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Gravitational Waves from More Attractive Dark Binaries

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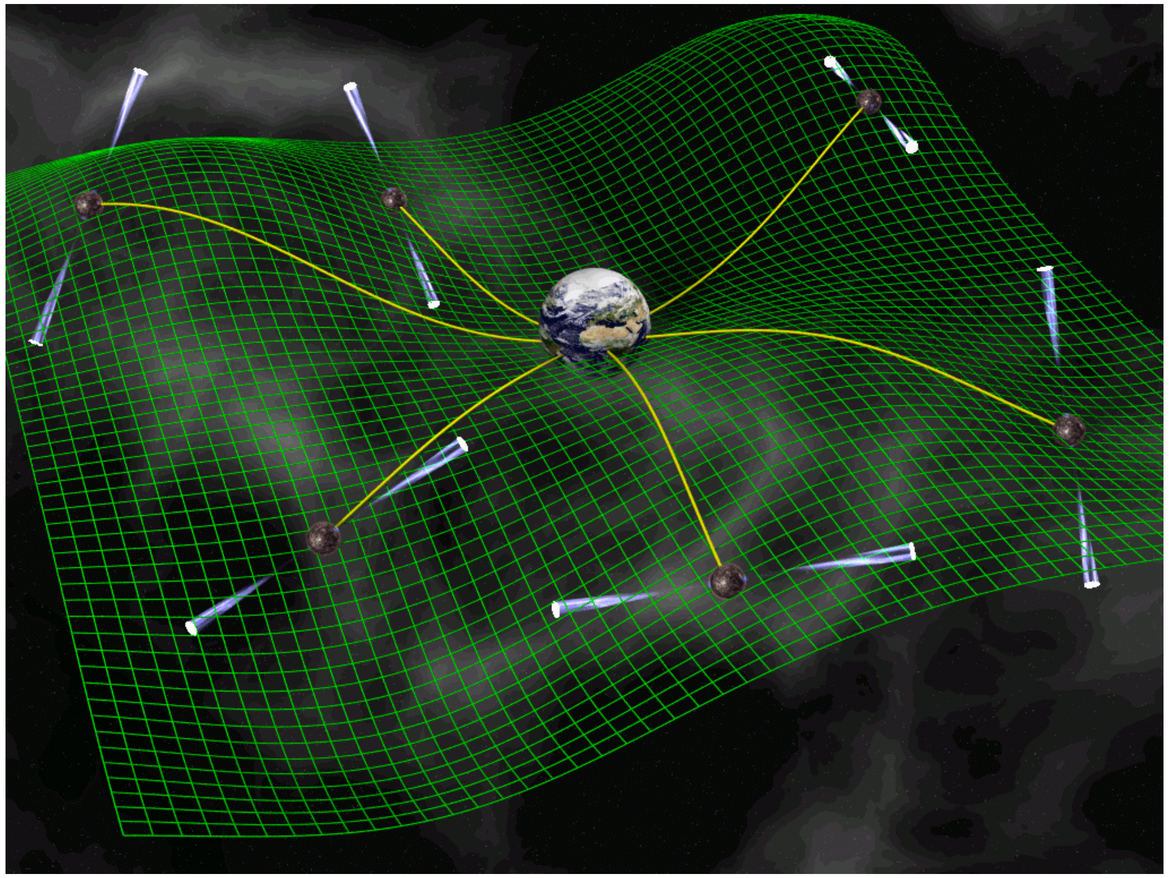
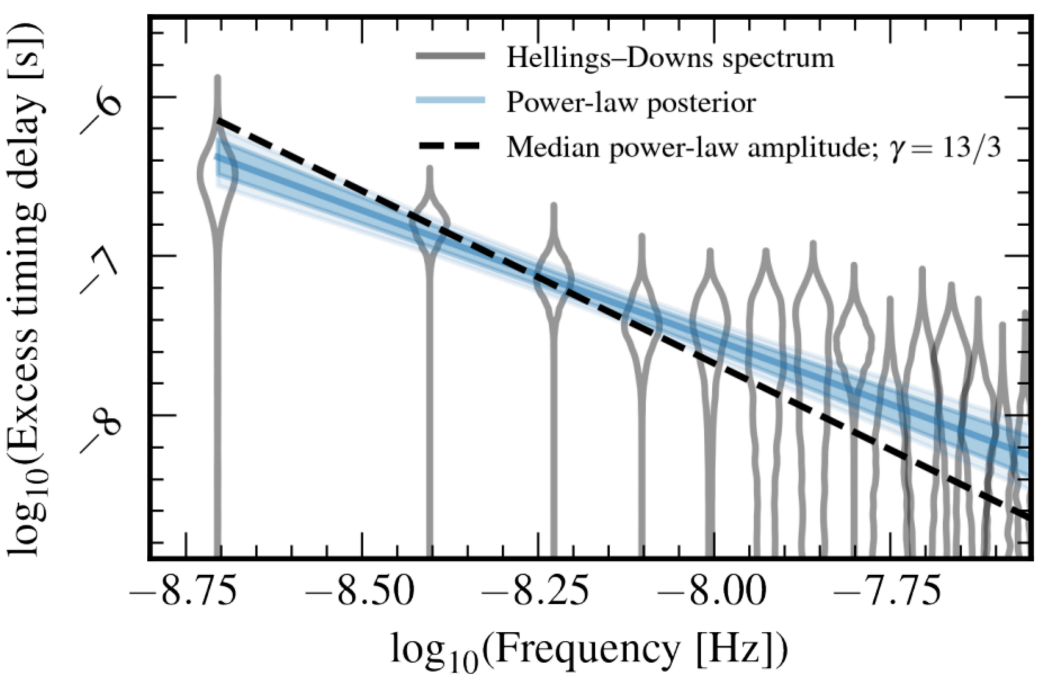
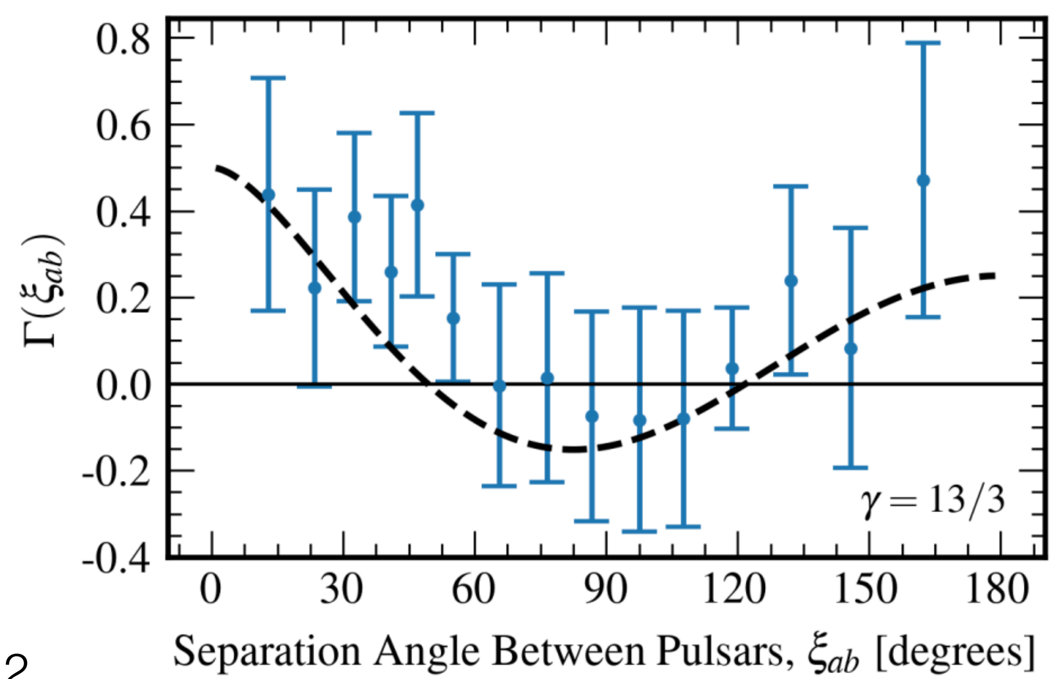
based on 2312.13378, w/ Yang Bai and Nicholas Orlofsky

Stochastic GW Background



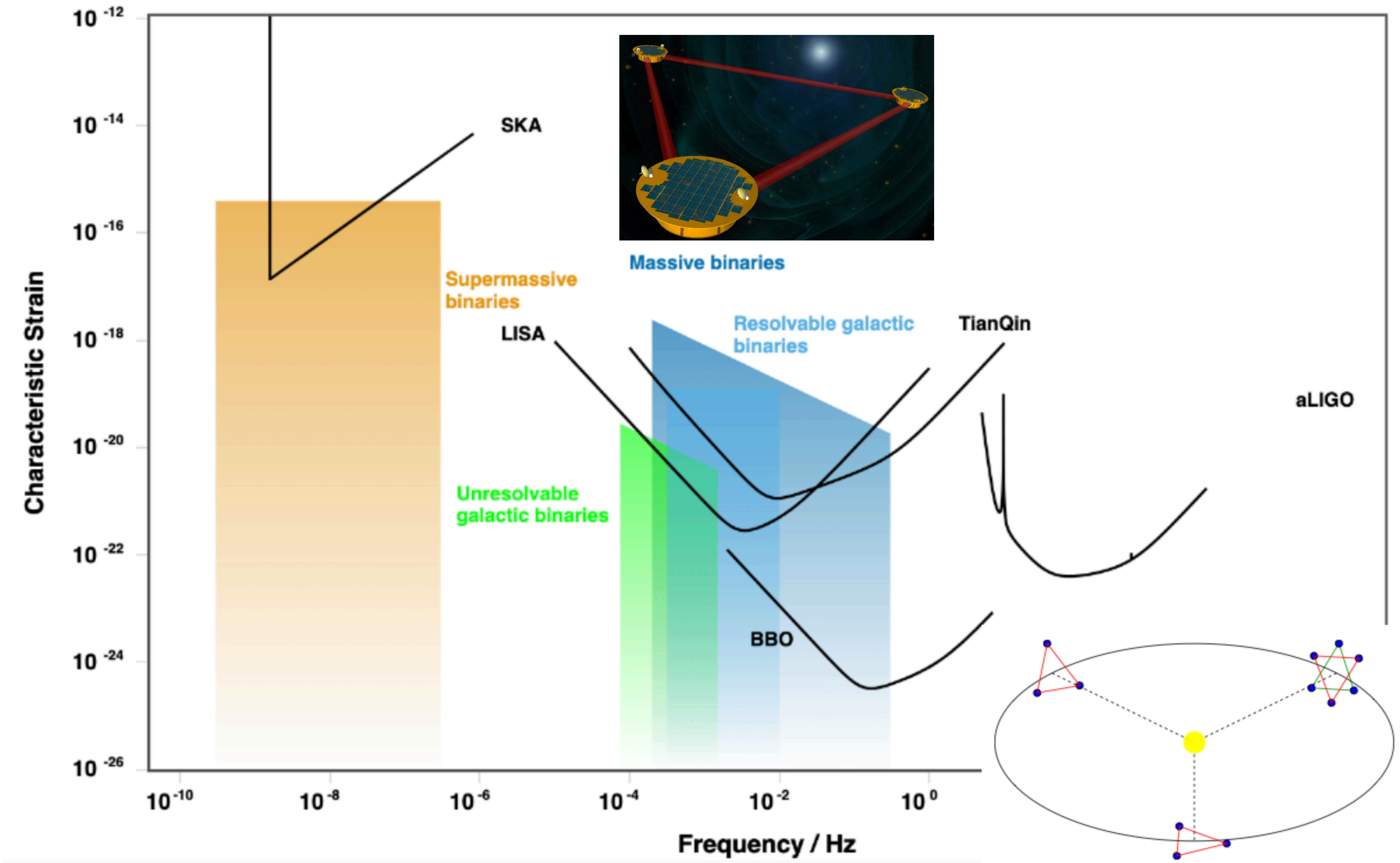
[NANOGrav Collaboration, 2306.16213]





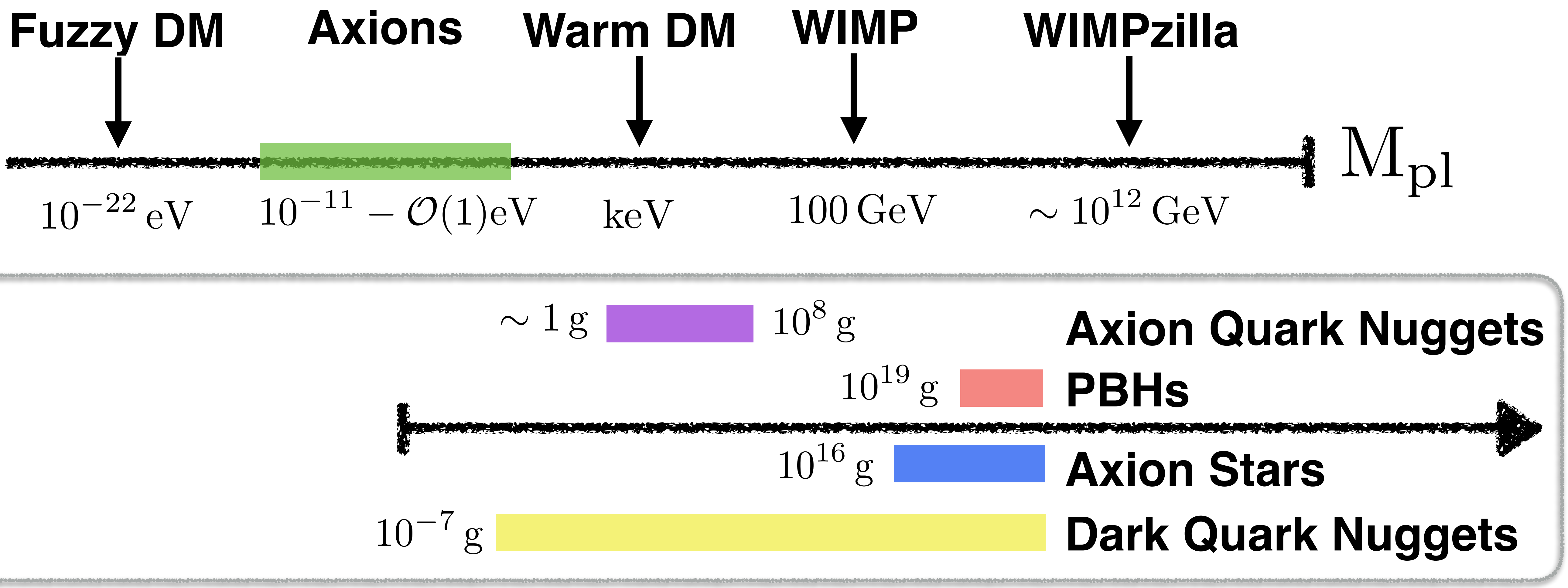
Stochastic GW Background

- New experiments & data analysis



- Possible sources
  - Supermassive black hole binaries, cosmic phase transition, topological defects, scalar-induced (secondary) GW…
  - New possibilities?

Macroscopic DM



- Macroscopic dark matters are interesting candidates born from field theory
- New interactions within the dark sector can change the merger evolution, and hence the SGWB

MDM from Phase Transitions

PHYSICAL REVIEW D VOLUME 30, NUMBER 2 15 JULY 1984

Cosmic separation of phases

Edward Witten*
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 9 April 1984)

A first-order QCD phase transition that occurred reversibly in the early universe would lead to a surprisingly rich cosmological scenario. Although observable consequences would not necessarily survive, it is at least conceivable that the phase transition would concentrate most of the quark excess in dense, invisible quark nuggets, providing an explanation for the dark matter in terms of QCD effects only. This possibility is viable only if quark matter has energy per baryon less than 938 MeV. Two related issues are considered in appendices: the possibility that neutron stars generate a quark-matter component of cosmic rays, and the possibility that the QCD phase transition may have produced a detectable gravitational signal.

MDM from Phase Transitions

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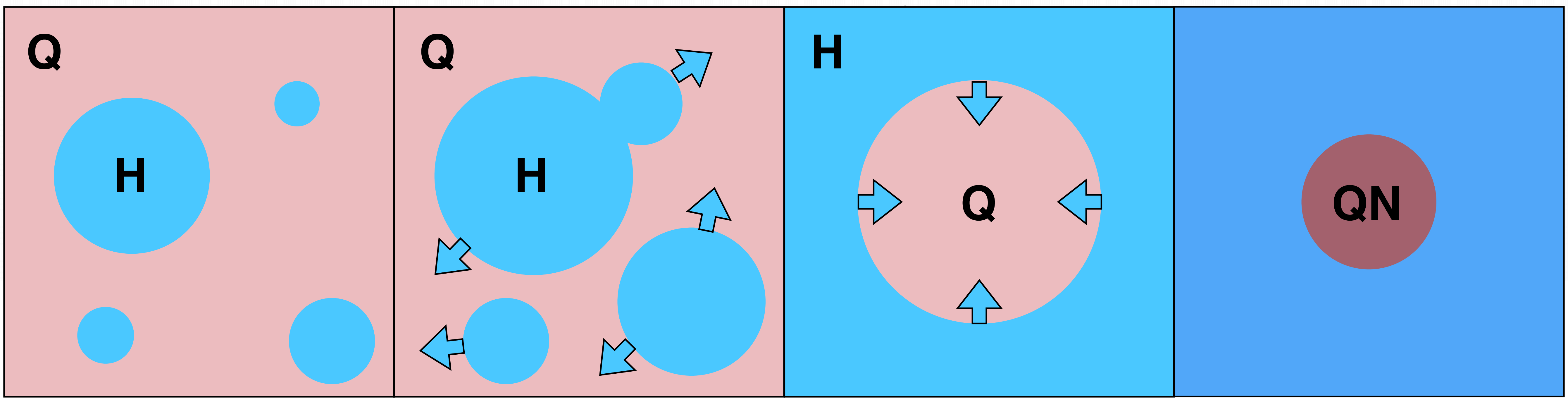


Q: quarks H: hadrons QN: quark nuggets

- relevant degrees of freedom: mass and density

MDM Being More Attractive

- **Orbital motion of the binary with a dark interaction**
  - ➡ Centripetal force, "opposite charge"

$$F = -\frac{Gm_1m_2}{r^2}\left(1 - \alpha e^{-m_{\rm med}r}(1 + m_{\rm med}r)\right)$$

$$\alpha = y^2 q_1 q_2/(4\pi G m_1 m_2)$$

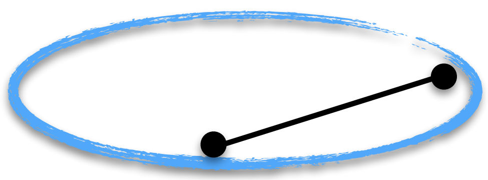
  - ➡ Massless mediator limit

$$F = -G' m_1 m_2/r^2, \quad G' = (1-\alpha)G \equiv \beta G$$

$$\omega^2 = \frac{G'm}{a^3}, \quad E = -\frac{G'm^2\eta}{2a} \quad e^2 = 1 + \frac{2EL^2}{G'^2 m^5 \eta^3}$$

a: semi-major axis
e: eccentricity

- **Dark force changes the energy emission**



Energy Spectrum from the Binary

- **Energy and angular momentum emission**
  - Energy emission through GW portal

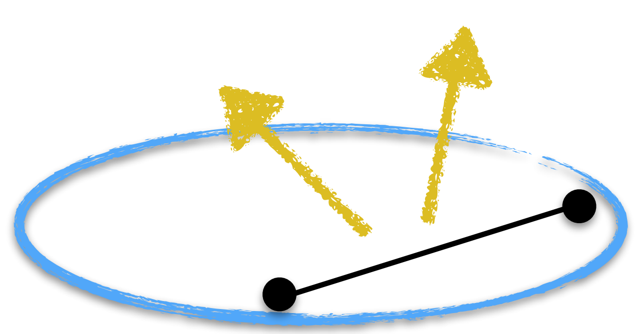
$$Q^{ij} \equiv M^{ij} - \frac{1}{3}\delta^{ij} M_{kk}\,, \quad M^{ij} = \int d^3x\, T^{00} x^i x^j$$

$$\dot{E}_{\rm GW} = \frac{G}{5}\langle \dddot{Q}_{ij}\dddot{Q}_{ij}\rangle\,, \quad \dot{L}^i_{\rm GW} = \frac{2G}{5}\epsilon^{ikl}\langle \ddot{Q}_{ka}\dddot{Q}_{la}\rangle$$

  - The binary is on an elliptical orbit

$$\langle \dot{E}_{\rm GW}\rangle = \frac{32GG'^3\eta^2 m^5}{5a^5(1-e^2)^{7/2}}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)$$

$$\langle \dot{L}_{\rm GW}\rangle = \frac{32GG'^{5/2}\eta^2 m^{9/2}}{5a^{7/2}(1-e^2)^2}\left(1+\frac{7}{8}e^2\right)$$



$$m = m_1 + m_2\,, \quad \eta = \frac{m_1 m_2}{m^2}$$

Energy Spectrum from the Binary

- **For DF domination**
  - Energy emission through the dark force portal

$$\langle \dot{E}_{\rm DF} \rangle = \frac{GG'^2}{12\pi}\eta^2 m^4 \left(\frac{gq_1}{\sqrt{G}m_1} - \frac{gq_2}{\sqrt{G}m_2} \right)^2 \frac{1}{a^4} \frac{2+e^2}{(1-e^2)^{5/2}}$$

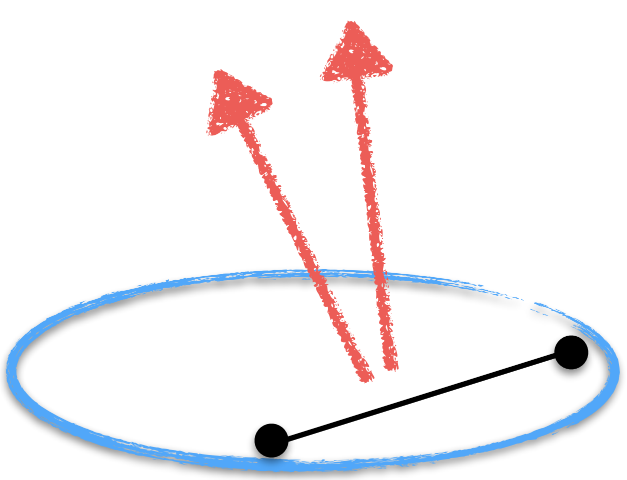
$$\langle \dot{L}_{\rm DF} \rangle = \frac{G'^{3/2}(gq_1 m_2 - gq_2 m_1)^2}{6\pi a^{5/2}(1-e^2)\sqrt{m}}$$

  - Orbit evolution and merger lifetime (DF dominates)

e decreases as *a* decreases

$$a(e) = a_0 \frac{g(e)}{g(e_0)}\,, \quad g(e) = \frac{e^{4/3}}{1-e^2}$$

$$\tau = \frac{4\pi a_0^3}{GG'\eta m^2} \left(\frac{gq_1}{\sqrt{G}m_1} - \frac{gq_2}{\sqrt{G}m_2} \right)^{-2} \frac{(1-e_0^2)^{5/2}(1-\sqrt{1-e_0^2})^2}{e_0^4}$$



Energy Spectrum from the Binary

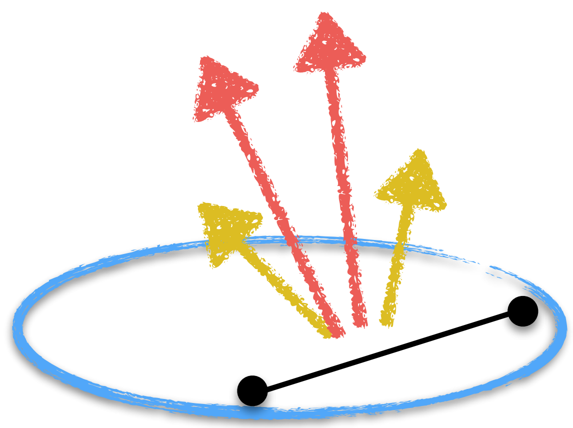
- **The GW energy spectrum of the binary**
  - ➡ GW frequency is related to orbital frequency: $f_{\mathrm{GW},s} = \omega/\pi$

$$\frac{dE_{\mathrm{GW}}}{df_{\mathrm{GW},s}} = \frac{\dot{E}_{\mathrm{GW}}}{\dot{f}_{\mathrm{GW,s}}} = \frac{\pi \dot{E}_{\mathrm{GW}}}{\dot{\omega}} = \frac{\pi \dot{E}_{\mathrm{GW}}}{-\frac{3\sqrt{2}}{G' m^{5/2} \eta^{3/2}} \sqrt{-E}\dot{E}}$$

$$\dot{E} = \dot{E}_{\mathrm{GW}} + \dot{E}_{\mathrm{DF}}$$

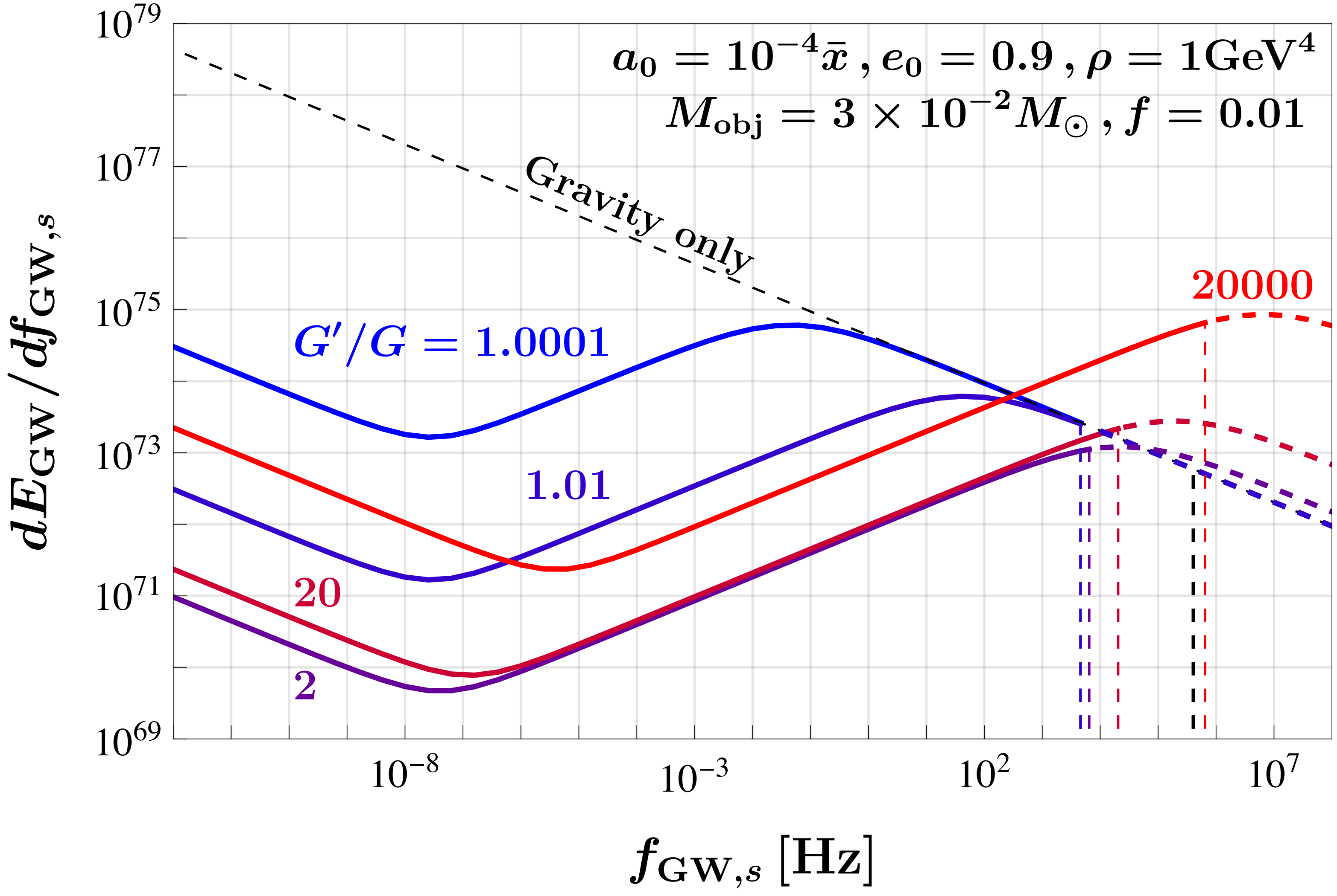
  - ➡ For convenience we assume same mass, opposite charge

$$\frac{dE_{\mathrm{GW}}}{df_{\mathrm{GW},s}} = \frac{\pi \sqrt{a} \left(37e^4 + 292e^2 + 96\right) G'^{3/2} M_{\mathrm{obj}}^{5/2}}{3\sqrt{2} \left(10\, a(1-e^2)(2+e^2)(\beta - 1) + \left(37e^4 + 292e^2 + 96\right) G' M_{\mathrm{obj}}\right)}$$



Energy Spectrum from the Binary

✤ **The GW energy spectrum of the binary**



Energy Spectrum from the Binary

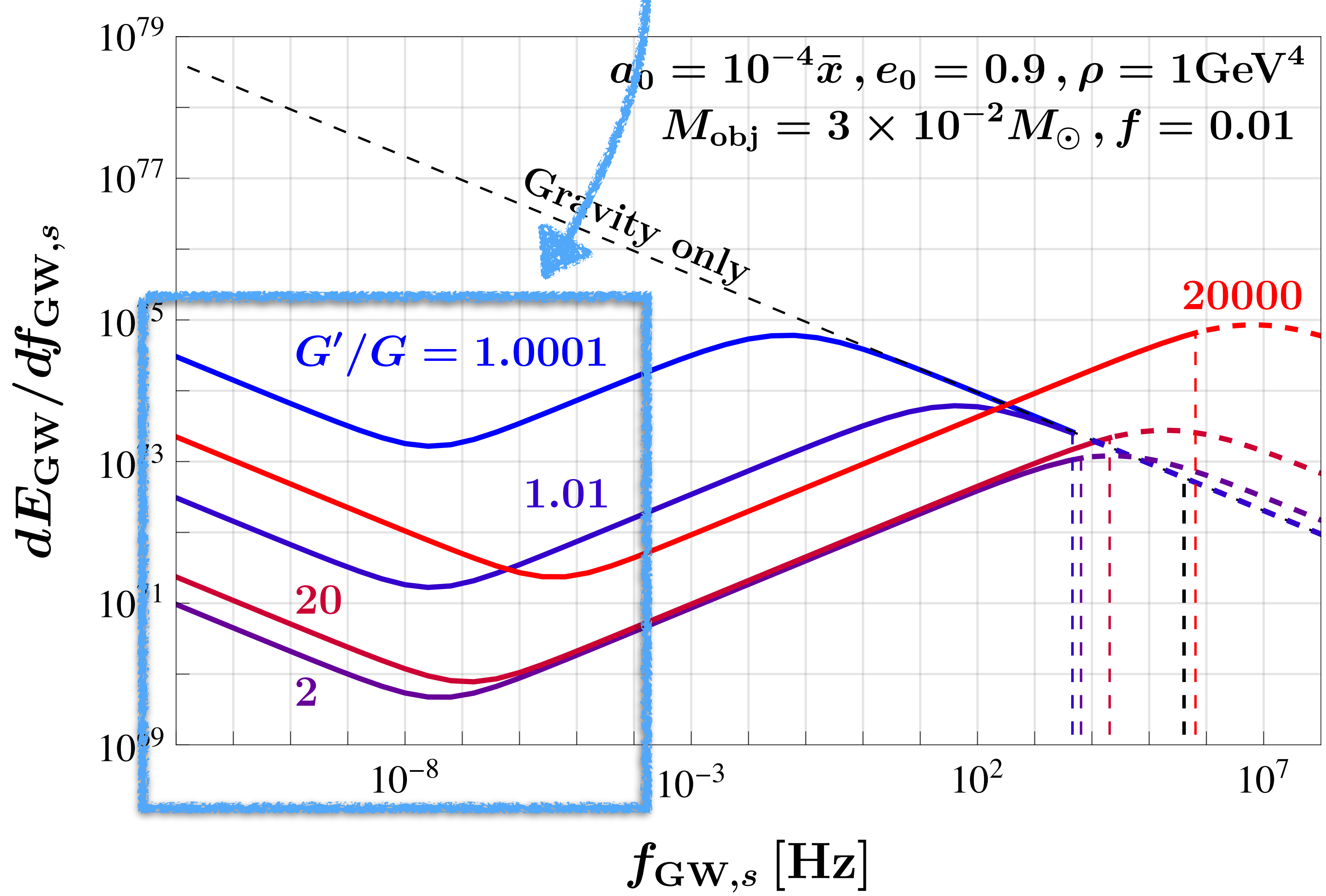
✤ **The GW energy spectrum of the binary**

$$\frac{dE_{\rm GW}}{df_{{\rm GW},s}} = \frac{\pi\sqrt{a}\,(37e^4+292e^2+96)\,G'^{3/2}M_{\rm obj}^{5/2}}{3\sqrt{2}\,(10\,a(1-e^2)(2+e^2)(\beta-1)+\sout{(37e^4+292e^2+96)\,G'M_{\rm obj}})}$$

frozen

$$a \propto f_{{\rm GW},s}^{-2/3}$$

$$a(e) = a_0\frac{g(e)}{g(e_0)}\,,\quad g(e) = \frac{e^{4/3}}{1-e^2}$$

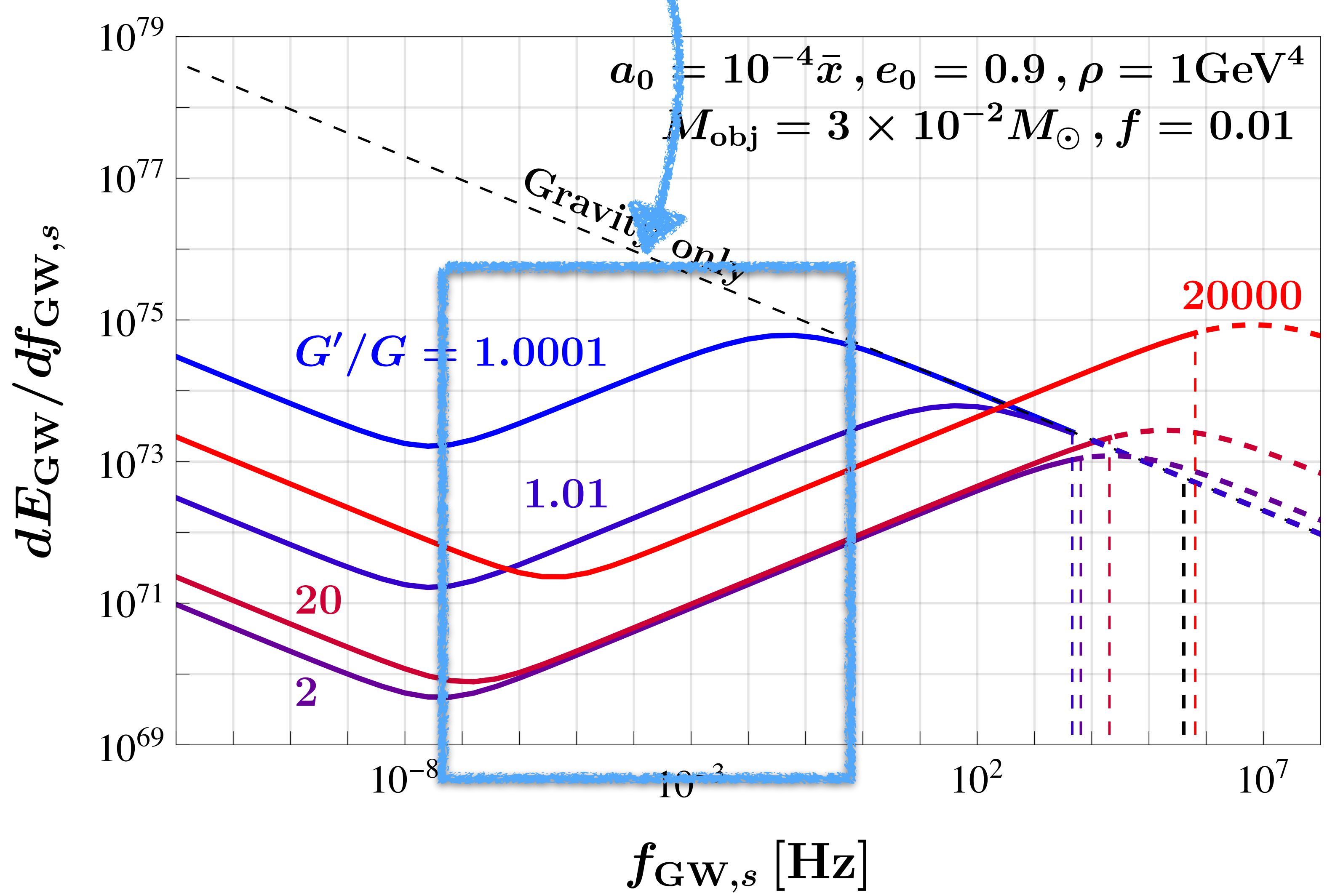


Energy Spectrum from the Binary

- The GW energy spectrum of the binary

$$\frac{dE_{\rm GW}}{df_{{\rm GW},s}} = \frac{\pi\sqrt{a}\,(37e^4+292e^2+96)\,G'^{3/2}M_{\rm obj}^{5/2}}{3\sqrt{2}\,(10\,a(1-e^2)(2+e^2)(\beta-1)+(37e^4+292e^2+96)\,G'M_{\rm obj})}$$

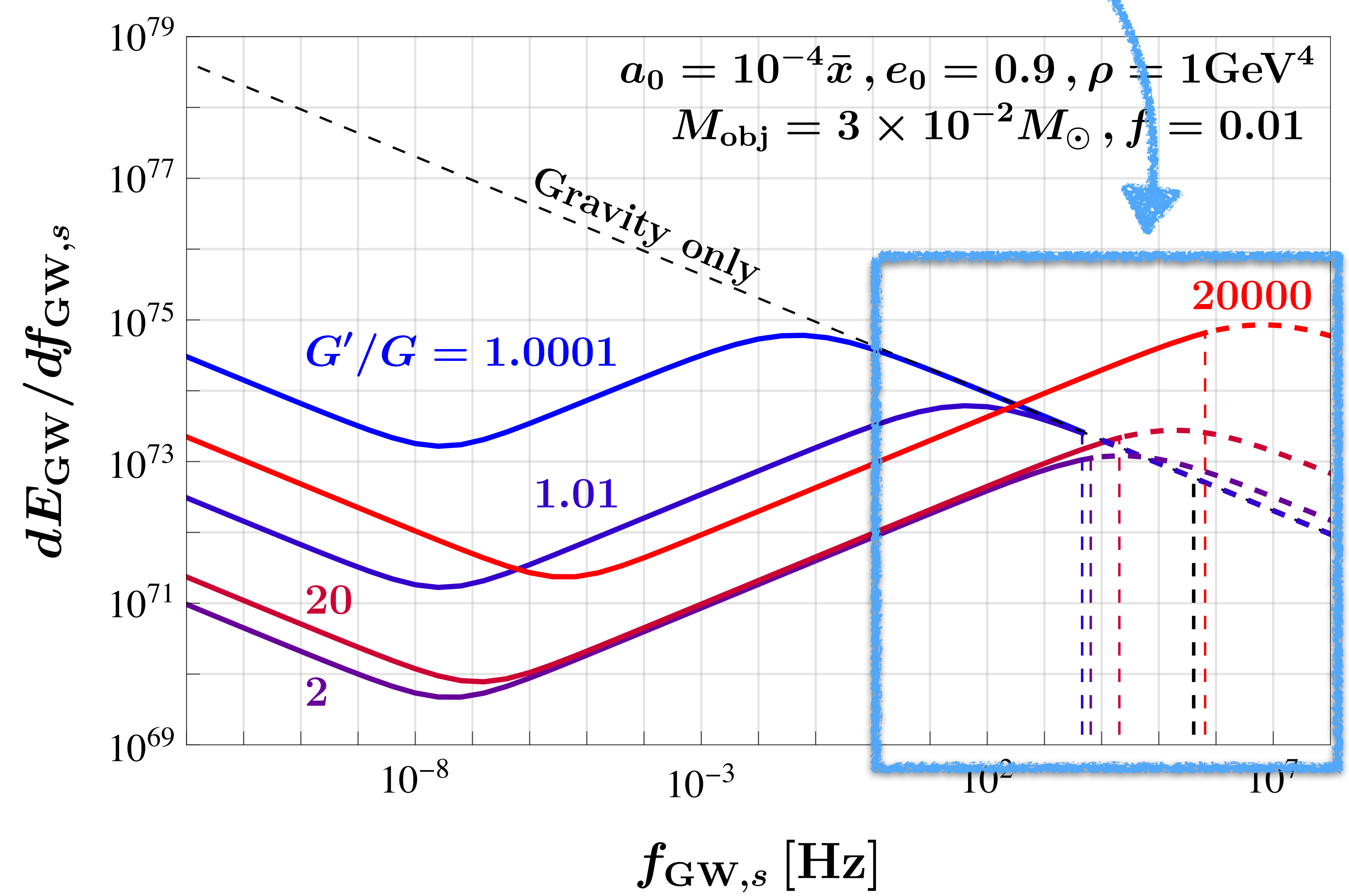
$$a \propto f_{{\rm GW},s}^{-2/3}$$



Energy Spectrum from the Binary

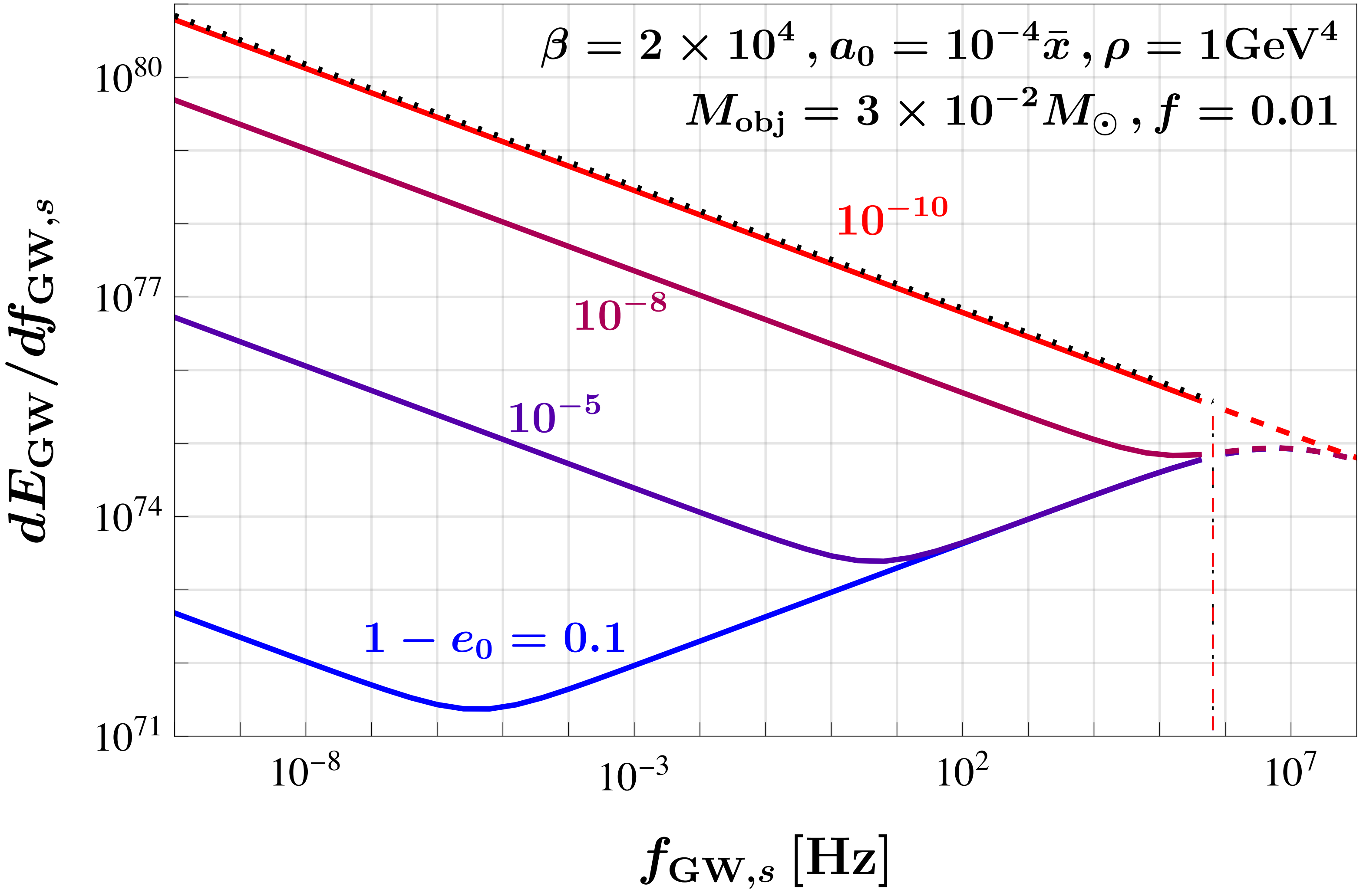
- **The GW energy spectrum of the binary**

$$\frac{dE_{\rm GW}}{df_{{\rm GW},s}} = \frac{\pi\sqrt{a}\left(37e^4+292e^2+96\right)G'^{3/2}M_{\rm obj}^{5/2}}{3\sqrt{2}\left(10a(1-e^2)(2+e^2)(\beta-1)+(37e^4+292e^2+96)\,G'M_{\rm obj}\right)}$$



Energy Spectrum from the Binary

- **The GW energy spectrum of the binary**
  - ➡ Small a_0 or e_0



SGWB from Dark Binaries

- **Convolution over cosmic history**
  - ➡ For primordial black holes (gravity only)

$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{f_{\rm GW}}{\rho_c}\int_0^{z_{\rm sup}} dz \frac{R(z)}{(1+z)H(z)}\frac{dE_{\rm GW}}{df_{{\rm GW},s}}((1+z)f_{\rm GW})$$

  - ➡ With additional interactions, orbital geometry becomes extremely important

$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{f_{\rm GW}}{\rho_c}\int_{\sqrt{1-c_2^2}}^{e_{0,\rm max}} de_0 \int d\tau \frac{n_{\rm obj}}{4}\mathcal{P}(\tau, e_0)\frac{dE_{\rm GW}}{df_{{\rm GW},s}}\left[(1+z(t))f_{\rm GW}\right]$$

orbital eccentricity

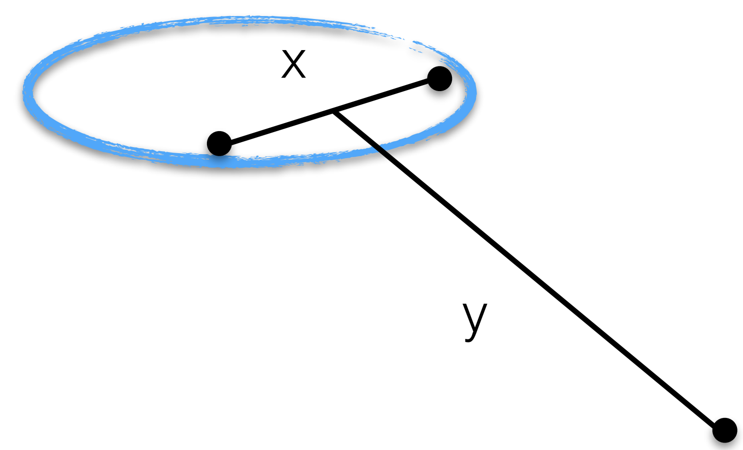
merger lifetime, related to semi-major axis

The Merger Rate

- **The merger rate depends on the geometry of the binary and its nearest neighbor**

$$R(x,y) = \frac{1}{2}\frac{n_{\rm obj}}{2}P = \frac{1}{2}\frac{3H_0^2}{8\pi G}\frac{f\,\Omega_{\rm DM}}{2M_{\rm obj}}P(x,y)$$

x: comoving distance between the binary
y: comoving distance to the nearest neighbor



➡ Assuming random formation

$$P(x,y)\,dxdy = \frac{9x^2y^2}{\bar{x}^6}e^{-(y/\bar{x})^3}dxdy$$

$$\bar{x} = \frac{1}{1+z_{\rm eq}}\left(\frac{8\pi G M_{\rm obj}}{3H_0^2 f\,\Omega_{\rm DM}}\right)^{1/3}$$

[Ioka, Chiba, Tanaka, Nakamura, astro-ph/9807018]

The Merger Rate

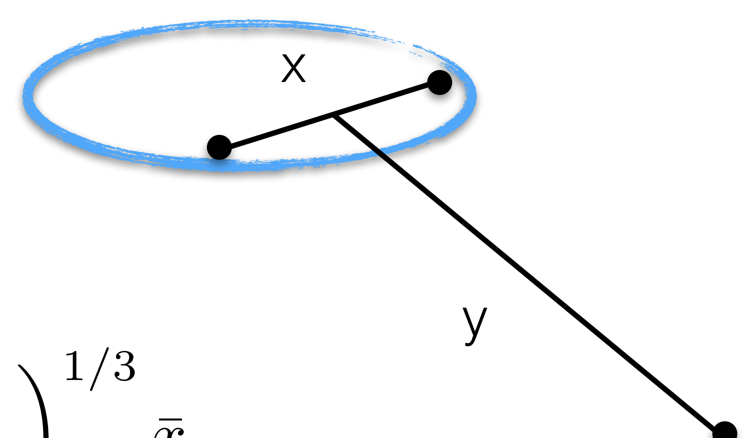
- **The merger rate**
  - In terms of the orbital parameters

[Ioka, Chiba, Tanaka, Nakamura, astro-ph/9807018]

$$a_0 = \frac{c_1}{\beta}\frac{1}{f}\frac{x^4}{\bar{x}^3}\,, b_0 = c_2\left(\frac{x}{y}\right)^3 a_0\,,$$

$$e_0 = \sqrt{1-\left(\frac{b_0}{a_0}\right)^2}\,.$$

$$y > x \Rightarrow e_0^2 > 1 - c_2^2$$



  - We take c1=0.4, c2=0.8
  - Related to the merger lifetime

$$a_0 = \left(\frac{4G^2(\beta-1)\beta M_{\rm obj}^2\tau}{h(e_0)}\right)^{1/3} = \left(\frac{\tau/\bar{\tau}}{h(e_0)}\right)^{1/3}\bar{x}$$

$$h(e_0) = \frac{(1-e_0^2)^{5/2}(1-\sqrt{1-e_0^2})^2}{e_0^4}$$

SGWB from Dark Binaries

- **With these discussed**

$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{f_{\rm GW}}{\rho_c} \int_{\sqrt{1-c_2^2}}^{e_{0,\rm max}} de_0 \int d\tau \frac{n_{\rm obj}}{4} \mathcal{P}(\tau, e_0) \frac{dE_{\rm GW}}{df_{{\rm GW},s}} \left[(1+z(t)) f_{\rm GW}\right]$$

$$t = t_{\rm dec} + \tau$$

- **Decoupling of the dark binaries**
  - ➡ Compare the average energy density with the dragging from the Hubble flow

$$G' \bar{\rho}_{\rm obj} \equiv G' \cdot f \frac{\rho_{\rm eq}}{2} \frac{\bar{x}^3 R_{\rm eq}^3}{x^3 R^3} = G \rho_r$$

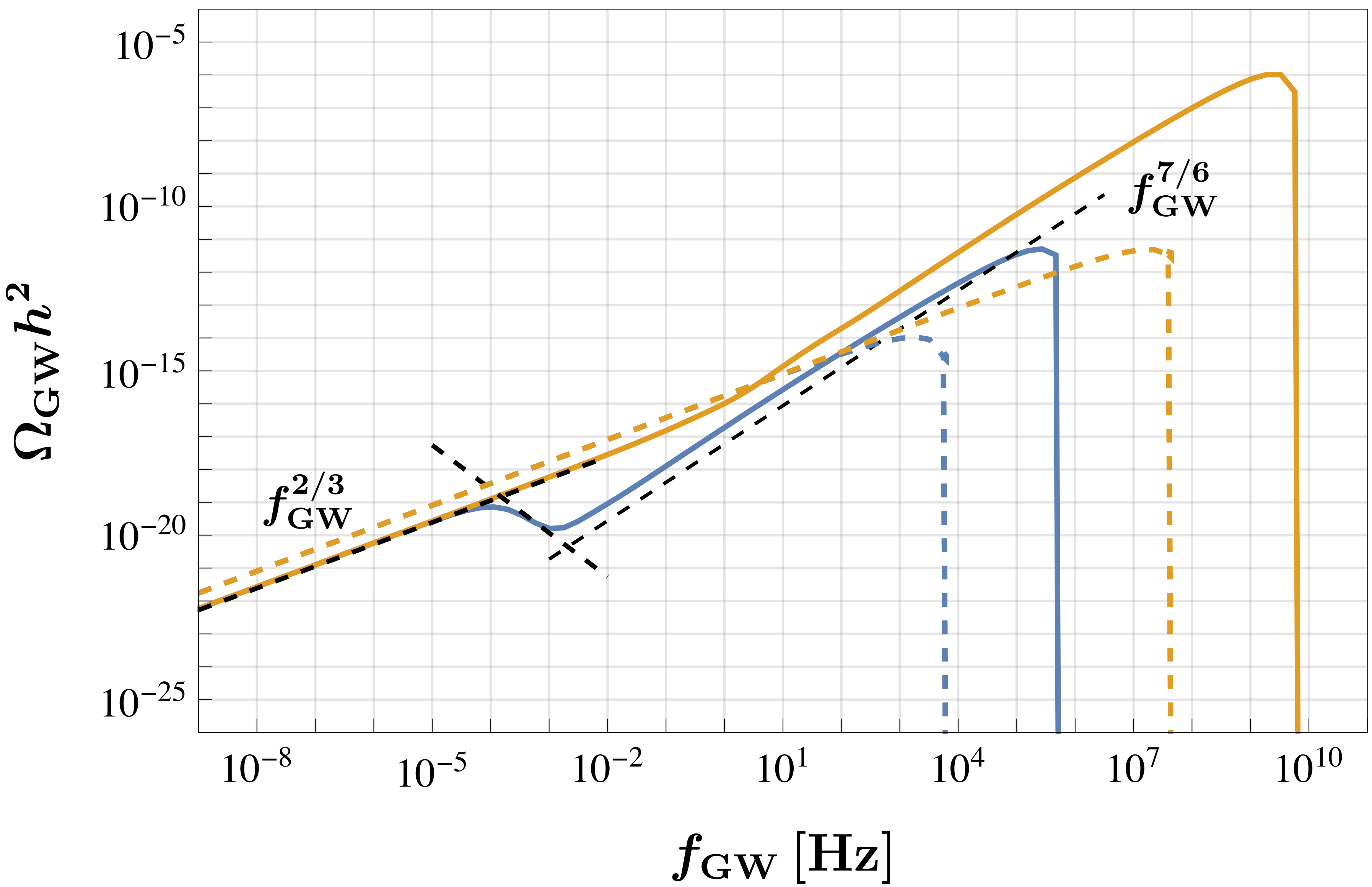
$$1 + z_{\rm dec} = \left(\frac{2\pi\, c_1^3 (1+z_{\rm eq})}{3 H_0^2 G M_{\rm obj} (\beta - 1) \Omega_{\rm DM}} \frac{h(e_0)}{\tau} \right)^{1/4}$$

  - ➡ Should decouple before matter-radiation equality

[Ioka, Chiba, Tanaka, Nakamura, astro-ph/9807018]

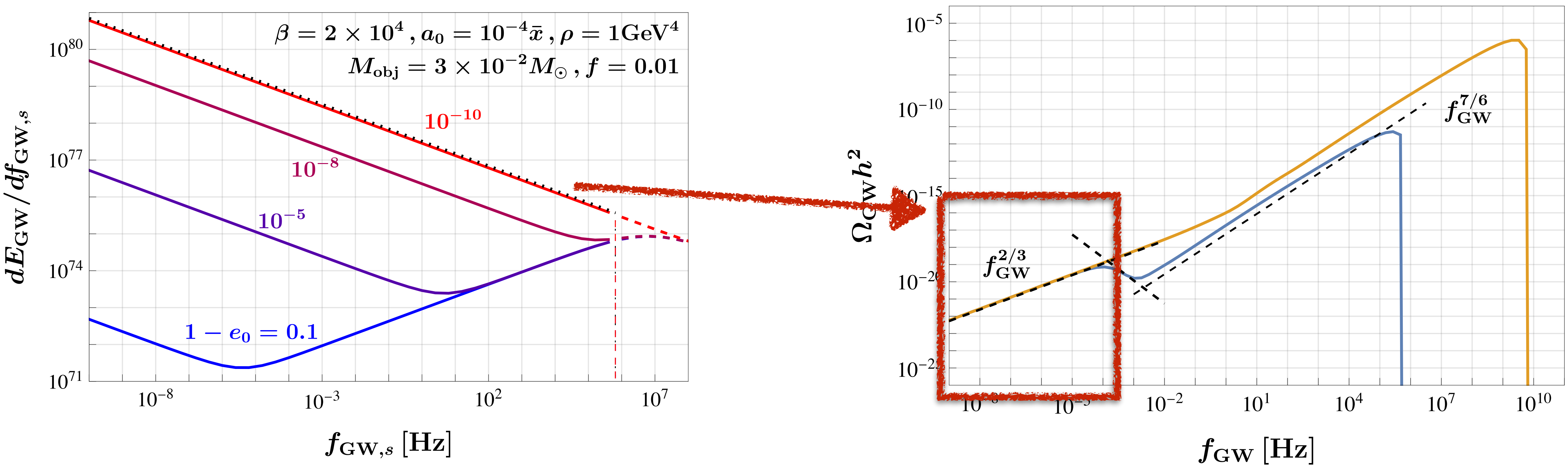
Spectral Shape

- A two- or three-stage power-law spectrum



Spectral Shape

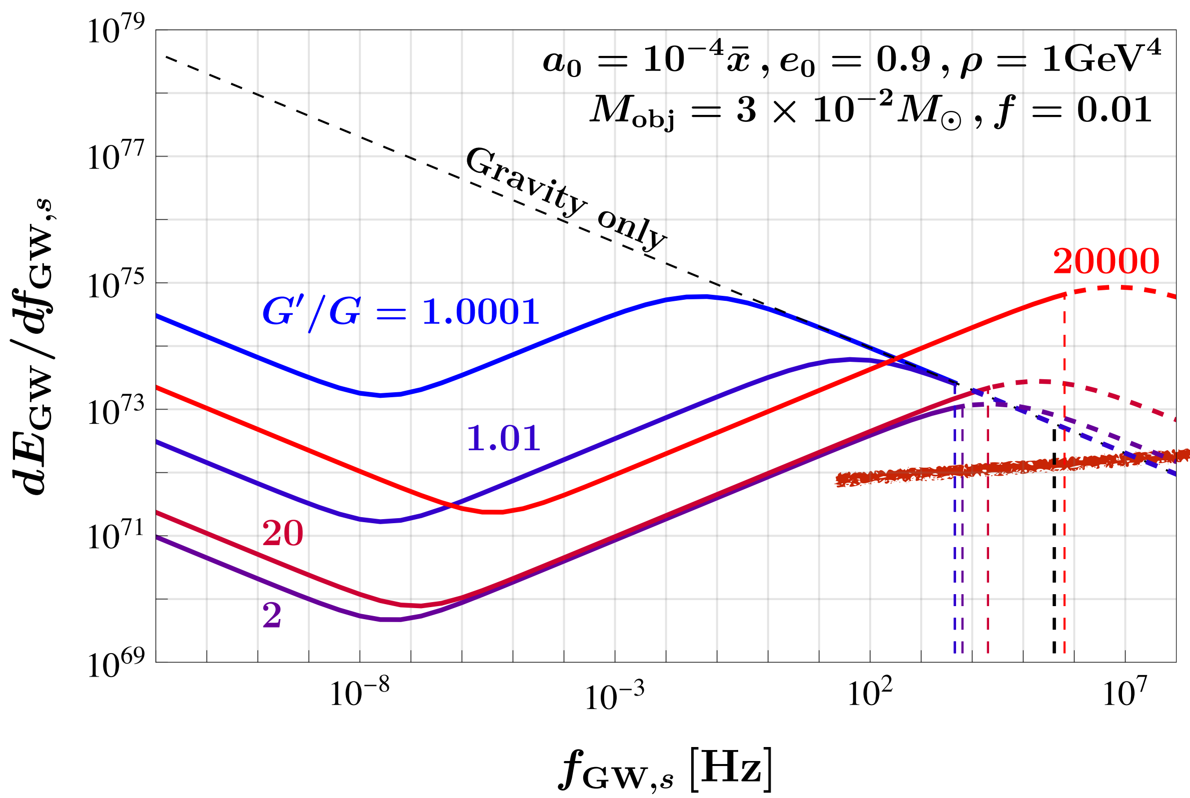
- **The low-frequency region**
  - A relic from the very early, GW-dominated mergers

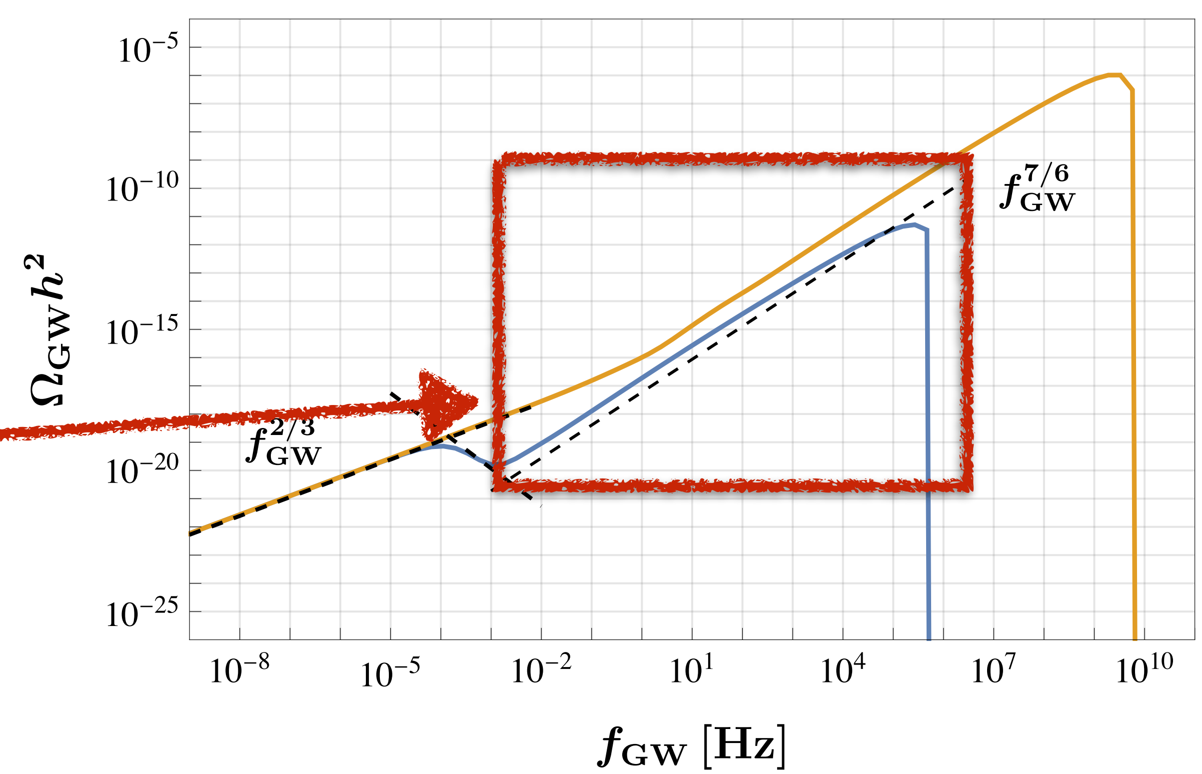


$$\Omega_{\rm GW} \approx \frac{85^{\frac{2}{5}} \pi^{\frac{8}{15}} \Gamma^{\frac{3}{5}}(\frac{7}{3}) f^{\frac{6}{5}} \left(G^{14} \beta^{17} H_0^4 M_{\rm obj}^{14} (1+z_{\rm eq}) \Omega_{\rm DM}^{17}\right)^{\frac{1}{15}}}{2^{\frac{77}{15}} 3^{\frac{4}{15}} c_1^{\frac{4}{5}} c_2^{\frac{4}{5}} (\beta - 1)^{\frac{2}{5}}} f_{\rm GW}^{2/3}$$

Spectral Shape

- **The high-frequency region**
  - Mainly from dark force dominated mergers at MD





$$\Omega_{\rm GW} \approx \frac{4\times 2^{\frac{5}{6}}\times 13^{\frac{1}{18}}\pi^{\frac{59}{36}} f^{\frac{13}{9}} G^{\frac{55}{36}}\beta^{\frac{67}{36}}(1+z_{\rm eq})^{\frac{1}{3}}\rho_{\rm obj}^{\frac{1}{12}} H_0^{\frac{1}{9}} M_{\rm obj}^{\frac{13}{9}}\Omega_{\rm DM}^{\frac{10}{9}}}{45\times 3^{7/36} e^{13/9} c_1^{1/3} c_2^{5/9}(\beta-1)^{8/9}\Omega_{\rm M}^{1/18}} f_{\rm GW}^{7/6}$$

Sensitivity at Experiments

- **Signal-to-noise ratio for multiple detectors where cross-correlation can be performed**

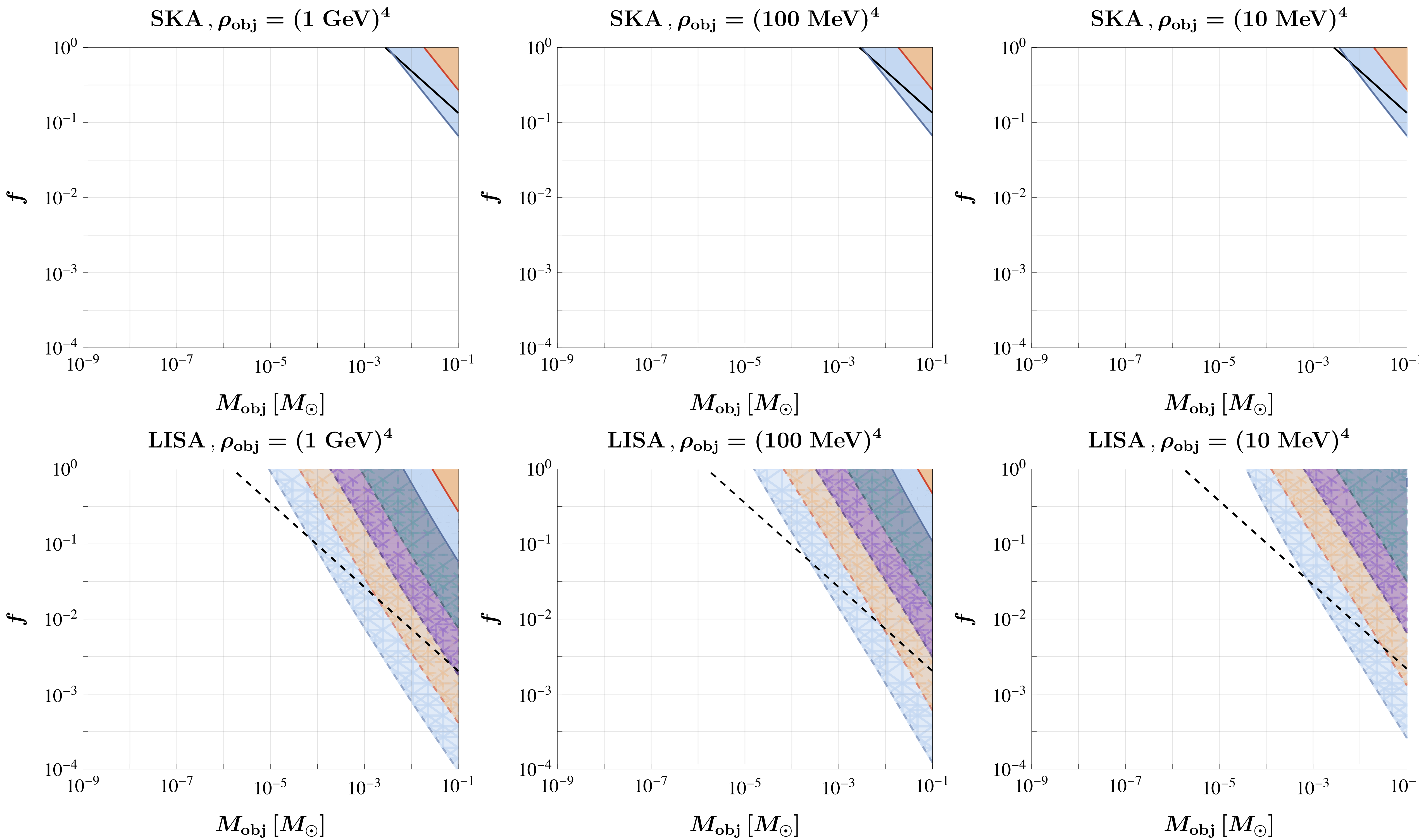
$$\varrho^2 = n_{\rm det} T_{\rm obs} \int df_{\rm GW} \left(\frac{\Omega_{\rm GW}}{\Omega_{\rm noise}} \right)^2$$

[Schmitz, 2002.04615]

  - ➡ SKA, BBO, DECIGO (if still alive), LIGO-Virgo network (HLV)
  - ➡ n_det=2 for cross-correlation, and 1 for auto-correlation (if applicable)
  - ➡ Nontrivial noise subtraction is required for auto-correlation
    - TDI interfereometry?

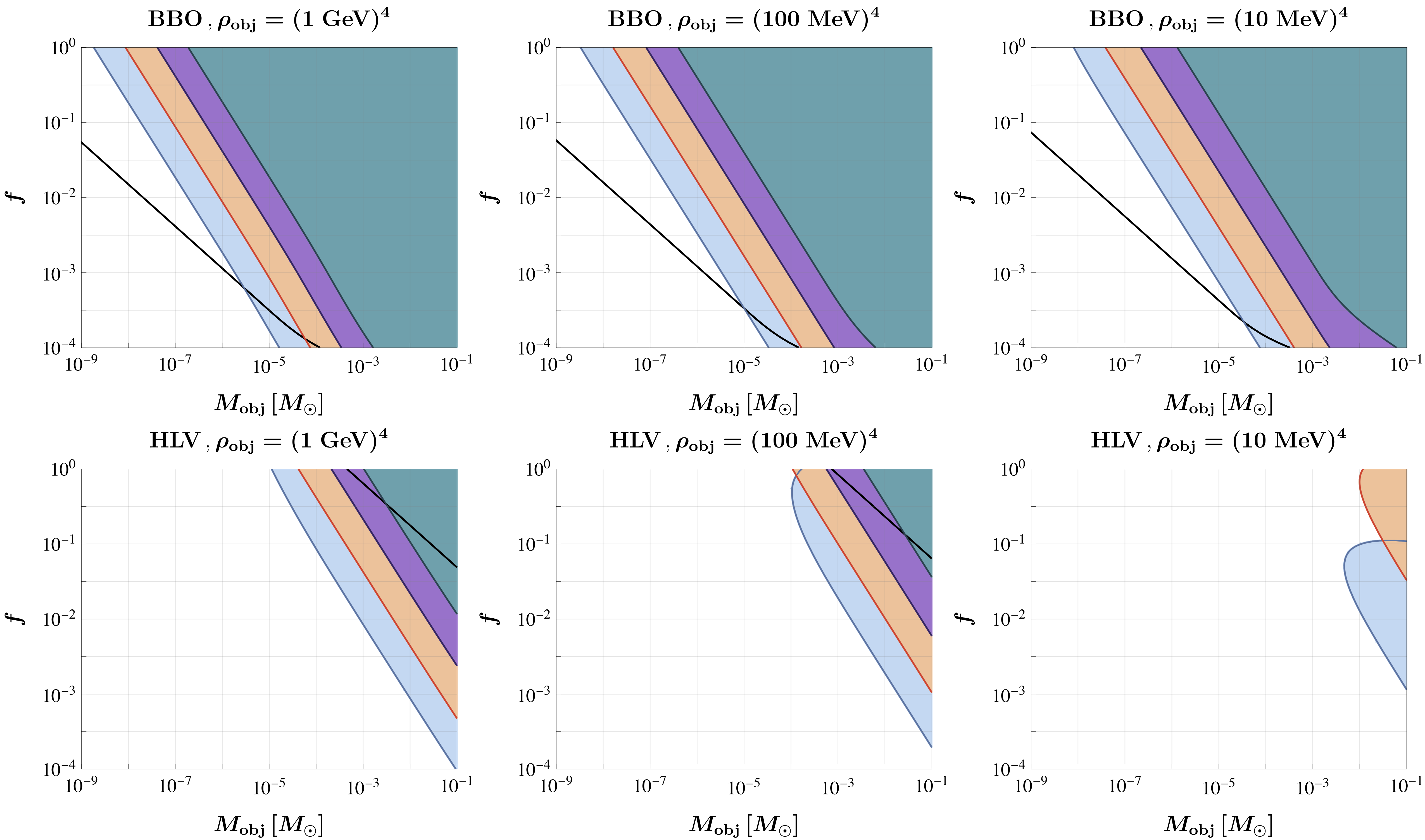
[Smith and Caldwell, 1908.00546]

Sensitivity at Experiments



$\varrho_{th} = 1\,,\ T_{obs} = 20\ \mathrm{yr\ (SKA)}/1\ \mathrm{yr\ (others)}$

Sensitivity at Experiments



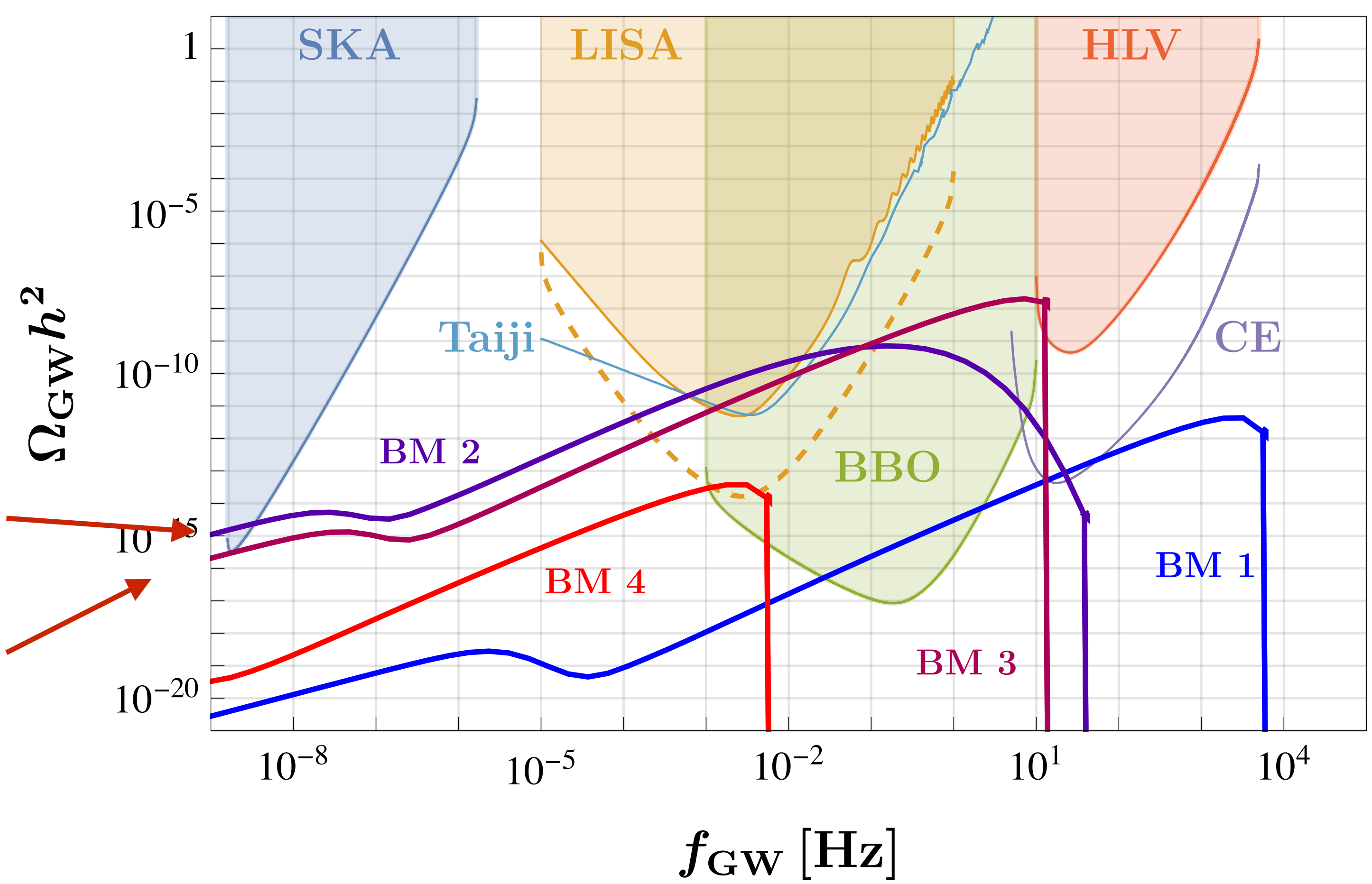
$\varrho_{\rm th} = 1\,,\ T_{\rm obs} = 20$ yr (SKA)/1 yr (others)

Sensitivity at Experiments

- **A larger interaction doesn't always come with a larger signal**

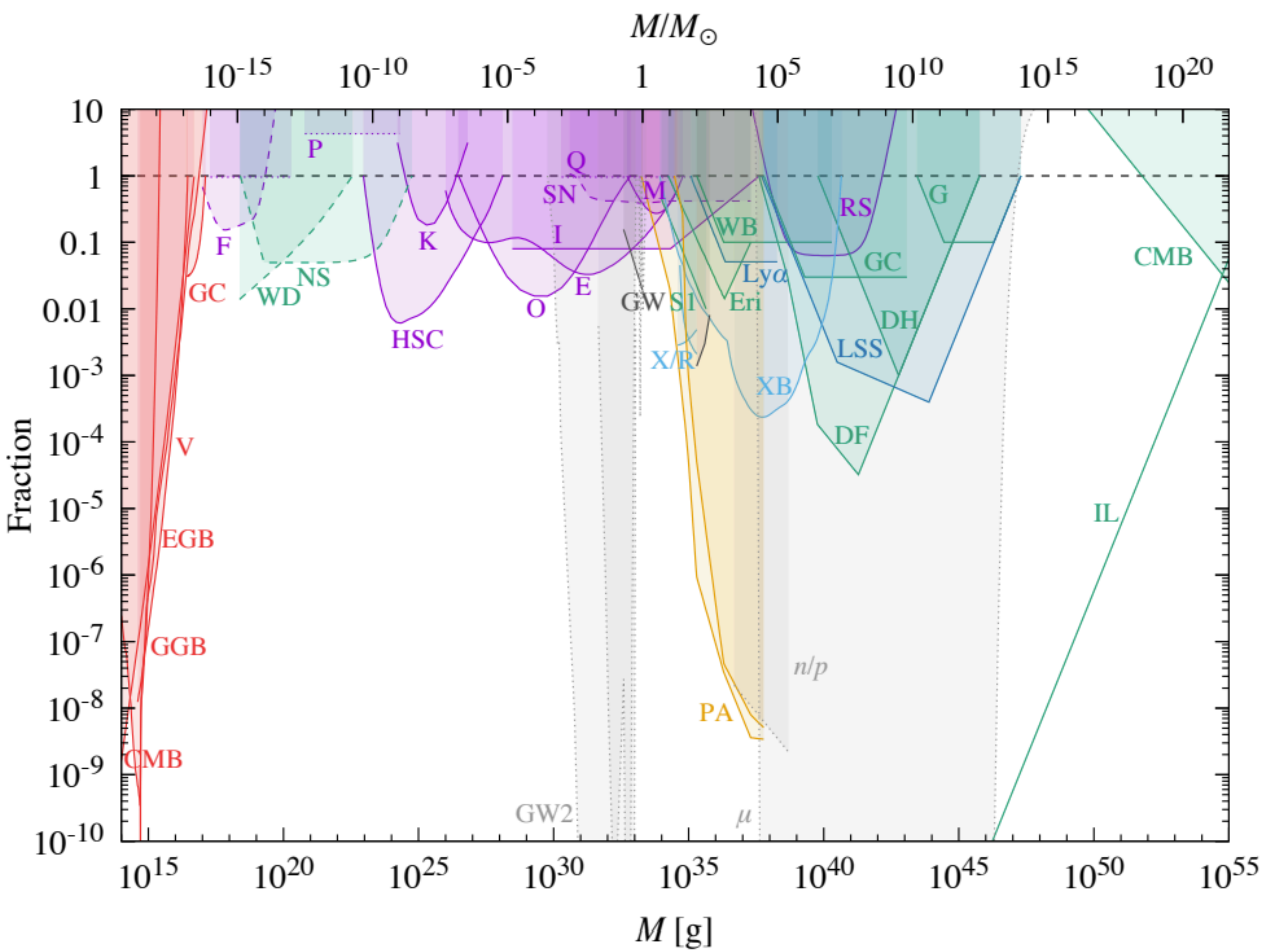
$(\beta, M_{\rm obj}, f, \rho_{\rm obj}^{1/4}) = (10^4, 0.1\,M_\odot, 0.25, 10\,{\rm MeV})$

$(10^3, 0.1\,M_\odot, 0.25, 10\,{\rm MeV})$



➡ Too large an interaction makes the merger happens too early and thus doesn't contribute to the corresponding frequency

Lensing



[Carr et.al., 2002.12778]

Constraints from Neff

- **Extra radiation d.o.f. is usually parameterized in terms of extra neutrino species**

$$\Delta\rho_{\rm rad} = \frac{\pi^2}{30}\frac{7}{4}\left(\frac{4}{11}\right)^{4/3}\Delta N_{\rm eff} T^4$$

  - ➡ Currently constrained by Planck to be $\Delta N_{\rm eff} \lesssim 0.3$

[Planck Collaboration, 1807.06209]

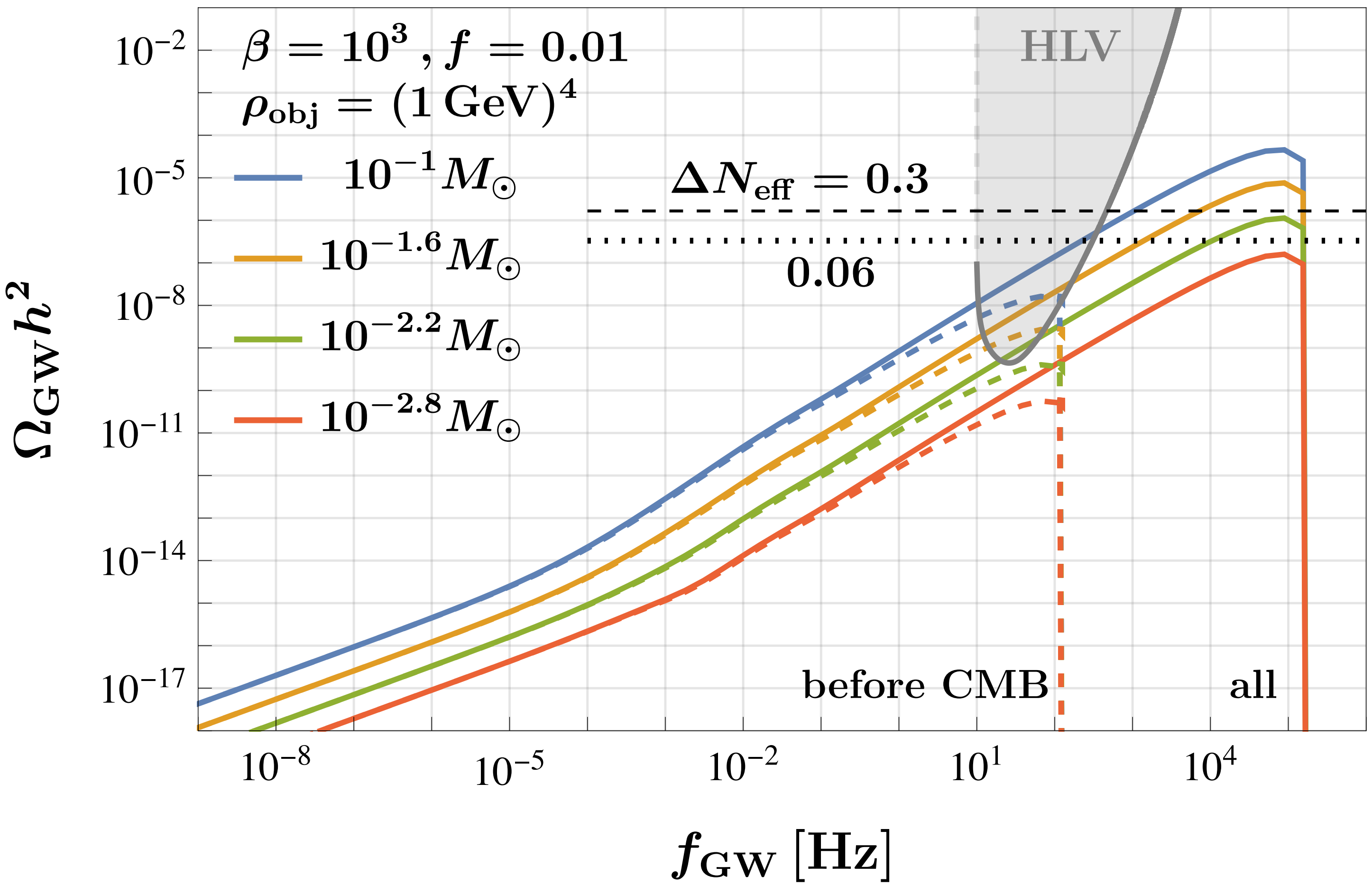
  - ➡ CMB-Stage 4 aims at $\Delta N_{\rm eff} \lesssim 0.06$

- **This constraint doesn't directly apply to SGWB from dark binaries**
  - ➡ There are contributions from after recombination

$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{f_{\rm GW}}{\rho_c}\int_{\sqrt{1-c_2^2}}^{e_{0,\max}} de_0 \int d\tau \frac{n_{\rm obj}}{4}\mathcal{P}(\tau, e_0)\frac{dE_{\rm GW}}{df_{{\rm GW},s}}\left[(1+z(t))f_{\rm GW}\right]$$

Constraints from Neff

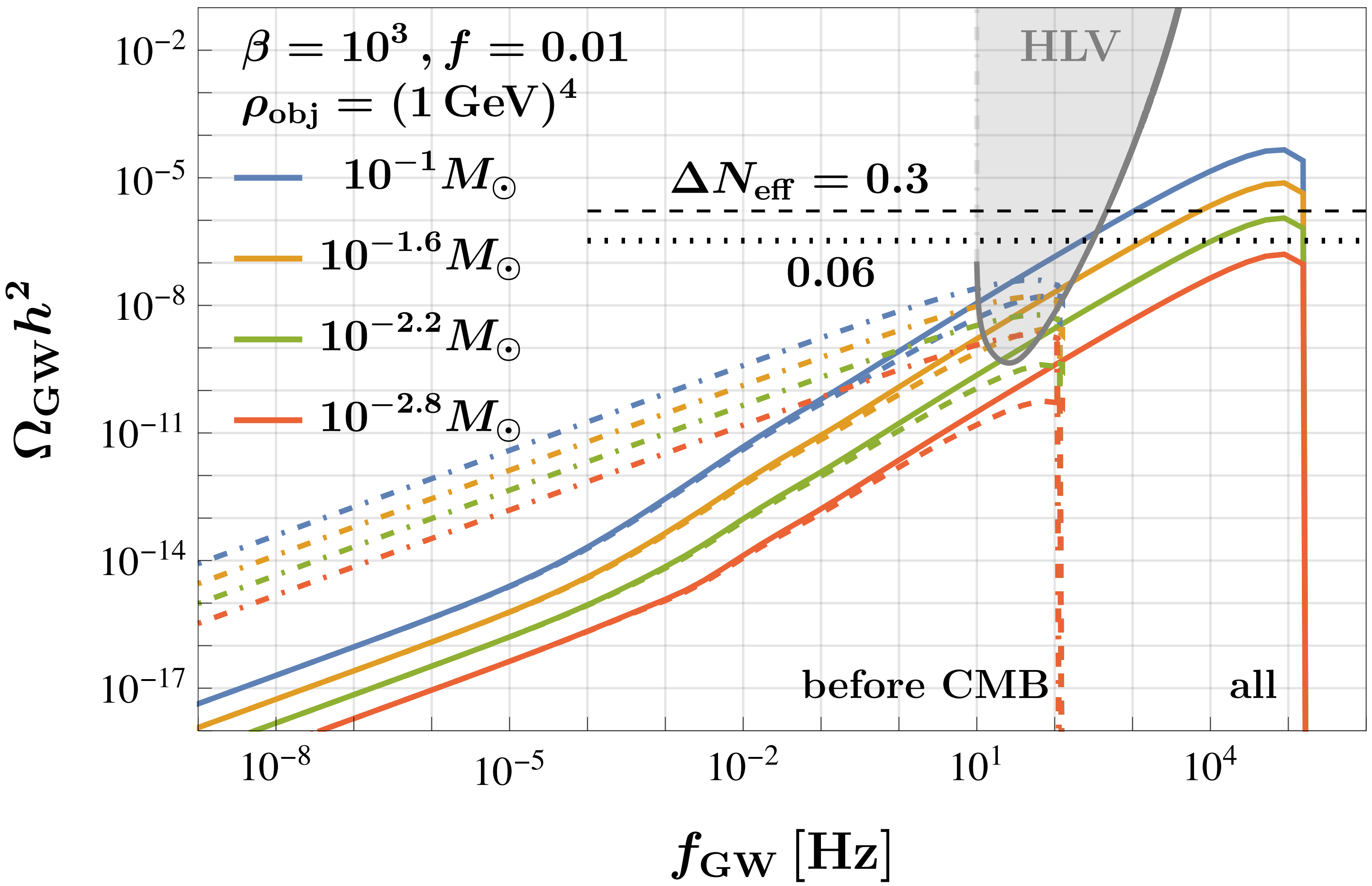
- A fair comparison should be against the radiation d.o.f. produced before the CMB time



- ➡ Amplitude of that part is ~10^3.5 smaller compared with the full spectrum for the GW

Constraints from Neff

- A fair comparison should be against the radiation d.o.f. produced before the CMB time



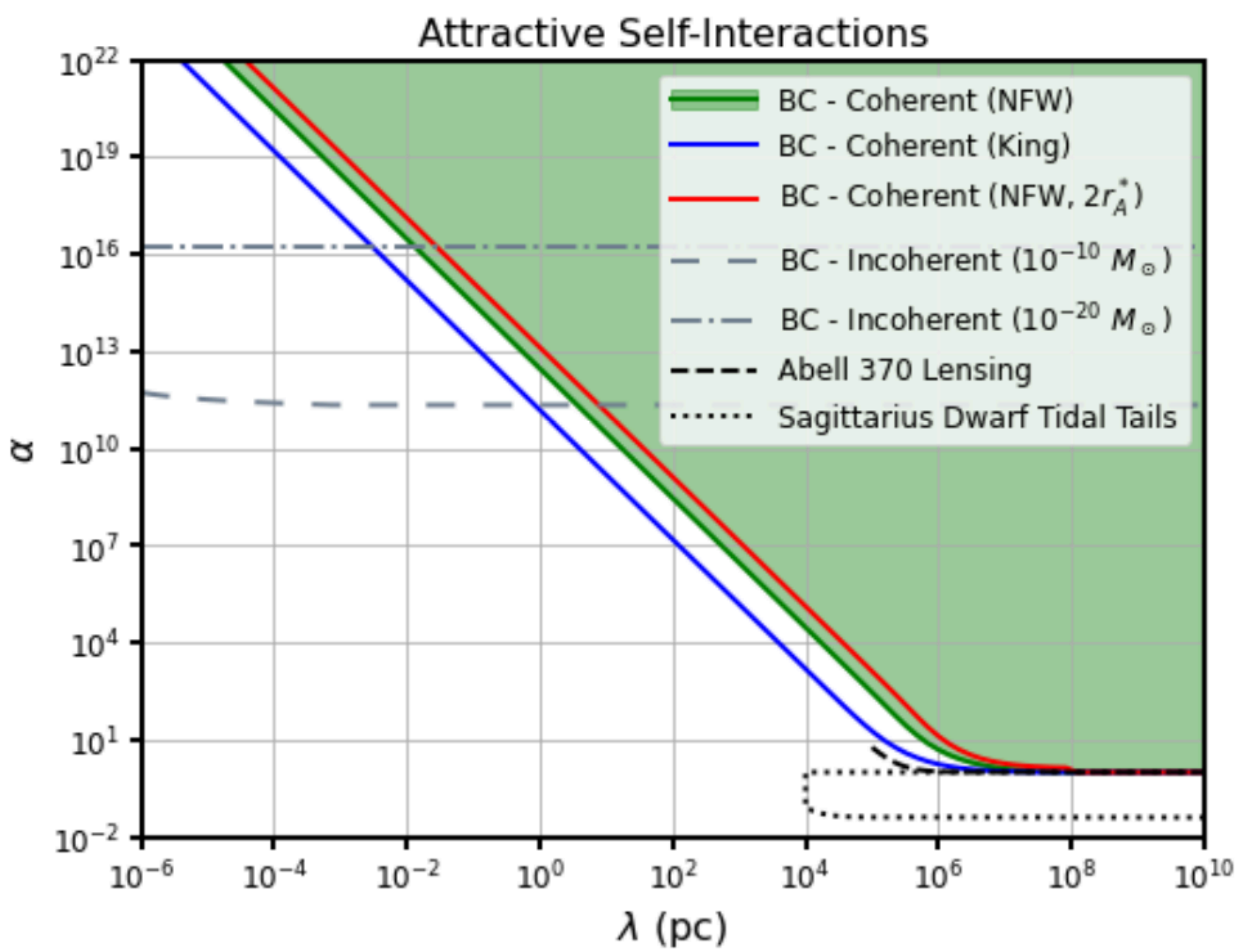
➡ DF emission is not a huge issue either

DF Mediator Mass

- **Typical distance**

$$\bar{x} = \frac{1}{1+z_{\rm eq}}\left(\frac{8\pi G M_{\rm obj}}{3H_0^2\, f\,\Omega_{\rm DM}}\right)^{1/3} \approx 0.1\,{\rm pc}\left(\frac{M_{\rm obj}}{M_\odot}\right)^{1/3}\left(\frac{1}{f}\right)^{1/3} \sim (6\times 10^{-23}\,{\rm eV})^{-1}\left(\frac{M_{\rm obj}}{M_\odot}\right)^{1/3}\left(\frac{1}{f}\right)^{1/3}$$

- **Cosmological constraints from bullet clusters**



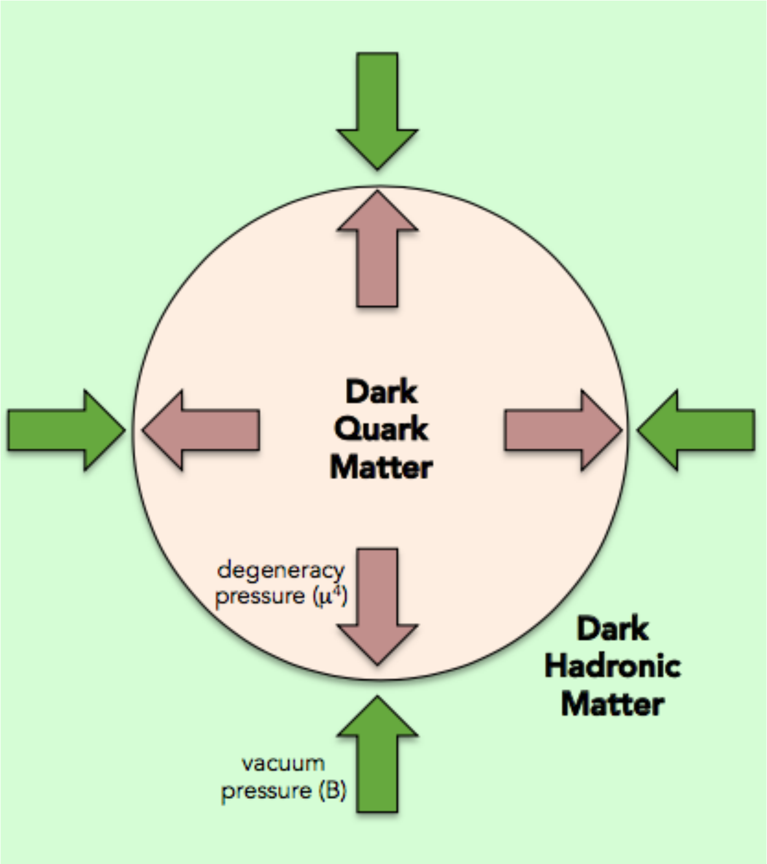
[Bogorad, Graham, Ramani, 2311.07648]

Model Building

- **We focus on the "dark quark nugget" model, with an additional scalar interaction**

[Bai, Long, *SL*, 1810.04360]

$$\mathcal{L}_{\rm dQCD} = \sum_{i=1}^{N_f} \left[\bar{\psi}_i i\gamma^\mu D_\mu \psi_i - m_{\psi_i} \bar{\psi}_i \psi_i\right] - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu\, a}$$



➡ From thermal dynamics:

$$n = g\frac{\mu^3}{6\pi^2} \quad \rho = g\frac{\mu^4}{8\pi^2} + B \quad P = g\frac{\mu^4}{24\pi^2} - B$$

$$g = 2N_d N_f \qquad n_{\rm B_d, nug} = \frac{1}{N_d} n = N_f \frac{\mu^3}{3\pi^2}$$

➡ The bag parameter: $B \sim \Lambda_d^4$

Model Building

- **The Yukawa sector**

$$\mathcal{L}_{\text{Yukawa}} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \sum_i (m_{\psi_i} + y_i\,\phi)\,\bar{\psi}_i\,\psi_i - V_0(\phi)\,, \quad V_0(\phi) = \frac{1}{2}m_{\text{med}}^2\phi^2$$

- **Finite density effect**
  - ➡ Effective potential for the mediator field

$$V_1 = g\,\frac{1}{2\pi^2}\int_0^{k_F} dk\,k^2\sqrt{k^2 + (m_\psi + y\,\phi)^2} \approx \frac{g}{8\pi^2}\,\mu^2\,\left[\mu^2 - (m_\psi + y\,\phi)^2\right]$$

$$m_{\text{in}}^2 = g\,y^2/(4\pi^2)\,\mu_{\text{eq}}^2$$

  - ➡ A screening effect if the mediator becomes too heavy inside the nugget: it doesn't "see" all the fermions inside

Model Building

- **Requiring the penetration depth to be large enough**

$$|m_{\rm in}|R < 1$$

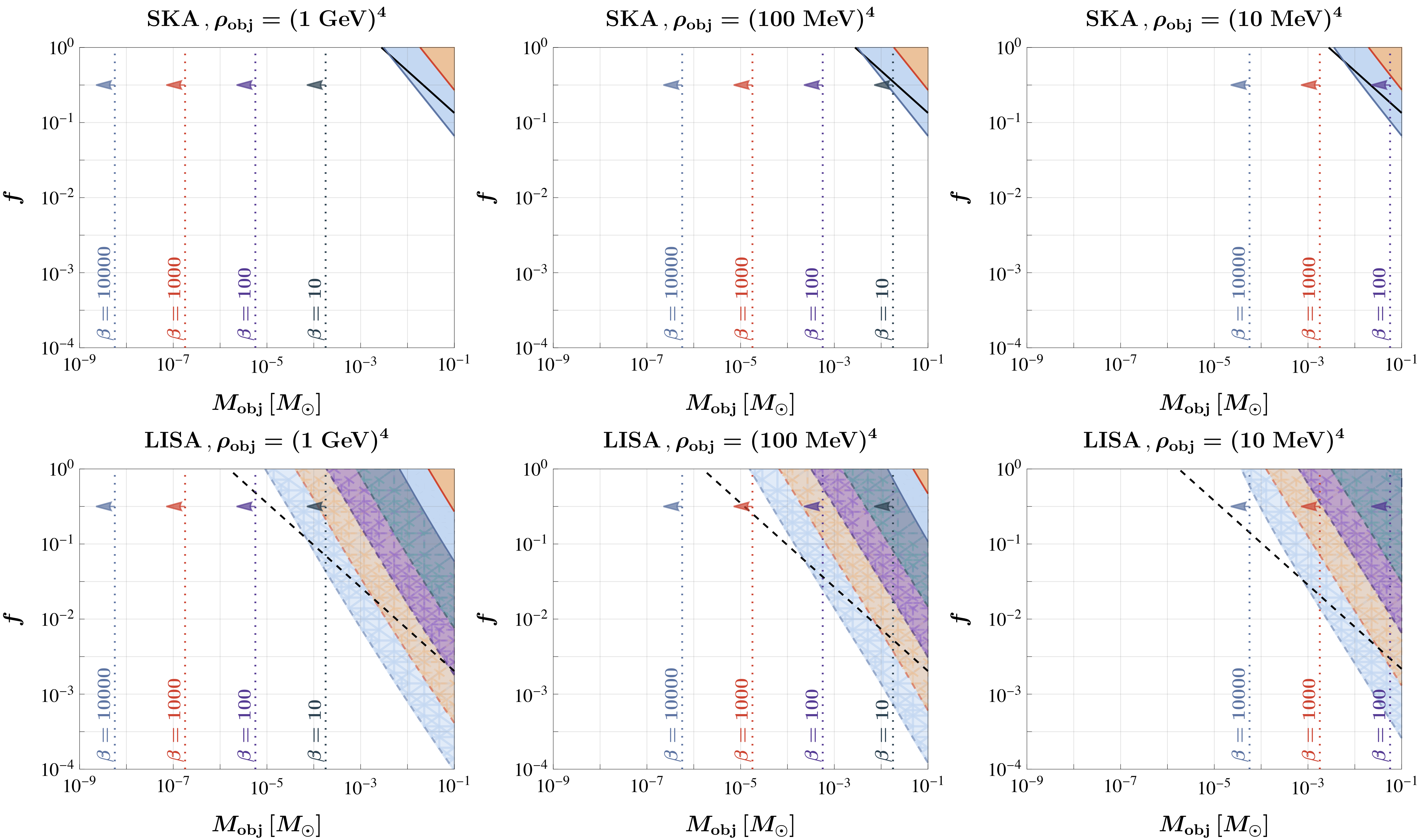
$$\Rightarrow y < 2^{19/12} 3^{-7/12} \pi^{5/6} g^{-1/4} \Lambda_d^{1/3} M^{-1/3} = (2\times 10^{-19}) \left(\frac{\Lambda_d}{1\,{\rm GeV}}\right)^{1/3} \left(\frac{M_\odot}{M}\right)^{1/3}$$

- **In terms of the strength against gravity *α***
  - The effective charge the mediator field couples to is

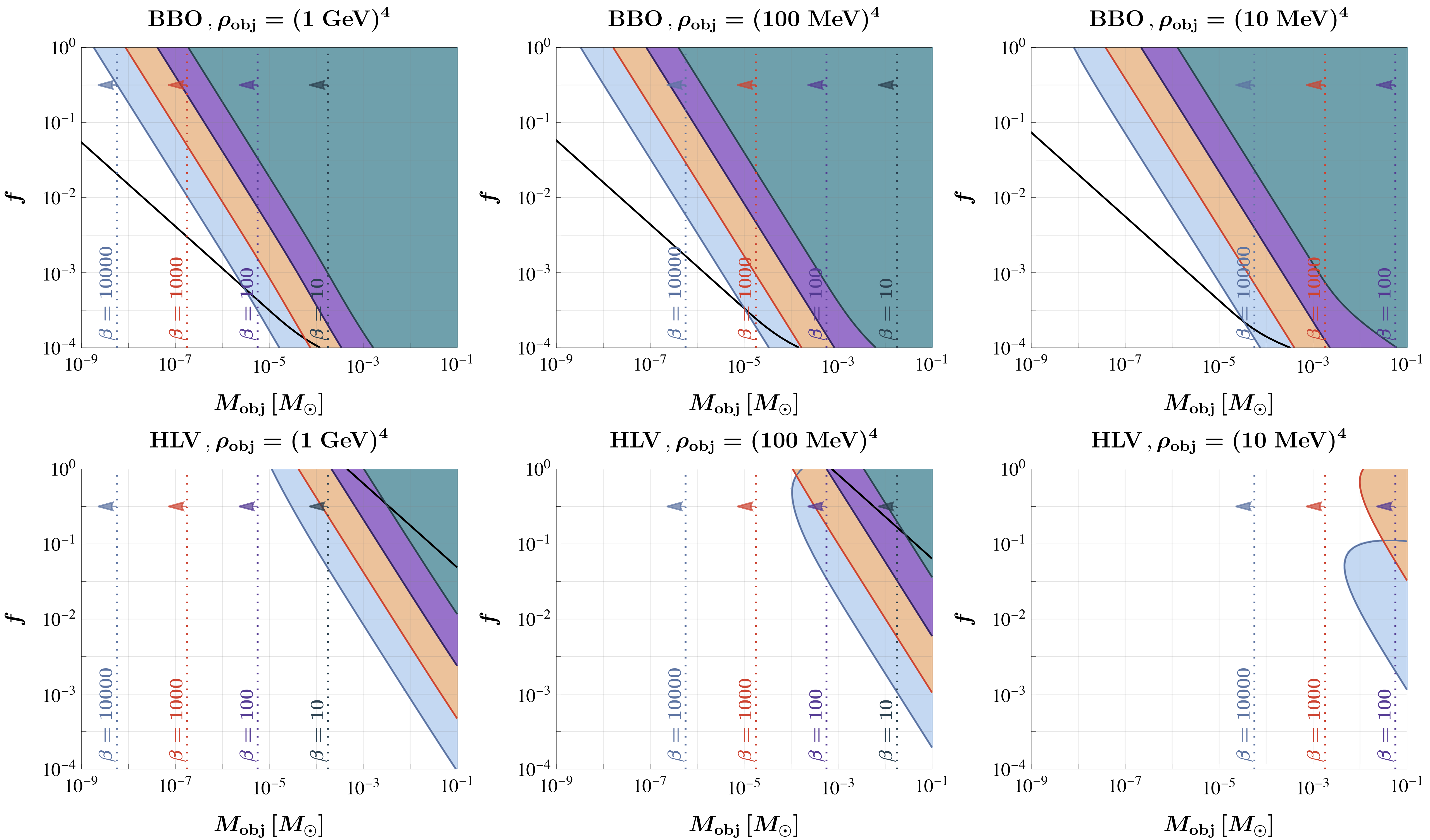
$$q_{\rm eff}\, y = \frac{g}{4\pi^2}\, m_\psi\, \mu_{\rm eq}^2\, y \left(\frac{4\pi}{3} R^3\right)$$

$$\alpha = \frac{3\, g\, y^2\, m_\psi^2}{128\pi^3\, G\, \Lambda_d^4} \lesssim (0.02) \left(\frac{m_\psi/\Lambda_d}{0.5}\right)^2 \left(\frac{1\,{\rm GeV}}{\Lambda_d}\right)^{4/3} \left(\frac{M_\odot}{M}\right)^{2/3}$$

Possible Model Constraints



Possible Model Constraints



Conclusion

- We calculated the SGWB from macroscopic DM binaries with an additional attractive force
- Visibility on ground- / satellite-based interferometers and PTAs is checked
- Solid constraints come from lensing and CMB. The macroscopic DM model itself can also provide (possibly strong) constants, though could be model dependent

Thank you!

Backup

Sensitivity at Experiments

- **A larger interaction doesn't always come with a larger signal**
  - Technically this is related to the exponential function in the merger rate

$$\mathcal{P} \propto \exp\left[-\left(\frac{3c_2^4 f^4 \beta^4 (\beta-1) H_0^2 G\, M_{\rm obj} (1+z_{\rm eq})^3 \Omega_{\rm DM} \tau}{2\pi c_1^3 (1-e_0^2)^2 h(e_0)}\right)^{1/4}\right]$$

  - Requiring the argument to be not too negative

$$\beta \left(\frac{f}{0.01}\right)^{4/5} \left(\frac{M_{\rm obj}}{10^{-10} M_\odot}\right)^{1/5} \left(\frac{\tau_{\rm min}}{t_{\rm now}}\right)^{1/5} \lesssim 8 \times 10^6$$

More on the Screening

- **The scalar field with the effective potential**

$$\partial_r^2 \phi + \frac{2}{r}\partial_r \phi = \frac{\partial V_{\rm eff}(\phi)}{\partial \phi}$$

$$V_{\rm eff}(\phi) = -\, a\, \phi\, \Theta(R-r) + \frac{1}{2}\left[m_{\rm in}^2\, \Theta(R-r) + m_{\rm med}^2\, \Theta(r-R)\right]\, \phi^2$$

- **The solution**

$$\phi_{\rm out} = c_1 e^{-m_{\rm med}\, r}\,, \qquad \phi_{\rm in} = a/m_{\rm in}^2 + c_2 (e^{-m_{\rm in}\, r} - e^{m_{\rm in}\, r})/r$$

$$\frac{q_{\rm eff}\, y}{4\pi} = c_1 = a\, \frac{e^{m_{\rm med}\, R}[m_{\rm in}\, R\, \cosh(m_{\rm in}\, R) - \sinh(m_{\rm in}\, R)]}{m_{\rm in}^2 [m_{\rm med}\, \sinh(m_{\rm in}\, R) + m_{\rm in}\, \cosh(m_{\rm in}\, R)]}$$

$$m_{\rm med} R \ll 1\,,\; m_{\rm in} R \ll 1 \Longrightarrow c_1 = a\, R^3/3$$

$$m_{\rm med} R \ll 1\,,\; m_{\rm in} R \gg 1 \Longrightarrow c_1 = a\, \frac{R^3}{(m_{\rm in}\, R)^2}$$

Decoupling of the binary

[Ioka, Chiba, Tanaka, Nakamura, astro-ph/9807018]

Consider a pair of black holes with the same mass M_{BH} and a comoving separation $x < \bar{x}$. These holes' masses produce a mean energy density over a sphere with the radius of the size of their separation as $\bar{\rho}_{BH} \equiv \rho_{eq}\bar{x}^3/(x^3R^3)$. $\bar{\rho}_{BH}$ becomes larger than the radiation energy density $\rho_r = \rho_{eq}/R^4$ if

$$R > R_m \equiv \left(\frac{x}{\bar{x}}\right)^3 . \tag{2.3}$$

After $R = R_m$ the binary decouples from the cosmic expansion and becomes a bound system. The tidal force from neighboring black holes gives the binary sufficiently large angular momentum to keep the holes from colliding with each other unless x is exceptionally small.

➡ Additional G'/G to account for the strength of the dark force

$$G'\bar{\rho}_{\rm obj} \equiv G' \cdot f\,\frac{\rho_{\rm eq}}{2}\,\frac{\bar{x}^3 R_{\rm eq}^3}{x^3 R^3} = G\rho_r$$

SGWB from Dark Binaries

- **With these discussed**

$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{f_{\rm GW}}{\rho_c} \int_{\sqrt{1-c_2^2}}^{e_{0,\rm max}} de_0 \int d\tau \frac{n_{\rm obj}}{4} \mathcal{P}(\tau, e_0) \frac{dE_{\rm GW}}{df_{{\rm GW},s}} \left[(1+z(t)) f_{\rm GW}\right]$$

$$t = t_{\rm dec} + \tau$$

- **Boundaries of the τ-integrations**
  - Formation of the macroscopic DM

$$t_{\rm dec} > t_{\rm form}\ ,\ t < t_0$$

we take $t_{\rm form} \sim \rho_{\rm obj}^{1/4}$

  - Decouple before matter-radiation equality

$$\tau < c_1^3 f\, \beta h(e_0) \bar{\tau}$$

  - For a given GW frequency to be produced at the source

$$\frac{G'm}{a_0^3} < \pi^2 (1+z(t))^2 f_{\rm GW}^2 < \frac{G'm}{(2R_{\rm obj})^3}\ .$$

$$f_{\rm GW,max} = \sqrt{\frac{G'\rho_{\rm obj}}{3\pi}}$$

SGWB from Dark Binaries

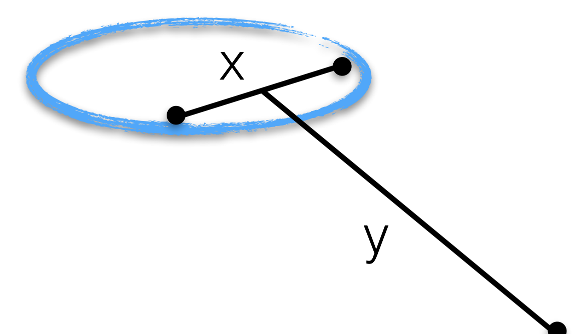
- **With these discussed**

$$\Omega_{\rm GW}(f_{\rm GW}) = \frac{f_{\rm GW}}{\rho_c} \int_{\sqrt{1-c_2^2}}^{e_{0,\rm max}} de_0 \int d\tau \frac{n_{\rm obj}}{4} \mathcal{P}(\tau, e_0) \frac{dE_{\rm GW}}{df_{{\rm GW},s}} \left[(1+z(t)) f_{\rm GW}\right]$$

$$t = t_{\rm dec} + \tau$$

- **Boundaries of the e0-integrations**
  - Lower boundary comes from the requirement of y>x

$$y > x \Rightarrow e_0^2 > 1 - c_2^2$$



  - Upper boundary comes indirectly from the phase space of the τ-integration

The Dipole Moment

- **The mediator couples differently to different quarks**

For the simplified case with the same dark quark masses and Yukawa couplings $m_{\psi_i} = m_\psi$ and $y_i = y$ for all i, the ratio of the effective charge $q_{\rm eff}$ over the object mass M is fixed and independent of M in the limit of $|m_{\rm in}|R < 1$. For a more general case with different values of m_{ψ_i} and y_i, there is a global $U(1)^{N_f}$ flavor symmetry. For a dark quark nugget with total N_ψ dark quark number, the number of dark quarks of each flavor i can be labelled by $(N_{\psi_1}, N_{\psi_2}, \cdots, N_{\psi_{N_f}})$ with $\sum_{i=1}^{N_f} N_{\psi_i} = N_\psi$. The effective charge to emit a ϕ particle is $q_{\rm eff}\, y_{(N_{\psi_1}, N_{\psi_2}, \cdots, N_{\psi_{N_f}})} = \frac{3}{2}\sum_{i=1}^{N_f} y_i\, m_{\psi_i}\, N_{\psi_i}/\mu_{\rm eq}$. The dark object mass is $M \approx 2^{3/4} 3^{1/4} \pi^{1/2} \mathfrak{g}^{-1/4} \Lambda_d\, N_\psi$ by ignoring the bare dark quark mass contributions. The ratio of the effective charge over mass is

$$\frac{q_{\rm eff}\, y}{M}(N_{\psi_1}, N_{\psi_2}, \cdots, N_{\psi_{N_f}}) = \frac{\sqrt{\frac{3}{2}\sqrt{\mathfrak{g}}}}{4\pi\, \Lambda_d^2}\, \frac{\sum_{i=1}^{N_f} y_i\, m_{\psi_i}\, N_{\psi_i}}{\sum_{i=1}^{N_f} N_{\psi_i}} \,. \tag{9}$$

Higher Harmonics

- **With an eccentric orbit, the binary should emit GW at all harmonics of the orbital frequency**
  - ➡ We are effectively assuming all energy are emitted through the n=2 channel
  - ➡ To account for the other modes

$$\frac{dE_{\rm GW}}{dt} = \frac{32GG'^3\eta^2 m^5}{5a^5}\frac{1+\frac{73}{24}e^2+\frac{37}{96}e^4}{(1-e^2)^{7/2}} = \frac{32GG'^3\eta^2 m^5}{5a^5}\sum_n g(n,e)\,,$$

$$g(n,e) = \frac{n^4}{32}\Bigg\{\left[J_{n-2}(ne) - 2eJ_{n-1}(ne) + \frac{2}{n}J_n(ne) + 2eJ_{n+1}(ne) - J_{n+2}(ne)\right]^2 + (1-e^2)\left[J_{n-2}(ne) - 2eJ_n(ne) + J_{n+2}(ne)\right]^2 + \frac{4}{3n^2}J_n^2(ne)\Bigg\}\,,$$

[Peters and Mathews, Phys. Rev. 131 (1963) 435-439]

[Enoki, Nagashima, astro-ph/0609377]

Higher Harmonics

- The following calculation is straight-forward

$$
\begin{aligned}
\frac{d^2 E_{\rm GW}}{dt\, df_{{\rm GW},s}} &= \frac{32 G G'^3 \eta^2 m^5}{5a^5} \sum_n g(n,e)\delta(f_{{\rm GW},s} - n f_{\rm orb})\,, \\
\frac{dE_{\rm GW}}{df_{{\rm GW},s}} &= \sum_n \int dt \frac{32 G G'^3 \eta^2 m^5}{5a^5} g(n,e)\delta(f_{{\rm GW},s} - n f_{\rm orb}) \\
&= \sum_n \int \frac{de}{de/dt} \frac{32 G G'^3 \eta^2 m^5}{5a^5} g(n,e)\delta(f_{{\rm GW},s} - n f_{\rm orb}) \\
&= \sum_n \int de \frac{32 G G'^3 \eta^2 m^5}{5a^5} \frac{a^3 (1-e^2)^{3/2}}{4eG^2(\mathcal{G}-1)\mathcal{G} M_{\rm obj}^2} \frac{g(n,e)}{n} \delta(f_{\rm orb} - f_{{\rm GW},s}/n) \\
&= \sum_n \frac{32 G G'^3 \eta^2 m^5}{5a^5} \frac{a^3 (1-e^2)^{3/2}}{4eG^2(\mathcal{G}-1)\mathcal{G} M_{\rm obj}^2} \frac{g(n,e)}{n} \frac{1}{\left|\frac{df_{\rm orb}}{de}\right|_{e=e_n}} \\
&\equiv \sum_n \left(\frac{E_{\rm GW}}{df_{{\rm GW},s}}\right)_n \,,
\end{aligned}
$$

Higher Harmonics