURE

Resumm
showers

See C. I
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linear physics

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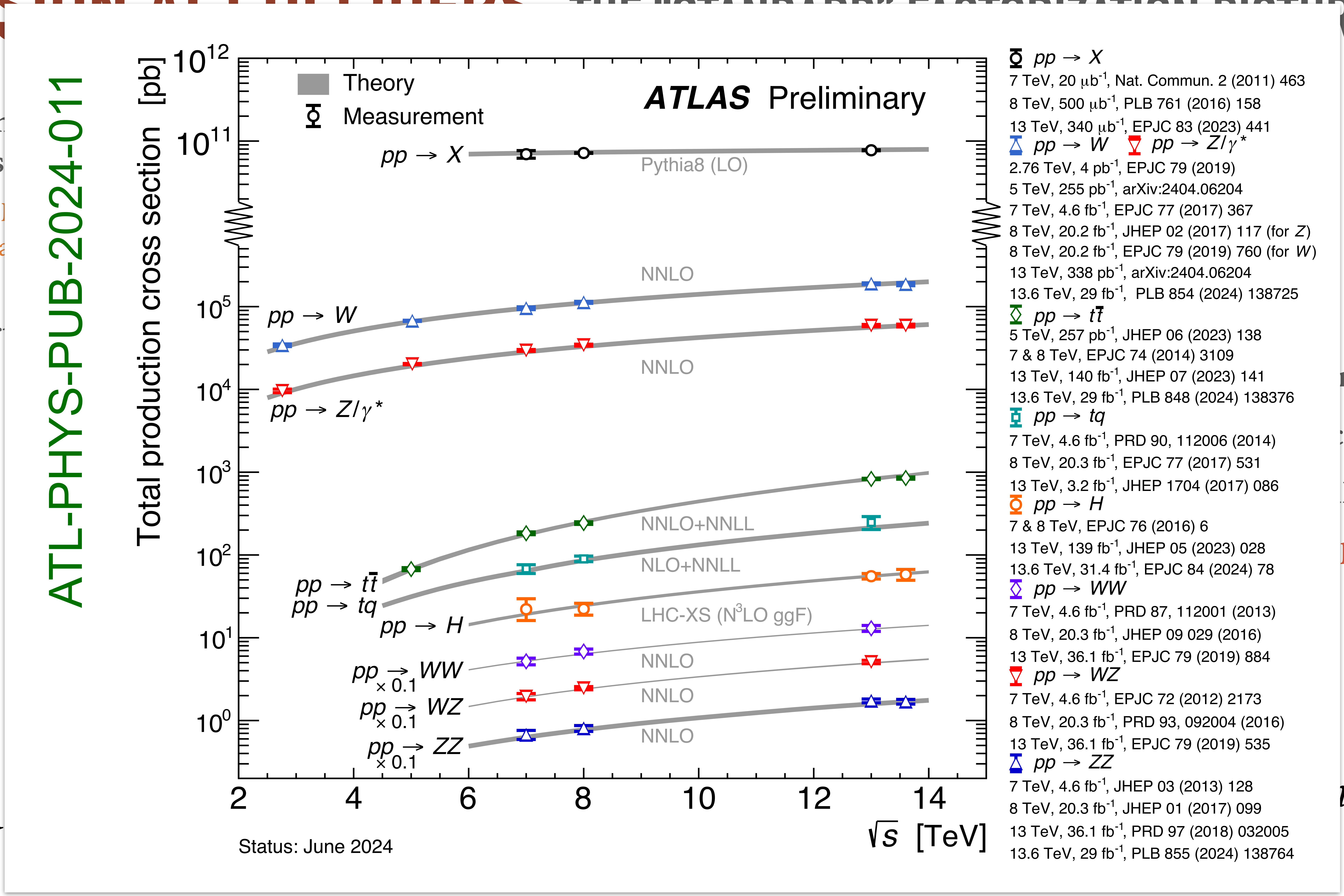


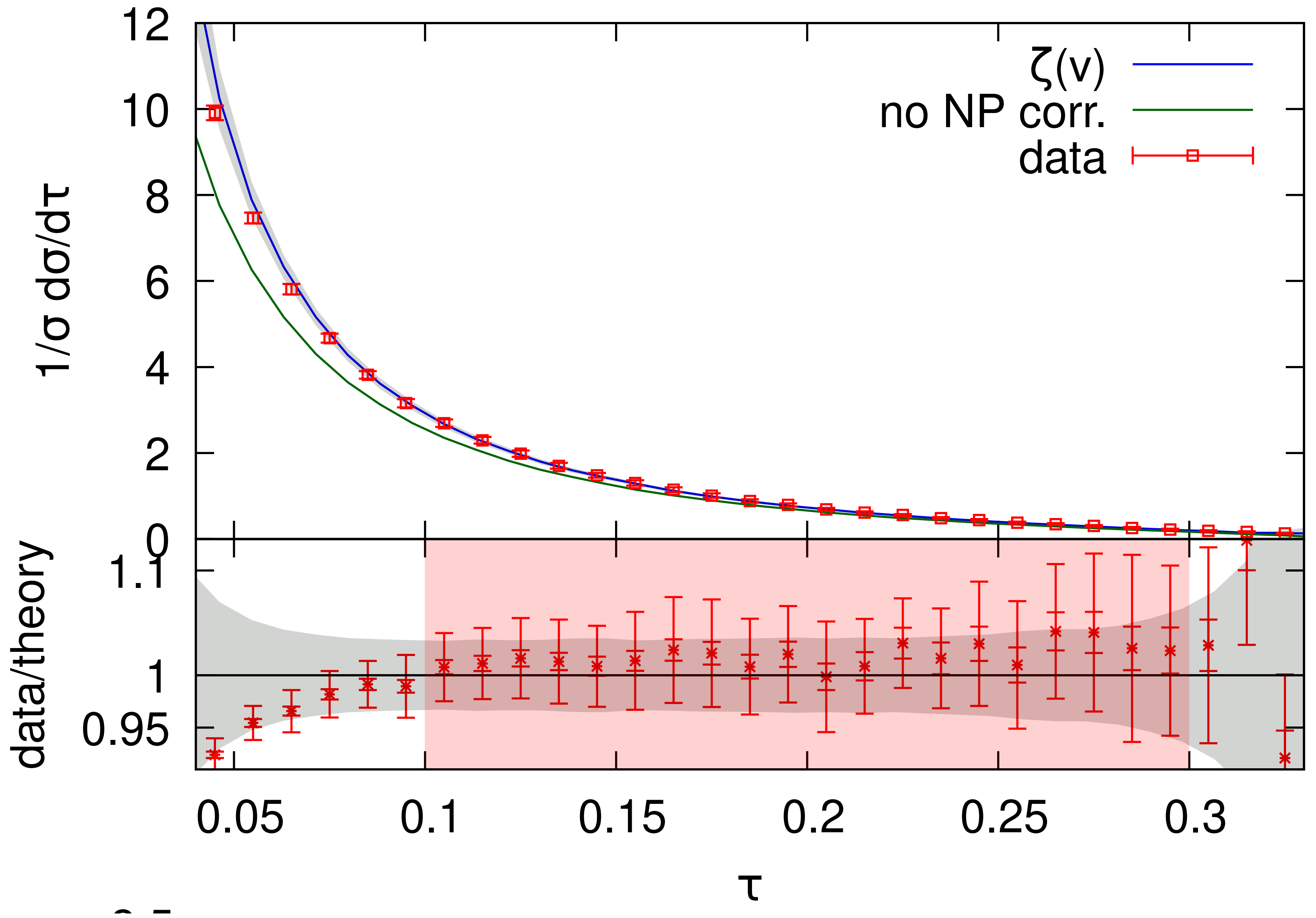
PRECISION AT COLLIDERS: IS IT UNDER CONTROL?

$$(1 + \mathcal{O}(\Lambda_{\rm QCD}^n / Q^n))$$

$\Lambda_{\rm QCD} \sim 1$ GeV $Q \sim 30 - 100$ GeV

if $n = 1$, can easily give % level corrections

Impact in $e^+e^- \to 3$ jets for α_S fits *(subtleties in 3 jets vs 2 jet case)* [Nason, Zanderighi '23]



Recently excluded for some observables

$q\bar{q} \to t\bar{t}$ [Makarov, Melnikov, Nason, Ozcelik '23]

Using short-distance ($\overline{\rm MS}$) top-mass scheme

Single top [Makarov, Melnikov, Nason, Ozcelik '23, '24]

Depending also on observable (positron momentum components!)

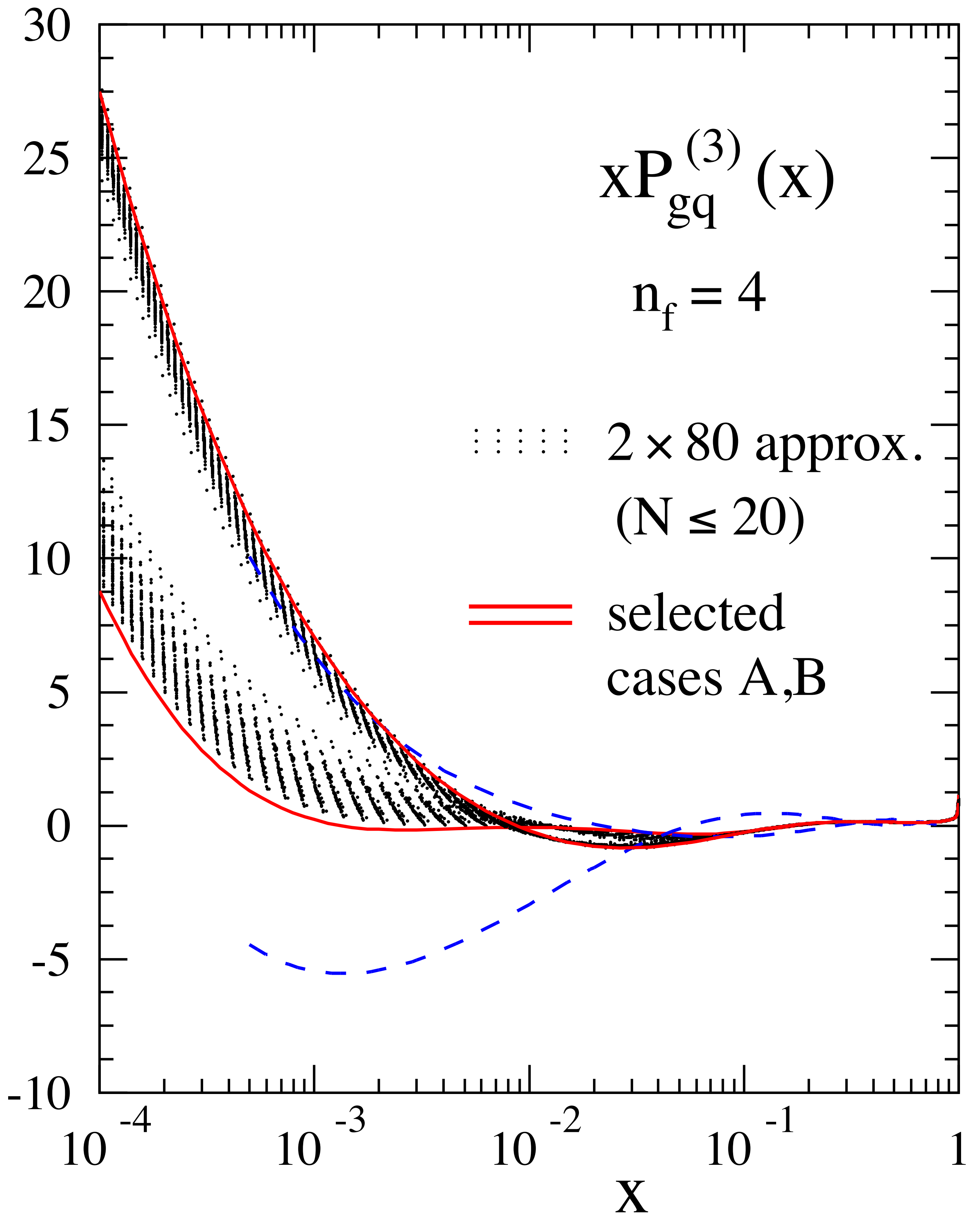
What about more subtle effects? $\Lambda_{\rm QCD}^2 \ln^2 \Lambda_{\rm QCD}$

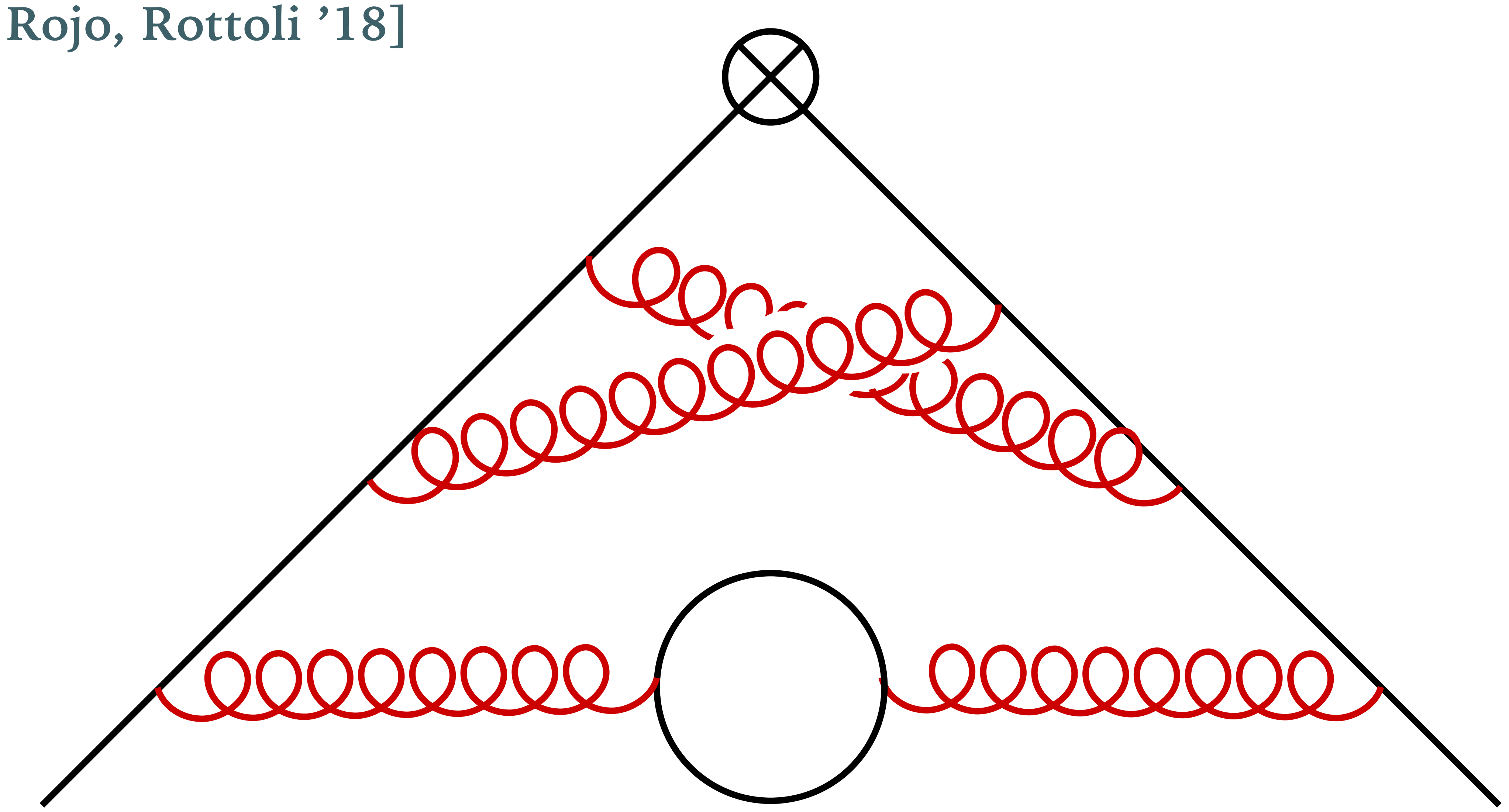
PRECISION AT COLLIDERS: PARTON DISTRIBUTION FUNCTIONS

$$\mathrm{d}x_1\mathrm{d}x_2 f(x_1) f(x_2)$$

Huge effort to understand **PDF evolution to N3LO**

Interesting **low-x behaviour** [Ball, Bertone, Bonvini, Marzani, Rojo, Rottoli '18]





[Davies, Vogt, Ruji, Ueda, Vermaseren,'16]
[Gehrmann, Manteuffel, Sotnikov, Yang '23, '24]
[Falcioni, Herzog, Moch, Pelloni, Vogt '23, '24]

HARD SCATTERING

$$\mathrm{d}\sigma_{\mathrm{part}}(x_1, x_2) \longrightarrow \sigma_{q\bar{q}\to gg} = \int [\mathrm{dPS}] \left|\mathcal{M}_{q\bar{q}\to gg}\right|^2$$

HARD SCATTERING

$$\mathrm{d}\sigma_{\mathrm{part}}(x_1, x_2) \longrightarrow \sigma_{q\bar{q}\to gg} = \int [\mathrm{dPS}]\, |\mathcal{M}_{q\bar{q}\to gg}|^2$$

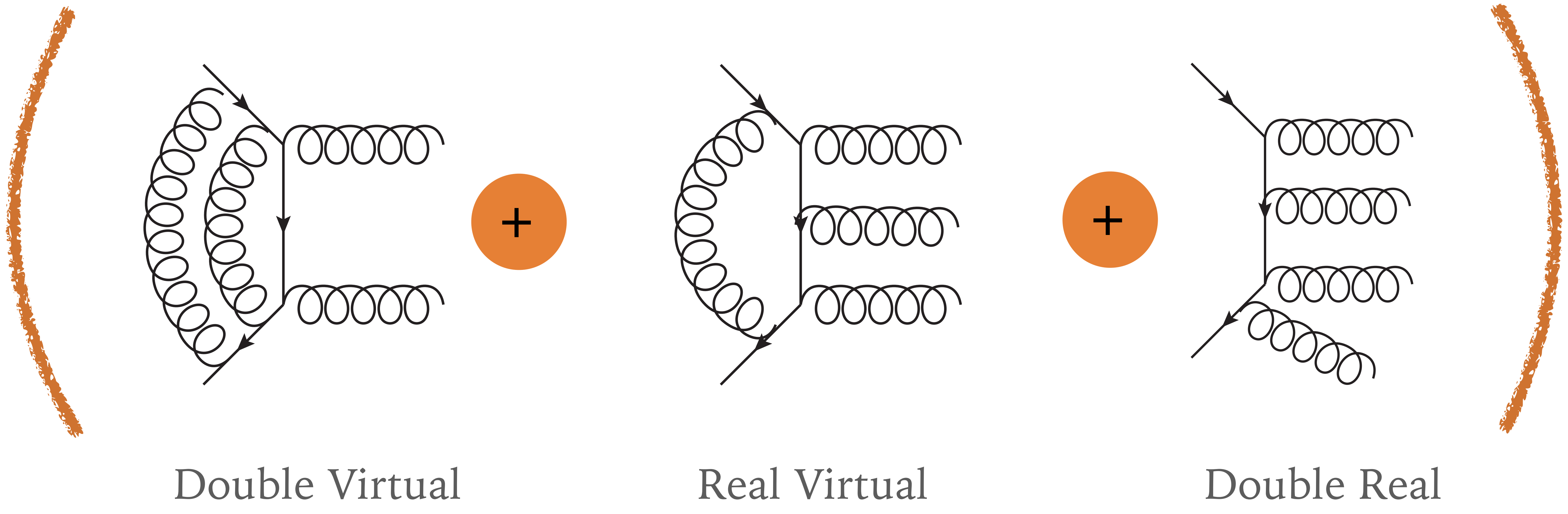
small "coupling constant" ~ 0.1

$$|\mathcal{M}_{q\bar{q}\to gg}|^2 = |\mathcal{M}^{LO}_{q\bar{q}\to gg}|^2 + \left(\frac{\alpha_s}{2\pi}\right)|\mathcal{M}^{NLO}_{q\bar{q}\to gg}|^2 + \left(\frac{\alpha_s}{2\pi}\right)^2 |\mathcal{M}^{NNLO}_{q\bar{q}\to gg}|^2 + \ldots$$

HARD SCATTERING

$$d\sigma_{\text{part}}(x_1, x_2) \longrightarrow \sigma_{q\bar{q}\to gg} = \int [\text{dPS}] \left|\mathcal{M}_{q\bar{q}\to gg}\right|^2$$

$$\left|\mathcal{M}_{q\bar{q}\to gg}\right|^2 = \left|\mathcal{M}^{LO}_{q\bar{q}\to gg}\right|^2 + \left(\frac{\alpha_s}{2\pi}\right)\left|\mathcal{M}^{NLO}_{q\bar{q}\to gg}\right|^2 + \left(\frac{\alpha_s}{2\pi}\right)^2 \left|\mathcal{M}^{NNLO}_{q\bar{q}\to gg}\right|^2 + \dots$$



~ O(5%) precision

Often not even enough in pure QCD

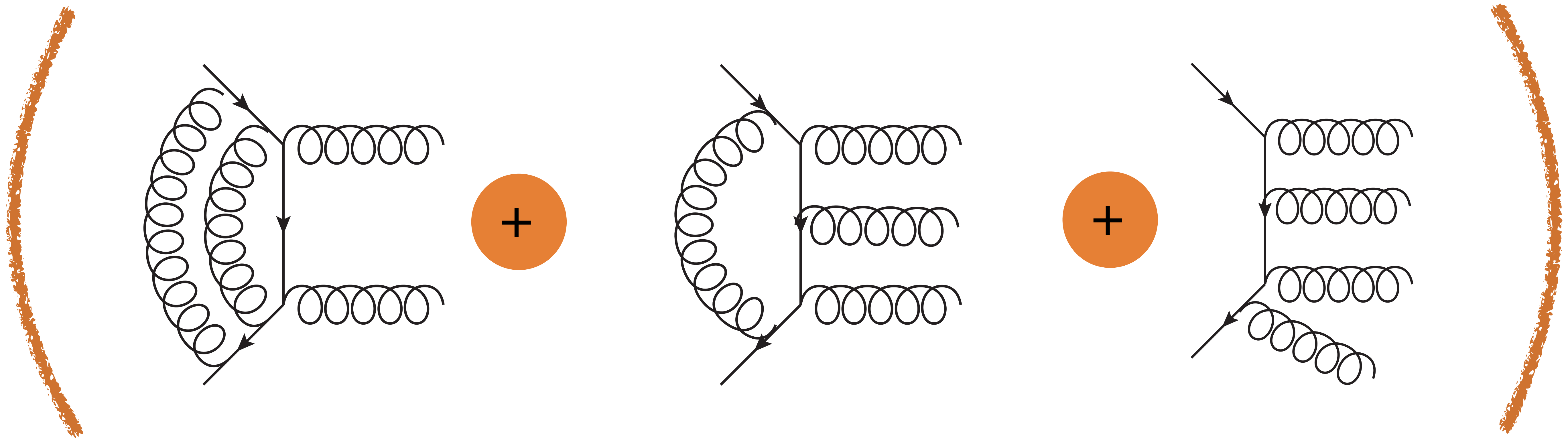
QCD+EW effects must be included!

See T. Armadillo's talk this afternoon

HARD SCATTERING

$$\mathrm{d}\sigma_{\mathrm{part}}(x_1, x_2) \longrightarrow \sigma_{q\bar{q}\to gg} = \int [\mathrm{dPS}] \left|\mathcal{M}_{q\bar{q}\to gg}\right|^2$$

$$\left|\mathcal{M}_{q\bar{q}\to gg}\right|^2 = \left|\mathcal{M}^{LO}_{q\bar{q}\to gg}\right|^2 + \left(\frac{\alpha_s}{2\pi}\right)\left|\mathcal{M}^{NLO}_{q\bar{q}\to gg}\right|^2 + \left(\frac{\alpha_s}{2\pi}\right)^2 \left|\mathcal{M}^{NNLO}_{q\bar{q}\to gg}\right|^2 + \ldots$$



Double Virtual

Real Virtual

Double Real

Cancellation of IR divergences

Well under control up to NNLO

Antennas, Stripper, Nested, Torino, Colorful, Geometric, slicing schemes

(Some more developed than others, but **conceptually under control**!)

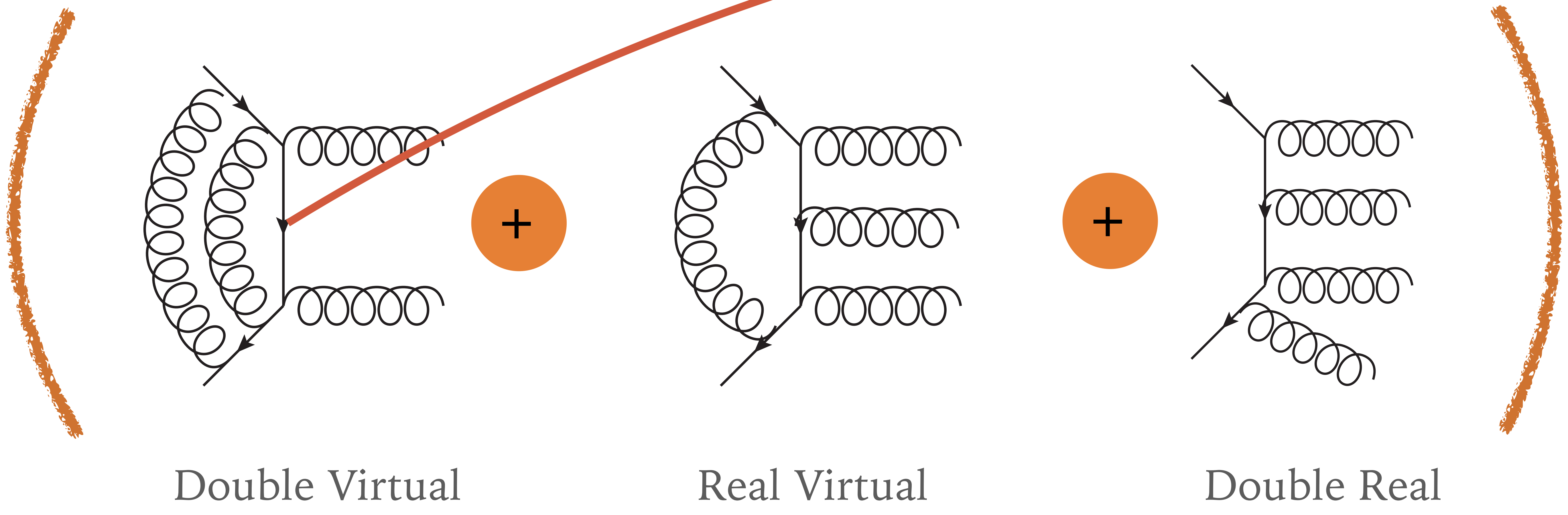
See L. Bonino's and G. Fontana's talks

HARD SCATTERING

$$d\sigma_{\text{part}}(x_1, x_2) \longrightarrow \sigma_{q\bar{q}\to gg} = \int [\text{dPS}]\, |\mathcal{M}_{q\bar{q}\to gg}|^2$$

$$|\mathcal{M}_{q\bar{q}\to gg}|^2 = |\mathcal{M}^{LO}_{q\bar{q}\to gg}|^2 + \left(\frac{\alpha_s}{2\pi}\right) |\mathcal{M}^{NLO}_{q\bar{q}\to gg}|^2 + \left(\frac{\alpha_s}{2\pi}\right)^2 |\mathcal{M}^{NNLO}_{q\bar{q}\to gg}|^2 + \dots$$

Two-loop amplitudes often bottleneck



Cancellation of IR divergences

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(Some more developed than others, but **conceptually under control**!)

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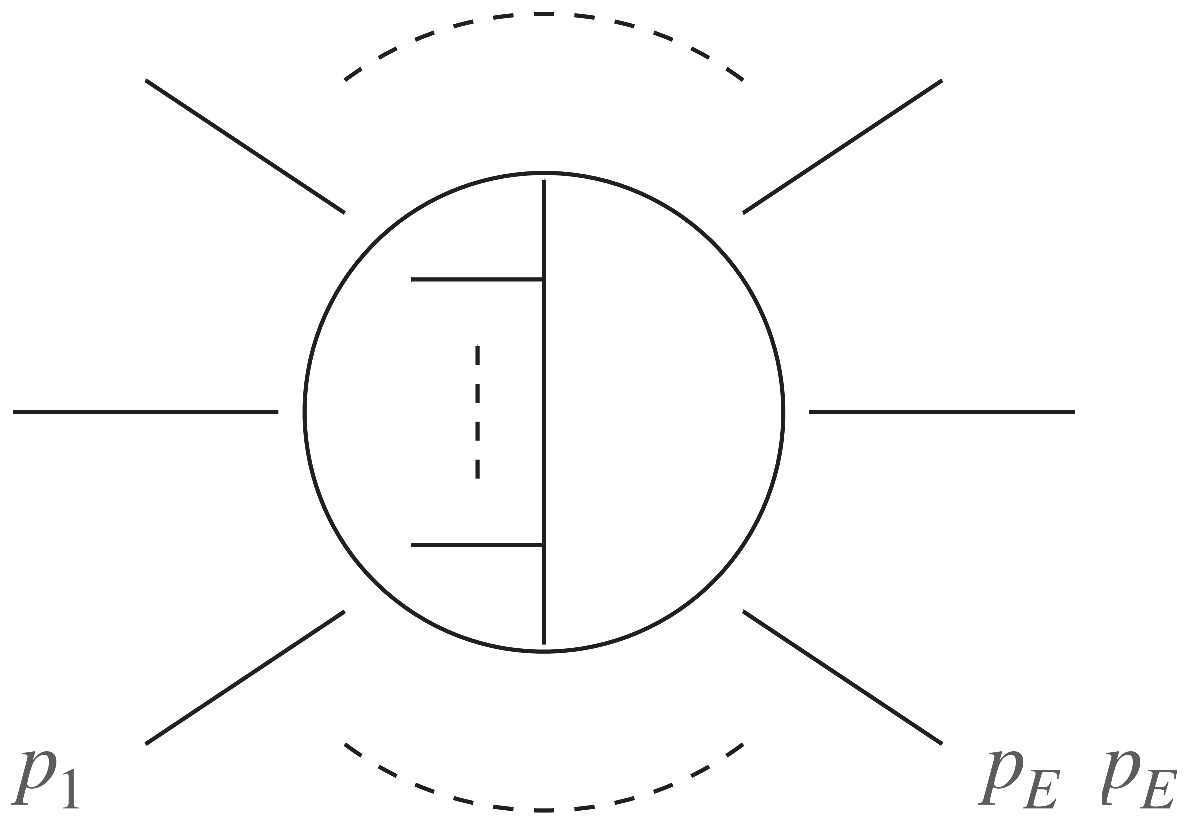
AMPLITUDES: THE STANDARD WAY AT ℓ LOOPS

$\mathscr{A}$ →



$$= \epsilon_1^{\mu_1} \cdots \epsilon_n^{\mu_n} \bar{v}(q) \, \Gamma_{\mu_1,\ldots,\mu_n} \, u(p)$$

↓

$$\mathscr{I} = \int \prod_{l=1}^{L} \frac{d^D k_l}{(2\pi)^D} \frac{S_1^{b_1} \ldots S_m^{b_m}}{D_1^{a_1} \ldots D_n^{a_n}}$$

scalar Feynman integrals

$$D_i = q_i^2 - m_i^2$$

$$S_i = \{\ell_j \cdot \ell_k, \ell_j \cdot p_k\}$$

←

$$0 = \int \prod_{l=1}^{L} \frac{d^D k_l}{(2\pi)^D} \frac{\partial}{\partial \ell_k^\mu} \left[v^\mu \frac{S_1^{b_1} \ldots S_m^{b_m}}{D_1^{a_1} \ldots D_n^{a_n}} \right]$$

Integration by parts identities (IBPs)

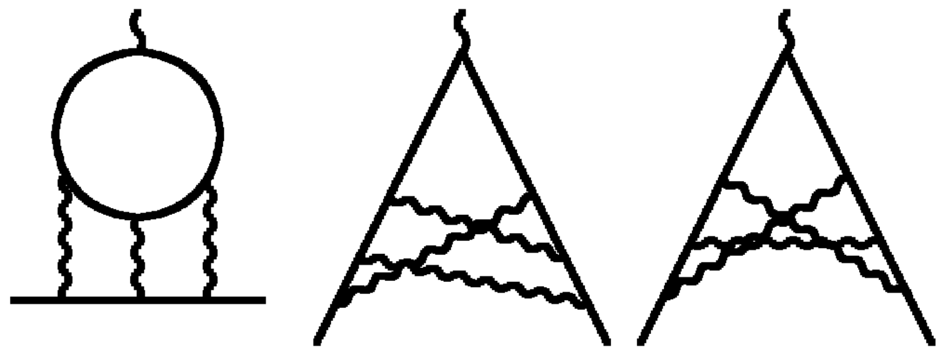
(+ Symmetries, Lorentz ids and all that)

[Chetyrkin, Tkachov; Laporta; ...]

ON THE DECOMPOSITION: IBPS AND MASTER INTEGRALS

Modern methods, first applied* systematically in 1997 to calculate **electron g-2 to 3 loops**

Reduction to 17 Master Integrals



$$= \frac{83}{72}\pi^2\zeta(3) - \frac{215}{24}\zeta(5) + \frac{100}{3}\left[\left(\mathrm{Li}_4\left(\frac{1}{2}\right) + \frac{\ln^4 2}{24}\right) - \frac{\pi^2\ln^2 2}{24}\right]$$
$$- \frac{239}{2160}\pi^4 + \frac{139}{18}\zeta(3) - \frac{298}{9}\pi^2\ln 2 + \frac{17101}{810}\pi^2 + \frac{28259}{5184}$$

[Laporta, Remiddi '97]

* as far as I know…

ON THE DECOMPOSITION: IBPS AND MASTER INTEGRALS

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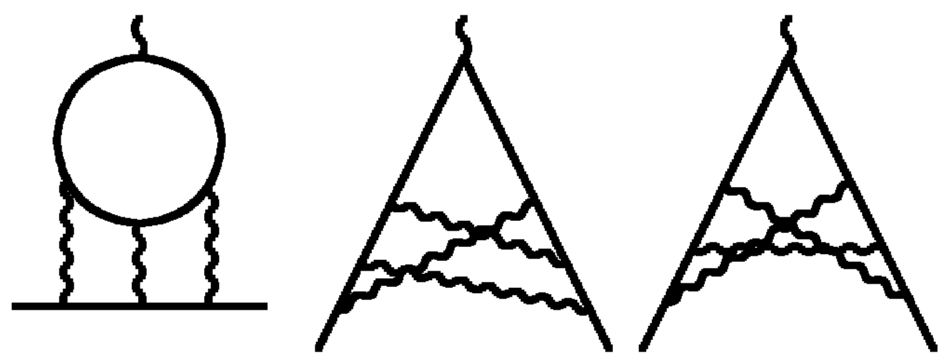

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[Laporta, Remiddi ’97]

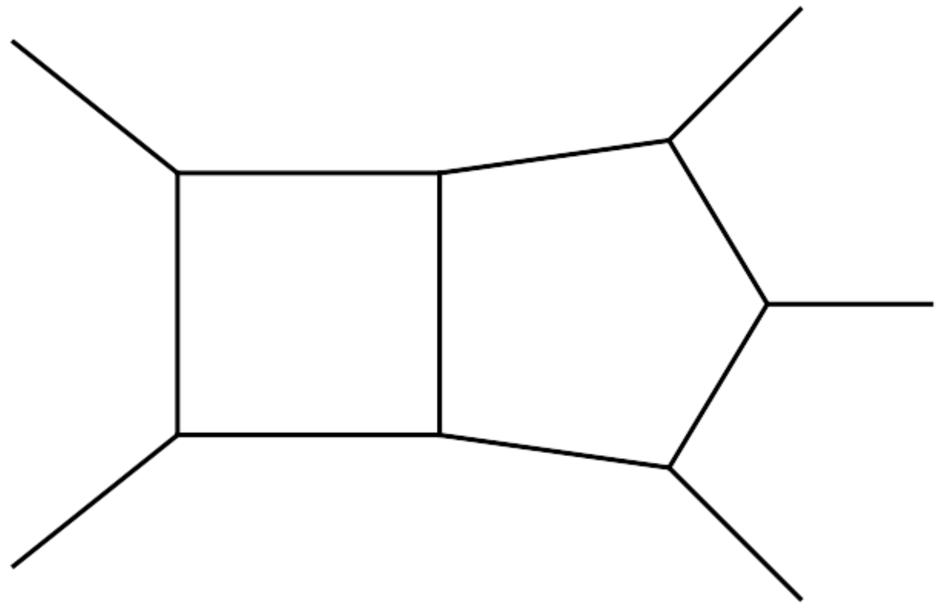
Since then, things have changed a lot!

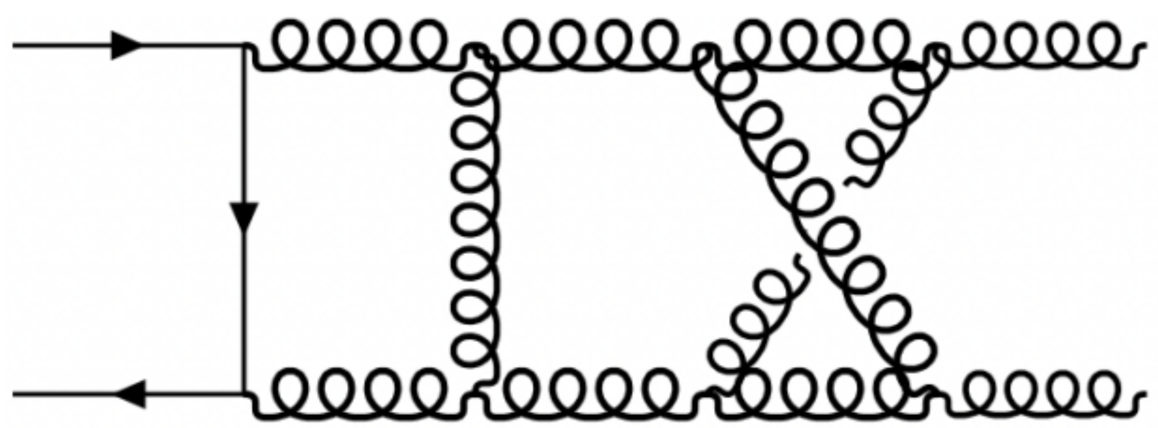
Complexity **increases factorially** with **# of legs** and **# of loops**

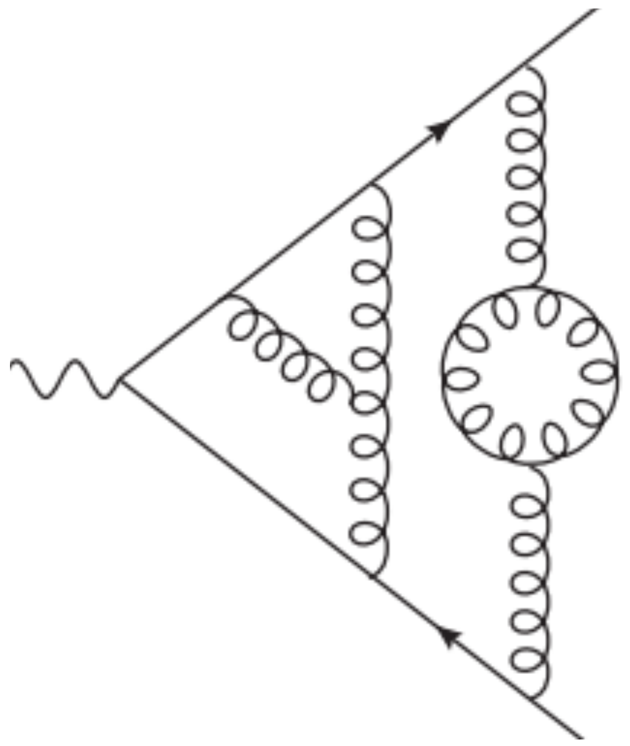
- **many scales** → huge rational functions to handle symbolically (typically TBs of RAM on large machines!)
- **many loops** → explosion in number of identities (typically $\geq 10^9$ for $2 \to 2$ at three loops, again TBs!)

* as far as I know…

ON THE DECOMPOSITION: NEW METHODS FOR IBPS







Finite-fields methods

Avoid intermediate expression swell

[von Manteufell, Schabinger, Peraro, Abreu, Page, Ita, Klappert, Lange,....]

Algebraic geometry methods

Reduce the number of IBPs generated

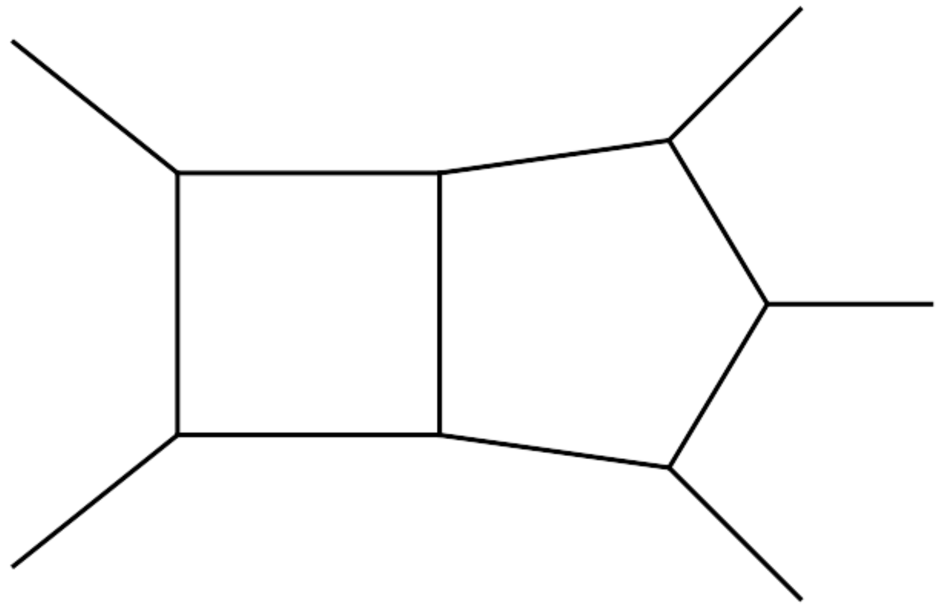
[Zhang, Bohem, Kosower, Peraro, Page, Abreu, Ita, von Manteuffel, Schabinger ...]

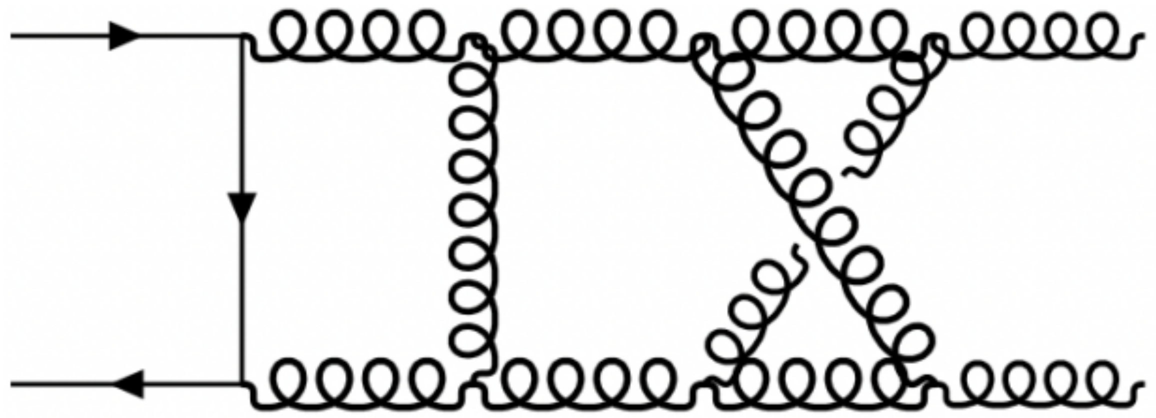
intersection theory

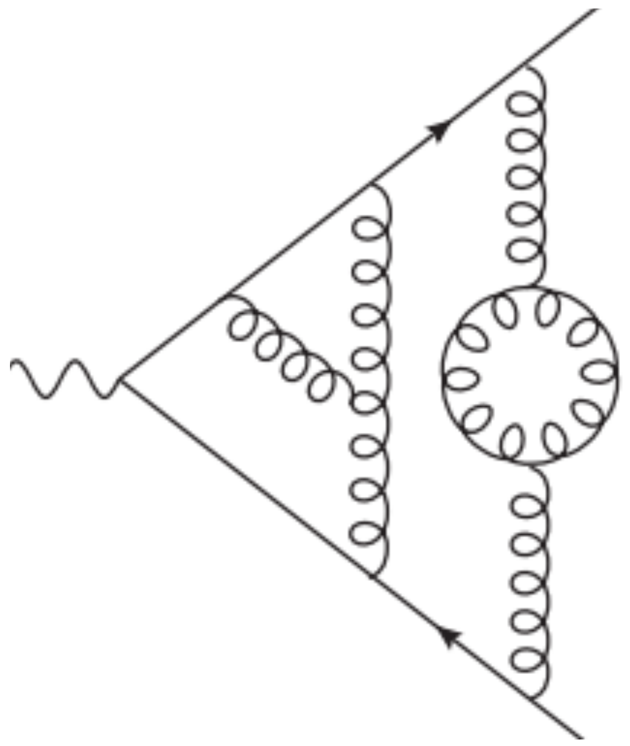
$$\langle\varphi|\mathcal{C}] = \sum_{i,j,k,l=1}^{|\chi|} \langle\varphi|\varphi_j\rangle\,(\mathbf{C}^{-1})_{ji}\,\mathbf{P}_{il}\,(\mathbf{H}^{-1})_{lk}\,[\mathcal{C}_k|\mathcal{C}]$$

[Mizera, Mastrolia, Frellesvig, Brunello, Crisanti, Mattiazzi, Gasparotto, Smith, Chen, Feng, Yang, Xu, Pokraka, Caron-Huot, Giroux, Weinzierl, Fontana, Peraro...]

ON THE DECOMPOSITION: STATE-OF-THE-ART FOR QCD CALCULATIONS







2 loop 5 point

Abreu, Agarwal, Badger, Buccioni, Chawdhry, Chicherin, Czakon, de Laurentis, Febres-Cordero, Gambuti, Gehrmann, Henn, Ita, Lo Presti, Manteuffel, Ma, Mitov, Page, Peraro, Pochelet, Schabinger, Sotnikov, Tancredi, Zhang, …

3 loop 4 point

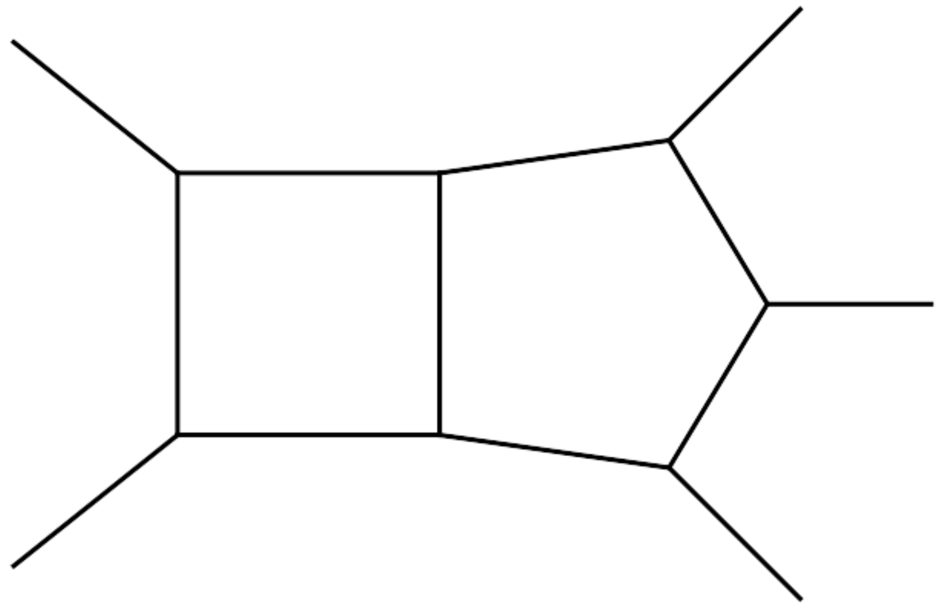
Bargiela, Bobadilla, Canko, Caola, Jakubcik, Gambuti, Gehrmann, Henn, Lim, Mella, Mistlberger, Wasser, Manteuffel, Syrrakos, Smirnov, Tancredi, …

4 loop 3 point

Henn, Lee, Manteuffel, Schabinger, Smirnov, Smirnov, Stainhauser,…

ON THE DECOMPOSITION: STATE-OF-THE-ART FOR QCD CALCULATIONS

$\mathcal{A}$



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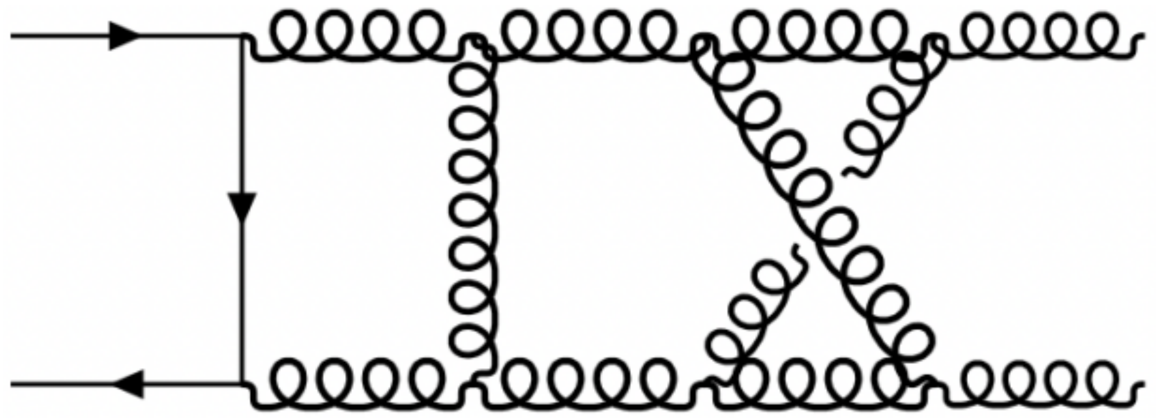

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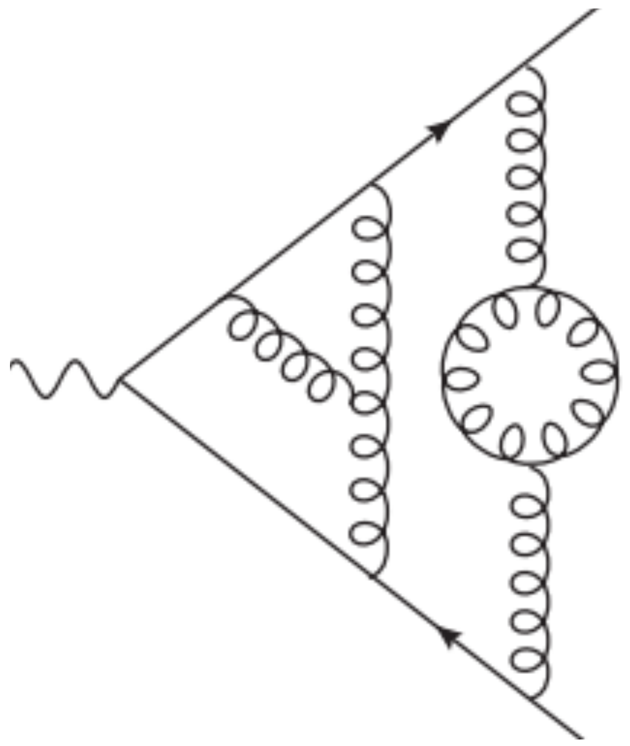
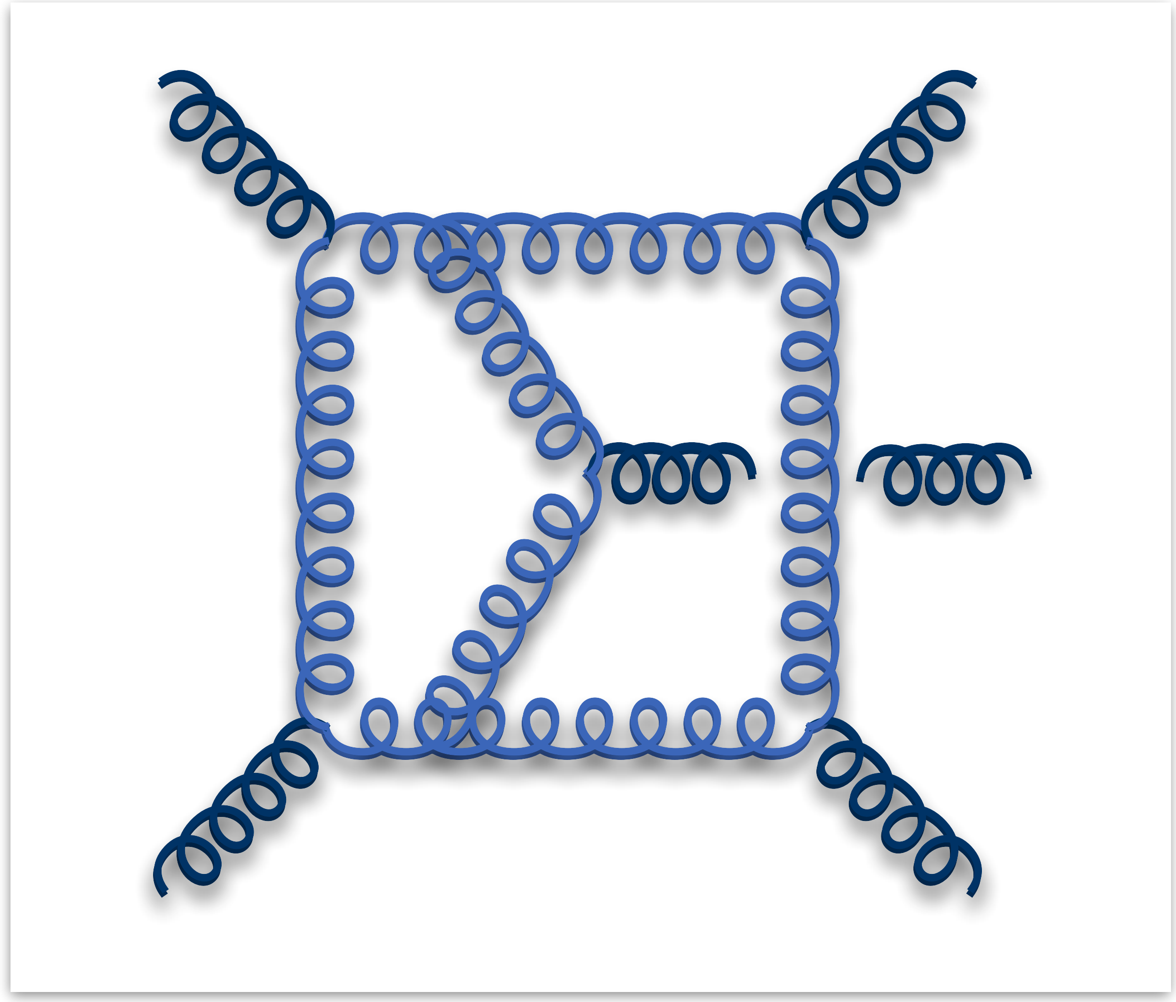


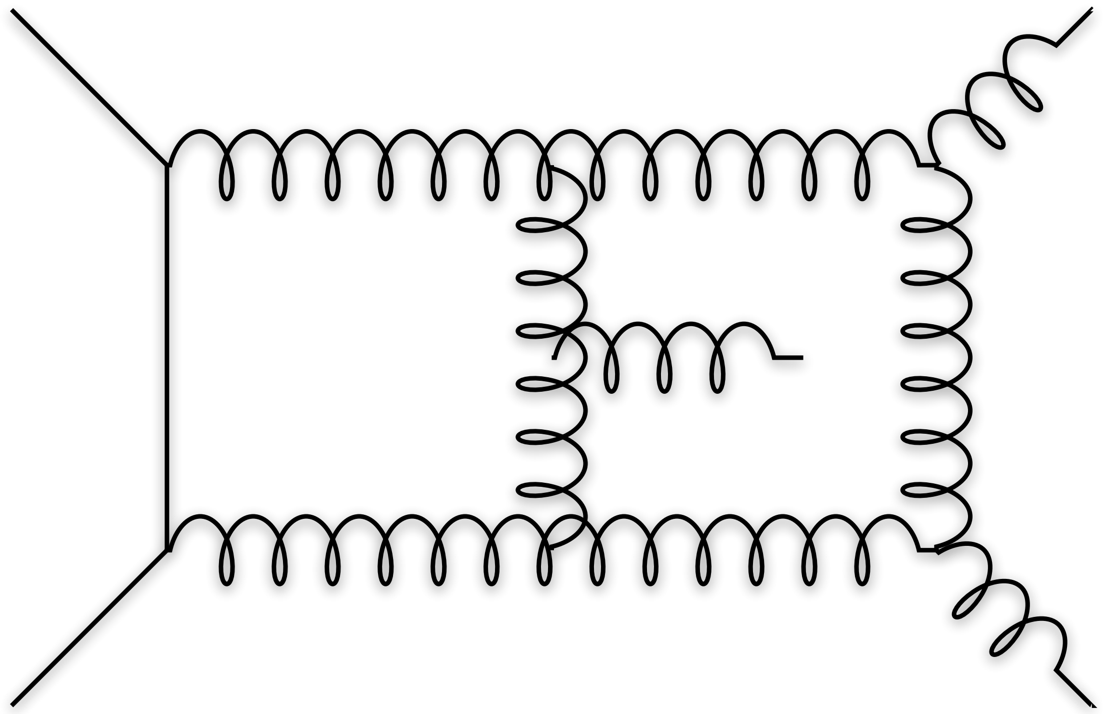
4 loop 3 point

Henn, Lee, Manteuffel, Schabinger, Smirnov, Smirnov, Stainhauser,…

All processes computed in Full Color
Including Planar and Non-Planar diagrams

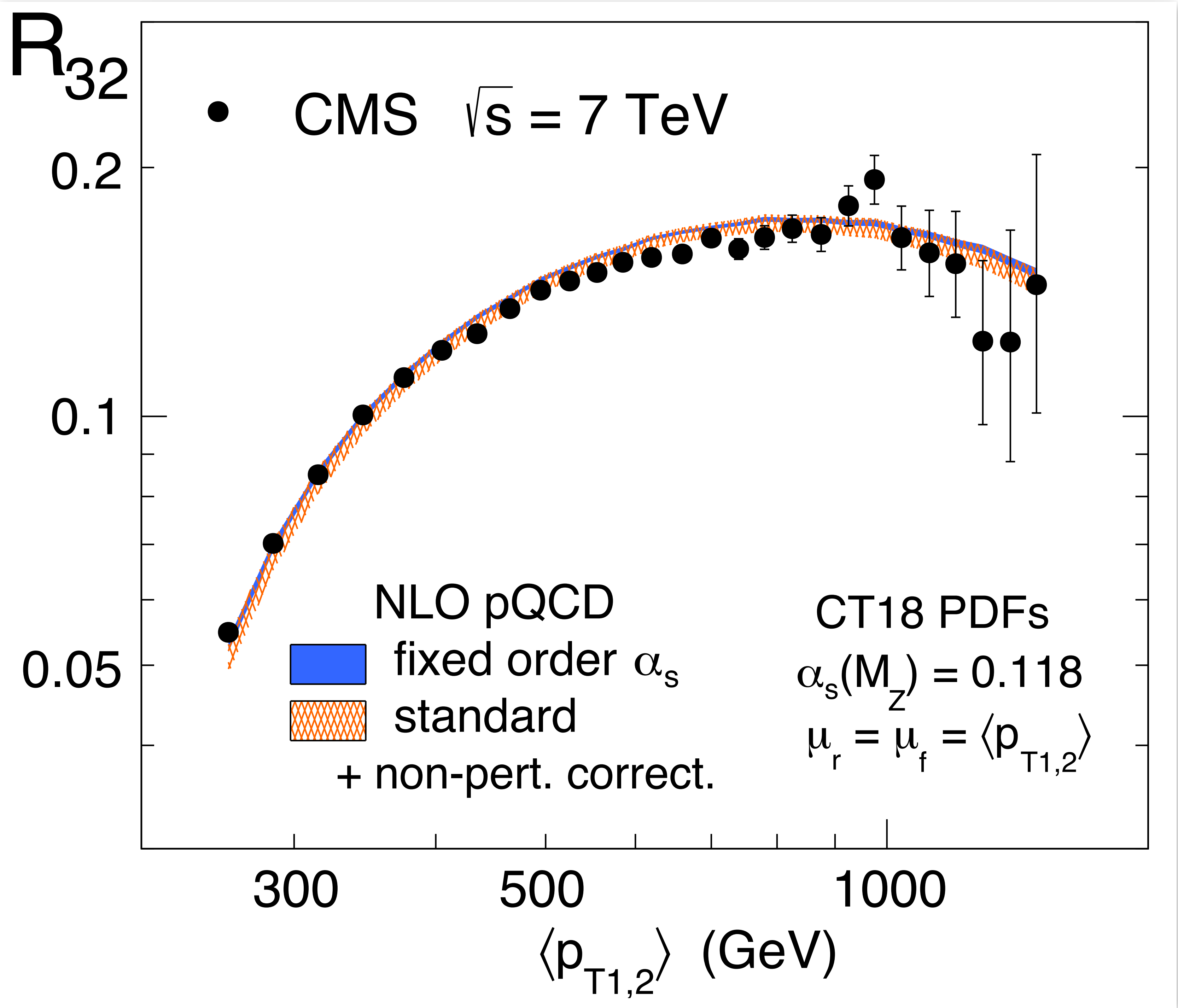
PROBING QCD AT THE HIGHEST ENERGIES



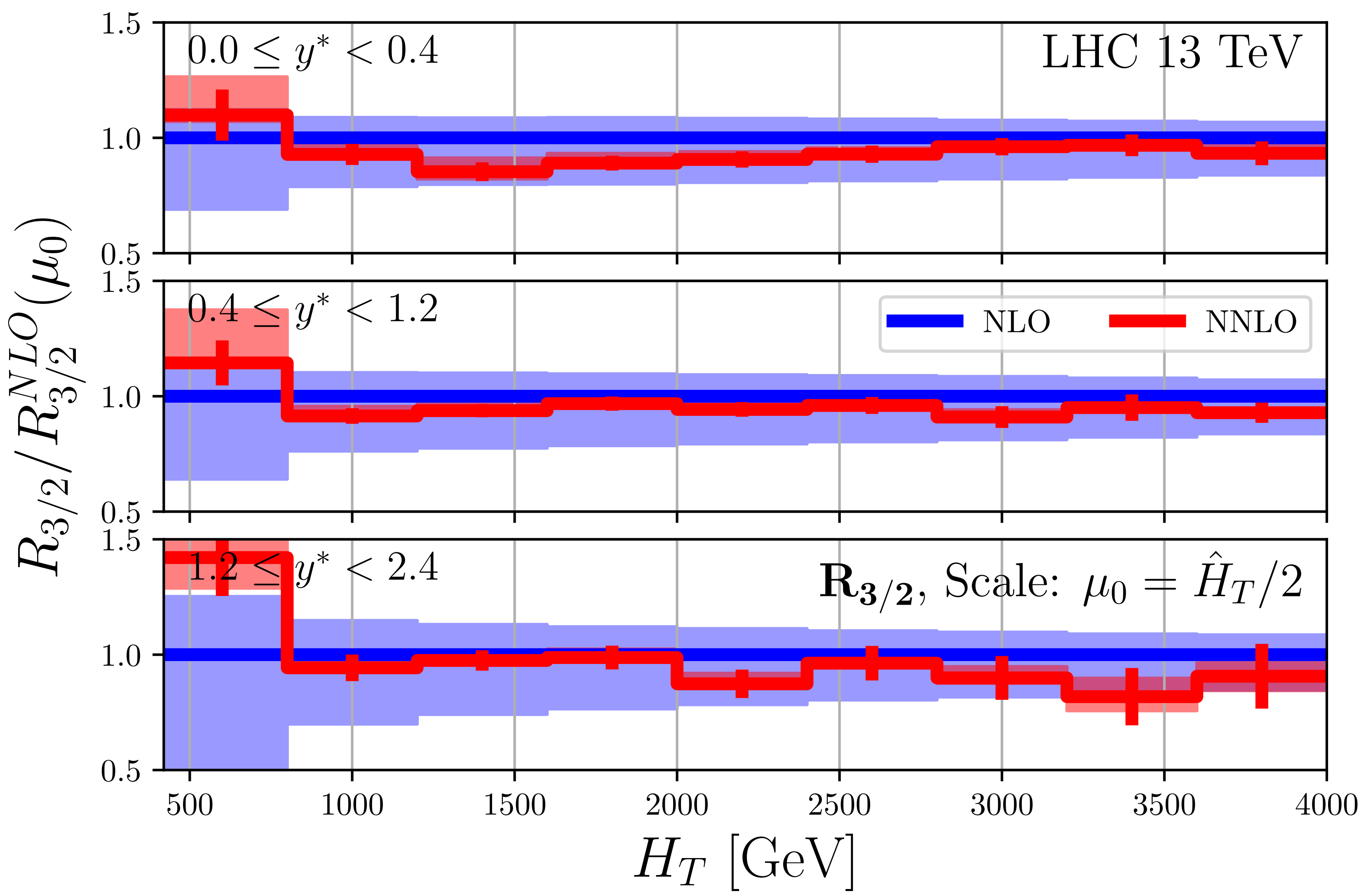


Recently, **full-color** calculation of all scattering amplitudes

[Agarwal, Buccioni, Devoto, Gambuti, Manteuffel, LT '23]
[De Laurentis, Ita, Klinkert, Sotnikov, '21, '23]



Multijet to fit α_S



3-jet production in **NNLO QCD**

[Czakon, Mitov, Poncelet '22, '23]

BEYOND ALL-MASSLESS: AMPLITUDES AND CROSS SECTIONS

Frontier of algebraic complexity: production of 2 massless and 1 massive particle at 2 loops

$$pp \to \{Vjj\,, Hjj\,, V\gamma\gamma\,, \cdots\}$$

BEYOND ALL-MASSLESS: AMPLITUDES AND CROSS SECTIONS

Frontier of algebraic complexity: production of 2 massless and 1 massive particle at 2 loops

$$pp \to \{Vjj\,, Hjj\,, V\gamma\gamma\,, \cdots\}$$

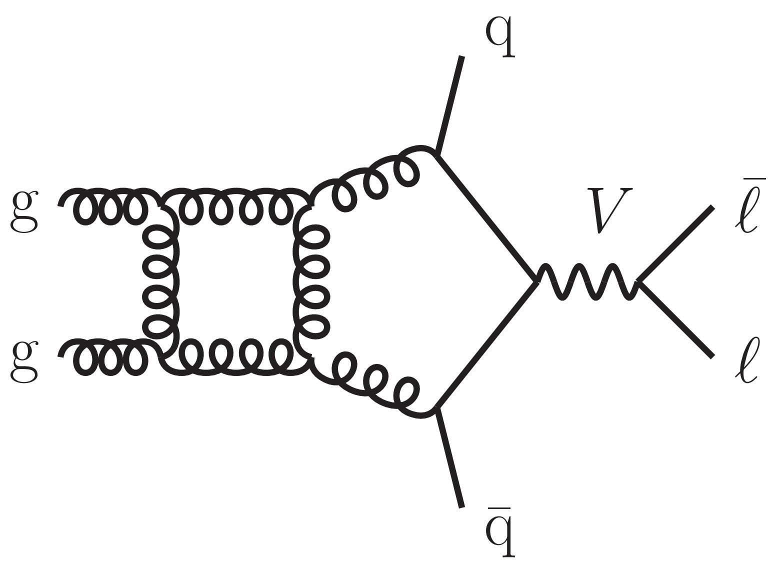
Many results for **LC (planar)** virtual corrections, e.g.

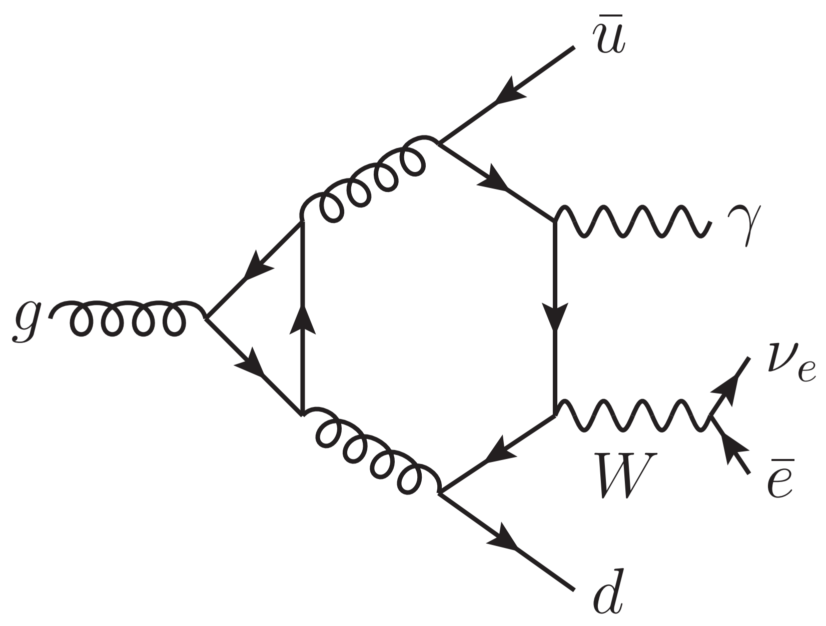
$pp \to Wb\bar{b}$ [Badger, Hartanto, Zoia '21]

$pp \to Hb\bar{b}$ [Badger, Hartanto, Krys, Zoia '21]

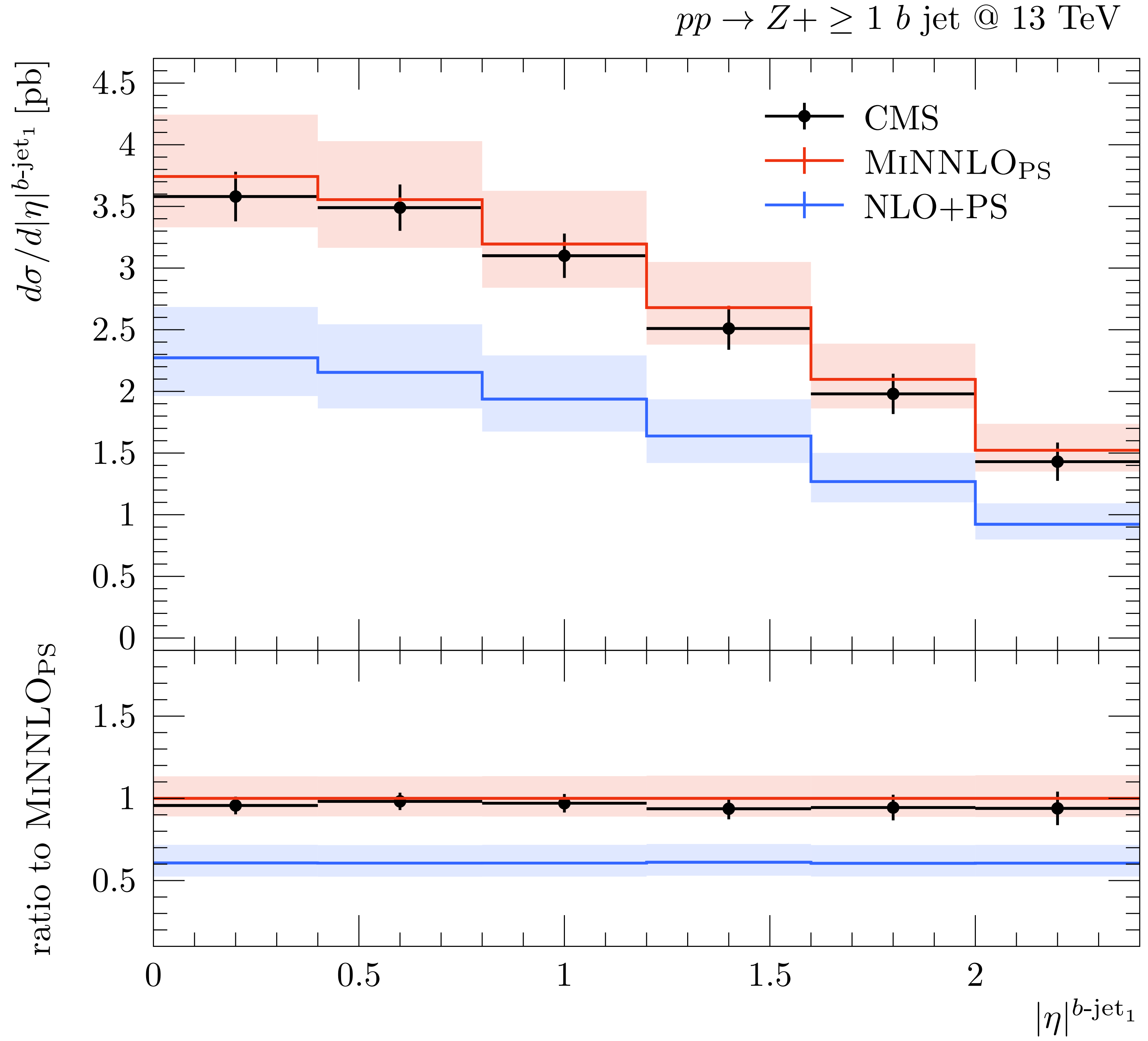
$pp \to W\gamma j$ [Badger, Hartanto, Krys, Zoia '22]

$pp \to Wjj$ [Abreu, Cordero, Ita, Klinkert, Page, Sotnikov '22]





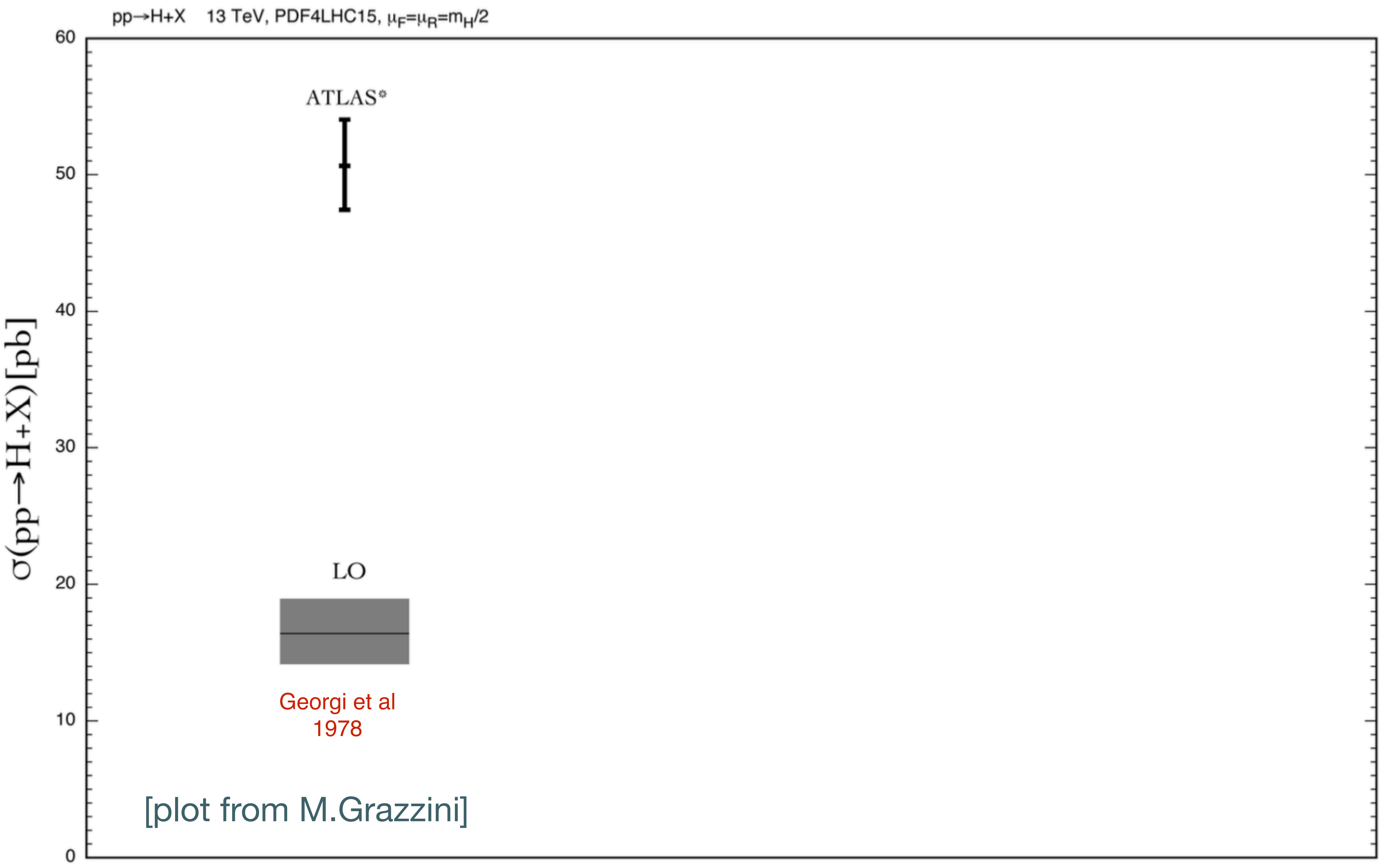
[Abreu, Chicherin, Ita, Page, Sotnikov, Tschernow, Zoia '23]



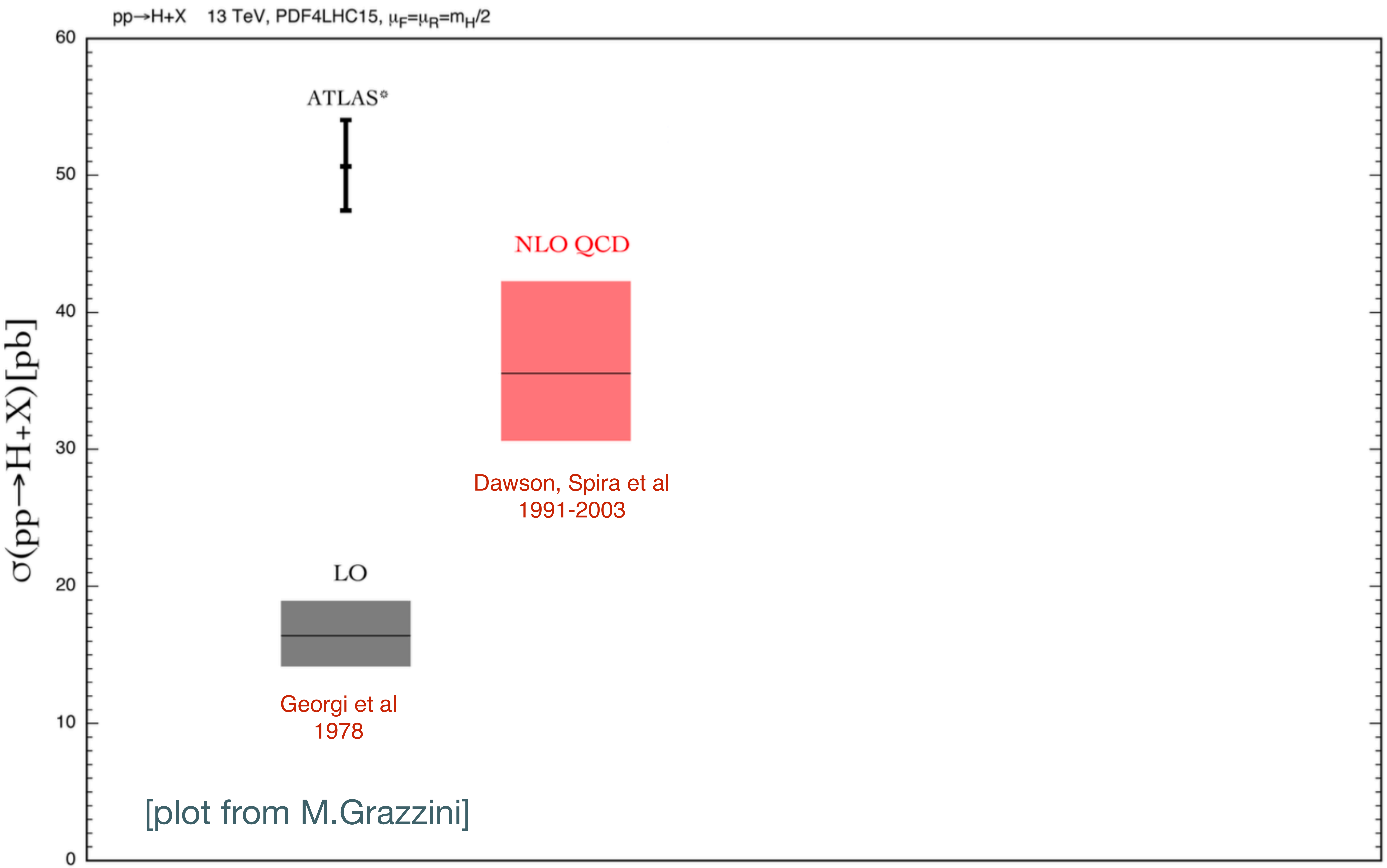
NNLO corrections to $pp \to Zb\bar{b}$ + PS matching

[Mazzitelli, Sotnikov, Wieseman '24]

THE NEED OF PRECISION: TOWARDS THE % LEVEL



THE NEED OF PRECISION: TOWARDS THE % LEVEL



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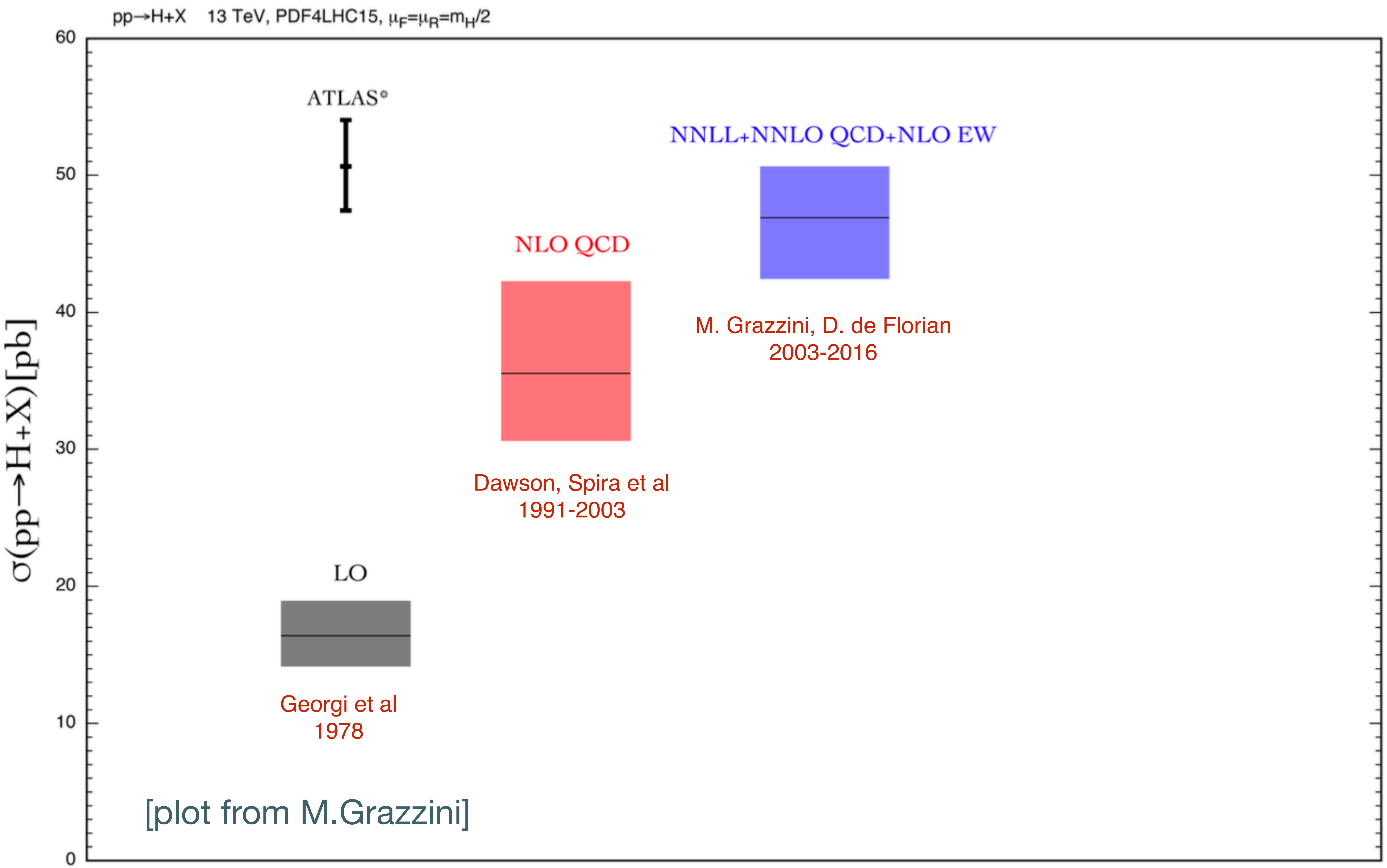

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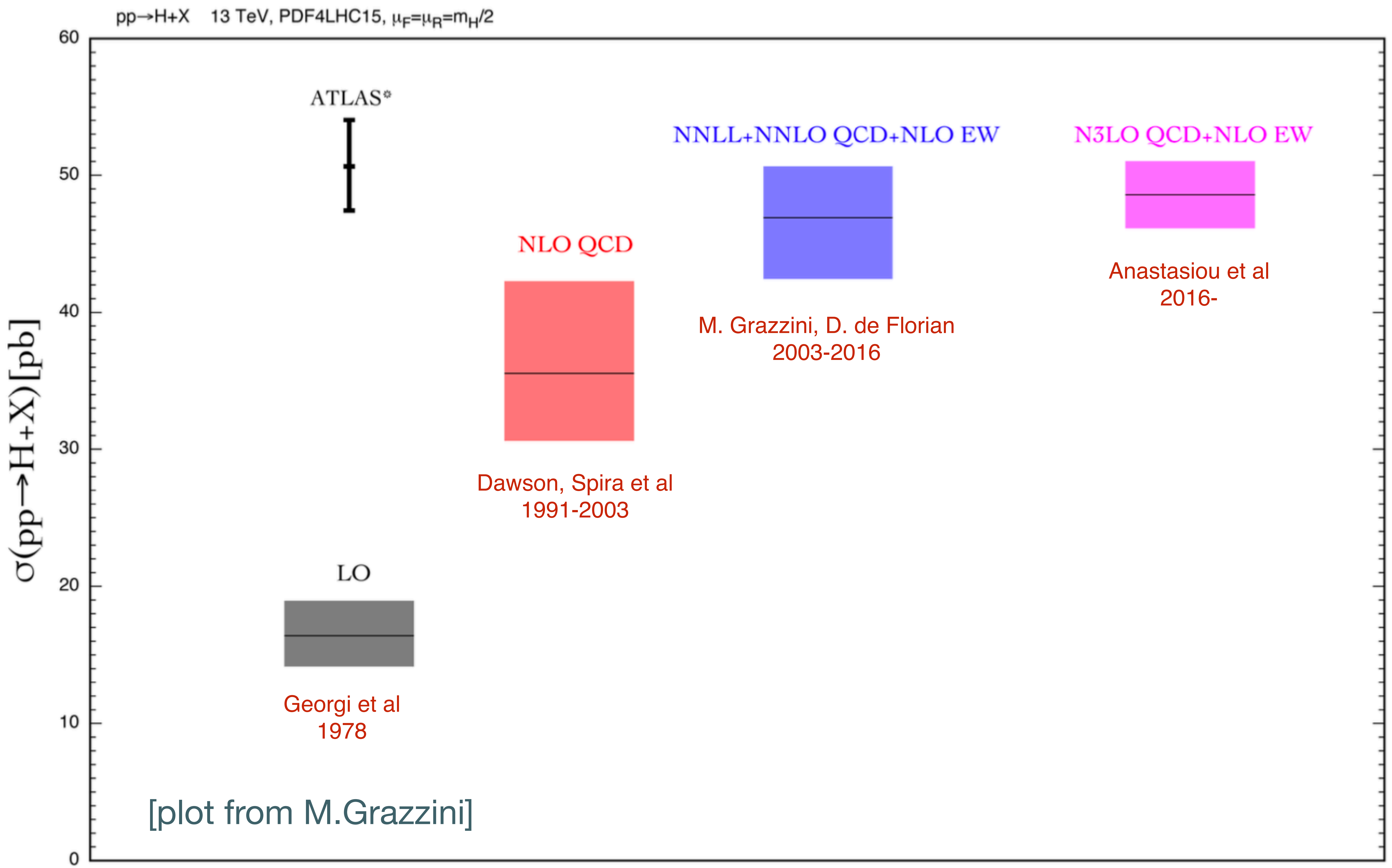
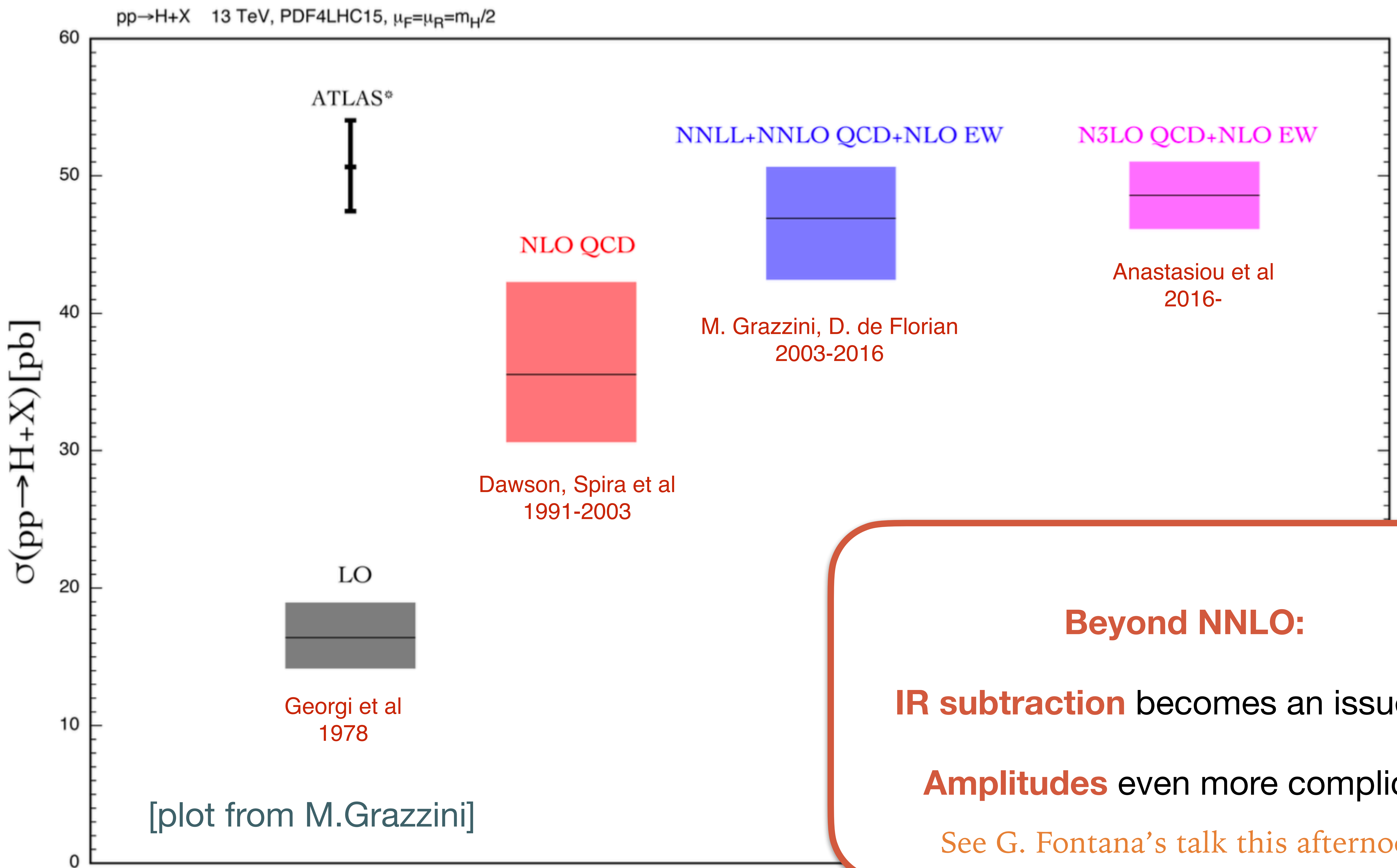


THE NEED OF PRECISION: TOWARDS THE % LEVEL



[plot from M.Grazzini]

Beyond NNLO:

IR subtraction becomes an issue again

Amplitudes even more complicated

See G. Fontana's talk this afternoon

TOWARDS N3LO: THE NEW FRONTIER

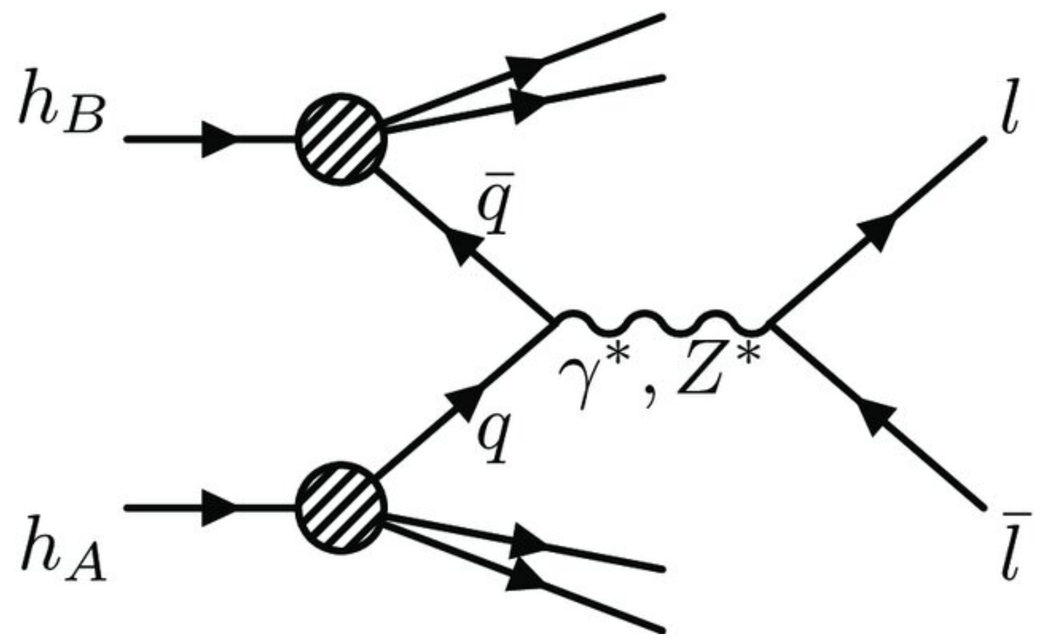
Our current ability of going to N3LO still rather limited and based on:

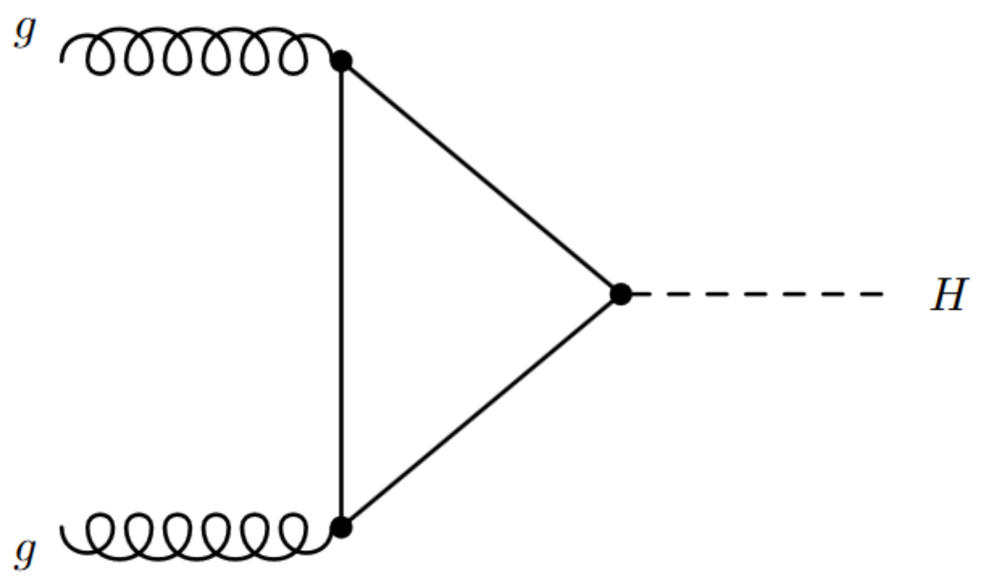
- either direct "analytic calculation" of some observables **(reverse unitarity)** [Anastasiou, Melnikov '02] and **Projection to Born** [Cacciari, Dreyer, Karlberg, Salam, Zanderighi '15]

- or **slicing techniques** based on factorization theorems **(q_T, N-jettines)** [Catani, Grazzini '07] [Boughezal, Focke, Liu, Petriello '16]

↓

First results for $2 \to 1$ processes **(DY, Higgs)**



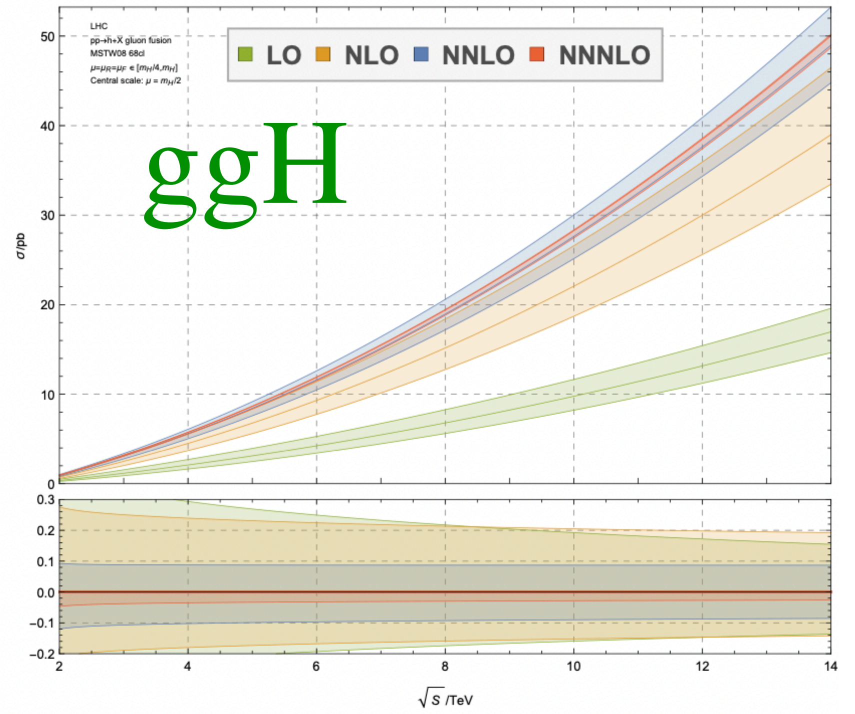


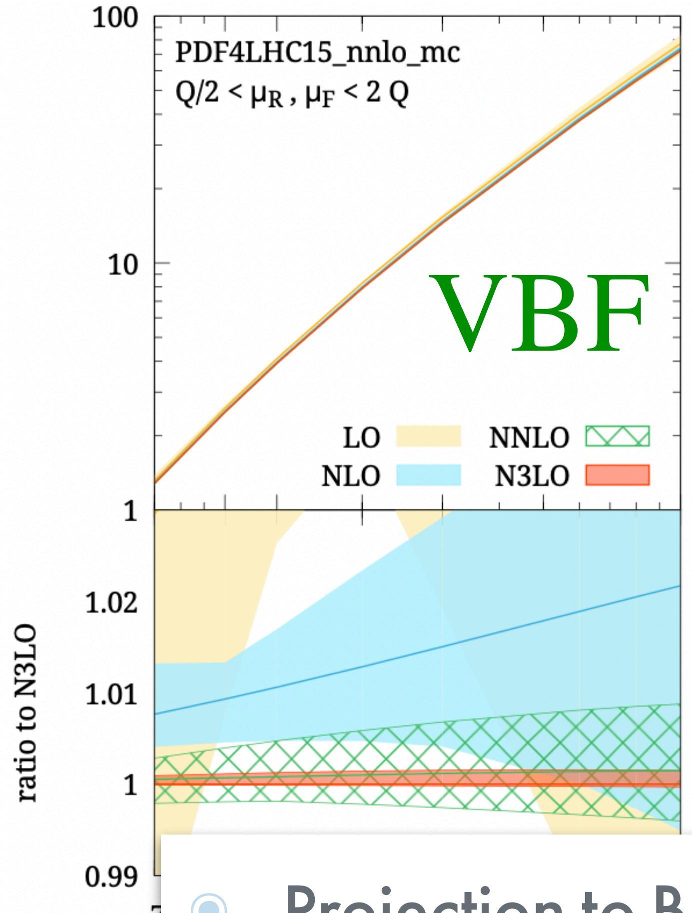
TOWARDS N3LO: THE NEW FRONTIER

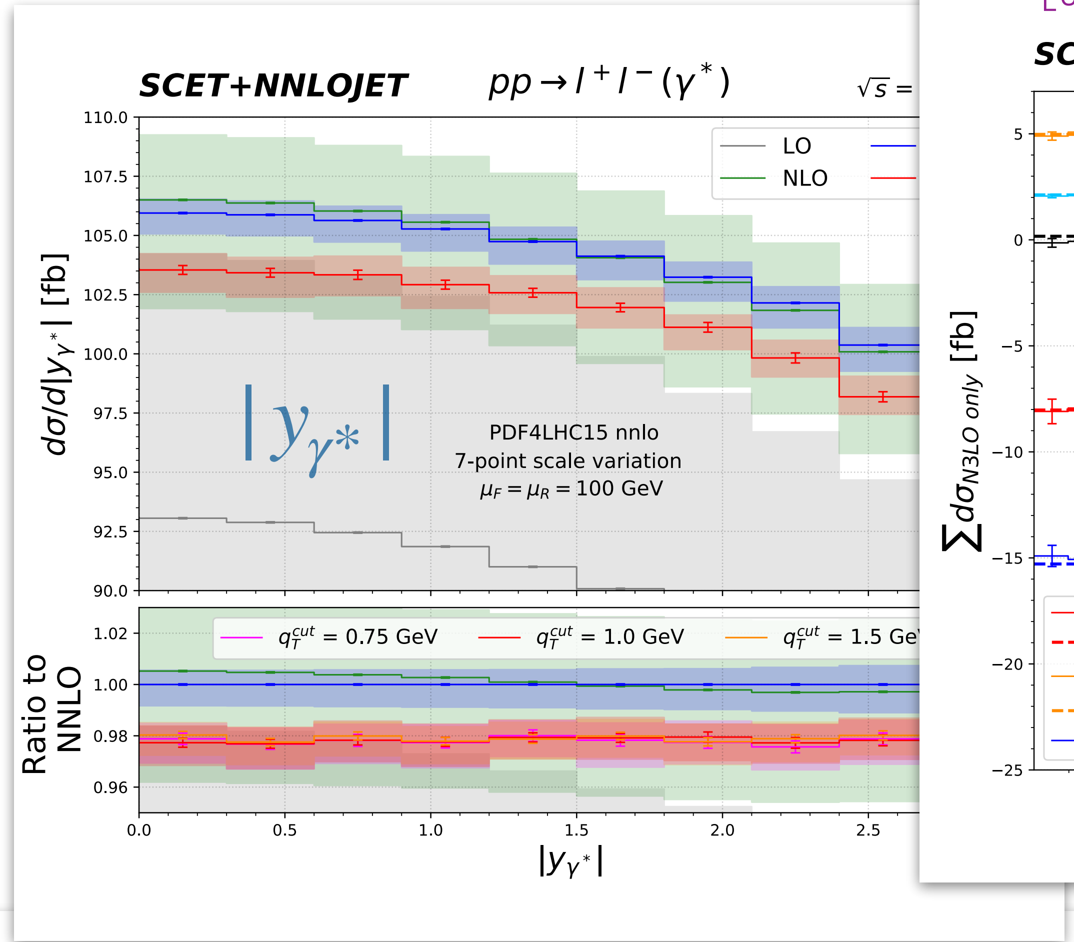
N3LO Inclusive Cross Sections

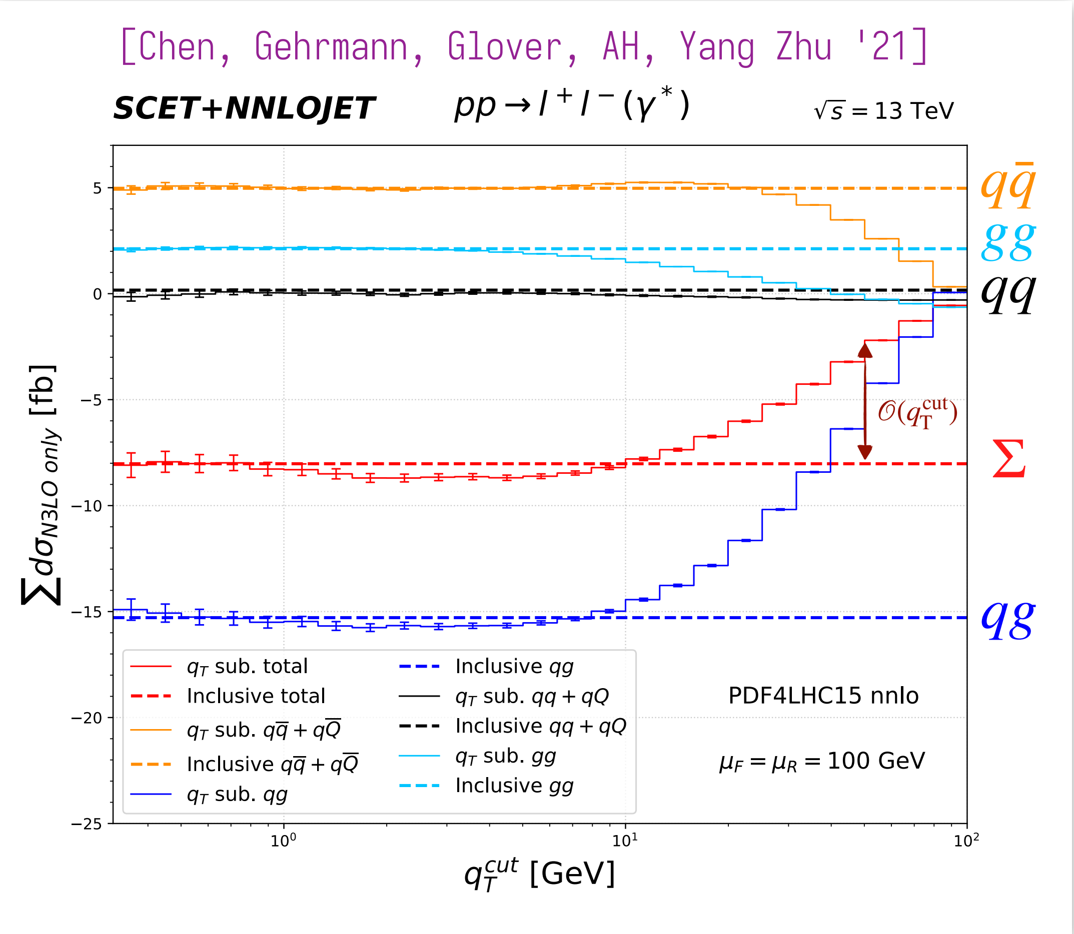
✓ nice convergence of pert. expansion

- $gg \to H$ ✓
  [C. Anastasiou, C. Duhr, F. Dulat, F. Herzog, B. Mistlberger '15]
- VBF-H ✓, VBF-HH ✓
  [F. Dreyer, A. Karlberg '16, '18]
- $b\bar{b} \to H$ ✓
  [C. Duhr, F. Dulat, B. Mistlberger '19]
- $pp \to \gamma^*$?, $pp \to W^\pm$?
  [C. Duhr, F. Dulat, B. Mistlberger '20]
- $gg \to HH$ ✓
  [L. Chen, H. Li, H. Shao, J. Wang '20]
- $pp \to \gamma^*/Z$?
  [C. Duhr, B. Mistlberger '21]
- $pp \to VH$?
  [J. Baglio, C. Duhr, B. Mistlberger, R. Szafron '2





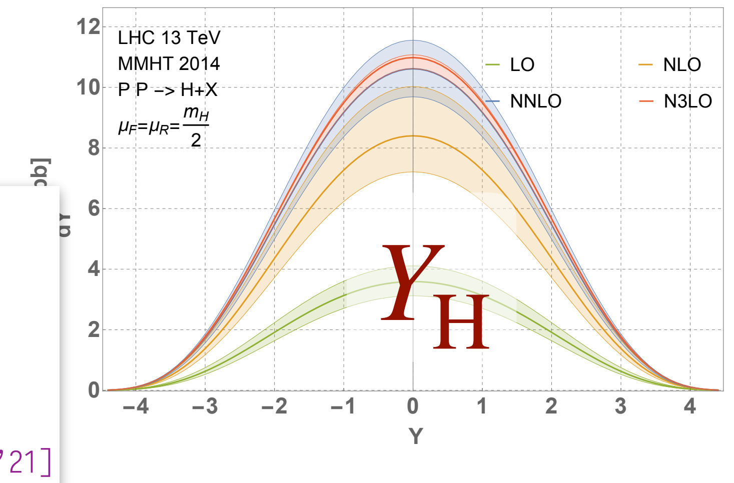




Fully Differential ggH @ N3LO

[Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21]

- only non-trivial observable:



- Projection-to-Born
  - $gg \to H$
    [Chen, Gehrmann, Glover, AH, Mistlberger, Pelloni '21]
- q_T subtraction
  - $gg \to H$
    [Billis, Dehnadi, Ebert, Michel, Tackmann '21]
  - $pp \to \gamma^*$
    [Chen, Gehrmann, Glover, AH, Yang, Zhu, '21]
  - $pp \to Z$
    [Camarda, Cieri, Ferrera '21]
    [NNLOJET + RadISH '22]
    [Neumann, Campbell '22]
  - $pp \to W$
    [Chen, Gehrmann, Glover, AH, Yang, Zhu '22]

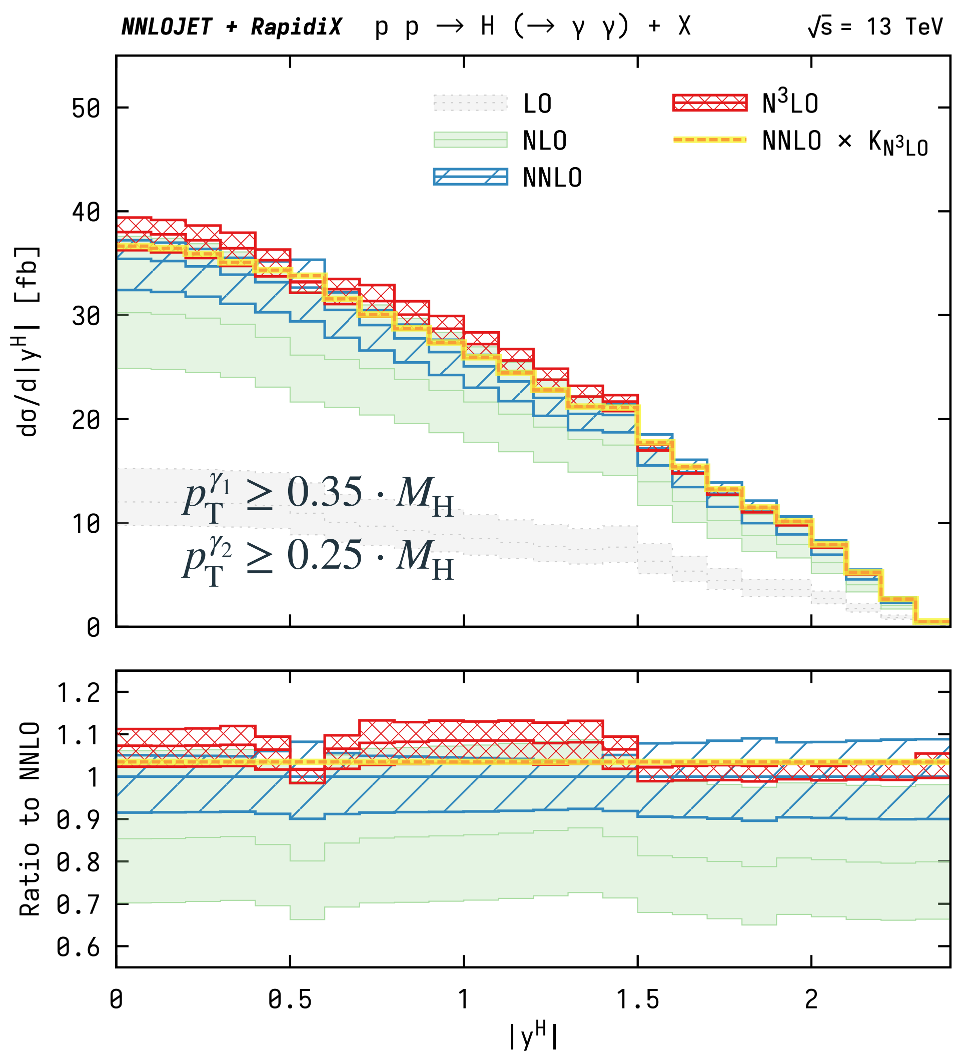
…: restore *differential* info

$$\ldots^{k}\text{LO} \ldots = \frac{d\sigma^{\text{N}^k\text{LO}}_{F,\,\text{inc.}}}{d\mathcal{O}_B} \ldots \left\{ \frac{d\sigma^{\text{N}^{k-1}\text{LO}}_{F+\text{jet}}}{d\mathcal{O}} - \frac{d\sigma^{\text{N}^{k-1}\text{LO}}_{F+\text{jet}}}{d\mathcal{O}}\Big|_{\mathcal{O}\to\mathcal{O}_B} \right\}$$

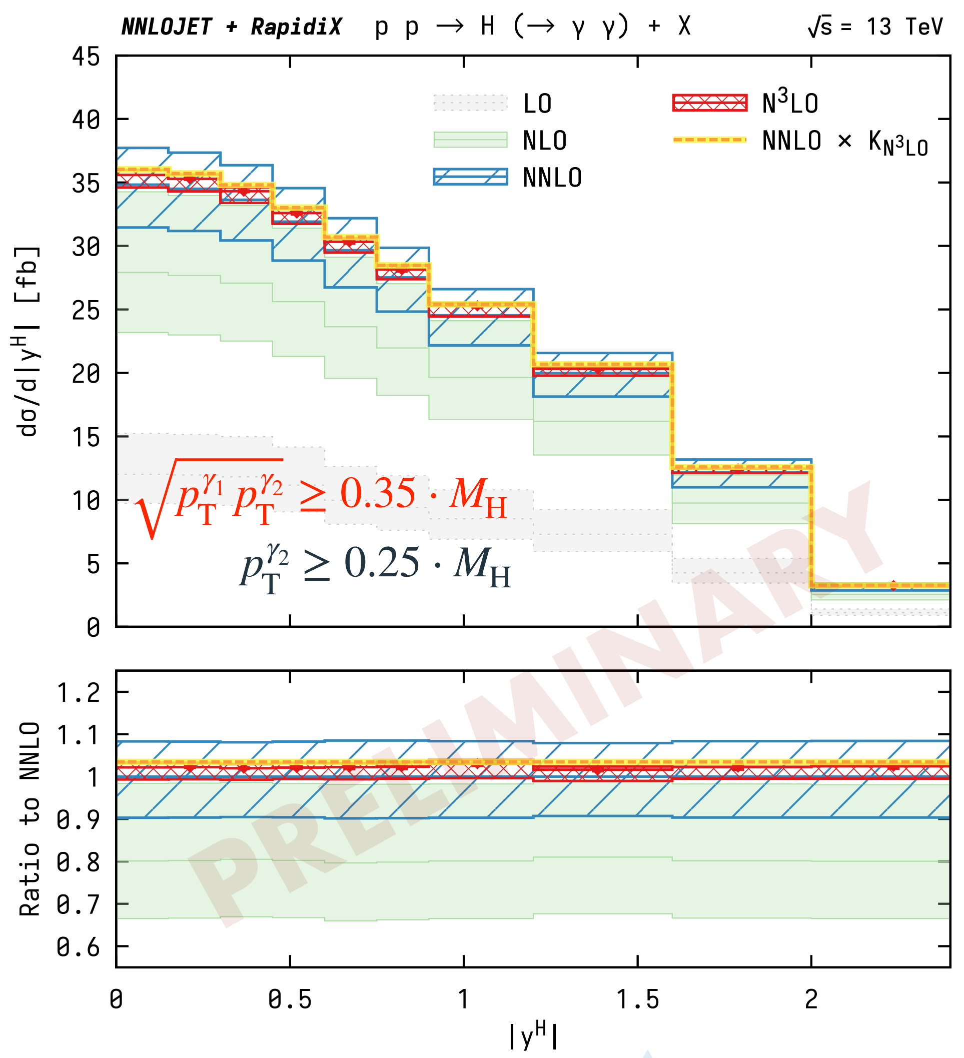
projection to Born

~~lin. fid. power corr.~~
[Salam, Slade '21]

ATLAS Cuts



Product Cuts



no instabilities & flat K-factor: N³LO ≃ NNLO × $K_{\text{N}^3\text{LO}}$

Slides by **A. Huss**, Ringberg '22

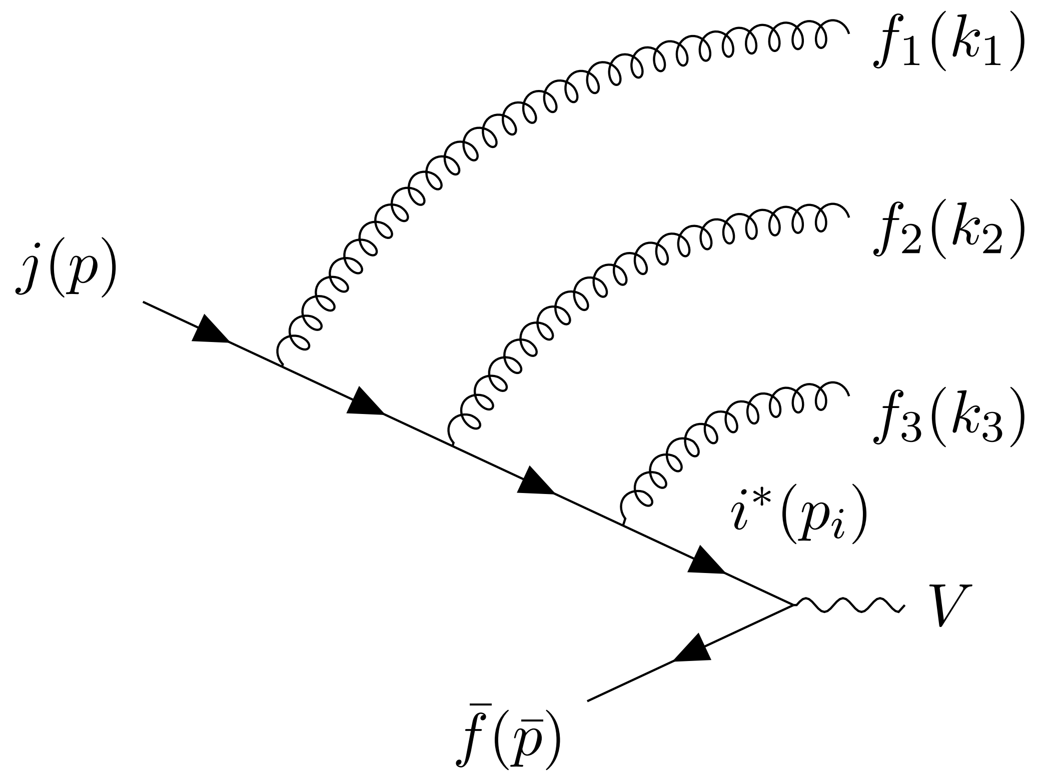
TOWARDS N3LO: THE NEW FRONTIER

Impressive effort to compute all **missing ingredients for slicing methods to N3LO**

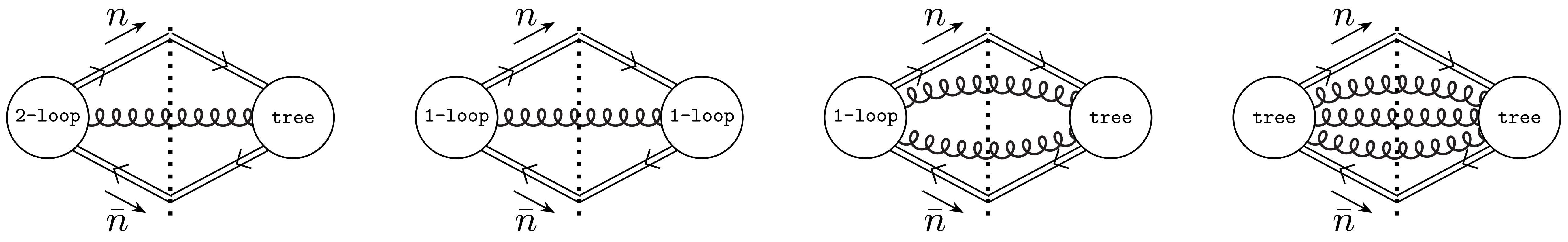
- N-jettiness **Beam Functions**

  [Ebert, Mistlberger, Vita '20]

  [Baranowski, Behring, Melnikov, Rietkerk, Tancredi, Wever '17,'19,'22]



- **zero-jettiness soft function** (for color singlet) [Baranowski, Delto, Melnikov, Pikelner, Wang '24]



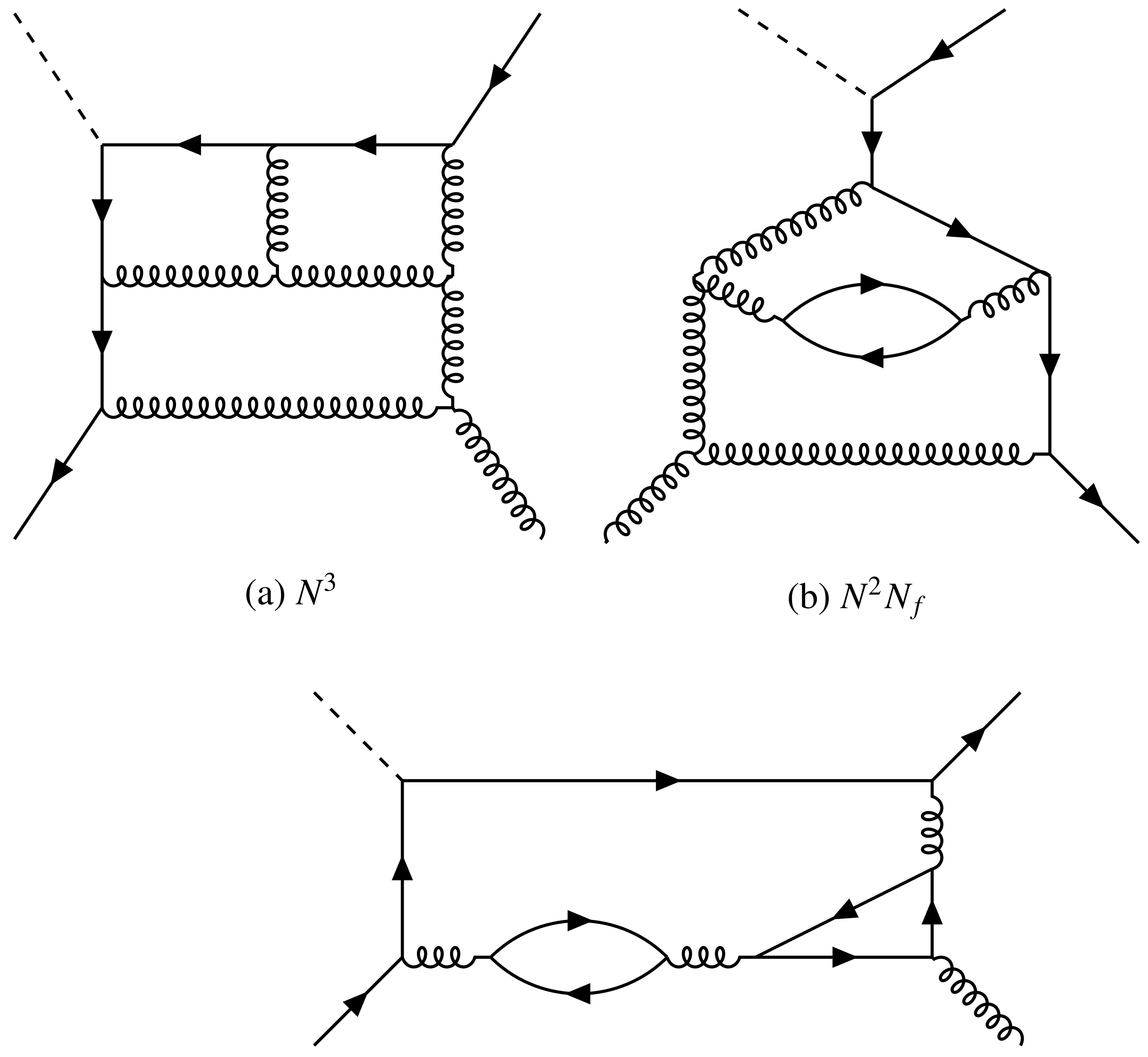
- recent progress on **generalization of Antenna's to N3LO** See G. Fontana's talk this afternoon

TOWARDS N3LO: THE NEW FRONTIER (FOR AMPLITUDES TOO)

First 3 loop amplitudes with **one off-shell external leg**

- **3 loop leading-color amplitudes** for $q\bar{q} \to \{Zj\ ,\ Wj\}$

  [Vita, Mastrolia, Schubert, Yundin, Syrrakos '14] [Canko, Syrrakos '21]
  [Gehrmann, Jakubcik, Mella, Syrrakos, Tancredi '23]



(a) N^3 (b) N^2N_f (c) NN_f^2

- **3 loop master integrals** for leading color amplitudes for $pp \to Hj$

  [Bobadilla, Henn, Lim,'23] [Canko, Syrrakos '23]
  [Bobadilla, Gehrmann, Henn, Jakubcik, Lim, Mella, Syrrakos, LT *to appear soon*]

CONCLUSIONS

1. Colliders remain some of the most flexible (multi-purpose) experiments to investigate fundamental questions in physics

2. Higher-order corrections crucial to precision physics studies: QCD, QCD-EW, pure EW …

3. QCD NNLO calculations for $2 \to 3$ have become a reality!

4. Breaking the QCD N3LO barrier seems to be also around the corner (including progress on PDF evolution): amplitudes are on the way, progress on IR subtraction.

5. Accounting for QCD-EW and pure EW to higher order becomes increasingly important

Exciting developments all over! Stay tuned…!


THANK YOU !