

# New physics during neutrinos decoupling: a DSMC approach

*Based on 2409.15129, 2409.07378, and on studies to appear*

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# Introduction I

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- Measurements of CMB and primordial nuclear abundances  $\Rightarrow$  learning about the state of the Universe at times as small as  $t \simeq 10^{-2}$  s  
*See also Benjamin's talk*
- Opportunity to **constrain/discover new physics** that may have been present at that epoch
- This requires knowing the dynamics of **neutrinos** in presence of new physics
- We need to solve the **neutrino Boltzmann equation** at MeV temperatures:

$$\partial_t f_{\nu_\alpha}(\mathbf{p}, t) - H p \partial_p f_{\nu_\alpha}(\mathbf{p}, t) = \mathcal{I}_{\text{coll}}[f_{\nu_\alpha}, \mathbf{p}] \quad (1)$$

# Introduction II

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Collision integral  $\mathcal{I}_{\text{coll}}$ :

$$\mathcal{I}_{\text{coll}} = \frac{1}{2E_{\nu_\alpha}} \sum \int \prod_{i=2} \frac{d^3 p_i}{(2\pi)^3 2E_i} \prod_{f=1} \frac{d^3 p_f}{(2\pi)^3 2E_f} |\mathcal{M}|^2 F[f] (2\pi)^4 \delta^{(4)} \left( \sum_{i=1} p_i - \sum_{f=1} p_f \right) \quad (2)$$

$\mathcal{I}_{\text{coll}}$  includes  $\nu - \nu$ ,  $\nu - \text{EM}$  interactions and (probably) **new physics** effects:

Lepton asymmetry	Decaying LLPs	NSI
AD mechanism	ALPs	$B - L_\alpha$ mediators
Decays of asymmetric HNLs	HNLs	Neutrinophilic particles
...	Majorons	...
	...	

See also [Isabel's talk](#)

The scope of the talk:

Model-independent and efficient approach to solve Eq. (1)

# Existing approaches to solve neutrino Boltzmann equation I

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## Complexities of the problem:

- $\Lambda$ CDM scenario:
  1. Stiffness (fast EM/decay rates, slow weak rates)
  2. Importance of high-energy neutrino tail (as  $|\mathcal{M}_{\nu\nu/\nu\text{EM}}|^2 \propto s^2$ )
- If we additionally add new physics:
  3. Complexity of new physics matrix elements  $\mathcal{M}$   
*Example: decays into jets,  $2 \rightarrow 3$  scatterings/specific non-standard neutrino interactions, many-body exclusive decays*
  4. Decays of heavy LLPs:
    - Inject high-energy neutrinos with  $E_\nu \gg 3T$
    - Inject mesons/muons that evolve non-trivial evolution

## Existing approaches to solve neutrino Boltzmann equation II

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**1. Integrated approach** [2001.04466] assumes that non-thermal distortions are always small:

$$f_{\nu\alpha}(p, t) \approx \frac{1}{\exp[p/T_{\nu\alpha}(t)] + 1} \quad (3)$$

**Transparency and performance** confronts **limited applicability**

**2. Discretization approach** [9506015] discretizes the comoving momentum space  $y(t) = p \cdot a(t) \rightarrow \{y_i\}$ , where  $i = \overline{1, n}$ :

$$\mathcal{I}_{\text{coll}} = \int G(\vec{x}) d^l \vec{x} = \prod_{k=1}^l \sum_{i_k=1}^n \tilde{G}, \quad l \geq 2 \quad (4)$$

**Allowing tracing distortions in  $f_{\nu\alpha}$**  confronts

- **heavy model dependence**
- **implementation complexity**, and
- **performance issue** ( $t_{\text{calc}} \propto \mathcal{O}(E_{\nu, \text{max}}^{l+1})$ )

# Direct simulation Monte Carlo I

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- Idea: instead of solving Eqs. (1) explicitly, start with  $N \gg 1$  particles – neutrinos, EM particles, new physics – and simulate their interactions
- In the physics of rarefied gases, the approach is known as **Direct Simulation Monte Carlo**, or **DSMC** [[Physics of Fluids 31, 067104 \(2019\)](#)]
- Immediate advantages of using Monte Carlo approach:
  - Free from  $E_{\nu, \max}$  dependence
  - Phase space of decays/scatterings using accelerator particle physics (independently of the matrix element): **MadGraph**, **PYTHIA**, **SensCalc**
  - Rarefied gases field: high performance in the case of huge  $N$

## Direct simulation Monte Carlo II

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Vanilla DSMC (utilizing so-called **No-Time-Counter method** [Prog.Astron.Aeron. 117, 211–226 (1989)]): at each time iteration,


0. Update the coordinates and velocities of particles due to external forces<sup>1</sup>
1. Split the system of volume  $V$  into cells containing  $N_{\text{cell}}$  particles
2. For each cell, per timestep  $\Delta t$ , sample

$$N_{\text{sample}} = \frac{1}{2} N_{\text{cell}} (N_{\text{cell}} - 1) \overbrace{\frac{(\sigma v)_{\text{max}}}{V_{\text{cell}}} \Delta t}^{\omega_{\text{cell}}^{\text{max}} \cdot \Delta t} \quad (5)$$

pairs of particles to interact

3. Iteratively: for each sampled pair, accept the interaction with the probability  $P_{\text{acc}} = (\sigma v)_{\text{pair}} / (\sigma v)_{\text{max}}$ , generate the kinematics and final state if accepted

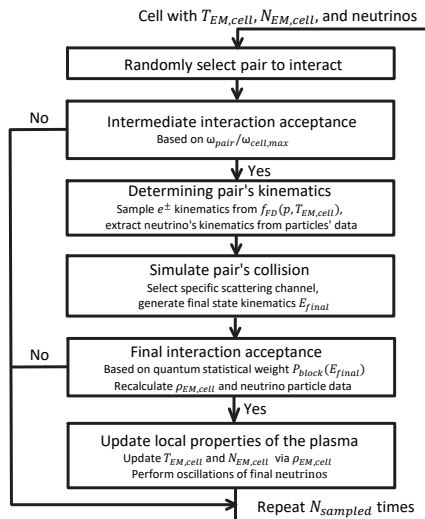
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<sup>1</sup>Good news: no need to include the spatial degree of freedom for neutrinos 

# Neutrino DSMC I

To apply it to neutrinos, DSMC requires fundamental modifications:

1. **Expansion of the Universe:** redshift particles' momenta and system volume
2. **EM plasma properties:** represent the EM particles globally and at cell level by  $T_{EM}$ ; update it after any interaction involving EM particles
3. **Quantum statistics:** final interaction approval decision based on the blocking factors  $1 - f_{final}(E_{final})$  for the final states
4. **Decaying particles:** introduce  $N_{LLP}$  LLPs, decay  $\Delta N_{LLP} = N_{LLP}(t)\Delta t/\tau$  particles per each timestep  $\Delta t$ , simulate decays e.g., in SensCalc/PYTHIA8





## Neutrino DSMC II

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- We have developed a **neutrino DSMC prototype** written in **Mathematica+C++**
- **Cross-checks:** comparing with the state-of-the-art approaches in the case of a few well-defined setups

### Already delivered

- 1) Includes the  $\Lambda$ CDM dynamics of neutrinos plus decaying LLPs
- 2) Performs as fast as discretization approach for injections of  $\nu$ s with  $\mathcal{O}(50 \text{ MeV})$
- 3) Independent of  $\nu$  energy

### To be done

- 1) Add finite electron mass
- 2) Improve performance by  $\mathcal{O}(10)$
- 3) Add NSI

*Code may be provided upon request*

# Injection of high-energy neutrinos: the questions I

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**Toy scenarios** with initial conditions intimately related to real cases:

<i>Case</i>	<b>Injection of neutrinos with <math>E_\nu \gg 3T</math></b>	<b>Injection of <math>\mu, \pi, K</math></b>	<b>Equilibrium shape of <math>f_{\nu_\alpha}, T_{\nu_\alpha} \neq T_{\text{EM}}</math></b>
<i>Examples</i>	Decays of heavy HNLs, majorons, $\nu$ philic, $B - L_\alpha, \dots$	Decays of LLPs with $m \gtrsim m_\mu$ : dark photons, Higgs-like scalars, ALPs, $\dots$	Heating of EM plasma “Equilibrium” $\nu$ heating
<i>Question</i>	Does $N_{\text{eff}}$ increase or decrease?	The same question	Is integrated approach accurate?
<i>Answer by DSMC</i>	<b>Decreases</b>	<b>Decreases</b>	<b>Inaccurate</b>

## Injection of high-energy neutrinos: the questions II

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- $N_{\text{eff}}$  is defined as

$$N_{\text{eff}} = \frac{8}{7} \left( \frac{11}{4} \right)^{\frac{4}{3}} \frac{\rho_{\text{UR}} - \rho_{\gamma}}{\rho_{\gamma}} \Big|_{1 \text{ eV} \ll T \ll m_e} \quad (6)$$

$\Lambda\text{CDM}$ :  $N_{\text{eff}} \approx 3.043$  [2402.18481]

Does  $N_{\text{eff}}$  increase or decrease in presence of high-energy  $\nu$ s at MeV temperatures?

- **Naive expectation**: we increase  $\rho_{\nu}$  by injections  $\Rightarrow N_{\text{eff}}$  **increases**
- **Discretization approach** studies – conflicting predictions:
  - [2103.09831], [2109.11176], [1202.2841]:  $N_{\text{eff}}$  **decreases**
  - [0008138], [2104.11752]:  $N_{\text{eff}}$  **increases**

No conclusive answer yet  $\Rightarrow$  open question

## Injection of high-energy neutrinos: the questions III

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How DSMC can resolve this question:

- To understand the behavior of  $\Delta N_{\text{eff}} \equiv N_{\text{eff}} - N_{\text{eff}}^{\Lambda\text{CDM}}$ , introduce its dynamical analog

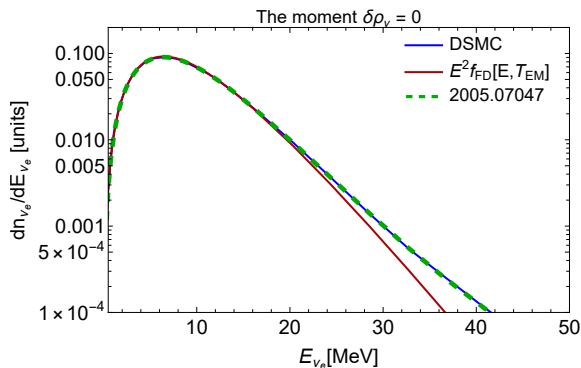
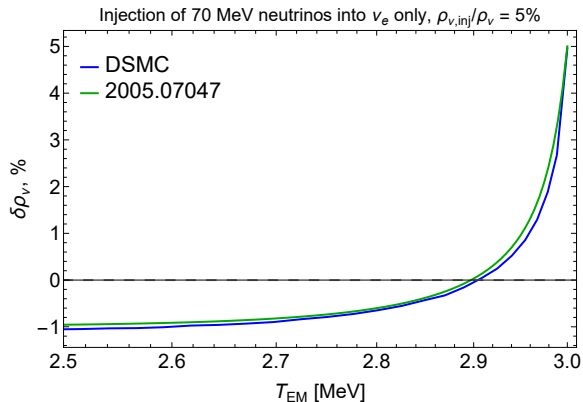
$$\delta\rho_\nu(t) = \left(\frac{\rho_{\text{EM}}}{\rho_\nu}\right)_{\Lambda\text{CDM}} \left(\frac{\rho_\nu}{\rho_{\text{EM}}}\right) - 1 \quad (7)$$

as well as the shape of the neutrino distribution

- $\delta\rho_\nu^{\text{fin}} > 0 (< 0)$ :  $\Delta N_{\text{eff}} > 0 (< 0)$
- Study the dynamics of  $\delta\rho_\nu$  under the instant injection of high-energy neutrinos with DSMC
- DSMC is completely independent approach  $\Rightarrow$  results will be conclusive

**Result:  $N_{\text{eff}}$  decreases**

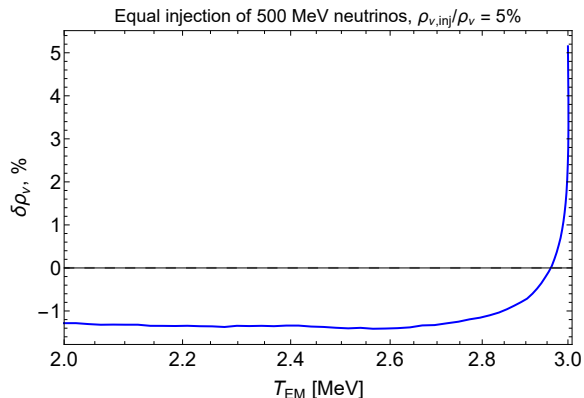
# Evolving of $\delta\rho_\nu$ : case studies I



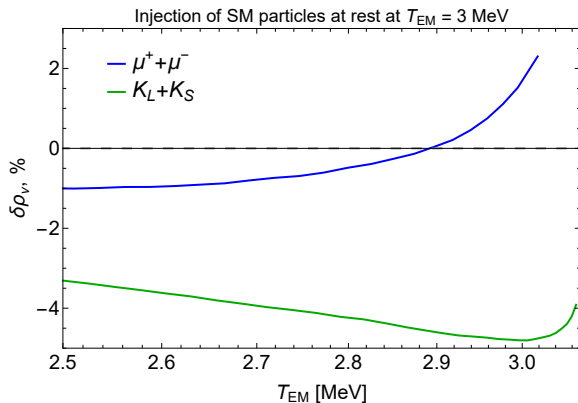
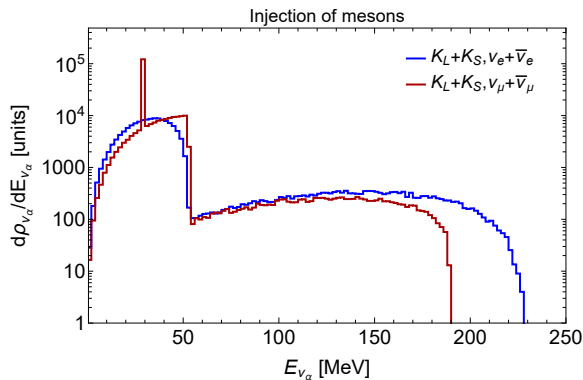
- Our simulations on high-energy neutrino injections agree with the discretization approaches from [2103.09831], [2005.07047]
- Distortions are important for, e.g.,  $\mathbf{n}/\mathbf{p}$  ratio evolution

## Evolving of $\delta\rho_\nu$ : case studies II

- The same is qualitatively true for any higher neutrino energy
- With the same running time, we have simulated the injection of 500 MeV neutrinos



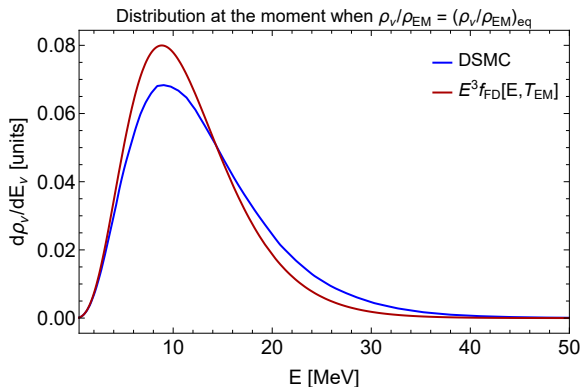
# Evolving of $\delta\rho_\nu$ : case studies III



- The same conclusion holds if injecting mesons and muons

# Why $N_{\text{eff}}$ decreases? I

- Interactions of high-energy  $\nu$ s when  $\delta\rho > 0$ : much faster than thermal interactions ( $\sigma_{\text{int}} \sim s$ )
- They will either pump the energy to the EM plasma or interact with thermal neutrinos
- The EM plasma thermalizes instantly  $\Rightarrow$  no fast inverse reactions
- Characteristic change in actual  $p^3 f_\nu(p)$  compared to  $p^3 f_{\text{FD}}$ :
  - **Overrepresented** at high  $p$
  - **Underrepresented** at low  $p$



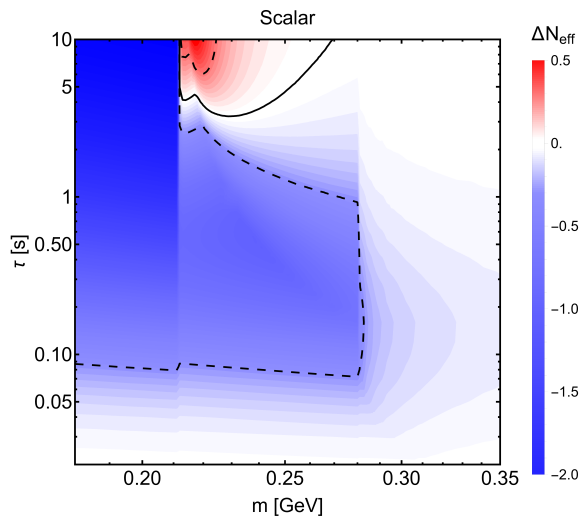
At  $\delta\rho_\nu = 0$ , distortions cause the shift  $\nu \rightarrow \text{EM} \Rightarrow \delta\rho_\nu^{\text{fin}} < 0$



## Why $N_{\text{eff}}$ decreases? II

### Case study: Higgs-like scalars $S$

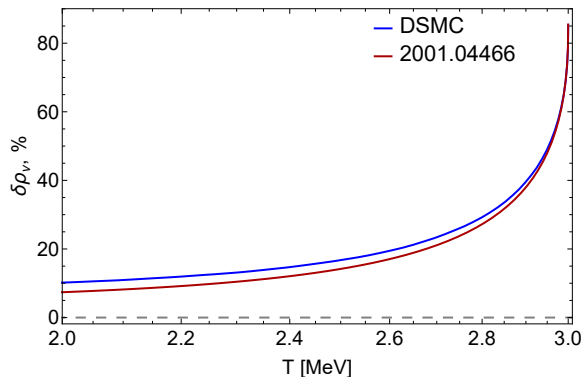
- For the same  $S$  mass,  $\Delta N_{\text{eff}}$  may be negative at small lifetimes and positive and large lifetimes
- Small lifetimes: effect of  $\nu$  distortions and disappearance of mesons and muons before decaying  $\Rightarrow \Delta N_{\text{eff}} < 0$
- Large lifetimes: these effects diminish  $\Rightarrow \Delta N_{\text{eff}}$  may become positive



*In preparation, w Kensuke Akita, Gideon Baur, Thomas Schwetz*

# Evolving equilibrium $\nu$ shape: I

- Start with the equilibrium shape of the neutrino spectrum but with  $T_{\nu_\alpha} \neq T_{EM}$
- During the evolution, the shape will depart from equilibrium
- Reason: developing distortions  
 *$\nu$  thermalization rates are energy-dependent*
- Confirmed also with the discretization approach

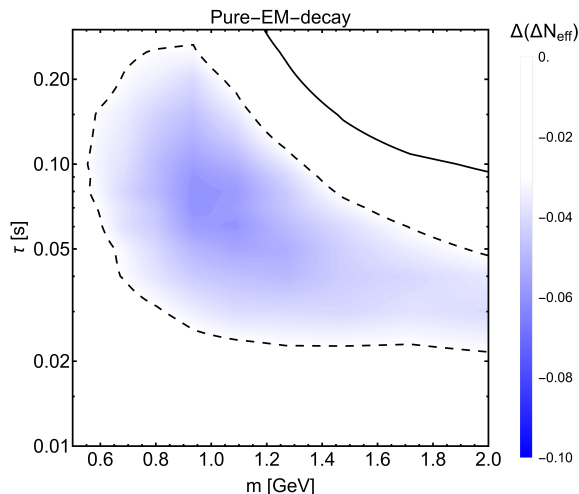


# Evolving equilibrium $\nu$ shape: II

- Toy scenario with a particle decaying purely electromagnetically
- Abundance

$$Y = (n/s)_{T=10 \text{ MeV}} = 2 \cdot 10^{-3} \quad (8)$$

The integrated approach is inaccurate even under scenarios without high-energy neutrinos



*In preparation, w/ Kensuke Akita, Gideon Baur, Thomas Schwetz*

# Conclusions

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- We have developed a **novel approach to solve the neutrino Boltzmann equation** that may handle a broad range of new physics scenarios
- **Immediate results:**
  1. Injections of high-energy neutrinos and metastable particles  $\pi, K, \mu, \dots$  at MeV temperatures always decrease  $N_{\text{eff}}$
  2. Neutrino spectral distortions significantly influence  $N_{\text{eff}}$  in any possible scenario with new physics
- Under improvement, the approach will become a universal robust tool linking models of new physics to the impact on CMB and BBN
- It may also be used to study other systems:
  - Dense medium of supernova
  - Spectral distortions of dark matter in the Early Universe
  - ...
- Next steps: improve the DSMC implementation, integrate it into a BBN framework that includes other interesting effects of new physics, particular case studies

# Backup slides

# Problems of the discretization approach I

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## Limitations of the discretization approach:

- Requires the presence of analytic matrix elements  
*Not the case for hadronic decays of LLPs with mass  $m \gg \Lambda_{QCD}$*
- Computational complexity:  $t_{\text{comp}} \propto \mathcal{O}(n^{l+1})$ , where  $n$  is the number of  $\mathbf{y}$  grid points
  - To achieve good energy resolution (in a model-independent way) to trace, simultaneously, the  $\Lambda\text{CDM}$  neutrino tail  $E_\nu \simeq T$  and the injections of  $\nu$ s from new physics, linear  $y$  grid is required
  - This means that  $n \propto E_{\nu,\text{max}}$ , and hence  $t_{\text{computation}} \propto \mathcal{O}(E_{\nu,\text{max}}^{l+1})$ .
  - Assuming the  $\Lambda\text{CDM}$  case, for which  $E_{\nu,\text{max}}^{\Lambda\text{CDM}} \simeq 20 - 50$  MeV, and some new physics scenario with  $E_{\nu,\text{max}}$  one gets

$$\frac{t_{\text{computation}}(E_{\nu,\text{max}})}{t_{\text{computation}}(\Lambda\text{CDM})} \sim \left( \frac{E_{\nu,\text{max}}}{E_{\nu,\text{max}}^{\Lambda\text{CDM}}} \right)^{l+1} \quad (9)$$

## Weeks to compute the impact of 1 GeV neutrinos

*Typical situation if GeV-scale particles are present in the plasma*

## Problems of the discretization approach II

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- Moreover,  $l$  may be higher than 2 for some scenarios (NSI, complicated decays, etc.)
- Implementation complexity: solver stability, energy conservation, grid adjustment, and this is all model-dependent

- Consider the Liouville equation for the  $N$ -particle probability distribution density  $F_N(\mathcal{R}, \mathcal{V}, t)$  with a short-range potential  $\Phi_{i,j}$  of binary interactions:

$$\frac{\partial F_N}{\partial t} + \sum_{i=1}^N \mathbf{v}_i \frac{\partial F_N}{\partial \mathbf{r}_i} + \sum_{1 \leq i < j \leq N} \Phi_{i,j} F_N = 0 \quad (10)$$

- DSMC approach approximately solves it using the following scheme:
  1. Apply the  $N - 1$  space variable reduction  $F_N \rightarrow \tilde{F}_N = \int F_N \prod_{s=2}^N d\mathbf{r}_s$
  2. Switch to the iteration scheme by considering the equation on the time intervals  $(t; t + \Delta t)$
  3. Decompose the space domain  $\mathcal{D}$  onto disconnected sub-domains  $\mathcal{D} = \cup_{l=1}^M \mathcal{D}^{(l)}$  (“cells”), populated by fixed amounts of particles during  $\Delta t$
  4. Split the evolution: evolve the system subsequently due to “ballistic motion” (free-streaming in the absence of collisions), binary collisions within  $\mathcal{D}^{(l)}$ , and then interchanging the particles between cells as a result of the first two steps



## What is DSMC [paper] II

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- Under an assumption that the system obeys ergodic conditions, the DSMC approach may be converted to an analog of the BGGKY system for  $3 + 3N$  phase space, which **reduces to the Boltzmann equation** in the limit  $N \rightarrow \infty$  and assuming the molecular chaos

## No-time-counter scheme I

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- The central part of the DSMC approach is to evolve particles' interaction inside the cell  $\mathcal{D}^{(l)}$
- Below, we describe the so-called **No-Time-Counter** (NTC) method often used for this purpose
  1. One defines the timestep of the simulation, defined by

$$\Delta t \sim \left( \chi_{\text{particle}} \cdot \sigma v \cdot \frac{N}{V_{\text{system}}} \right)_{\text{max}}^{-1} \quad (11)$$

$\chi_{\text{particle}}$  is the particles' weight (how many physical particles are represented by each particle),  $N$  is the total number of interacting particles,  $V_{\text{system}}$  is the system's volume,  $\sigma$  is the interaction cross-section,  $v$  is the relative velocity.  $\Delta t$  must be sufficiently small to resolve the equilibration

## No-time-counter scheme II

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2. Within a cell (containing  $n_{\text{cell}}$  particles), one samples randomly

$$N_{\text{sampled}} = \chi_{\text{particle}} \frac{N_{\text{cell}}(N_{\text{cell}} - 1)}{2} \frac{\omega_{\text{cell,max}} \Delta t}{V_{\text{cell}}} \quad (12)$$

pairs, where  $N_{\text{cell}}$  is the number of particles in cell,  $\omega_{\text{cell,max}} = (\sigma v)_{\text{max}}$  is the maximum interaction cross-section within the cell, and  $V_{\text{cell}} = V_{\text{system}}/n_{\text{cells}}$  is cell's volume

3. Once the pair is chosen, one accepts its interaction with the probability

$$P_{\text{acc}} = \omega / \omega_{\text{cell,max}}, \quad \omega = (\sigma v)_{\text{pair}} \quad (13)$$

4. If the interaction is accepted, one simulates the possible final states for the given pair and its scattering kinematics
  - In standard DSMC applications, a very low number of particles per cell is typically enough to simulate the interactions adequately –  $\mathcal{O}(10 - 20)$
  - Has been tested for various systems, including relativistic ones [paper], [0902.1762]
  - Detailed discussions and comparison with other DSMC methods may be found in [paper]

# Details of neutrino DSMC I

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## – Particles species:

- 3 types of neutrinos  $\nu_\alpha = \nu_e, \nu_\mu, \nu_\tau$
- EM particles – electrons and photons

## – Interactions:

- Interaction processes (plus charge conjugated ones):

$$\nu_\alpha + \nu_\beta \rightarrow \nu_\alpha + \nu_\beta, \quad \nu_\alpha + \bar{\nu}_\alpha \leftrightarrow e^+ + e^-, \quad (14)$$

$$\nu_\alpha + e^\pm \rightarrow \nu_\alpha + e^\pm, \quad \nu_\alpha + \bar{\nu}_\beta \leftrightarrow \nu_{\alpha'} + \bar{\nu}_{\beta'} \quad (15)$$

- The cross-sections and collision kinematics are calculated according to the matrix elements (see, e.g., [2006.07387])
- Photons are not involved in any direct interaction but share the same temperature with electrons

## – Neutrino oscillations:

- In each neutrino interaction, each neutrino  $\nu_\beta$  may interact as  $\nu_\alpha$  with the weight  $P_{\text{osc}}(\beta \rightarrow \alpha)$ , where  $P_{\text{osc}}$  is the oscillation probability
- At the end of each iteration, the flavors of neutrinos are re-sampled according to  $P_{\text{osc}}$

## Details of neutrino DSMC II

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### – Aspects of simulating the interactions:

- Due to full isotropy and homogeneity in the system, the spatial degrees of freedom are dropped – the interactions are considered in 0D space
- To speed up the calculations, however, particles are split into fictitious cells; interactions are only allowed between the particles of the same cell (so no boundary effects, etc.)
- The expansion of the Universe is accounted for by redshifting all energies/momenta and volume
- *No-time-counter* (NTC) method is used to simulate the pairs' interaction in each cell; the local thermalization of the EM plasma, neutrino oscillations, and the Pauli principle are accounted for
- After simulating interactions in each cell, the properties of the EM plasma and neutrinos are averaged over cells (instant EM thermalization and neutrino oscillations)

# NTC method: modifications for studying neutrinos I

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Our system is much more complicated than a typical system simulated with NTC. A few key changes are required

– **Feature 1: EM plasma equilibration**

- Electrons and photons immediately acquire thermal distribution parametrized by some temperature  $T$
- Once one of the particles forming the pair is an electron, one samples its energy by using the FD distribution with the local cell's temperature  $T_{\text{EM}}$
- After each interaction in the cell involving electrons:
  1. One computes the change in the total energy of the EM plasma in the cell,  $E_{\text{EM,tot}}$
  2. One recomputes the local EM plasma temperature  $T_{\text{EM}}$  using the relation

$$\rho_{\text{EM,cell}} \equiv \frac{E_{\text{EM,tot}}}{V_{\text{cell}}} = 4\rho_{\text{Fermi-Dirac}}(T_{\text{EM}}) + 2\rho_{\text{Bose-Einstein}}(T_{\text{EM}}), \quad (16)$$

where 4 accounts for helicities and particle-antiparticle and 2 for helicities

3. Having  $T_{\text{EM}}$ , one samples the number of electrons  $N_e$  by using the relation

$$n_e \equiv \frac{N_e}{V_{\text{cell}}} = 4n_{\text{Fermi-Dirac}}(T_{\text{EM}}) \quad (17)$$

- After simulating interactions within cells, we use the relations (16), (17) to rebuild the global population of EM particles

# NTC method: modifications for studying neutrinos II

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## – Feature 2: Expansion of the Universe:

- At each timestep, one first calculates the value of the Hubble factor via the relation

$$H = \sqrt{\frac{8\pi}{3m_{\text{Pl}}^2} \rho}, \quad \rho = \frac{E_{\text{total}}}{V_{\text{system}}} \quad (18)$$

- The timestep  $\Delta t$  for the iteration (determined by the maximal collision rate) must be  $\ll H^{-1}$ :

$$\Delta t = \min \left[ 0.01 H^{-1}, \left( \chi_{\text{particle}} \cdot (\sigma v)_{\text{max}} \cdot \frac{N}{V_{\text{system}}} \right)_{\text{max}}^{-1} \right] \quad (19)$$

- The system's and cell's volumes and all energies/momenta are redshifted:

$$V \rightarrow V(1 + 3H\Delta t), \quad E \rightarrow E(1 - H\Delta t) \quad (20)$$

# NTC method: modifications for studying neutrinos III

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## – Feature 3: Quantum statistics:

- After interaction has been accepted and kinematics generated, there is the final decision to accept the final states based on the Pauli blocking factors  $\mathbf{P}_{\text{block}}$
- The blocking factors are approximated by the relations

$$P_{\text{block}}(E_{\text{final}}) \approx 1 - f_{\text{FD}}(E_{\text{final}}, T), \quad (21)$$

where  $\mathbf{T}$  is the local temperature of the given species in the cell, and  $\mathbf{E}_{\text{final}}$  the final energy of the given particle



# NTC method: modifications for studying neutrinos IV

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## – Feature 4: number of particles per cell

- In the plasma we consider, there are distinct 4 species
- We describe some of their aspects by temperatures, which makes sense only if there is a significant amount of particles
- Therefore, we take  $\mathcal{O}(100 - 1000)$  particles per cell, which highly exceeds the amounts taken in the traditional NTC simulations

## – Feature 5: decaying LLPs

- Start with the number  $N_{\text{LLP},0}$  of LLPs defined by the initial number density
- Assuming the exponential decay law  $N_{\text{LLP}}(t) = \exp[-t/\tau]N_{\text{LLP},0}$ , per each timestep, decay  $\Delta N_{\text{LLP}} = N_{\text{LLP}}(t) - N_{\text{LLP}}(t + \Delta t)$  LLPs

## Cross-checks I

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- We have performed a number of **cross-checks**: in short,
  1. The system tends to the equilibrium state defined by the relations

$$\left(\frac{\rho_\nu}{\rho_{\text{EM}}}\right)_{\text{eq}} = \frac{\sum_{\nu, \bar{\nu}} \frac{7}{8}}{\sum_{s_e, e^\pm} \frac{7}{8} + \sum_{s_\gamma}} = \frac{6 \cdot \frac{7}{8}}{4 \cdot \frac{7}{8} + 2}, \quad (22)$$

$$\left(\frac{n_\nu}{n_{\text{EM}}}\right)_{\text{eq}} = \frac{\sum_{\nu, \bar{\nu}}}{\sum_{s_e, e^\pm} + \sum_{s_\gamma} \frac{4}{3}} = \frac{6}{4 + \frac{8}{3}}, \quad (23)$$

$$f_{\nu, \text{eq}}(E) = f_{\text{Fermi-Dirac}}(E, T) = \frac{1}{\exp\left[\frac{E}{T}\right] + 1} \quad (24)$$

2. Assuming the thermal-like initial conditions

$$f_{\nu_\alpha, \text{ini}} = f_{\text{Fermi-Dirac}}(E, T_{\nu_\alpha}), \quad T_{\nu_\alpha} \neq T_{\text{EM}}, \quad (25)$$

the equilibration of the system approximately follows the theoretical approach from [2001.04466], with and without expansion of the Universe

# Injection of monochromatic neutrinos I

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- Let us consider the injection of monochromatic neutrinos with energy  $E_{\text{inj}}$
- From the qualitative grounds (supplemented by Boltzmann code studies) discussed in [2103.09831], we expect very different behavior of the ratio  $\rho_\nu/\rho_{\text{EM}}$  during the thermalization depending on  $E_{\text{inj}}$ :
  - If  $E_{\text{inj}} \simeq T$ , the ratio would behave similarly to the one under the assumptions of simply increasing the neutrino temperature  $T_\nu > T_\gamma$ . In particular, when tending to the equilibrium value (24), it will always be higher than it
  - If  $E_{\text{inj}} \gg T$ , the ratio would behave in a different fashion: it will fastly drop below the equilibrium value and then slowly tend to it from below. If the injection occurs at  $T \sim \text{MeV}$  scale, it may result in a decrease of  $N_{\text{eff}}$
- Appendix A of [2103.09831] discusses the evolution of  $\delta\rho_\nu$  under the injection of 70 MeV neutrinos, and it is shown that  $\delta\rho_\nu$  falls below zero
- The same effect is observed for heavy HNLs, which decay into high-energy neutrinos: although they inject most of their energy into neutrinos, the resulting  $N_{\text{eff}}$  would decrease

## Injection of monochromatic neutrinos II

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- The findings of [2103.09831] are qualitatively reproduced in [2109.11176], which also studied the effects of heavy HNLs into  $N_{\text{eff}}$
- However, other works [0008138], [2104.11752] that studied HNLs have found no drop  $\delta\rho_\nu < 0$
- The results of all these papers are based on independent Boltzmann codes. It would be complicated to analyze them: in addition to the complexity of the scheme, only the one from [2103.09831] is public
- As the DSMC is a fully independent and transparent approach, it is suited well to resolve the discrepancy

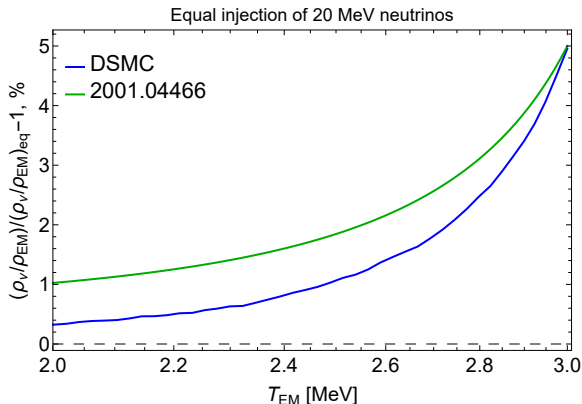
## Injection of monochromatic neutrinos III

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- We consider the SM plasma at  $T = 3$  MeV; for easier reproducibility, we turn off neutrino oscillations
- We consider several injection setups:
  - Injection of neutrinos with  $E_{\text{inj}} = 20$  MeV equally into all flavors,  $\rho_{\nu,\text{inj}}/\rho_{\nu,\text{thermal}} = 5\%$ . The evolution for this setup should not show the drop of  $\delta\rho_{\nu} < 0$ , as neutrinos have too low energy. However, already for this setup, there should be significant deviations from the temperature-based description of the thermalization from [2001.04466]
  - Injection of neutrinos with  $E_{\text{inj}} = 70$  MeV, considering various injection patterns and amounts  
(*A particular realization of this setup is discussed in [2103.09831]*)
  - Injection of high-energy neutrinos with  $E_{\text{inj}} = 500$  MeV,  $\rho_{\nu,\text{inj}}/\rho_{\nu,\text{thermal}} = 5\%$ , to study how the performance of the code depends on  $E_{\text{inj}}$
- Apart from [2103.09831], for comparison for the injection of 70 MeV neutrinos, we will use results of an independent Boltzmann solver developed by Kensuke Akita based on [2005.07047]

# Injection of monochromatic neutrinos IV

- As expected, the injection of 20 MeV neutrinos would not result in a drop  $\delta\rho_\nu < 0$
- However, there is already a sizable deviation from the temperature-based approach from [2001.04466]



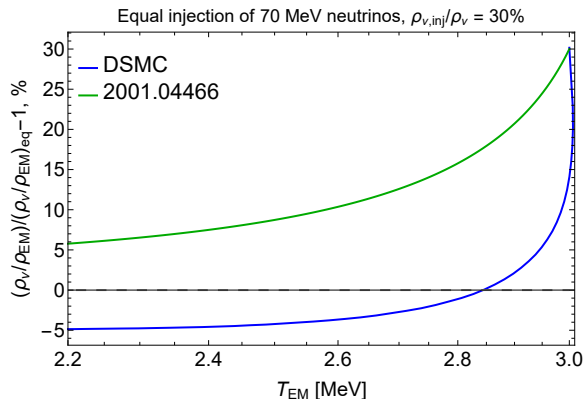
- It is caused by the energy dependence of the weak interaction cross-section,  $\sigma \propto s$ , which enhances the energy transfer rate for this setup:  $E_{\nu, \text{inj}} > \langle E_\nu \rangle = 3.15T$

# Injection of monochromatic neutrinos V

- Let us now consider the injection of 70 MeV neutrinos. We will first start with a large injection fraction

$$\rho_{\nu,\text{inj}}/\rho_{\nu,\text{eq}} = 30\%$$

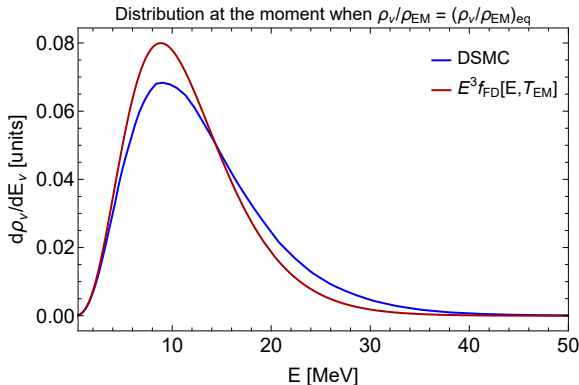
- Compared to the 20 MeV case, the situation changes: the ratio  $\delta\rho_\nu$  drops below 0 and freezes there due to the expansion of the Universe



# Injection of monochromatic neutrinos VI

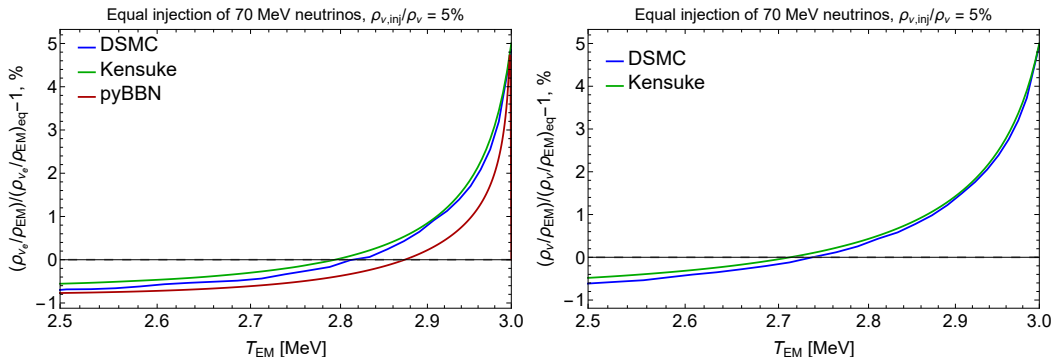
Mechanism:

- The main thermalization mechanism is annihilations  $\nu\bar{\nu} \rightarrow e^+e^-$  or  $\nu\nu \rightarrow \nu\nu$ , where one of the  $\nu$ s is high-energy, and another one may be from the initial thermal population
- Annihilations knock out thermal neutrinos to the EM plasma and to the high-energy neutrino tail
- At the moment where  $\rho_\nu = \rho_{EM}$ , the neutrino distribution is still non-thermal: 1) its high-energy tail is hotter than the EM plasma, 2) its low-energy part is underrepresented
- Since weak interactions prefer high-energy interacting particles, the first feature still leads to the shift of the  $\nu \leftrightarrow EM$  energy exchange to the EM sector



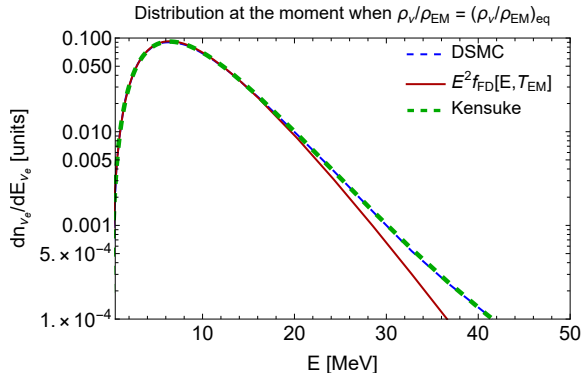
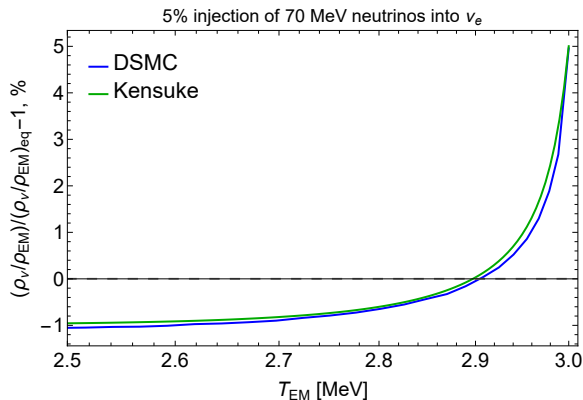


## Injection of monochromatic neutrinos VII



- We will now consider the injection  $\rho_{\nu_{\alpha},\text{inj}}/\rho_{\nu,\text{eq}} = 5/3\%$ , similar to the one studied in [\[2103.09831\]](#)
- The qualitative effect of  $\delta\rho_{\nu} < 0$  remains
- We obtain a close agreement with Kensuke's code, but there is a quantitative disagreement with [\[2103.09831\]](#) (possibly a different setup has been used there?)

# Injection of monochromatic neutrinos VIII



- The case of the injection  $\rho_{\nu_e, inj}/\rho_{\nu, eq} = 5\%$ ,  $\rho_{\nu_{\mu, \tau}, inj} = 0$  shows a similar behavior  $\delta\rho_\nu < 0$
- Again, the behavior closely agrees with Kensuke's code;

## Injection of monochromatic neutrinos IX

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- Let us now proceed to the case of the injection of 500 MeV neutrinos
- The Boltzmann codes discussed above are not suitable for such cases, as the computation time would grow enormously. For instance, if keeping the linearity of the momentum grid, such that the number of grid points  $N_{\text{grid}} \sim E_{\text{inj}}$ , the computation time would increase (compared to the 70 MeV case) by a large factor

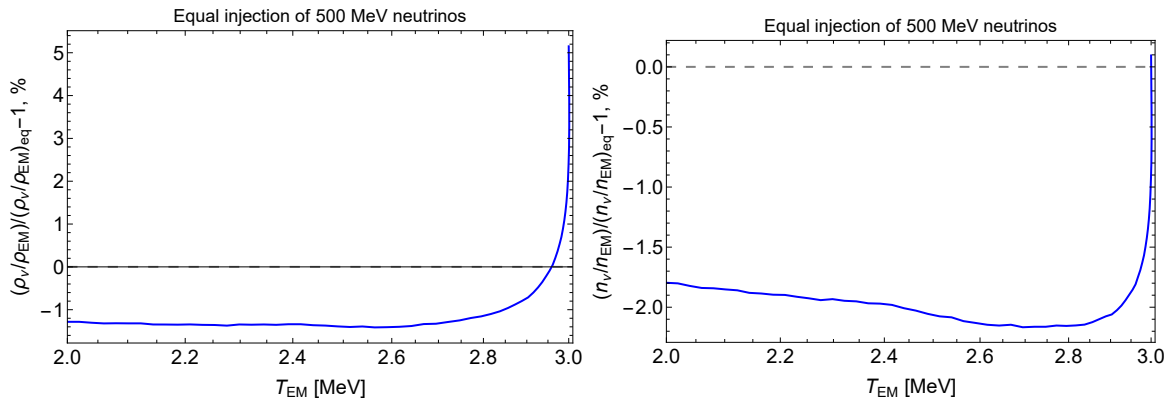
$$\frac{t_{500 \text{ MeV}}}{t_{70 \text{ MeV}}} \sim \left(\frac{500}{70}\right)^3 \times \chi_{\text{timestep}} \approx 360 \times \chi_{\text{timestep}}, \quad (26)$$

where  $\chi_{\text{timestep}}$  is a slowdown due to a decrease in the timestep required to resolve the equilibration

*Switching to the logarithmic grid may somewhat improve the timing, but additional studies would require whether it may be made accurate enough*

- In contrast, with DSMC, the dropdown in the computation time would be caused almost solely by  $\chi_{\text{timestep}}$

# Injection of monochromatic neutrinos X



- To summarize, an independent study of the injection of high-energy neutrinos with DSMC has confirmed that the injection of high-energy neutrinos around their decoupling would lead to a decrease in  $N_{\text{eff}}$

# Indirect injection of neutrinos: decays of SM particles I

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- In many new physics scenarios, neutrinos are produced secondarily – by decays of unstable SM particles  $m$ : muons, pions, kaons, etc.
- In this case, they have a continuous spectrum determined by the properties of the decaying particles
- Depending on the type of  $m$ , it may either lose all the kinetic energy before decaying ( $\pi^\pm, \mu^\pm, K^\pm$ ), or decay in-flight (the case of  $K^0$ )
- The phase space of decaying particles must be carefully implemented to obtain the neutrinos kinematics. Fortunately, we do not need to do this from scratch: we may use one of the modules of **SensCalc** [2305.13383]  
*modifications: 1) depending on the decaying particle type, remove its kinetic energy and add it directly to the EM sector, 2) allow  $\pi^\pm, \mu^\pm, K^\pm, K_L$  to decay*
- The same module simulates decays of various new physics particles: HNLs, vector mediators, dark scalars, ALPs, etc. We will study these physics cases in future works

## Indirect injection of neutrinos: decays of SM particles II

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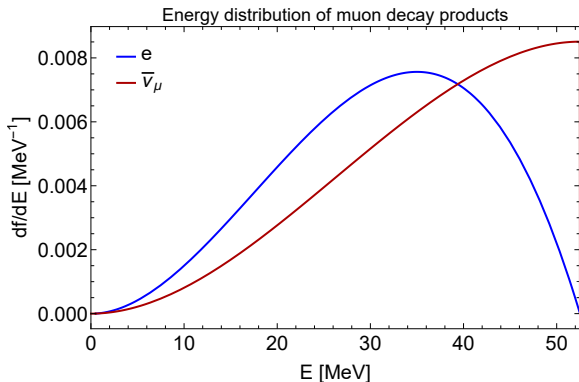
- Consider neutrinos appearing from decays of charged pions and muons
- Independently of the initial kinetic energies of  $\pi, \mu, E_{\text{kin}}$ , they will thermalize before decaying
- The past study [1706.01920] of the impact of neutrinos from decays of muons and pions (in the context of Higgs-like scalars) in the limit  $E_{\text{kin}} \ll m_{\mu/\pi}$  came to the conclusion that it would increase  $N_{\text{eff}}$ . It did not account for the impact of  $E_{\nu, \text{inj}} \gg \langle E_{\nu, \text{thermal}} \rangle$
- Let us study this with DSMC

# Indirect injection of neutrinos: decays of SM particles III

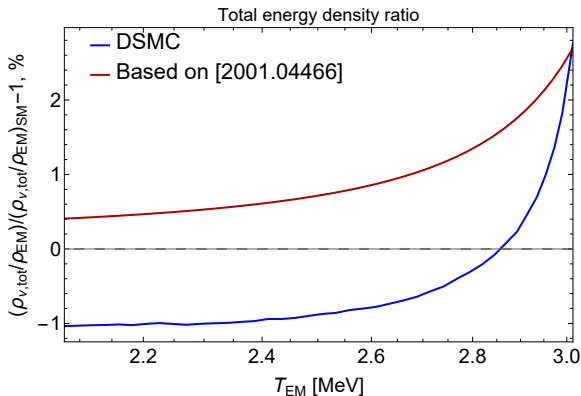
- Consider for simplicity here decaying muons



- The spectrum of neutrinos from decays of  $\mu$  is continuous, with  $E_{\nu, \max} \approx m_\mu/2 \approx 52.5$  MeV
- On average, only 30% of  $\mu$ 's energy goes directly to the EM plasma; for  $\pi^\pm$ , the ratio is even smaller



# Indirect injection of neutrinos: decays of SM particles IV



- Since  $E_{\nu,\text{inj}} \gg \langle E_{\nu,\text{thermal}} \rangle$  around neutrino decoupling, we see the same feature  $\delta\rho_{\nu} < 0$



# Injection of neutrinos and large lepton asymmetry I

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- Another interesting case is when, in addition to injections of neutrinos, the thermal plasma is characterized by a large lepton asymmetry stored in the neutrino sector
- We parametrize the asymmetry in terms of the neutrino chemical potential  $\mu_\nu$ . The correction to  $N_{\text{eff}}$  caused by  $\mu_\nu \neq 0$  is always positive independently of the sign of  $\mu_\nu$
- However, it may be possible to diminish this correction by injections of high-energy neutrinos

## Cross-check 1: Dynamical equilibrium I

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- **Cross-check 1: Dynamical equilibrium.** Independently of the initial conditions, the system of neutrinos and EM particles tends to the equilibrium state having the following properties:
  - The ratios of energy and number densities agree with the exact result

$$\left(\frac{\rho_\nu}{\rho_{\text{EM}}}\right)_{\text{eq}} = \frac{\sum_{\nu, \bar{\nu}} \frac{7}{8}}{\sum_{s_e, e^\pm} \frac{7}{8} + \sum_{s_\gamma}} = \frac{6 \cdot \frac{7}{8}}{4 \cdot \frac{7}{8} + 2}, \quad (28)$$

$$\left(\frac{n_\nu}{n_{\text{EM}}}\right)_{\text{eq}} = \frac{\sum_{\nu, \bar{\nu}}}{\sum_{s_e, e^\pm} + \sum_{s_\gamma} \frac{4}{3}} = \frac{6}{4 + \frac{8}{3}} \quad (29)$$

where  $s$  is the polarization

- In particular, the shape of the neutrino energy distribution function tends to the FD distribution (times the phase space factor) with the temperature  $T_\nu = T_{\text{EM}}$ :

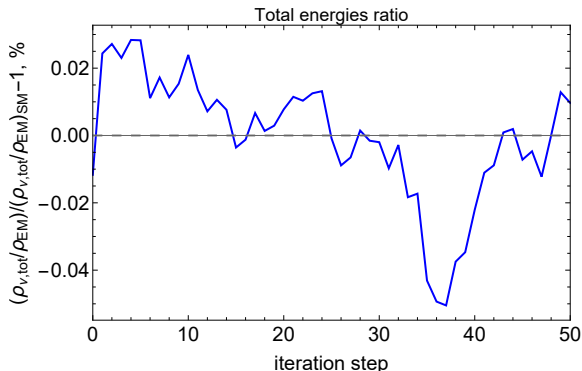
$$f_{\text{eq}}(E) \sim \frac{E^2}{\exp\left[\frac{E}{T}\right] + 1} \quad (30)$$

## Cross-check 1: Dynamical equilibrium II

- Here and below, we will consider the quantities of the type

$$\delta\rho = \left( \frac{\rho_{EM}}{\rho_\nu} \right)_{eq} \frac{\rho_\nu}{\rho_{EM}} - 1 \quad (31)$$

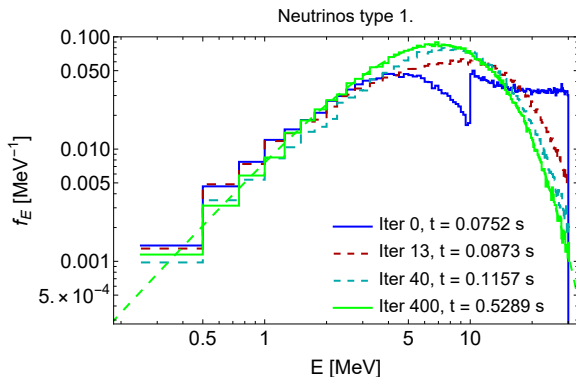
to characterize the macroscopic equilibration



- The first check: if the initial condition is full equilibrium, the system must stay in it
- The DSMC simulation reproduces this behavior  
*The plot shows the evolution of the system with the initial condition chosen such that neutrinos have the Fermi-Dirac distributions with the temperatures  $T_{\nu_\alpha} = T_{EM}$*

## Cross-check 1: Dynamical equilibrium III

- The second test: we start with an arbitrary initial distribution of neutrinos and study its relaxation
- In the absence of the expansion of the Universe, the final state must be described by Eqs. (24), (30)



- We reproduce this behavior:
  - The blue curve: the initial neutrino distribution
  - The green solid curve: the distribution after reaching the equilibration as obtained by the simulation. It matches the theoretical distribution (30) with the temperature defined by  $\rho_{\nu,\text{sim}} \propto T^4$  (dashed green)

## Cross-check 2: initial conditions parametrized by temperatures I

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- **Cross-check 2: “Equilibrium” rates.** Assuming the initial conditions parametrized solely by the neutrino and EM temperatures  $T_{\nu\alpha}, T_{\text{EM}}$ , the energy and number transfer rates *at the first iterations* must agree with the theoretical predictions obtained in [2001.04466]:

$$\frac{\delta\rho_{\nu e}}{\delta t} = \frac{G_F^2}{\pi^5} [4(g_{eL}^2 + g_{eR}^2) F(T_\gamma, T_{\nu e}) + F(T_{\nu\mu}, T_{\nu e}) + F(T_{\nu\tau}, T_{\nu e})], \quad (32)$$

$$\frac{\delta\rho_{\nu\mu}}{\delta t} = \frac{G_F^2}{\pi^5} [4(g_{\mu L}^2 + g_{\mu R}^2) F(T_\gamma, T_{\nu\mu}) - F(T_{\nu\mu}, T_{\nu e}) + F(T_{\nu\tau}, T_{\nu\mu})], \quad (33)$$

$$\frac{\delta\rho_{\nu\tau}}{\delta t} = \frac{G_F^2}{\pi^5} [4(g_{\mu L}^2 + g_{\mu R}^2) F(T_\gamma, T_{\nu\tau}) - F(T_{\nu\tau}, T_{\nu e}) - F(T_{\nu\mu}, T_{\nu\tau})], \quad (34)$$

$$F(T_1, T_2) = 32 \cdot 0.884 \cdot (T_1^9 - T_2^9) + 56 \cdot 0.829 \cdot T_1^4 T_2^4 (T_1 - T_2). \quad (35)$$

## Cross-check 2: initial conditions parametrized by temperatures II

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- **Cross-check 2: “Equilibrium” rates.** Assuming the initial conditions parametrized solely by the neutrino and EM temperatures  $T_{\nu_\alpha}, T_{\text{EM}}$ , the energy and number transfer rates *at the first iteration* agree with the theoretical predictions obtained in [2001.04466]:

$$\frac{\delta n_{\nu_e}}{\delta t} = 0.852 \cdot 8 \frac{G_F^2}{\pi^5} \left[ 4 (g_{eL}^2 + g_{eR}^2) (T_\gamma^8 - T_{\nu_e}^8) + (T_{\nu_\mu}^8 - T_{\nu_e}^8) + (T_{\nu_\tau}^8 - T_{\nu_e}^8) \right], \quad (36)$$

$$\frac{\delta n_{\nu_\mu}}{\delta t} = 0.852 \cdot 8 \frac{G_F^2}{\pi^5} \left[ 4 (g_{\mu L}^2 + g_{\mu R}^2) (T_\gamma^8 - T_{\nu_\mu}^8) - (T_{\nu_\mu}^8 - T_{\nu_e}^8) + (T_{\nu_\tau}^8 - T_{\nu_\mu}^8) \right], \quad (37)$$

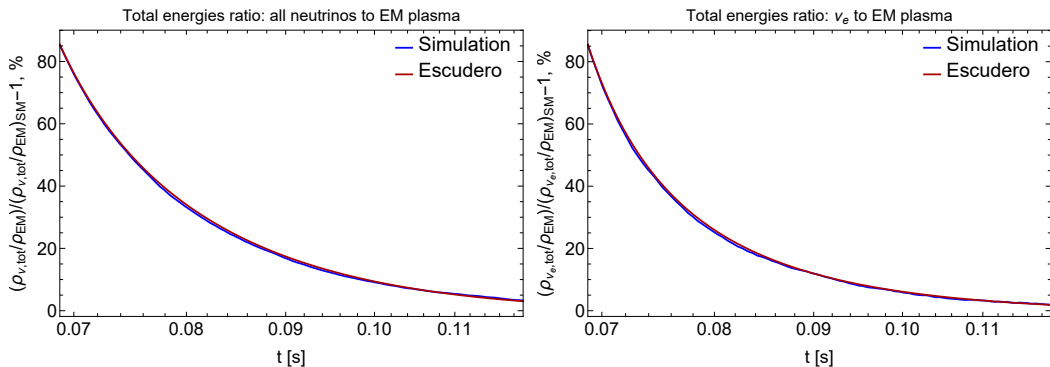
$$\frac{\delta n_{\nu_\tau}}{\delta t} = 0.852 \cdot 8 \frac{G_F^2}{\pi^5} \left[ 4 (g_{\mu L}^2 + g_{\mu R}^2) (T_\gamma^8 - T_{\nu_\tau}^8) - (T_{\nu_\tau}^8 - T_{\nu_e}^8) - (T_{\nu_\tau}^8 - T_{\nu_e}^8) \right], \quad (38)$$

## Cross-check 2: initial conditions parametrized by temperatures III

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- We split this cross-check into two stages
- In the first one, we have made a simplified simulation where one fully characterizes neutrinos by their temperatures  $T_{\nu_\alpha}$ . The simulation, therefore, determines only the temperature exchange rate
- At each iteration, it must agree perfectly with the approach described in [\[2001.04466\]](#)
- To check this, we have implemented the approach from [\[2001.04466\]](#) as well. Here, we turn off the expansion of the Universe

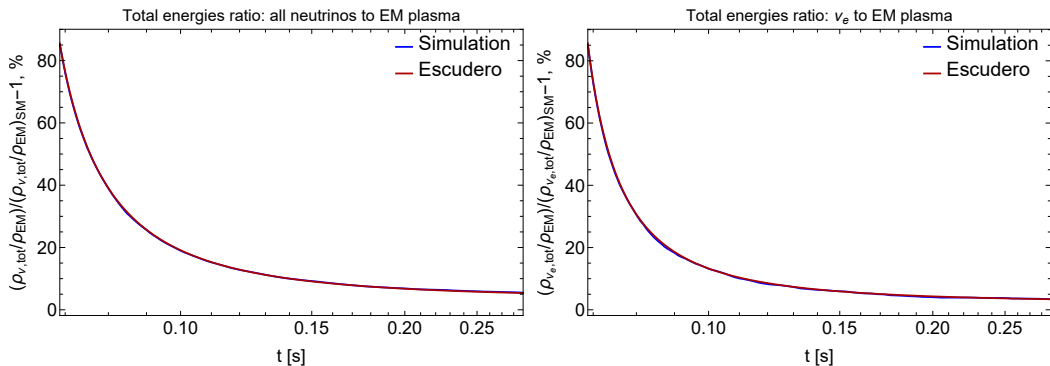
## Cross-check 2: initial conditions parametrized by temperatures IV



- The agreement is perfect if we include a factor of 2 in Eq. (12) for the number of sampled pairs  $N_{\text{sampled}}$
  - This factor is universal; we are studying its origin
- The simulation assumes the initial conditions according to  $T_{\nu_\alpha} = 3.5 \text{ MeV}$ ,  $T_{EM} = 3 \text{ MeV}$*



## Cross-check 3: including the Hubble expansion I



- **Cross-check 3: Hubble expansion.** In the presence of the expansion, the thermalization must respect the neutrino decoupling
- To make this cross-check, we again consider the two simulations, simplified “equilibrium” and realistic ones, and compare their predictions with the approach from [2001.04466]. Results agree in the same fashion as in the case  $H = 0$