中国科学院大学
University of Chinese Academy of Sciences

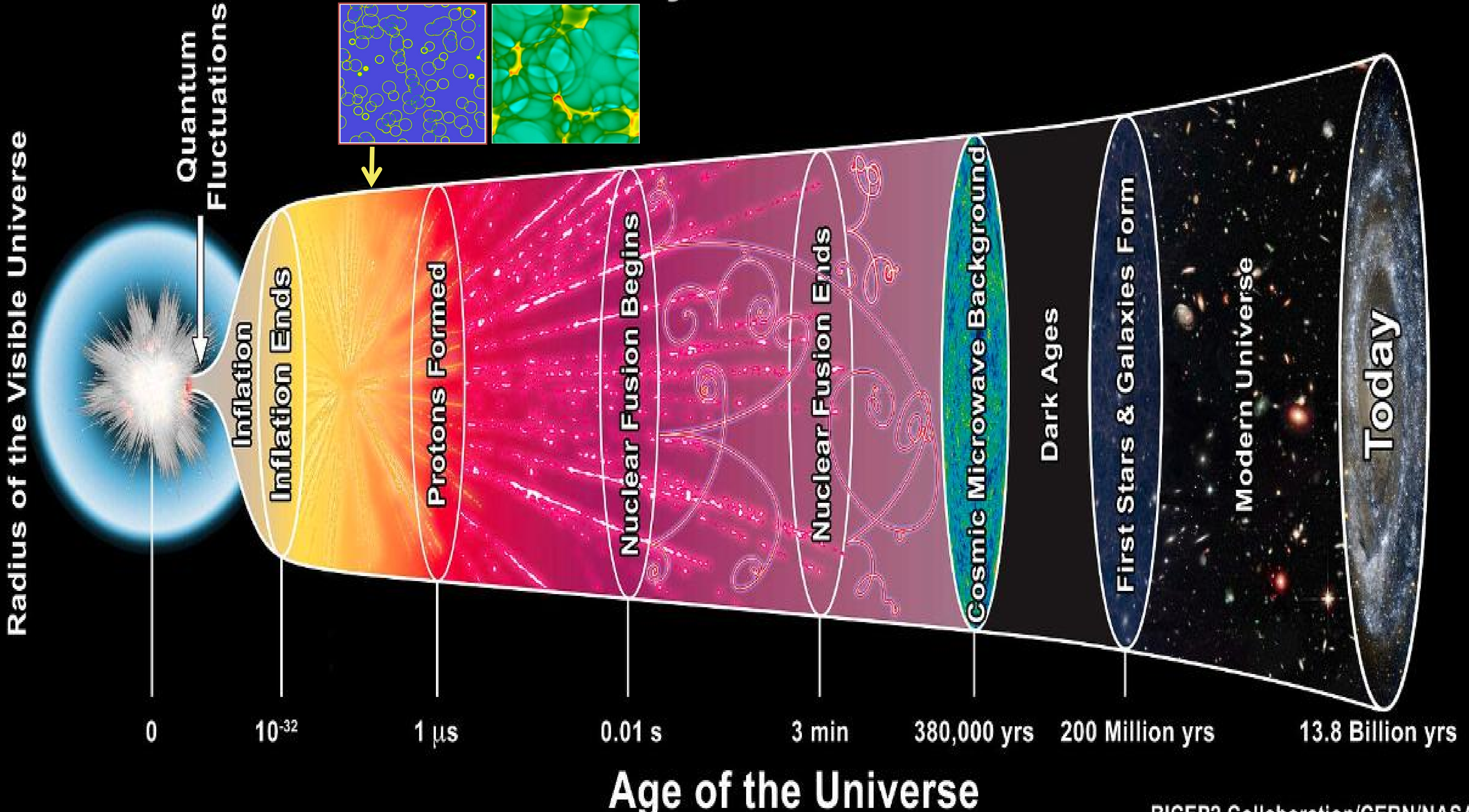
ICTP-AP
International Centre
for Theoretical Physics Asia-Pacific
国际理论物理中心-亚太地区

Dissipative Effects as New Observables for Cosmological Phase Transitions

Talk based on HG [2310.10927]

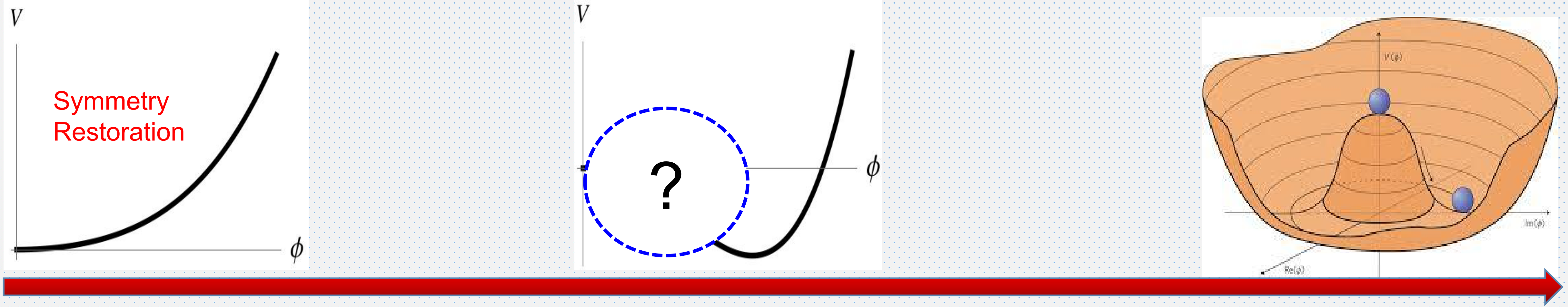
Huaike Guo

Jan 15, 2024

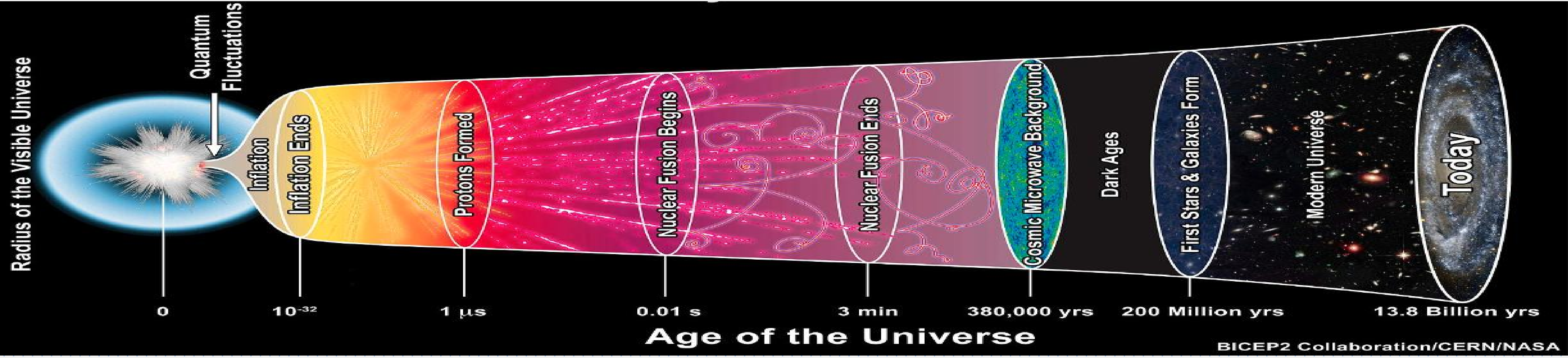


BICEP2 Collaboration/CERN/NASA

Electroweak Phase Transition



Temperature drops



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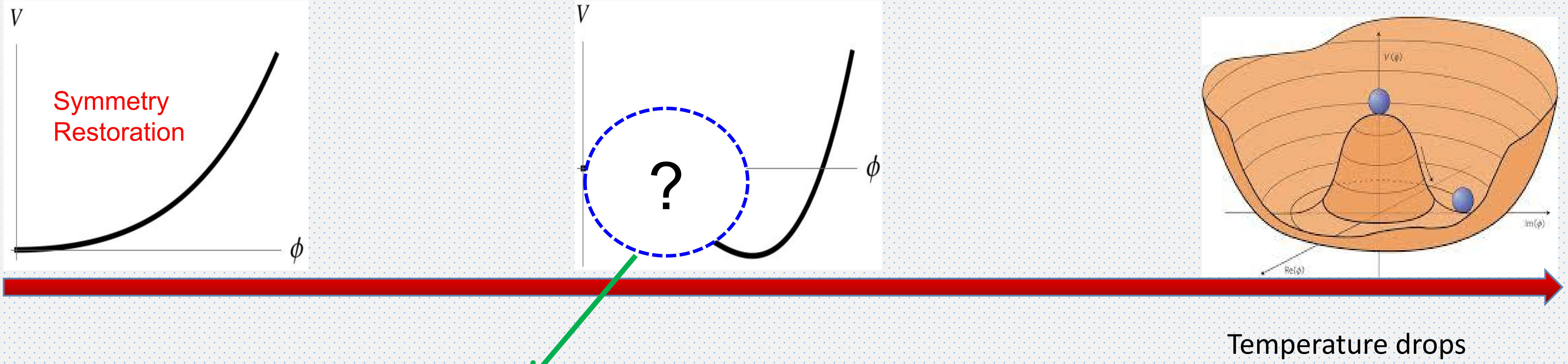
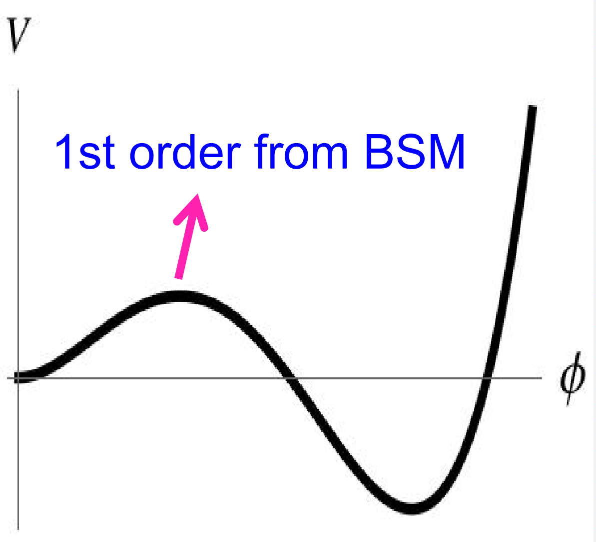
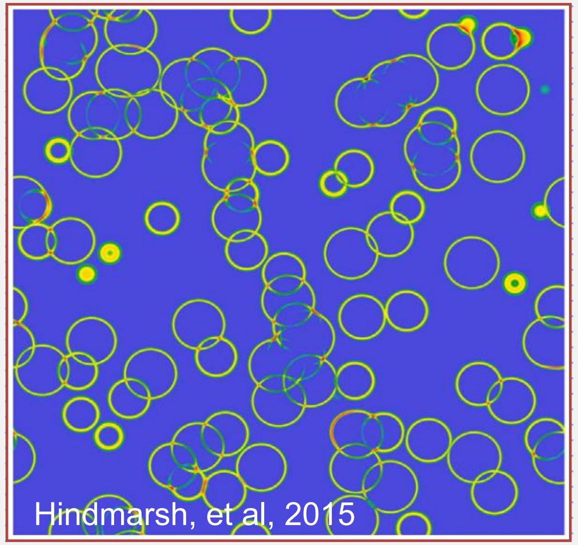
Electroweak Phase Transition






Hindmarsh, et al, 2015

→ Electroweak Baryogenesis

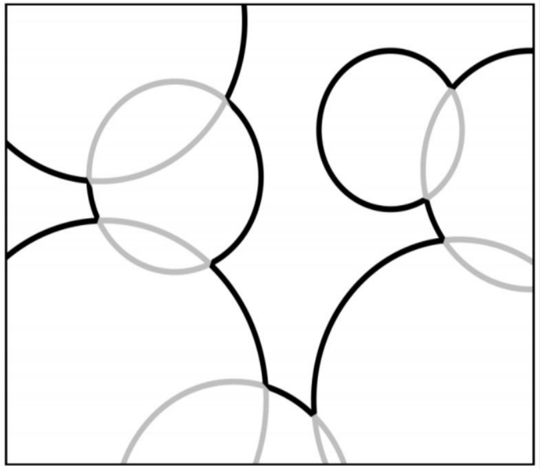
- Modified Higgs potential (Higgs physics, GW)
- Extra CP-violation (EDM, LHC)
- New particles, symmetries (LHC, GW)

Gravitational Wave Sources

The current understanding:

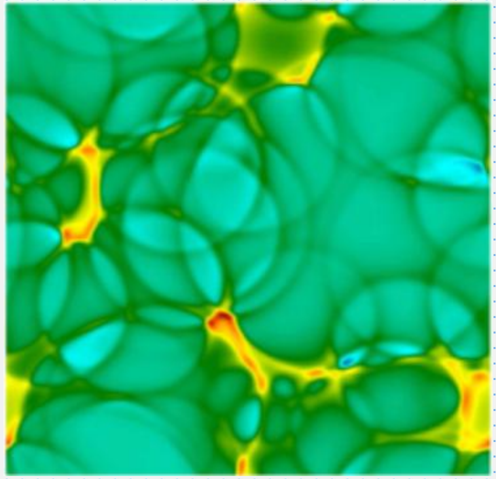
$$\Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}$$

energy near the wall



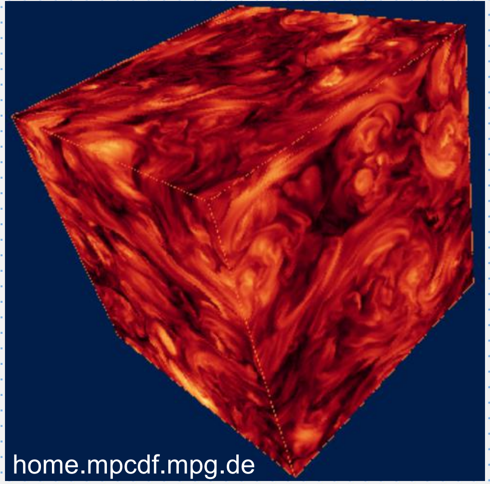
Bubble Collisions

fluid kinetic energy



Sound Waves

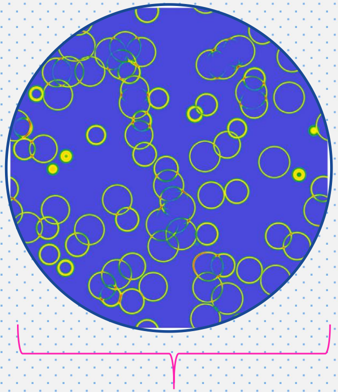
turbulent fluid + magnetic field



home.mpcdf.mpg.de

Magnetohydrodynamic Turbulence

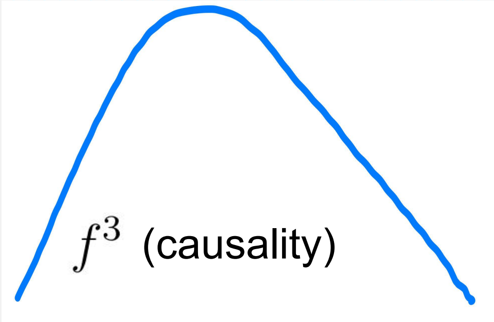
Properties



Horizon size: 1/H*

$$f_{\rm now} = 1.65 \times 10^{-5} \boxed{\left(\frac{f_{\rm PT}}{\beta}\right)\left(\frac{\beta}{H_*}\right)} \left(\frac{T_*}{100{\rm GeV}}\right)\left(\frac{g_*}{100}\right)^{1/6} {\rm Hz}$$

~100-1000



Cai, Pi, Sasak, PRD [1909.13728]

nHz (~100MeV) QCD scale

~mHz : (~100GeV) weak scale

~100Hz (~PeV - EeV) high scale

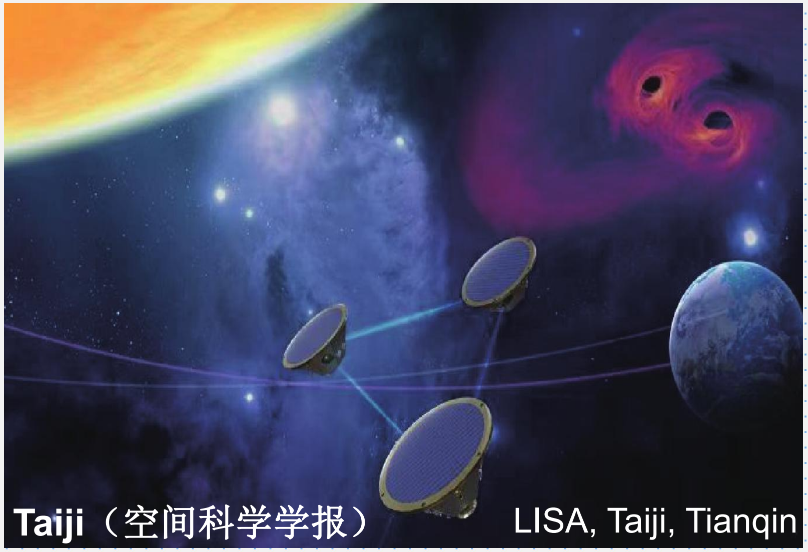
EPTA

NANOGrav Physics Frontiers Center

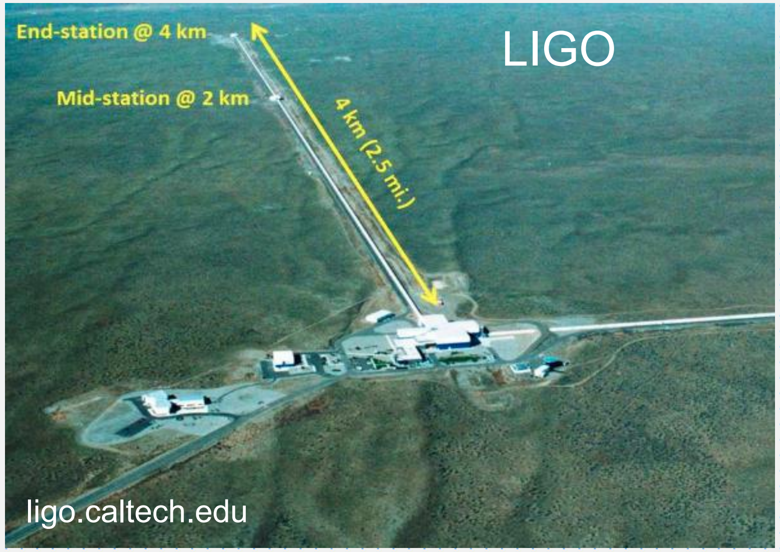
PPTA

InPTA Indian Pulsar Timing Array

中国脉冲星测时阵列（CPTA）



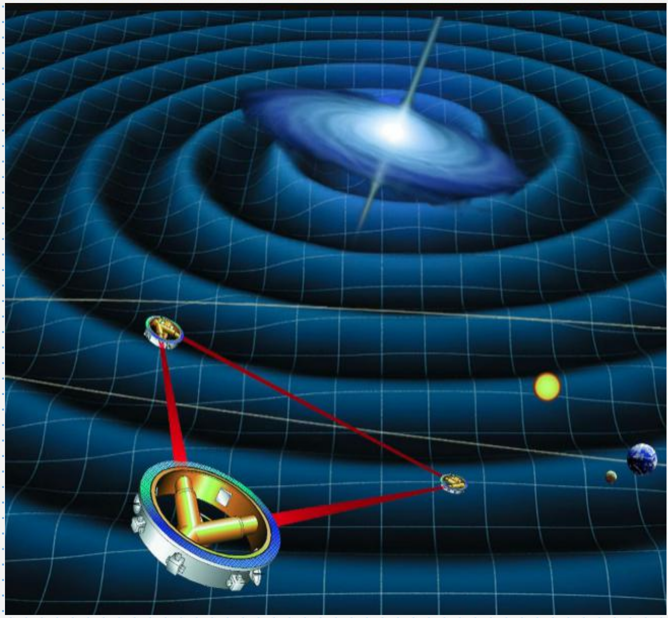
Taiji（空间科学学报） LISA, Taiji, Tianqin



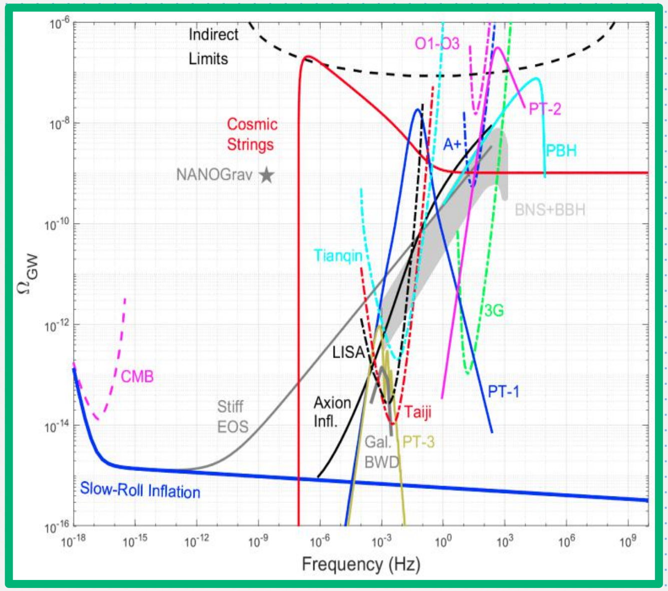
ligo.caltech.edu

From Theory to Experiment

theorist



LIGO, LISA/Taiji/Tianqin, PTA, ...



Gravitational Wave Spectrum

α

β

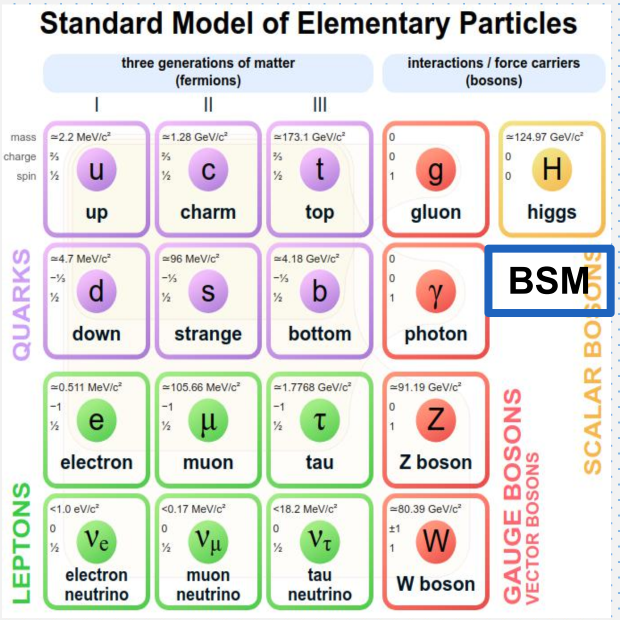
v_w

T_*

g_s

...

Phase Transition Parameters



Particle Physics Model

experimentalist

Problem: parameter degeneracy

Many models can lead to the same PT parameter values

Models	Strong 1st order phase transition	GW signal	Cold DM	Dark Radiation and small scale structure
SM charged				
Triplet [20–22]	✓	✓	✓	✗
complex and real Triplet [23] (Georgi-Machacek model)	✓	✓	✓	✗
Multiplet [24]	✓	✓	✓	
2HDM [25–30]	✓	✓		✗
MLRSM [31]	✓	✓	✗	✗
NMSSM [32–36]	✓	✓	✓	✗
SM uncharged				
S_r (xSM) [37–49]	✓	✓	✗	✗
2 S_r's [50]	✓	✓	✓	✗
S_c (cxSM) [49, 51–54]	✓	✓	✓	✗
$\mathrm{U(1)_D}$ (no interaction with SM) [55]	✓	✓	✓	✗
$\mathrm{U(1)_D}$ (Higgs Portal) [56]	✓	✓	✓	
$\mathrm{U(1)_D}$ (Kinetic Mixing) [57]	✓	✓	✓	
Composite SU(7)/SU(6) [58]	✓	✓	✓	
$\mathrm{U(1)_L}$ [59]	✓	✓	✓	✗
$\mathrm{SU(2)_D} \to$ global $SO(3)$ by a doublet [60–62]			✓	✗
$\mathrm{SU(2)_D} \to \mathrm{U(1)_D}$ by a triplet [63–65]			✓	✓
$\mathrm{SU(2)_D} \to Z_2$ by two triplets [66]			✓	✗
$\mathrm{SU(2)_D} \to Z_3$ by a quadruplet [67, 68]			✓	✗
$\mathrm{SU(2)_D} \times \mathrm{U(1)_{B\text{-}L}} \to Z_2 \times Z_2$ by a quintuplet and a S_c [69]			✓	✗
$\mathrm{SU(2)_D}$ with two dark Higgs doublets [70]	✓	✓	✗	✗
$\mathrm{SU(3)_D} \to Z_2 \times Z_2$ by two triplets [62, 71]			✓	✗
$\mathrm{SU(3)_D}$ (dark QCD) (Higgs Portal) [72, 73]	✓	✓	✓	
$G_{\mathrm{SM}} \times G_{\mathrm{D,SM}} \times Z_2$ [74]	✓	✓	✓	
$G_{\mathrm{SM}} \times G_{\mathrm{D,SM}} \times G_{\mathrm{D,SM}} \cdots$ [75]	✓	✓	✓	
Current work				
$\mathrm{SU(2)_D} \to \mathrm{U(1)_D}$ (see the text)	✓	✓	✓	✓

Ghosh,HG,Han,Liu, JHEP [2012.09758]

Solutions: New Observables

- Anisotropy

  Geller, Hook, Sundrum, Yuhsin Tsai, PRL [1803.10780]
  Li, Huang, Wang, Zhang, PRD [2112.01409]
  Li, Yan, Huang, PRD [2211.03368]

- Primordial magnetic field

  Di,Wang,Zhou,Bian,Cai, PRL [2012.15625]
  Yang,Bian,PRD [2102.01398], ...

- Primordial black holes and solitons

  Hong, Jung, Xie, PRD [2008.04430]
  Kawana,Xie,PLB [2106.00111]
  Liu,Bian,Cai,Guo,Wang, PRD [2106.05637]
  Lu,Kawana,Xie, PRD [2202.03439]

- Curvature perturbations

  Liu,Bian,Cai,Guo,Wang,PRL[2208.14086]
  Jiang,Liu,Sun,Wang, PLB [1512.07538]

Anything directly readable from GW spectrum?

Dissipative Effects as New Observables

GW depends on (large) bulk velocity of the system

$$h \sim 10^{-22} \frac{M/M_\odot}{r/100\mathrm{Mpc}} \left(\frac{v}{c}\right)^2$$

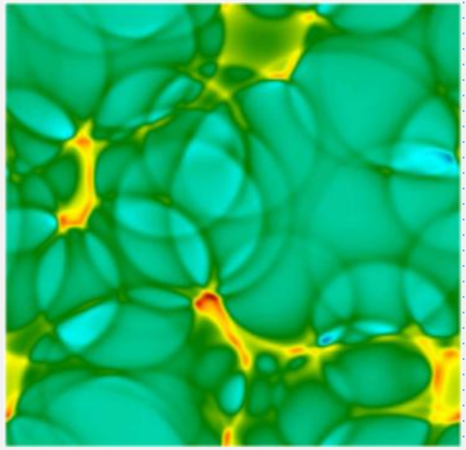
Dissipative effects dissipate away the bulk kinetic energy (leaves imprint)

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right) = -\nabla p + \boxed{\mu\nabla^2\mathbf{v} + (\zeta + \frac{1}{3}\mu)\nabla(\nabla\cdot\mathbf{v})}$$

Navier–Stokes equations

studiousguy.com

Sound Waves



Usually the dominant source (Hindmarsh,Huber,Rummukainen,Weir, PRL [1304.2433])

$$T^{ij} \propto (p+e)v^i v^j$$

$$h^2\Omega_{\rm sw}(f) = 2.65\times 10^{-6}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 v_w S_{\rm sw}(f)\Upsilon(\tau_{\rm sw})$$

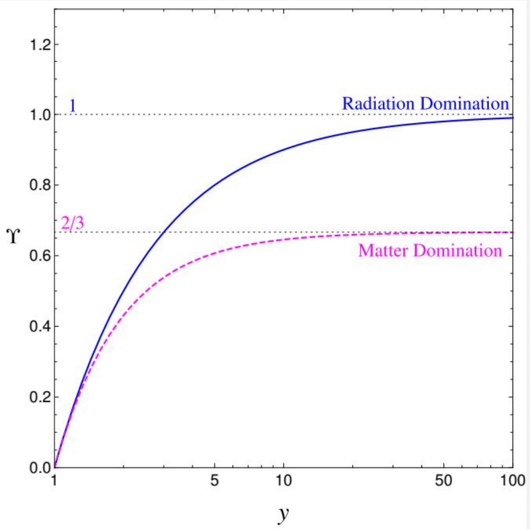
$$S_{\rm sw}(f) = \left(\frac{f}{f_{\rm sw}}\right)^3\left[\frac{7}{4+3(f/f_{\rm sw})^2}\right]^{7/2} \qquad f_* = \frac{2\beta}{\sqrt{3}v_w} \approx \frac{3.4}{R_*}$$

Hindmarsh,Huber,Rummukainen,Weir, PRD [1504.03291]

$$\Omega_{\rm SW}(f \gtrsim f_{\rm peak}) \propto f^{-4}$$

$$\Omega_{\rm SW}(f \lesssim f_{\rm peak}) \propto f^{3}$$

Slight different fit obtained by the same group, PRD [1704.05871]



$$\Upsilon = 1-(1+2\tau_{\rm sw}H_{\rm pt})^{-1/2}$$ (radiation domination)

HG, Sinha, Vagie, White, JCAP [2007.08537]

Sound Waves: Recent Development

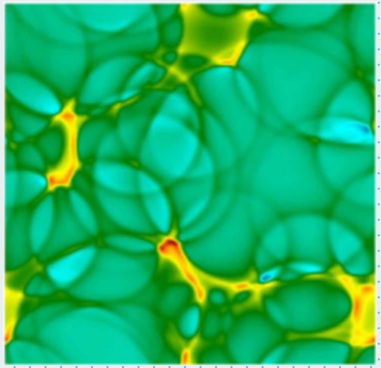
Analytical Modelling

- Refine the sound shell model
- Synergy with simulations

Sound Shell Model

Hindmarsh, PRL [1608.04735]
Hindmarsh, Hijazi, JCAP [1909.10040]
HG, Sinha, Vagie, White, JCAP [2007.08537]
Cai, Wang, Yuwen, PRD Letter [2305.00074]
Pol, Procacci, Caprini [2308.12943]

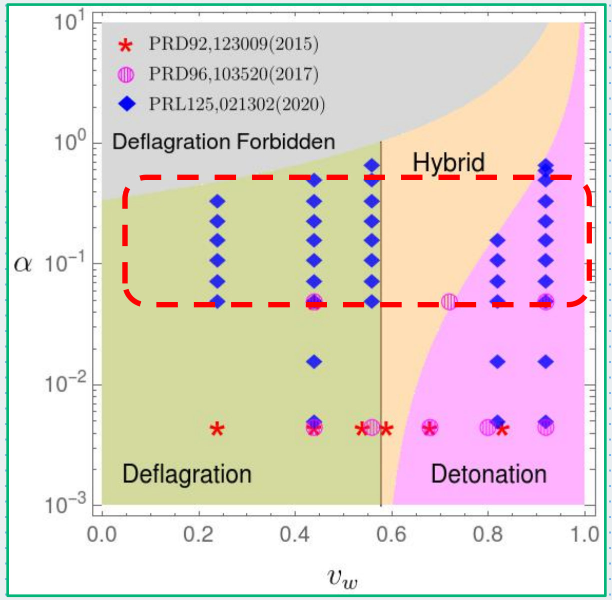
$$v^i_{\mathbf{q}} = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}}$$

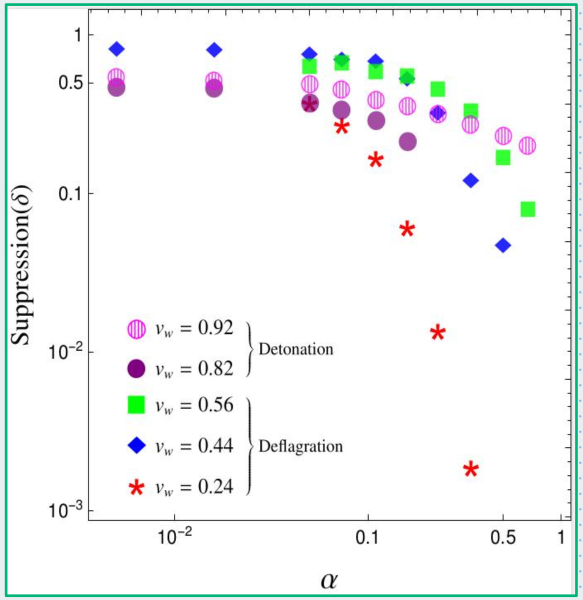


Numerical Simulation

- Suppression found for strong transitions with small vw
- Need to cover more parameter space (very strong PT)

$$h^2\Omega_{\rm sw}(f) = 2.65\times 10^{-6}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_{\rm sw}\alpha}{1+\alpha}\right)^2 v_w S_{\rm sw}(f)\Upsilon(\tau_{\rm sw})$$





Cutting, Hindmarsh, Weir, PRL [1906.00480]

All based on perfect fluid approximation

Sound Waves: Modelling

Sound Shell Model

Hindmarsh, PRL [1608.04735]
Hindmarsh, Hijazi, JCAP [1909.10040]
HG, Sinha, Vagie, White, JCAP [2007.08537]
Cai, Wang, Yuwen, PRD Letter [2305.00074]
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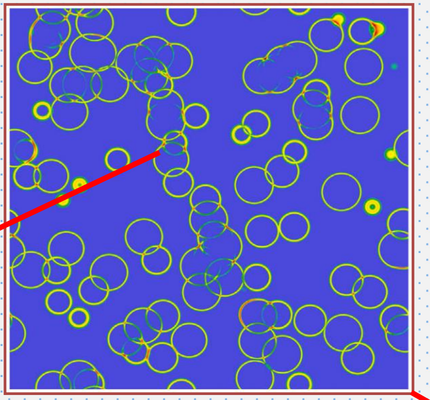
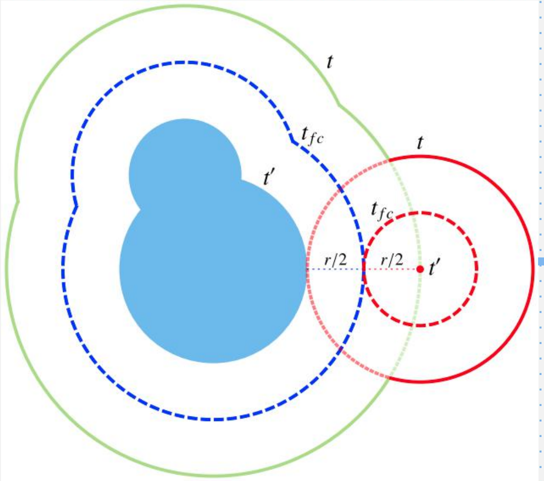




$$v^i(\eta, \mathbf{x}) = \int \frac{d^3q}{(2\pi)^3}\left[v^i_{\mathbf{q}} e^{-i\omega\eta + i\mathbf{q}\cdot\mathbf{x}} + v^{i*}_{\mathbf{q}} e^{i\omega\eta - i\mathbf{q}\cdot\mathbf{x}}\right]$$

linear superposition
(core of SSM)

$$v^i_{\mathbf{q}} = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}}$$

forced fluid motion

freely propagating sound

$$\left.\frac{\mathrm{d}\Omega_{\mathrm{GW}}}{\mathrm{d}\ln k}\right|_{\mathrm{SW}} = \left.\frac{\mathrm{d}\Omega_{\mathrm{GW}}}{\mathrm{d}\ln k}\right|^{\mathrm{forced}}_{\mathrm{SW}} + \left.\frac{\mathrm{d}\Omega_{\mathrm{GW}}}{\mathrm{d}\ln k}\right|^{\mathrm{free}}_{\mathrm{SW}}.$$

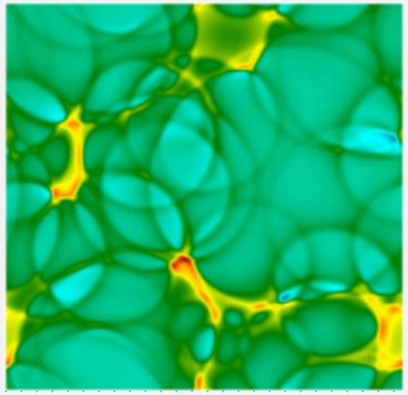
Cai, Wang, Yuwen, PRD Letter [2305.00074]

Neglect possible forced motion in the following.

Effects of Dissipation

- Disturbed fluid comes into rest eventually

$$v^i(\eta, \mathbf{x}) = \int \frac{d^3q}{(2\pi)^3} \left[v^i_{\mathbf{q}} e^{-i\omega\eta + i\mathbf{q}\cdot\mathbf{x}} + c.c.\right]$$



$$v^i_{\mathbf{q}}(\eta) \propto \exp\left[-\int \Gamma(\mu, \zeta, \xi) d\eta\right]$$

$$\boxed{\Gamma \propto q^2}$$

shear viscosity

bulk viscosity

$$\Delta T^{ij} = -\boxed{\mu}\left(\frac{\partial U_i}{\partial x^j} + \frac{\partial U_j}{\partial x^i} - \frac{2}{3}\delta_{ij}\nabla\cdot\mathbf{U}\right) - \boxed{\zeta}\,\delta_{ij}\nabla\cdot\mathbf{U},$$

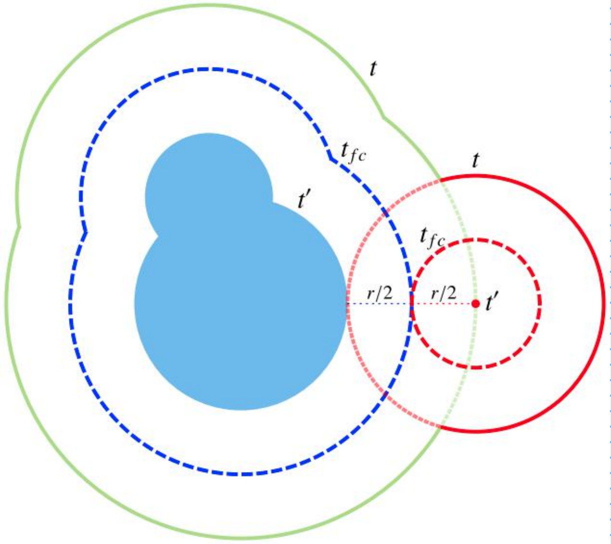
$$\Delta T^{i0} = -\boxed{\chi}\left(\frac{\partial T}{\partial x^i} + T\dot{U}_i\right). \qquad (1)$$

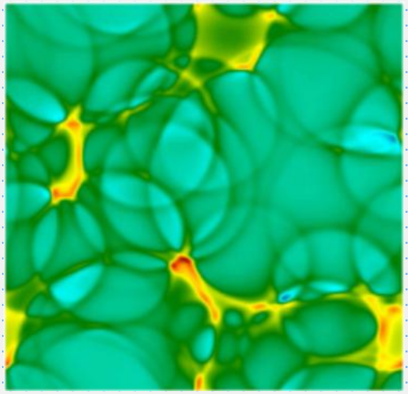
thermal conduction

Weinberg, ApJ, 1971

Euler equation -> Navier–Stokes equations

Sound Shell Model with Dissipation





$$v^i_{\mathbf{q}} = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}} \quad \Longrightarrow \quad v^i_{\mathbf{q}}(\eta) = \sum_{n=1}^{N_b} v^{i(n)}_{\mathbf{q}} \boxed{\exp\left[-\int_{\eta_d^{(n)}}^{\eta} \Gamma d\bar{\eta}\right]} \theta(\eta - \eta_d^{(n)})$$

bubble destruction time

Velocity Power Spectrum

- Velocity spectrum is generally non-stationary

$$\langle \tilde{v}^i_{\mathbf{q}}(\eta_1)\tilde{v}^{j*}_{\mathbf{k}}(\eta_2)\rangle = 2\pi^2 q^{-3}\delta^3(\mathbf{q}-\mathbf{k})\hat{q}^i\hat{k}^j \times \boxed{\mathcal{P}_v(q,\eta_1,\eta_2)}\cos[\omega(\eta_1-\eta_2)]$$

joint bubble lifetime, destruction time distribution

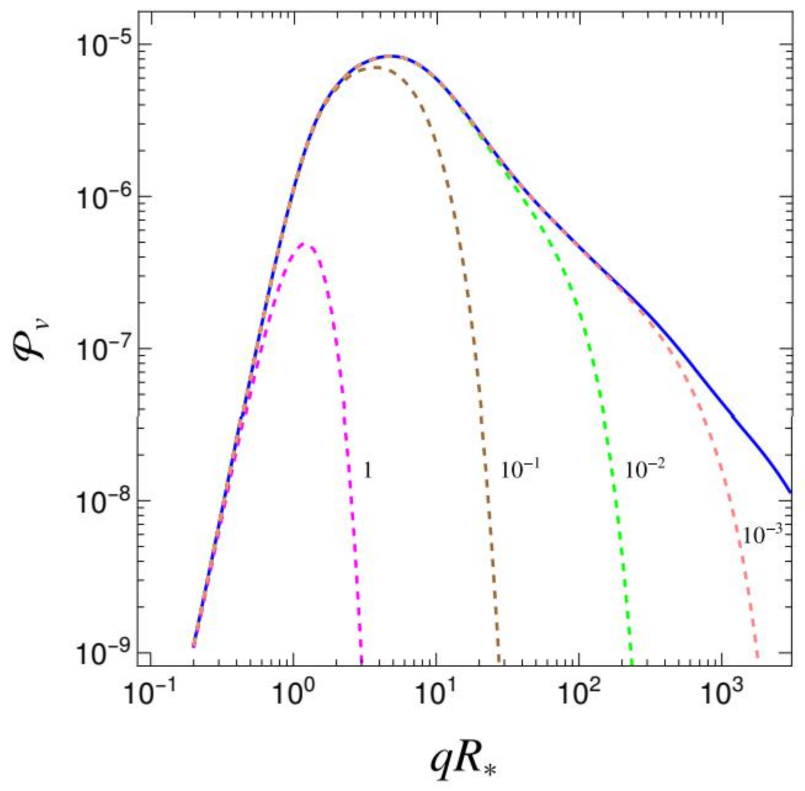
$$\mathcal{P}_v(q,\eta_1,\eta_2) = \frac{q^3}{\pi^2}\int d\eta_{\rm lt}\int d\eta_d \left[\boxed{P(\eta_{\rm lt},\eta_d)}\frac{N_b}{V}\right] \times \eta_{\rm lt}^6|A(q\eta_{\rm lt})|^2 \boxed{\exp\left[-\int_{\eta_d}^{\eta_1}\Gamma dt - \int_{\eta_d}^{\eta_2}\Gamma dt\right]}$$

Effective damping length

$$\int_{\eta_d}^{\eta_1}\Gamma dt = q^2 d_D^2(\eta_d,\eta_1)$$

$$q^2[d_D^2(\eta_d,\eta_1) + d_D^2(\eta_d,\eta_2)] \equiv q^2 d_D^2(\eta_d,\eta_1,\eta_2)$$

assuming const dD



Extreme situation: dD=const, then spectrum is stationary

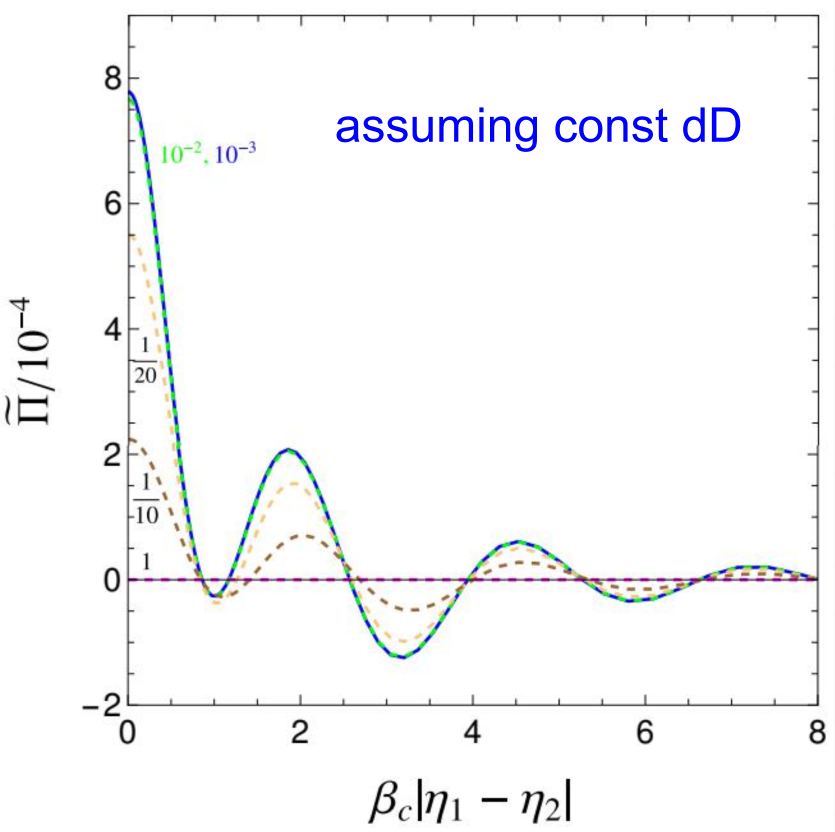
Correlator of Stress Tensor

- Anisotropic stress spectrum is thus also generally non-stationary

Simplified case for illustration: common destruction time eta_d = eta*

$$\mathcal{P}_v(q,\eta_1,\eta_2) = \exp\left[-q^2 d_D^2(\eta_*,\eta_1,\eta_2)\right]\mathcal{P}_v(q)$$

$$\tilde{\Pi}^2 = \frac{\pi}{2}\frac{1}{\bar{U}_f^4}\int d^3\tilde{q}\,\mathcal{P}_v(\tilde{q})\mathcal{P}_v(\tilde{\tilde{q}})\frac{(1-\mu^2)^2}{\tilde{q}\tilde{\tilde{q}}^5}\boxed{e^{-(q^2+\bar{q}^2)d_D^2}} \times \cos\left[c_s\tilde{q}\frac{\beta_c(\eta_1-\eta_2)}{\beta_c R_*}\right]\cos\left[c_s\tilde{\tilde{q}}\frac{\beta_c(\eta_1-\eta_2)}{\beta_c R_*}\right] \quad (9)$$



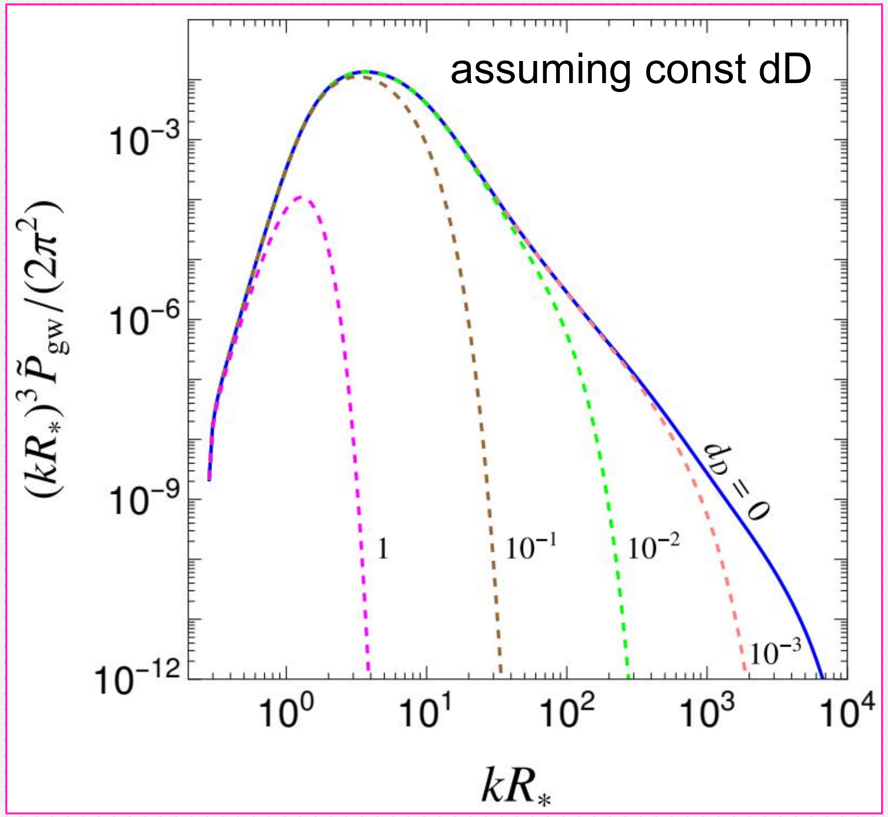
assuming const dD

Only in extreme cases, can it be stationary.

Gravitational Wave Spectrum

- Suppression of GW amplitude
- Starts at large wavenumber (small scale)
- Can makes peak frequency lower for larger damping length

$$
\begin{aligned}
\mathcal{P}_{\rm GW}(\eta,k) \;=\; & \frac{32G^2[(\bar{\rho}+\bar{p})\,\bar{U}_f^2]^2}{3a^2H^2}(kR_*)^3 \int_{\tilde{y}_s}^{\tilde{y}} d\tilde{y}_1 \int_{\tilde{y}_s}^{\tilde{y}} d\tilde{y}_2 \\
& \times \left(\frac{\partial\tilde{y}}{\partial\tilde{\eta}}\right)^2 \frac{\partial G(\tilde{y},\tilde{y}_1)}{\partial\tilde{y}} \frac{\partial G(\tilde{y},\tilde{y}_2)}{\partial\tilde{y}} \frac{a(\eta_s)^8}{a^2(\eta_1)a^2(\eta_2)} \\
& \times \boxed{\frac{\tilde{\Pi}^2(kR_*,k\eta_1,k\eta_2)}{k^2}}, \qquad (8)
\end{aligned}
$$



Realistic Cases

- The fatorized form (Upsilon) might not exist
- The suppressions due to expansion and dissipation are mixed up

$$\mathcal{P}_{\mathrm{GW}}(y, kR_{*c}) = \frac{[16\pi G\,(\bar{\bar{\epsilon}} + \bar{\bar{p}})\,\bar{U}_f^2]^2}{24\pi^2 H^2 H_s^2}\,\frac{1}{y^4}(kR_{*c})^3 \times \int dy_-\,\tilde{\Pi}^2\left(kR_{*c}, \beta_c|\eta_1 - \eta_2|\right) \boxed{\left[\int dy_+ \frac{\mathcal{G}_2(\tilde{y}, \tilde{y}_1, \tilde{y}_2)}{\tilde{\tilde{k}}^2} \left\{ \begin{array}{c} y_1^{-2} y_2^{-2} \\ y_1^{-3/2} y_2^{-3/2} \end{array} \right\}\right]}$$

$$\left[\int dy_+ \cdots\right] = \frac{1}{2}\Upsilon(y)\cos\left(\tilde{\tilde{k}} y_-\right)$$

$$\boxed{h^2\Omega_{\mathrm{sw}}(f) = 2.65 \times 10^{-6} \left(\frac{100}{g_*}\right)^{\frac{1}{3}} \left(\frac{H_*}{\beta}\right) \left(\frac{\kappa_{\mathrm{sw}}\alpha}{1+\alpha}\right)^2 v_w S_{\mathrm{sw}}(f)\boxed{\Upsilon(\tau_{\mathrm{sw}})}}$$

Lifetime of Sound Waves

- Expansion of the universe provides an effective lifetime
- Dissipation effects, when strong, provide a shorter effective lifetime
- Onset of MHD turbulence serves as a cut-off (dissipation causes changes)

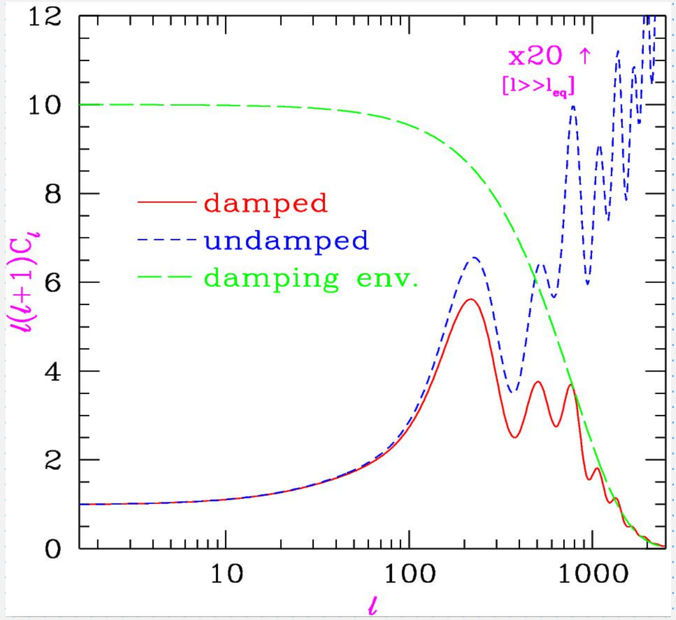
Realistic cases: intertwining of these effects (makes GW spectrum model dependent)

Model dependent spectrum carries information about each model (break parameter degeneracy)

Microscopic Origin

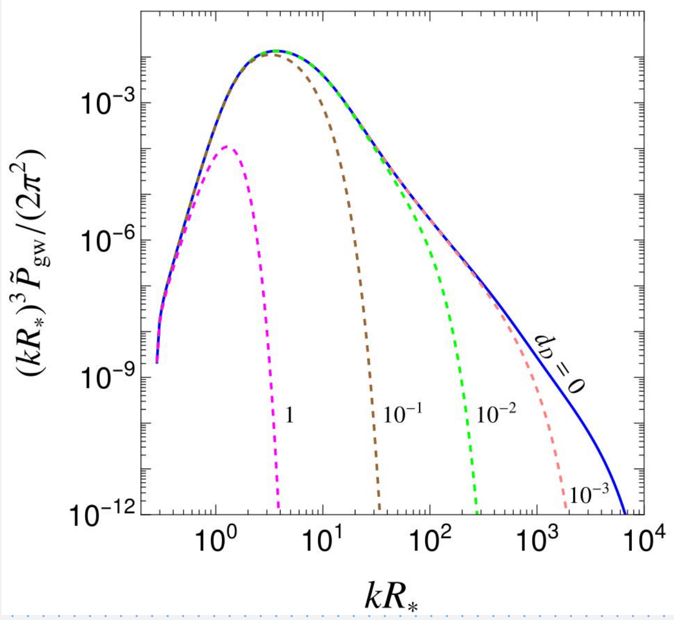
- Viscosity in the early universe is very small
- But can be significant for phase transitions in the dark sector
- Can also be stronger when BSM physics are included (from very weak interactions)
- Viscosity and transport coeffieicnts calculable from semi-classical kinetic theory or Green-Kubo relations

Analogy: Silk damping of CMB Anisotropy



Hu,White, ApJ [9609079]

damping of GW



HG [2310.10927]

Summary

- Dissipative effects can serve as new obervables for cosmic phase transitions
- New portals to probe microscopic particle (very weak) interactions
- Experimental searches of new spectrum are desired

Thanks!