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Dark Phase Transition as the Origin of nano-Hz Gravitational Waves

Yuichiro Nakai

T. D. Lee Institute & Shanghai Jiao Tong U.

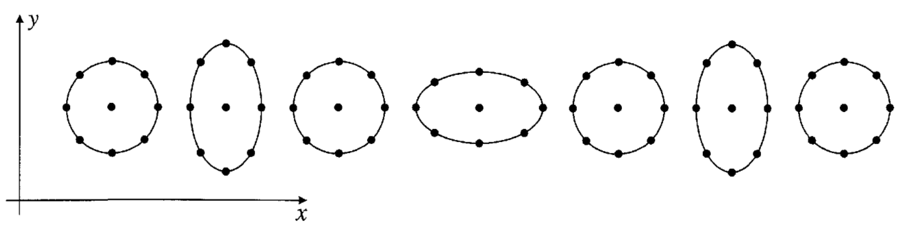
based on Fujikura, YN, Yamada, JHEP 02 (2020) 111

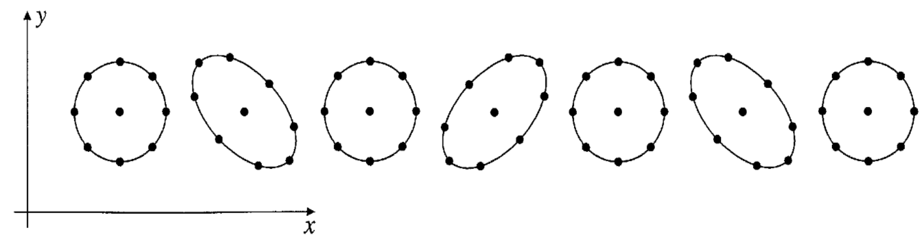
YN, Suzuki, Takahashi, Yamada, PLB 816 (2021) 136238

Fujikura, Girmohanta, YN, Suzuki, PLB 846 (2023) 138203

Gravitational Waves

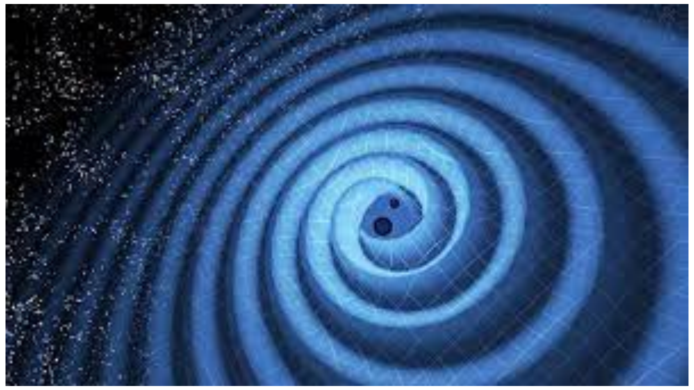
Gravitational waves (GWs) are small ripples over background spacetime predicted in Einstein equation.

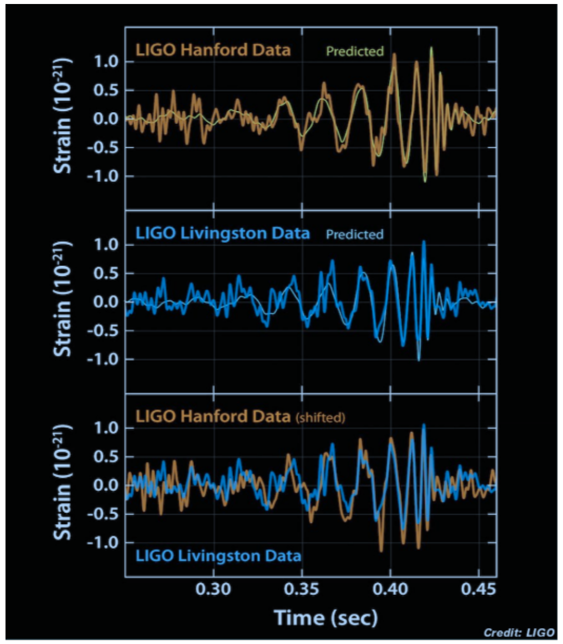


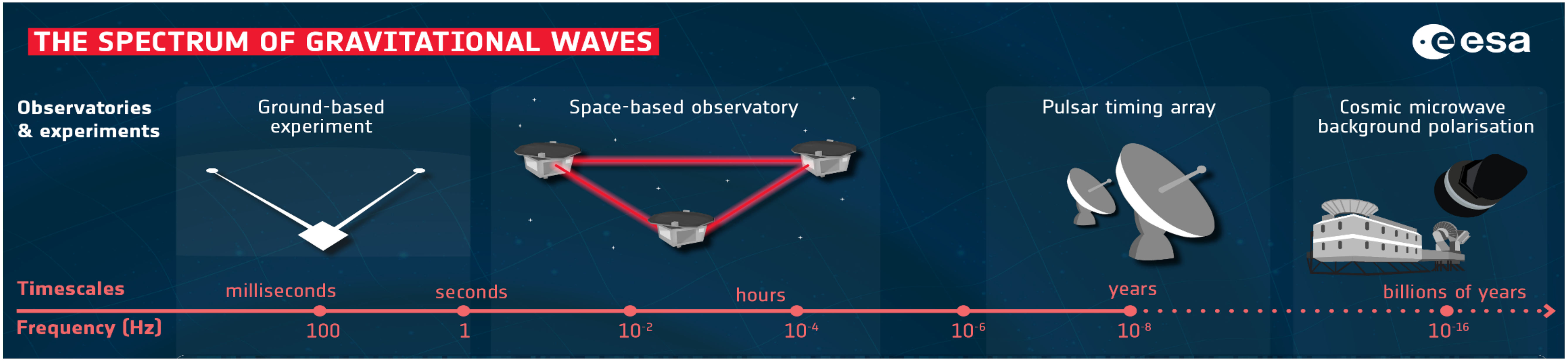


Detection of GWs from black hole & neutron star binaries

➡ **GW astronomy** has started !







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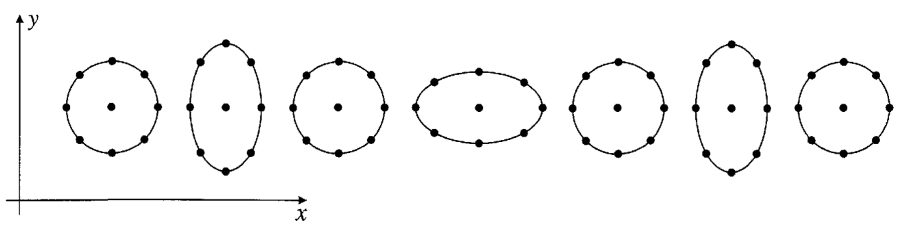

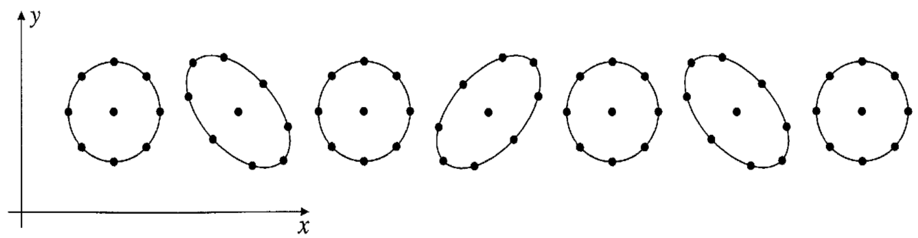

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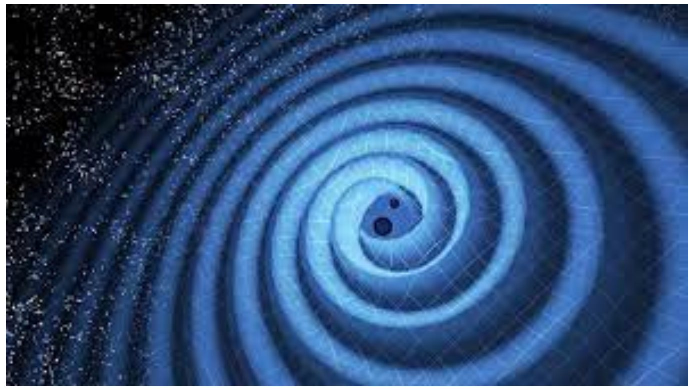
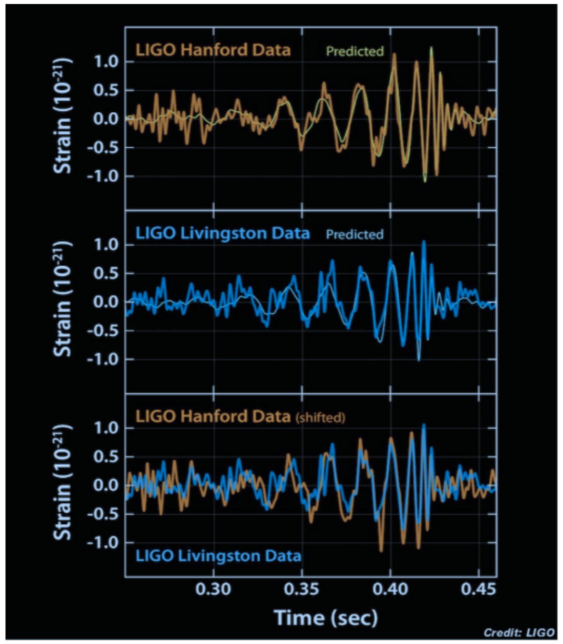
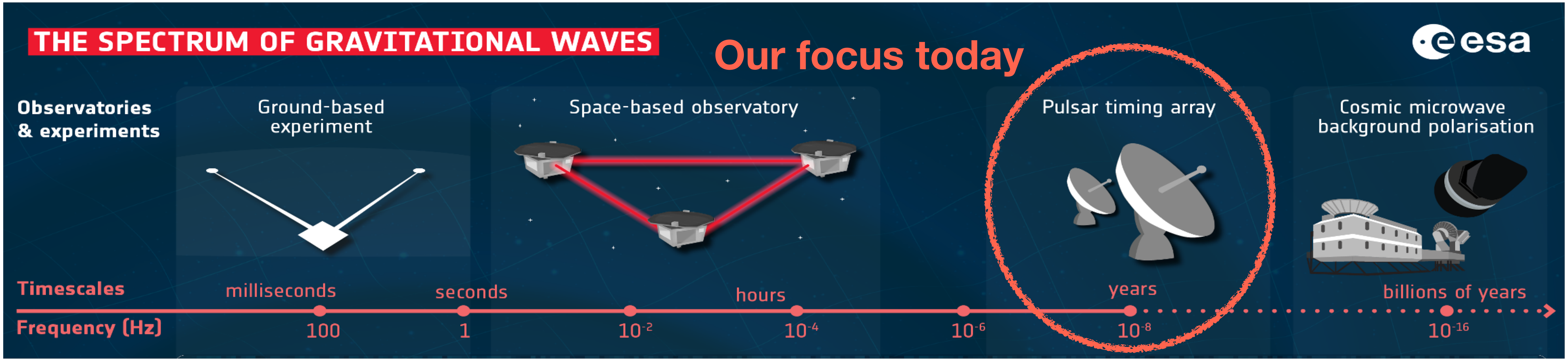
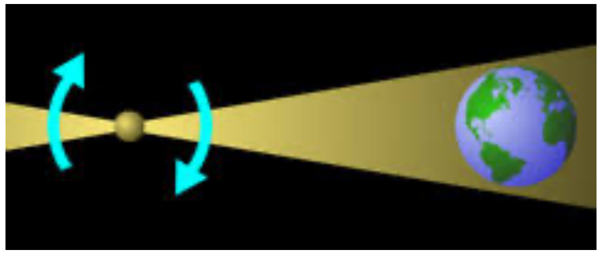


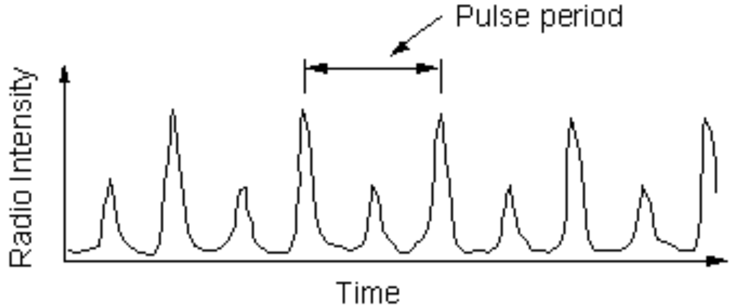




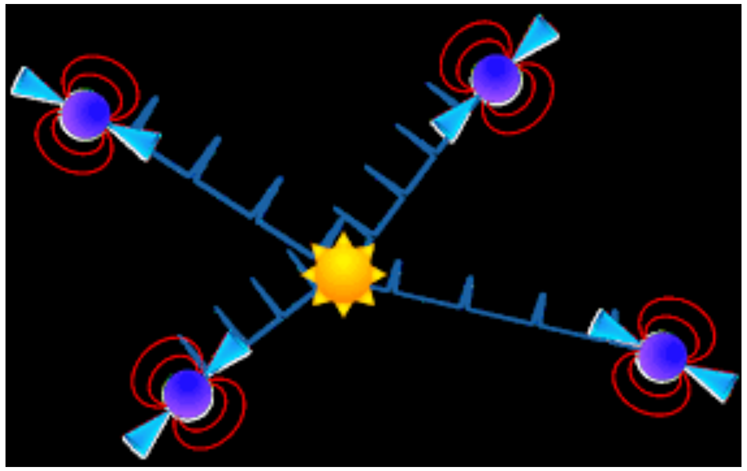
Pulsar Timing Array

- (Millisecond) pulsars are precise clocks.

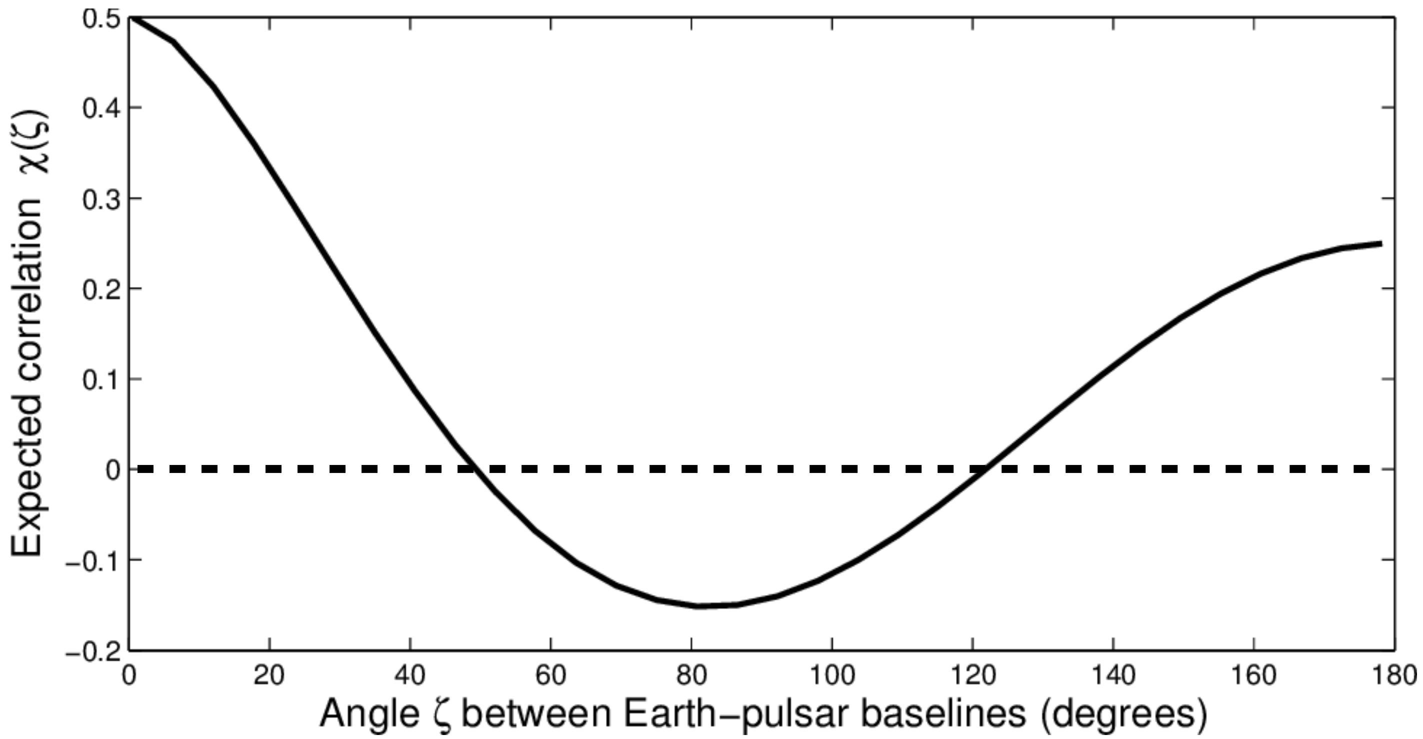


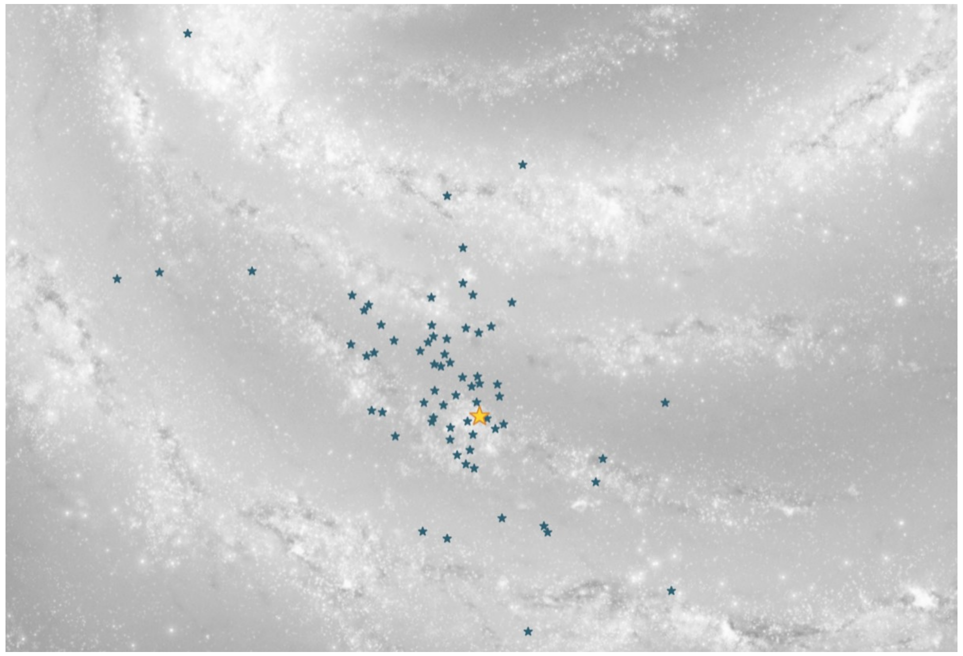


- Earth-pulsar system as GW antenna.



- GWs slightly shift the pulse-arrival time by a specific angular correlation (Hellings & Downs curve).

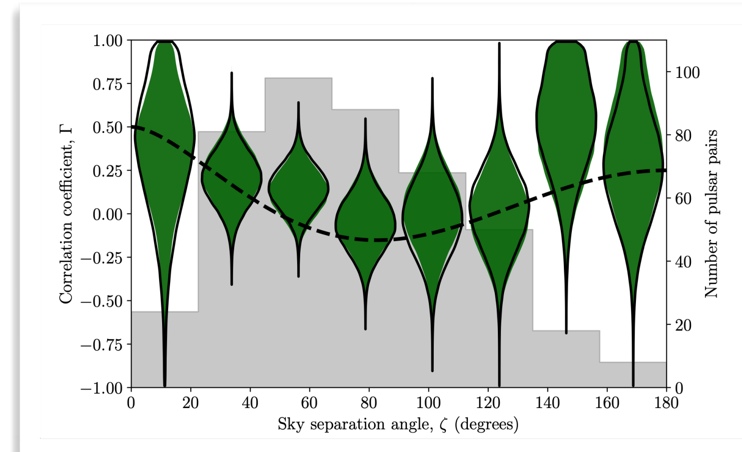




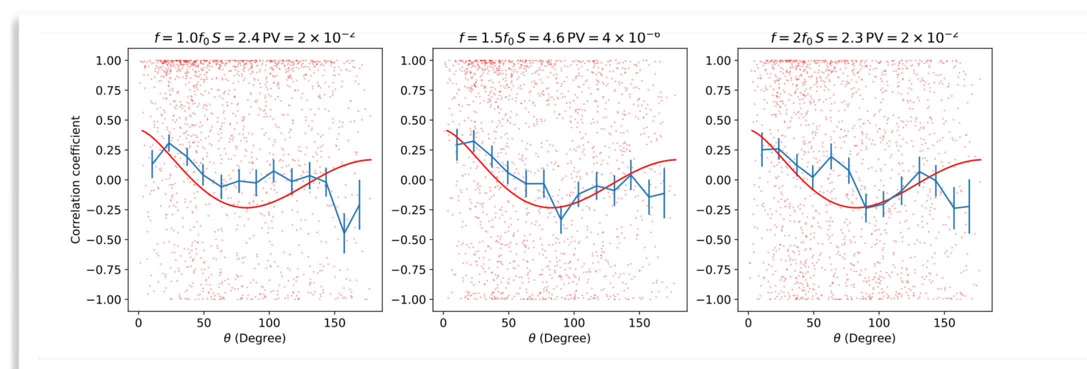
Recent Observations

CPTA, EPTA, InPTA, NANOGrav, PPTA have reported evidence for nano-Hz stochastic gravitational waves !

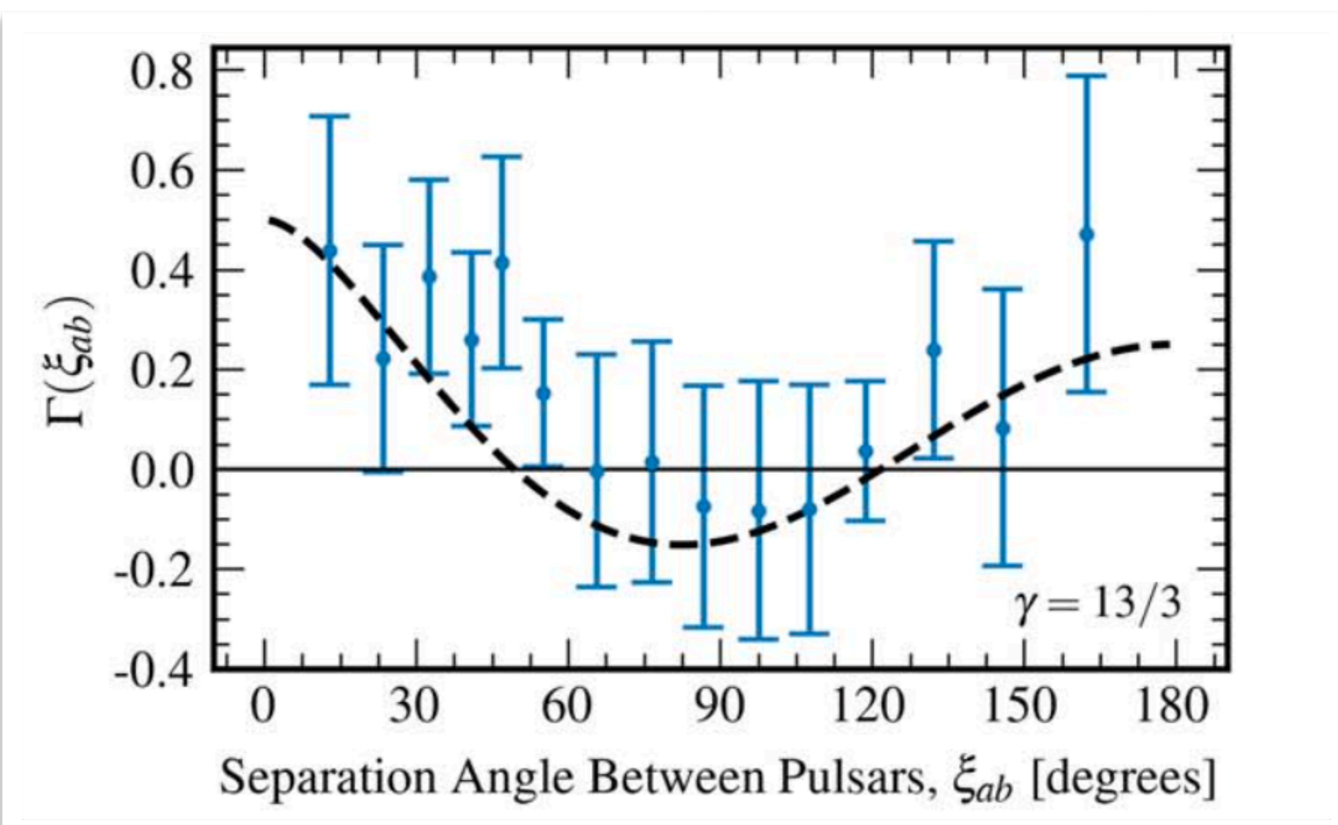
PPTA



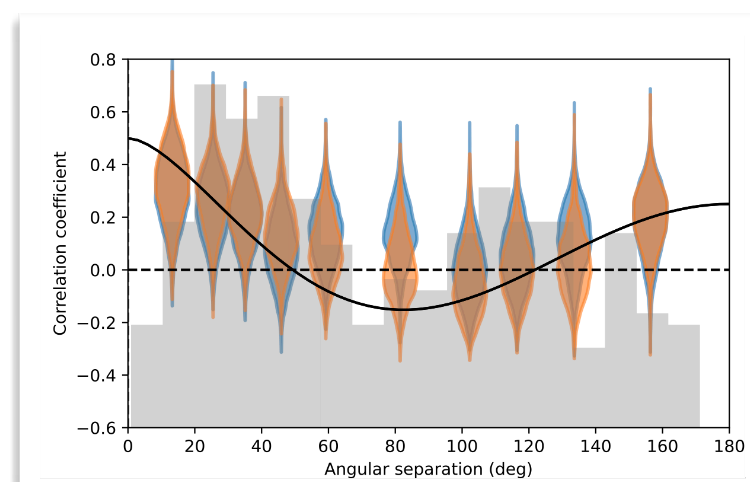
CPTA



NANOGrav



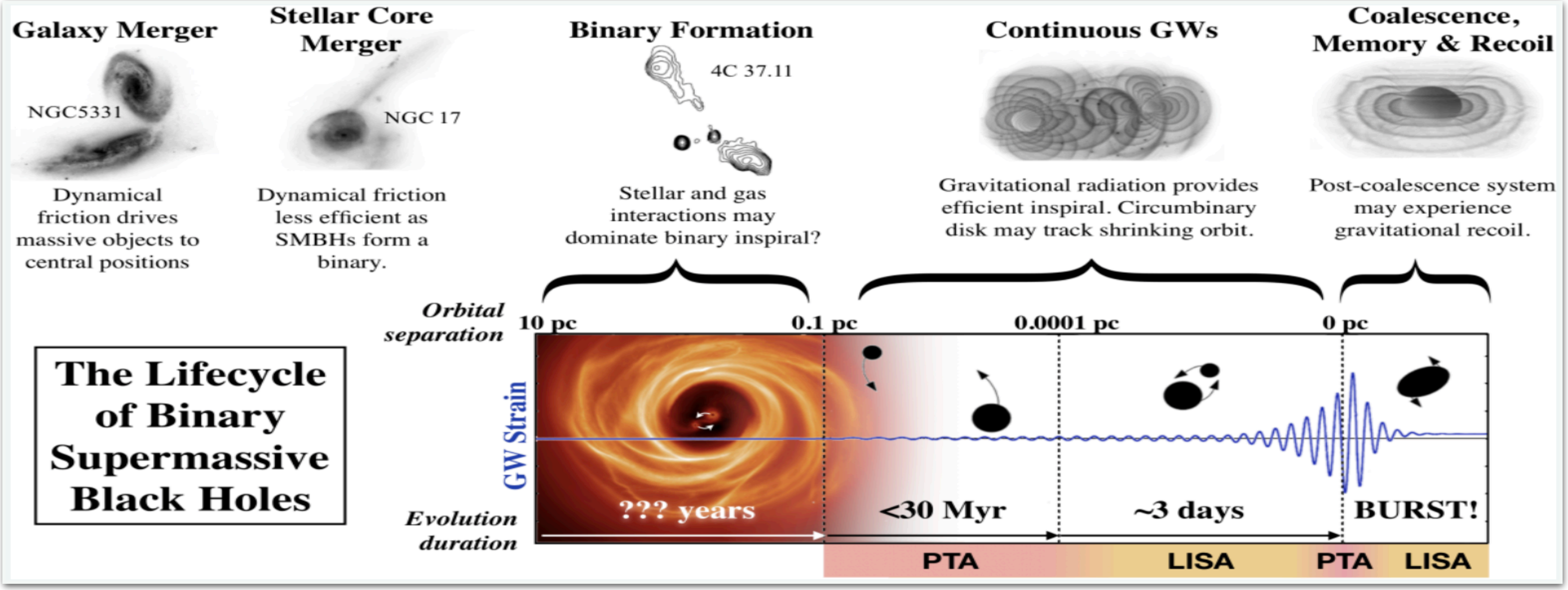
EPTA+InPTA



Possible Sources

- ✓ Supermassive black hole binaries
  (However, final parsec problem ?)
- ✓ Cosmological phase transitions
- ✓ Defects: cosmic strings, domain walls

...



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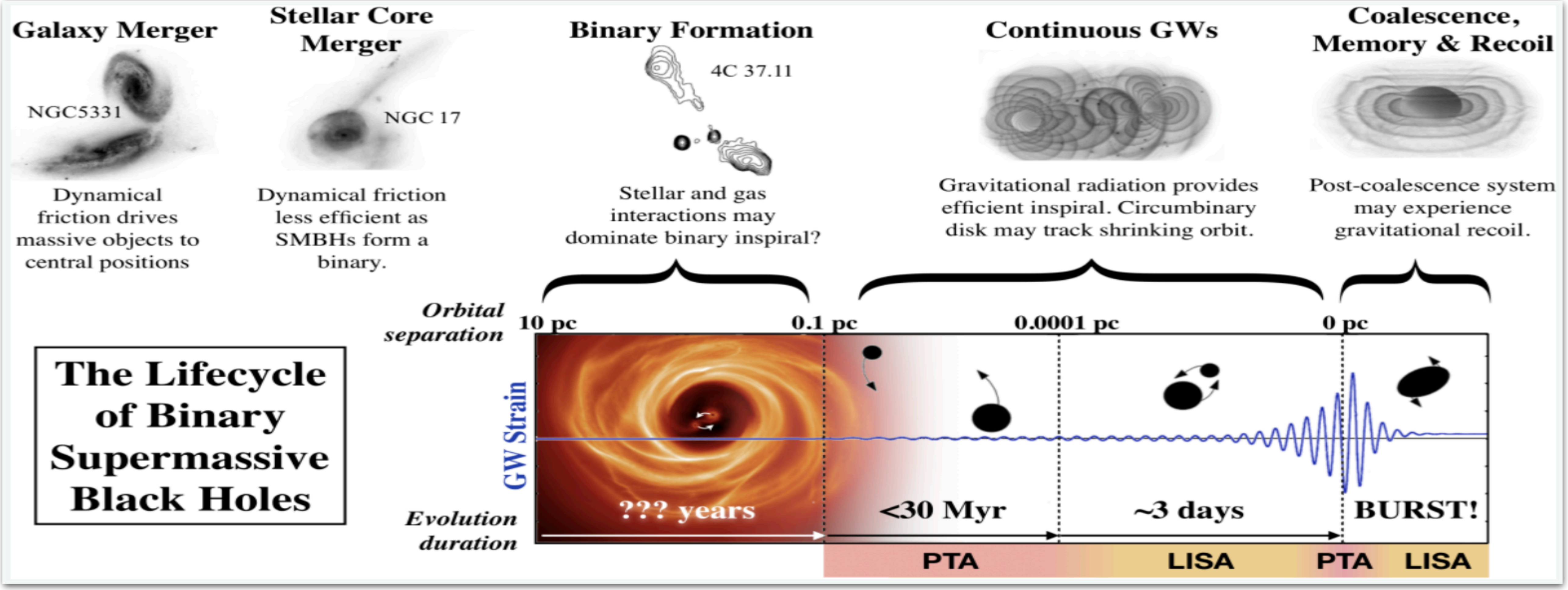
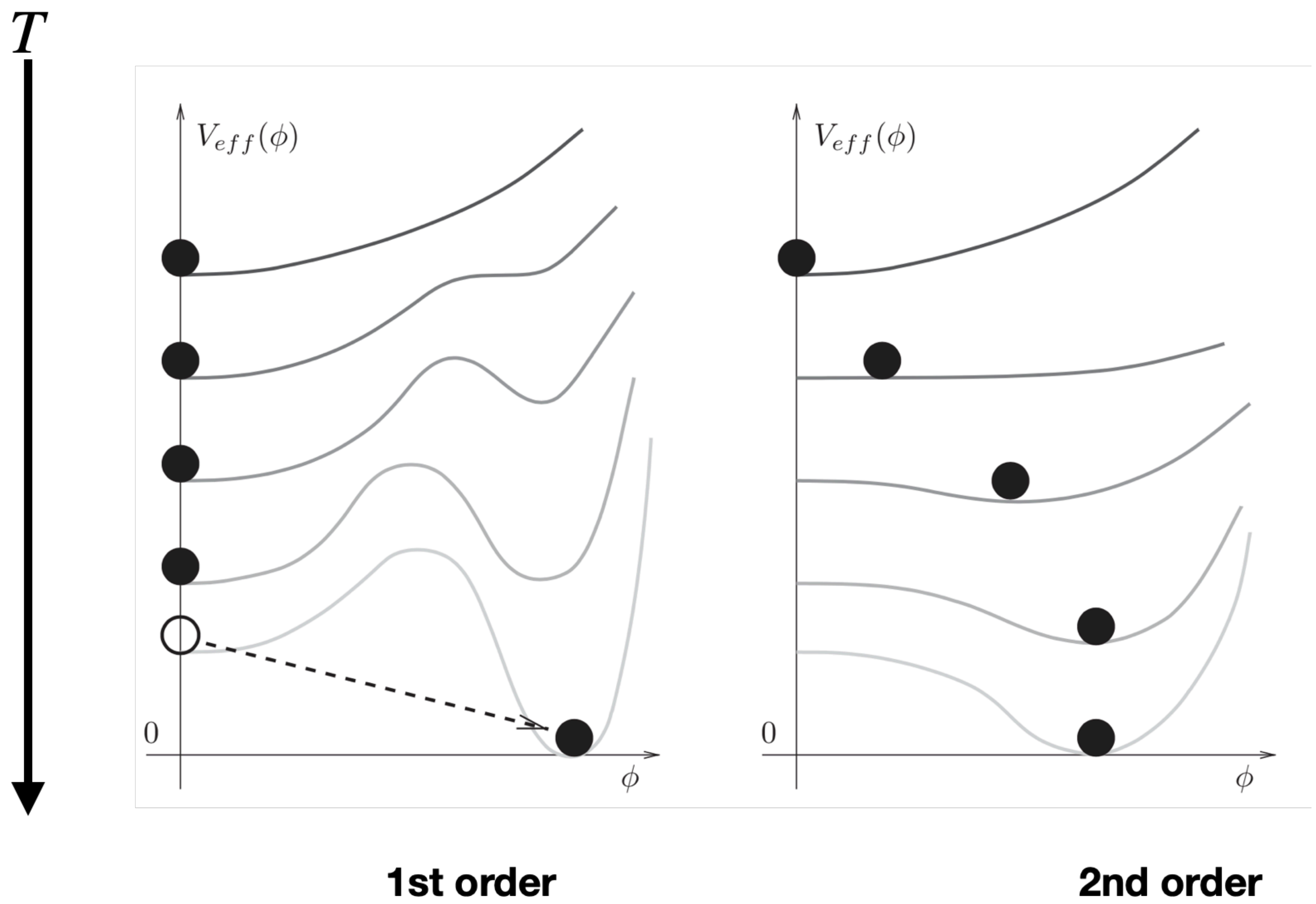


Phase Transition

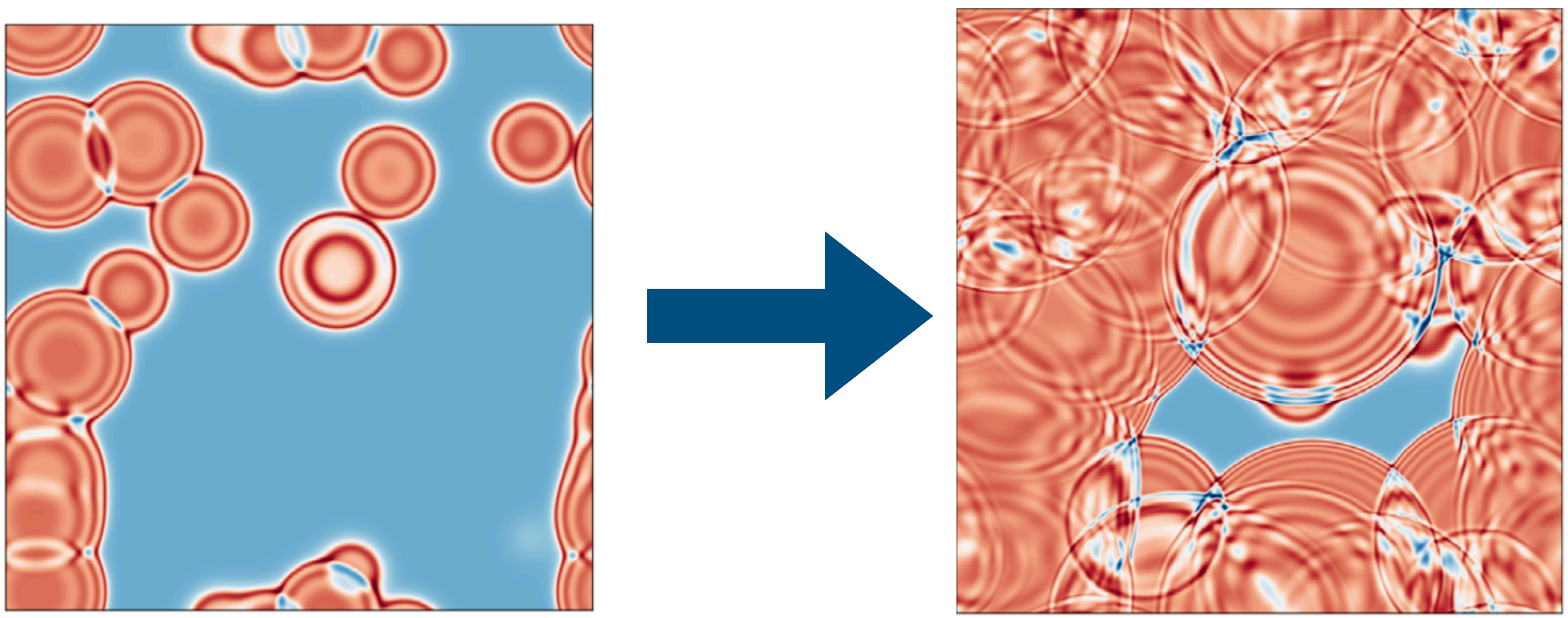
Phase transition occurs when there is a mismatch of true ground state at zero and non-zero temperatures.



1st order phase transition proceeds via nucleation, expansion and merger of bubbles of the true ground state.

GW Generation

The collision of bubbles and subsequent fluid flows produce shear stresses that source GWs.



Observed frequency f_0 is redshifted and associated with the epoch when GWs are produced.

$$f_0 \simeq 10^{-8}\ \mathrm{Hz} \left(\frac{T_*}{1\ \mathrm{GeV}} \right)$$

Dark Phase Transition

- Peak frequency in the nHz implies a phase transition temperature $T_* \sim \mathcal{O}(10 - 100)$ MeV

- QCD phase transition is not 1st order,
  1st order electroweak phase transition: $f_{\rm peak}^{\rm (EW)} \gtrsim 10^{-4}$ Hz.

Ellis, Lewicki, No (2019)

➡ **Phase transition in a dark sector**

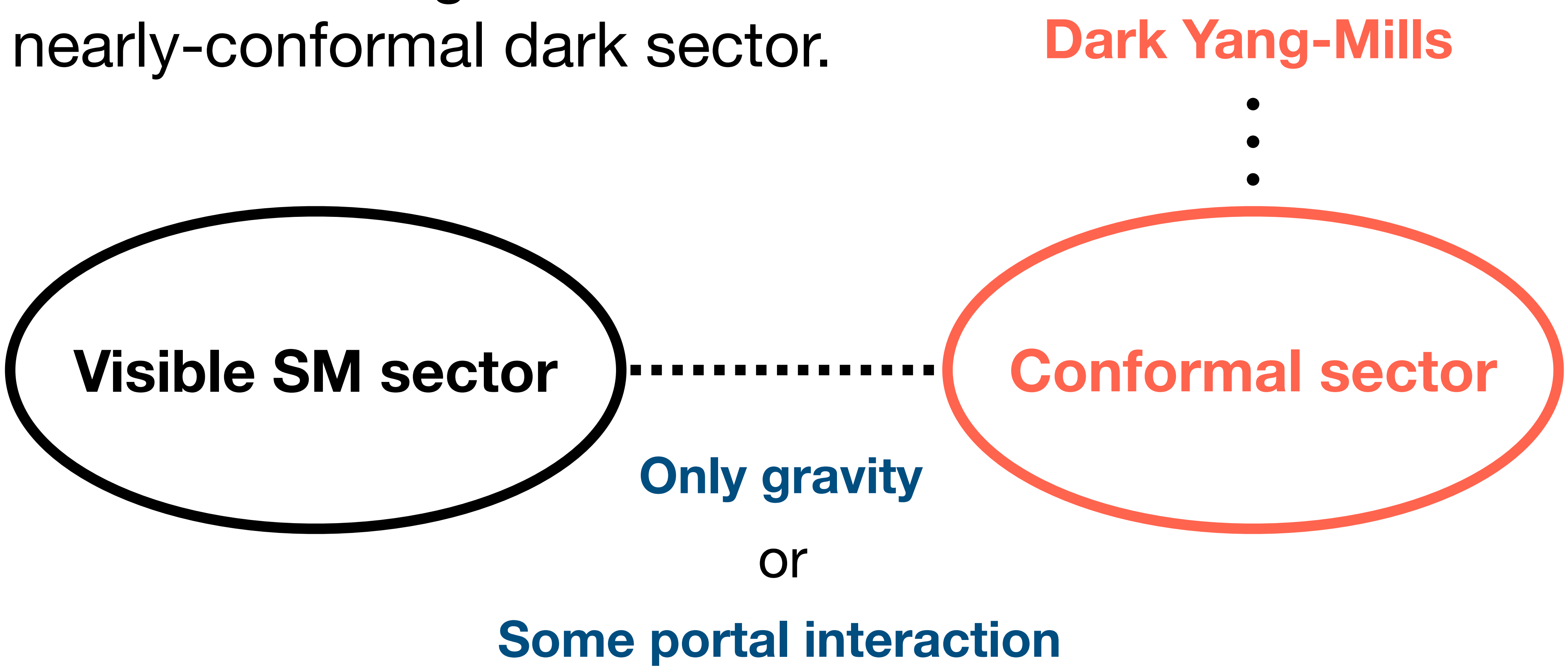
YN, Suzuki, Takahashi, Yamada (2021)

- Generically, it is not easy to reach the strength required by the PTA signal explanation.

It is valuable to find a particle physics model that can generate the reported signal.

Dark Conformal PT

Consider a confining 1st order PT
of a nearly-conformal dark sector.



- Confinement of dark Yang-Mills drives **spontaneous breaking of conformal invariance**.
- Confinement-deconfinement phase transition generates GWs.

Dark Conformal PT

Consider a confining 1st order PT
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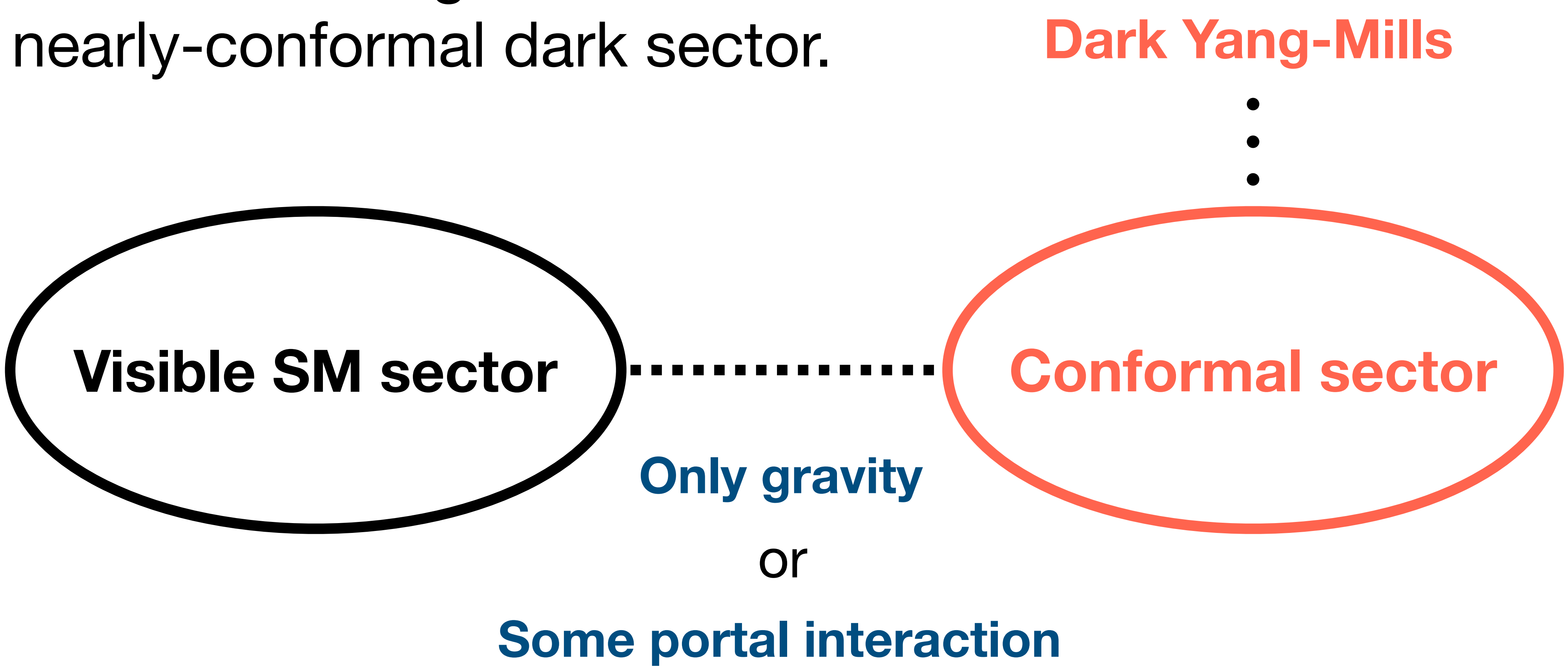


To give a concrete weakly-coupled description of our scenario …

We consider <u>a holographic model</u> with **a warped extra dimension bounded by two 3-branes** !

Randall, Sundrum (1999)

Warped Extra Dimension

- 5D universe bounded by two branes.

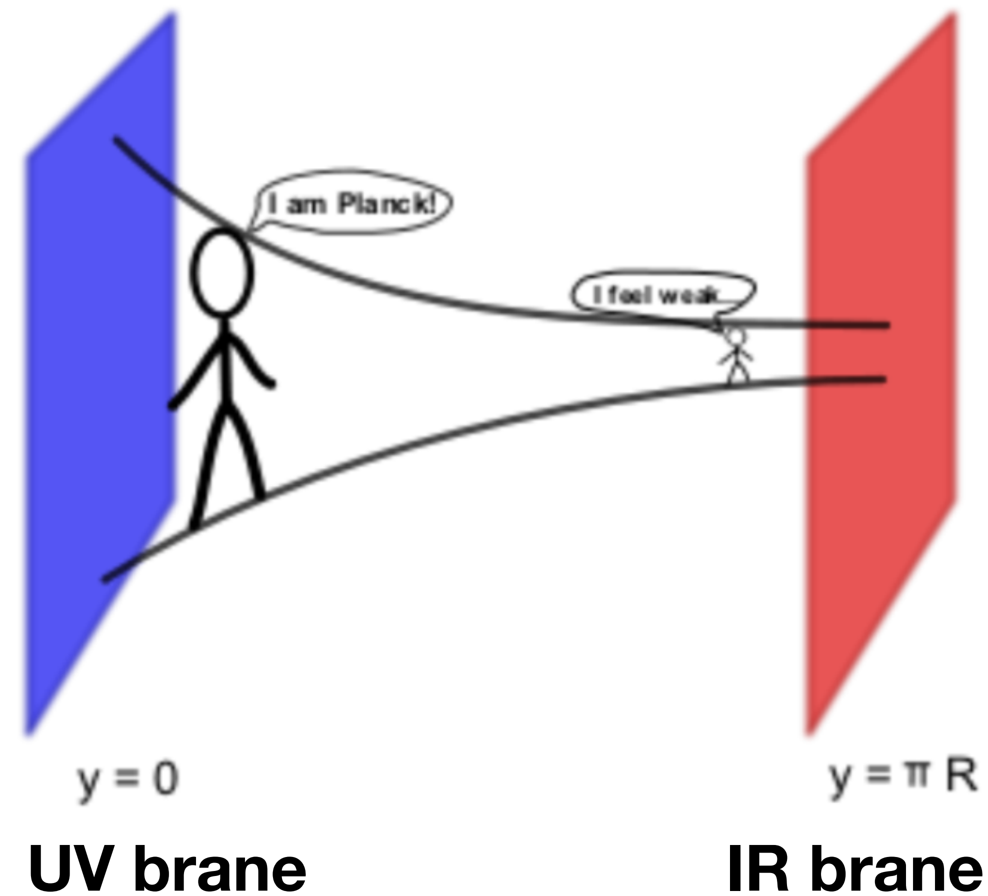
- 5th dimension highly curved

  ⟶ Anti-de-Sitter (AdS) space

Metric : $ds^2 = \boxed{e^{-k|y|}} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$

Warp factor

(4D flat : $ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu$)



- All fundamental mass parameters on IR brane are exponentially redshifted.

Unlike the ordinary Randall-Sundrum model …

SM particles locate on UV brane and dark sector particles are localized toward IR brane.

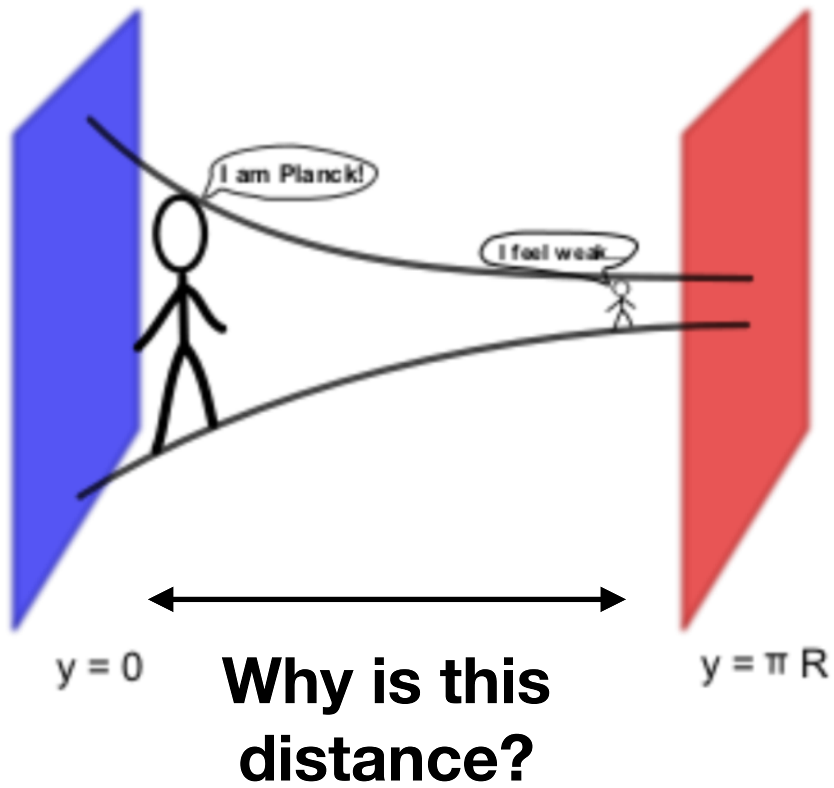
Radion

- A modulus field, called **radion**, parameterizes <u>the distance between IR and UV branes</u>.

- To stabilize the distance, we introduce **a 5D Yang-Mills field**.

Fujikura, YN, Yamada (2020)

➡ Radion potential



According to AdS/CFT correspondence …

✓ The existence of the IR brane (in absence of its stabilization mechanism)

⟷ Dilation invariance of the CFT is spontaneously broken (Scale)

✓ Radion ⟷ A massless Nambu-Goldstone boson called **dilaton**

Radion Effective Action

The geometry of the RS spacetime :

$$ds^2 = G_{AB} dx^A dx^B = e^{-2kT(x)\,|y|} g_{\mu\nu} dx^\mu dx^\nu - T^2(x) dy^2$$

y ∈ (–1/2, 1/2),   UV and IR branes are placed at y = 0 and y = 1/2 respectively

T(x) determines the size of the extra dimension and is a modulus field associated with a fluctuation along the extra dimension.

A pure gravity action of RS :

$$S = \int d^4x dy \left[\sqrt{G} \left(\frac{1}{2} M_5^3 R - \Lambda_{\mathrm{bulk}} \right) - \Lambda_{\mathrm{IR}} \sqrt{-g_{\mathrm{IR}}}\, \delta(y - y_{\mathrm{IR}}) - \Lambda_{\mathrm{UV}} \sqrt{-g_{\mathrm{UV}}}\, \delta(y) \right]$$

Λ_{bulk} : bulk cosmological constant, $\Lambda_{\mathrm{IR}}, \Lambda_{\mathrm{UV}}$: IR and UV brane tensions

The RS geometry is realized when $\Lambda_{\mathrm{bulk}}|_{\mathrm{RS}}/k = \Lambda_{\mathrm{IR}}|_{\mathrm{RS}} = -\Lambda_{\mathrm{UV}}|_{\mathrm{RS}} = -6M_5^3 k$

But, in general…

$$\Lambda_{\mathrm{IR}} = -6M_5^3 k + \delta\Lambda_{\mathrm{IR}}, \quad \Lambda_{\mathrm{UV}} = 6M_5^3 k + \delta\Lambda_{\mathrm{UV}}$$

Radion Effective Action

The Kaluza-Klein (KK) reduction of the pure gravity action

➡ **4D effective action of radion** $\mu \equiv k e^{-kT(x)/2}$

$$S_{\rm radion} = \int d^4x \left[\frac{3N^2}{4\pi^2} \left(\partial\mu(x)\right)^2 - V(\mu) \right]$$

$$V(\mu) = \delta\Lambda_{\rm UV} + \mu^4 \delta\Lambda_{\rm IR}/k^4$$

The radion kinetic term is <u>not canonically normalized.</u>

$$N \equiv 2\pi (M_5/k)^{3/2}$$

Terms with higher powers of the Ricci scalar coming from <u>quantum gravity effects</u> can be neglected for

$$N \gtrsim 4 \cdot 5^{3/4}/\sqrt{3\pi} \simeq 4.4$$

Harling and G. Servant (2018)

Radion Potential

Introduce **a SU(N_H) pure Yang-Mills field** in the bulk of the extra dimension.

$$S_{\rm Yang-Mills} = \int d^5x\sqrt{G}\left(-\frac{1}{4g_5^2}F_{AB}F^{AB}\right)$$

KK decomposition and integrating over the extra dimension

➡ 4D effective action for the zero-mode gauge field

RGE of 4D gauge coupling:

Radion ↓

$$\frac{1}{g_4^2(Q,\mu)} = \frac{\log\frac{k}{\mu}}{kg_5^2} - \frac{b_{\rm YM}}{8\pi^2}\log\left(\frac{k}{Q}\right) \quad \text{for} \quad Q \lesssim \mu$$

$b_{\rm YM} = 11N_H/3$

Gauge coupling becomes strong at low-energies and the theory confines !

Radion Potential

The confinement generates a vacuum energy.

$$V_H = \frac{1}{4}\langle T^\mu_\mu \rangle \simeq -\frac{b_{\rm YM}}{8}\left(\Lambda_H(\mu)\right)^4$$

Radion can be stabilized by the balance between the vacuum energy and the IR brane tension.

$$V_{r,{\rm eff}}(\mu) = \begin{cases} V_0 + \frac{\lambda}{4}\mu^4 - \frac{b_{\rm YM}}{8}\Lambda^4_{H,0}\left(\frac{\mu}{\mu_{\rm min}}\right)^{4n} & \text{for } \mu > \mu_c\,, \\ V_0 + \frac{\lambda}{4}\mu^4 - \frac{b_{\rm YM}}{8}\gamma_c^4\mu_c^4 & \text{for } \mu < \mu_c \end{cases}$$

$$n = \frac{8\pi^2}{b_{\rm YM}\cdot k g_5^2}$$

n < 1 is required.

$\lambda \equiv 4\delta\Lambda_{IR}/k^4$

$V_0 \equiv \delta\Lambda_{UV}$ determined by the condition that the potential energy at the minimum is vanishingly small.

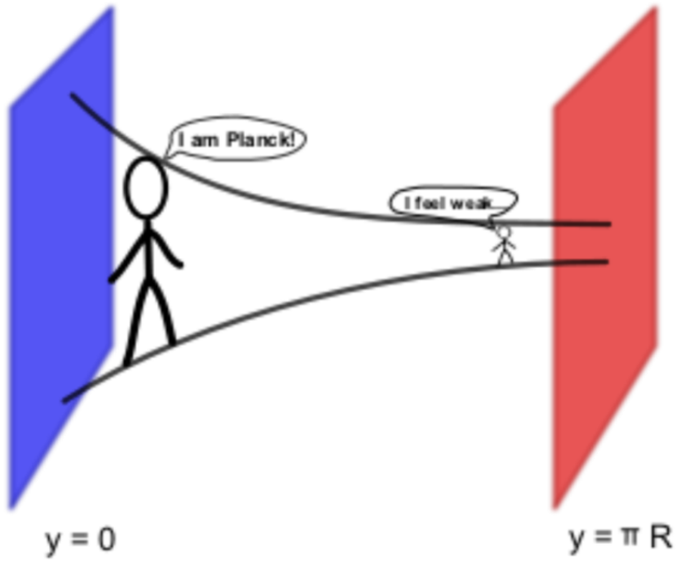
$$\Rightarrow \quad \mu_{\rm min} = \left(\frac{n b_{\rm YM}}{2\lambda}\right)^{\frac{1}{4}}\Lambda_{H,0}$$

Finite Temperature

Geometry of the 5D space-time admits two different phases, one of which is energetically favorable over the other, depending on the temperature.

At low temperature …

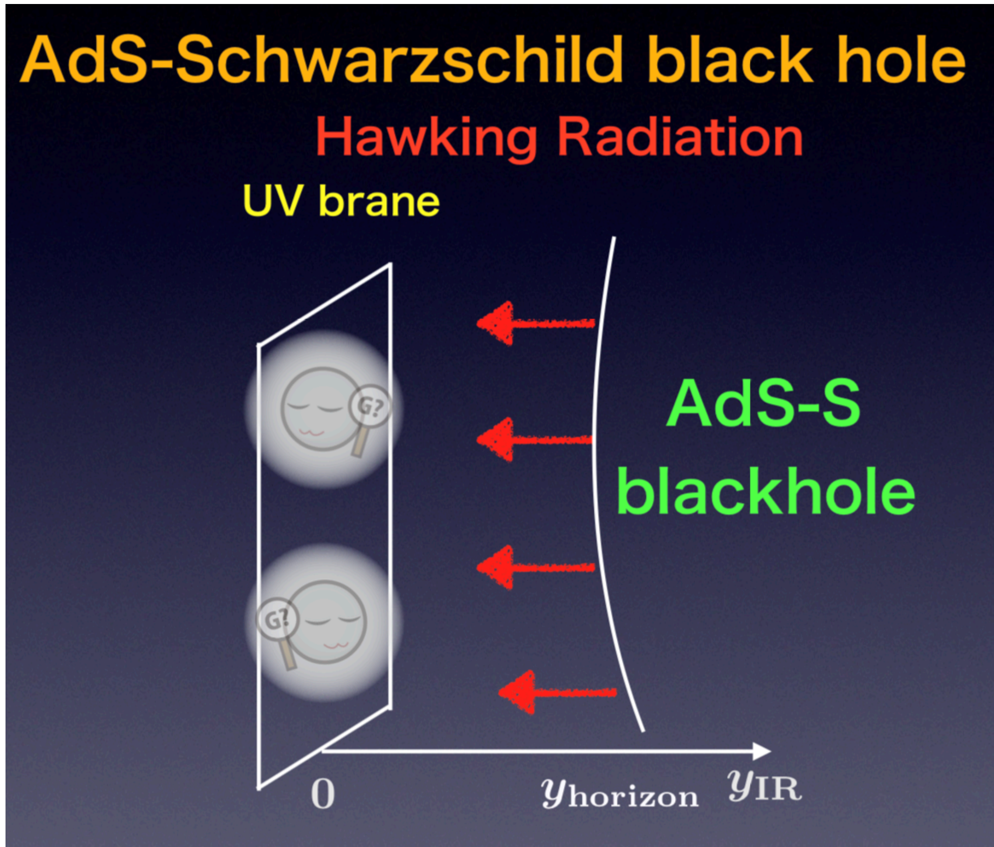
The Universe is described by the compact RS model.



At high temperature …

The system is described by the de-compactified AdS-Schwarzschild (AdS-S) black hole with the IR brane replaced by an event horizon.

Creminelli, Nicolis, Rattazzi (2001)



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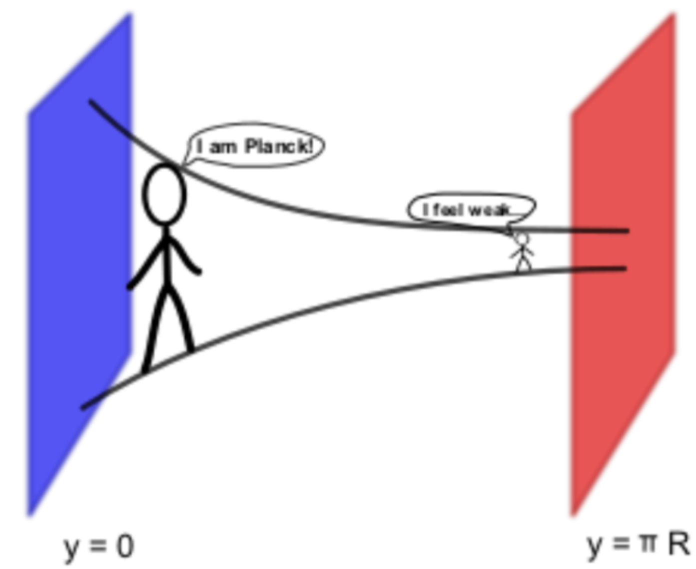

Phase transition

At high temperature …

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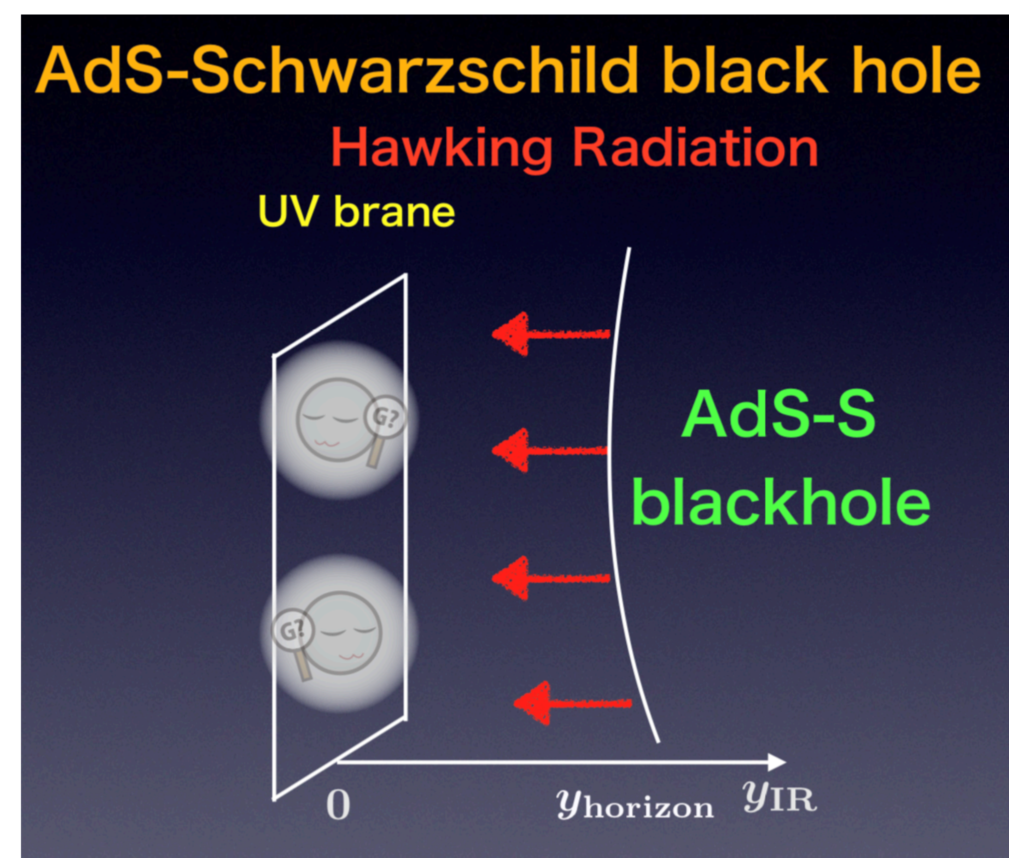


Creminelli, Nicolis, Rattazzi (2001)

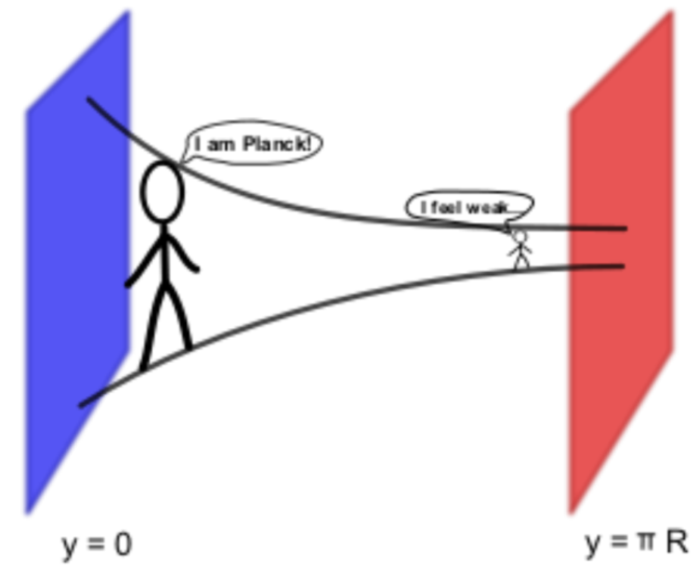
Finite Temperature

In the dual 4D picture,

At low temperature …

The confined phase of the CFT

⟷ The compact RS model

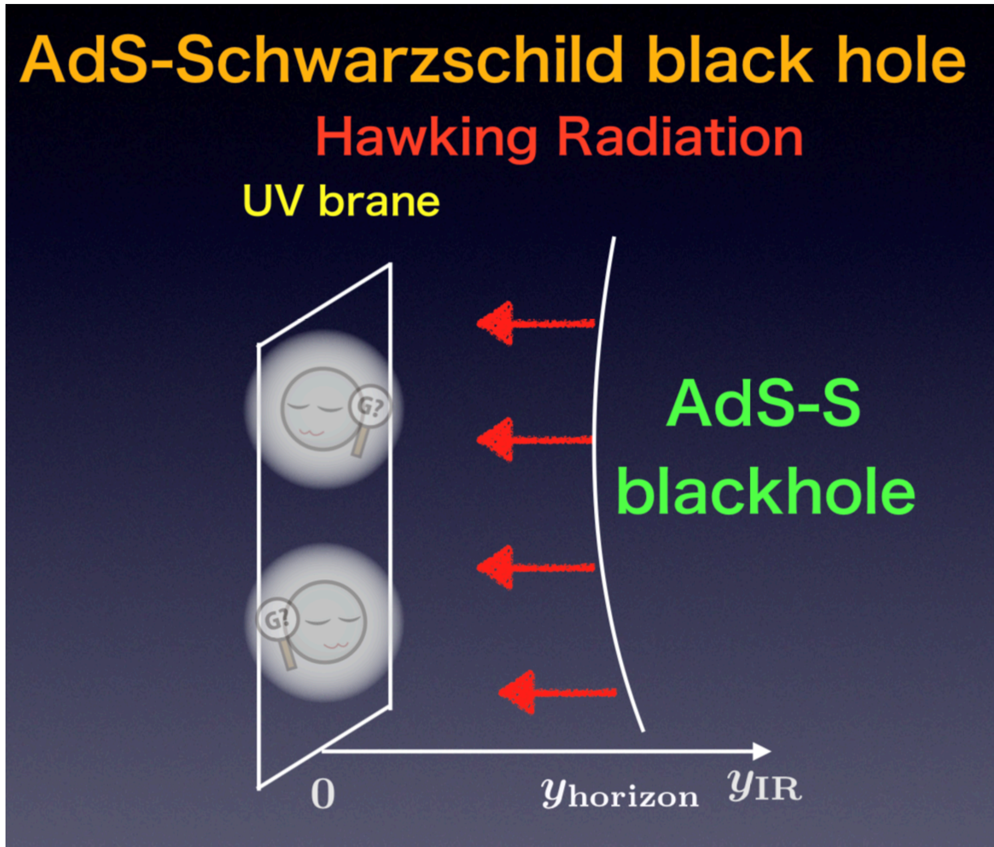


Confinement-deconfinement phase transition

At high temperature …

The de-confined phase of the CFT

⟷ The AdS-Schwarzschild (AdS-S) black hole



AdS-S Spacetime

At high temperature, the system is described by the AdS-S spacetime with the IR brane replaced by the event horizon.

$$ds^2 = k^2\rho^2\left(1-\frac{\rho_H^4}{\rho^4}\right)dt^2 - k^2\rho^2\sum_{i=1}^{3}dx_i^2 - \frac{d\rho^2}{k^2\rho^2\left(1-\frac{\rho_H^4}{\rho^4}\right)}$$

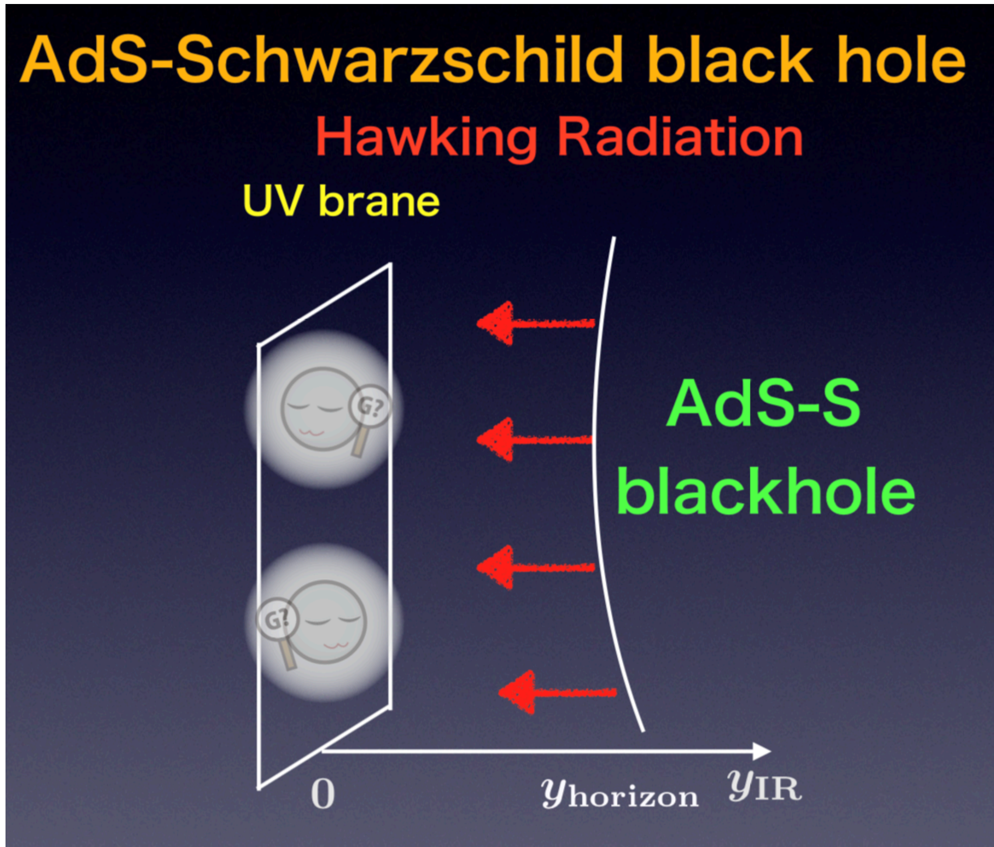
T_H ($\equiv k^2\rho_H/\pi$) : the Hawking temperature parameterized by the position of the event horizon

The free energy of the AdS-S spacetime :

$$F_{\mathrm{AdS-S}}(T_H) = \frac{3}{8}\pi^2N^2T_H^4 - \frac{1}{2}\pi^2N^2T_H^3T$$

Creminelli, Nicolis, Rattazzi (2001)

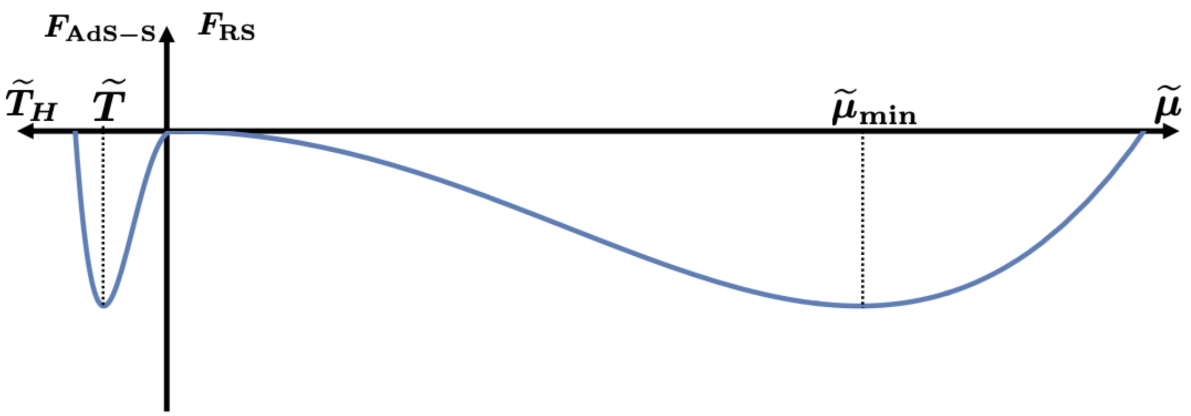
The minimum is given by $T_H = T$.



Phase Transition

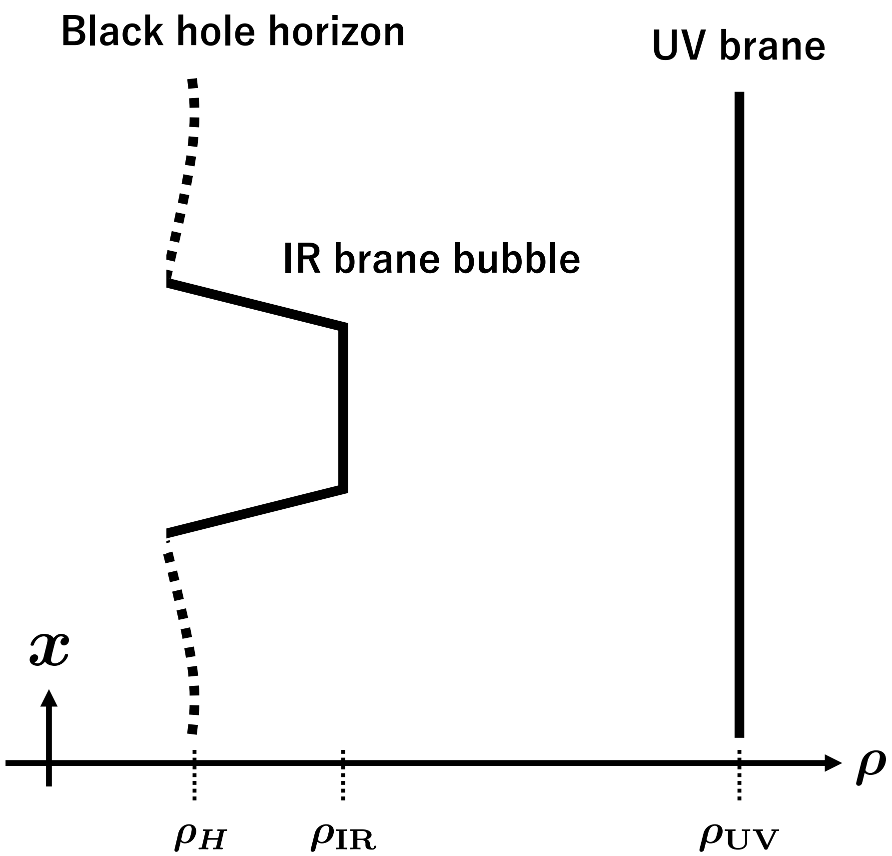
As the temperature cools down, the phase transition from the AdS-S spacetime to the RS spacetime takes place.

Both the AdS-S spacetime and the RS spacetime are locally stable.



A phase transition takes place as T is lowered below T_c .

$$T_c = \left(8\frac{V_{r,\rm eff}(\mu_{\rm min})}{\pi^2 N^2}\right)^{1/4}$$



The phase transition proceeds via the "IR brane bubble nucleation"

Transition Rate

- Bubble nucleation can start when the tunneling rate Γ per unit time and volume compete with the Hubble rate at that time H_* .

$$\Gamma \sim A e^{-S_E}\Big|_{T=T_*} \sim H_*^4$$

Visible ↓ Dark ↓

$$H_*^2 = \frac{1}{3M_{\rm Pl}^2}\left[\rho_{\rm rad}(T_*) + \rho_{\rm DR}(T_{*i}^{(D)})\right] \simeq \frac{\rho_{\rm rad}(T_*)}{3M_{\rm Pl}^2}$$

- The O(4)-symmetric bounce action after <u>canonically normalizing the radion kinetic term</u> :

$$\boldsymbol{S_4 \sim \frac{9N^4}{8\pi^2}\frac{\mu_t^4}{V(\mu_{\rm min})\left(\frac{T}{T_c}\right)^4 - V(\mu_t)}}$$

Tunneling point

$$\boldsymbol{\frac{\partial S_4}{\partial \mu_t} = 0}$$

A large N dependence

GW Generation

Two key quantities for the GW spectrum :

Inverse duration of the phase transition

$$\beta \equiv - \frac{dS_E}{dt}\bigg|_{t=t_*}$$

↑ The cosmic time when GWs are produced

Vacuum energy density of the dark sector released to the total radiation bath

$$\alpha' \equiv \frac{\rho_{\rm vac}}{\rho_{\rm rad}(T_*) + \rho_{\rm DR}(T_{*i}^{(D)})} \simeq \frac{V(\mu_t) - V(\mu_{\rm min})}{\pi^2 g_*(T_*)\, T_*^4/30}$$

↑ The effective number of relativistic degrees of freedom for the visible sector

Main contributions to GW signals :

Bubble collisions & **the sound wave of the plasma**

- Which is dominant depends on the strength of interactions between nucleated bubbles and the thermal plasma.
- We consider both cases, where the most dominant contribution comes from bubble collisions or the sound wave of the plasma.

Two scenarios

- **Secluded dark sector**

Most of the vacuum energy is injected into **dark radiation** after the phase transition.

Such a dark radiation component acts as extra relativistic neutrino species during the recombination epoch.

$$\rho_{\mathrm{DR},0} \equiv \frac{7}{8}\Delta N_{\mathrm{eff}} \left(\frac{4}{11}\right)^{4/3} \rho_{\gamma,0} \qquad \alpha' \sim 0.07 \left(\frac{\Delta N_{\mathrm{eff}}}{0.5}\right)$$

- **Decaying dark sector**

All the dark sector energy quickly goes into the visible sector after the phase transition.

The portal coupling can be consistent with BBN and laboratory experiments.

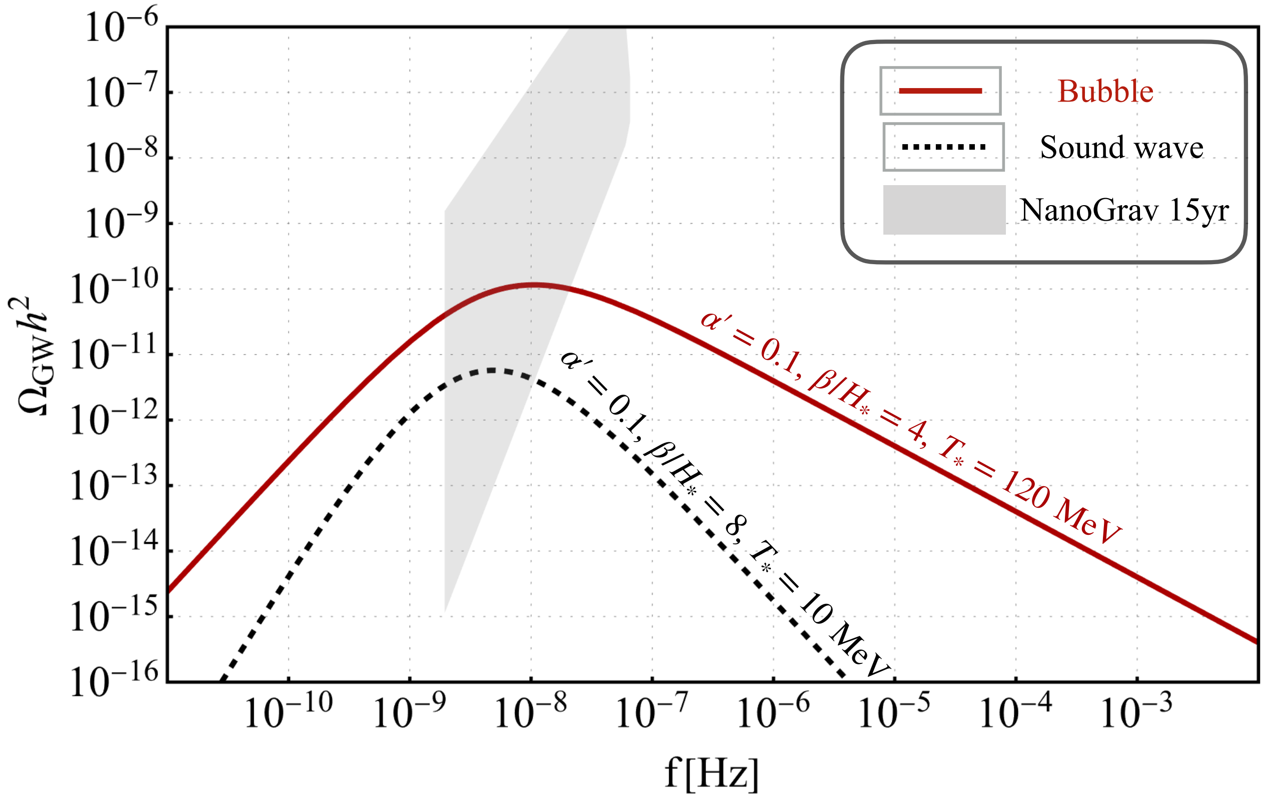
Bringmann, Depta, Konstandin, Schmidt-Hoberg, Tasillo (2023)

Fitting the Data

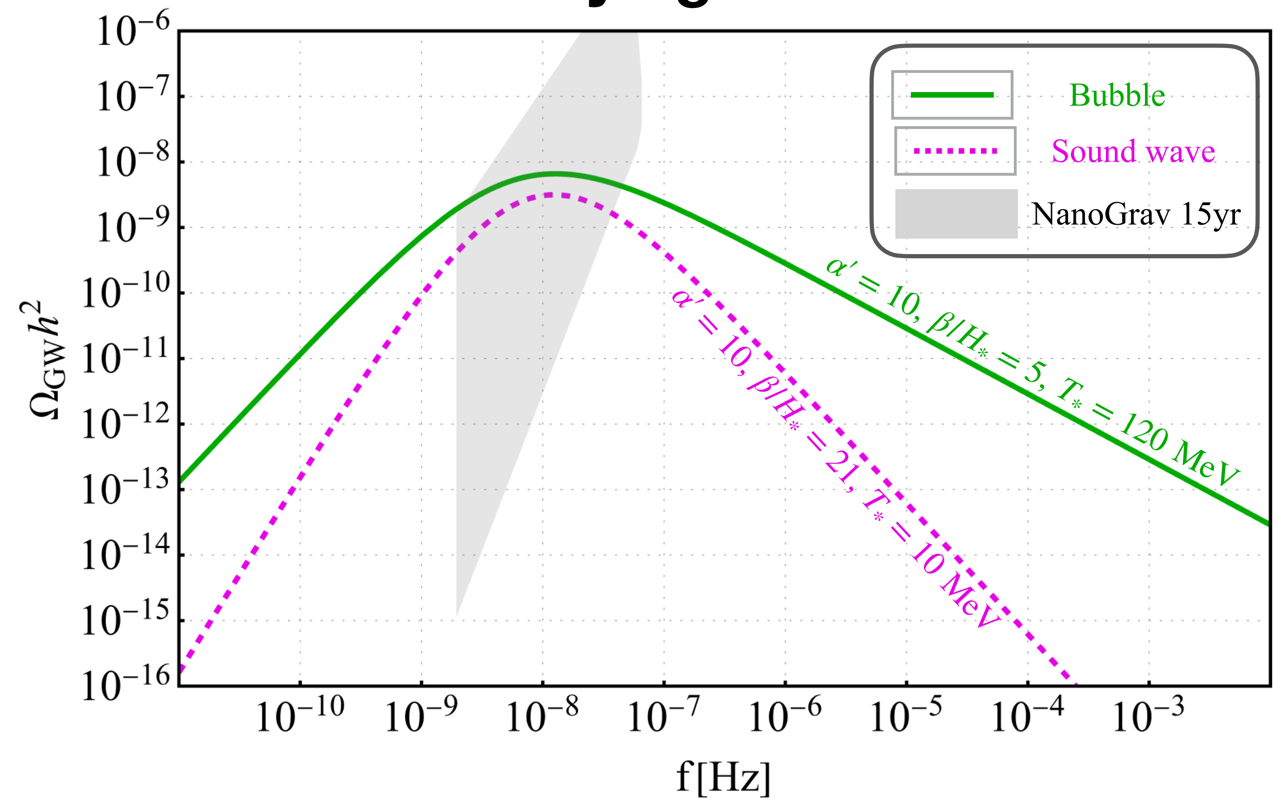
A GW background produced by the dark conformal phase transition

Fujikura, Girmohanta, YN, Suzuki (2023)

Secluded dark sector



Decaying dark sector



- Secluded dark sector case can explain the NANOGrav signal together with SMBHB contribution and ameliorate the Hubble tension.
- Decaying dark sector case can explain the NANOGrav signal by itself.

Supercooling

The phase transition takes place via a supercooling phase.

The vacuum energy dominates the energy density of the Universe and **mini-inflation** takes place before the phase transition is completed.

The e-folding number of mini-inflation :

$$N_e \simeq \log\left(\frac{T_c}{T_n}\right)$$

→ Dilution of dark matter and baryon asymmetry if they are produced before the phase transition.

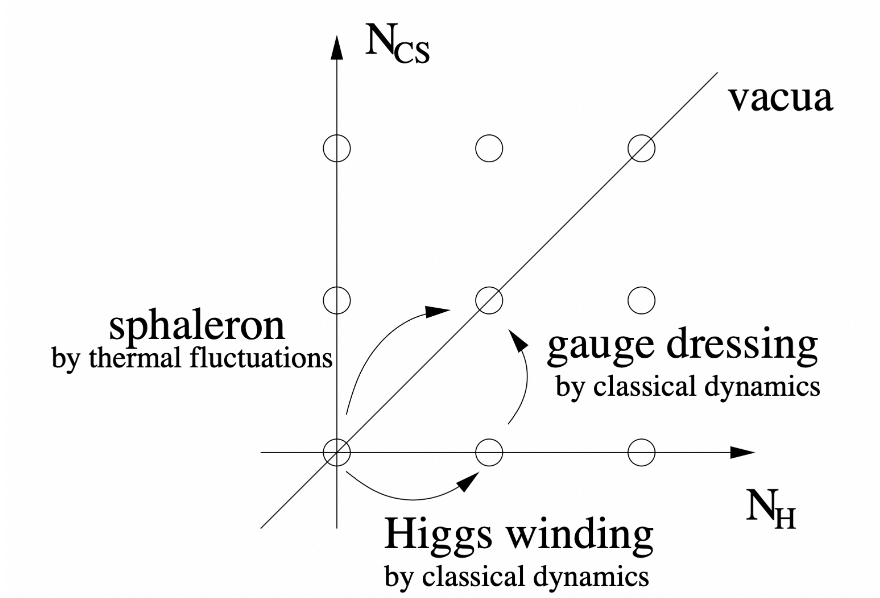
The dilution factor ~10^{-6}

We need a very large amount of dark matter and baryon asymmetry before the phase transition or need to produce them after the phase transition.

Baryogenesis & DM

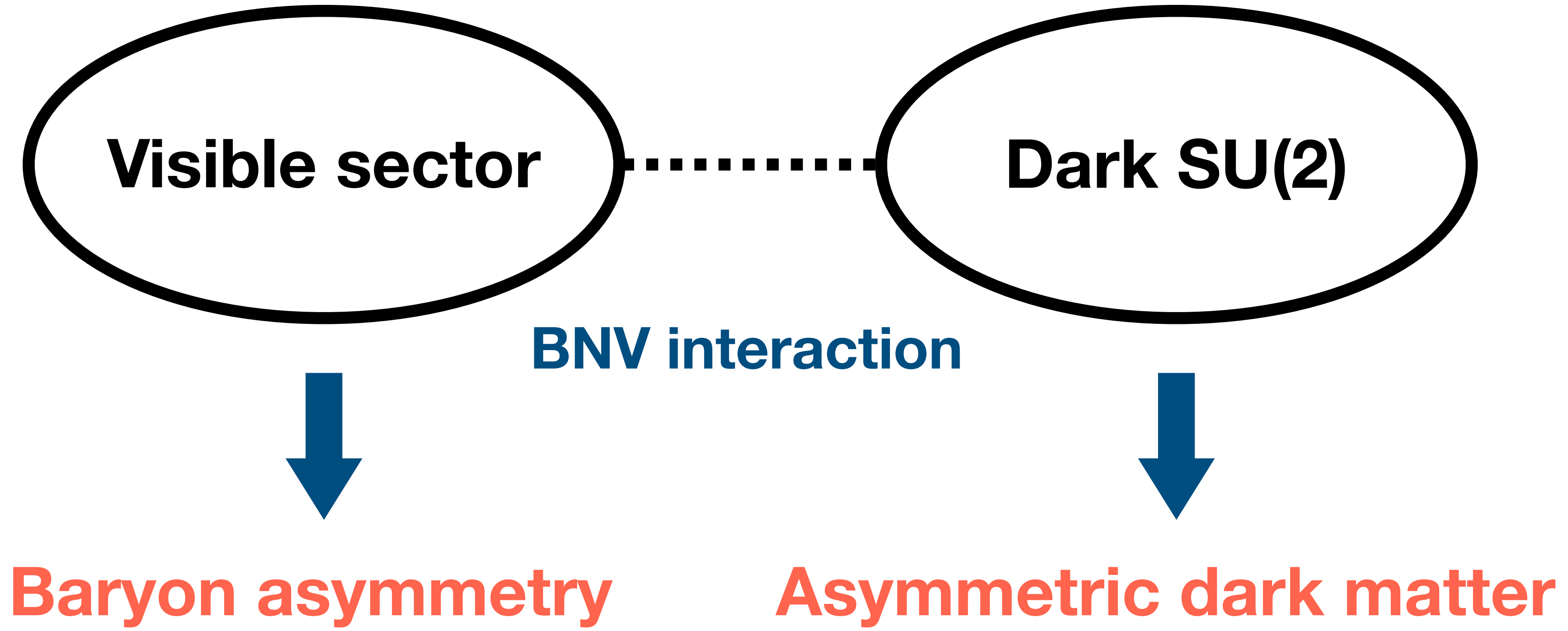
Supercooled phase transition naturally provides a setting of **cold baryogenesis**.

Konstandin, Servant (2011)



Introduce a dark SU(2) and its doublet Higgs field with a CPV coupling.

Doublet/singlet fermions provide dark matter.



Fujikura, Girmohanta, YN, Zhang, work in progress

Summary

✓ Dark phase transition is a promising interpretation of the observed PTA signal.

✓ **Conformal phase transition** can realize a supercooled phase transition to explain the data.

✓ Secluded dark sector case can explain the signal together with SMBHB contribution and ameliorate the Hubble tension.

✓ Decaying dark sector case can explain the signal by itself.

✓ Supercooled phase transition naturally provides a setting of cold baryogenesis, and asymmetric dark matter may solve the baryon-dark matter coincidence problem.

Thank you.

Backup Material

Radion Stabilization

(i) $\Lambda_H(\mu) < m_{KK} = \pi\mu$

Confinement scale: $\Lambda_H(\mu) = \Lambda_{H,0}\left(\frac{\mu}{\mu_{\min}}\right)^n$ $\quad n = \frac{8\pi^2}{b_{\mathrm{YM}} \cdot k g_5^2}$

(ii) $\Lambda_H(\mu) > m_{KK} = \pi\mu$

The description of the 4D effective theory breaks down.

The confinement scale is independent of the radion VEV.

Confinement scale: $\Lambda_H(\mu) = \Lambda_H(\mu_c) \equiv \gamma_c \mu_c \quad \gamma_c = \pi$

Confinement scale of (i) and (ii) are the same at $\mu = \mu_c$