NNLO QCD corrections to polarized and unpolarized semi-inclusive DIS

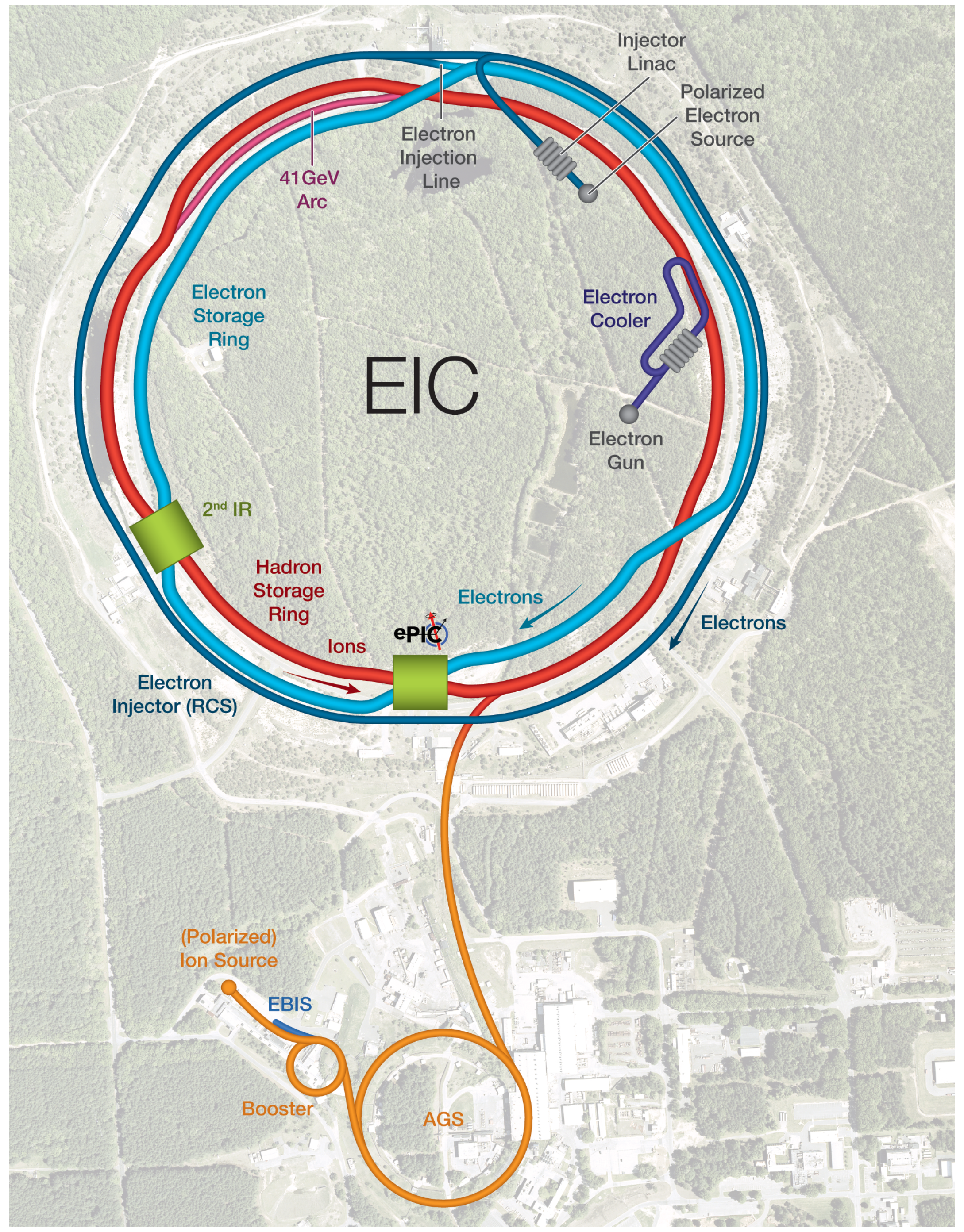
35th *Rencontres de Blois* — 22/10/2024

Based on 2401.16281 and 2404.08597

Leonardo Bonino

Introduction

Electron-Ion Collider (EIC)

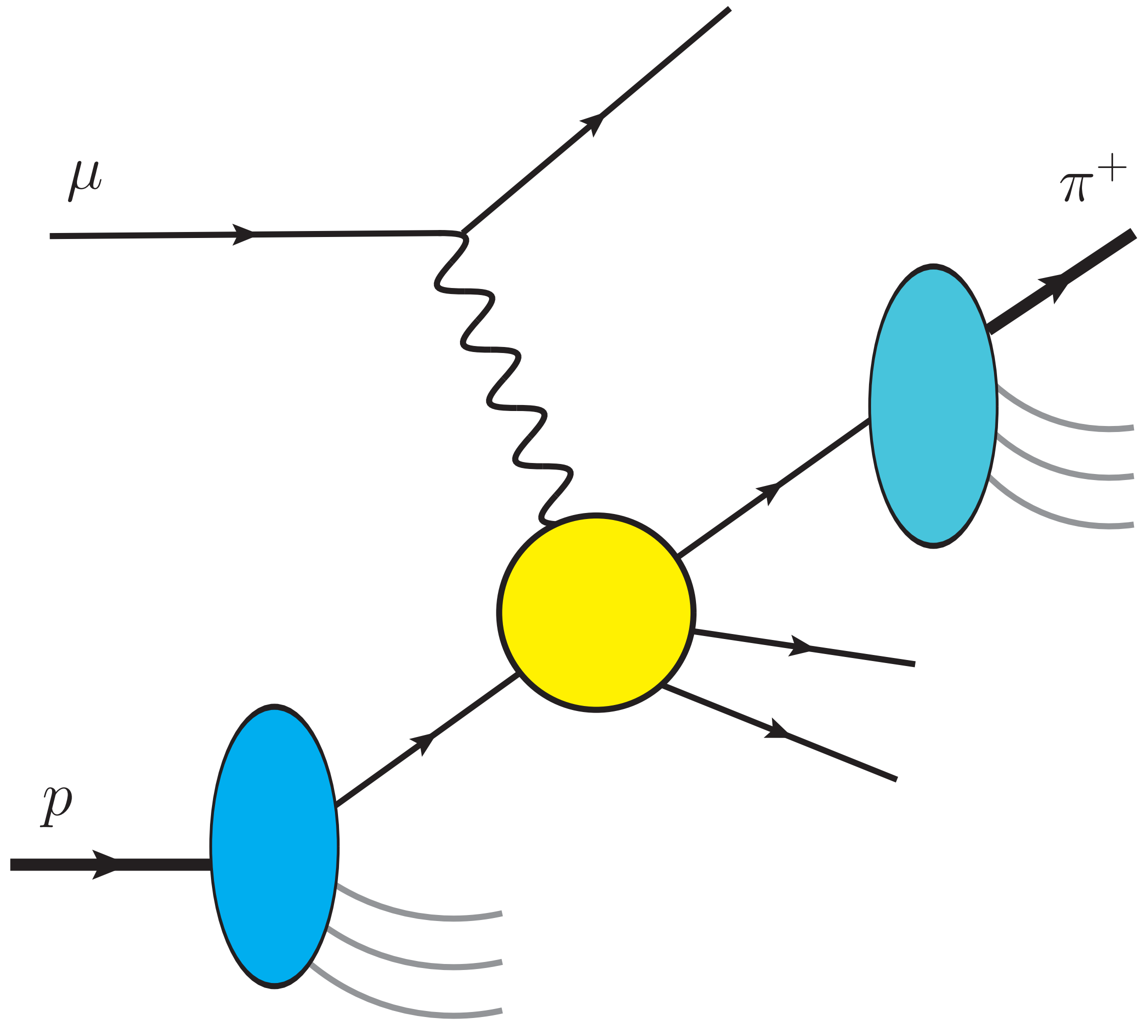


BNL EIC now firmly on its path toward construction

- $e^- N$ collision (DIS) with high-luminosity $10^{33} - 10^{34}\mathrm{cm}^{-2}\mathrm{s}^{-1}$
- Center-of-mass energy range: $20 - 140\,\mathrm{GeV}$
- Full final state identification and polarised collisions (SIDIS)
- Theoretical accuracy needed: NNLO

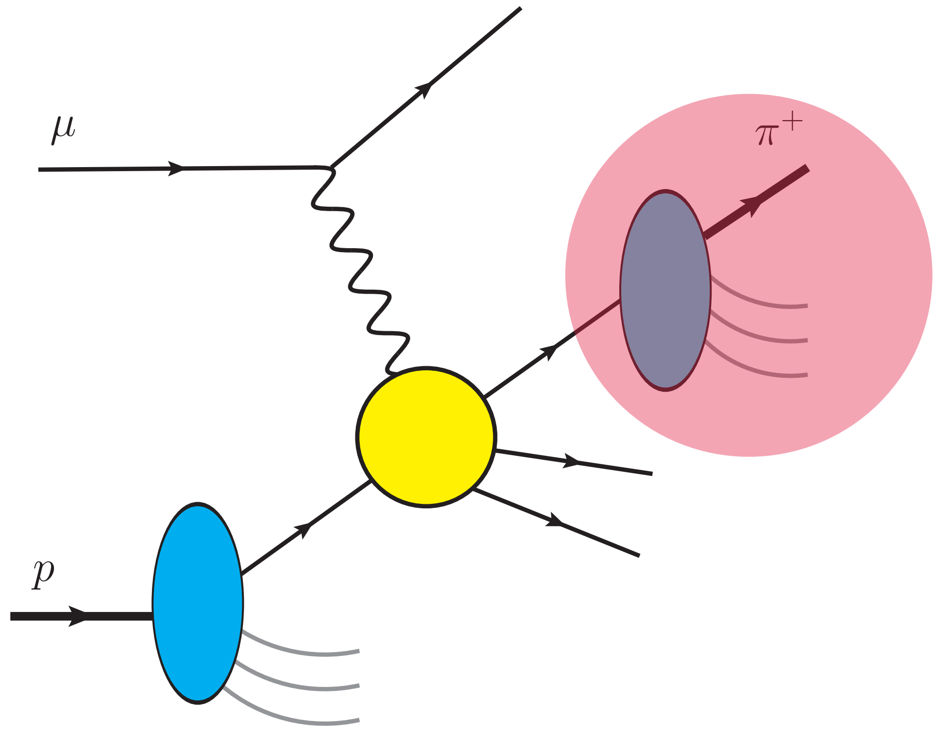
SIDIS measurements @ EIC: [Aschenauer, Borsa, Sassot, Van Hulse '19]

- Useful for full flavour decomposition for (pol)PDFs
- Useful for full flavour separation for FFs
- Reduction in uncertainties of strange distributions by more then 60%



SIDIS = semi-inclusive deep-inelastic scattering

Introduction



Global aNNLO fits to FFs

aNNLO
FFs

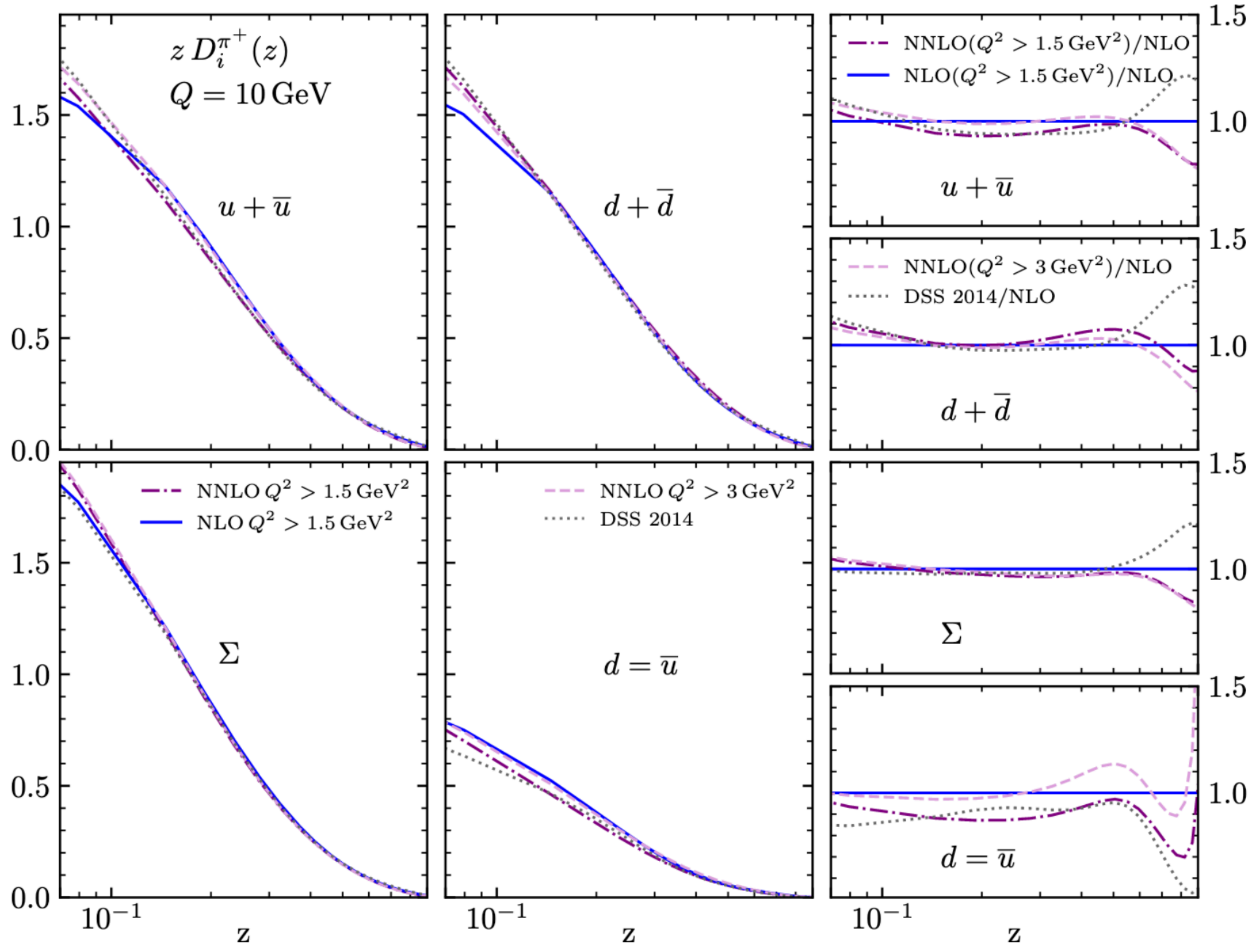
[Abele, de Florian, Vogelsang '21] aNNLO (approximate NNLO) corrections to $q \to q$ channel from threshold resummation formalism

- Same for polarised case

[Borsa, Sassot, de Florian, Vogelsang '22] aNNLO results in FFs global fits: SIA (e^+e^-) + COMPASS + HERMES

- with aNNLO surpassing quality of NLO fit only for $Q^2 \geq 2\,\mathrm{GeV}^2$
- aNNLO for SIDIS missing significant contributions?

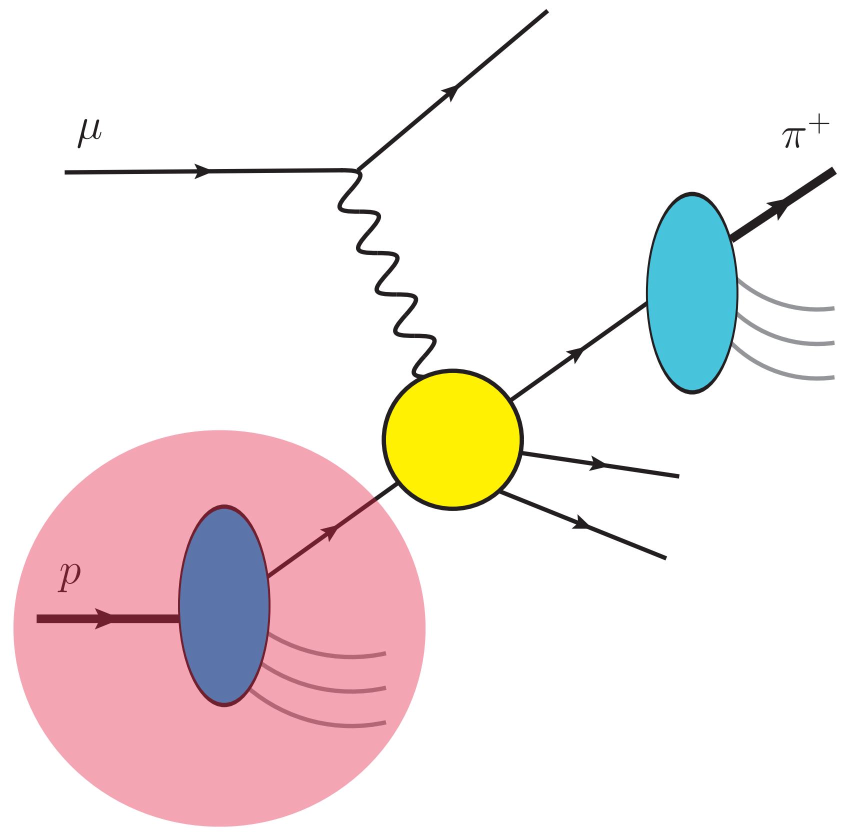
aNNLO global fits with neural-network from Mont Blanc collaboration [Abdul Khalek, Bertone, Khoudli, Nocera '22]



Experiment	$Q^2 \geq 1.5\,\mathrm{GeV}^2$			$Q^2 \geq 2.0\,\mathrm{GeV}^2$			$Q^2 \geq 2.3\,\mathrm{GeV}^2$			$Q^2 \geq 3.0\,\mathrm{GeV}^2$		
	#data	NLO	NNLO	#data	NLO	NNLO	#data	NLO	NNLO	#data	NLO	NNLO
SIA	288	1.05	0.96	288	0.91	0.87	288	0.90	0.91	288	0.93	0.86
COMPASS	510	0.98	1.14	456	0.91	1.04	446	0.91	0.92	376	0.94	0.93
HERMES	224	2.24	2.27	160	2.40	2.08	128	2.71	2.35	96	2.75	2.26
TOTAL	1022	1.27	1.33	904	1.17	1.17	862	1.17	1.13	760	1.16	1.07

Introduction

Global aNNLO fits to polPDFs



Helicity parton distribution functions $\Delta f(x, Q^2) = f^+(x, Q^2) - f^-(x, Q^2)$

Gluon spin contribution to the proton spin: $\Delta G = \int_0^1 \Delta g(x) \mathrm{d}x$

[de Florian, Sassot, Stratmann, Vogelsang '08 '14] NLO global analysis

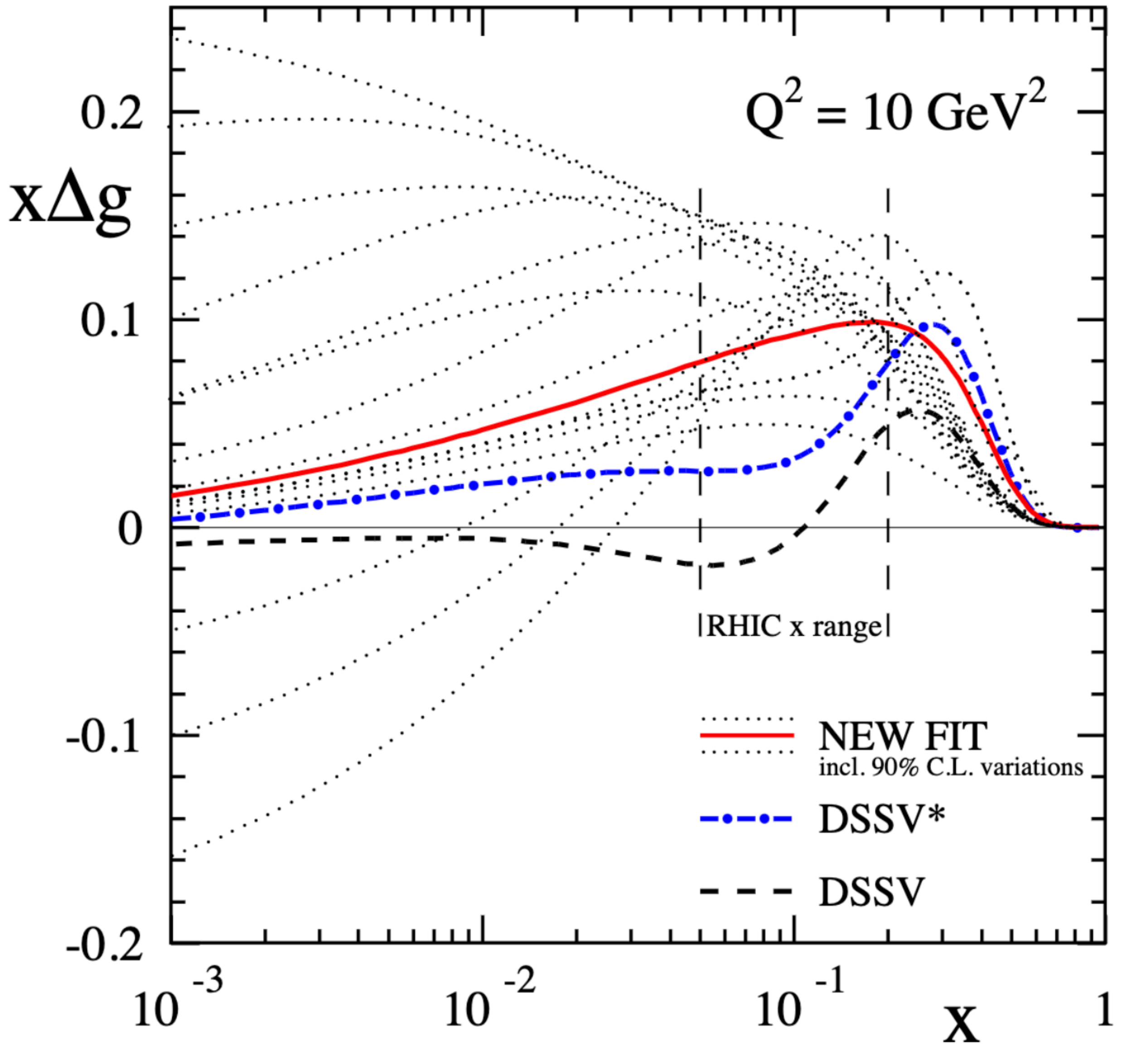
[Borsa, de Florian, Sassot, Stratmann, Vogelsang '24] aNNLO global analysis

- [DSSV '14] evidence for polarisation of gluons in the proton (STAR)
- DIS + SIDIS + proton-proton ($x > 0.12$ cut on SIDIS data)

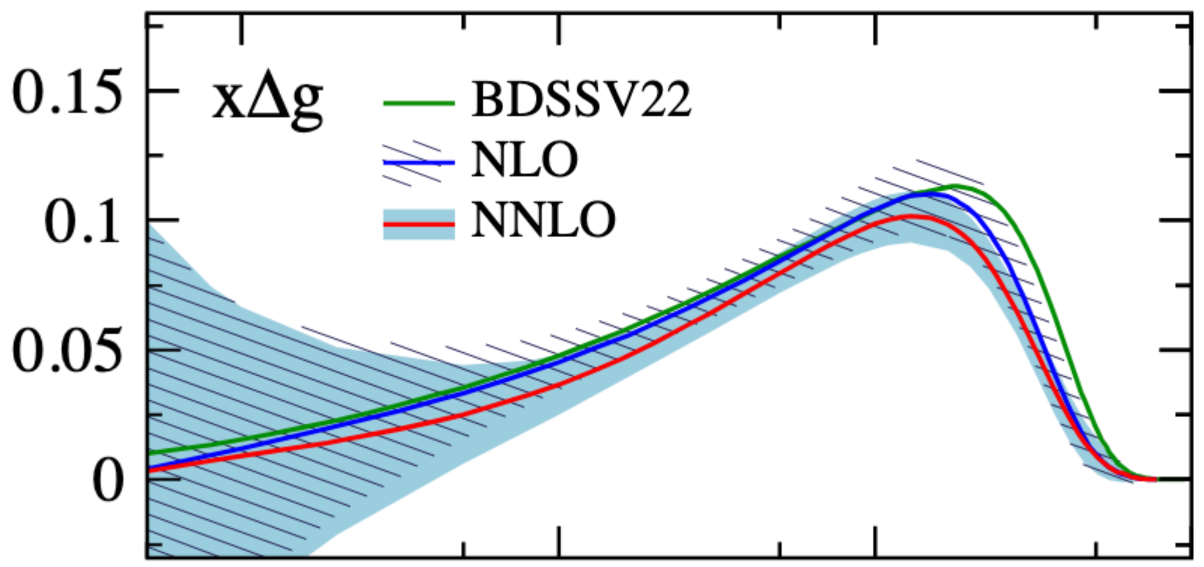
[Bertone, Chiefa, Nocera (MAP) '24] aNNLO global analysis (neural-network)

- DIS + SIDIS only → Δg almost unconstrained

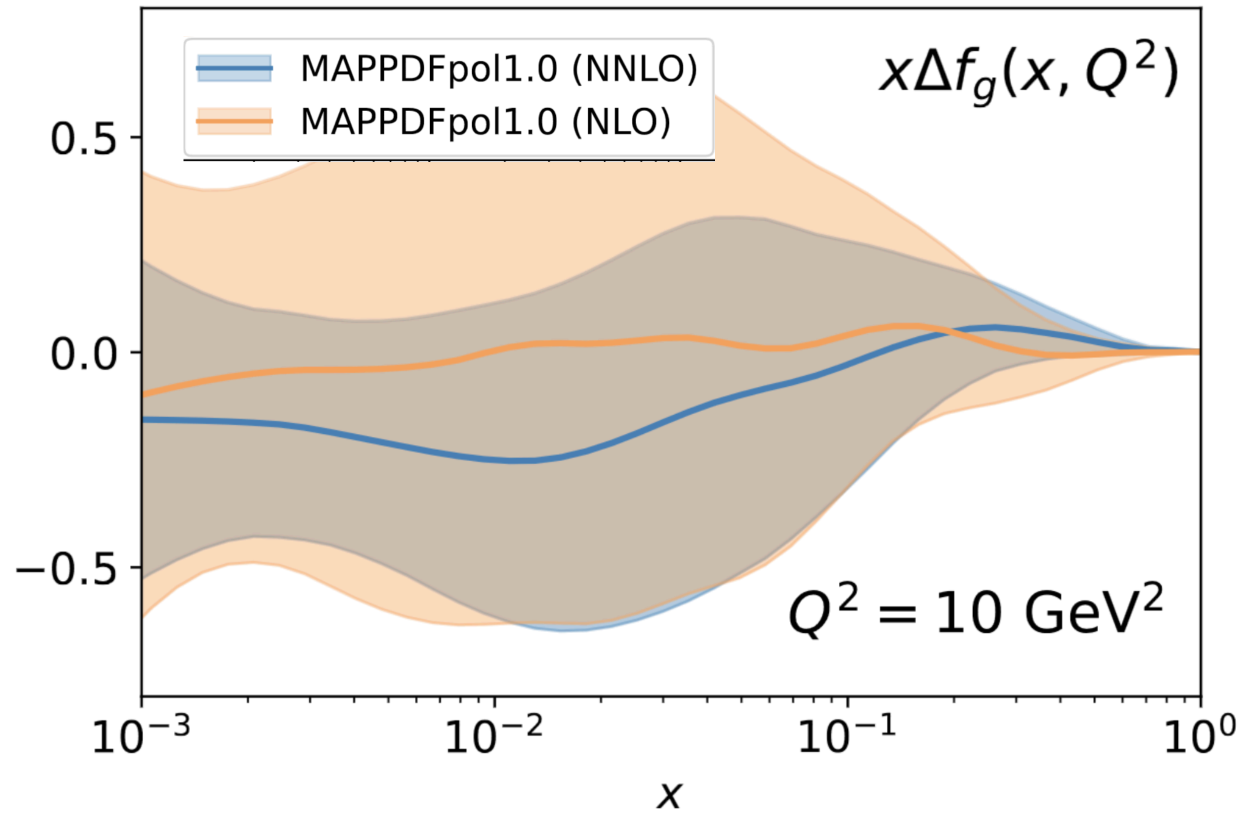
Improvements with inclusion of the full NNLO corrections?



DSSV



BDSSV



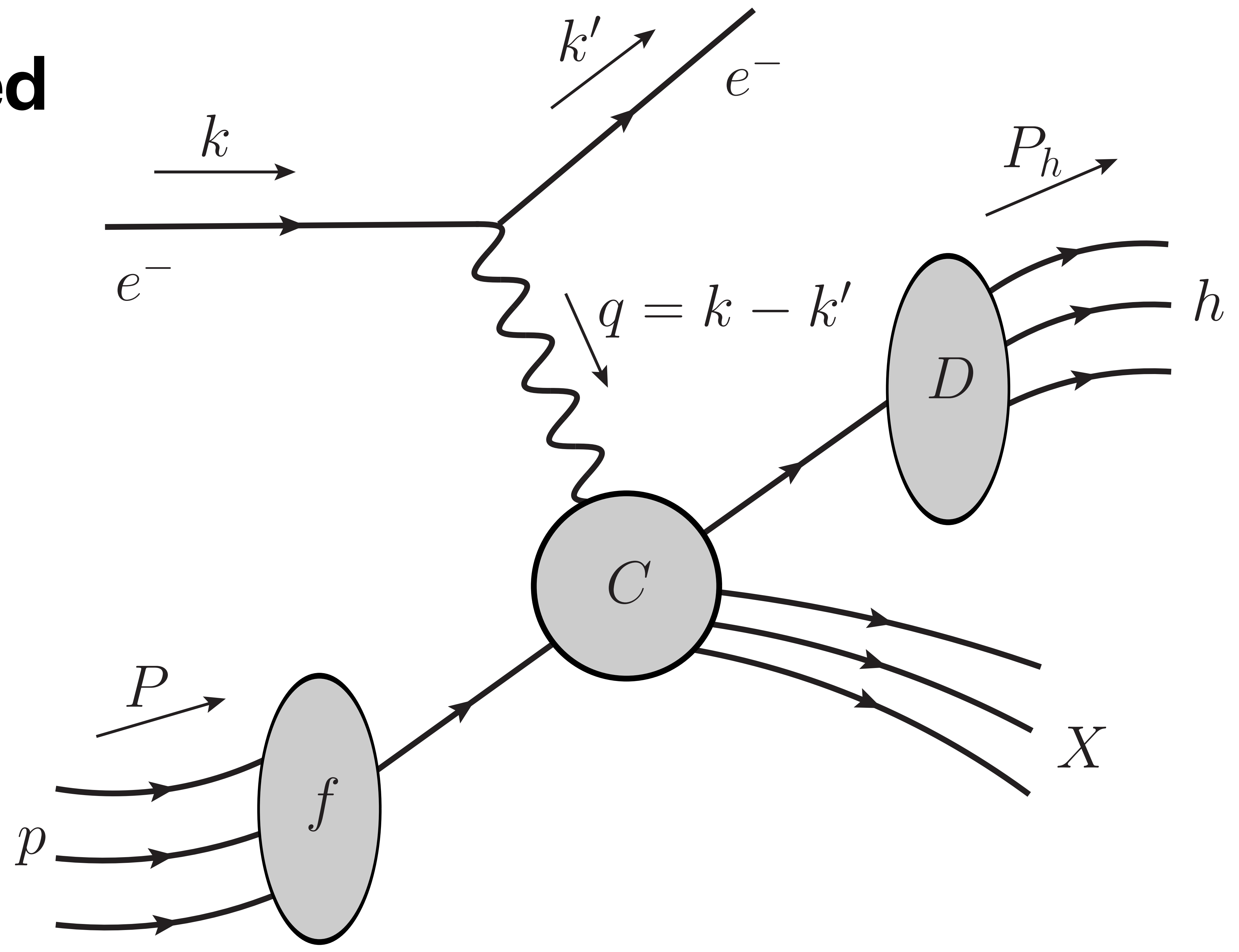
MAP

Kinematics of (pol)SIDIS

Variables involved

$\ell(k)\, p(P) \to \ell(k')\, h(P_h)\, X$ with $Q \ll M_Z$

- $Q^2 = -q^2$: invariant mass of γ^* ($q = k - k'$)
- $x = \dfrac{Q^2}{2P \cdot q}$: Bjorken scaling variable
- $z = \dfrac{P \cdot P_h}{P \cdot q}$: fractional energy of the observed final state hadron h
- $y = \dfrac{P \cdot q}{P \cdot k}$: energy transfer (inelasticity)
- $W^2 = (k+P)^2$: mass squared of recoiling system X against scattered lepton



$$Q^2 = -q^2 = xys$$

Semi-inclusive ($z-$differential) deep ($Q^2 \gg M_p^2$) inelastic ($W^2 \gg M_p^2$) scattering

$\sqrt{s}$: center of mass energy of lepton-nucleon system

Kinematics of SIDIS

Unpolarised differential cross section

Spin-averaged triple-differential cross section $\frac{\mathrm{d}^3\sigma^h}{\mathrm{d}x\mathrm{d}y\mathrm{d}z} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y}\mathscr{F}_T^h(x,z,Q^2) + \frac{1-y}{y}\mathscr{F}_L^h(x,z,Q^2)\right]$

- Transverse ($\mathscr{F}_T^h$) and longitudinal ($\mathscr{F}_L^h$) ‘fragmentation’ structure functions (SF)
  - Defined in terms of the hadronic tensor $W_{\mu\nu}$ (from F_1 and F_2)
  - Relevant SF for neutral-current semi-inclusive deep inelastic scattering on unpolarized nucleon

Typical observable: SIDIS hadron multiplicity $\frac{\mathrm{d}M^h}{\mathrm{d}z} = \frac{\mathrm{d}^3\sigma^h/\mathrm{d}x\mathrm{d}y\mathrm{d}z}{\mathrm{d}^2\sigma^{DIS}/\mathrm{d}x\mathrm{d}y}$

- Double-differential (DIS) cross section known up to N3LO

$$\frac{\mathrm{d}^2\sigma^{DIS}}{\mathrm{d}x\mathrm{d}y} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y}\mathscr{F}_T(x,Q^2) + \frac{1-y}{y}\mathscr{F}_L(x,Q^2)\right]$$

Kinematics of SIDIS

Unpolarised structure functions

Structure functions satisfy the factorisation theorem ($Q \gg \Lambda_{QCD}$) $\rightarrow$ initial μ_F and final μ_A factorisation scales

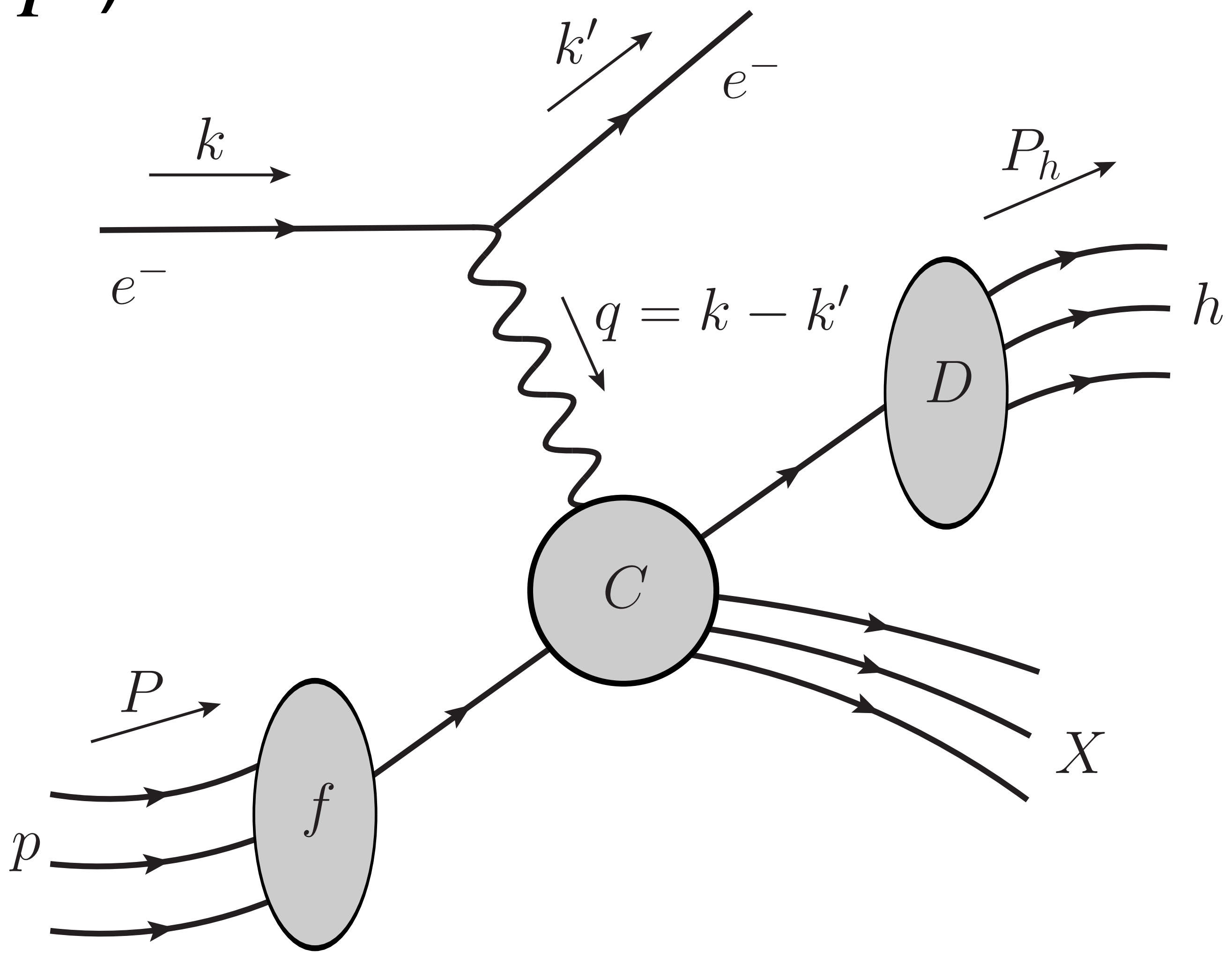
$$\mathcal{F}_i^h(x,z,Q^2) = \sum_{p,p'} \int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}} \int_z^1 \frac{\mathrm{d}\hat{z}}{\hat{z}} f_p\left(\frac{x}{\hat{x}},\mu_F^2\right) D_{p'}^h\left(\frac{z}{\hat{z}},\mu_A^2\right) \mathcal{C}^i_{p'p}\left(\hat{x},\hat{z},Q^2,\mu_R^2,\mu_F^2,\mu_A^2\right), \quad i = T,L$$

- Initial and final states collinear divergences reabsorbed in PDFs f_p and FFs $D_{p'}^h$ by mass factorisation ($\overline{\mathrm{MS}}$)
- The coefficient functions $\mathcal{C}^i_{p'p}$ encode the hard scattering part of the process ($p \rightarrow p'$)

$$\mathcal{C}^i_{p'p} = C^{i,(0)}_{p'p} + \frac{\alpha_s(\mu_R^2)}{2\pi} C^{i,(1)}_{p'p} + \left(\frac{\alpha_s(\mu_R^2)}{2\pi}\right)^2 C^{i,(2)}_{p'p} + \mathcal{O}(\alpha_s^3)$$

- Un-physical renormalised and mass-factorised (finite) objects

Our focus: all coefficients $C^{i,(2)}_{p'p}$

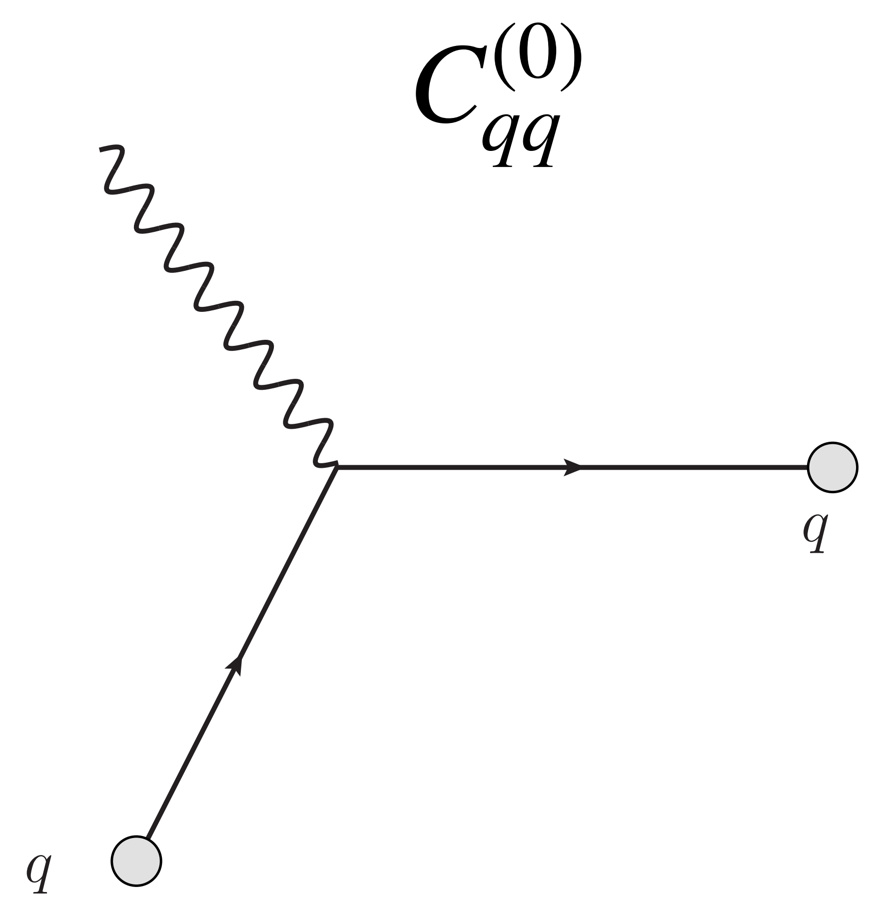


QCD corrections

LO and NLO results

- LO: only $q \to q$ channel contributes:
  $C_{qq}^{T,(0)} = e_q^2 \delta(1-\hat{x})\delta(1-\hat{z})\,, \quad C_{qq}^{L,(0)} = 0$
- NLO: [Altarelli, Ellis, Martinelli, Pi '79] [Baier, Fey '79]
  - Also gluons in initial ($g \to q$) or final state ($q \to g$) contribute
  - Opening of longitudinal channels

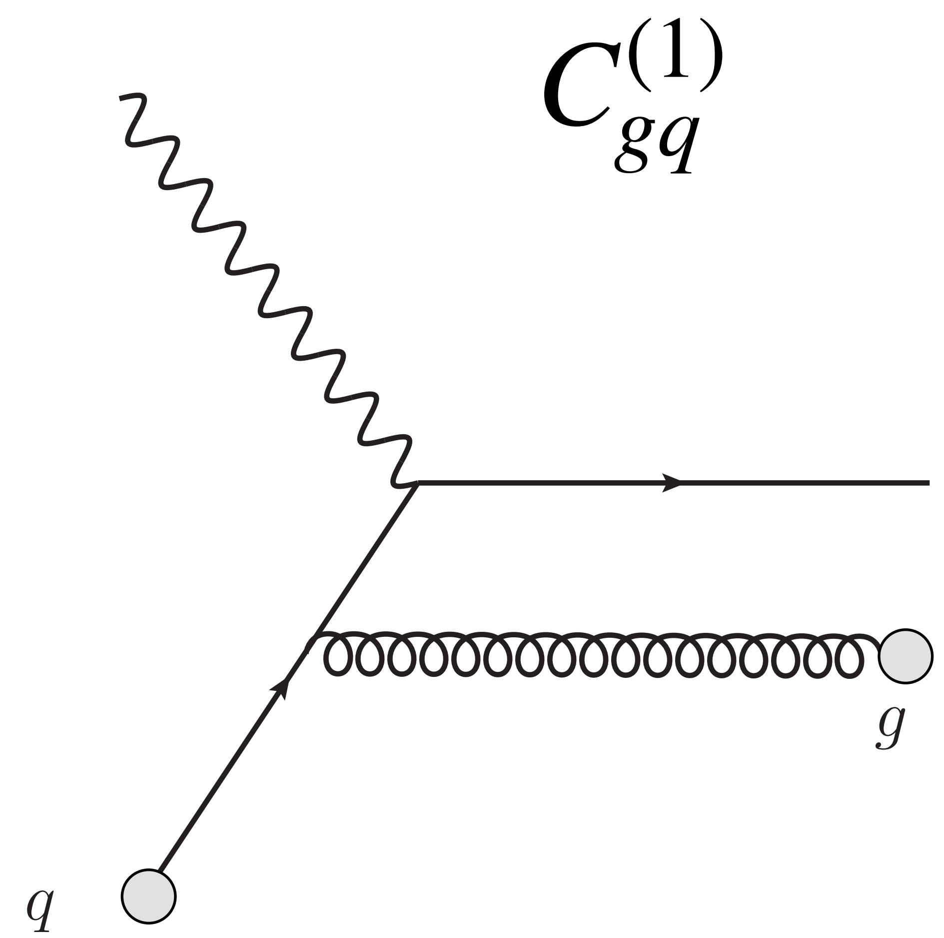
LO

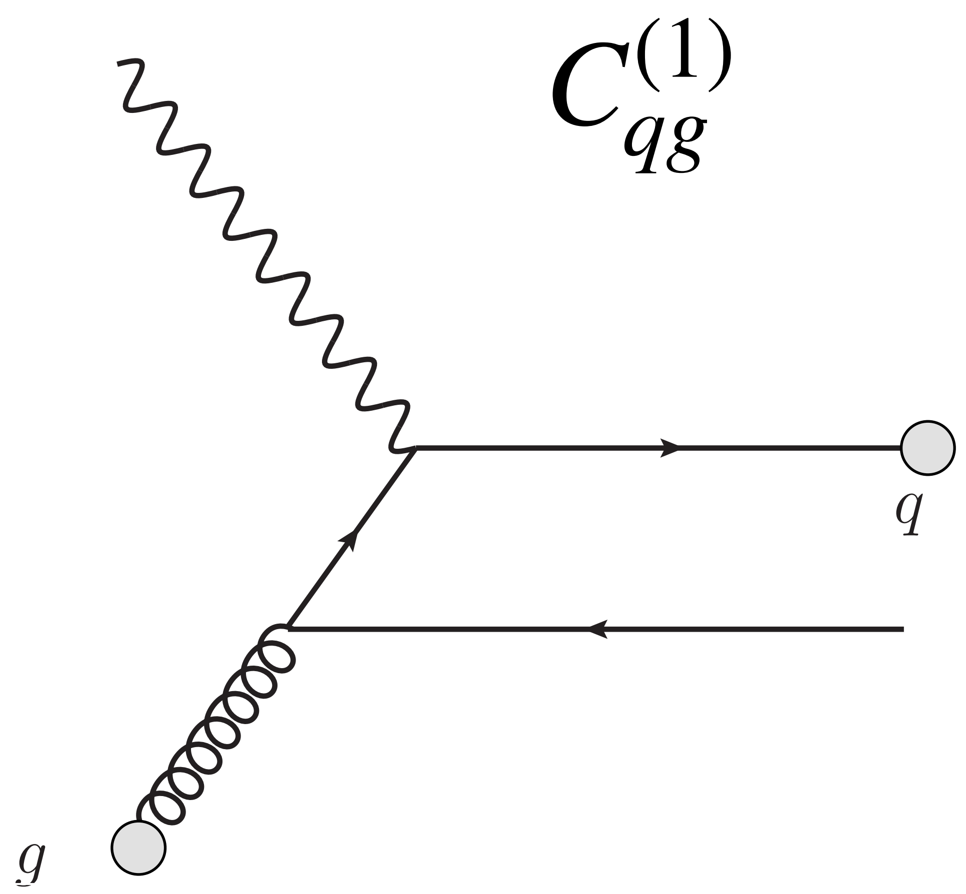


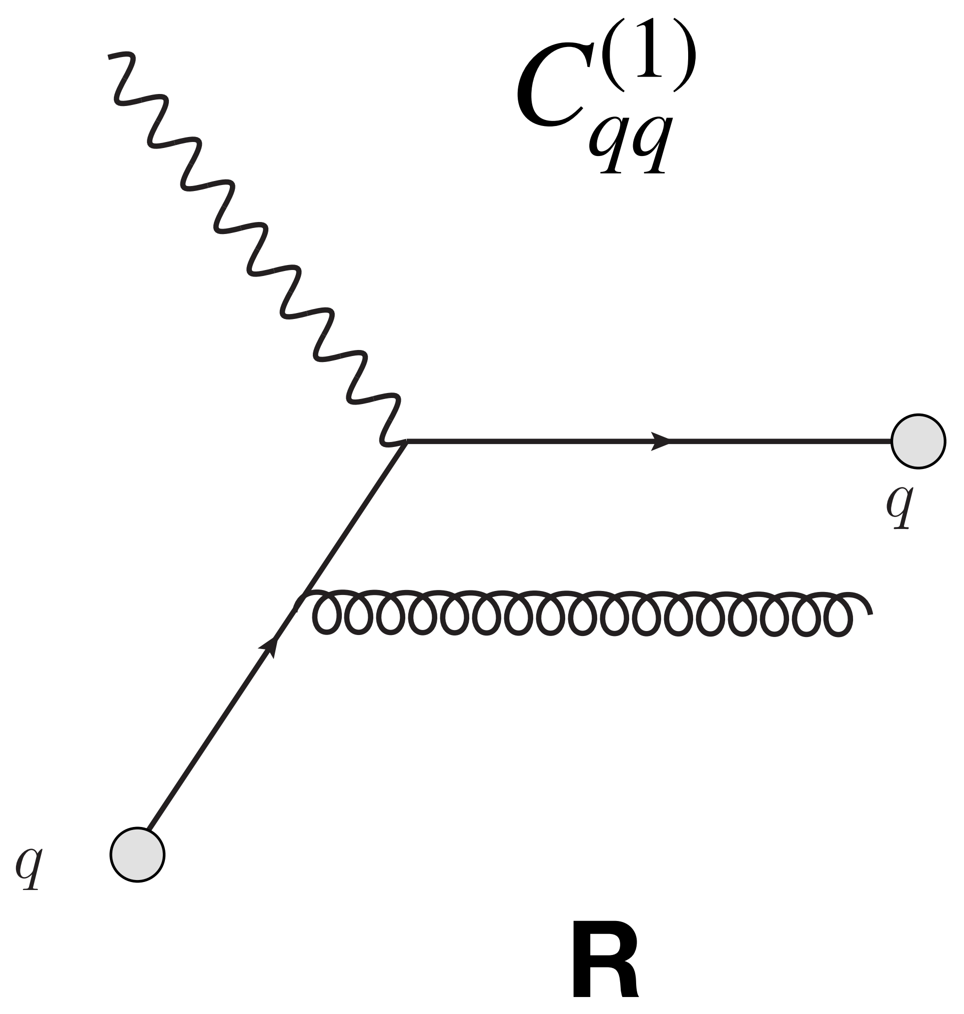
← Final-state $\hat{z}$

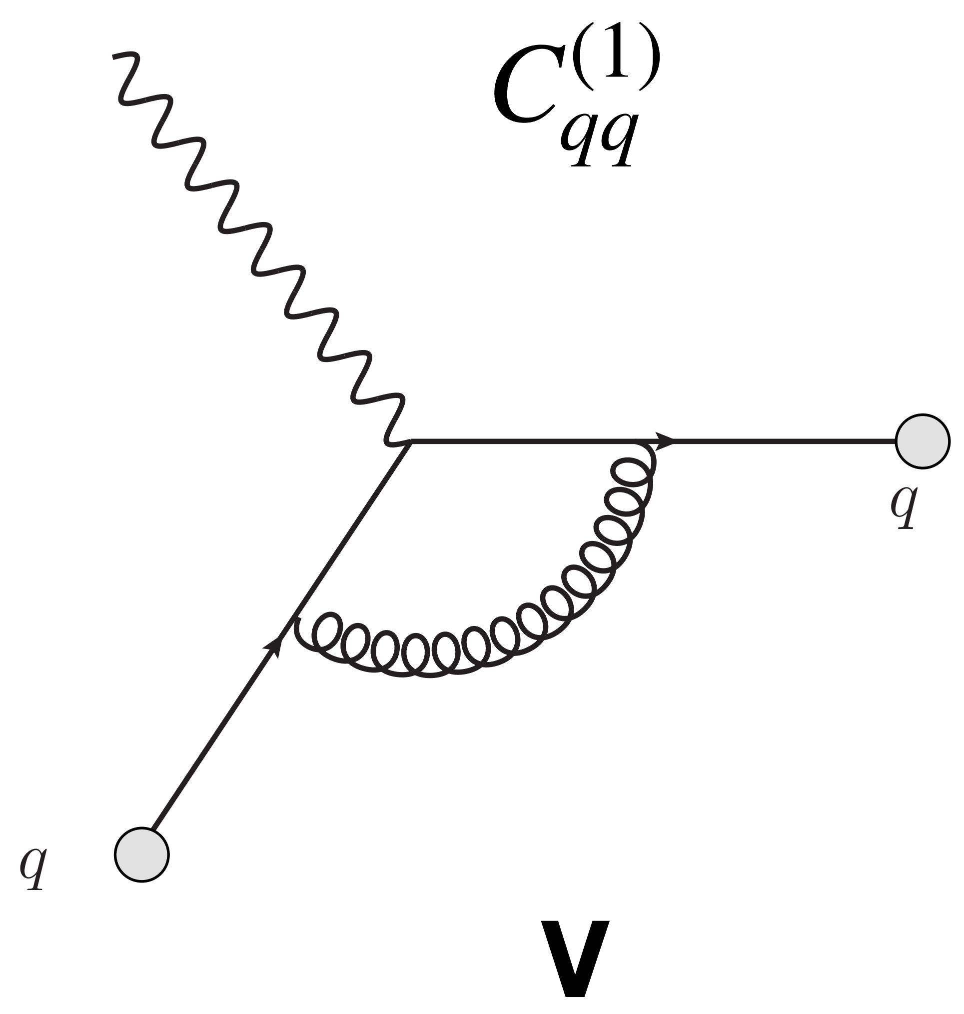
Initial-state $\hat{x}$ →

NLO









QCD corrections

NNLO corrections: channel decomposition

[LB, Gehrmann, Stagnitto 2401.16281]: all NNLO corrections

Four new channels @ NNLO

$$C_{qq}^{i,(2)} = C_{\bar{q}\bar{q}}^{i,(2)} = e_q^2 C_{qq}^{i,\mathrm{NS}} + \left(\sum_j e_{q_j}^2\right) C_{qq}^{i,\mathrm{PS}},$$

$$C_{\bar{q}q}^{i,(2)} = C_{q\bar{q}}^{i,(2)} = e_q^2 C_{\bar{q}q}^{i},$$

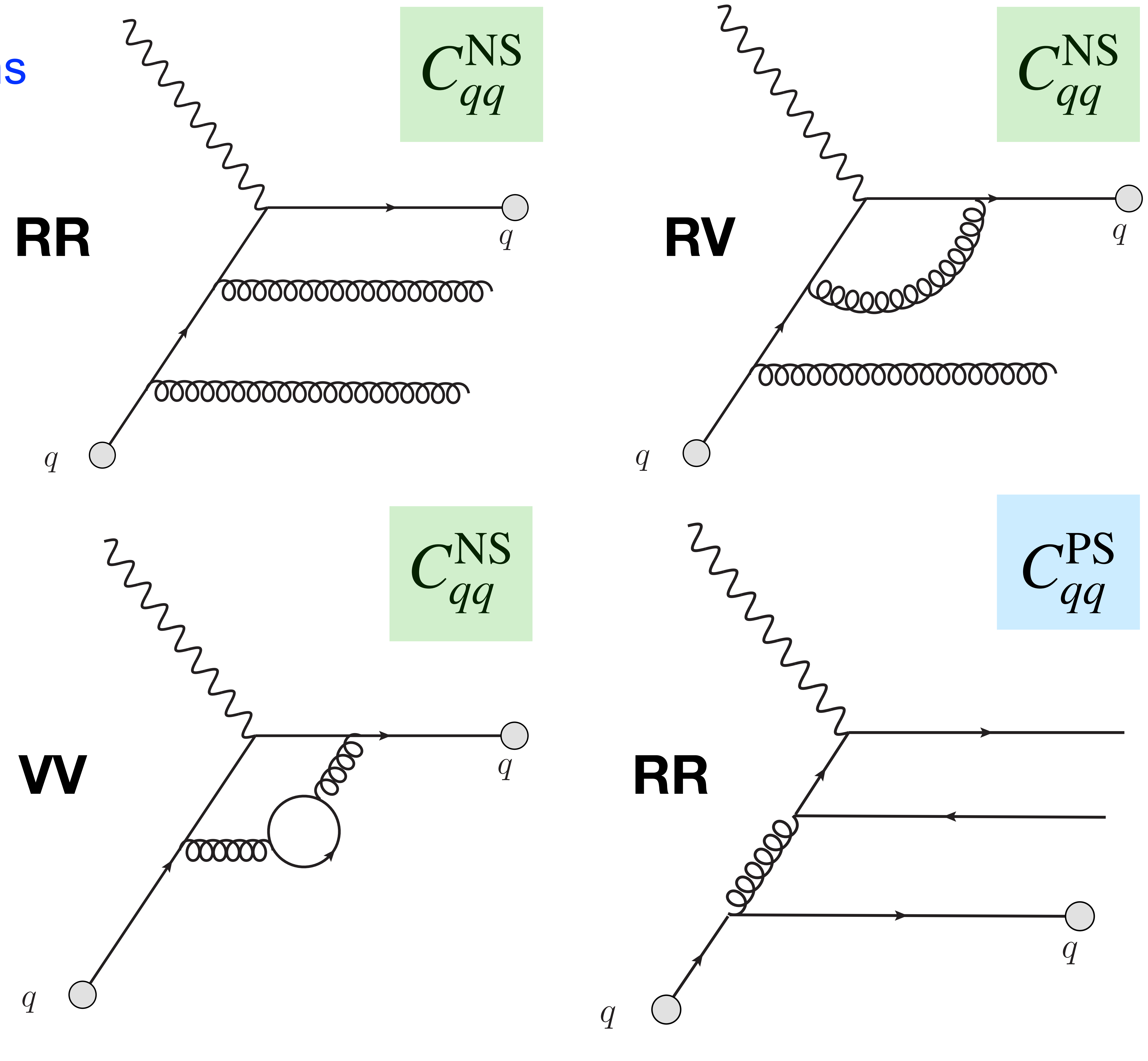
$$C_{q'q}^{i,(2)} = C_{\bar{q}'\bar{q}}^{i,(2)} = e_q^2 C_{q'q}^{i,1} + e_{q'}^2 C_{q'q}^{i,2} + e_q e_{q'} C_{q'q}^{i,3},$$

$$C_{\bar{q}'q}^{i,(2)} = C_{q'\bar{q}}^{i,(2)} = e_q^2 C_{q'q}^{i,1} + e_{q'}^2 C_{q'q}^{i,2} - e_q e_{q'} C_{q'q}^{i,3},$$

$$C_{gq}^{i,(2)} = C_{g\bar{q}}^{i,(2)} = e_q^2 C_{gq}^{i},$$

$$C_{qg}^{i,(2)} = C_{\bar{q}g}^{i,(2)} = e_q^2 C_{qg}^{i},$$

$$C_{gg}^{i,(2)} = \left(\sum_j e_{q_j}^2\right) C_{gg}^{i},$$



QCD corrections

NNLO corrections: channel decomposition

[LB, Gehrmann, Stagnitto 2401.16281]

Four new (RR) channels @ NNLO

$$C_{qq}^{i,(2)} = C_{\bar{q}\bar{q}}^{i,(2)} = e_q^2 C_{qq}^{i,\mathrm{NS}} + \left(\sum_j e_{q_j}^2\right) C_{qq}^{i,\mathrm{PS}},$$

New $$C_{\bar{q}q}^{i,(2)} = C_{q\bar{q}}^{i,(2)} = e_q^2 C_{\bar{q}q}^{i},$$

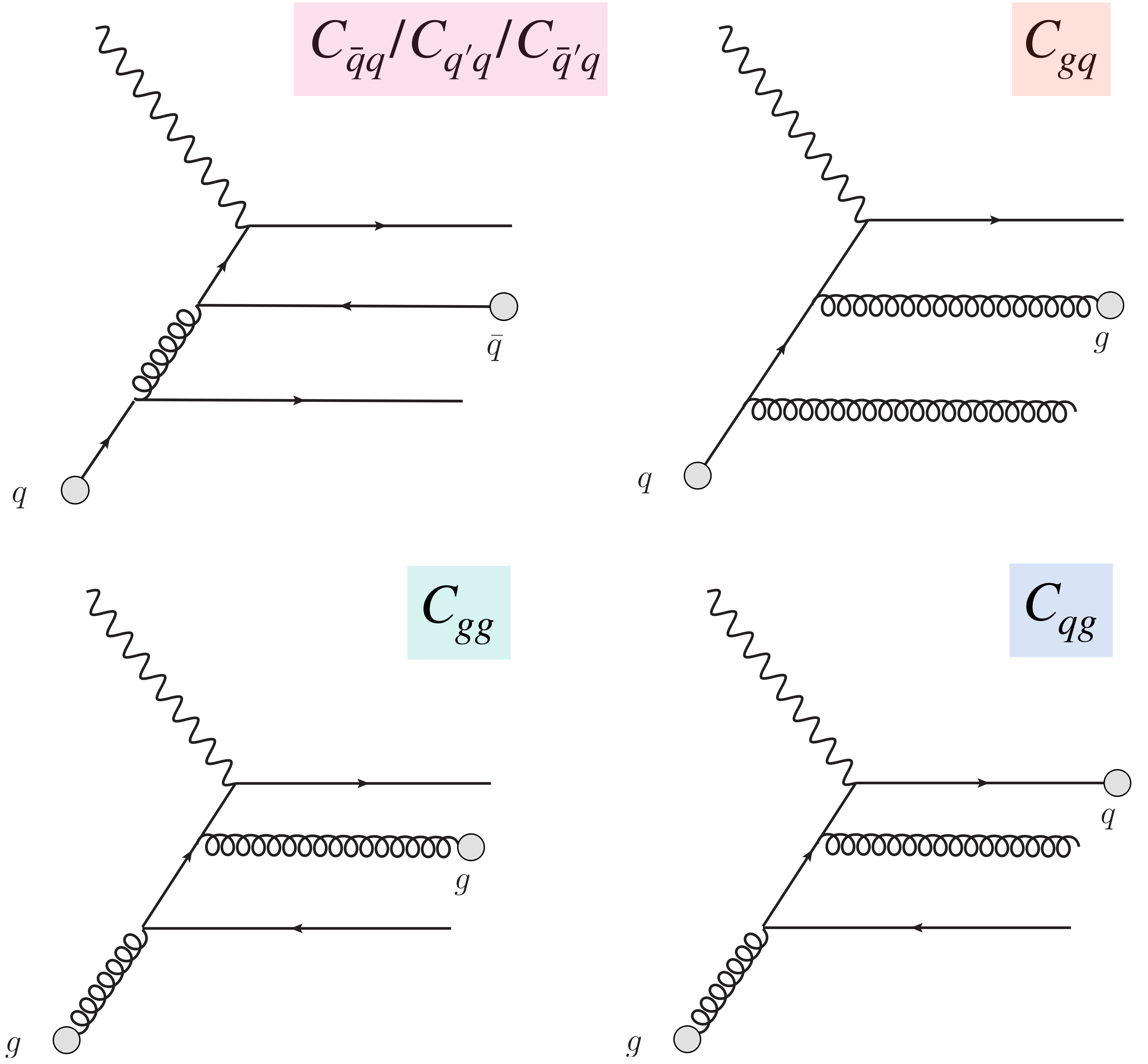
New $$C_{q'q}^{i,(2)} = C_{\bar{q}'\bar{q}}^{i,(2)} = e_q^2 C_{q'q}^{i,1} + e_{q'}^2 C_{q'q}^{i,2} + e_q e_{q'} C_{q'q}^{i,3},$$

New $$C_{\bar{q}'q}^{i,(2)} = C_{q'\bar{q}}^{i,(2)} = e_q^2 C_{q'q}^{i,1} + e_{q'}^2 C_{q'q}^{i,2} - e_q e_{q'} C_{q'q}^{i,3},$$

$$C_{gq}^{i,(2)} = C_{g\bar{q}}^{i,(2)} = e_q^2 C_{gq}^{i},$$

$$C_{qg}^{i,(2)} = C_{\bar{q}g}^{i,(2)} = e_q^2 C_{qg}^{i},$$

New $$C_{gg}^{i,(2)} = \left(\sum_j e_{q_j}^2\right) C_{gg}^{i},$$



QCD corrections

Check with approximate NNLO results

Our results in agreement with:

- [Abele, de Florian, Vogelsang '21]: aNNLO corrections to $q \to q$ channel from threshold resummation formalism
  - $\hat{x} \to 1$ and $\hat{z} \to 1$: large double-logarithmic terms
- [Goyal, Moch, Pathak, Rana, Ravindran '23]: leading colour contribution to non-singlet $q \to q$ channel

Further checks:

- Scale dependent terms are found to be as predicted by RGE
- Integrated specific subprocess contributions over final-state momentum $\hat{z}$ and recover respective contribution to inclusive result

Numerical results

K-factors up to NNLO

$$\frac{d^3\sigma^{\pi^+}}{dxdydz} = \frac{4\pi\alpha^2}{Q^2}\left[\frac{1+(1-y)^2}{2y}\mathcal{F}_T^{\pi^+}(x,z,Q^2) + \frac{1-y}{y}\mathcal{F}_L^{\pi^+}(x,z,Q^2)\right]$$

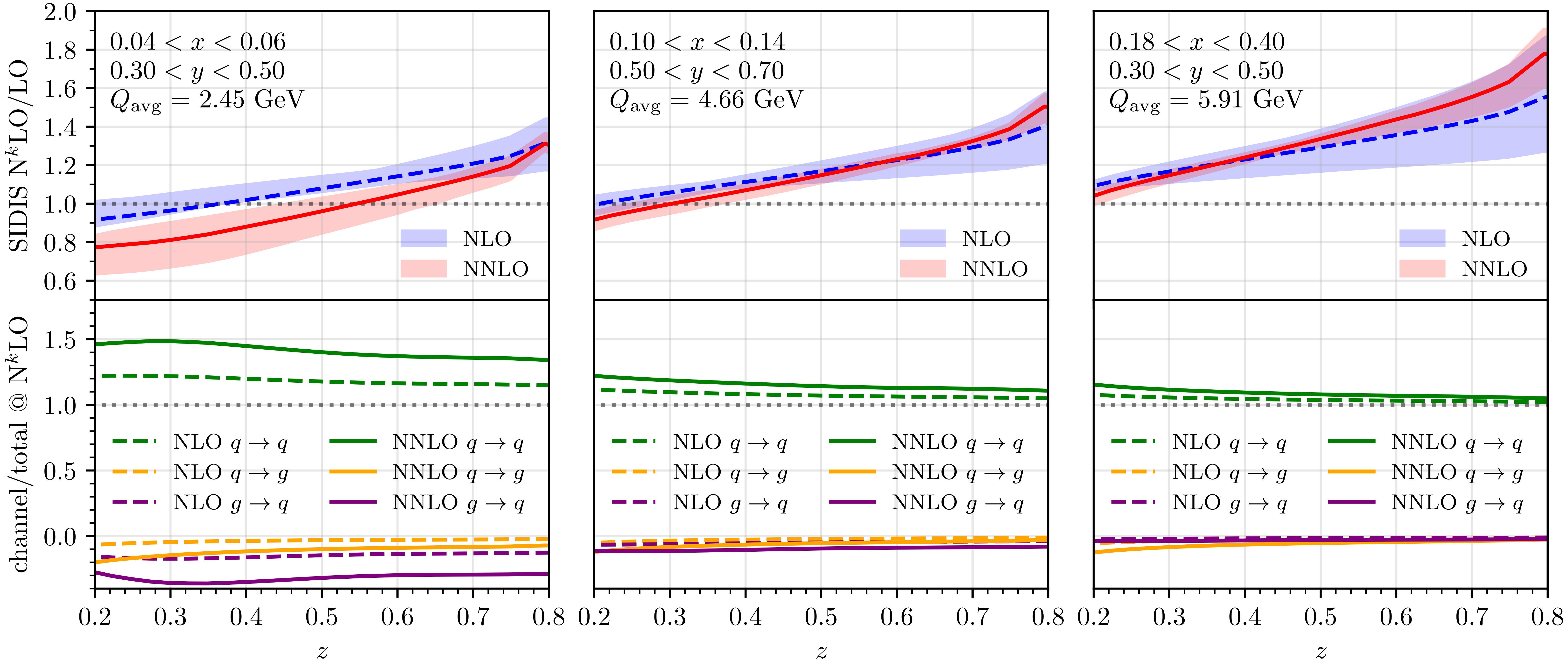
SIDIS $\mathrm{N^kLO/LO}$ with channels decomposition for π^+

- **NNPDF3.1** PDF set [NNPDF '17]
- **BDSSV** FF set [Borsa, Sassot, De Florian, Stratmann, Vogelsang '22]
- 7-point scales variation ($\mu_A = \mu_F$)
- Only dominant channels presented

COMPASS kinematics and cuts:

$\sqrt{s} = 17.35\,\mathrm{GeV}$

- $Q^2 > 1\,\mathrm{GeV}^2$ and $W > 5\,\mathrm{GeV}$



- Good perturbative convergence increasing Q^2
- Gluonic channels play a role for lower Q^2

Comparison with data

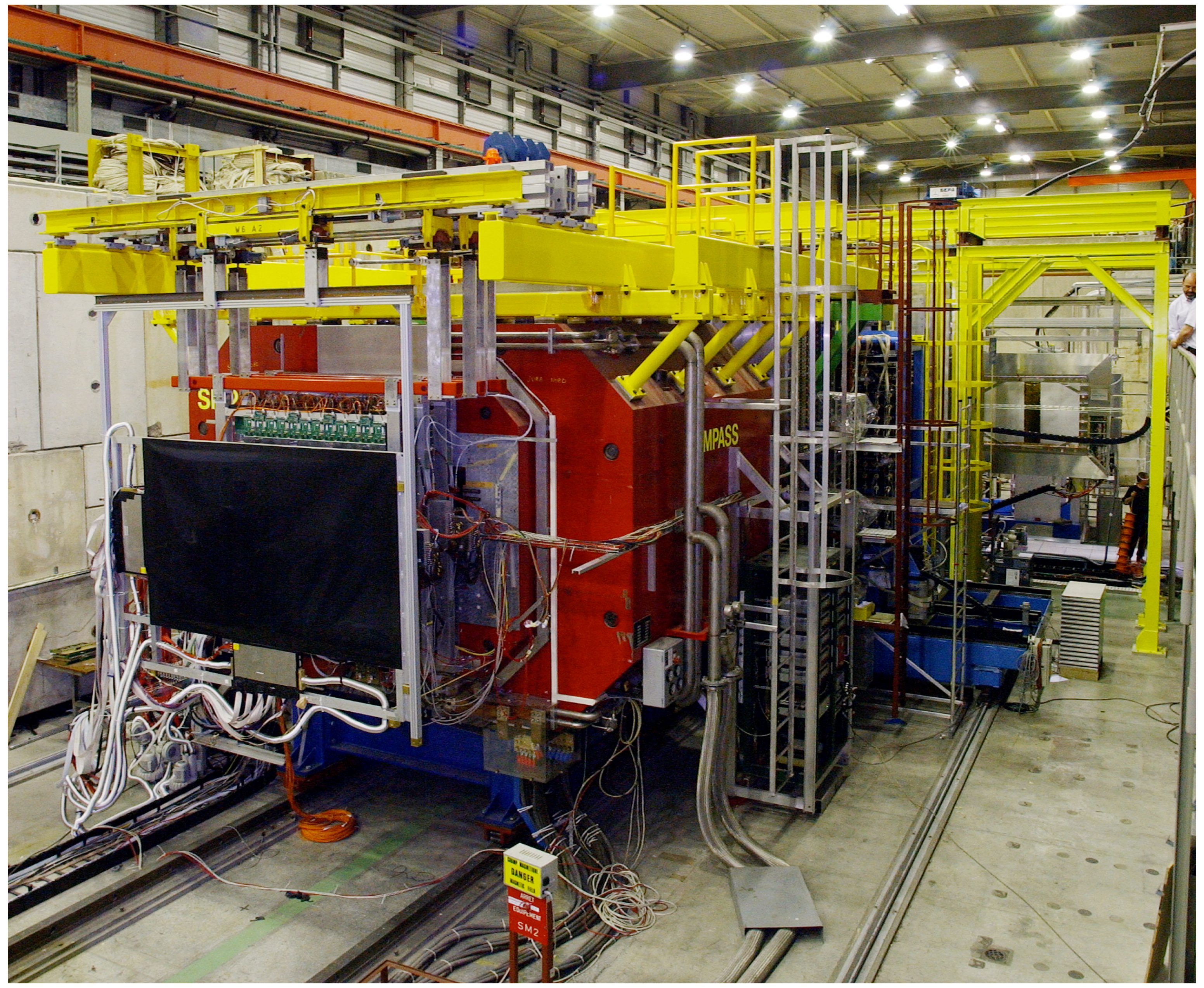
The COMPASS experiment

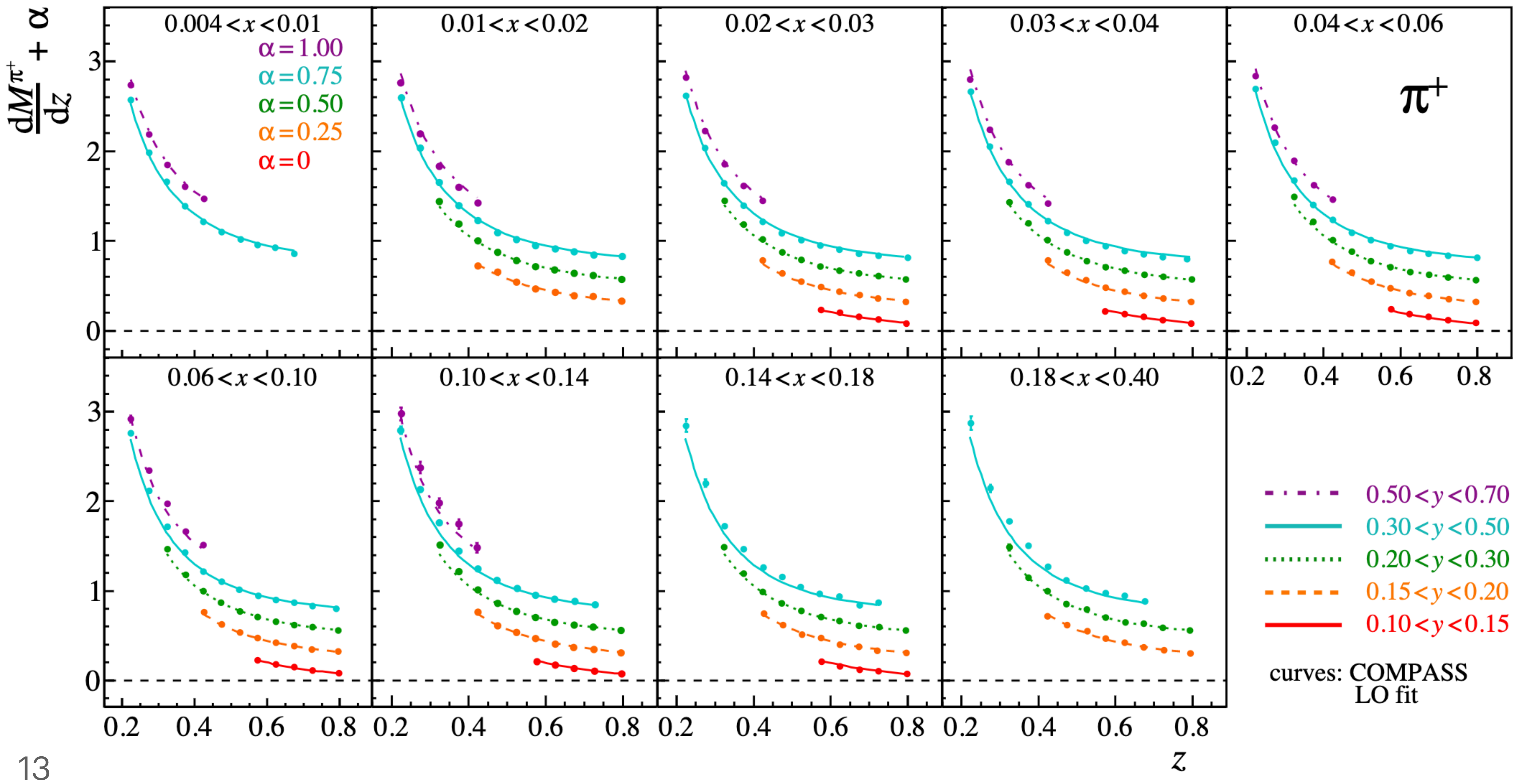
"COmmon Muon Proton Apparatus for Structure and Spectroscopy" @ CERN

COMPASS [COMPASS '16] kinematics

- $160\,\mathrm{GeV}$ μ-beam on fixed isoscalar target ($^6\mathrm{LiD}$)
- $\sqrt{s} \approx 17.35\,\mathrm{GeV}$
- Events accepted if $Q^2 > 1\,\mathrm{GeV}^2$ and $W > 5\,\mathrm{GeV}$

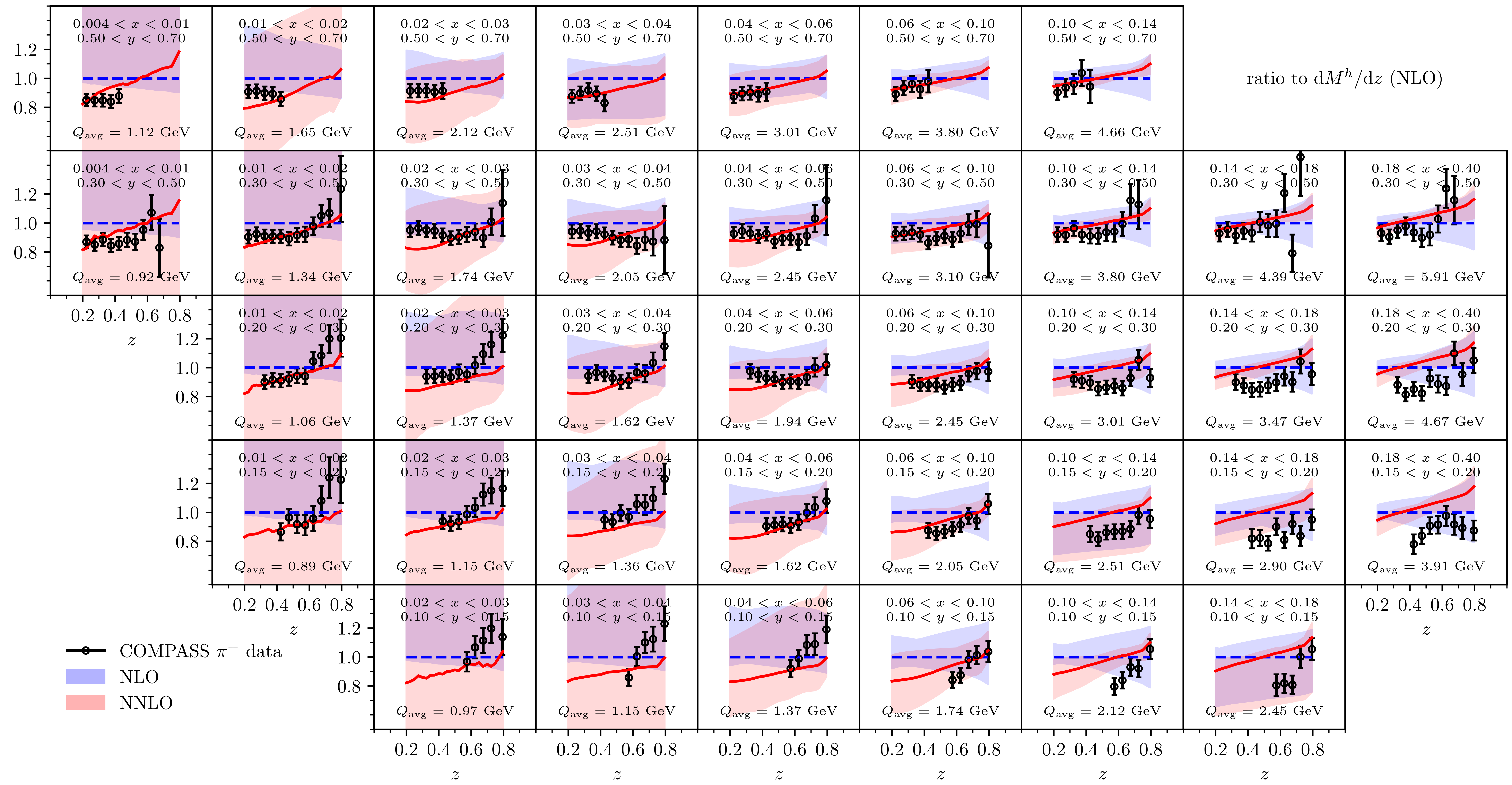
Focus on π^+ production $\longrightarrow$





Comparison with data

π^+ production: ratio to NLO



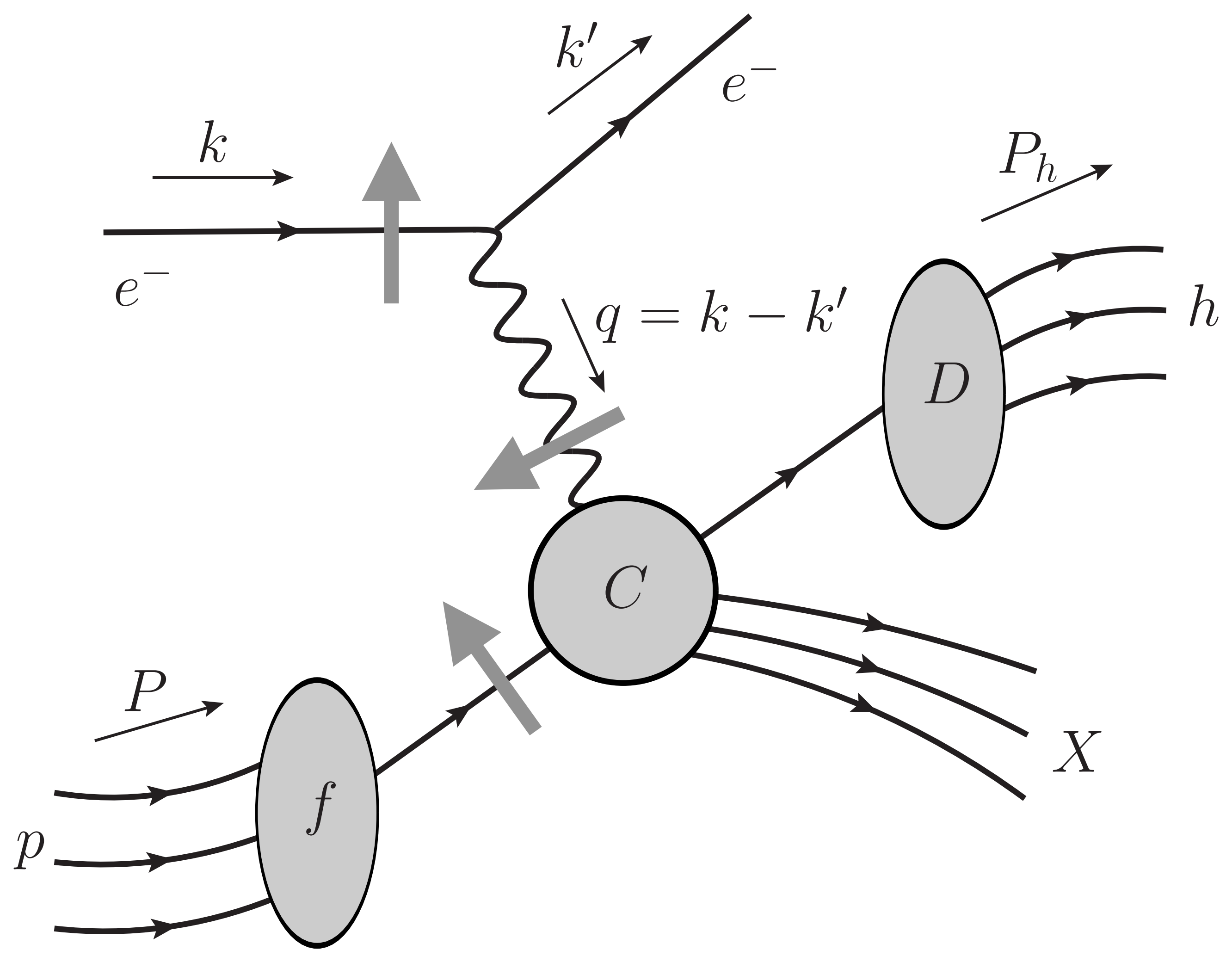
- SIDIS/DIS with DIS from APFEL++ [Bertone '17]
- x and y bins integrated
- Ratio to NLO

$$\frac{\mathrm{d}M^{\pi^+}}{\mathrm{d}z} = \int_x \int_y \frac{\mathrm{d}^3\sigma^{\pi^+}/\mathrm{d}x\mathrm{d}y\mathrm{d}z}{\mathrm{d}^2\sigma^{DIS}/\mathrm{d}x\mathrm{d}y}$$

NNLO improves description of data in many bins

Kinematics of polSIDIS

Structure function and asymmetry



Scattering of longitudinally polarised lepton and nucleon

- Same kinematics and channel decomposition as unpolarised SIDIS
- Relevant SF for neutral-current polarised semi-inclusive deep inelastic scattering ($g_2^h \approx 0$)

$$2g_1^h(x,z,Q^2) = \sum_{p,p'} \int_x^1 \frac{\mathrm{d}\hat{x}}{\hat{x}} \int_z^1 \frac{\mathrm{d}\hat{z}}{\hat{z}} \Delta f_p\left(\frac{x}{\hat{x}}, \mu_F^2\right) D_{p'}^h\left(\frac{z}{\hat{z}}, \mu_A^2\right) \Delta\mathscr{C}_{p'p}\left(\hat{x}, \hat{z}, Q^2, \mu_R^2, \mu_F^2, \mu_A^2\right)$$

- $\Delta\mathscr{C}_{p'p} = \Delta C_{p'p}^{(0)} + \frac{\alpha_s(\mu_R^2)}{2\pi}\Delta C_{p'p}^{(1)} + \left(\frac{\alpha_s(\mu_R^2)}{2\pi}\right)^2 \Delta C_{p'p}^{(2)} + \mathscr{O}(\alpha_s^3)\,.$

Same channel decomposition and same result from [Goyal, Lee, Moch, Pathak, Rana, Ravindran ’24] ✓

Our focus: all coefficients $\Delta C_{p'p}^{(2)}$

[LB, Gehrmann, Löchner, Schönwald, Stagnitto 2404.08597]

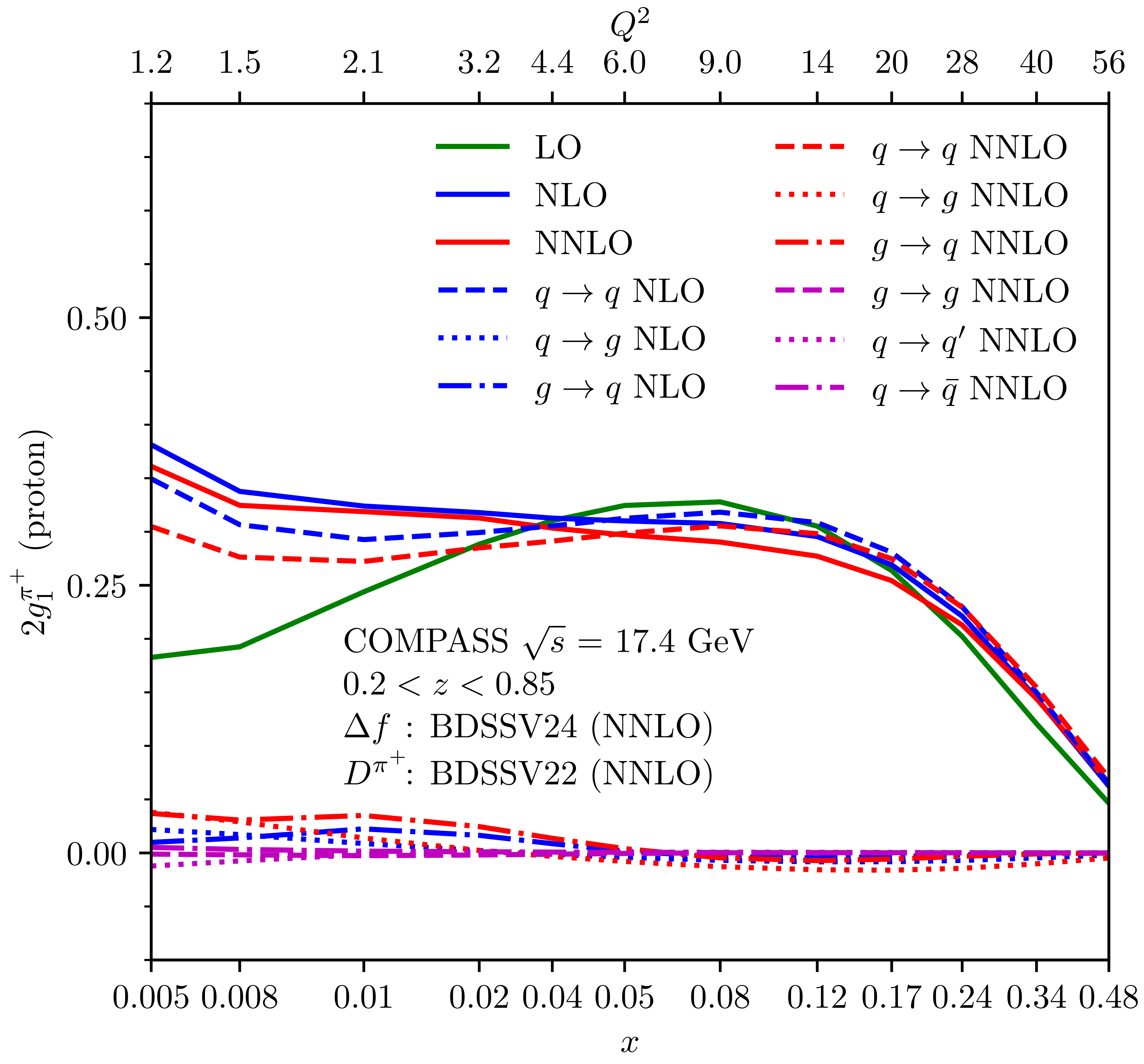
Typical observable: SIDIS double spin asymmetry $A_1^h(x,z,Q^2) = \frac{g_1^h(x,z,Q^2)}{F_1^h(x,z,Q^2)}$ with $2F_1^h = \mathscr{F}_T^h$

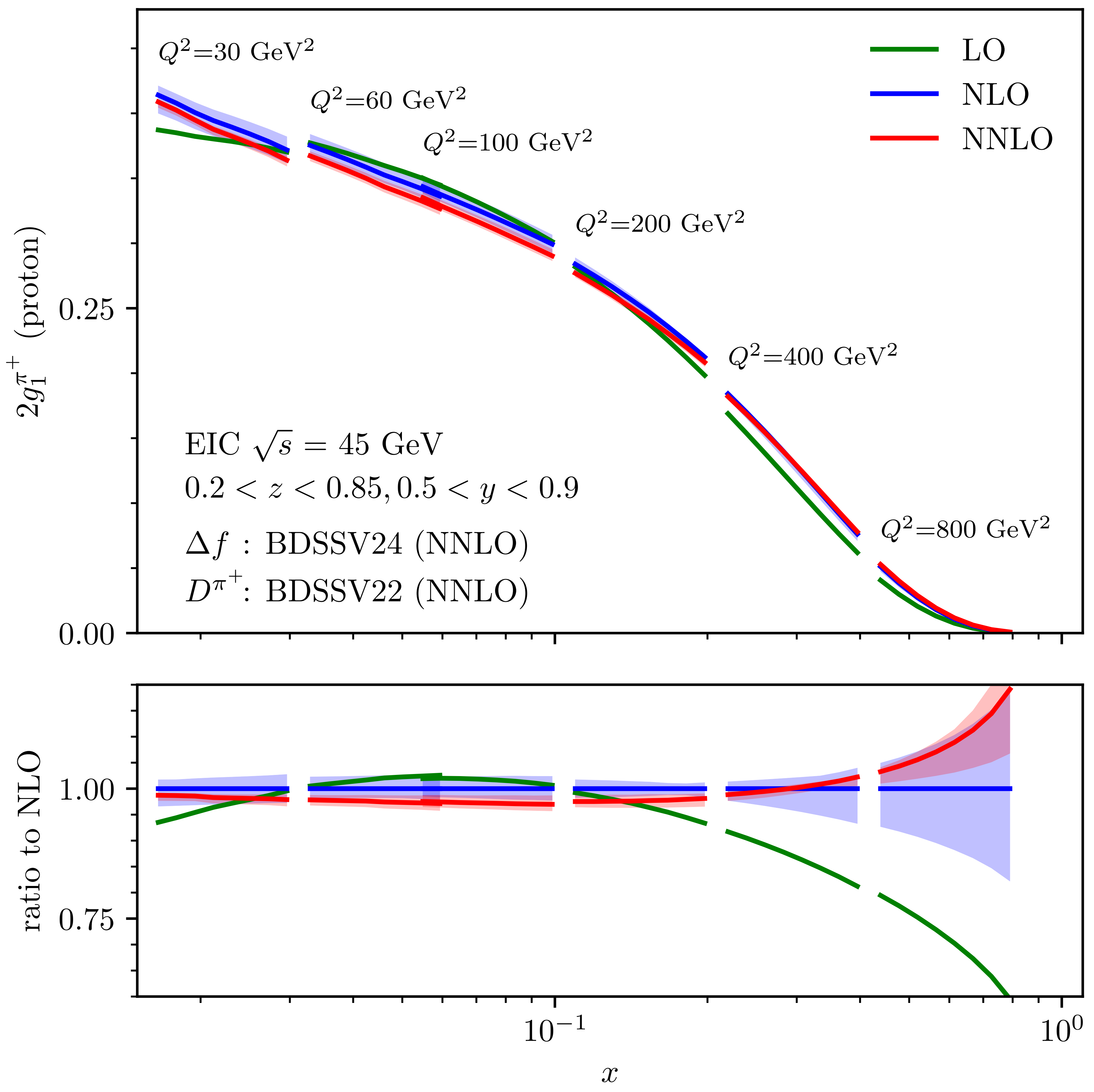
- $A_1^h = \frac{\sigma_{1/2}^h - \sigma_{3/2}^h}{\sigma_{1/2}^h + \sigma_{3/2}^h}$ is measured, with $\sigma_{1/2}^h$ photo–absorption xs for photons with spin antiparallel to target nucleon spin

Numerical results

$g_1^{\pi^+}$ with channels decomposition for COMPASS kinematics [COMPASS '10]

- aNNLO polPDFs and FFs from BDSSV
- $g \to q$ and $q \to g$ more important at NNLO
- Sizeable corrections at small x and Q^2



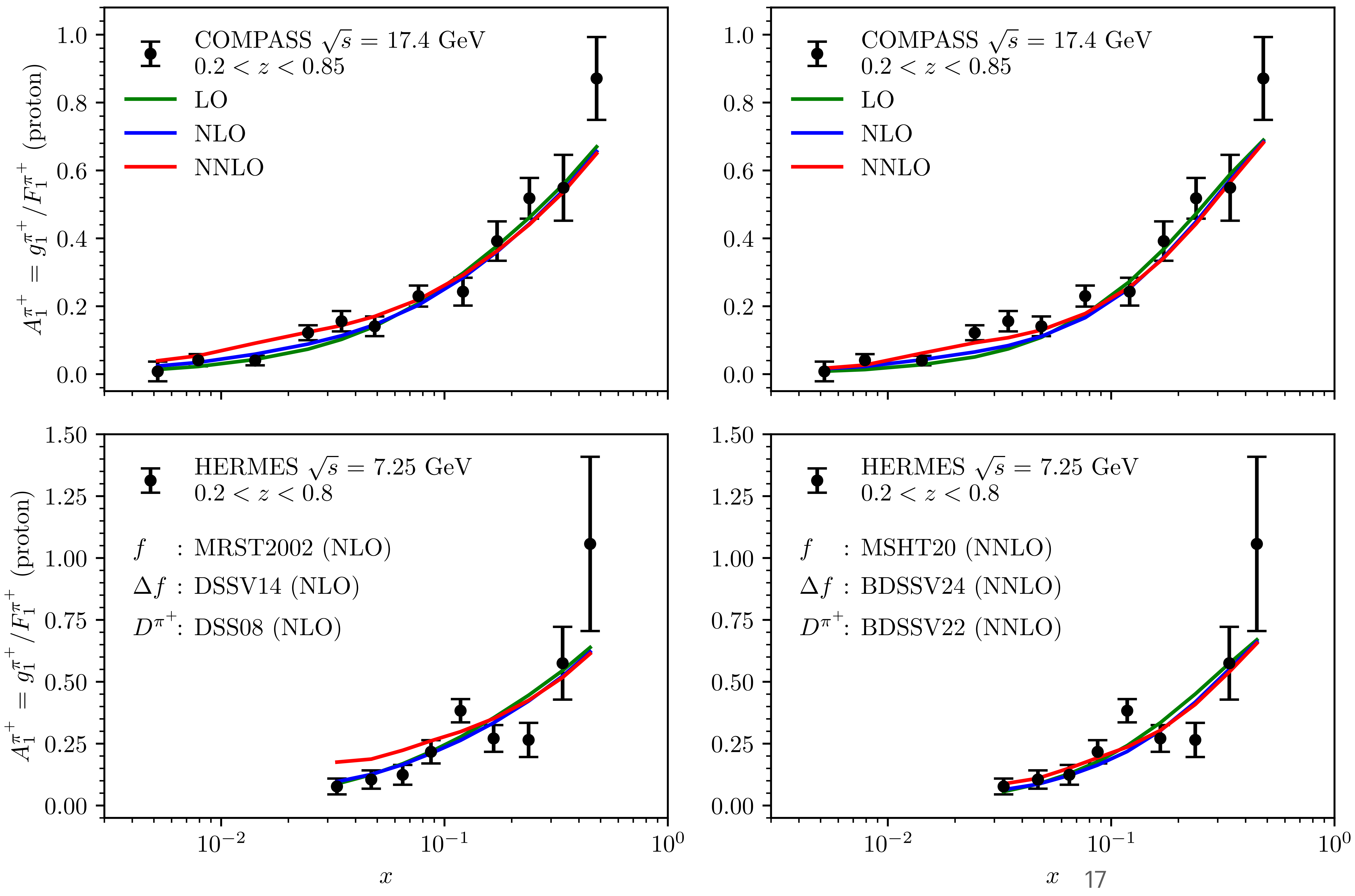


$g_1^{\pi^+}$ with for EIC $\sqrt{s} = 45\,\text{GeV}$

- Collider-like kinematic cuts on y
- aNNLO polPDFs and FFs from BDSSV
- 7 point scale variation
- Good perturbative convergence

Comparison with data

π^+ production at COMPASS and HERMES



$$A_1^{\pi^+}(x,z,Q^2) = \frac{g_1^{\pi^+}(x,z,Q^2)}{F_1^{\pi^+}(x,z,Q^2)}$$

- [COMPASS '10] [HERMES '18]
- Comparing DSSV (NLO) and BDSSV (aNNLO) sets
- Effects in small x region due to (mainly) polPDFs

Improvement with NNLO? Only a global full-NNLO fit will tell ...

Conclusions

- Full set of NNLO QCD corrections to polarised and unpolarised SIDIS now available
- aNNLO $\neq$ full NNLO: gluonic channels and small-x region fundamental
- Promising improvement in description of data for π^+ for SIDIS, polSIDIS?
- Universal ingredients to study light and heavy hadron production at colliders
    - Improvement in global fits for FFs and polPDFs
    - NNLO frontier needed for EIC phenomenology

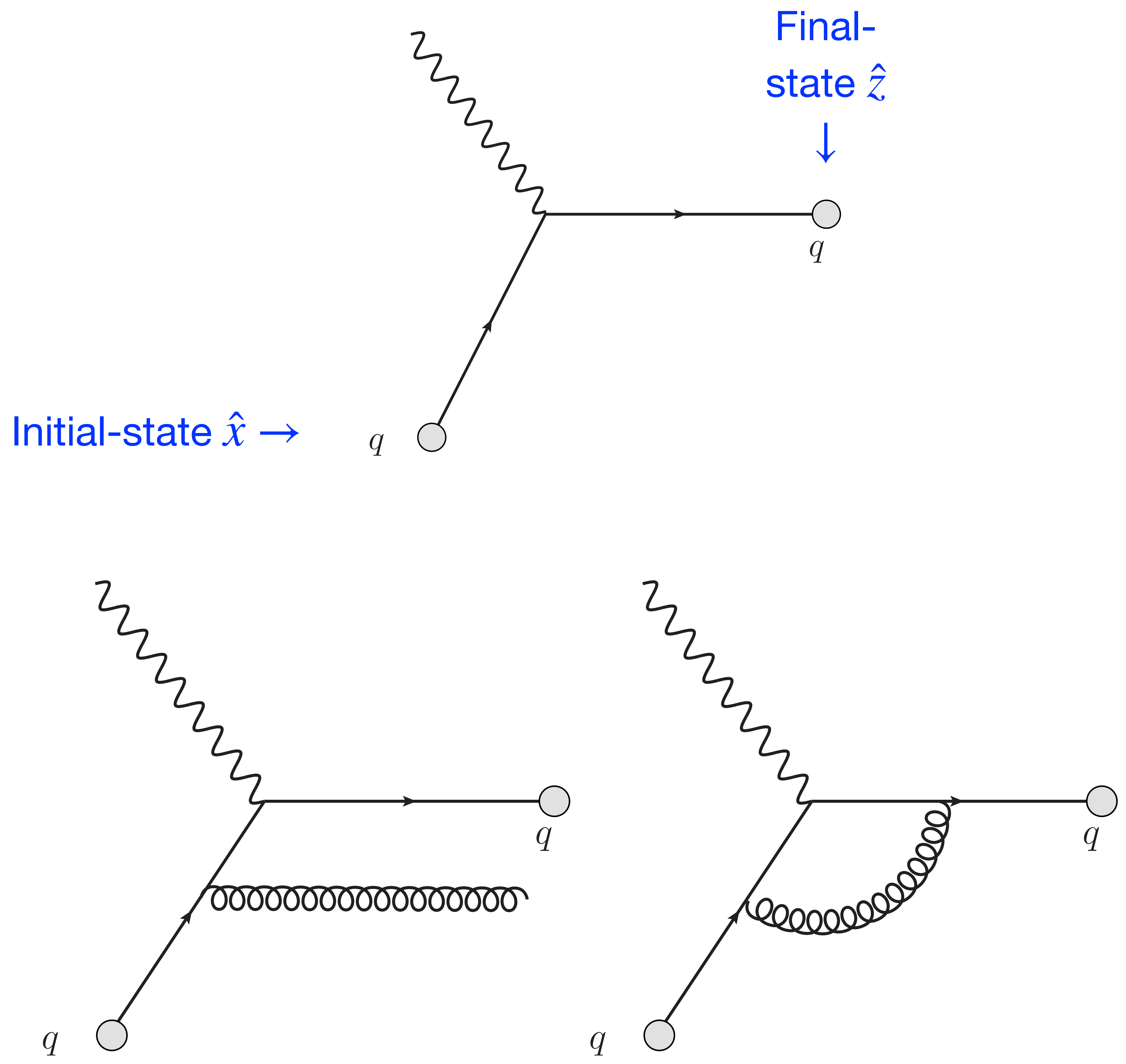
QCD corrections

LO and NLO results

- LO: $C_{qq}^{T,(0)} = e_q^2\delta(1-\hat{x})\delta(1-\hat{z})\,, \quad C_{qq}^{L,(0)} = 0$
- NLO: [Altarelli, Ellis, Martinelli, Pi '79] [Baier, Fey '79]
  Screenshots from [Anderle, Ringer, Vogelsang '12]

$$\begin{aligned}
C_{qq}^{T,(1)}(\hat{x},\hat{z}) \;=\; & e_q^2 C_F\Bigg[-8\delta(1-\hat{x})\delta(1-\hat{z}) \\
& +\delta(1-\hat{x})\left[\tilde{P}_{qq}(\hat{z})\ln\frac{Q^2}{\mu_F^2}+L_1(\hat{z})+L_2(\hat{z})+(1-\hat{z})\right] \\
& +\delta(1-\hat{z})\left[\tilde{P}_{qq}(\hat{x})\ln\frac{Q^2}{\mu_F^2}+L_1(\hat{x})-L_2(\hat{x})+(1-\hat{x})\right] \\
& +\frac{2}{(1-\hat{x})_+(1-\hat{z})_+}-\frac{1+\hat{z}}{(1-\hat{x})_+}-\frac{1+\hat{x}}{(1-\hat{z})_+} \\
& +2(1+\hat{x}\hat{z})\Bigg], \qquad (49)
\end{aligned}$$

$$C_{qq}^{L,(1)}(\hat{x},\hat{z}) \;=\; 4e_q^2 C_F\hat{x}\hat{z},$$



QCD corrections

Details of the calculation

VV: two-loops form-factors in space-like kinematics

RV: one-loop squared matrix elements in terms of one-loop bubble and box integrals (known in exact form in ϵ)

⚠️ Paying attention to analytical continuations

$$C^{\mathrm{RV}}_{j\leftarrow i} \propto \int \mathrm{d}\Phi_2(k_j, k_k; k_i, q)\, \delta\left(z - x\frac{(k_i + k_j)^2}{Q^2}\right) \left|\mathcal{M}^{\mathrm{RV}}\right|^2 \propto \mathcal{J}(x,z) \, \left|\mathcal{M}^{\mathrm{RV}}\right|^2 (x,z)$$

- Fixed $\hat{x}$ and $\hat{z}$ → phase space integral fully constrained → only expansion in end-point distributions $\hat{x} = 1$ and $\hat{z} = 1$
- From (photon) fragmentation antenna functions [Gehrmann, Schürmann '22]

RR: integrations over three-particle phase space with multi-loop techniques

$$C^{\mathrm{RR}}_{j\leftarrow i} \propto \int \mathrm{d}\Phi_3(k_j, k_k, k_l; k_i, q)\, \delta\left(z - x\frac{(k_i + k_j)^2}{Q^2}\right) \left|\mathcal{M}^{\mathrm{RR}}\right|^2$$

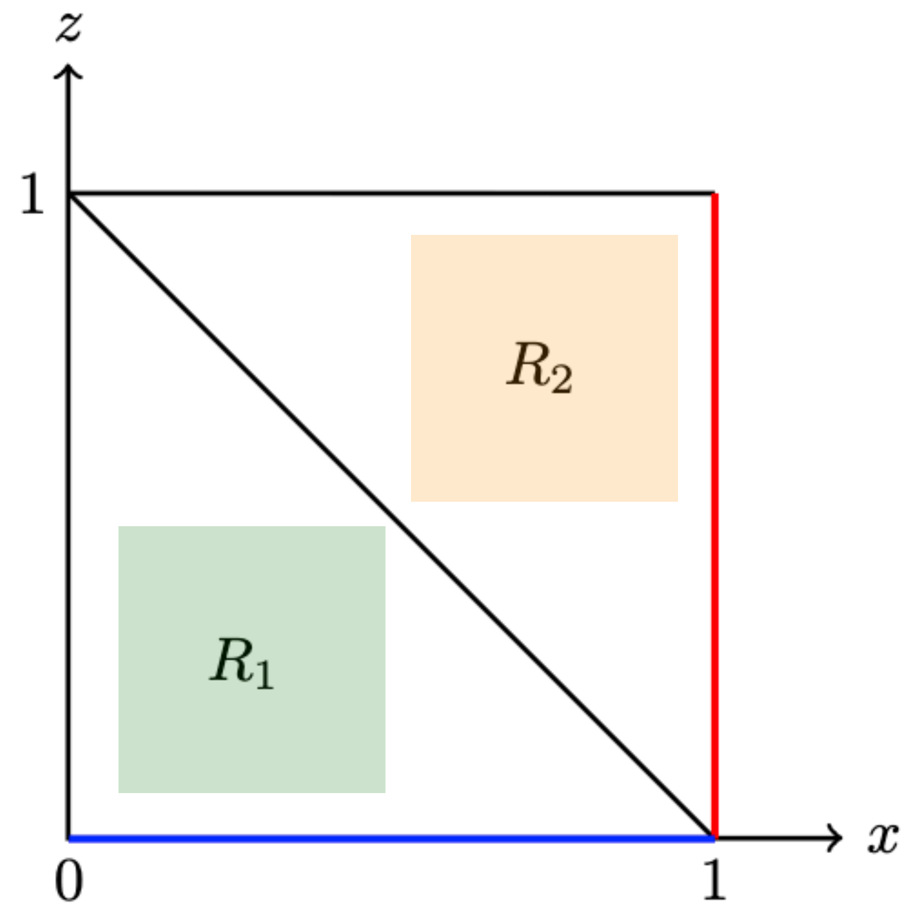
- Reduction to master integrals (MI) with IBP identities → 21 MI
- From fragmentation antenna functions [LB, Marcoli, Gehrmann, Schürmann, Stagnitto 2406.09925]

family	master	deepest pole	at $x=1$	at $z=1$
	$I[0]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
A	$I[5]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[2,3,5]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
B	$I[7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[-2,7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[-3,7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[2,3,7]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
C	$I[5,7]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[3,5,7]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
D	$I[1]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[1,4]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[1,3,4]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
E	$I[1,3,5]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
G	$I[1,3,8]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
H	$I[1,4,5]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
I	$I[2,4,5]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
J	$I[4,7]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[3,4,7]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
K	$I[3,5,8]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
L	$I[4,5,7]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
M	$I[4,5,8]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$

Table 1. Summary of the double real radiation master integrals.

QCD corrections

Analytic continuation in RV contribution



Avoid ambiguities associated with the analytic continuation of box integrals: segment the (x, z)-plane into four sectors → expressions real and continuous across boundaries [Gehrmann, Schürmann ’22]

- Example: $\text{Box}(s_{12}, s_{23})$

$$\begin{aligned}
&\text{Box}(s_{ij}, s_{ik}) \\
&= \frac{2(1-2\epsilon)}{\epsilon} A_{2,LO} \frac{1}{s_{ij}s_{ik}} \\
&\quad \times \Bigg[\left(\frac{s_{ij}s_{ik}}{s_{ij}-s_{ijk}}\right)^{-\epsilon} {}_2F_1\left(-\epsilon, -\epsilon; 1-\epsilon; \frac{s_{ijk}-s_{ij}-s_{ik}}{s_{ijk}-s_{ij}}\right) \\
&\quad + \left(\frac{s_{ij}s_{ik}}{s_{ik}-s_{ijk}}\right)^{-\epsilon} {}_2F_1\left(-\epsilon, -\epsilon; 1-\epsilon; \frac{s_{ijk}-s_{ij}-s_{ik}}{s_{ijk}-s_{ik}}\right) \\
&\quad - \left(\frac{-s_{ijk}s_{ij}s_{ik}}{(s_{ij}-s_{ijk})(s_{ik}-s_{ijk})}\right)^{-\epsilon} {}_2F_1\left(-\epsilon, -\epsilon; 1-\epsilon; \frac{s_{ijk}(s_{ijk}-s_{ij}-s_{ik})}{(s_{ijk}-s_{ij})(s_{ijk}-s_{ik})}\right) \Bigg]
\end{aligned}$$

R_1

$$\begin{aligned}
a_1(s_{12}, s_{23}) &= \frac{s_{123}-s_{12}-s_{23}}{s_{123}-s_{12}} = -\frac{z}{1-x-z}\,, \\
a_2(s_{12}, s_{23}) &= \frac{s_{123}-s_{12}-s_{23}}{s_{123}-s_{23}} = z\,, \\
a_3(s_{12}, s_{23}) &= \frac{s_{123}\,s_{13}}{(s_{13}+s_{23})(s_{12}+s_{13})} = -\frac{x\,z}{1-x-z}
\end{aligned}$$

R_2

$$\begin{aligned}
\tilde{a}_1(s_{12}, s_{23}) &= 1 - \frac{1}{a_1(s_{12}, s_{23})} = \frac{1-x}{z}\,, \\
\tilde{a}_3(s_{12}, s_{23}) &= 1 - \frac{1}{a_3(s_{12}, s_{23})} = \frac{(1-x)(1-z)}{xz}
\end{aligned}$$

QCD corrections

RR contribution

[Gehrmann, Schürmann ’22]: limited set of RR ($\mathcal{X}_4^0$) antenna functions relevant to photon fragmentation

- Now full set of initial-final RR antenna functions [LB, Gehrmann, Schürmann, Stagnitto in preparation]
- 12 denominators (4 cut propagators) → 21 MI in 12 families
- Subset of integrated $2 \to 3$ antenna functions contribute to RR corrections of CF $C^{\mathrm{RR}}_{j \leftarrow i} \propto \mathcal{X}^{0,id.j}_{i,jkl}$
- MI solved with differential equations, boundary conditions from z-integration and comparing to inclusive result

$$
\begin{aligned}
D_1 &= (q-k_j)^2\,, \\
D_2 &= (p+q-k_j)^2\,, \\
D_3 &= (p-k_l)^2\,, \\
D_4 &= (q-k_l)^2\,, \\
D_5 &= (p+q-k_l)^2\,, \\
D_6 &= (q-k_j-k_l)^2\,, \\
D_7 &= (p-k_j-k_l)^2\,, \\
D_8 &= (k_j+k_l)^2\,, \\
D_9 &= k_j^2\,, \\
D_{10} &= k_l^2\,, \\
D_{11} &= (q+p-k_j-k_l)^2\,, \\
D_{12} &= (p-k_j)^2 + Q^2\frac{z}{x}\,,
\end{aligned}
$$

set of denominator factors

family	master	deepest pole	at $x=1$	at $z=1$
	$I[0]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
A	$I[5]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[2,3,5]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
B	$I[7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[-2,7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[-3,7]$	ϵ^0	$(1-x)^{1-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[2,3,7]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
C	$I[5,7]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{1-2\epsilon}$
	$I[3,5,7]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
D	$I[1]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[1,4]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[1,3,4]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
E	$I[1,3,5]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
G	$I[1,3,8]$	ϵ^{-2}	$(1-x)^{-2\epsilon}$	$(1-z)^{-1-2\epsilon}$
H	$I[1,4,5]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
I	$I[2,4,5]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
J	$I[4,7]$	ϵ^0	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
	$I[3,4,7]$	ϵ^{-1}	$(1-x)^{-2\epsilon}$	$(1-z)^{-2\epsilon}$
K	$I[3,5,8]$	ϵ^{-2}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
L	$I[4,5,7]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$
M	$I[4,5,8]$	ϵ^{-1}	$(1-x)^{-1-2\epsilon}$	$(1-z)^{-2\epsilon}$

Table 1. Summary of the double real radiation master integrals.

Example: $q + p \to k_j + k_l + k_k$ and $z = x(k_j+p)^2/(-q^2)$

$$I[-3,7] \propto \int d^dk_j d^dk_l \delta(D_9)\delta(D_{10})\delta(D_{11})\delta(D_{12})\frac{D_3}{D_7}$$

Details of the calculation

Larin and MSbar schemes

Projectors to extract g_1^h contain γ_5 and $\varepsilon^{\mu\nu\rho\sigma}$ [Zijlstra, van Neerven '94]

- Consistent treatment in d-reg → Larin prescription $\gamma_\mu \gamma_5 = \frac{i}{3!}\varepsilon_{\mu\nu\rho\sigma}\gamma^\nu\gamma^\rho\gamma^\sigma$ [Larin, Vermaseren '91] [Larin '93]

Different ME^2 but same integration of RR and RV contributions

- same MIs as for unpol SIDIS
- NLO V and NNLO VV from vector form factors

Mass factorisation in Larin → Larin-scheme coefficient functions $\Delta\mathscr{C}^L$

- Scheme transformation to $\overline{\mathrm{MS}}$ → $g_1 = \Delta\mathscr{C}^{\overline{\mathrm{MS}}} \otimes \Delta f^{\overline{\mathrm{MS}}} = (\Delta\mathscr{C}^L \otimes Z^{-1}) \otimes (Z \otimes \Delta f^L) = \Delta\mathscr{C}^L \otimes \Delta f^L$ [Matiounine, Smith, van Neerven '98] [Moch, Vermaseren, Vogt '14] → same for g_1^h
- Check NNLO scale dependence from RGE in both schemes

More checks against aNNLO and g_1 [Zijlstra, van Neerven '94], same result from [Goyal, Lee, Moch, Pathak, Rana, Ravindran '24] ✓

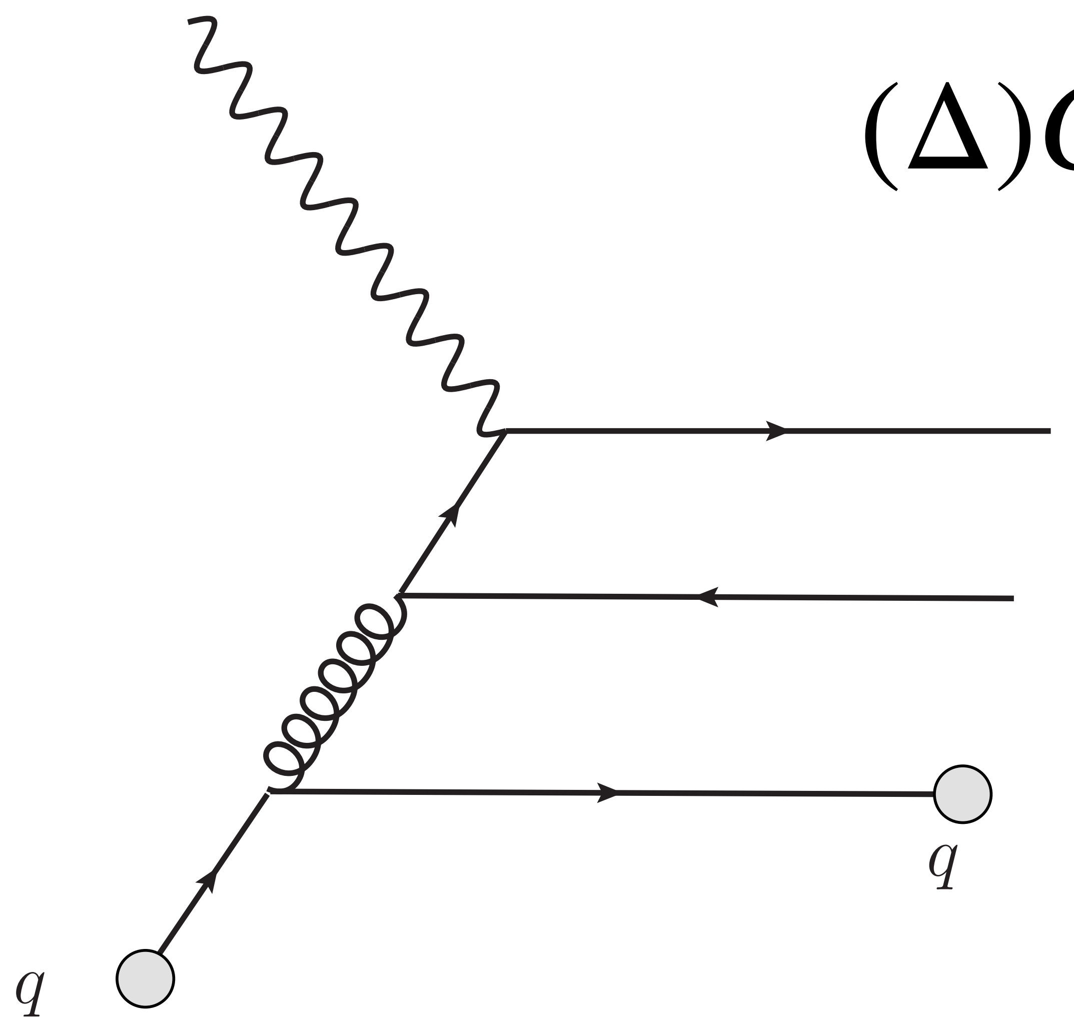
QCD corrections

Analytical results

Pure singlet qq channel

$$s_{x/z} = \sqrt{\frac{x}{z}}\ , \ \mathrm{Ti}_2(y) = \int_0^y \frac{\arctan x}{x} \mathrm{d}x$$

$(\Delta)C_{qq}^{\mathrm{PS}}$



$$\begin{aligned}
\Delta\mathscr{C}_{qq}^{\mathrm{PS}} = C_F \Bigg\{ & s_{x/z} P_1(x,z) \Big(-4\ln(s_{x/z})\arctan(s_{x/z}) + 4\ln(z s_{x/z})\arctan(z s_{x/z}) + 2\,\mathrm{Ti}_2(s_{x/z}) \\
& - 2\,\mathrm{Ti}_2(-s_{x/z}) - 4\,\mathrm{Ti}_2(z s_{x/z}) \Big) + \delta(1-z)\Big((x-3)\ln(x) + 2(x-1) - (x+2)\frac{1}{2}\ln(x)^2 \Big) \\
& - 8\frac{xz - x - z + 1}{z} + 3\frac{xz - x + z - 1}{z}\ln(x) + 3\frac{xz + x - z - 1}{z}\ln(z) \\
& - 2\frac{xz + x + z + 1}{z}\ln(x)\ln(z) \Bigg\}, \qquad (13)
\end{aligned}$$

$$\begin{aligned}
\mathscr{C}_{qq}^{T,\mathrm{PS}} = C_F \Bigg\{ & s_{x/z} P_2(x,z) \Big(-\frac{1}{8}\ln(s_{x/z})\arctan(s_{x/z}) + \frac{1}{8}\ln(z s_{x/z})\arctan(z s_{x/z}) - \frac{1}{8}\mathrm{Ti}_2(z s_{x/z}) \\
& + \frac{1}{16}\mathrm{Ti}_2(s_{x/z}) - \frac{1}{16}\mathrm{Ti}_2(-s_{x/z}) \Big) - 2\frac{xz + x + z + 1}{z}\ln(x)\ln(z) - \frac{1}{16}\ln(x) P_3(x,z) \\
& + \frac{1}{16}\ln(z) P_4(x,z) - \frac{5}{8} P_5(x,z) \Bigg\}, \qquad (14)
\end{aligned}$$

$$\begin{aligned}
P_1(x,z) &= \frac{x^2 z + x z^2 + x + z}{xz}, \\
P_2(x,z) &= \frac{5x^4z^2 + 18x^3z^3 + 18x^3z + 5x^2z^4 + 52x^2z^2 + 5x^2 + 18xz^3 + 18xz + 5z^2}{x^2z^2}, \\
P_3(x,z) &= \frac{5x^3z^2 - 5x^3z - 5x^2z^3 - 34x^2z^2 + 34x^2z + 5x^2 - 5xz^3 - 34xz^2 + 34xz + 5x + 5z^2 - 5z}{xz^2}, \\
P_4(x,z) &= \frac{5x^3z^2 + 5x^3z - 5x^2z^3 + 34x^2z^2 + 34x^2z - 5x^2 + 5xz^3 - 34xz^2 - 34xz + 5x - 5z^2 - 5z}{xz^2}, \\
P_5(x,z) &= \frac{x^3z^2 - x^3z + x^2z^3 + 6x^2z^2 - 6x^2z - x^2 - xz^3 - 6xz^2 + 6xz + x - z^2 + z}{xz^2}. \qquad (16)
\end{aligned}$$

QCD corrections

Our results

~ 1 MB files with expressions in FORM format available in ancillary files of arXiv submissions

https://arxiv.org/abs/2401.16281

```
ancillary.inc
******************************************************************************
** SIDIS coefficient functions up to NNLO from:
**
** Semi-inclusive deep-inelastic scattering at NNLO in QCD
** L. Bonino, T. Gehrmann and G. Stagnitto
**
** FORM readable format
**
** Notation, according to eq.(6) of the paper:
**           C[order][component][a2b][label] with
**           - order: 1 = NLO, 2 = NNLO
**           - component: L = Longitudinal, T = transverse
**           - a2b: means a -> b, for a and b partons
**           - label (it can be none, NS, PS, 1, 2, 3)
**
** Symbols:
**           NC = 3: number of colours
**           NF = 5: number of active flavours
**
** Scales:
**          LMUR = ln(muR^2/Q2)
**          LMUF = ln(muF^2/Q2)
**          LMUA = ln(muA^2/Q2)
**          with Q2 = -q2, invariant mass of t
**               muR: renormalisation scale
**               muF: initial-state factorisat
**               muA: final-state factorisatio
**
** Functions:
**             Li2(a) = PolyLog(2,a)
**             Li3(a) = PolyLog(3,a)
**             sqrtxz1 = sqrt(1 - 2*z + z*z +
**             poly2   = 1 + 2*x + x*x - 4*x*z
**             sqrtxz2 = sqrt(poly2)
**             sqrtxz3 = sqrt(x/z)
**             InvTanInt(x) = int_0^x dt arcta
**             T(region): Heaviside Theta func
**
** Distributions:
**           Dd([1-x]) is the Dirac delta fun
**           Dn(a,[1-x]) = (ln^a(1-x)/(1-x))_
**           same for z
**
** Kinematic regions in (x,z)-plane: as defin
**           ui = Ui for i = 1,2,3,4
**           ri = Ri, ti = Ti for i = 1,2
**           Ri, Ti and Ui defined in eq. (5.
**
** Constants: pi, zeta3 = Zeta(3) with Zeta R
**
******************************************
```

```
ancillary_pol.inc
******************************************************************************
** Polarized SIDIS coefficient functions up to NNLO from:
**
** Polarized semi-inclusive deep-inelastic scattering at NNLO in QCD
** L. Bonino, T. Gehrmann, M. Loechner, K. Schoenwald and G. Stagnitto
**
** FORM readable format
**
** Notation, according to eq.(8) of the paper:
**           DC[order][a2b][label] with
**           - order: 1 = NLO, 2 = NNLO
**           - a2b: means a -> b, for a and b partons
**           - label (it can be none, NS, PS, 1, 2, 3)
**
** Symbols:
**           NC = 3: number of colours
**           NF = 5: number of active flavours
**
** Scales:
**          LMUR = ln(muR^2/Q2)
**          LMUF = ln(muF^2/Q2)
**          LMUA = ln(muA^2/Q2)
**          with Q2 = -q2, invariant mass of the photon
**               muR: renormalisation scale
**               muF: initial-state factorisation scale
**               muA: final-state factorisation scale
**
** Functions:
**             Li2(a) = PolyLog(2,a)
**             Li3(a) = PolyLog(3,a)
**             sqrtxz1 = sqrt(1 - 2*z + z*z + 4*x*z)
**             poly2   = 1 + 2*x + x*x - 4*x*z
**             sqrtxz2 = sqrt(poly2)
**             sqrtxz3 = sqrt(x/z)
**             InvTanInt(x) = int_0^x dt arctan(t)/t : Arctangent integral
**             T(region): Heaviside Theta function
**
** Distributions:
**           Dd([1-x]) is the Dirac delta function of argument [1-x]
**           Dn(a,[1-x]) = (ln^a(1-x)/(1-x))_+ (plus-prescription) for a = 0,1,2,3
**           same for z
**
** Kinematic regions in (x,z)-plane: as defined in 2201.06982
**           ui = Ui for i = 1,2,3,4
**           ri = Ri, ti = Ti for i = 1,2
**           Ri, Ti and Ui defined in eq. (5.9), (5.12) and (5.16)
**
** Constants: pi, zeta3 = Zeta(3) with Zeta Riemann Zeta function
**
******************************************************************************
```

https://arxiv.org/abs/2404.08597