

Two-loop mixed QCD-EW corrections to charged-current Drell-Yan

In collaboration with R. Bonciani, S. Devoto, N. Rana, A. Vicini



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35th Rencontres de Blois - Blois (France) - 22nd October 2024

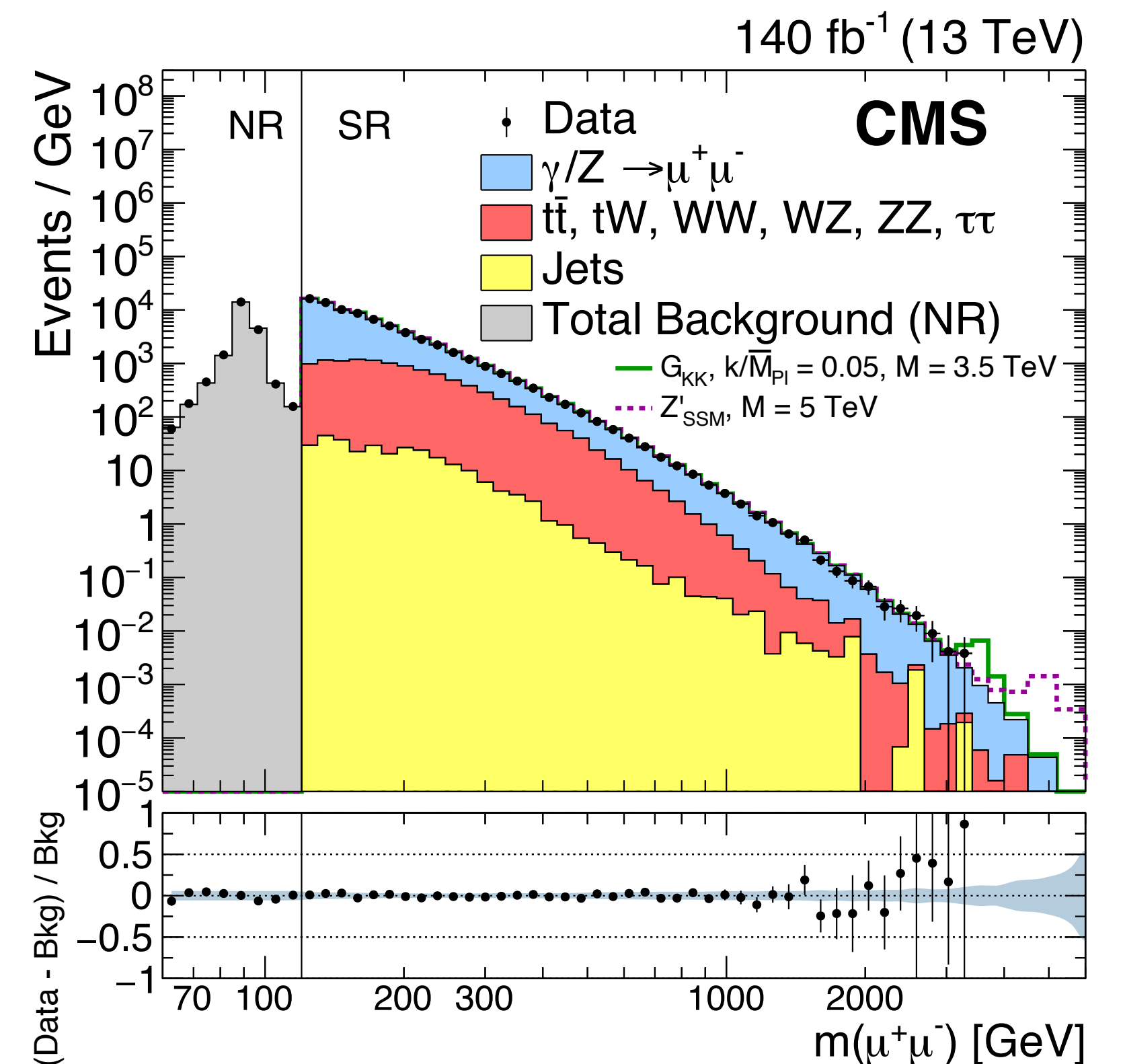
Motivations

- ▶ The **inclusive production of a fermion pair** is a standard candle process at LHC;
- ▶ At HL-LHC the **statistical errors** will reach $\mathcal{O}(0.5\%)$ across the entire invariant mass range

Bin range (GeV)	% error 140 fb ⁻¹	% error 3 ab ⁻¹
91-92	0.03	6×10^{-3}
120-400	0.1	0.02
400-600	0.6	0.13
600-900	1.4	0.30
900-1300	3.2	0.69

arXiv:2106.11953

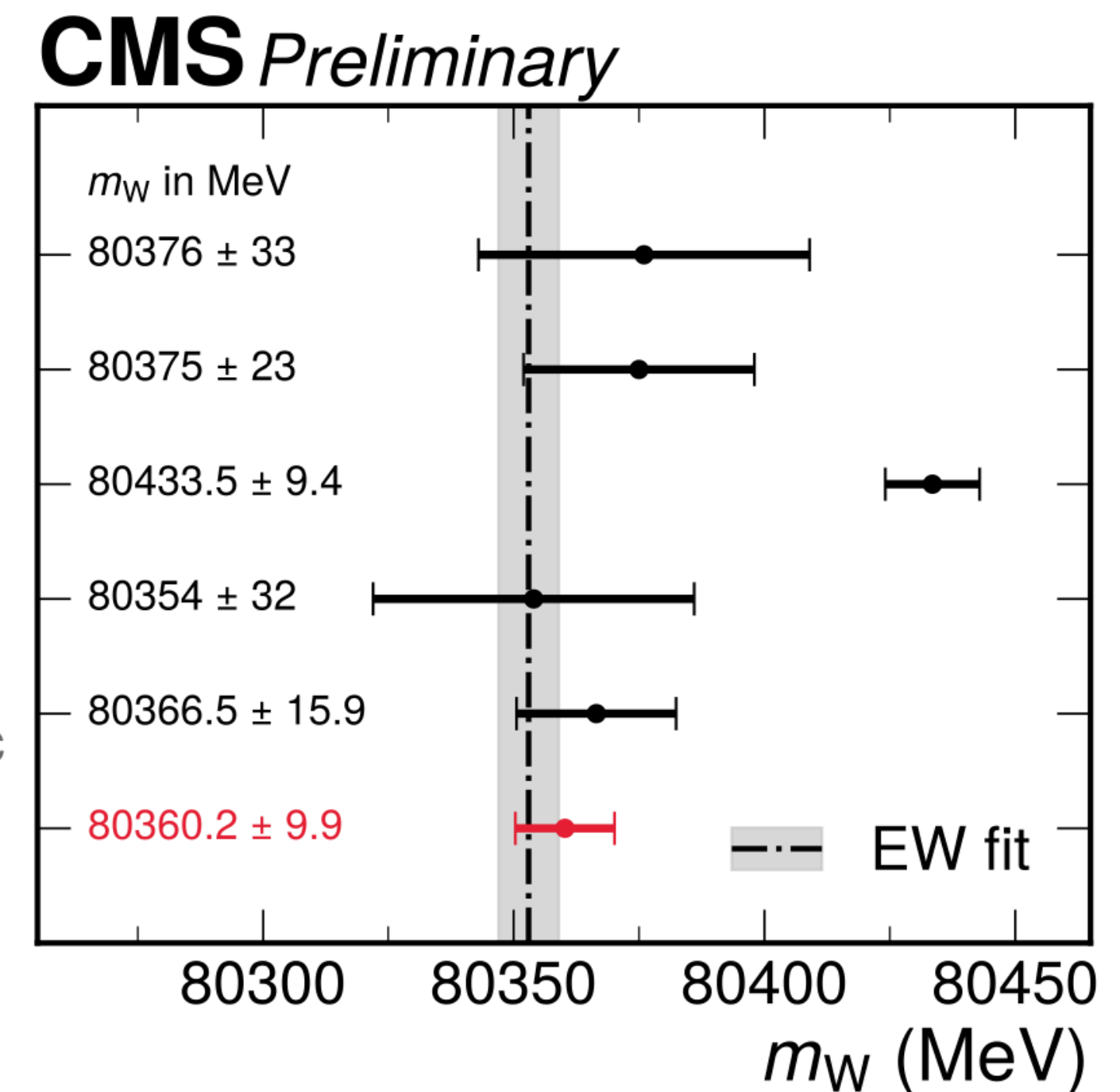
- ▶ The **theory systematics** (e.g. input scheme, PDFs, EW corrections, ...) must be kept under control at this level of precision.



Motivations

- ▶ High precision predictions for the Drell-Yan are important for the m_W **measurement**;
- ▶ Neutral-current result for **calibration** purposes;
- ▶ Charged-current for the actual measurement⁺.

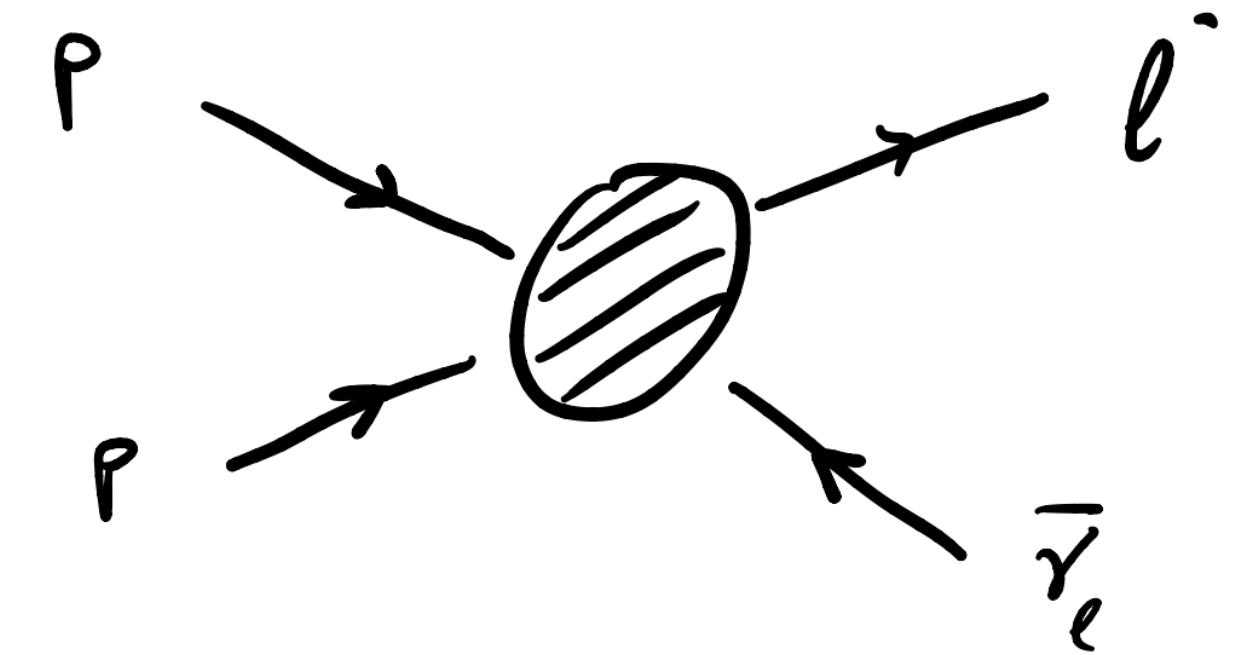
LEP combination
Phys. Rep. 532 (2013) 119
D0
PRL 108 (2012) 151804
CDF
Science 376 (2022) 6589
LHCb
JHEP 01 (2022) 036
ATLAS
arxiv:2403.15085, subm. to EPJC
CMS
This Work



- ▶ The **computational challenges** are similar to the ones for FCC-ee.

Higher orders

$$\begin{aligned}
 \sigma_{ij} = & \sigma_{ij}^{(0,0)} \\
 & + \alpha_s \sigma_{ij}^{(1,0)} + \alpha \sigma_{ij}^{(0,1)} + \\
 & + \alpha_s^2 \sigma_{ij}^{(2,0)} + \alpha_s \alpha \sigma_{ij}^{(1,1)} + \alpha^2 \sigma_{ij}^{(0,2)} + \\
 & + \alpha_s^3 \sigma_{ij}^{(3,0)} + \dots
 \end{aligned}$$



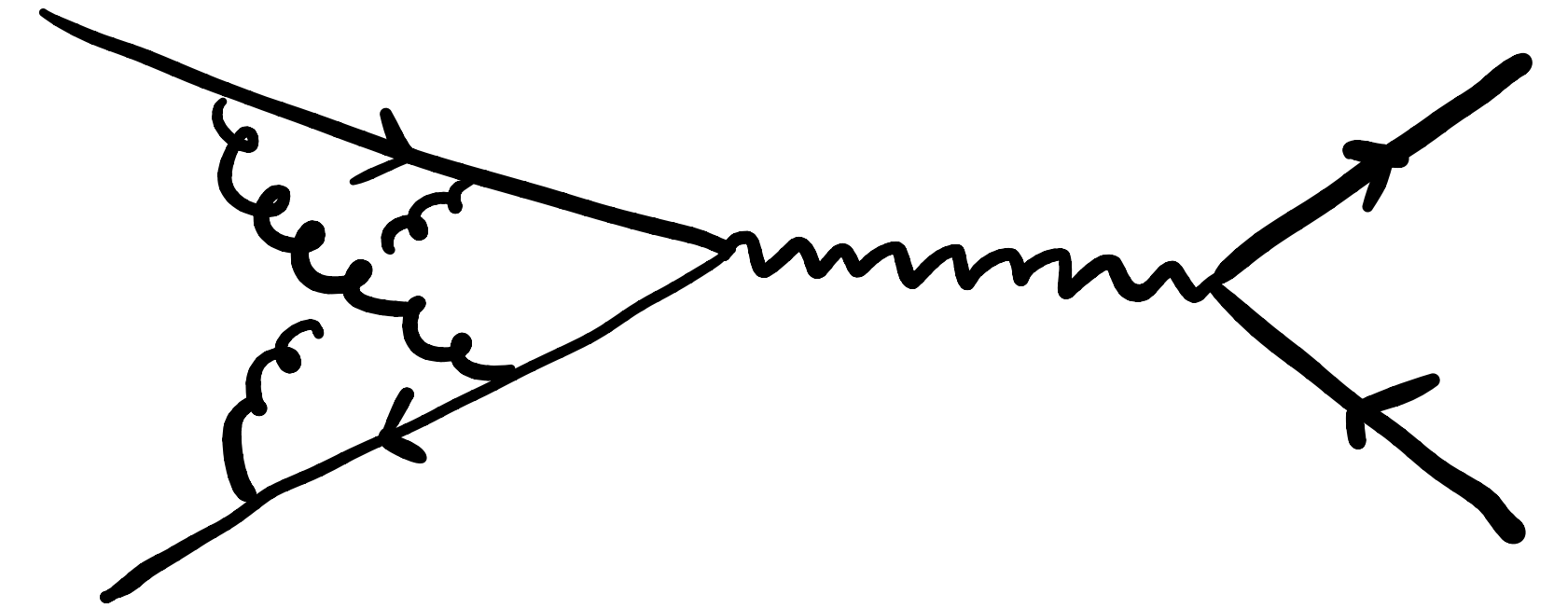
$$\sigma_{tot} = \sum_{i,j \in q, \bar{q}, g, \gamma} \int_0^1 dx_1 dx_2 f_i(x_1, \mu_F) f_j(x_1, \mu_F) \sigma_{ij}(\mu_F, \mu_R)$$

Parton Distribution Functions

Partonic cross-section

Higher orders

$$\begin{aligned}\sigma_{ij} = & \sigma_{ij}^{(0,0)} \\ & + \alpha_s \sigma_{ij}^{(1,0)} + \alpha \sigma_{ij}^{(0,1)} + \\ & + \alpha_s^2 \sigma_{ij}^{(2,0)} + \alpha_s \alpha \sigma_{ij}^{(1,1)} + \alpha^2 \sigma_{ij}^{(0,2)} + \\ & + \alpha_s^3 \sigma_{ij}^{(3,0)} + \dots\end{aligned}$$



QCD Corrections

NLO:

[G.Altarelli, R.Ellis, G.Martinelli Nucl.Phys.B 157 (1979)];

NNLO:

[R.Hamberg, T.Matsuura, W.van Nerveen, Nucl. Phys. B 359 (1991)];

[C.Anastasiou, L.J.Dixon, K.Melnikov, F.Petriello, hep-ph:0306192];

[S.Catani, L.Cieri, G.Ferrera, D.de Florian, M.Grazzini
arXiv:0903.2120];

N3LO:

[C.Duhr, F.Dulat, B.Mistlberger arXiv:2007.13313];

[X.Chen, T.Gehrmann, N.Glover, A.Huss, T.Yang, and H.Zhu
arXiv:2107.09085];

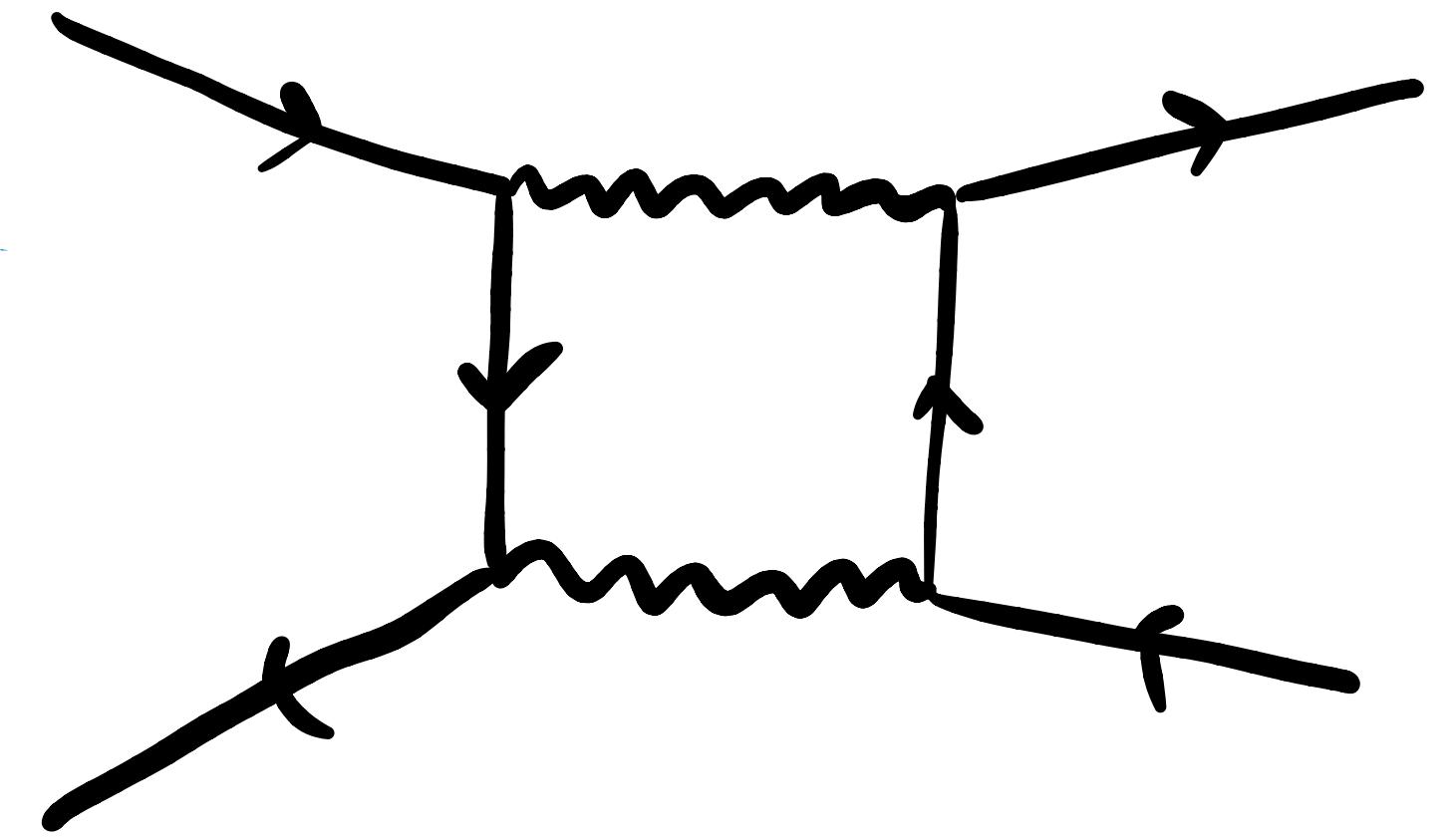
[S.Camarda, L.Cieri, G.Ferrera arXiv:2103.04974];

[X.Chen, T.Gehrmann, N.Glover, A.Huss, P.Monni, E.Re, L.Rottoli,
P.Torrielli arXiv:2203.01565];

[T.Neumann, J.Campbell arXiv:2207.07056]

Higher orders

$$\begin{aligned}\sigma_{ij} = & \sigma_{ij}^{(0,0)} \\ & + \alpha_s \sigma_{ij}^{(1,0)} + \alpha \sigma_{ij}^{(0,1)} + \\ & + \alpha_s^2 \sigma_{ij}^{(2,0)} + \alpha_s \alpha \sigma_{ij}^{(1,1)} + \alpha^2 \sigma_{ij}^{(0,2)} + \\ & + \alpha_s^3 \sigma_{ij}^{(3,0)} + \dots\end{aligned}$$



EW Corrections

NLO:

[U.Baur, O.Brein, W.Hollik, C.Schappacher, D.Wackeroth, hep-ph:0108274];

[S.Dittmaier, M.Kramer, hep-ph:0109062];

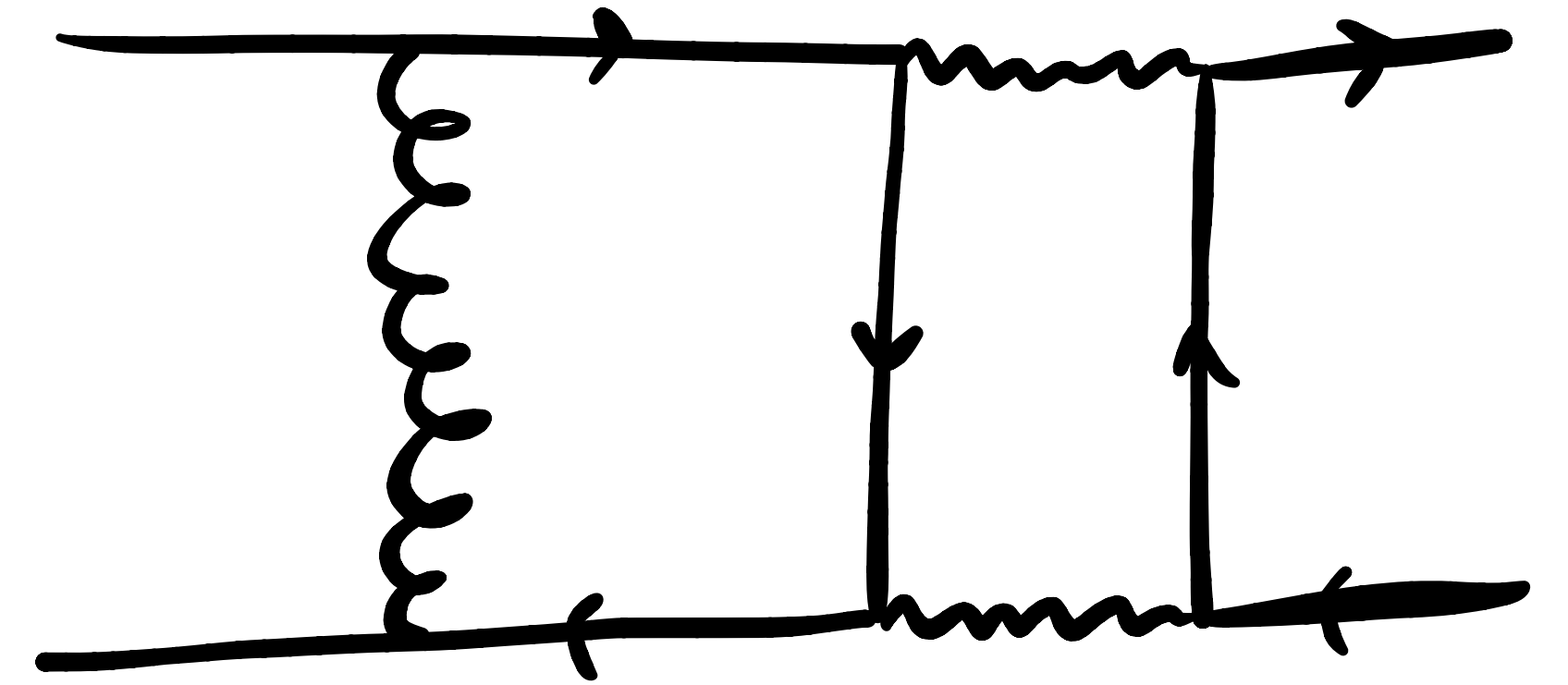
[U.Baur, D.Wackeroth, hep-ph:0405191];

NNLO (Sudakov approximation):

[B. Jantzen, J.H.Kühn, A.A.Penin, V.A.Smirnov, hep-ph:0509157];

Higher orders

$$\begin{aligned}\sigma_{ij} = & \sigma_{ij}^{(0,0)} \\ & + \alpha_s \sigma_{ij}^{(1,0)} + \alpha \sigma_{ij}^{(0,1)} + \\ & + \alpha_s^2 \sigma_{ij}^{(2,0)} + \alpha_s \alpha \sigma_{ij}^{(1,1)} + \alpha^2 \sigma_{ij}^{(0,2)} + \\ & + \alpha_s^3 \sigma_{ij}^{(3,0)} + \dots\end{aligned}$$



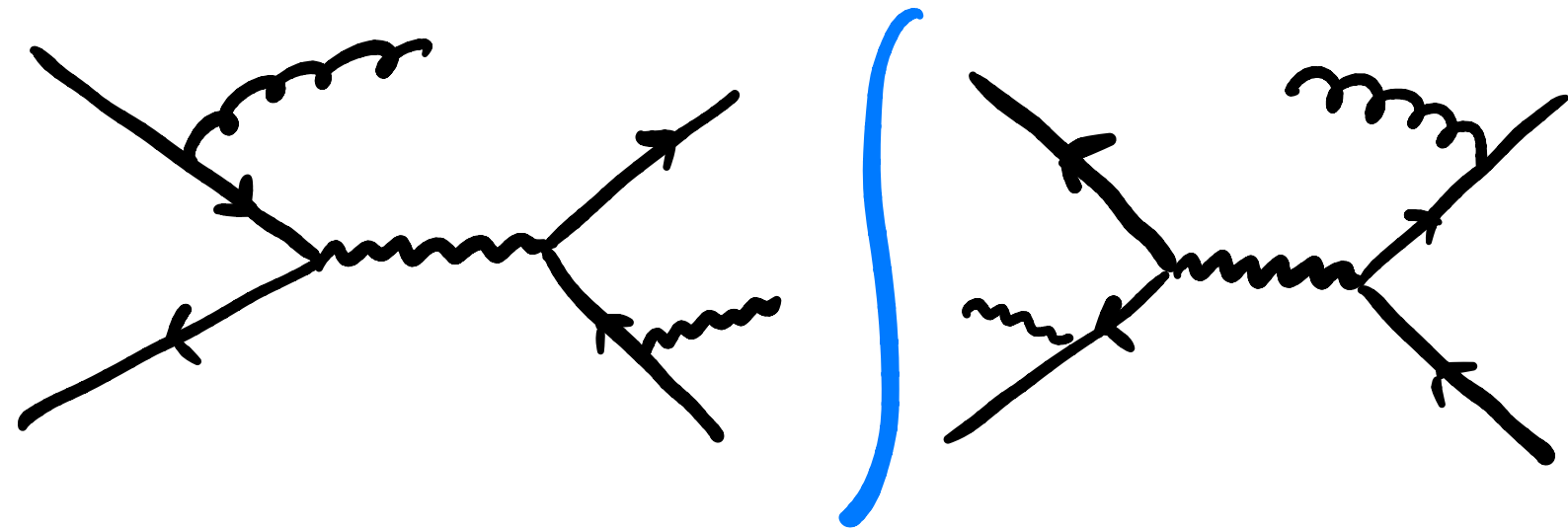
Mixed corrections

- ▶ Naively they have **similar magnitude of N3LO QCD**: $\alpha_s^3 \simeq \alpha_s \alpha$;
- ▶ In specific phase-space points, fixed order EW corrections can become **very large because of logarithmic enhancement** (weak and QED Sudakov type);
- ▶ They reduce the **input scheme dependence**.

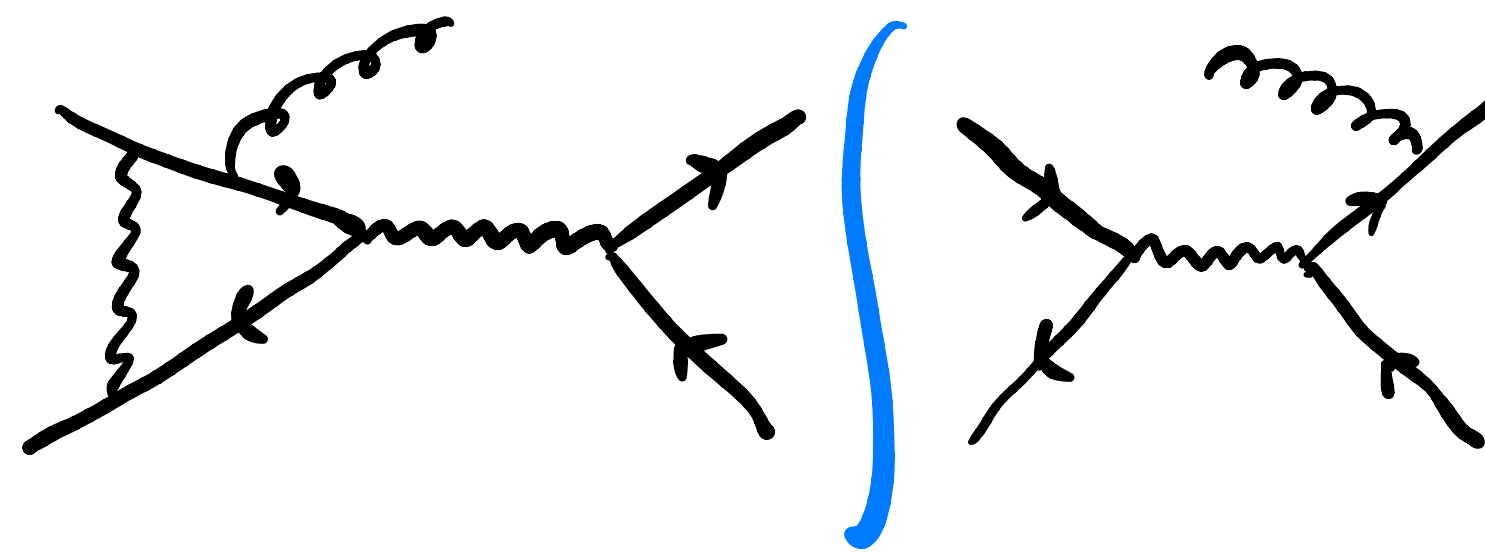
Extremely important for high precision phenomenology (per-cent and sub per-cent level)

Mixed QCD-EW corrections

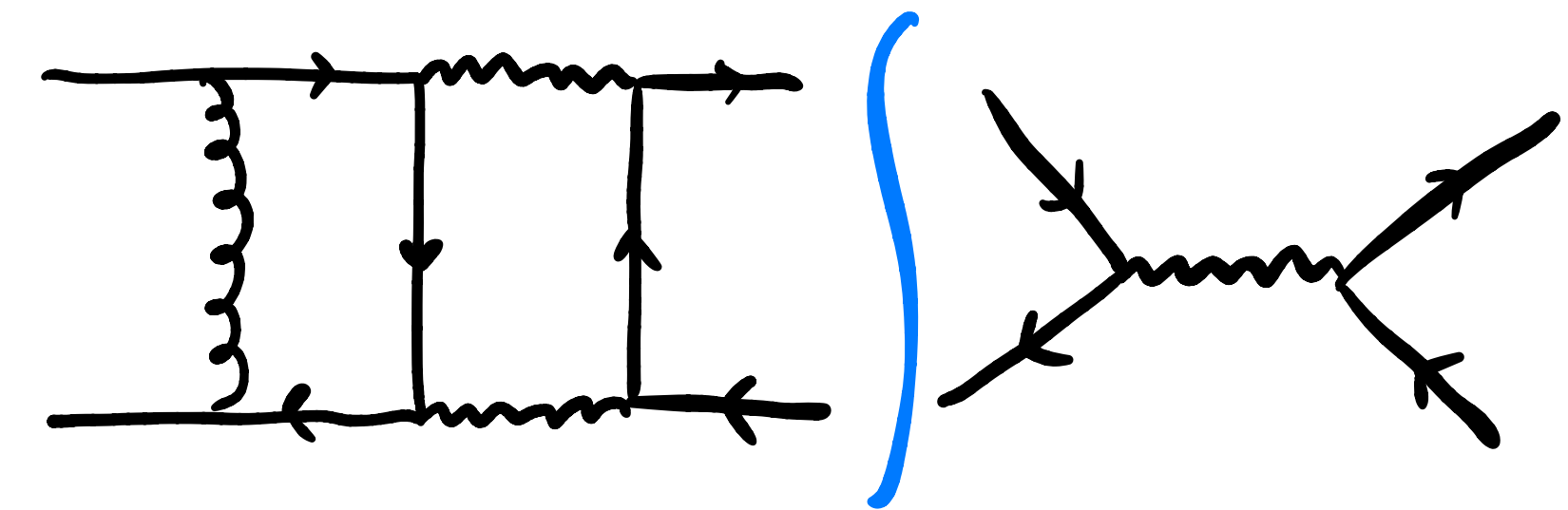
Double Real



Real-Virtual



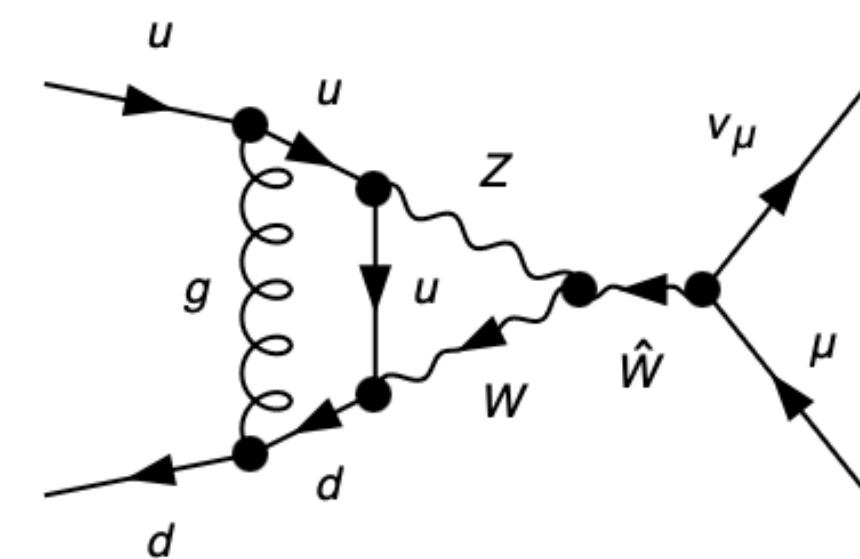
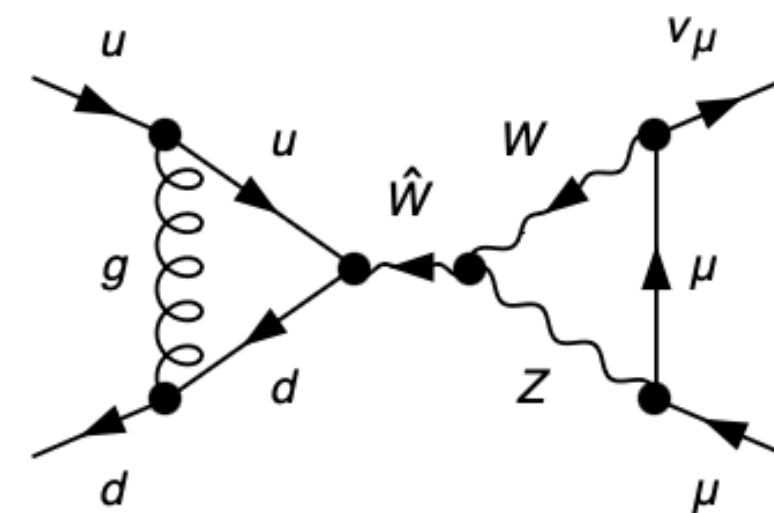
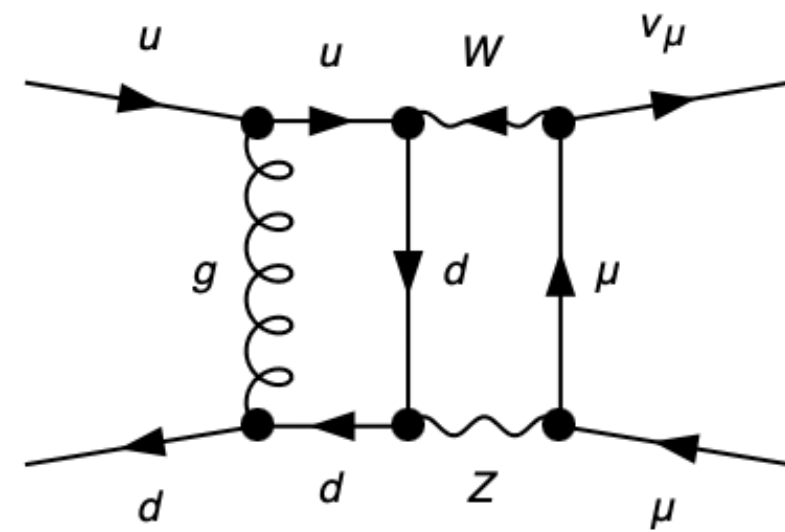
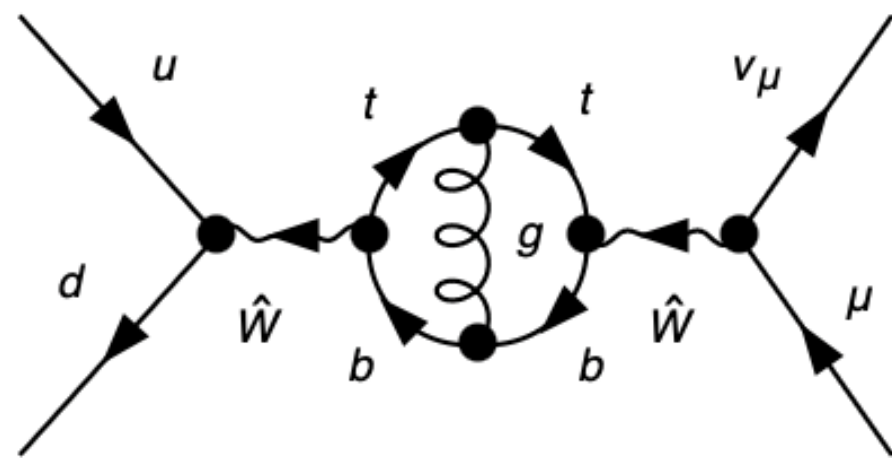
Pure Virtual



- ▶ Each of the three pieces carries its own challenges;
- ▶ The **pure virtual contributions are usually the main bottleneck**;
- ▶ Each individual contribution is divergent in the dimensional regulator ϵ .

The 2L amplitude

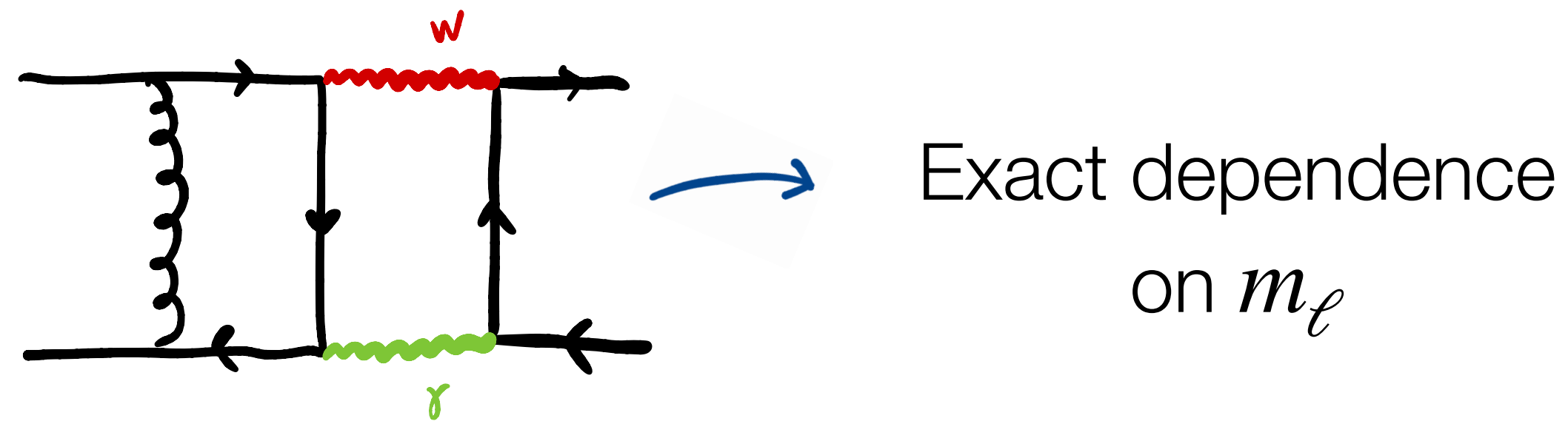
- ▶ The diagrams are generated using **FeynArts**;



- ▶ The computation of the interference terms between the 2L diagrams and the born has been done with in-house **Mathematica** routines;
- ▶ We treated γ^5 in d dimensions using the **naive anti commuting scheme**;

IR subtraction

- ▶ IR singularities are handled by the **qT-subtraction formalism**;
- ▶ The qT-subtraction requires the final state emitters (leptons) to be **massive**! I.e. that the final state collinear divergences are regularised by $\log(m_\ell^2/s)$;
- ▶ However, retaining the exact dependance on the lepton mass is extremely challenging. For this reason, we kept the lepton mass only when the photon originates IR divergences;



We are introducing a **mismatch** $\mathcal{O}(m_\ell^2/s)$



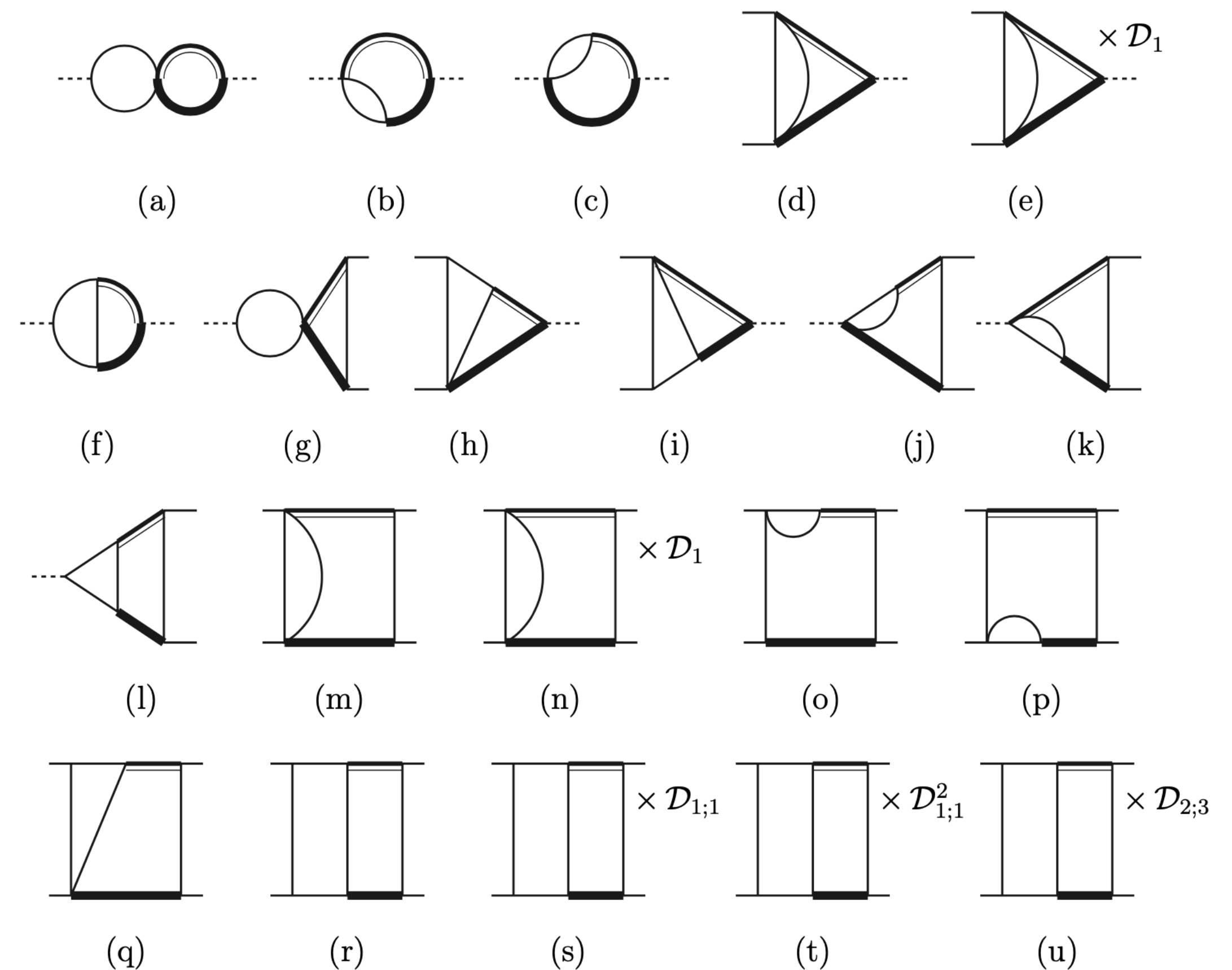
Reduction to Master Integrals

- ▶ We identified **11 integral families** with either 0, 1 or 2 masses. We reduced them to Master Integrals using **Kira** in combination with **Firefly**. The complete reduction took $\mathcal{O}(16h)$.

- ▶ We ended up with **274 masters integrals** to evaluate.

- ▶ The most complicated topology was a **two-loop box with two internal different masses**;

- ▶ We evaluated all the masters using the method of **differential equations**, using a **semi-analytical approach**.



SeaSyde

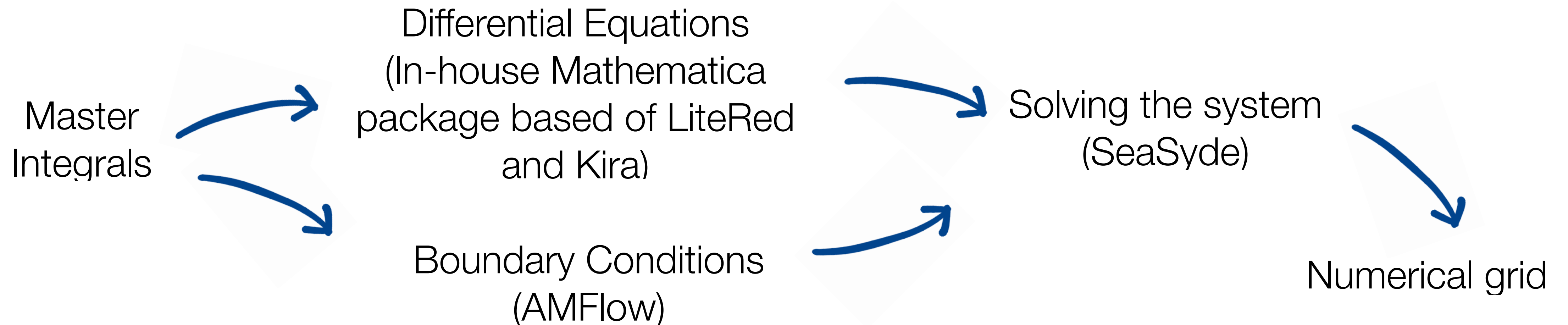
[TA, R. Bonciani, S. Devoto, N.Rana,
A.Vicini, arXiv:2205.03345]

<https://github.com/TommasoArmadillo/SeaSyde>



- ▶ **SeaSyde** (**S**eries **E**xpansion **A**pproach for **S**ystems of **D**ifferential **E**quations) is a general package for solving a system of differential equations using the series expansion approach;
- ▶ Seasyde can handle **complex kinematic variables** by introducing an original algorithm for the analytic continuation in the complex plane, thus being able to handle **complex internal masses**;
- ▶ **SeaSyde** can deal with arbitrary system of differential equations, covering also the case of **elliptic integrals**.

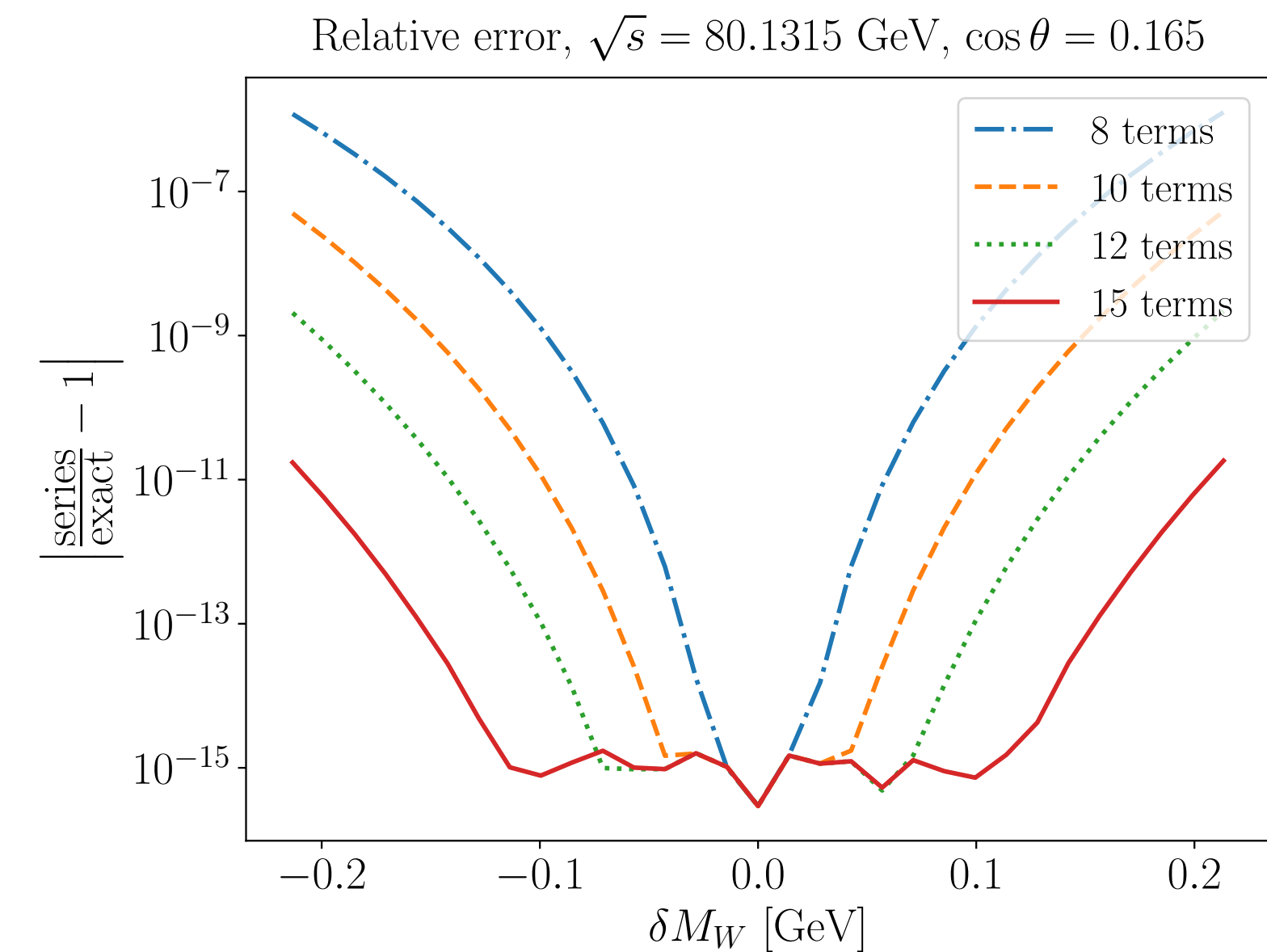
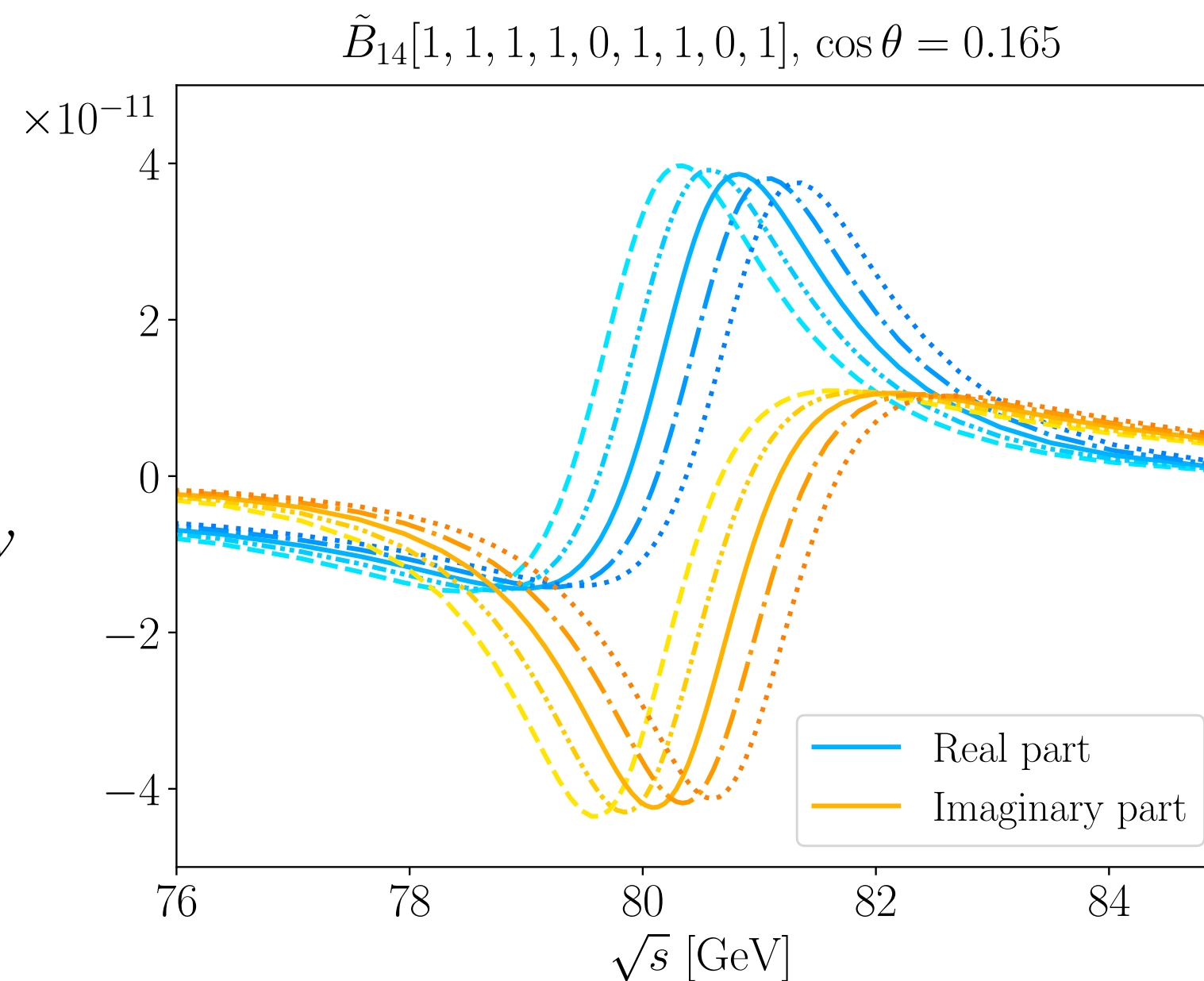
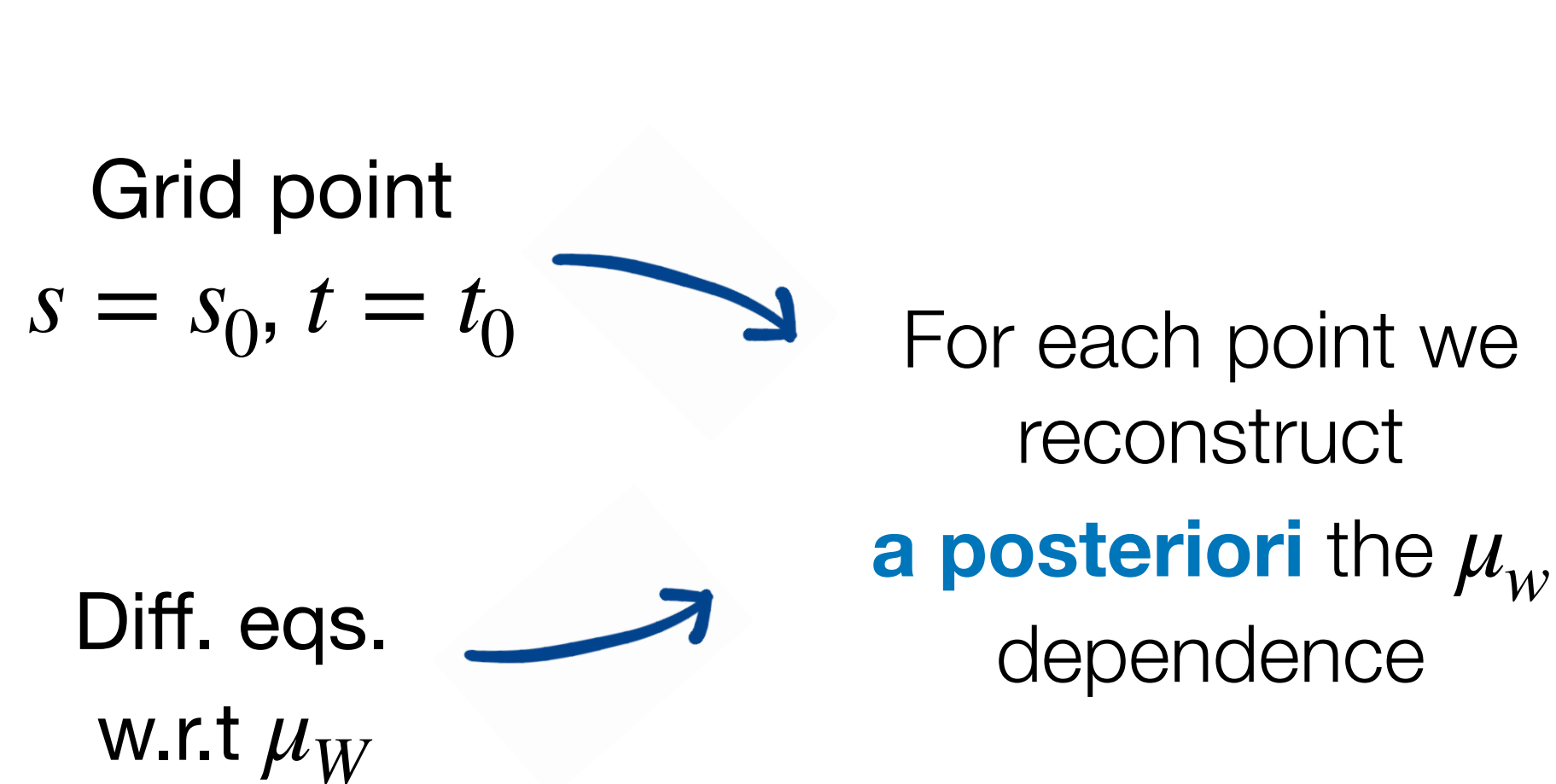
Creating a grid



- ▶ The computation of a **grid with 3250 points** for the two-loop box with two internal and different masses (56 equations) required ~ 3 weeks on 26 cores.
- ▶ This approach is **completely general and easy to automate**, and can be applied, in principle, to **any integral family**.

The expansion in $\delta\mu_W$

- ▶ In m_W **determination studies** we need $\mathcal{O}(10^2)$ templates with different values of μ_W . If we need 3 weeks for a single grid, this is **not feasible**.



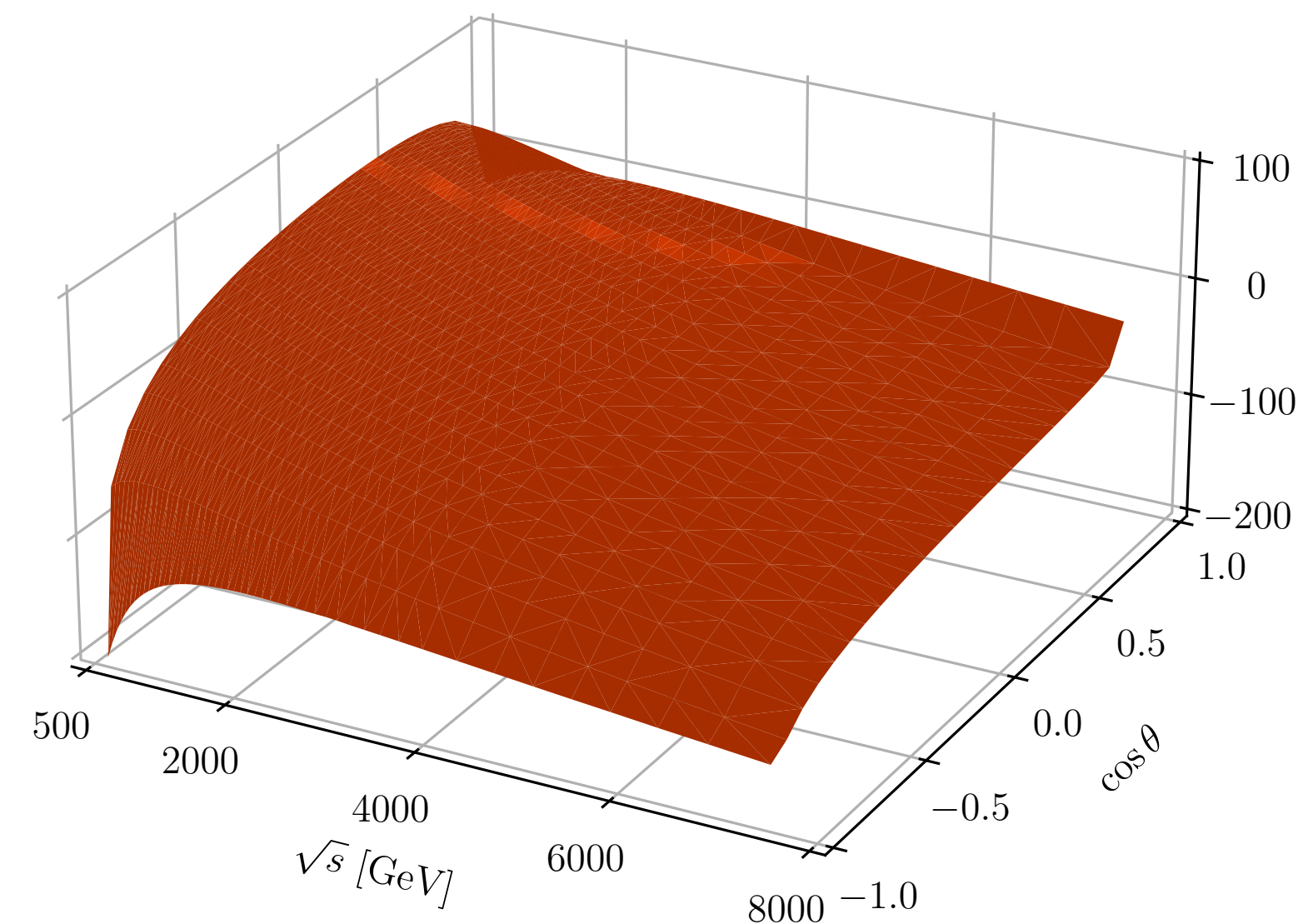
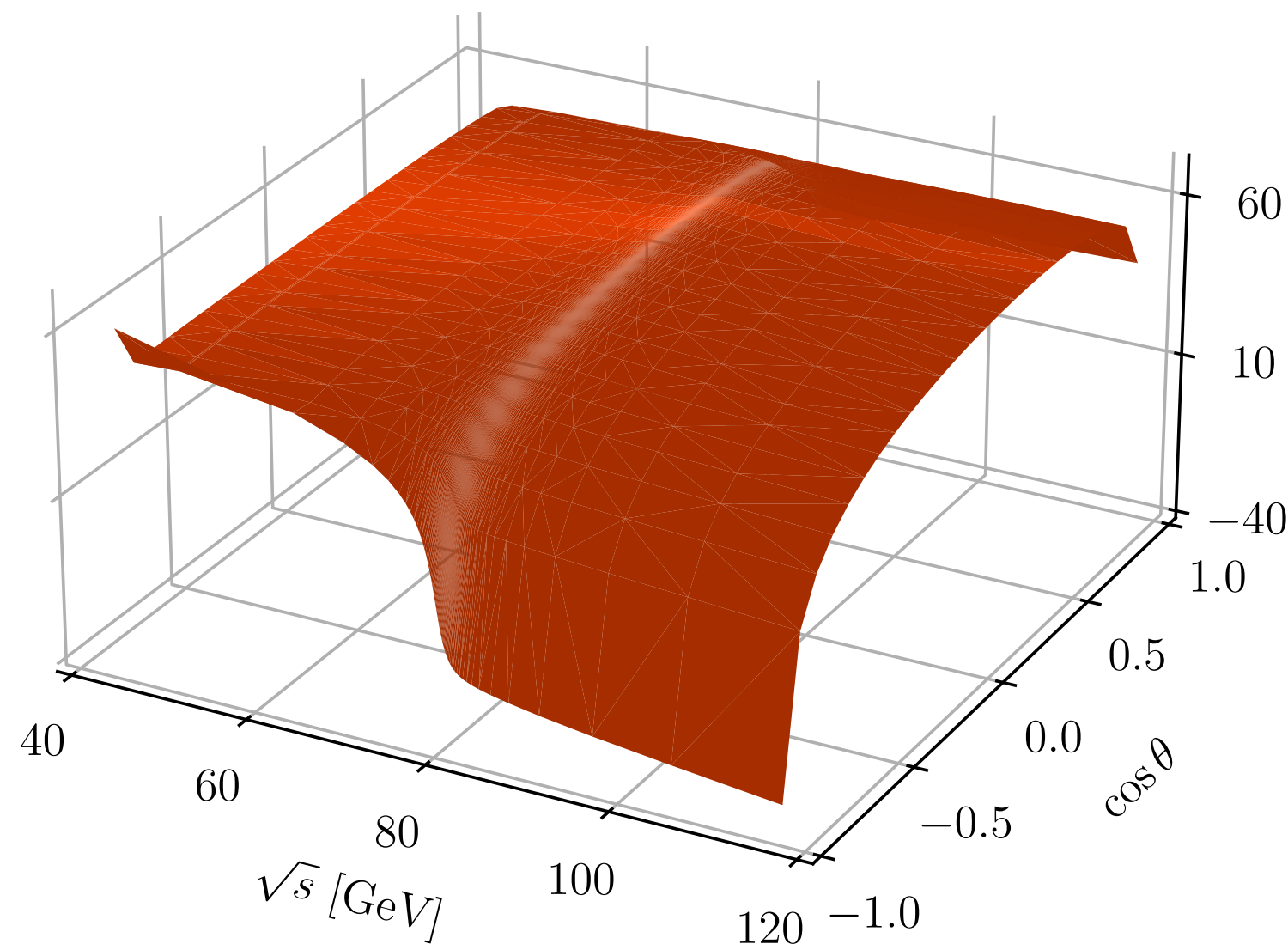
- ▶ Every point of our grid becomes a **series expansion** in $\delta\mu_W = \mu_W - \bar{\mu}_W$, which can be evaluated in a negligible amount of time for arbitrary (but reasonable) values of the W mass;
- ▶ The calculation of the $\delta\mu_W$ expansion for the entire grid took ~ 1.5 days.

The hard function

- ▶ We present our final result in the form of the **hard function** $H^{(1,1)}$, which can be passed to a Monte-Carlo generator, e.g. **MATRIX**

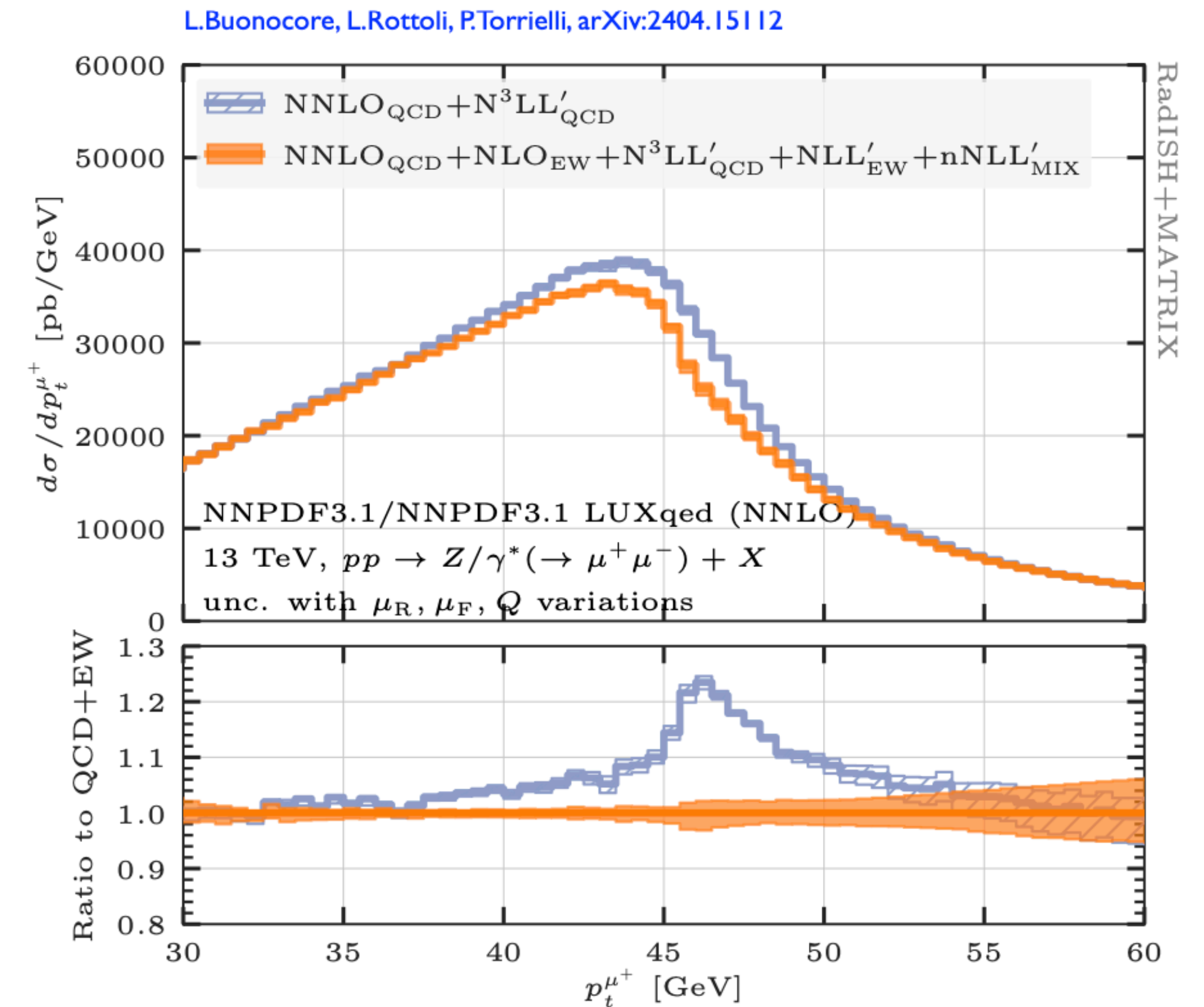
$$H^{(1,1)} = \frac{1}{16} \left[2\text{Re} \left(\frac{\langle \mathcal{M}^{(0,0)} | \mathcal{M}_{fin}^{(1,1)} \rangle}{\langle \mathcal{M}^{(0,0)} | \mathcal{M}^{(0,0)} \rangle} \right) \right]$$

- ▶ We can interpolate the value of $H^{(1,1)}$ in the entire phase-space. Thanks to its smoothness the error is, at worst, at the 10^{-3} level.



Combining with resummation

- ▶ A precise prediction to CC-DY is vital for determination of SM parameters, such as m_W .
- ▶ Recently, a joined QCD-QED resummation has been carried out in the **Radish** formulation at N3LL'-QCD + NLL'-EW + nNLL'-mixed accuracy, including QED effects from all charged legs;
- ▶ The next step will be matching with the exact $\mathcal{O}(\alpha\alpha_s)$ corrections needed to reach **full NNLL-mixed**;
- ▶ This will reduce the theoretical uncertainties due to QCD-QED radiation.



Summary & Outlook

- ▶ We presented the calculation of the pure **virtual contribution to the mixed QCD-EW corrections to Charged-current Drell-Yan**;
- ▶ We showed how to reconstruct a posteriori the **exact dependence on the W mass**;
- ▶ These corrections are currently being implemented in **MATRIX** and **Radish frameworks**;
- ▶ Finally, the techniques employed in this calculation are completely general, and can be applied to **other relevant 2->2 process** at **NNLO QCD-EW** level or even **NNLO EW**.

THANK YOU

IR subtraction

- ▶ The UV renormalised and IR subtracted scattering amplitude is given by:

UV renormalised amplitude

$$\left| \mathcal{M}_{fin}^{(1,1)} \right\rangle = \left| \mathcal{M}^{(1,1)} \right\rangle - \mathcal{F}^{(1,1)} \left| \mathcal{M}^{(0,0)} \right\rangle - \tilde{\mathcal{F}}^{(0,1)} \left| \mathcal{M}_{fin}^{(1,0)} \right\rangle - \tilde{\mathcal{F}}^{(1,0)} \left| \mathcal{M}_{fin}^{(0,1)} \right\rangle$$

Subtraction operators:

$$\mathcal{F}^{(1,1)} = C_F \left[\frac{Q_u^2 + Q_d^2}{2} \left(\frac{4}{\epsilon^4} + \frac{1}{\epsilon^3} (12 + 8i\pi) + \frac{1}{\epsilon^2} (9 - 28\zeta_2 + 12i\pi) + \frac{1}{\epsilon} \left(-\frac{3}{2} + 6\zeta_2 - 24\zeta_3 - 4i\pi\zeta_2 \right) \right) + \left(-\frac{2}{\epsilon^2} - \frac{1}{\epsilon} (3 + 2i\pi) + \zeta_2 \right) \frac{4}{\epsilon} \Gamma_l^{(0,1)} \right]$$

$$\Gamma_l^{(0,1)} = -\frac{1}{4} \left[Q_l^2 (1 - i\pi) + Q_l^2 \log \left(\frac{m_l^2}{s} \right) + 2Q_u Q_l \log \left(\frac{2p_1 \cdot p_4}{s} \right) - 2Q_d Q_l \log \left(\frac{2p_2 \cdot p_4}{s} \right) \right]$$

- ▶ We verified **analytically** the cancellation of the poles $1/\epsilon^4$, $1/\epsilon^3$ and $1/\epsilon^2$;
- ▶ We verified **numerically** the cancellation of the $1/\epsilon$ pole up to the 6th significant digit.

SeaSyde

[TA, R. Bonciani, S. Devoto, N.Rana,
A.Vicini, arXiv:2205.03345]

<https://github.com/TommasoArmadillo/SeaSyde>



- Our goal in the end is to fit the W mass to the data, hence, we need to employ a gauge invariant definition of the mass. For this reason, it is important to perform the calculations in the **complex-mass scheme**.

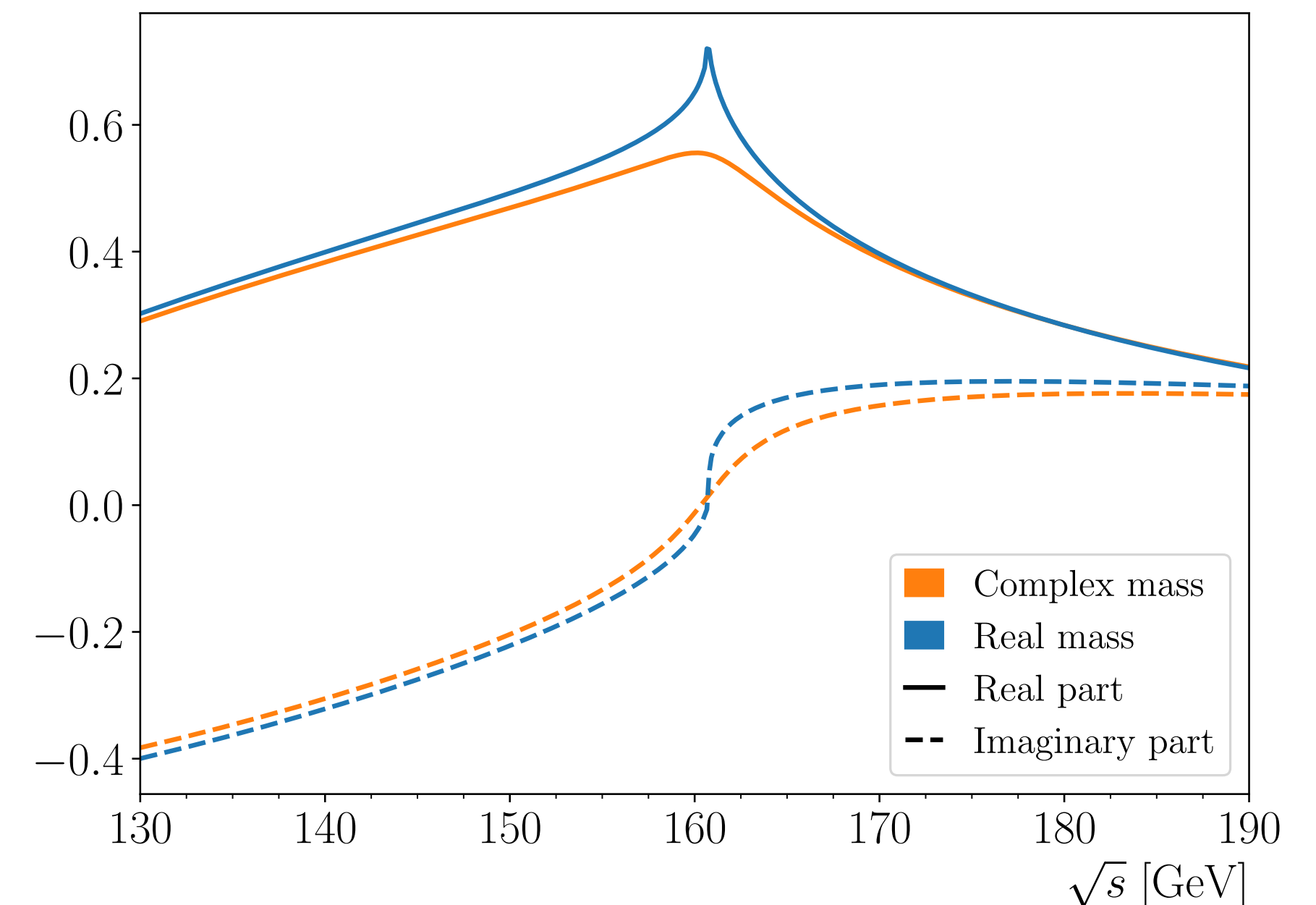
$$\mu_V^2 = m_V^2 - i\Gamma_V m_V$$

- The complex mass scheme **regularises** the behaviour at the resonance:

$$\frac{1}{s - \mu_V^2 + i\delta}$$

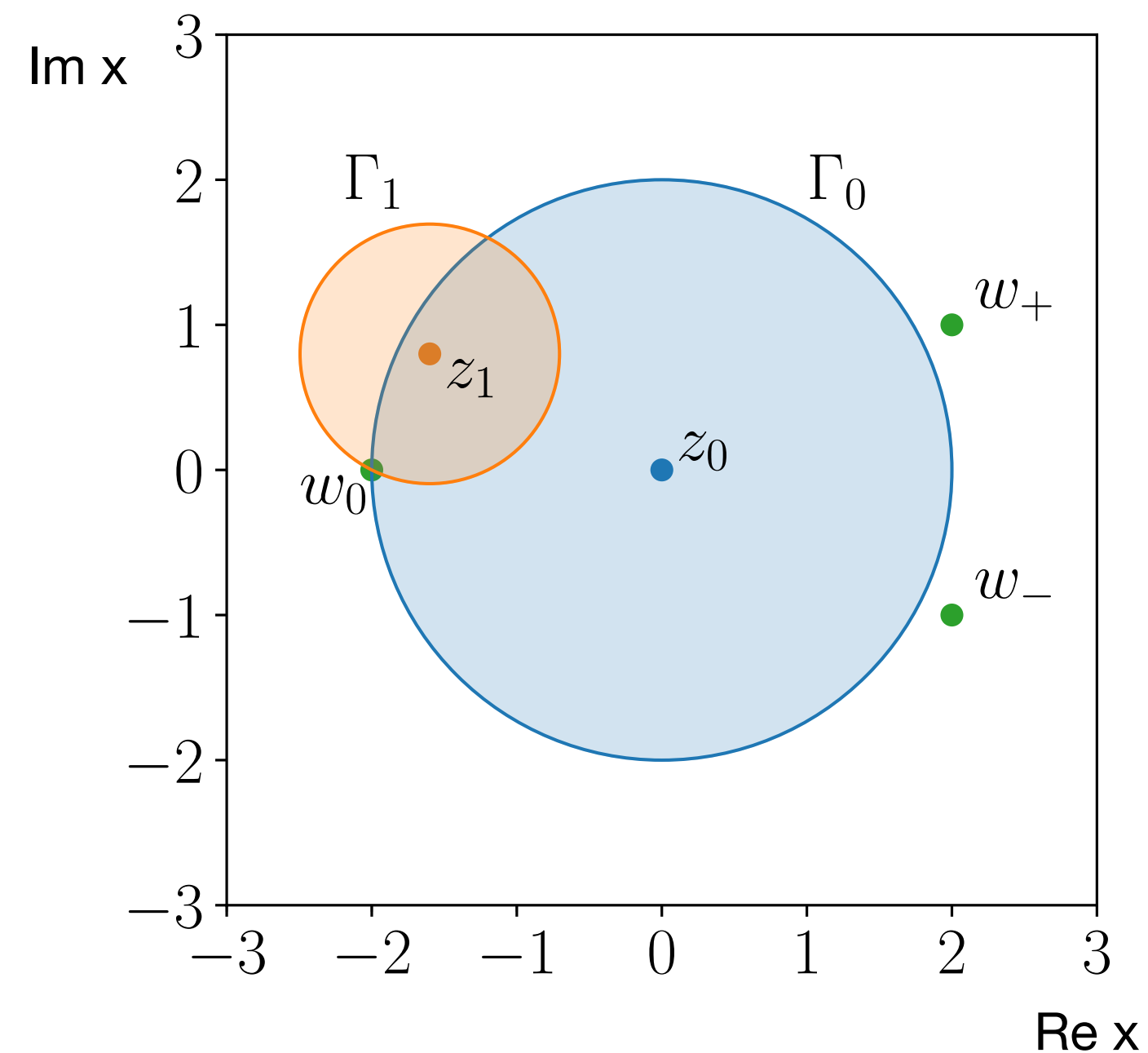
- If we utilise **adimensional variables**, they become complex-valued:

$$\tilde{s} = \frac{s}{m_V^2} \rightarrow \frac{s}{\mu_V^2}$$



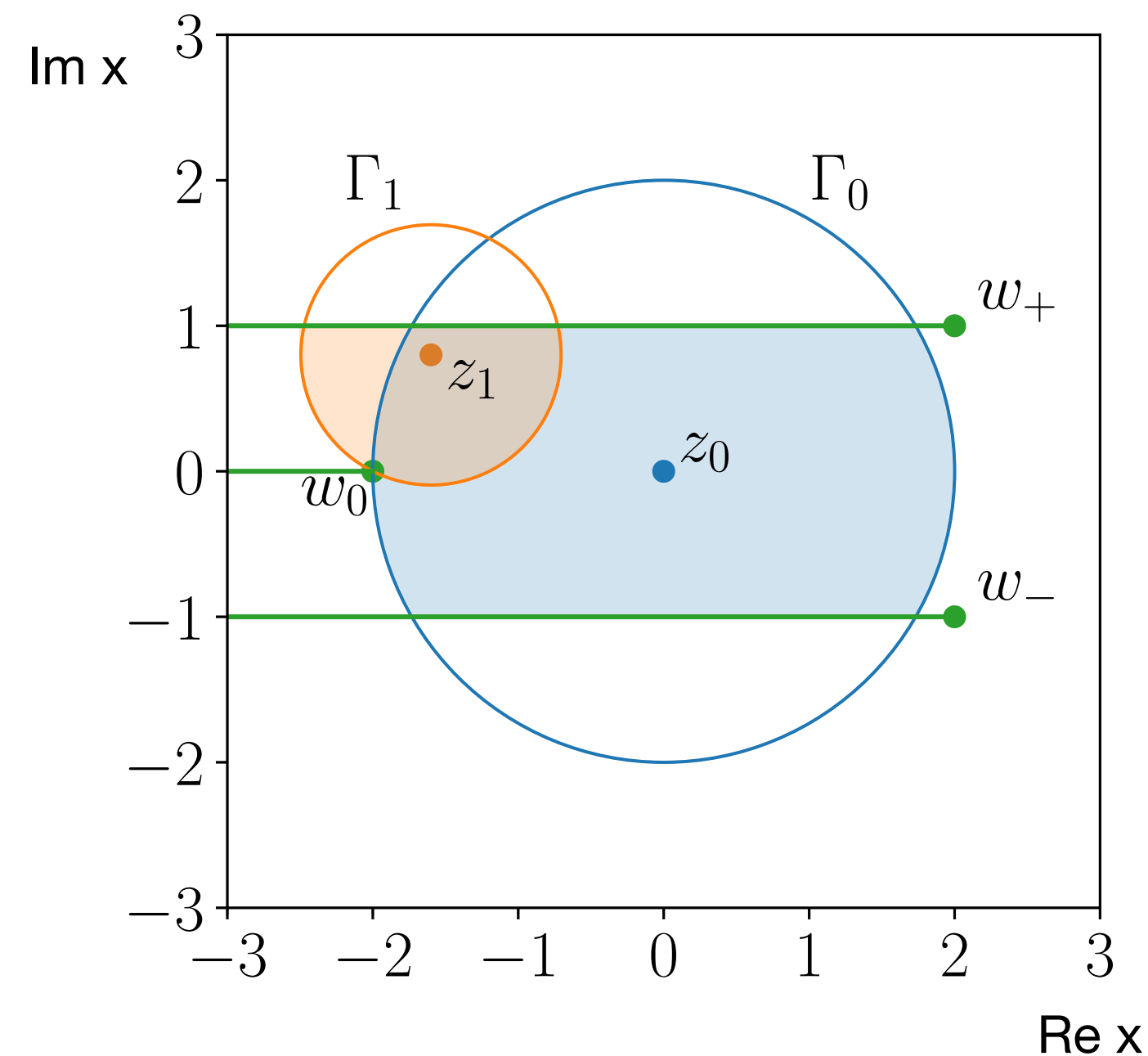


- ▶ Power series have a limited **radius of convergence** which is determined by the position of the **nearest singularity**.
- ▶ We need to be able to extend the solution beyond the radius of convergence, to the entire **complex plane**.



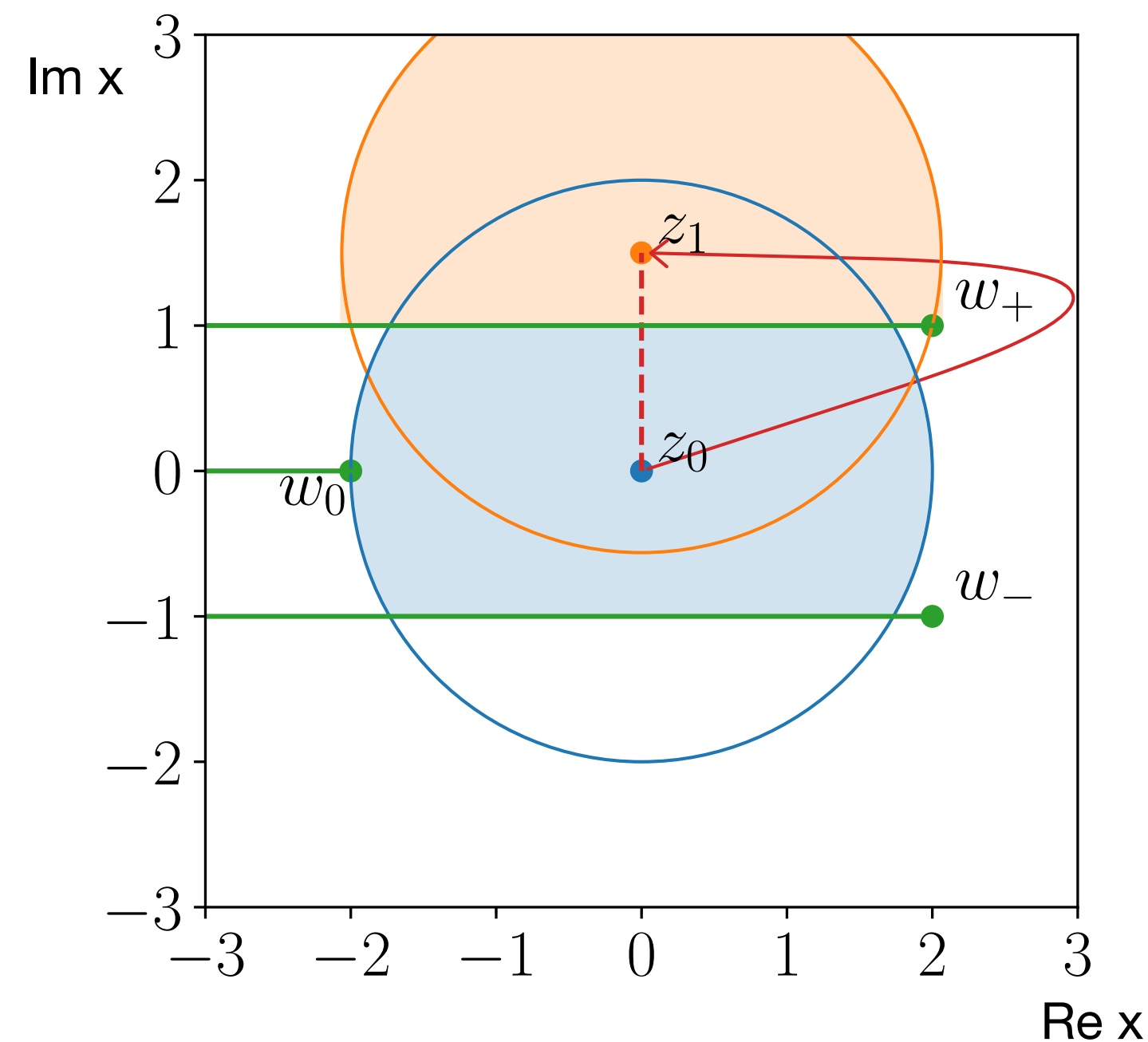
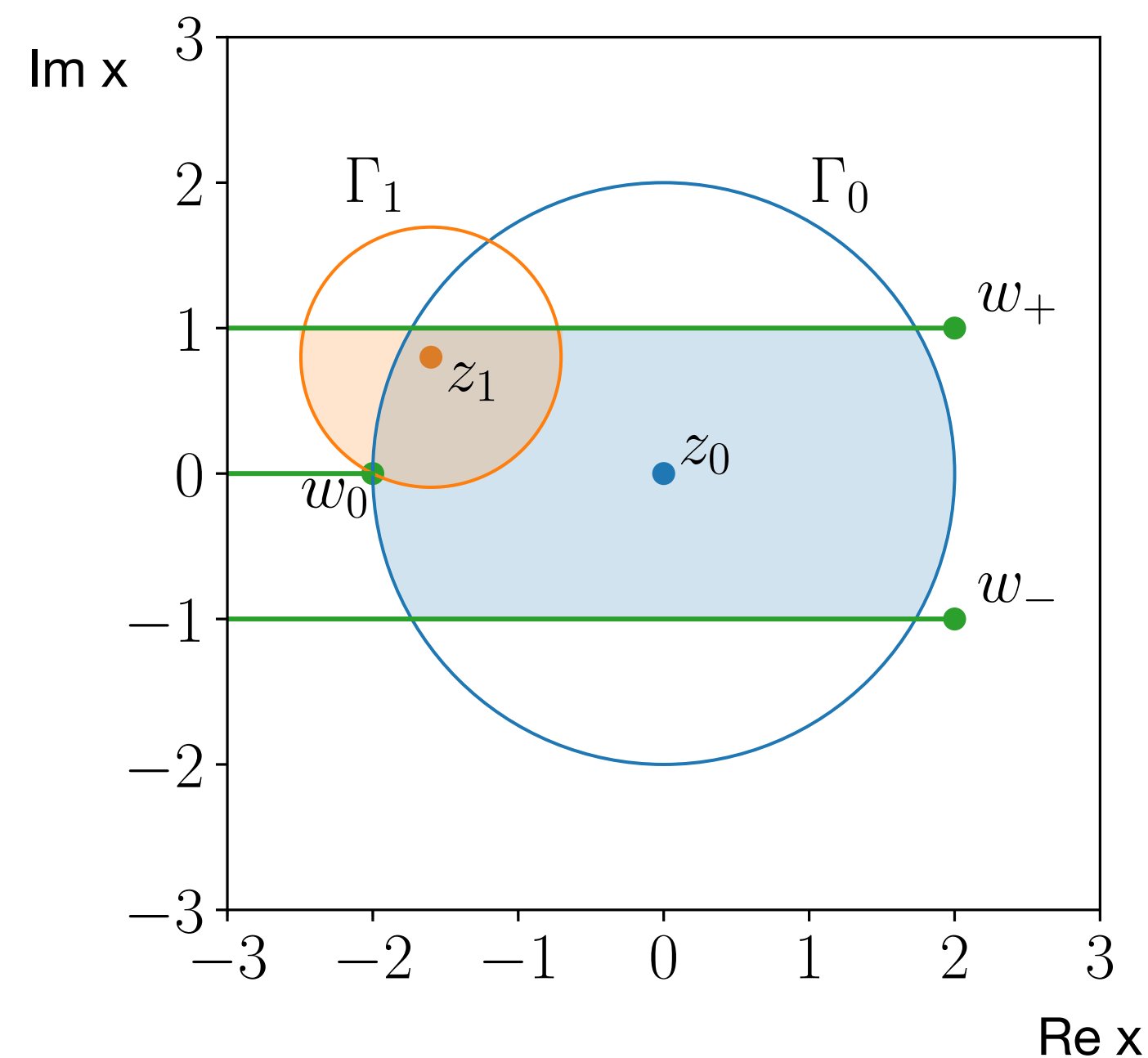


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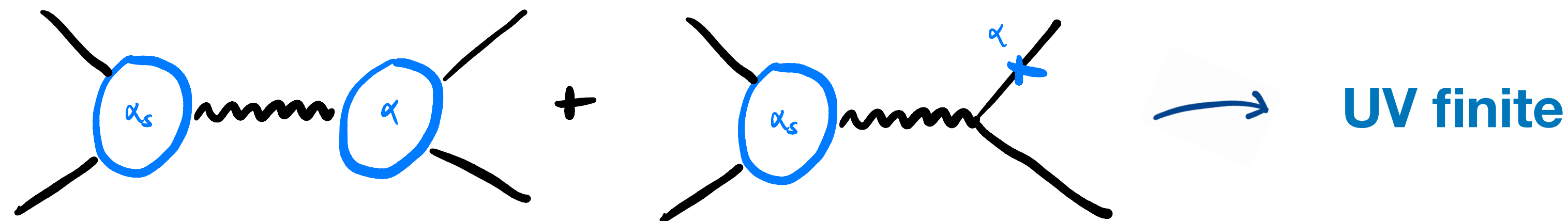
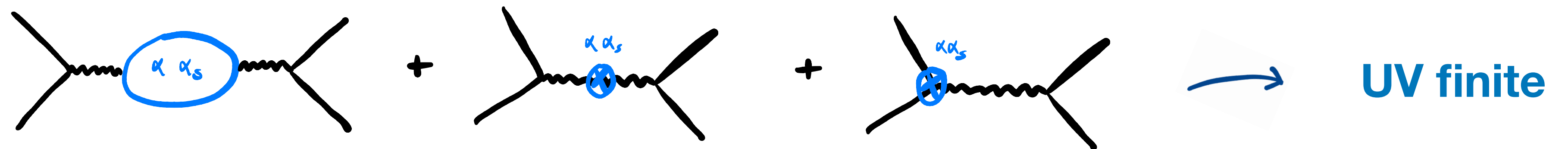
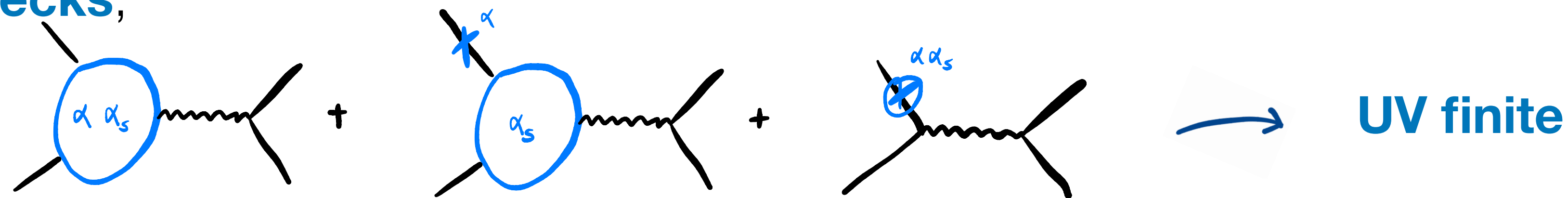
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*For simplicity, we are not showing all the intermediate circles.

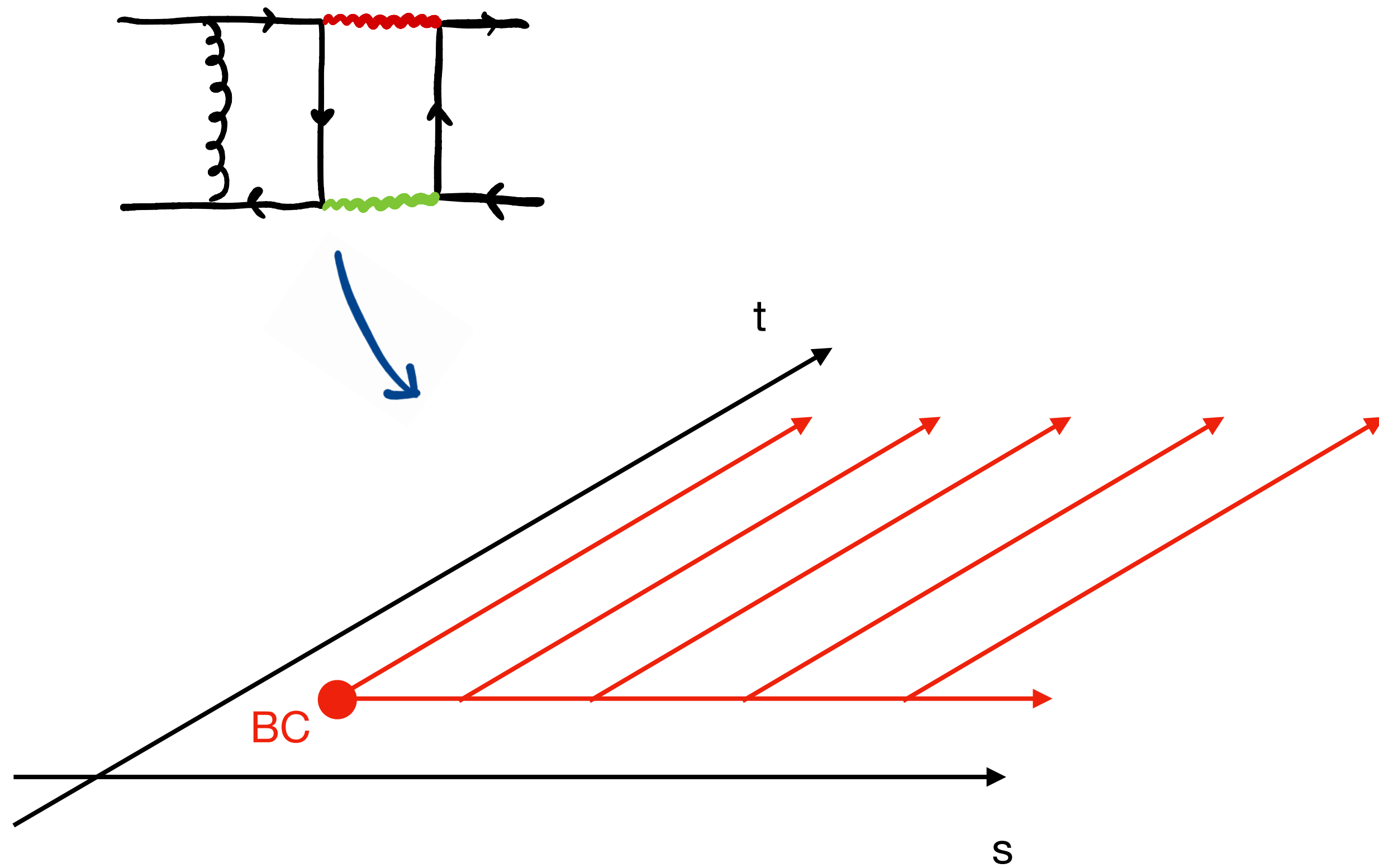
UV renormalisation

- ▶ In the computation we employed the **Background Field Method**. This let us identify some subsets of diagrams which are **UV finite**, which is useful for performing intermediate non trivial **cross-checks**;



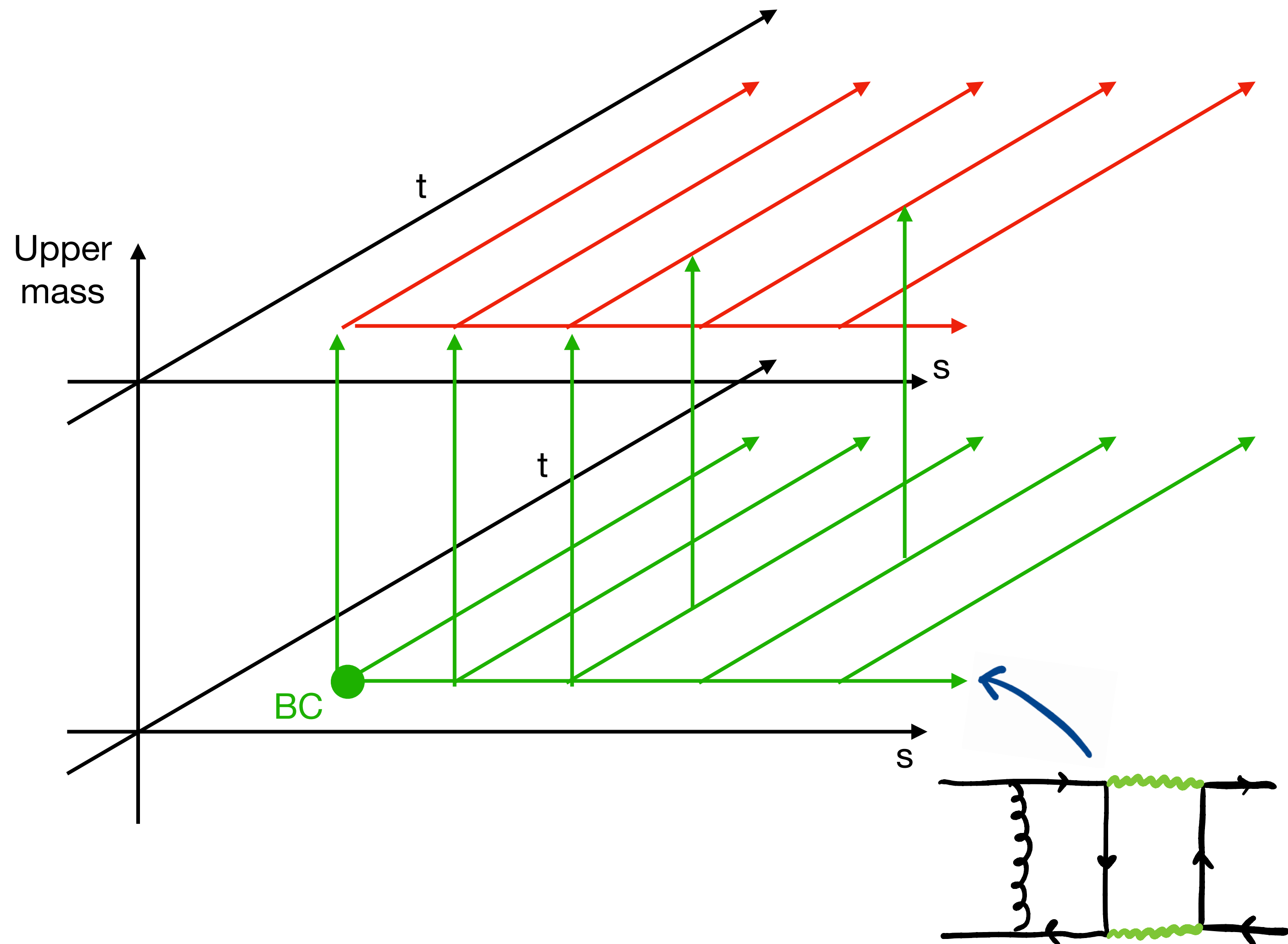
- ▶ All the counter-terms were computed in the **on-shell scheme**.

Creating a grid



- ▶ This approach is completely general and **easy to automate**;
- ▶ We have to solve a 56×56 system of differential equations w.r.t. to the Mandelstam variables s and t ;
- ▶ Since we are not putting the system in canonical form, these are usually quite complicated and the solution might require some time;
- ▶ The computation of a grid with 3250 points required ~ 3 weeks on 26 cores.

Mass evolution



- ▶ We can re-use the grid from the **Neutral-current Drell-Yan**;
- ▶ We have to solve a **36x36** system of differential equations w.r.t. to the Mandelstam variables s and t ;
- ▶ Then, for every point, we have to solve a **56x56, but easier, system** w.r.t. one mass;
- ▶ We used this as a **cross-check**.

Background-Field Method (BGF)

- ▶ We chose to perform the calculation using the **background-field method**:

$$\mathcal{L}_{SM} = \mathcal{L}_C(\hat{V} + V) + \mathcal{L}_{GF}(V) + \mathcal{L}_{FP}$$

$$\mathcal{L}_C = \mathcal{L}_{YM} + \mathcal{L}_H + \mathcal{L}_F$$

- ▶ The fields are split into background fields \hat{V} and quantum ones V ;
 - ▶ The quantum fields are the variables of integration in the functional integral, i.e. they appear only in loops.
 - ▶ Even though we have more fields, the expressions are usually simpler
- ▶ $\mathcal{L}_{GF}(V)$ breaks gauge-invariance only of the quantum fields, for this reasons very simple and **QED-like Ward identities** are satisfied at any order in perturbation theory for the background ones.



A SIMPLE EXAMPLE

$$\begin{cases} f'(x) + \frac{1}{x^2 - 4x + 5} f(x) = \frac{1}{x + 2} \\ f(0) = 1 \end{cases}$$

$$\begin{cases} rc_0 = 0 \\ \frac{1}{5}c_0 + c_1(r + 1) = 0 \\ \frac{4}{25}c_0 + \frac{1}{5}c_1 + c_2(2 + r) = 0 \\ \frac{11}{125}c_0 + \frac{4}{25}c_1 + \frac{1}{5}c_2 + c_3(3 + r) = 0 \\ \dots \end{cases}$$

$$f_{hom}(x) = x^r \sum_{k=0}^{\infty} c_k x^k$$

$$f_{hom}(x) = 5 - x - \frac{3}{10}x^2 + \frac{11}{150}x^3 + \dots$$



A SIMPLE EXAMPLE

$$\begin{cases} f'(x) + \frac{1}{x^2 - 4x + 5} f(x) = \frac{1}{x + 2} \\ f(0) = 1 \end{cases}$$

$$\begin{aligned} f_{part}(x) &= f_{hom}(x) \int_0^x dx' \frac{1}{(x' + 2)} f_{hom}^{-1}(x') \\ &= \frac{1}{2}x - \frac{7}{40}x^2 + \frac{2}{75}x^3 + \dots \end{aligned}$$

$$\begin{aligned} f(x) &= c f_{hom}(x) + f_{part}(x) \\ &= 1 + \frac{3}{10}x - \frac{47}{200}x^2 + \frac{3}{250}x^3 + \dots \end{aligned}$$



A SIMPLE EXAMPLE

$$\begin{cases} f'(x) + \frac{1}{x^2 - 4x + 5} f(x) = \frac{1}{x + 2} \\ f(0) = 1 \end{cases}$$

$$\begin{aligned} f(x) &= c f_{hom}(x) + f_{part}(x) \\ &= 1 + \frac{3}{10}x - \frac{47}{200}x^2 + \frac{3}{250}x^3 + \dots \end{aligned}$$

- ▶ This procedure can be generalised to **systems of differential equations**;
- ▶ The method has been firstly implemented in the Mathematica package **DiffExp** for a **real kinematic variable** [F.Moriello, arXiv:1907.13234], [M.Hidding, arXiv:2006.05510]
- ▶ The great advantage of this approach is that we can reach **arbitrary precision** just by adding more terms in the serie

Recent developments

Theoretical developments:

- **2-loop virtual Master Integrals with internal masses:** [U. Aglietti, R. Bonciani, arXiv:0304028, arXiv:0401193],[R. Bonciani, S. Di Vita, P. Mastrolia, U. Schubert, arXiv:1604.08581], [M.Heller, A.von Manteuffel, R.Schabinger arXiv:1907.00491], [M.Long,R,Zhang,W.Ma,Y,Jiang,L.Han,,Z.Li,S.Wang, arXiv:2111.14130],[X.Liu, Y.Ma, arXiv:2201.11669]
- **Altarelli-Parisi splitting functions including QCD-QED effects** [D. de Florian, G. Sborlini, G. Rodrigo, arXiv:1512.00612]
- **Renormalisation** [G.Degrassi, A.Vicini, hep-ph/0307122],[S.Dittmaier,T.Schmidt,J.Schwarz, arXiv:2009.02229], [S.Dittmaier, arXiv:2101.05154]

On-shell Z and W production:

- **pole approximation of the NNLO QCD-EW corrections** [S.Dittmaier, A.Huss, C.Schwinn, arXiv:1403.3216, 1511.08016]
- **analytical total Z production cross section including NNLO QCD-QED corrections** [D. de Florian, M.Der, I.Fabre, arXiv:1805.12214]
- **fully differential on-shell Z production including exact NNLO QCD-QED corrections** [M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:1909.08428] [S.Hasan, U.Schubert, arXiv:2004.14908]
- **analytical total Z production cross section including NNLO QCD-EW corrections** [R. Bonciani, F. Buccioni, R.Mondini, A.Vicini, arXiv:1611.00645], [R. Bonciani, F. Buccioni, N.Rana, I.Triscari, A.Vicini, arXiv:1911.06200], [R. Bonciani, F. Buccioni, N.Rana, A.Vicini, arXiv:2007.06518, arXiv:2111.12694]
- **fully differential Z and W production including NNLO QCD-EW corrections** [F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2005.10221], [A. Behring, F. Buccioni, F. Caola, M.Delto, M.Jaquier, K.Melnikov, R.Roentsch, arXiv:2009.10386, 2103.02671]

Complete Drell-Yan:

- **neutrino-pair production including NNLO QCD-QED corrections** [L. Cieri, D. de Florian, M.Der, J.Mazzitelli, arXiv:2005.01315]
- **2-loop amplitudes** [M.Heller, A.von Manteuffel, R.Schabinger, arXiv:2012.05918],[TA, R.Bonciani, S. Devoto, N.Rana, A.Vicini, arXiv:2201.01754]
- **NNLO QCD-EW corrections to neutral-current DY including leptonic decay** [R.Bonciani, L.Buonocore, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, A.Vicini, arXiv:2106.11953],[F.Buccioni, F.Caola, H.Chawdhry, F.Devoto, M.Heller,A.von Manteuffel, K.Melnikov, R.Röntsch, C.Signorile-Signorile, arXiv:2203.11237]
- **NNLO QCD-EW corrections to charged-current DY including leptonic decay (2-loop contributions in pole approximation).** [L.Buonocore, M.Grazzini, S.Kallweit, C.Savoini, F.Tramontano, arXiv:2102.12539]