Quantum entanglement at colliders

J. A. Aguilar Saavedra
Instituto de Física Téorica, UAM/CSIC, Madrid

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Basic question:

why?

Why?

☑ Quantum mechanics is a fundamental pillar of modern physics!
We have to test QM at all times!

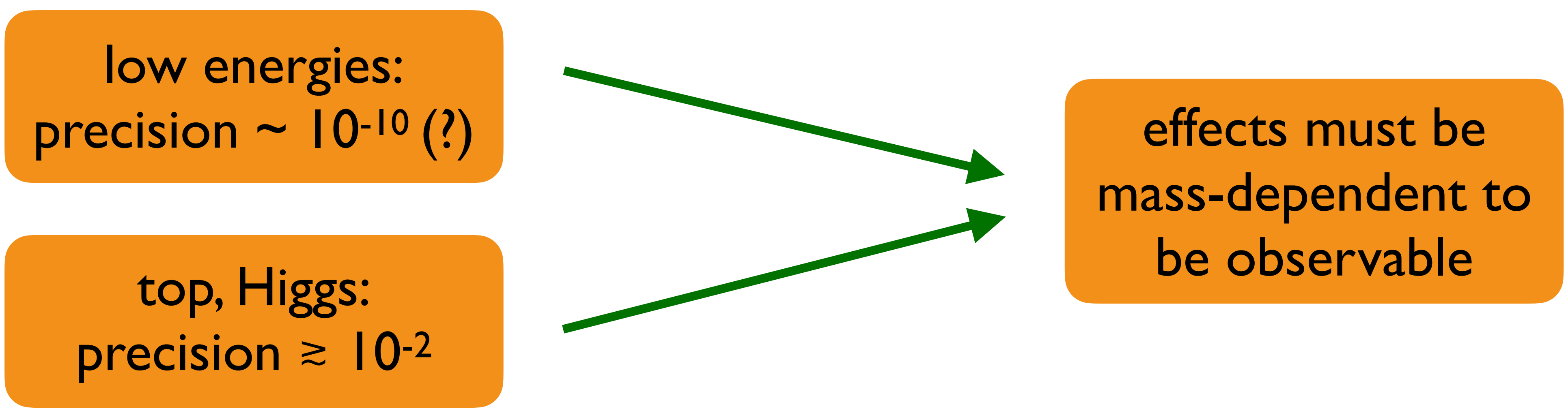
This effort is analogous to the many tests of general relativity at all scales, with increasing precision.

→ Gravitational waves: though they are predicted by GR, it is still worth looking for them!

Why?

Q: Should we see any breaking of QM at the LHC?

A: it is not clear that we should see any effect at LHC even if QM has to be corrected (e.g. with non-linear terms)



… and it remains to be shown that effects should precisely be seen in entanglement measurements!

> Even if we do not have a clear candidate beyond QM, we have to keep testing it. Remember Michelson-Morley experiment!

Why?

In quantum information jargon, entanglement is usually studied for:

- qubits: systems with 2 possible states.

  → Example: spin of top quarks, tau leptons

- qutrits: idem, with 3 states.

  → Example: spin of W, Z bosons

- …

  → Example: orbital angular momentum

All the tests, formalism, etc. developed there can be applied to particles produced at LHC and other colliders

Why?

LHC offers a variety of processes to test QM at the energy frontier.

- Top pair production

Afik, Nova 2003.02280, 2203.05582, 2209.03969
Fabbrichesi et al. 2102.11883
Maltoni et al. 2110.10112
JAAS , Casas 2205.00542
Dong et al. 2305.07075
Han, Low, Wu 2310.17696

- Higgs decays $H \to WW$

Barr 2106.01377
JAAS 2208.14033
Fabbri, Howarth, Maurin 2307.13783

- Higgs decays $H \to ZZ$

JAAS, Bernal, Casas, Moreno 2209.13441
JAAS 2403.3942

- Diboson EW production

Ashby-Pickering, Barr, Wierzchucka 2209.13990
Fabbrichesi, Floreanini, Gabrielli, Marzola 2302.00683

- VBF

Morales 2306.17247

- Other

JAAS 2307.06991, 2401.10988, 2402.14725
JAAS, Casas 2401.06854
JAAS 2402.14725
Morales 2403.18023

updated as of 3/2024

Why?

☑ Novel entanglement tests that were not possible before.

What is genuinely new in particle physics with respect to experiments with electrons and photons? Particle decay.

- Post-decay entanglement:

JAAS 2307.06991
JAAS, Casas 2401.06854
JAAS 2401.10988

A and B entangled
$A \to A_1\ A_2$

⇒

A_1, A_2 and B entangled
A_1 and B entangled

- Entanglement and post-selection:

JAAS 2308.07412

A and B entangled
$A \to A_1\ A_2$
Measurement on B

time ↓

⇒

$\asymp$ spin selection on A, *which already has decayed*

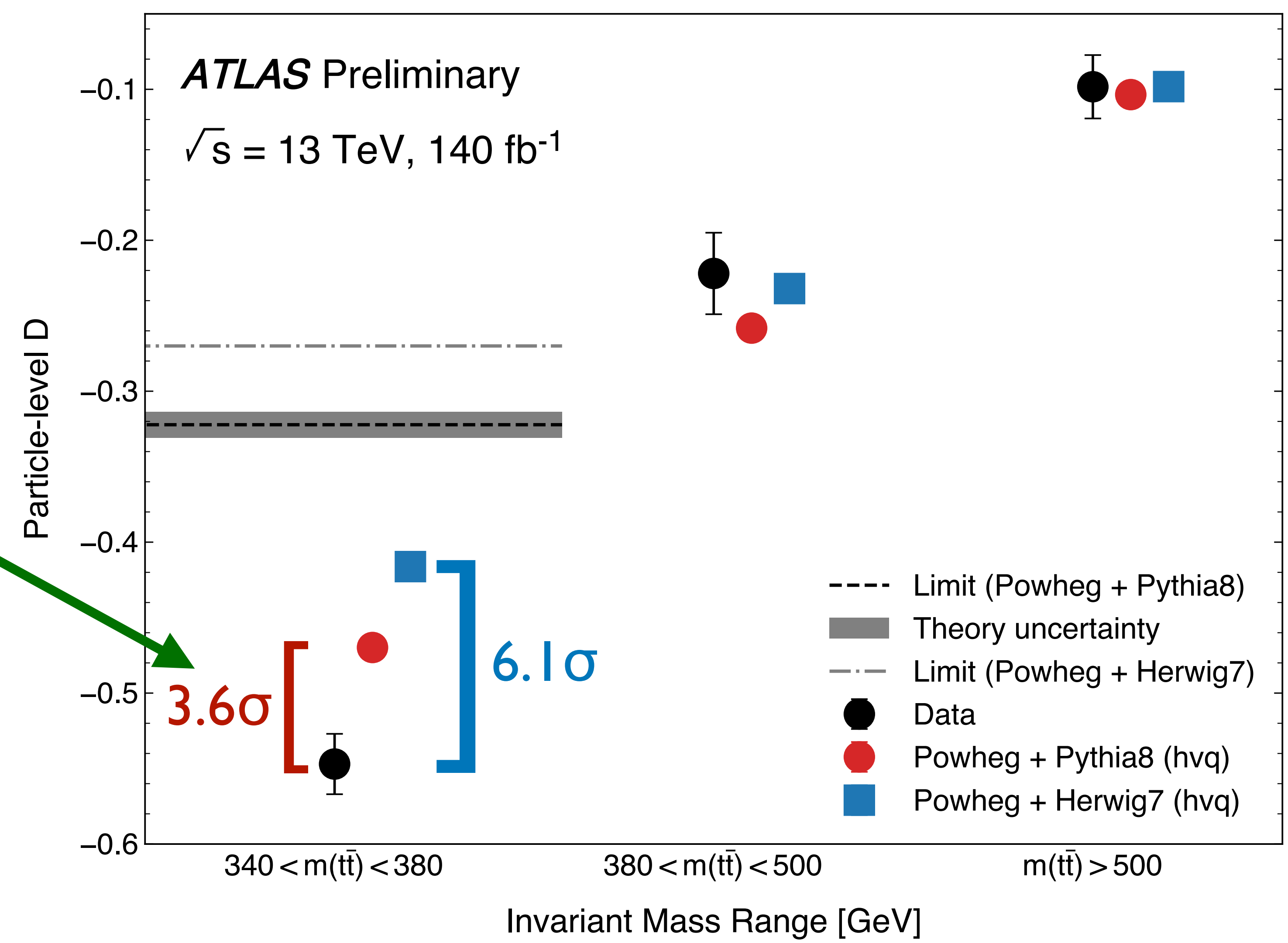
Why?

Entanglement measurements are quite demanding, and provide a stress test of our current understanding of

- theoretical modeling
- experimental systematic uncertainties

Example: ATLAS entanglement measurement in top pair production

Even if ATLAS does not make such claim, everybody can see that predictions and measurement are quite off and digitise the plot



Why?

☑ Looking for new physics

Yes, but only if we use dedicated observables.

Example: ATLAS and CMS measured spin-correlation coefficients C_{kk}, C_{rr}, C_{nn} in t t-bar production.

If we consider entanglement observables [explanations later]

$$C_{kk} + C_{rr} + C_{nn} \equiv 3D$$
$$C_{kk} + C_{rr} - C_{nn} \equiv 3D_3$$

and measure them indirectly from C_{kk}, C_{rr}, C_{nn}, it is unlikely to have any sensitivity gain.

The way to improve sensitivity is to consider observables that directly measure D and D_3 from distributions.

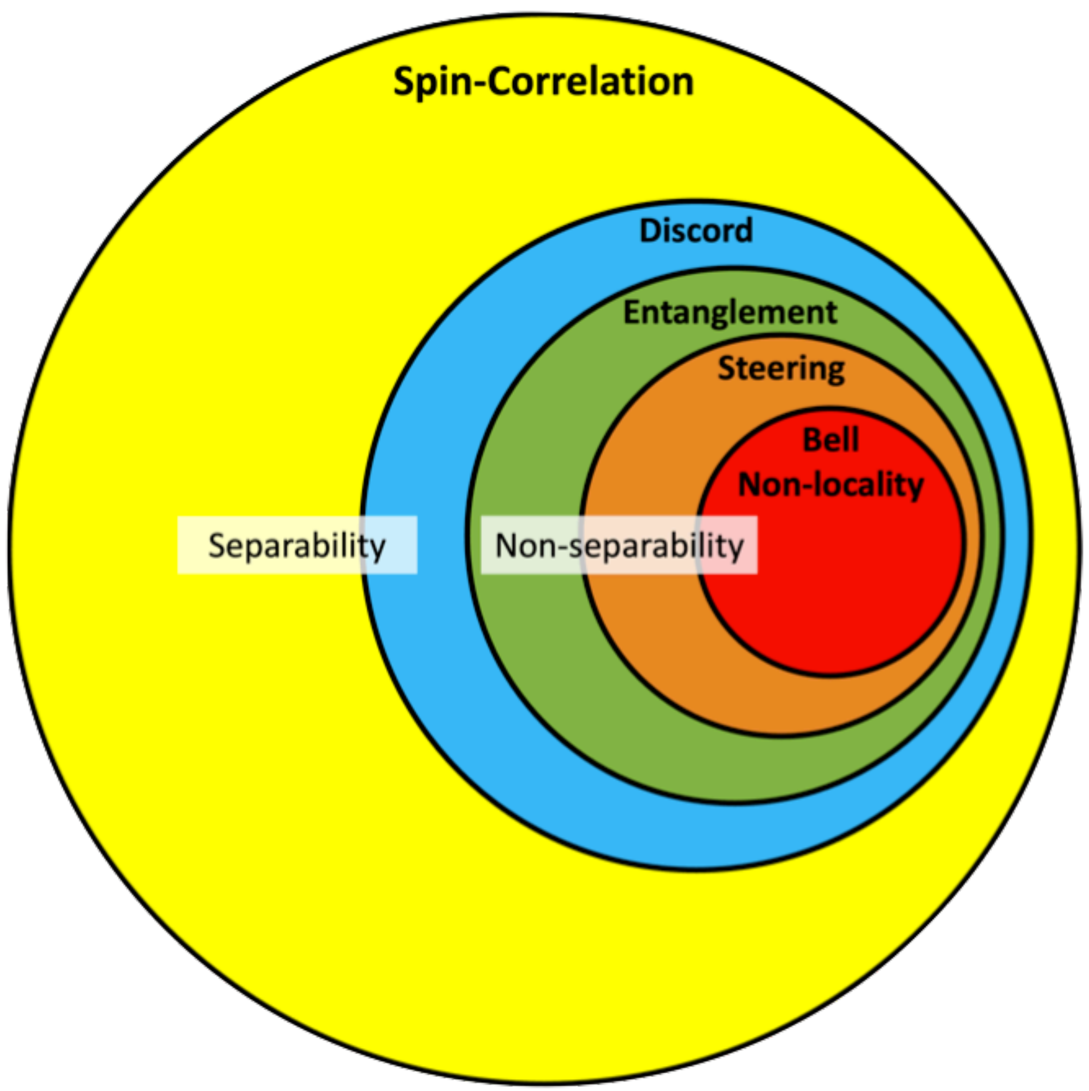
[an observable for D is known since long]

So, what is to
be looked for?

What?

There are many levels of quantum correlations

more stringent tests



Captured from Yoav Afik talks

- Spin correlation: statistical correlation between spins, classical
- Discord: quantum correlations yet in separable states
- Entanglement: subsystems are not separable
- Steering: measurement in one subsystem influences the other
- Bell non-locality: correlation cannot be described by local hidden variables

What?

Example: top pair production

- q_L q_L-bar → *t t*-bar at threshold gives a spin configuration $|\leftarrow\rangle \otimes |\leftarrow\rangle$ that is obviously separable [in the q direction]

- q_R q_R-bar → *t t*-bar at threshold gives a spin configuration $|\rightarrow\rangle \otimes |\rightarrow\rangle$ that is separable too [in the q direction]

⇒ q q-bar → *t t*-bar gives 50% of each [density operator], separable.

We do have a *classical* spin correlation

- *g g* → *t t*-bar at threshold gives $\frac{1}{\sqrt{2}}(|\uparrow\rangle \otimes |\downarrow\rangle - |\downarrow\rangle \otimes |\uparrow\rangle)$

This one is entangled [actually, it is maximally entangled, violates Bell inequalities, etc.]

The mathematical formulation for e.g. entanglement in mixed states are complicated, so I skip it. If curious, please ask.

OK, but how?

How?

If we want to study quantum information stuff with the spin of elementary particles, we have to measure it. All of it! → density operator

As we all know, top quarks, *W*/*Z* bosons, … even τ leptons decay before one can pass them through a Stern-Gerlach experiment to measure spin.

But: the spin leaves its imprint in angular distributions.

$$\rho = \frac{1}{2}\begin{pmatrix} 1+P_3 & P_1 - iP_2 \\ P_1 + iP_2 & 1-P_3 \end{pmatrix}$$

density operator for spin 1/2

take daughter particle as spin analyser

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega} = \frac{1}{4\pi}(1+\alpha\vec{P}\cdot\hat{n})$$

constant

momentum direction of daughter particle

$$\hat{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$$

How?

Top pair: two spin-1/2 particles, simplest example of quantum correlation

$$\rho = \frac{1}{4}\left(1 \otimes 1 + \sum_i B_i^+ \sigma_i \otimes 1 + \sum_i B_i^- 1 \otimes \sigma_i + \sum_{ij} C_{ik} \sigma_i \otimes \sigma_j\right)$$

normalisation

$$\hat{n}_a = (\sin\theta_a \cos\varphi_a, \sin\theta_a \sin\varphi_a, \cos\theta_a)$$
$$\hat{n}_b = (\sin\theta_b \cos\varphi_b, \sin\theta_b \sin\varphi_b, \cos\theta_b)$$

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_a d\Omega_b} = \frac{1}{(4\pi)^2}\left[1 + \alpha_a \vec{B}^+ \cdot \hat{n}_a + \alpha_b \vec{B}^- \cdot \hat{n}_b + \alpha_a \alpha_b \hat{n}_a^T C \hat{n}_b\right]$$

3 coefficients corresponding to top polarisation

3 coefficients corresponding to anti-top polarisation

9 spin correlations

Measured by ATLAS and CMS since some time

How?

For two qubits [e.g. spin-1/2 fermions] sufficient entanglement conditions are

$$|C_{11} + C_{22}| > 1 + C_{33} \quad \text{or} \quad |C_{11} - C_{22}| > 1 - C_{33}$$

Afik, Nova 2003.02280
Maltoni et al. 2110.10112
JAAS , Casas 2205.00542

And Bell-like inequalities are violated if

$$|C_{ii} + C_{jj}| > \sqrt{2} \quad \text{or} \quad |C_{ii} - C_{jj}| > \sqrt{2}$$

Maltoni et al. 2110.10112
JAAS , Casas 2205.00542

For $H \to VV$ [spin 1, extra symmetry] sufficient entanglement conditions are

$$C_{212-1} \neq 0 \quad \text{or} \quad C_{222-2} \neq 0$$

JAAS, Bernal, Casas, Moreno 2209.13441

And [optimised] sufficient condition for violation of Bell-like inequalities

$$I_3 = -\frac{4}{3\sqrt{3}} C_{1010} + \frac{2}{\sqrt{3}} \left(C_{111-1} + C_{1-111}\right) + \frac{1}{2} C_{2020} - \frac{1}{3} \left(C_{212-1} + C_{2-121}\right) + \frac{1}{12} \left(C_{222-2} + C_{2-222}\right) > 2$$

JAAS, Bernal, Casas, Moreno 2209.13441

For different dimensions, fall back into Peres-Horodecki criterion [backup]

Note

Bell inequalities have been explored for *t t*-bar pairs and dibosons.

They are much harder to observe than entanglement, and usually require luminosity beyond Run 3.

Violation of Bell inequalities in collider measurement does not imply QM:

- Free-will loophole
- We are not actually measuring *spins*, but rather we measure commuting observables [distributions] and *assume* QM to obtain spin state

Abel, Dittmar, Dreiner, ´92

Therefore, violation of Bell inequalities at colliders in *t t*-bar pairs and dibosons implies something like:

The spin state is such that if we were able to perform proper spin measurements we could rule out local hidden-variable theories

How?

To take away:

- ☑ You need to measure elements of spin density operator of composite system
- ☑ The spin can be accessed through distributions of decay products
- ☑ But for that you need to reconstruct rest frame, leaving only top, W and Z as candidates at LHC
- ☑ For τ leptons it is possible too, at e^+e^- colliders
- ☑ Orbital angular momentum cannot directly be accessed but this is another story…

JAAS 2402.14725
JAAS 2403.13942

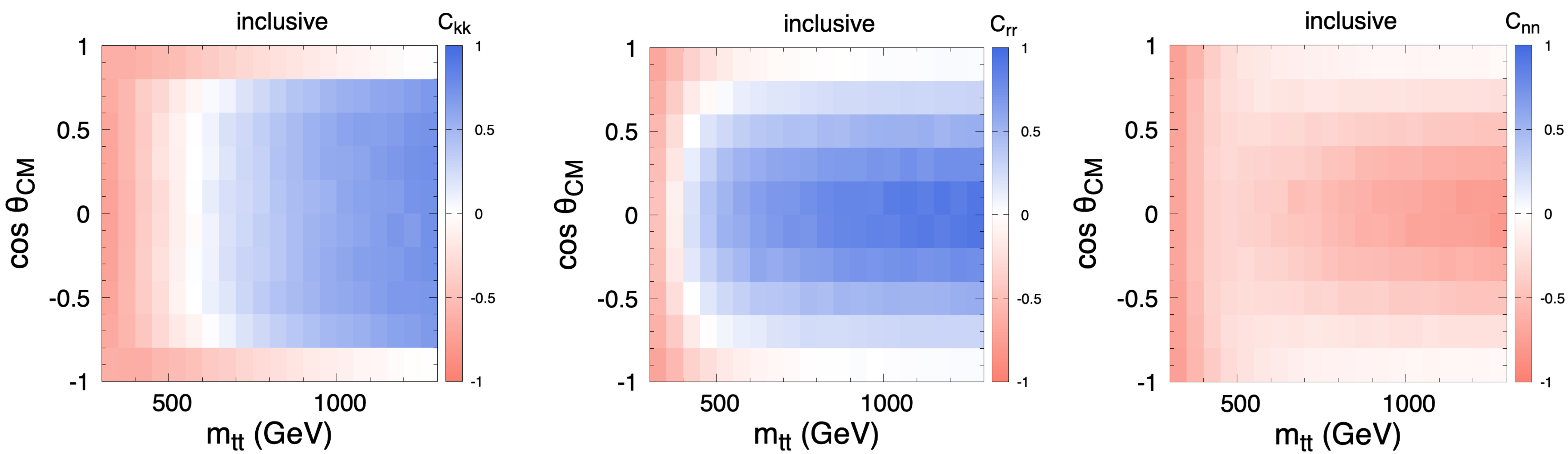
Top pair entanglement

Top pair entanglement

There is a dependence of the C_{ij} coefficients on the kinematics.

Use the helicity basis to parameterise C_{ij}:

K: top helicity | R: ⊥ in production plane | N: ⊥ to K and N



Most convenient entanglement criterion: $|C_{11} + C_{22}| > 1 + C_{33}$

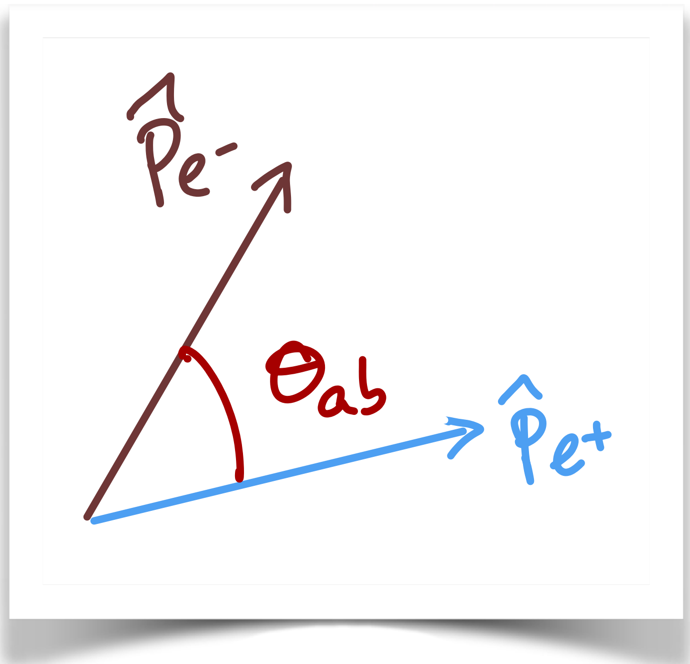
with 3 → N because $C_{nn} < 0$

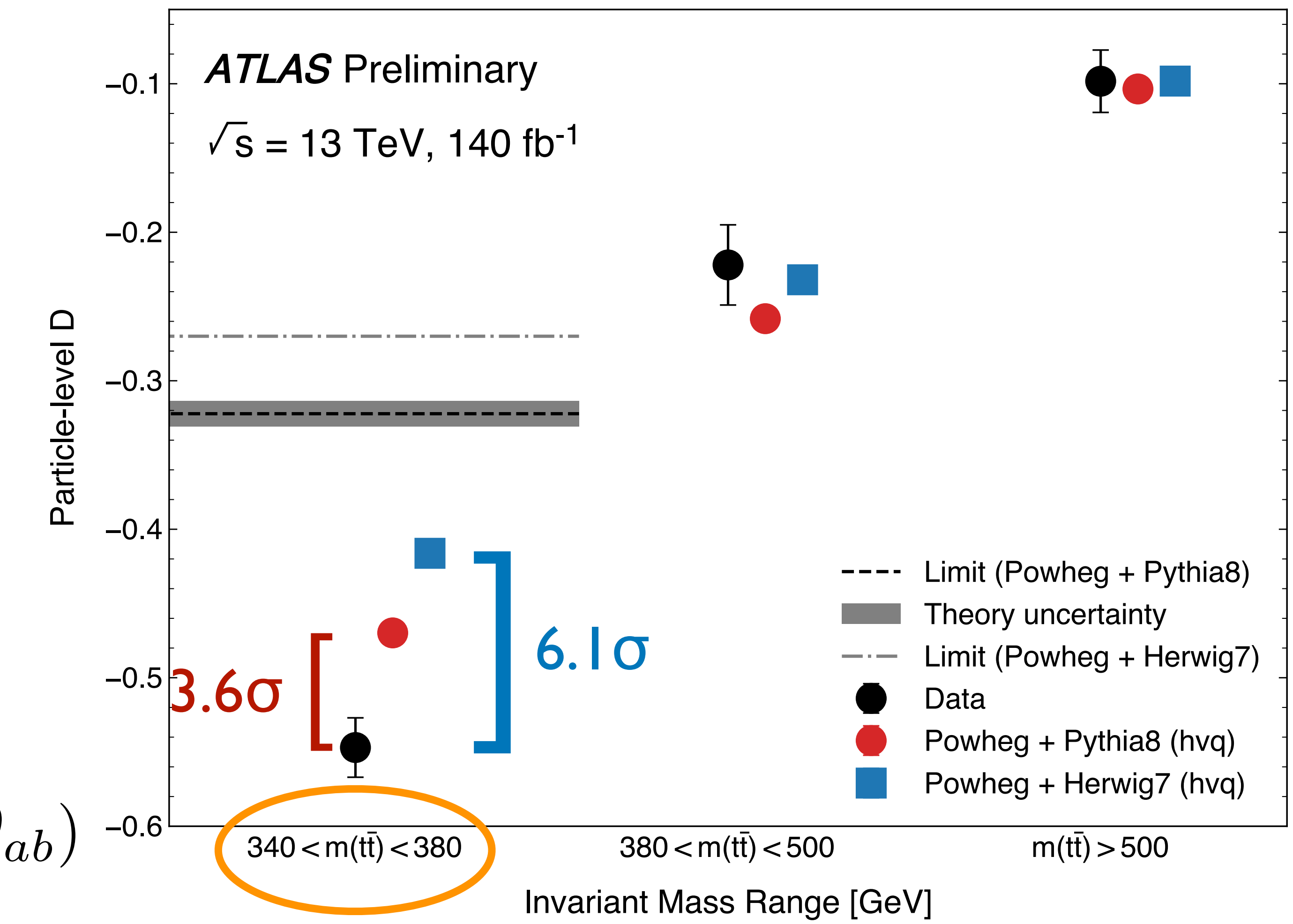
Near threshold: $|C_{kk} + C_{rr}| = - C_{kk} - C_{rr}$

Boosted central: $|C_{kk} + C_{rr}| = C_{kk} + C_{rr}$

Top pair entanglement

ATLAS has performed a measurement at threshold using the *D* observable, related to the angle between the two leptons





$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta_{ab}} = \frac{1}{2}\left(1 + \alpha_a\alpha_b D\cos\theta_{ab}\right)$$

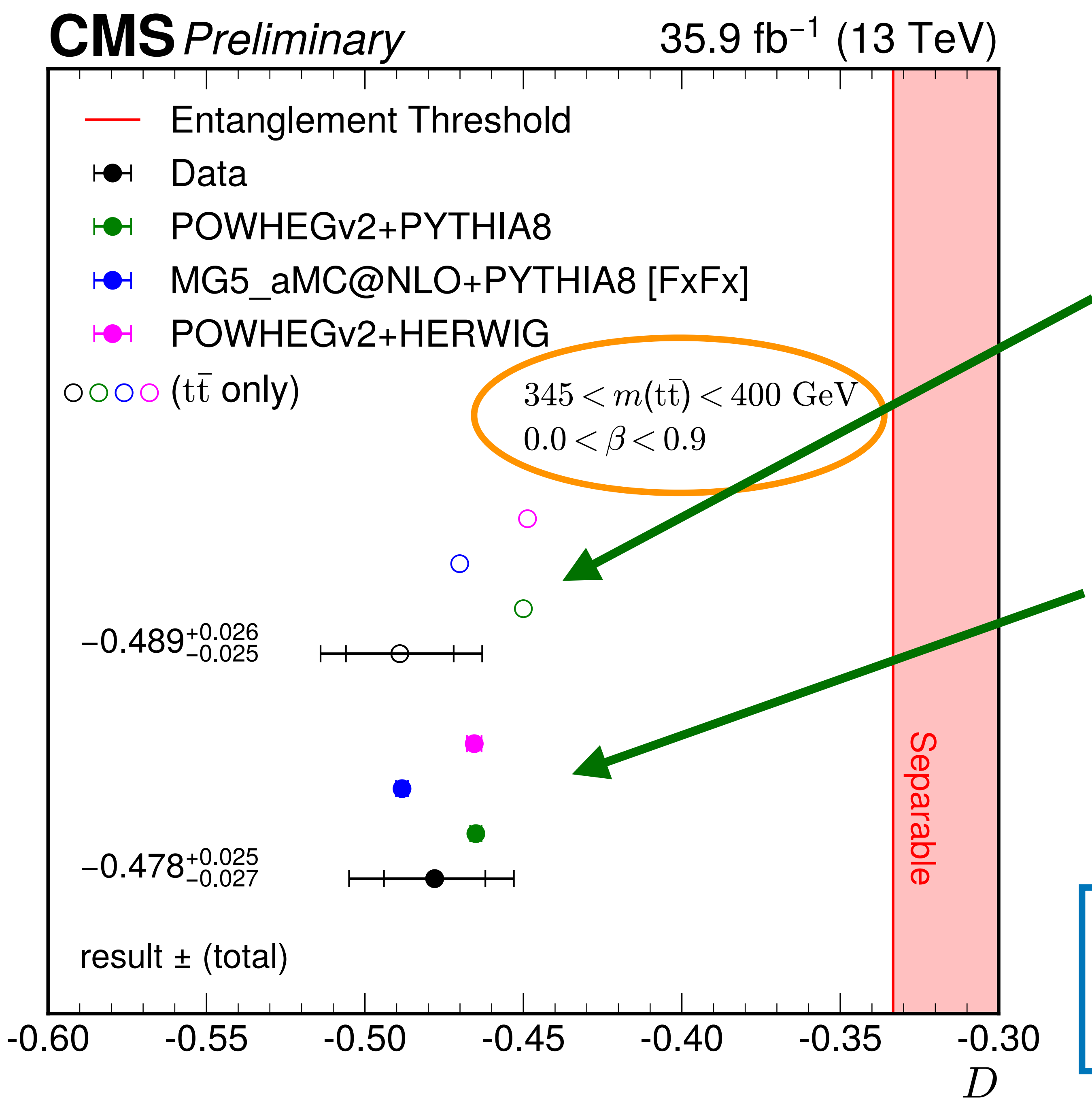
$$D = \frac{1}{3}(C_{11} + C_{22} + C_{33})$$

Entanglement test near threshold: $-3D - 1 > 0$

Bottom line: we know there are spin correlations since a decade, but entanglement is a stronger condition

Top pair entanglement

CMS has measured entanglement using the same observable, in a slightly different kinematical region



t t-bar NLO

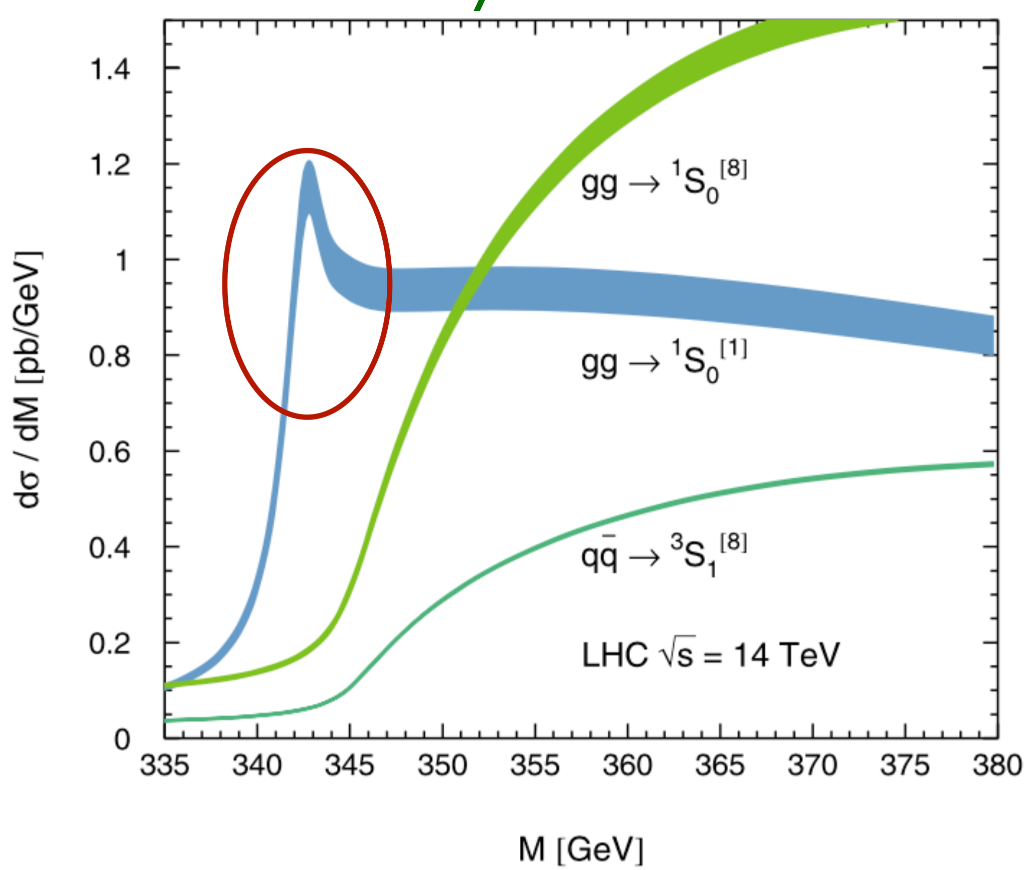
t t-bar NLO ...
+ toponium!

Overall agreement much better than ATLAS, even without toponium

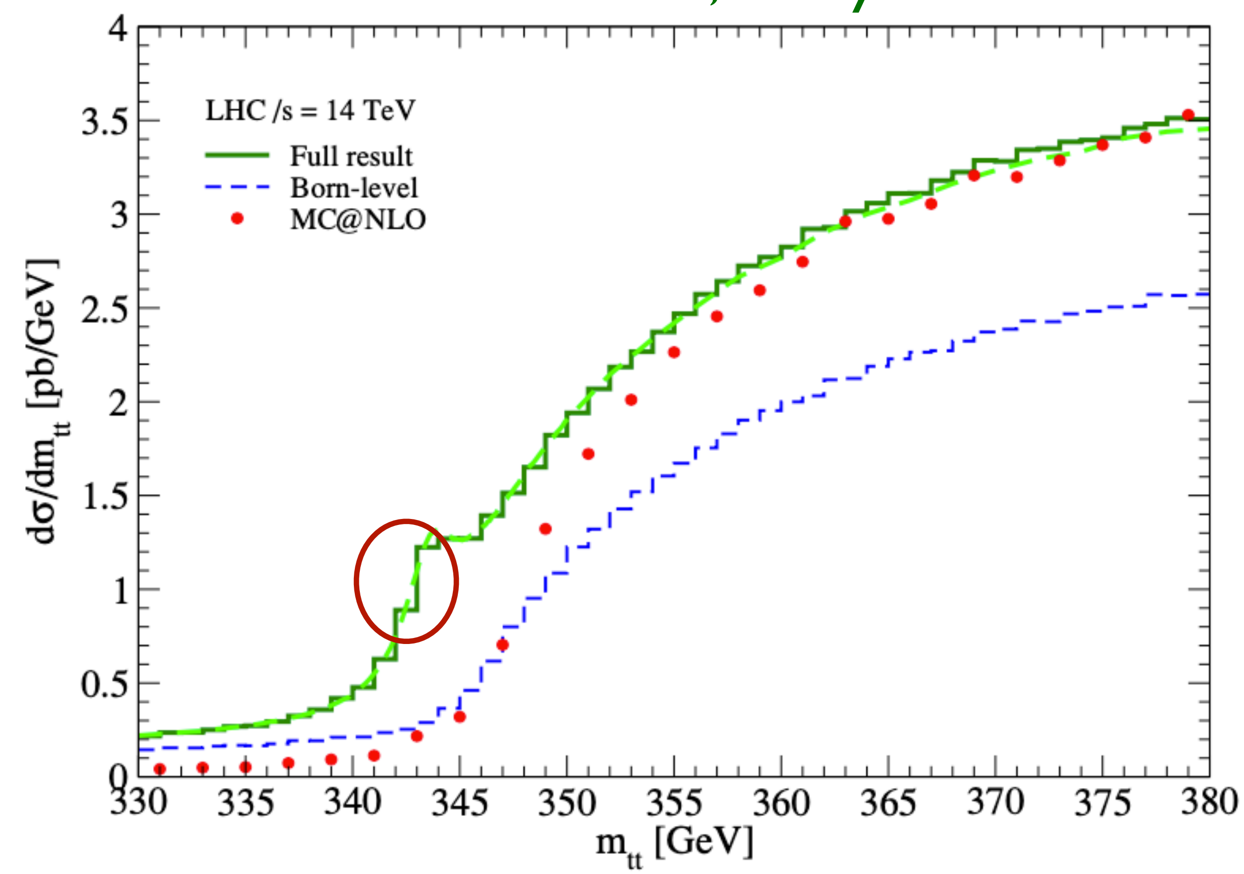
Toponium!

Non-perturbative corrections in the colour-singlet channel produce a pseudo-bound-state near threshold.

Kiyo et al. 0812.0919



Sumino, Yokoya 1007.0075



The toponium resonance produced in pp collisions has

$J^P = 0^-$

$m \simeq 2m_t - 2$ GeV

$\Gamma \simeq 2\Gamma_t$

The toponium contribution is very well approximated by a pseudo-scalar with these parameters.

Toponium!

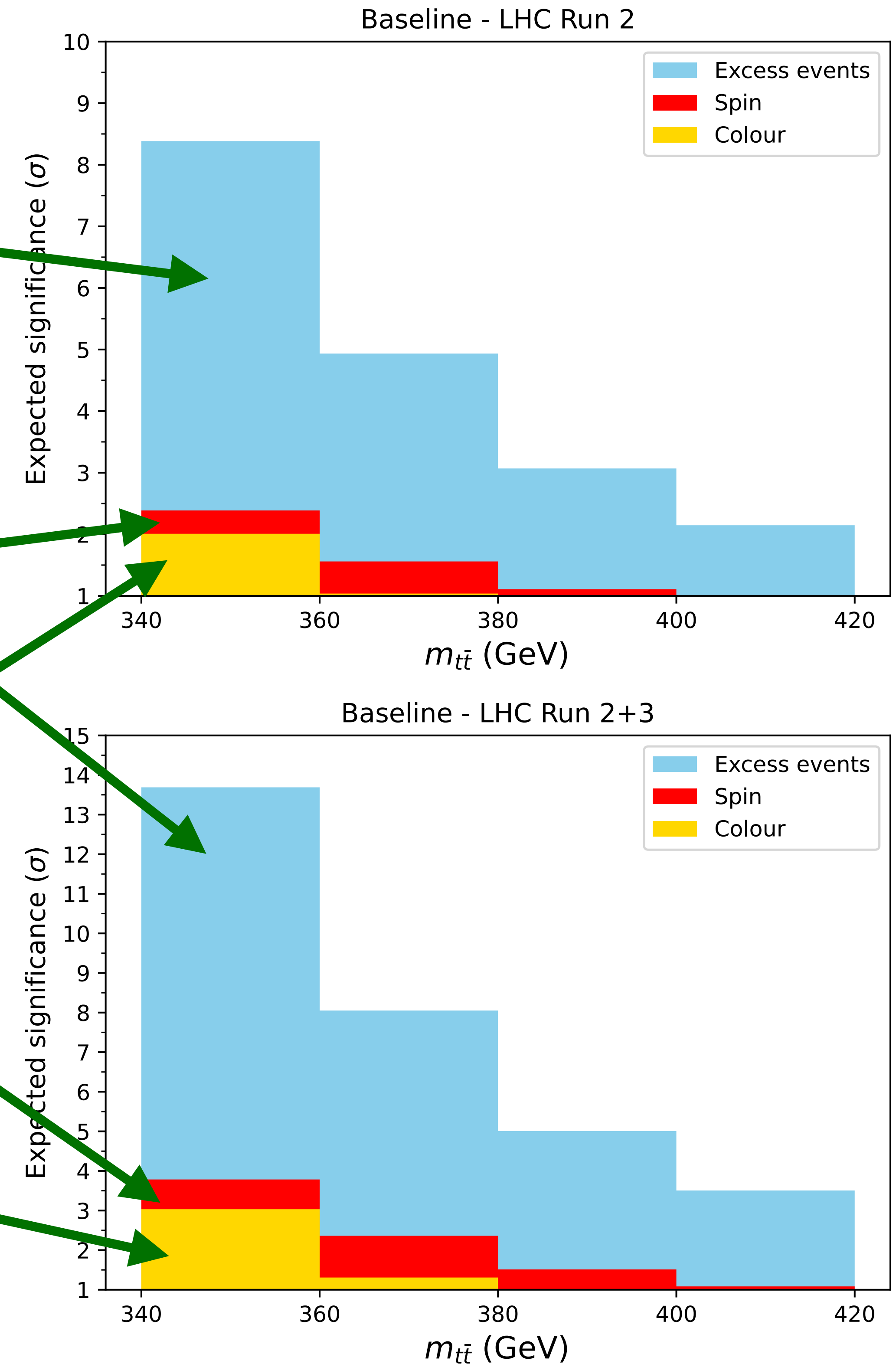
The presence of toponium can be spotted by

JAAS 2407.20330

an event excess near threshold

that modifies spin observables like a 0^-

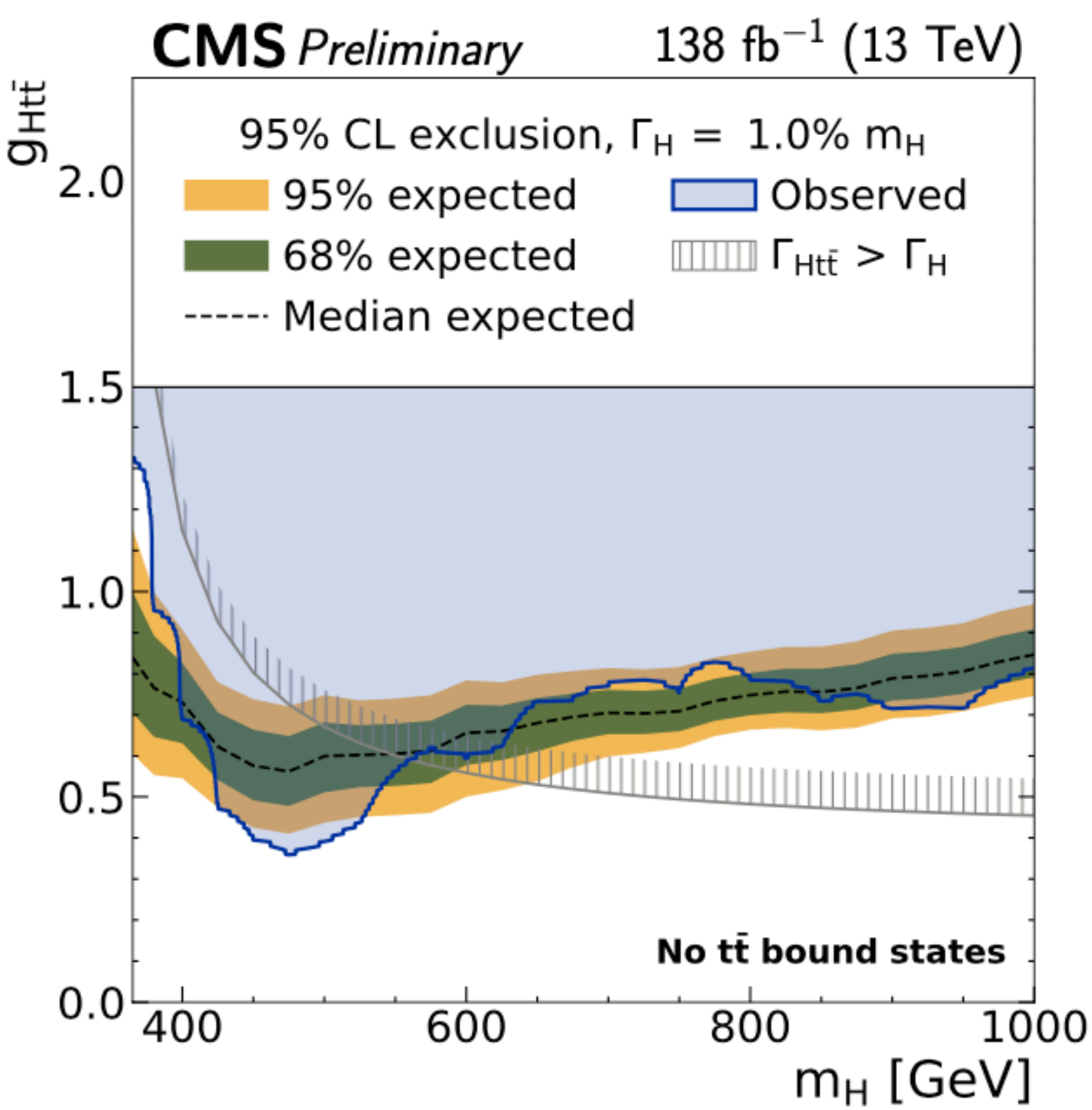
and looks like a colour singlet



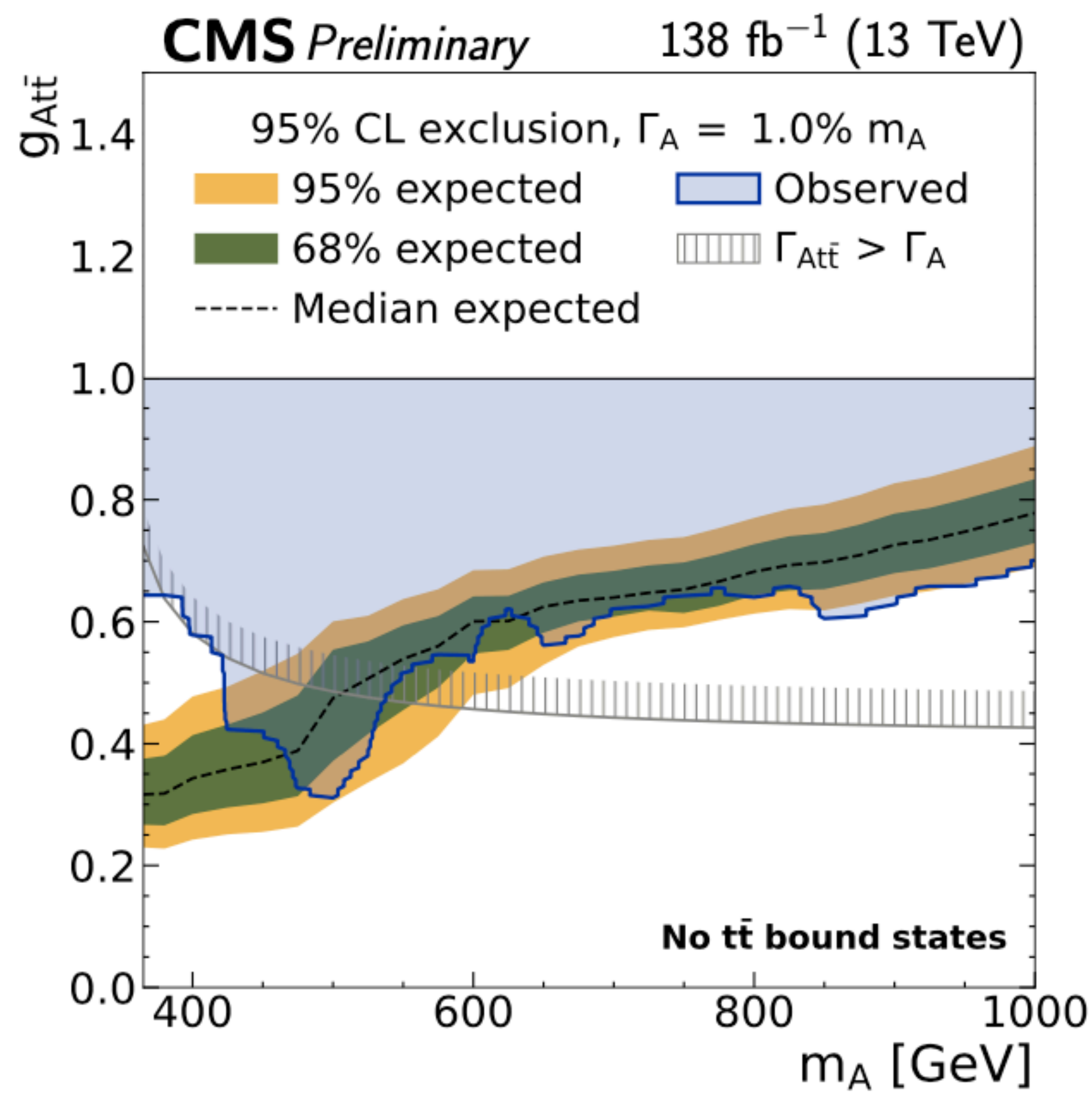
Toponium!

Regular searches for new scalars in *t t*-bar final states are also sensitive to toponium

scalar 0^{+}

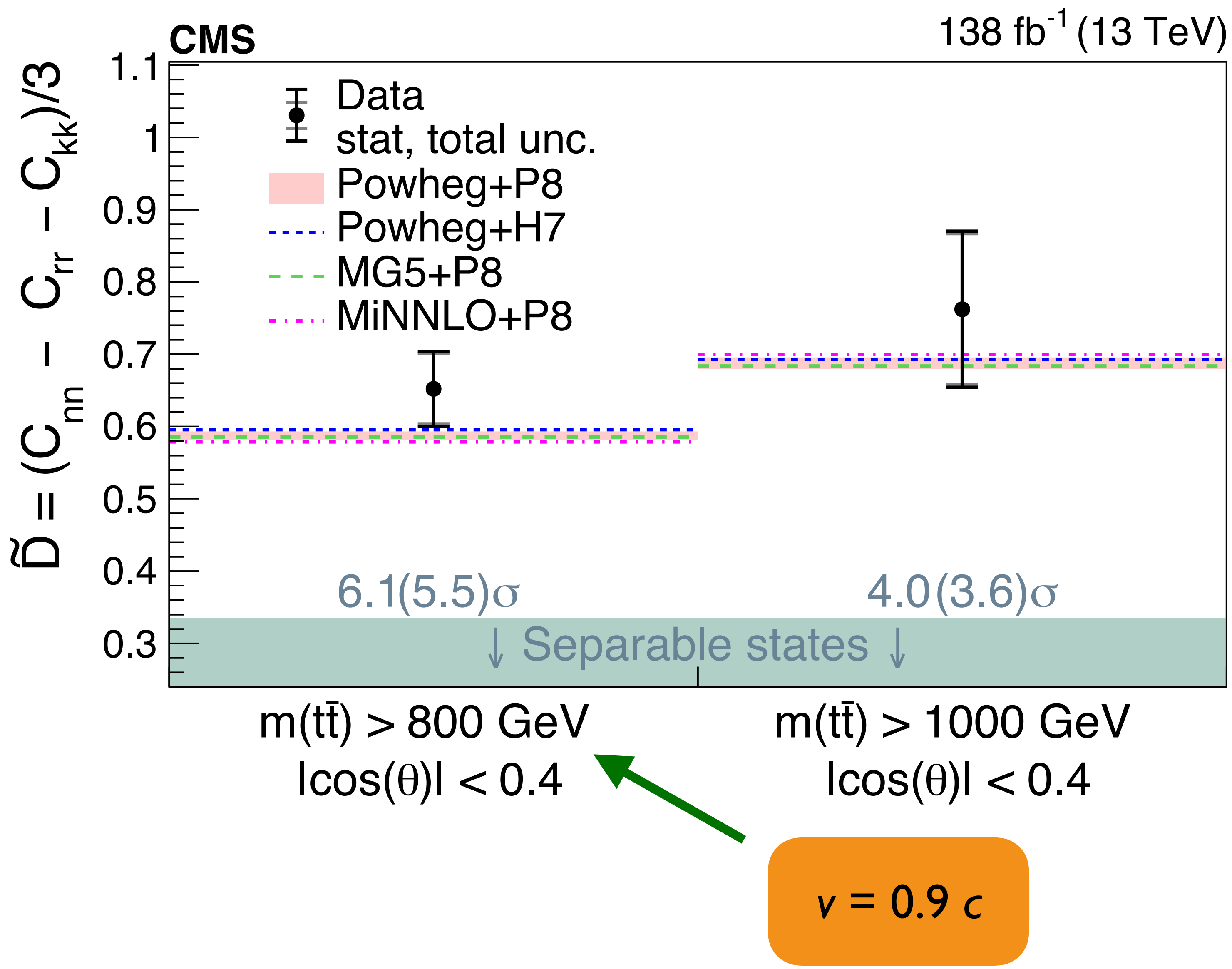


pseudo-scalar 0^{-}



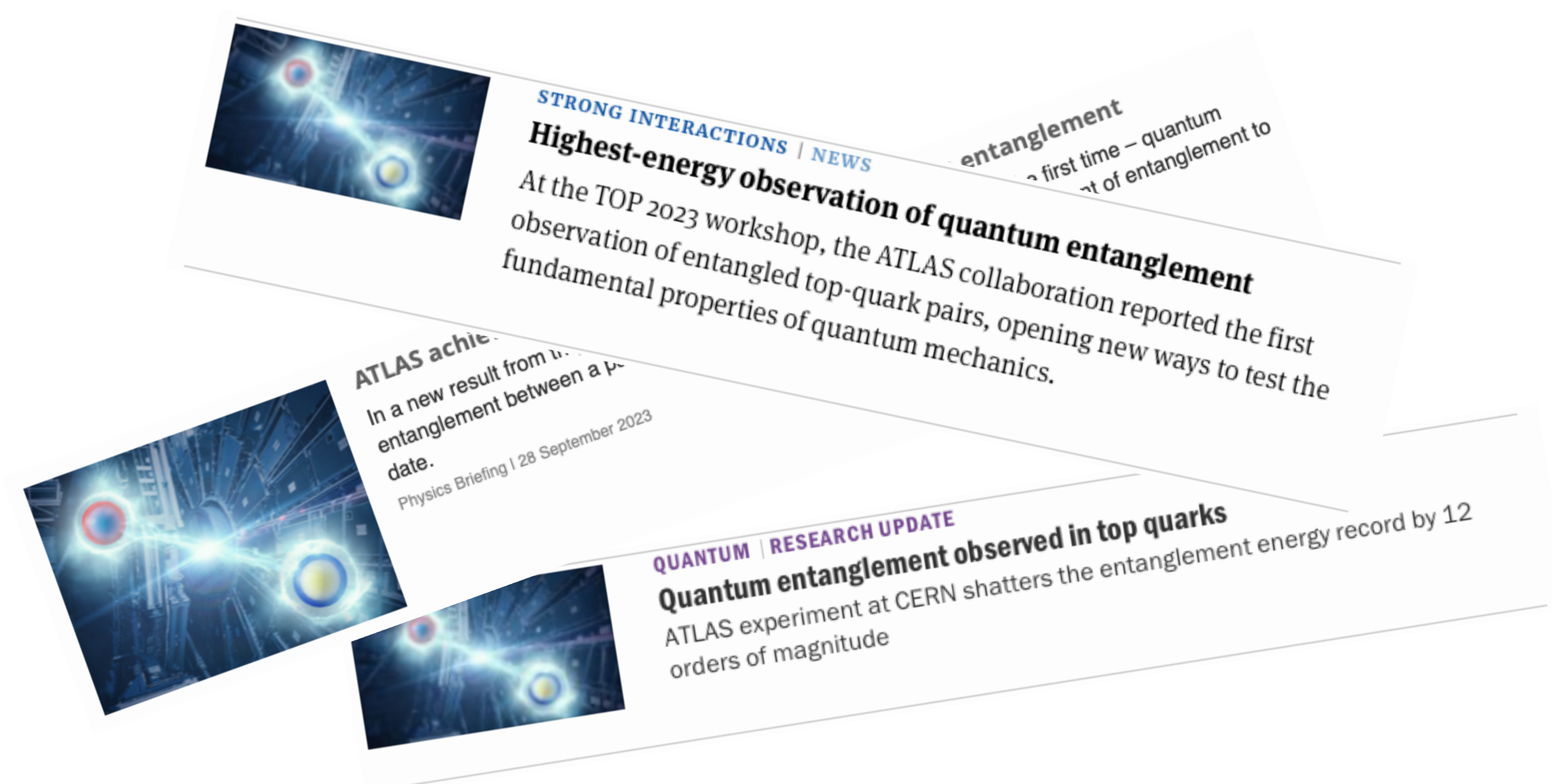
Top pair entanglement

Not covered: CMS measurement in boosted central region



Top pair entanglement

Testing basic properties of quantum mechanics with different particles, and higher energies, is very nice.



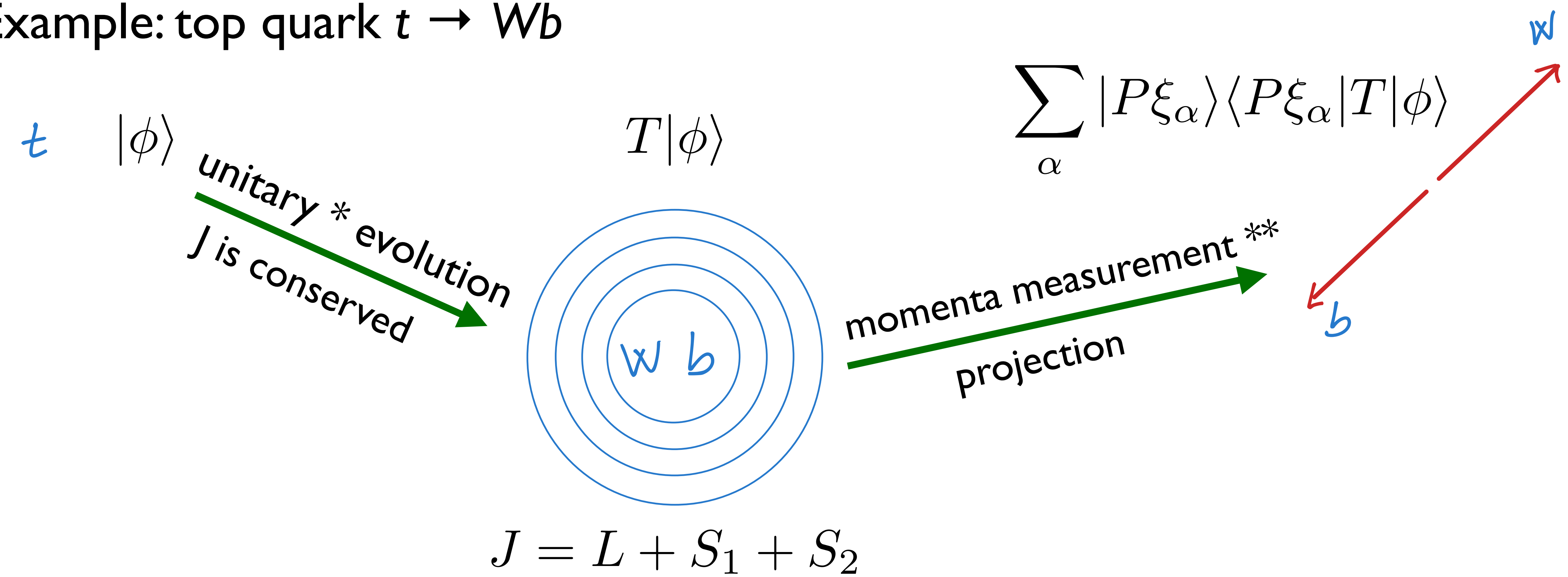
But as I have stressed, there are some tests that can be performed at colliders that cannot be [and have not been] done anywhere else: entanglement and decay.

Novel tests: decay and entanglement

Deconstructing particle decay

What does decay *mean* in a particle detector?

Example: top quark $t \to Wb$



* Strictly speaking, this is part of the unitary evolution, S = 1 + i T.

** This also involves the identification of the final state, *Wb* / …

The measurement of momenta influences the spin state but in general it does not collapse it as a Stern-Gerlach experiment would do.

Deconstructing particle decay

Consider a system of two particles *A*, *B*, with spin state described by

$$\rho = \sum_{ijkl} \rho_{ij}^{kl} |\phi_i \chi_k\rangle\langle\phi_j \chi_l| \qquad |\phi_i\rangle \in \mathcal{H}_A \,, \quad |\chi_k\rangle \in \mathcal{H}_B$$

$\mathcal{H}$ are the spin spaces

Let *A* decay *A* → A_1 A_2 … with amplitudes

$$M_{\alpha j} = \langle P\, \xi_\alpha | T | \phi_j \rangle \qquad |\xi_\alpha\rangle \in \mathcal{H}_{A_1} \otimes \mathcal{H}_{A_2} \otimes \ldots$$

Then, the spin state of A_1 A_2 … and *B* is described by

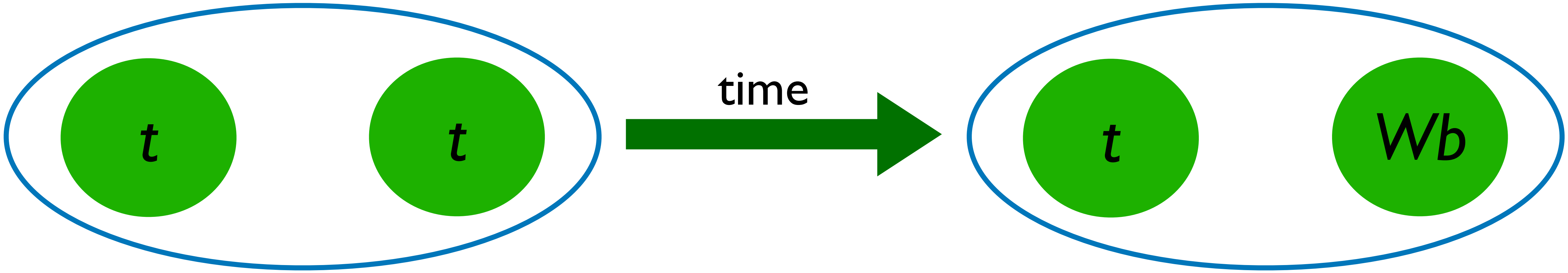
these come from the projector

$$\rho' = \frac{1}{\sum_{\alpha k} (M \rho^{kk} M^\dagger)_{\alpha\alpha}} \sum_{\alpha\beta kl} (M \rho^{kl} M^\dagger)_{\alpha\beta} |\xi_\alpha\, \chi_k\rangle\langle\xi_\beta\, \chi_l|$$

➡ in particular, the entanglement properties between *A* and *B* can be inherited by { the decay products of *A* } vs *B*

Post-decay entanglement

When *t* *t*-bar are entangled and *t*-bar decays into W^- *b*-bar, *t* is entangled with the W^- *b*-bar pair



Potential problem:

The *b* spin is, in principle, not measurable.

When we have several entangled particles and trace over [unobserved] degrees of freedom, entanglement may be \`lost´.

→ but *b*-bar has RH helicity up to small mass effects, trace maintains entanglement between *t* and W^-

Post-decay entanglement

Example: threshold region $m_{tt} \leq 390$ GeV, $\beta \leq 0.9$, beamline basis z = (0,0,1)

θ 👉 angle between W^{-} momentum in *t*-bar rest frame and ẑ axis *or any fixed axis*

Negativity: entanglement measure

phase space region	N(ρ)
θ = 0	0.13
cos θ > 0.9	0.12
cos θ > 0.5	0.10
cos θ > 0	0.07
all θ	0

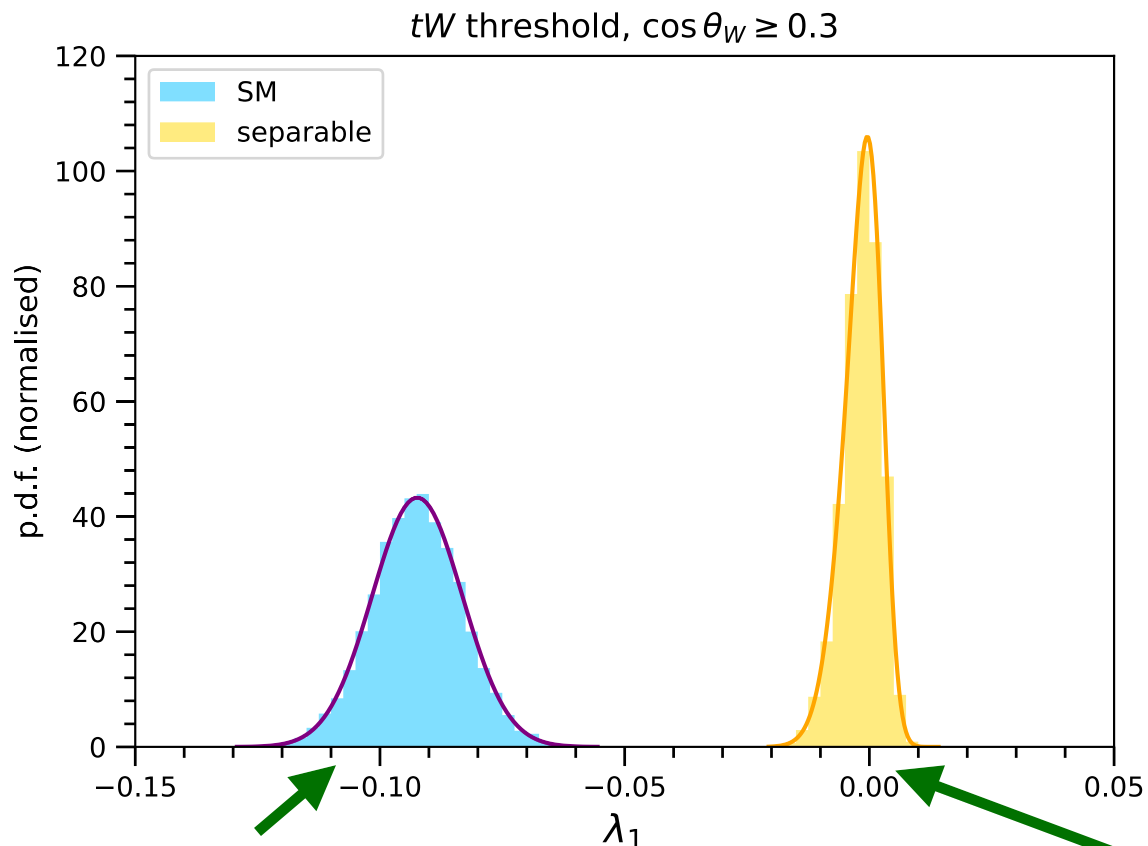
The amount of entanglement is the same in any direction but the quantum state is not, so integration washes out entanglement

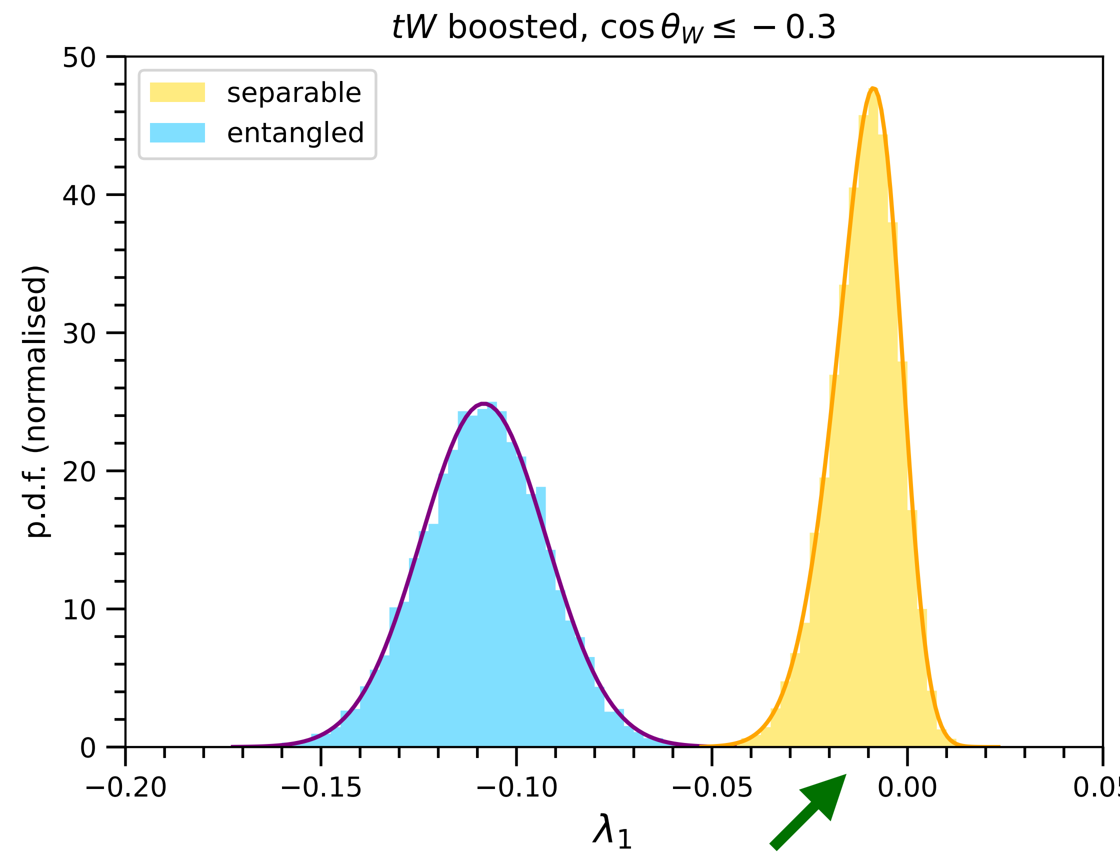
The projection is at work here: the spin quantum state depends on *t*-bar decay kinematics

Post-decay entanglement

Entanglement indicator:
lowest eigenvalue λ_1 of the ρ^{T2} matrix for *tW*

$\lambda_1 < 0 \Leftrightarrow$ Entanglement





stat uncertainty

Bias: even if $\lambda_1 > 0$, in a small sample we may find it negative

Run 2	Significance [stat + 10% sys + bias]
Threshold	7.0 σ
Boosted	5.0 σ

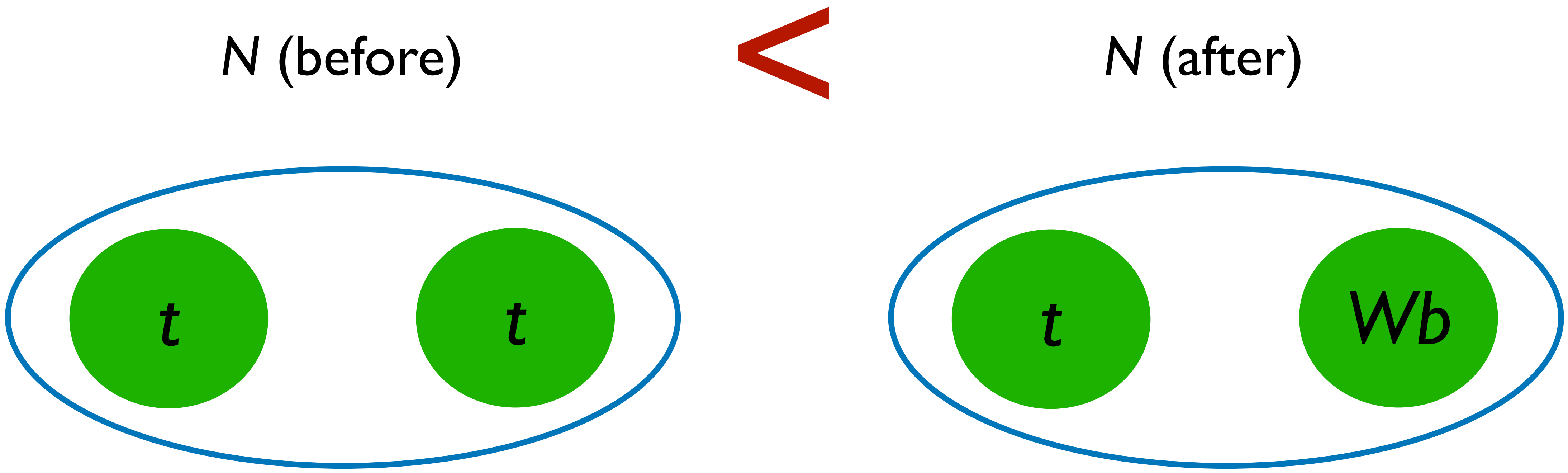
these numbers can possibly be improved by combining several regions...

Entanglement autodistillation

Entanglement decreases by measurements [collapse], interaction with environment [decoherence] …

Methods are known [distillation] to manipulate a sub-system and, if lucky, increase entanglement

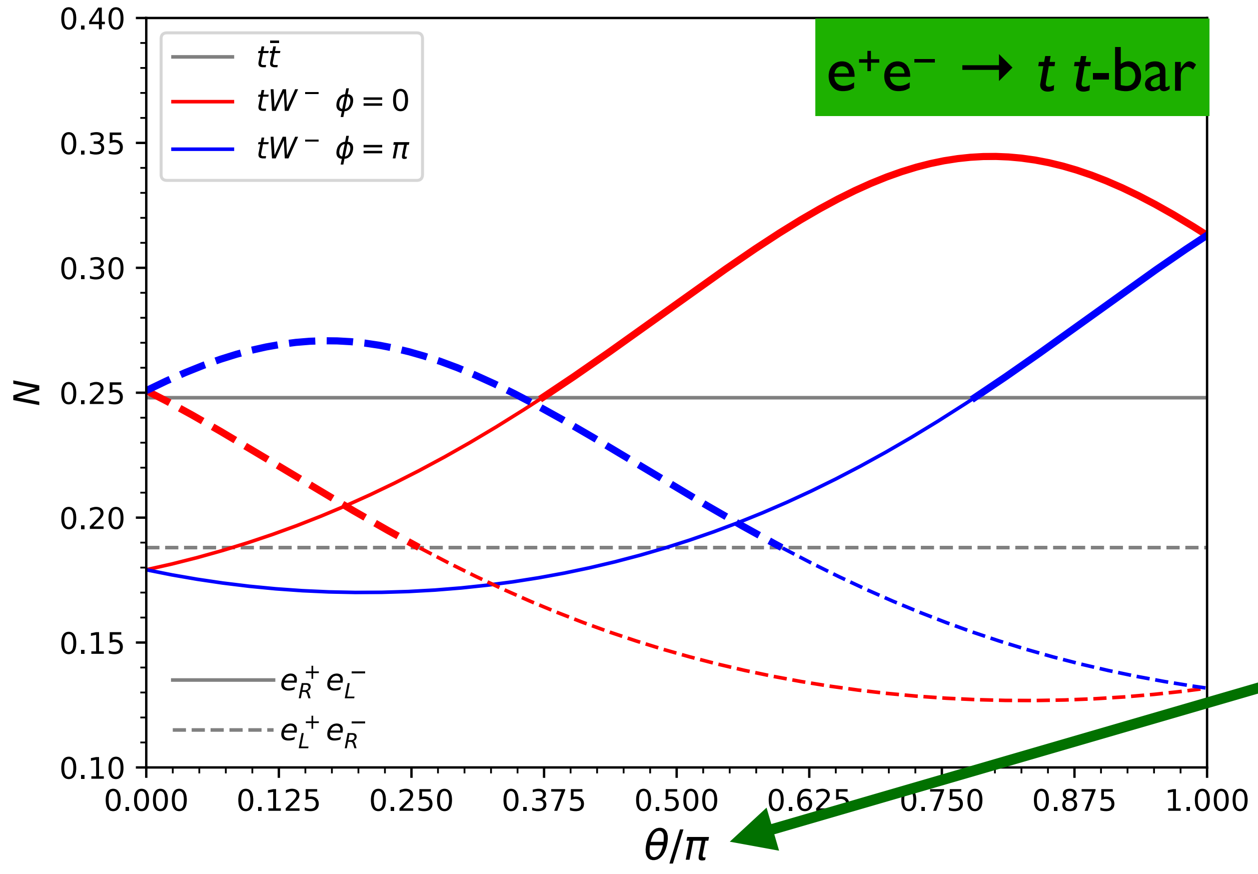
Most remarkably, the decay can increase entanglement spontaneously.



Entanglement autodistillation

Since the *b* spins are, in principle, not measurable, we can use the *t*-*W* entanglement as a proxy to probe the entanglement increase.

And this could be observed in e+ e− colliders [needs that tops are polarised]



Unique quantum effect that requires large luminosity to be observed

polar angle between *W* momentum in top rest frame and top direction in c.m. frame

Novel tests: decay and entanglement

To take away

- Particle decay and subsequent momenta projection is a very special kind of "measurement"
- Unique QM effects:
  - post-selection
  - autodistillation
- Post-decay entanglement never tested, test is possible at LHC with current data

End

Why?

Entanglement observables involve spin correlations, which are sensitive to new physics.

→ we can parameterise deviations from SM in terms of dim-6 operators, which provide a definite framework for comparisons

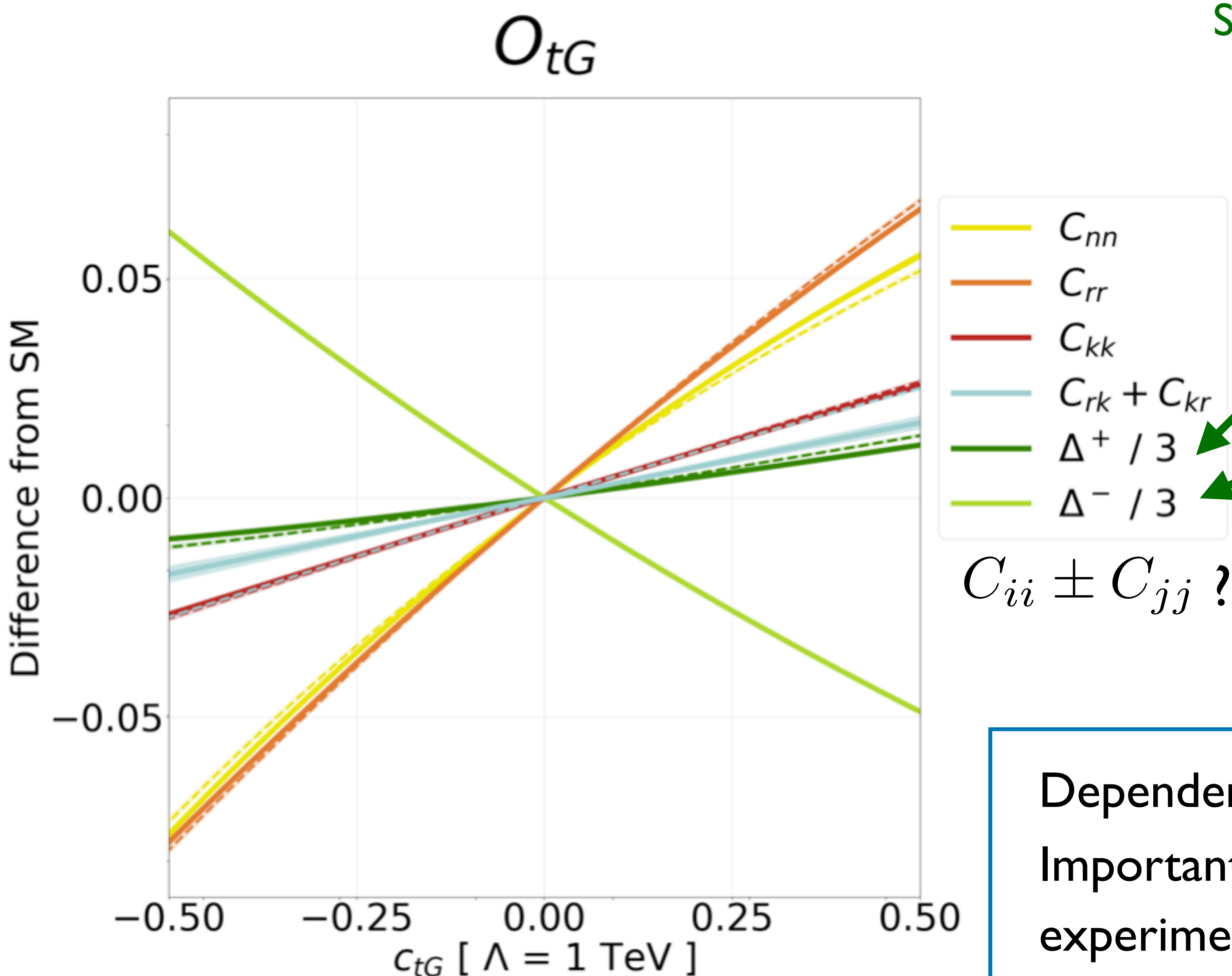
Spin correlations are measured with angular distributions, with a relation that may be modified by new physics

→ we can also introduce dim-6 operators for the decay of top, W, Z, but typically there are better ways to constrain them

Why?

t t-bar example: top chromomagnetic dipole operator

Severi, Vryonidou, 2210.09330



$C_{ii} \pm C_{jj}$???

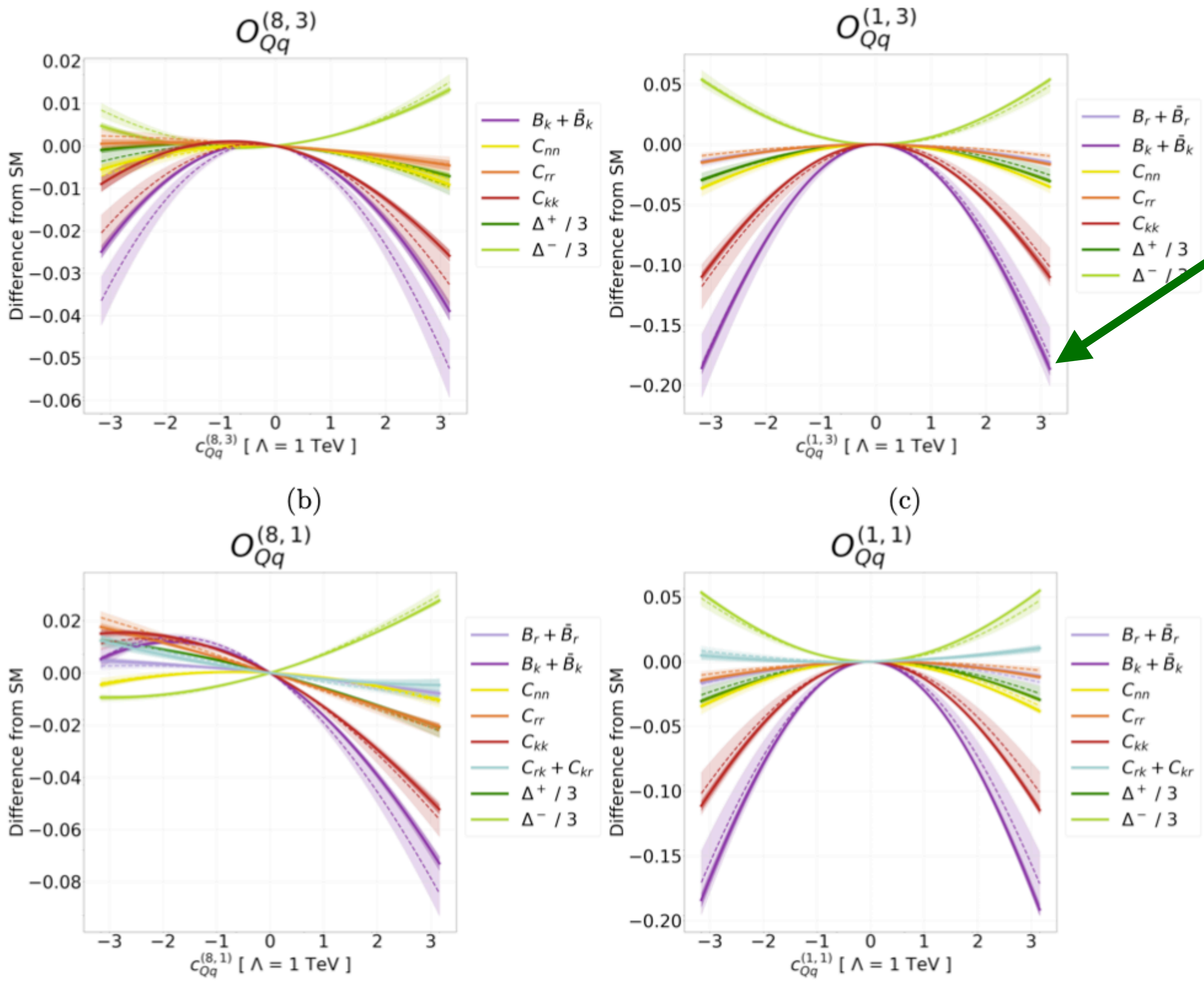
this is a new observable

this is essentially the old good D

Dependence on c is the first step. Important missing piece: expected experimental error bars for these quantities

Why?

t t-bar example: some four-fermion operators



polarisation in helicity direction

Polarisation seems to outperform the rest of observables [note that experimental uncertainties are likely smaller] but this statement is basis-dependent (!)

Why?

H → *ZZ* example: test anomalous *HZZ* interaction   Fabbrichesi et al. 2304.02403

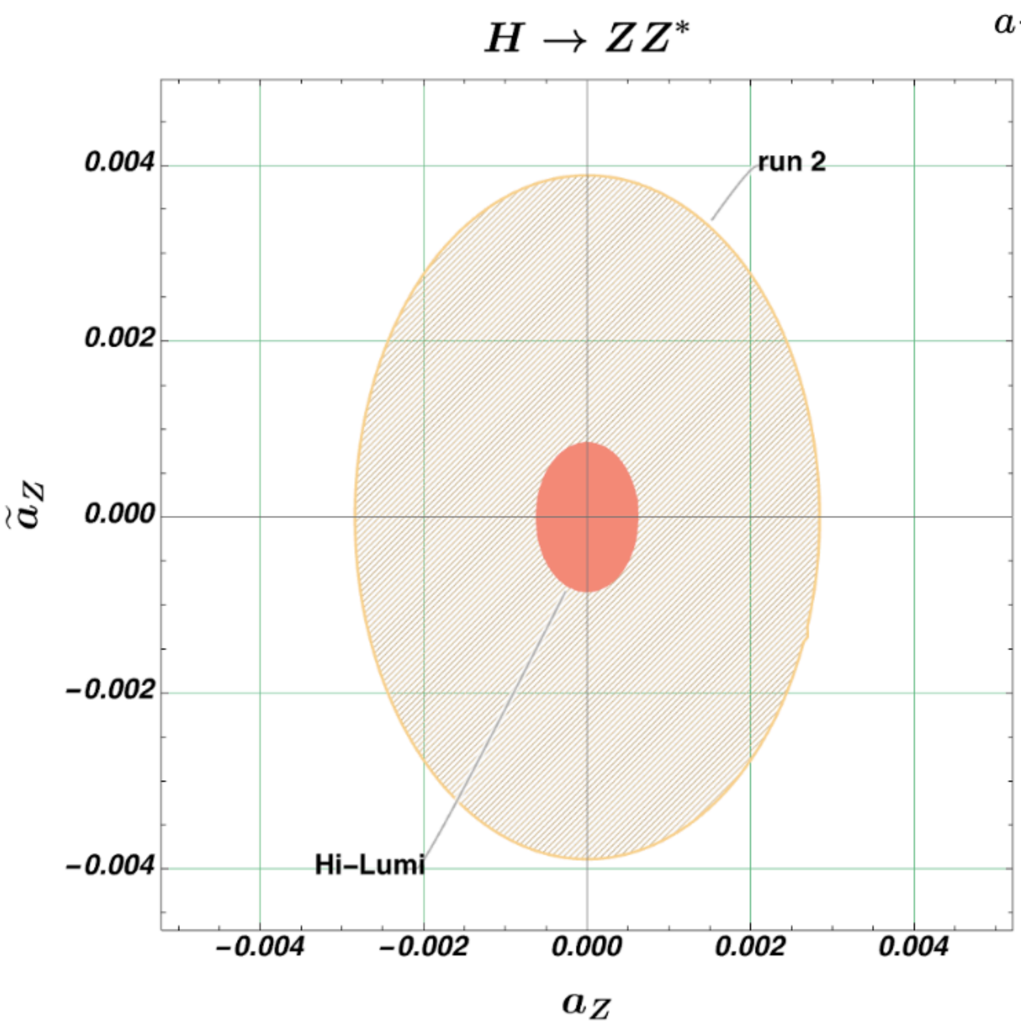
entanglement witness →

$$\mathscr{C}_2 = 2\max\Big[0,\ -\frac{2}{9} - 12\sum_a f_a^2 + 6\sum_a g_a^2 + 4\sum_{ab} h_{ab}^2\,,\ -\frac{2}{9} - 12\sum_a g_a^2 + 6\sum_a f_a^2 + 4\sum_{ab} h_{ab}^2\Big]$$

nothing to do with entanglement →

$$\mathscr{C}_{\rm odd} = \frac{1}{2}\sum_{\substack{a,b\\ a<b}} \big|h_{ab} - h_{ba}\big|$$

parameters of *ZZ* density matrix



- Why not using *ZZ* density matrix elements instead ofx $\mathscr{C}_2$?
- Why use $\mathscr{C}_{\rm odd}$ and not dedicated triple-product observables?
- Same applies to EW diboson production   Aoude et al. 2307.09675

What?

Any operator cannot be a density operator. A valid density operator has several characteristics:

- Unit trace
- Hermitian
- Positive semidefinite: eigenvalues ≥ 0

A density operator describing a composite system is separable if it can be written as

$$\rho_{\text{sep}} = \sum_n p_n \rho_n^A \otimes \rho_n^B$$

Note: in general, one has something like

$$\rho = \sum_{ijkl} p_{ij}^{kl} |\psi_i\rangle\langle\psi_j| \otimes |\psi_k\rangle\langle\psi_l|$$

What?

Necessary criterion for separability:

Peres, quant-ph/9604005
Horodecki, quant-ph/9703004

taking the transpose in subspace of B [for example] the resulting density operator is valid.

Example: composite system A ⊗ B with dim $\mathcal{H}_A$ = n, dim $\mathcal{H}_B$ = m

P_{ij} are m x m matrices, $(P_{ij})^{kl} = p_{ij}^{kl}$

$$\rho = \begin{pmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & & \\ \vdots & & \ddots & \\ P_{n1} & & & P_{nn} \end{pmatrix} \quad \Rightarrow \quad \rho^{T_2} = \begin{pmatrix} P_{11}^T & P_{12}^T & \cdots & P_{1n}^T \\ P_{21}^T & P_{22}^T & & \\ \vdots & & \ddots & \\ P_{n1}^T & & & P_{nn}^T \end{pmatrix}$$

(n∗m) × (n∗m) matrix

Not easily tractable!

What?

To take away:

- It is quite complicated to prove [analytically] that a composite system is in a separable state.
- Numerically, it can be done but there may be a bias [see later]
- However, we are interested in showing that the system is entangled.
- To prove that, in some systems there are simple sufficient conditions that do the work

ρ^{T2} non-positive $\Rightarrow$ ρ^{T2} not valid $\Rightarrow$ system entangled

Showing this for a single vector is enough

simple conditions

What?

A useful formulation of Bell-like inequalities for spin-1/2 systems is provided by the so-called CHSH inequalities for two systems A (Alice) and B (Bob).

Clauser, Horne, Shimony, Holt, '69

Alice measures two spin observables *A*, *A´*. Bob measures two spin observables *B*, *B´*. [Both normalised to unity]. Then, clasically:

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle| \leq 2$$

these are spin correlation observables!

One can show violation of CHSH inequalities if one finds spin observables *A*, *A´* for Alice and *B*, *B´* for Bob such that the inequality is violated.

in a given quantum state!

What?

The CHSH inequalities involve spin correlations. Therefore, for a particle of spin 1/2, they involve the C_{ij} spin-correlation coefficients [already measured for top pair production]

It can be shown that the maximum of the l.h.s.

$$|\langle AB\rangle - \langle AB'\rangle + \langle A'B\rangle + \langle A'B'\rangle|$$

is given by

$$2\sqrt{\lambda_1 + \lambda_2}$$

where λ_1 and λ_2 are the two largest eigenvalues of the positive definite matrix $C^T C$

Horodecki, Horodecki, Horodecki, ' 95

What?

Simpler but equally effective: Take judicious choice of [non-commuting] spin observables

$$A \to 2S_i \qquad B \to \frac{1}{\sqrt{2}}(2S_i + 2S_j)$$

$$A' \to 2S_j \qquad B' \to \frac{1}{\sqrt{2}}(-2S_i + 2S_j)$$

$$|\langle AB \rangle - \langle AB' \rangle + \langle A'B \rangle + \langle A'B' \rangle|$$

$$|C_{ii} + C_{jj}|$$

$$|C_{ii} - C_{jj}|$$

$$A \to 2S_i \qquad B \to \frac{1}{\sqrt{2}}(-2S_i - 2S_j)$$

$$A' \to 2S_j \qquad B' \to \frac{1}{\sqrt{2}}(2S_i - 2S_j)$$

CHSH violation is probed by testing if $|C_{ii} \pm C_{jj}| > \sqrt{2}$

These estimators are optimal when off-diagonal C_{ij} vanish

What?

For spin-1 systems there is an inequality that is stronger than CHSH. For any observables A_1, A_2 [on system A], B_1, B_2 [on system B] CGLMP PRL '02

$$
\begin{aligned}
I_3 = & P(A_1 = B_1) + P(B_1 = A_2 + 1) + P(A_2 = B_2) + P(B_2 = A_1) \\
& - [P(A_1 = B_1 - 1) + P(B_1 = A_2) + P(A_2 = B_2 - 1) + P(B_2 = A_1 - 1)] \leq 2
\end{aligned}
$$

if the systems are classical.

There is a well-known choice of A_1, A_2, B_1, B_2 that is believed to maximise I_3 for the spin-singlet state

$$|\psi\rangle = \frac{1}{\sqrt{3}} (|+-\rangle - |00\rangle + |-+\rangle)$$

However, it is not optimal for the mixed spin state of the *VV* pair resulting from *H* decay

$$\rho = \int d\beta \; \mathcal{P}(\beta) |\psi_\beta\rangle\langle\psi_\beta| \qquad |\psi_\beta\rangle = \frac{1}{\sqrt{1+\beta^2}} (|+-\rangle - \beta|00\rangle + |-+\rangle)$$

How?

With spin-1 particles V=W,Z it is the same but more complicated

$$\rho = \frac{1}{9}\left(1_{9\times 9} + A^1_{LM} T^L_M \otimes 1_{3\times 3} + A^2_{LM} 1_{3\times 3} \otimes T^L_M + C_{L_1 M_1 L_2 M_2} T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right)$$

8 polarisations for V_1

8 polarisations for V_2

64 spin correlations

where T^L_M [L = 1,2] are irreducible tensors

$$T^1_1 = \sqrt{\frac{3}{2}}\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad T^1_0 = \sqrt{\frac{3}{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$T^2_2 = \sqrt{3}\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad T^1_2 = \sqrt{\frac{3}{2}}\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

$$T^2_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T^1_{-1} = -(T^1_1)^\dagger$$
$$T^2_{-2} = -(T^2_2)^\dagger$$
$$T^2_{-1} = -(T^2_1)^\dagger$$

Alternative: Gell-Mann matrices

How?

… which translates into

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + B^1_{L_1} A^1_{L_1 M_1} Y^{M_1}_{L_1}(\Omega_1) + B^2_{L_2} A^2_{L_2 M_2} Y^{M_2}_{L_2}(\Omega_2)$$

$$+ B^1_{L_1} B^2_{L_2} C_{L_1 M_1 L_2 M_2} Y^{M_1}_{L_1}(\Omega_1) Y^{M_2}_{L_2}(\Omega_2)\Big]$$

$$\eta_\ell = \begin{cases} \frac{g_L^2 - g_R^2}{g_L^2 + g_R^2} & Z \\ 1 & W^- \\ -1 & W^+ \end{cases}$$

$$\Omega_1 = (\theta_1, \varphi_1)$$
$$\Omega_2 = (\theta_2, \varphi_2)$$

$$B_1 = -\sqrt{2\pi}\eta_\ell\,, \quad B_2 = \sqrt{\frac{2\pi}{5}}$$

Simpler than it looks because spherical harmonics are orthogonal functions

constants you can calculate

$$\int \frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} Y^{M_1}_{L_1}(\Omega_1)^* Y^{M_2}_{L_2}(\Omega_2)^* = \frac{1}{(4\pi)^2} B_{L_1} B_{L_2} C_{L_1 M_1 L_2 M_2}$$

data follow this distribution

calculate the average of this quantity on your data

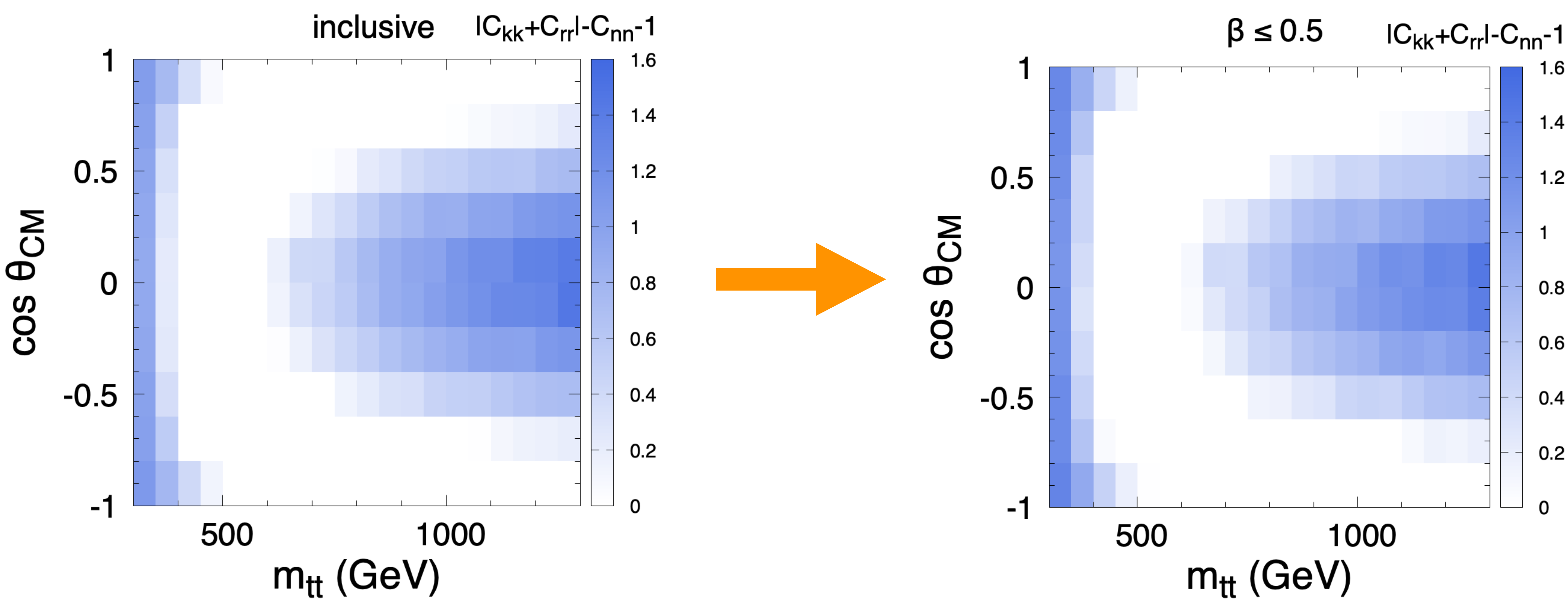
the quantity you want

Not yet measured neither in Higgs decays nor EW diboson production

Top pair entanglement

Improvement: consider events that are more central: upper cut on t t-bar velocity β in LAB frame

JAAS, Casas,2205.00542



- opposite contributions from *qq* and *gg* sub-processes
- the upper cut reduces the *qq* fraction
- can relax upper cut on m_{tt}, reducing systematics

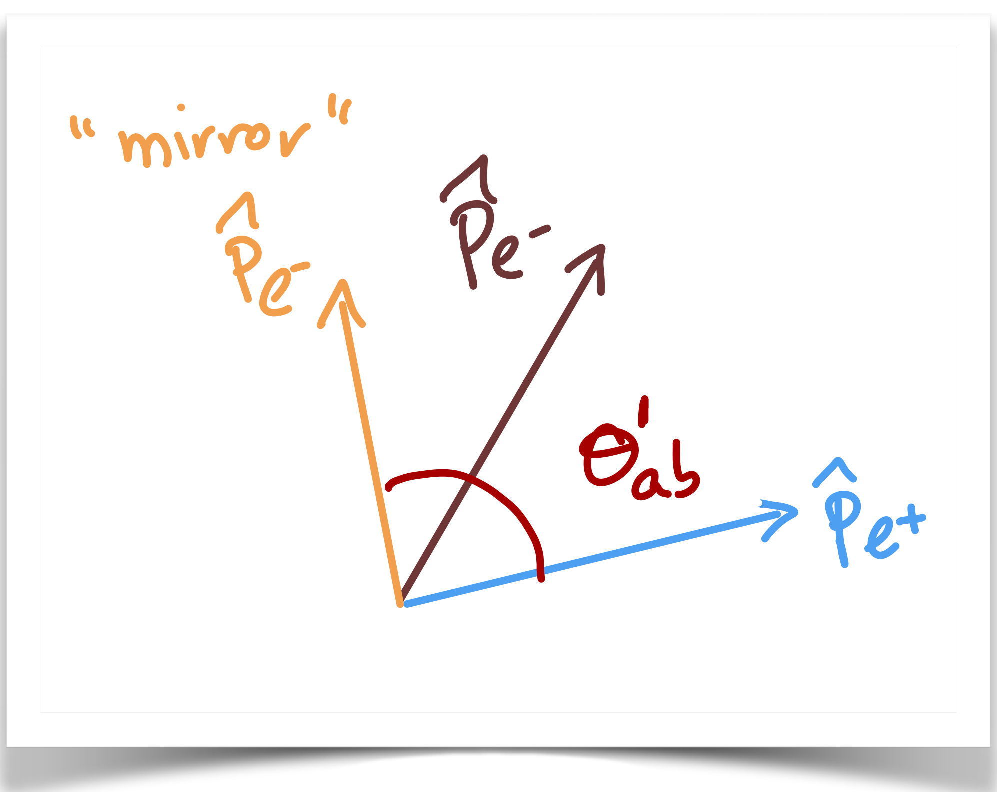
Top pair entanglement

What about the boosted central region?

The relevant quantity to test is $C_{kk} + C_{rr} - C_{nn}$ and there was no specific observable for this combination [one can however measure C’s and sum]

We can design a new observable

JAAS, Casas,2205.00542



Use the mirror image of ℓ^- momentum, reflected in the K-R plane

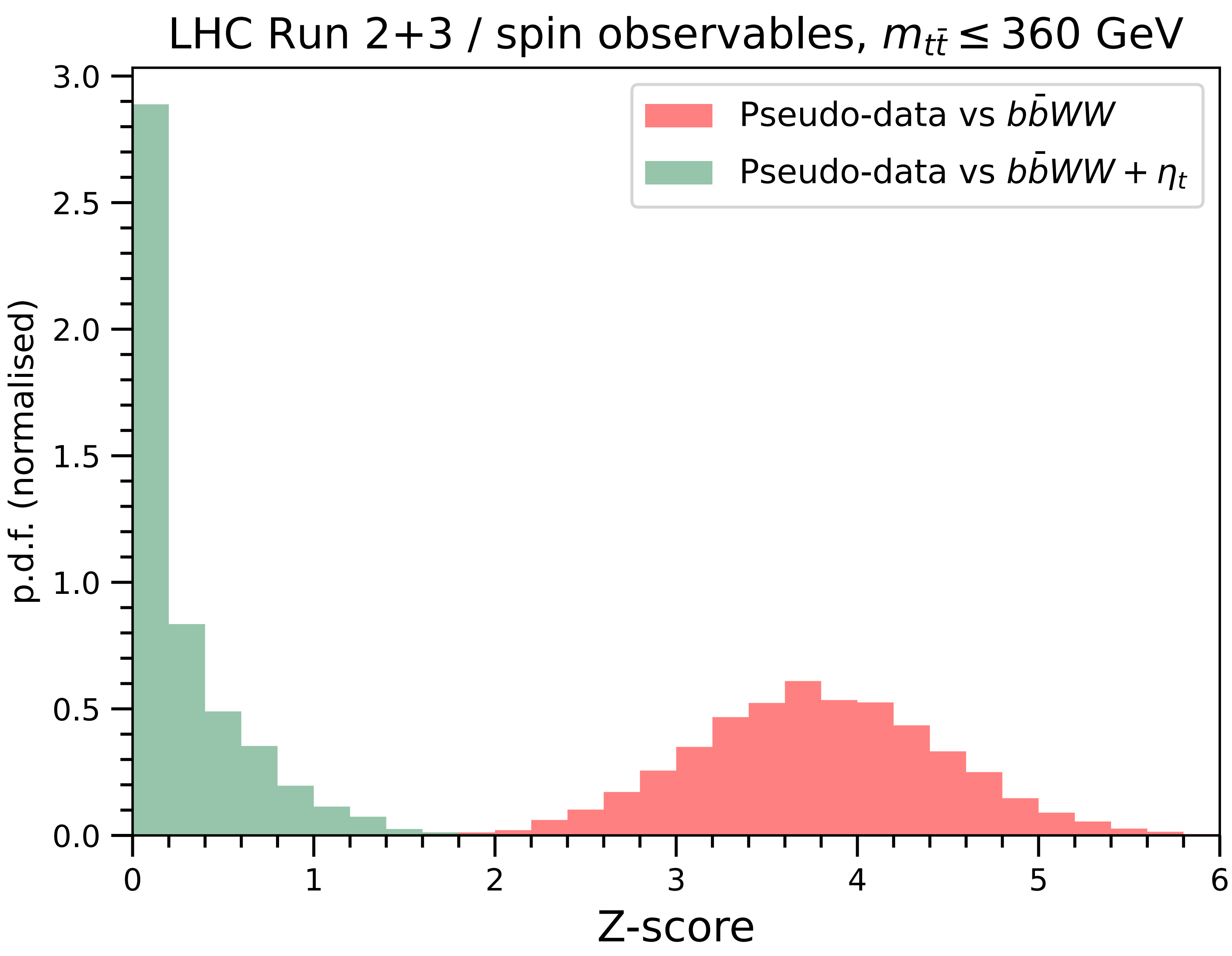
$$\frac{1}{\sigma}\frac{d\sigma}{d\cos\theta'_{ab}} = \frac{1}{2}(1 + \alpha_a\alpha_b D_3 \cos\theta'_{ab})$$

$$D_3 = \frac{1}{3}(C_{11} + C_{22} - C_{33})$$

Entanglement test for boosted region: $3D_3 - 1 > 0$

Toponium!

In order to `establish toponium´ we must observe that data agrees with toponium and disagrees with perturbative QCD



Novel tests: qutrits

$H \to VV$ is a decay $0 \to 1 + 1$. Angular momentum conservation implies that many A and C coefficients are zero. The non-zero ones are

$$A^1_{10} = -A^2_{10}\,, \quad A^1_{20} = A^2_{20}$$
$$C_{1010}\,, \quad C_{2020}\,, \quad C_{1020}\,, \quad C_{2010}$$
$$C_{111-1} = C^*_{1-111}\,, \quad C_{222-2} = C^*_{2-222}\,, \quad C_{212-1} = C^*_{2-121}\,,$$
$$C_{112-1} = C^*_{1-121}\,, \quad C_{211-1} = C^*_{2-111}$$

$$\rho = \frac{1}{9}\left(1_{9\times 9} + A^1_{LM} T^L_M \otimes 1_{3\times 3} + A^2_{LM} 1_{3\times 3} \otimes T^L_M + C_{L_1 M_1 L_2 M_2} T^{L_1}_{M_1} \otimes T^{L_2}_{M_2}\right)$$

and the 9×9 ρ matrix is sparse [relations among coefficients used below]

$$\rho = \frac{1}{3}\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 - C_{2020} & 0 & C_{212-1} & 0 & C_{222-2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & C^*_{212-1} & 0 & -1 + 2C_{2020} & 0 & C_{212-1} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & C^*_{222-2} & 0 & C^*_{212-1} & 0 & 2 - C_{2020} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}$$

Separability → (Peres-Horodecki) $C_{212-1} = 0\,, \quad C_{222-2} = 0$

← ($H \to VV$ special case)

Novel tests: qutrits

Prospects for $H \to ZZ \to 4\ell$ JAAS, Bernal, Casas,Moreno, 2209.13441

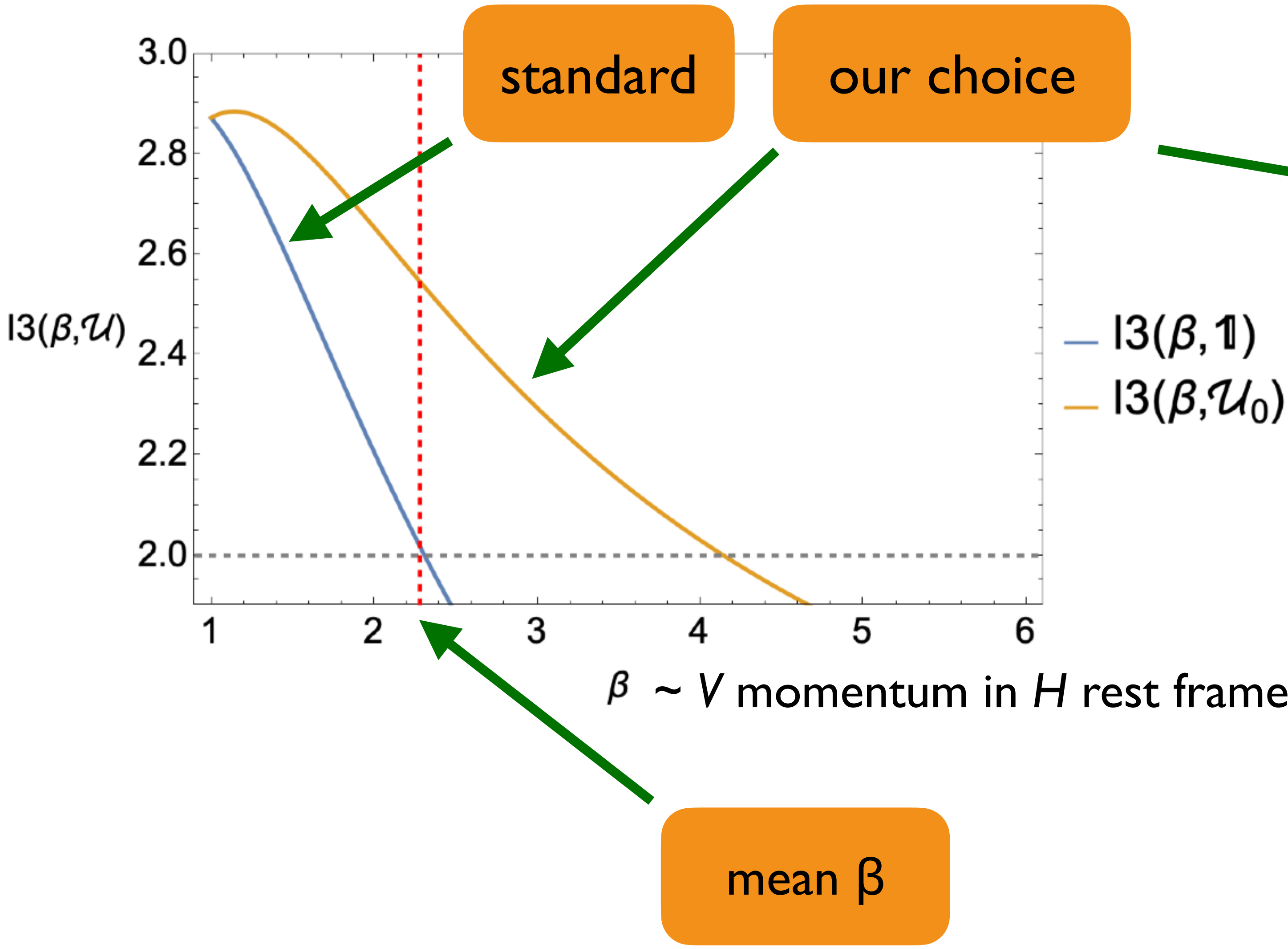
- Parton level, no detector simulation, approximate eff [0.25] injected
- Background not included [1/4 size of signal]
- Only statistical uncertainties, estimated with pseudo-experiments

	$C_{212\text{-}1}$	$C_{222\text{-}2}$	Significance
Run 2 + 3 : 300 fb^{-1}	-0.98 ± 0.31	0.60 ± 0.37	3σ
HL-LHC : 3 ab^{-1}	-0.95 ± 0.10	0.60 ± 0.12	many σ

Novel tests: qutrits

We saw earlier that for a spin singlet there is a `standard´ Bell operator that is believed to be optimal. But this is not the case for $H \rightarrow VV$

[V not at rest in H rest frame]



$$I_3 = \frac{1}{36}\Big[\Big(18+16\sqrt{3}\Big) - \sqrt{2}\Big(9-8\sqrt{3}\Big)\,A^1_{2,0} - 8\Big(3+2\sqrt{3}\Big)\,C_{2,1,2,-1} + 6\,C_{2,2,2,-2}\Big]$$

	I_3	Significance
Run 2 + 3 : 300 fb^{-1}	2.66 ± 0.46	1.4σ
HL-LHC : 3 ab^{-1}	2.63 ± 0.15	4.2σ

Novel tests: qutrits

The *ZZ* final state is clean and easy to reconstruct…

… but the *WW* final state is clearly superior in terms of both statistics and spin analysing power

$$\frac{1}{\sigma}\frac{d\sigma}{d\Omega_1 d\Omega_2} = \frac{1}{(4\pi)^2}\Big[1 + B^1_{L_1} A^1_{L_1 M_1} Y^{M_1}_{L_1}(\Omega_1) + B^2_{L_2} A^2_{L_2 M_2} Y^{M_2}_{L_2}(\Omega_2) + B^1_{L_1} B^2_{L_2} C_{L_1 M_1 L_2 M_2} Y^{M_1}_{L_1}(\Omega_1) Y^{M_2}_{L_2}(\Omega_2)\Big]$$

$$B_1 = -\sqrt{2\pi}\,\eta_\ell \quad \longrightarrow \quad \eta_\ell = \pm 1\ (W)\ ;\ 0.13\ (Z)$$

- Coefficients A_{lM}, $C_{lM2M'}$ have a suppression 1/10 for *Z* → Δ_{stat} 3x penalty
- Coefficients $C_{lMlM'}$ have a suppression 1/100 for *ZZ* → Δ_{stat} 10x penalty

Efforts needed towards realistic reconstruction methods for *WW*!

Novel tests: qutrits

Full reconstruction of $H \to WW \to \ell\nu qq$ possible by using *c*-tagging to distinguish jets

Fabbri, Howarth, Maurin, 2307.13783

Penalties of full reconstruction:

- 1/2 BR because $W \to ud$ is not usable
- 1/2 BR because $W \to cs$ is assumed on shell, $W \to \ell\nu$ off shell
- 0.4 efficiency for charm tagging

to reduce bkg

Still 20% more statistics than $WW \to 2\ell\, 2\nu$

- Detector simulation and unfolding
- Background included
- Only statistical uncertainties

standard operator [could be better]

	Entanglement	Bell inequalities
Run 2 : 139 fb^{-1}	?	1.8σ
Run 2 + 3 : 300 fb^{-1}	??	2.7σ
HL-LHC : 3 ab^{-1}	???	many σ

Novel tests: qutrits

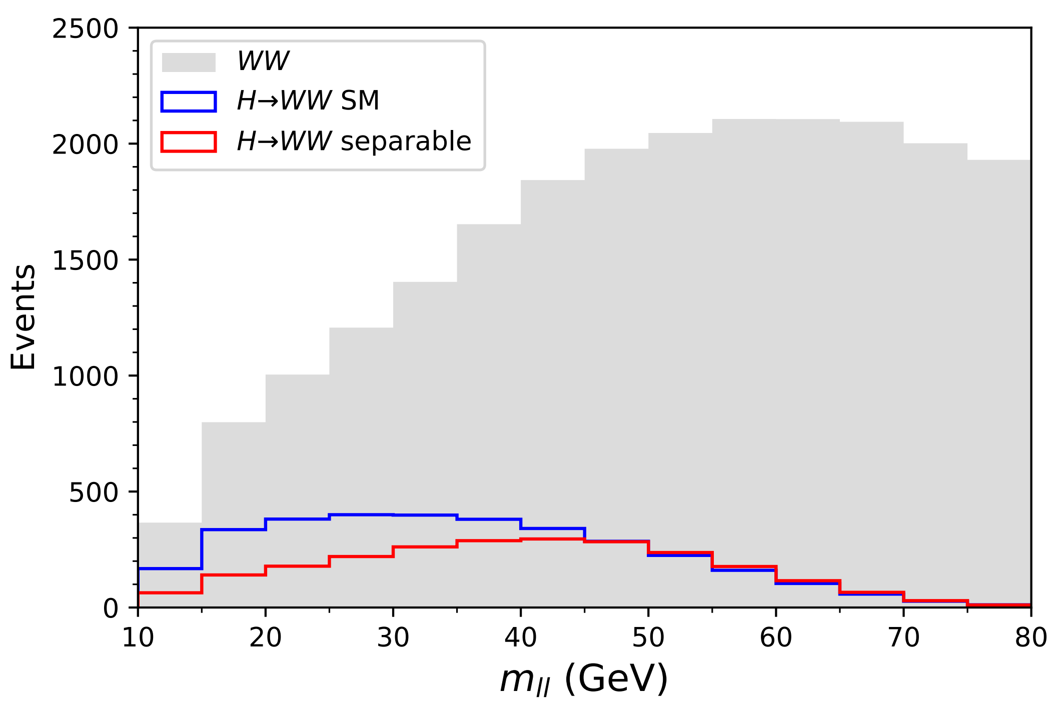
For $H \to WW \to 2\ell\, 2\nu$, entanglement conditions can be recast into a binary test using lab-frame dilepton kinematical distributions. JAAS, 2209.14033

SM vs separability hypothesis

$$\equiv \begin{aligned} C_{212-1} &= 0 \\ C_{222-2} &= 0 \end{aligned}$$

$$\hat{h}_0 \hat{h}_-^* = h_{16} + i\,(h_{17} - h_{26}) + h_{27}$$

$$\hat{h}_+ \hat{h}_-^* = h_{44} + i\,(h_{45} - h_{54}) + h_{55}$$



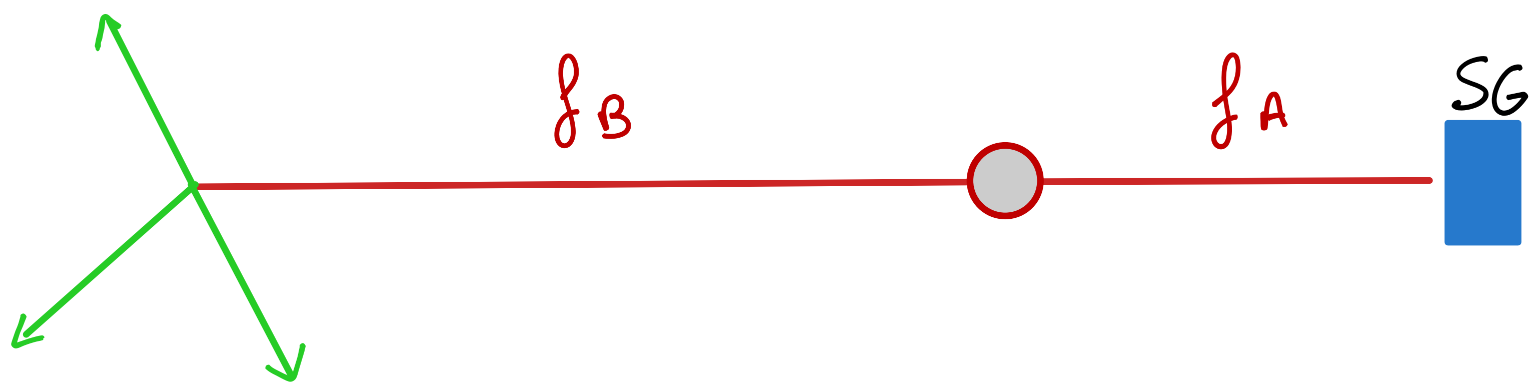
	Run 2 Significance
stat only	7.1σ
stat + modeling syst	6.1σ

A post-selection experiment

Assume fermion pairs f_A f_B produced in an entangled state, say

$$\frac{1}{\sqrt{2}}\left[\,|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\,\right]$$

We perform a Stern-Gerlach experiment on f_A, and after that, f_B decays



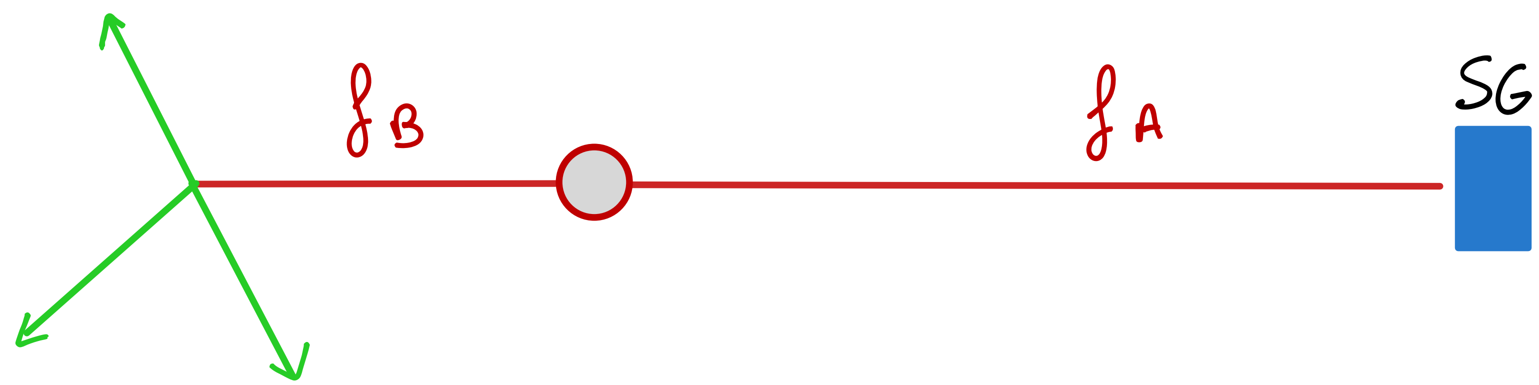
We select the subset of f_B for which the result of the SG experiment on f_A gives $|\uparrow\rangle$

Then, the decay distribution of those pre-selected f_B corresponds to having spin $|\downarrow\rangle$

A post-selection experiment

Remarkably, the same happens time-backwards:

f_B decays and after that, we perform a Stern-Gerlach experiment on f_A



We select the subset of f_B for which the result of the SG experiment on f_A gives $|\uparrow\rangle$

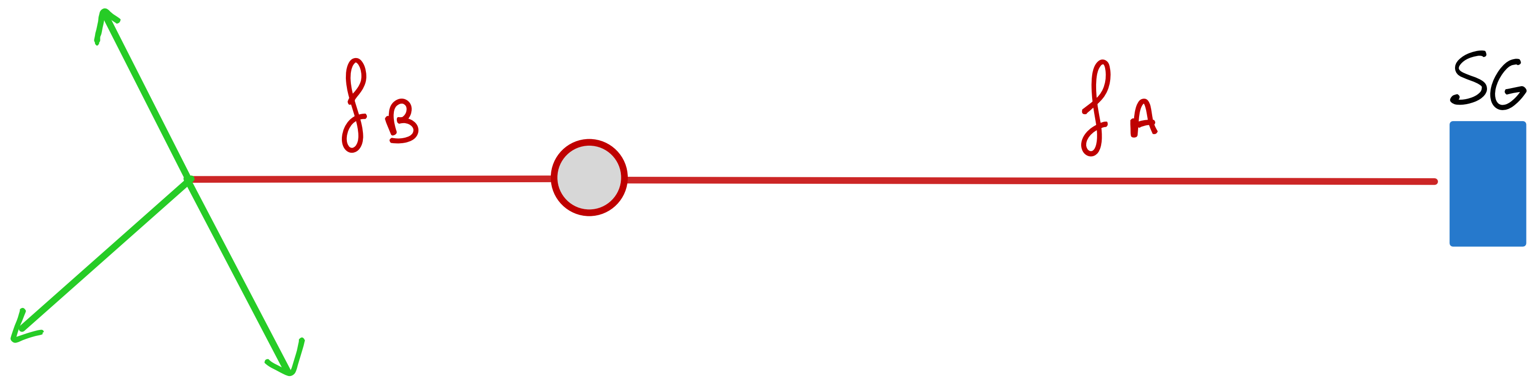
Then, the decay distribution of those f_B that had decayed before the outcome of the SG experiment corresponds to having spin $|\downarrow\rangle$

Magic? Spooky EPR action to the past? Not really. It is due to the projection.

A post-selection experiment

The initial state is $\frac{1}{\sqrt{2}}\left[\,|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\,\right]$ and if we do a SG on f_A before f_B decays, we get up or down with equal probability.

The decay of f_B projects f_A into a state $a_+|\uparrow\rangle + a_-|\downarrow\rangle$ with a_+, a_- depending on the decay configuration. The probability to have SG up or down is not the same.



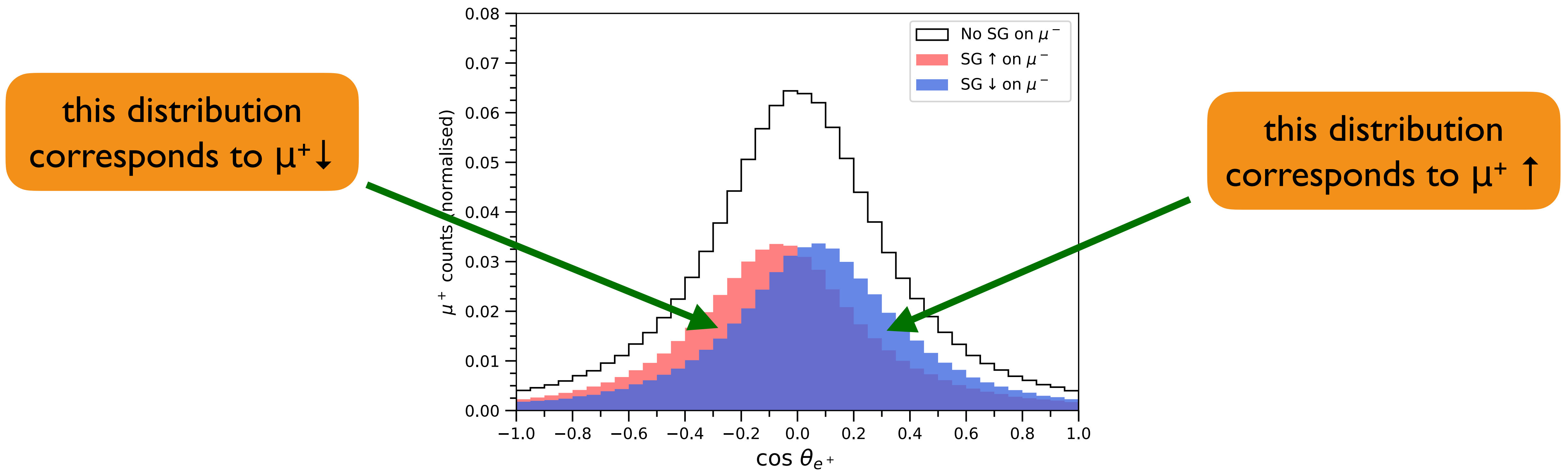
Because of this projection, if we post-select events where SG gives $|\uparrow\rangle$, we recover f_B decay distributions just as if f_B had spin $|\downarrow\rangle$ when it decayed.

This is a genuine entanglement effect. We can set our SG in any direction and even violate Bell inequalities.

A post-selection experiment

This experiment can be performed with low-energy $\mu^+\mu^-$ pairs produced in Drell-Yan or from the decay of a η meson

The muon polarisation can be measured from the daughter electron



Related: neutral kaon post-tag [Bernabéu, di Domenico 1912.04798] but the correlation presented [# decays vs time] does not seem a genuine quantum correlation <u>in my opinion</u> [the discussion is complicated] now this is the end