Relic QCD axions from the Early Universe

Giovanni Villadoro

The Abdus Salam International Centre for Theoretical Physics

Phys. Rev. Lett. 131 (2023) 1, 011004

with A. Notari and F. Rompineve

The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$

The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$

- dynamical explanation of $\theta_{\text{strong}} \ll 1$

The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$

- dynamical explanation of $\theta_{\text{strong}} \ll 1$
- contribute to part (or all) of Ω_{dm}

The (Minimal) QCD Axion

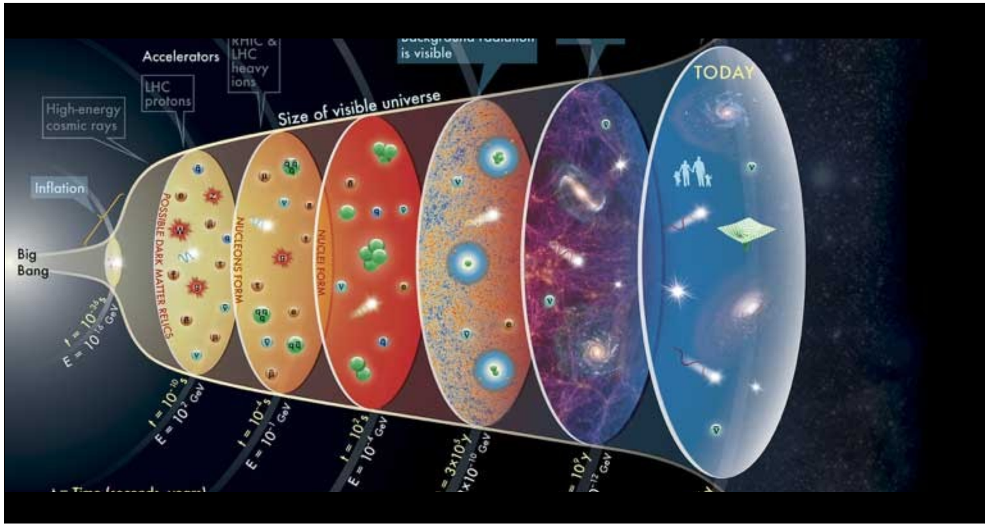
$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$

- dynamical explanation of $\theta_{\text{strong}} \ll 1$
- contribute to part (or all) of Ω_{dm}
- contribute to Ω_{rad}

The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \dots$$

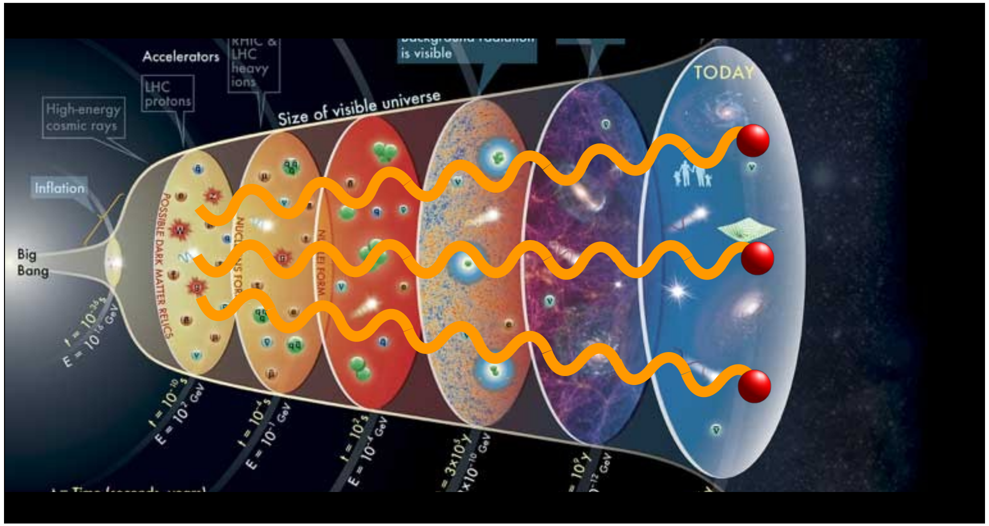
- dynamical explanation of $\theta_{\text{strong}} \ll 1$
- contribute to part (or all) of Ω_{dm}
- contribute to Ω_{rad}



The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$

- dynamical explanation of $\theta_{\text{strong}} \ll 1$
- contribute to part (or all) of Ω_{dm}
- contribute to Ω_{rad}



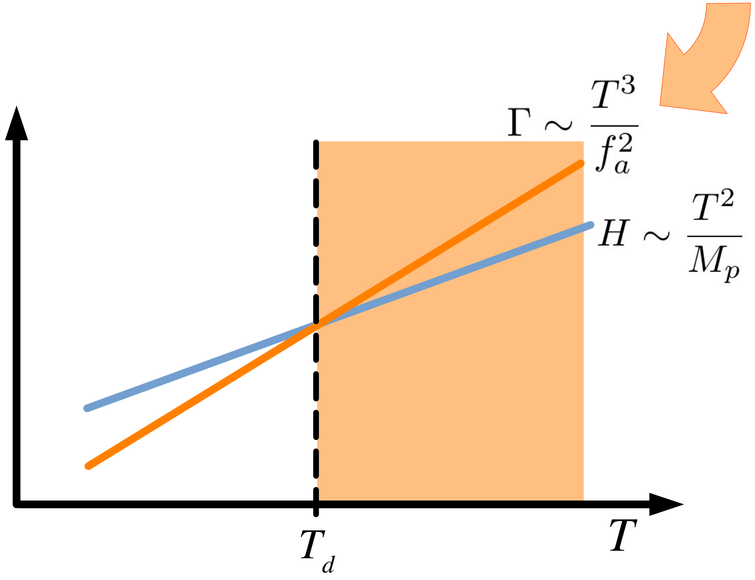
The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$

$$\Gamma \sim \frac{T^3}{f_a^2}$$

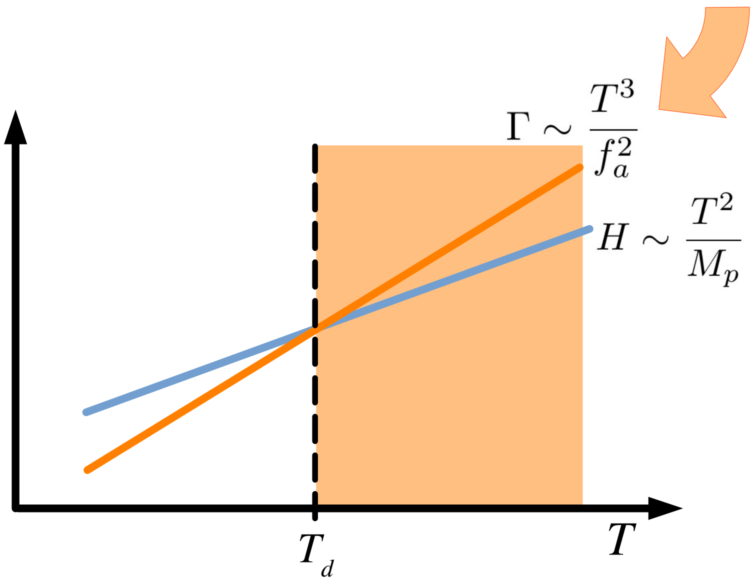
The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$



The (Minimal) QCD Axion

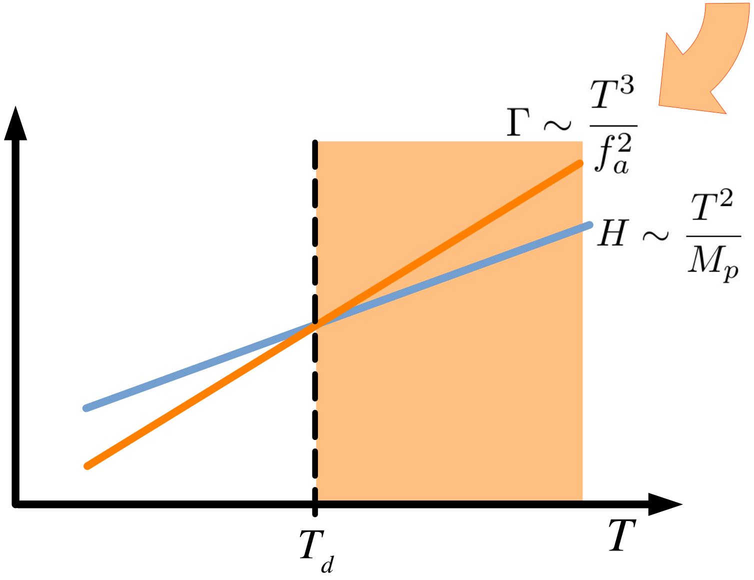
$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$



$$T_d \sim \frac{f_a^2}{M_p} \sim \Lambda_{QCD}\left(\frac{f_a}{10^8\ \text{GeV}}\right)^2$$

The (Minimal) QCD Axion

$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \dots$$

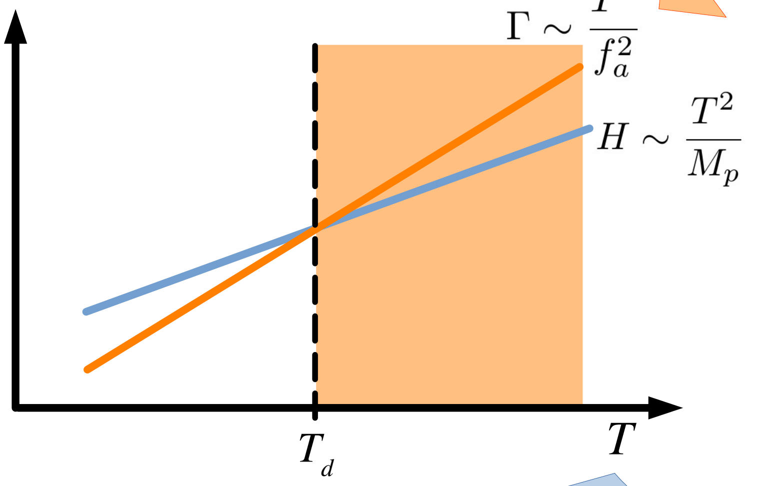


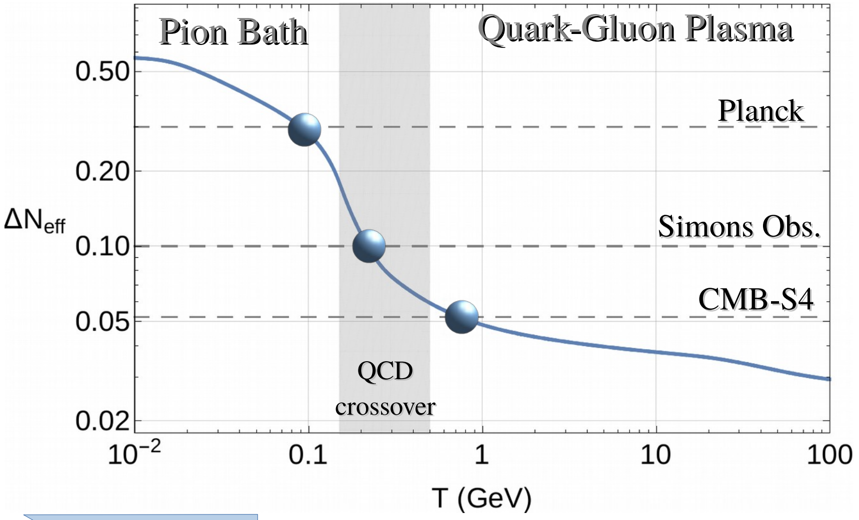
$$T_d \sim \frac{f_a^2}{M_p} \sim \Lambda_{QCD}\left(\frac{f_a}{10^8\ \mathrm{GeV}}\right)^2$$

$$\Delta N_{\rm eff} \equiv N_{\rm eff} - N_\nu = \frac{8}{7}\left(\frac{11}{4}\right)^{\frac{4}{3}}\left(\frac{\rho_a}{\rho_\gamma}\right)_{\rm CMB} \simeq\ 0.027\,[106.75/g_{*,S}(T_{\rm d})]^{4/3}$$

The (Minimal) QCD Axion

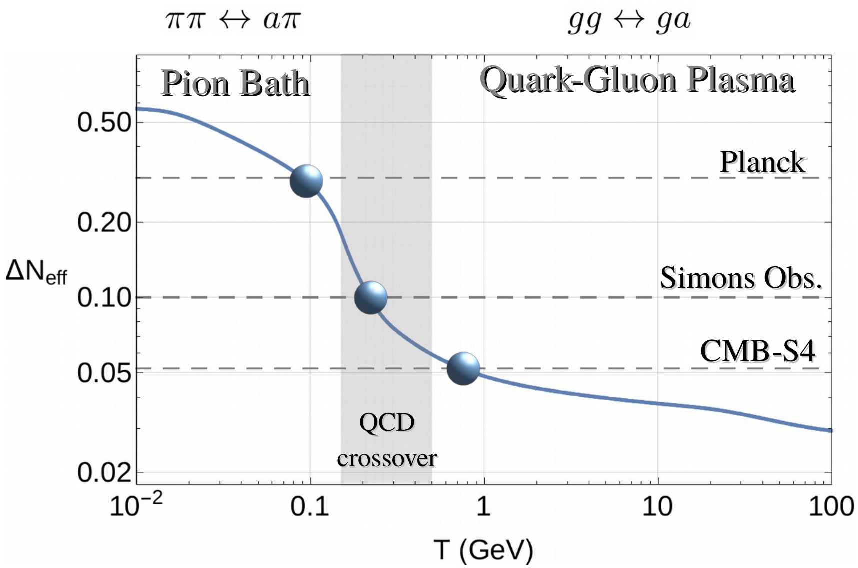
$$\mathcal{L}_{SM}^{\theta=0} + \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G\tilde{G} + \ldots$$





$$\Delta N_{\rm eff} \equiv N_{\rm eff} - N_\nu = \frac{8}{7}\left(\frac{11}{4}\right)^{\frac{4}{3}}\left(\frac{\rho_a}{\rho_\gamma}\right)_{\rm CMB} \simeq 0.027\,[106.75/g_{*,S}(T_{\rm d})]^{4/3}$$

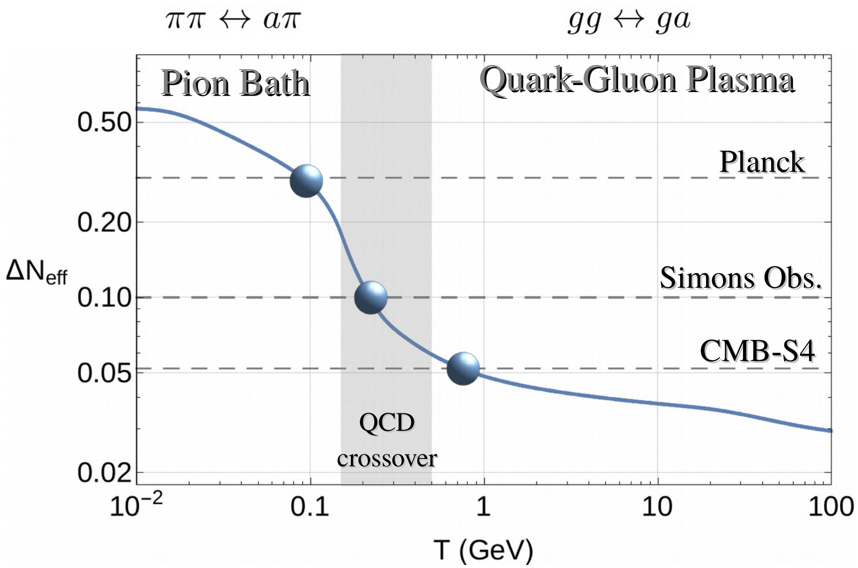
Axion $\Delta N_{\rm eff}$ has long history:



Arias-Arogon, Baumann, Bernal, Berezhiani, Chang, Choi, D'Eramo, Di Luzio, Di Valentino, Dunsky, Ferreira, Giusarma, Graf, Green, Guo, Hall, Hajkarim , Hannestad, Harigaya, Khlopov, Lattanzi, Martinelli, Masso, Melchiorri, Mena, Merlo, Mirizzi, Notari, Piazza, Raffelt, Rompineve, Rota, Sakharov, Silk, Slosar, Steffen, Wallisch, Wong, Yun, Zsembinszki ...

Axion $\Delta N_{\rm eff}$ has long history:

Arias-Arogon, Baumann, Bernal, Berezhiani, Chang, Choi, D'Eramo, Di Luzio, Di Valentino, Dunsky, Ferreira, Giusarma, Graf, Green, Guo, Hall, Hajkarim , Hannestad, Harigaya, Khlopov, Lattanzi, Martinelli, Masso, Melchiorri, Mena, Merlo, Mirizzi, Notari, Piazza, Raffelt, Rompineve, Rota, Sakharov, Silk, Slosar, Steffen, Wallisch, Wong, Yun, Zsembinszki ...

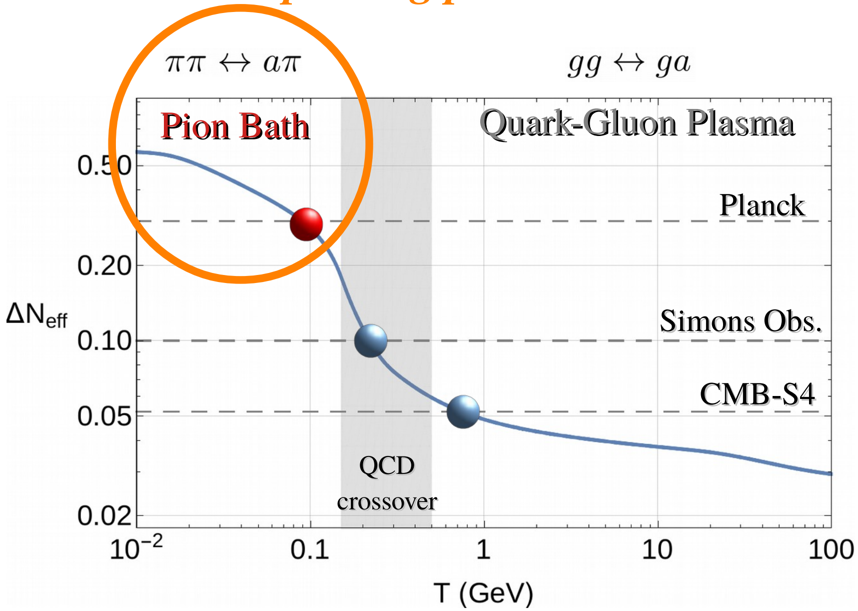


Boltzmann Eq.

$$\frac{dY}{d\log x} = (Y^{\rm eq} - Y)\frac{\overline{\Gamma}}{H}\left(1 - \frac{1}{3}\frac{d\log g_{*,S}}{d\log x}\right)$$

Axion ΔN_{eff} has long history:

Improving present bounds



Arias-Arogon, Baumann, Bernal, Berezhiani, Chang, Choi, D'Eramo, Di Luzio, Di Valentino, Dunsky, Ferreira, Giusarma, Graf, Green, Guo, Hall, Hajkarim , Hannestad, Harigaya, Khlopov, Lattanzi, Martinelli, Masso, Melchiorri, Mena, Merlo, Mirizzi, Notari, Piazza, Raffelt, Rompineve, Rota, Sakharov, Silk, Slosar, Steffen, Wallisch, Wong, Yun, Zsembinszki ...

Boltzmann Eq.

$$\frac{dY}{d\log x} = (Y^{\text{eq}} - Y)\frac{\overline{\Gamma}}{H}\left(1 - \frac{1}{3}\frac{d\log g_{*,S}}{d\log x}\right)$$

Boltzmann Equation and Thermalization Rate Γ

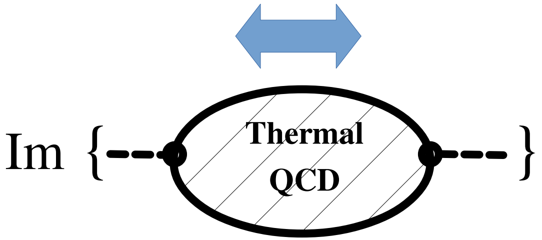
$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^{<} - f_{\mathbf{p}}\,\Gamma^{>}$$

Boltzmann Equation and Thermalization Rate Γ

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^{<} - f_{\mathbf{p}}\,\Gamma^{>}$$

$$\Gamma^{>} = e^{\frac{E}{T}}\Gamma^{<} = \frac{\Gamma^{>}_{\rm top}}{2Ef_a^2}$$

$$\Gamma^{>}_{\rm top} \equiv \int d^4x\, e^{ik^{\mu}x_{\mu}} \left\langle \frac{\alpha_s}{8\pi} G\tilde{G}(x^{\mu})\, \frac{\alpha_s}{8\pi} G\tilde{G}(0) \right\rangle$$

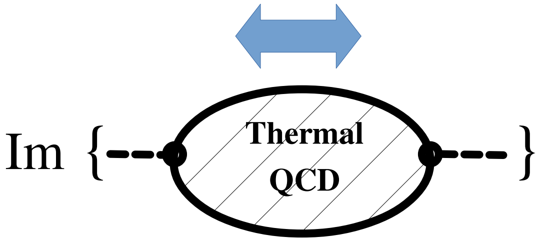


Boltzmann Equation and Thermalization Rate Γ

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^{<} - f_{\mathbf{p}}\,\Gamma^{>}$$

$$\Gamma^{>} = e^{\frac{E}{T}}\Gamma^{<} = \frac{\Gamma^{>}_{\text{top}}}{2Ef_a^2}$$

$$\Gamma^{>}_{\text{top}} \equiv \int d^4x\, e^{ik^\mu x_\mu}\, \left\langle \frac{\alpha_s}{8\pi} G\tilde{G}(x^\mu)\, \frac{\alpha_s}{8\pi} G\tilde{G}(0) \right\rangle$$



$$\Gamma^{<} = \frac{1}{2E} \int \left(\prod_{i=1}^{3} \frac{d^3\mathbf{k}_i}{(2\pi)^3 2E_i} \right) f_1^{\text{eq}} f_2^{\text{eq}} (1 + f_3^{\text{eq}}) (2\pi)^4 \delta^{(4)}(k_1^\mu + k_2^\mu - k_3^\mu - k^\mu) |\mathcal{M}|^2_{2 \leftrightarrow 2}$$

1. The Thermalization Rate Γ

$$\pi\pi \leftrightarrow a\pi$$

LO χPT rate
(Chang Choi '93)

$$|\mathcal{M}^{\rm LO}|^2 = \theta_{a\pi}^2 \frac{s^2+t^2+u^2-3m_\pi^4}{f_\pi^4}$$

$$\theta_{a\pi} = \frac{f_\pi}{2f_a}\left(\frac{m_d-m_u}{m_d+m_u} + c_u - c_d\right)$$

1. The Thermalization Rate Γ

$$\pi\pi \leftrightarrow a\pi$$

LO χPT rate
(Chang Choi '93)

$$|\mathcal{M}^{\rm LO}|^2 = \theta_{a\pi}^2 \frac{s^2 + t^2 + u^2 - 3m_\pi^4}{f_\pi^4}$$

NLO χPT rate
(Di Luzio, Martinelli, Piazza '21)

$$\theta_{a\pi} = \frac{f_\pi}{2f_a}\left(\frac{m_d - m_u}{m_d + m_u} + c_u - c_d\right)$$

→ breaks down at T ~ 60 MeV

1. The Thermalization Rate Γ

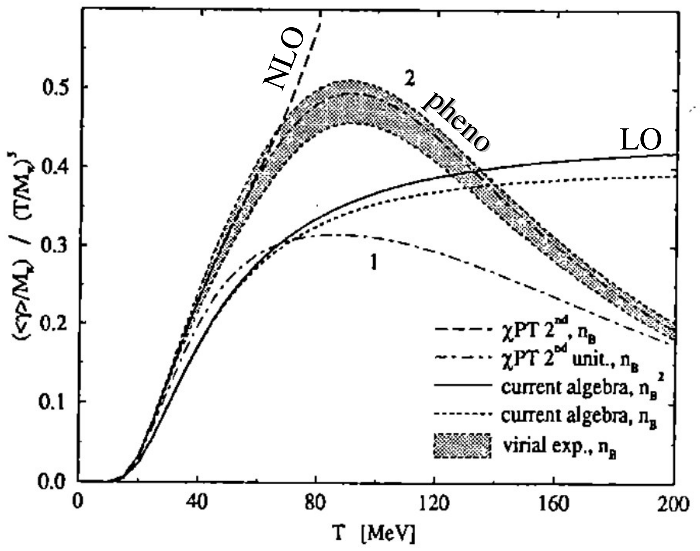
$$\pi\pi \leftrightarrow a\pi$$

LO χPT rate
(Chang Choi '93)

NLO χPT rate
(Di Luzio, Martinelli, Piazza '21)

<u>$\rightarrow$ breaks down at T ~ 60 MeV</u>

$$\pi\pi \leftrightarrow \pi\pi$$



Schenk '94

1. The Thermalization Rate Γ

$$\pi\pi \leftrightarrow a\pi$$

LO χPT rate
(Chang Choi '93)

NLO χPT rate
(Di Luzio, Martinelli, Piazza '21)

→ breaks down at T ~ 60 MeV

Strategy:

@ all orders in χPT

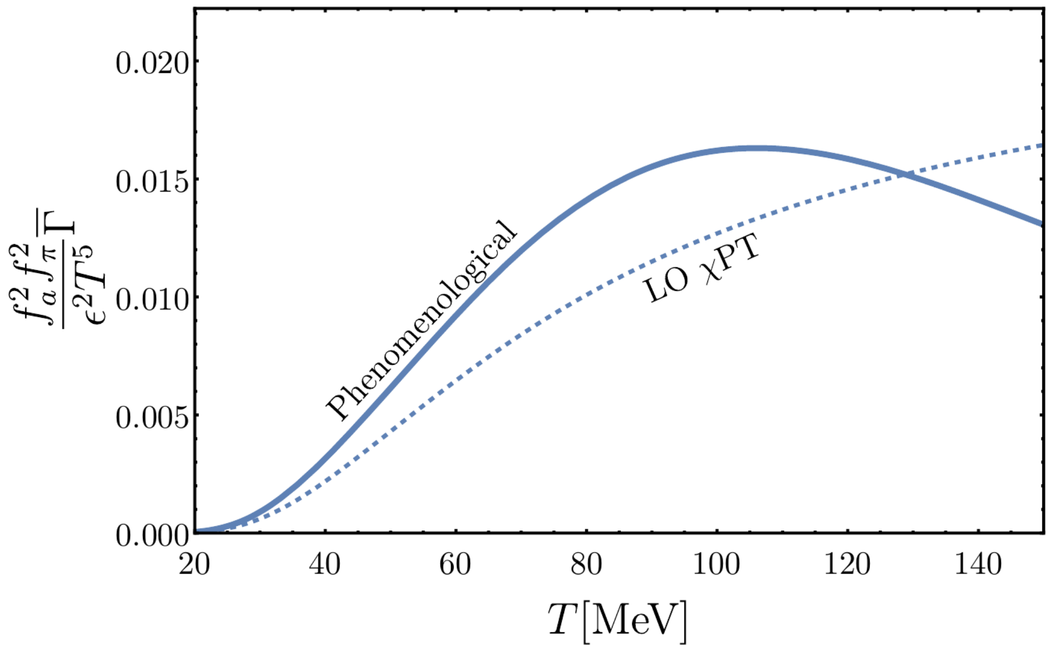
$$\mathcal{M}_{a\pi^i \to \pi^j \pi^k} = \theta_{a\pi} \cdot \mathcal{M}_{\pi^0 \pi^i \to \pi^j \pi^k} + \mathcal{O}\left(\frac{m_\pi^2}{s}\right)$$

e.g. @ LO

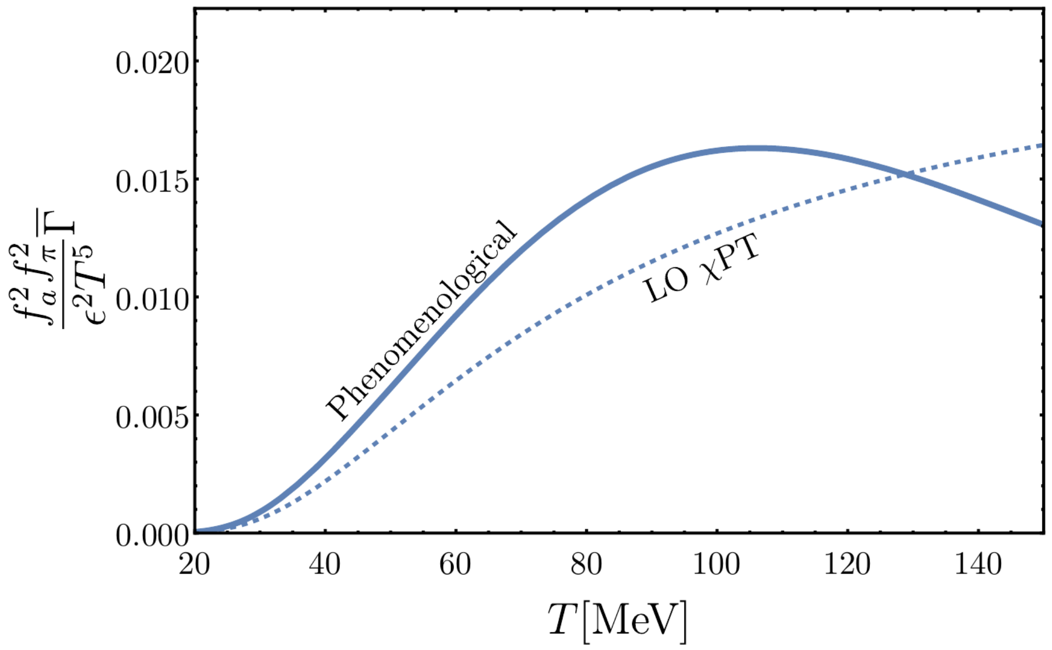
$$|\mathcal{M}^{\mathrm{LO}}|^2 = \theta_{a\pi}^2 \frac{s^2 + t^2 + u^2 - 3m_\pi^4}{f_\pi^4}$$

$$|\mathcal{M}_{\pi\pi}^{\mathrm{LO}}|^2 = \frac{s^2 + t^2 + u^2 - 4m_\pi^4}{f_\pi^4}$$

1. The Thermalization Rate Γ



1. The Thermalization Rate Γ

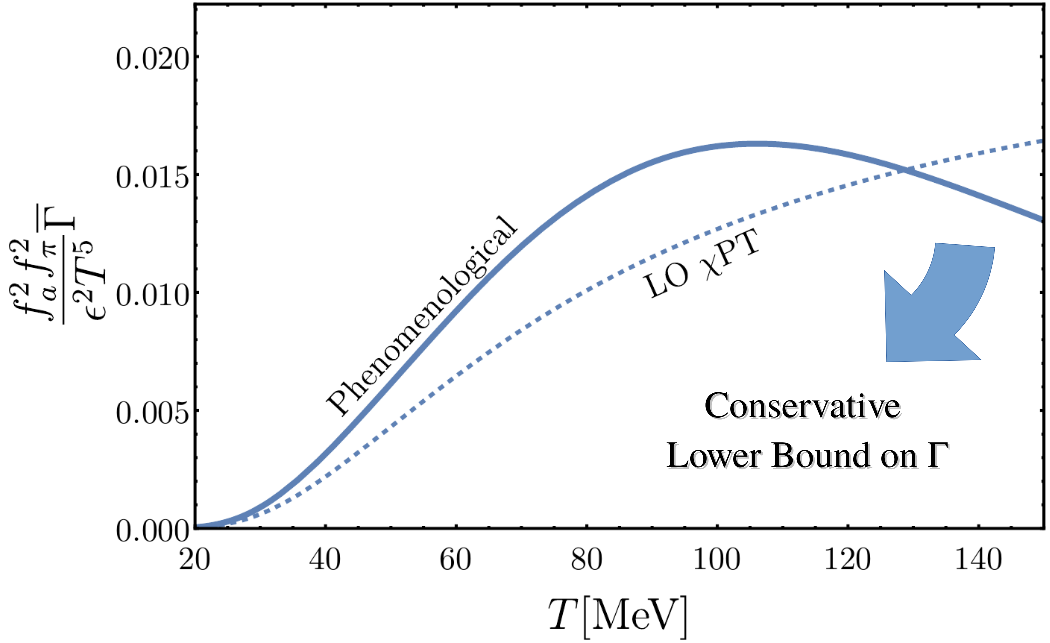


In reasonable agreement with:

Di Luzio, Camalich, Martinelli, Oller, Piazza '22

(using NLO+unitarization)

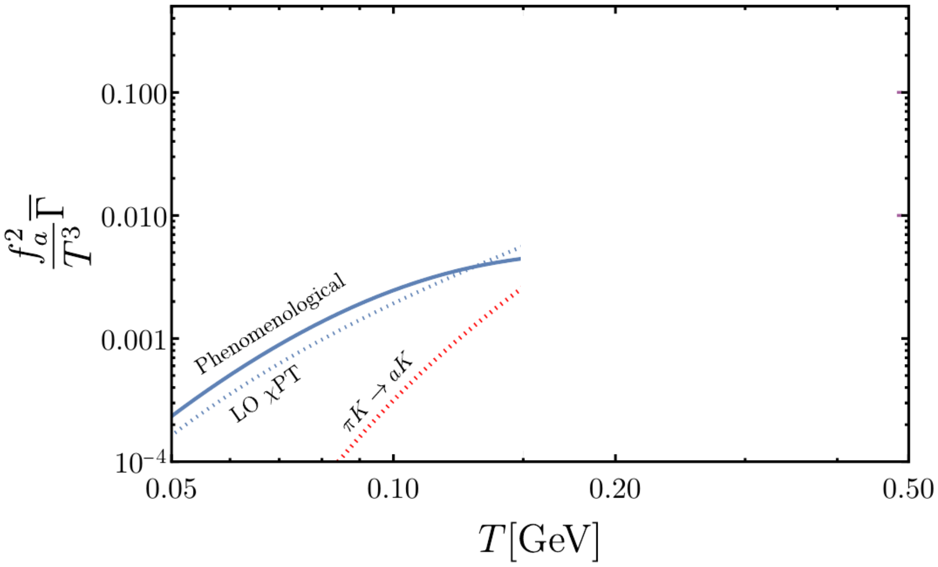
1. The Thermalization Rate Γ



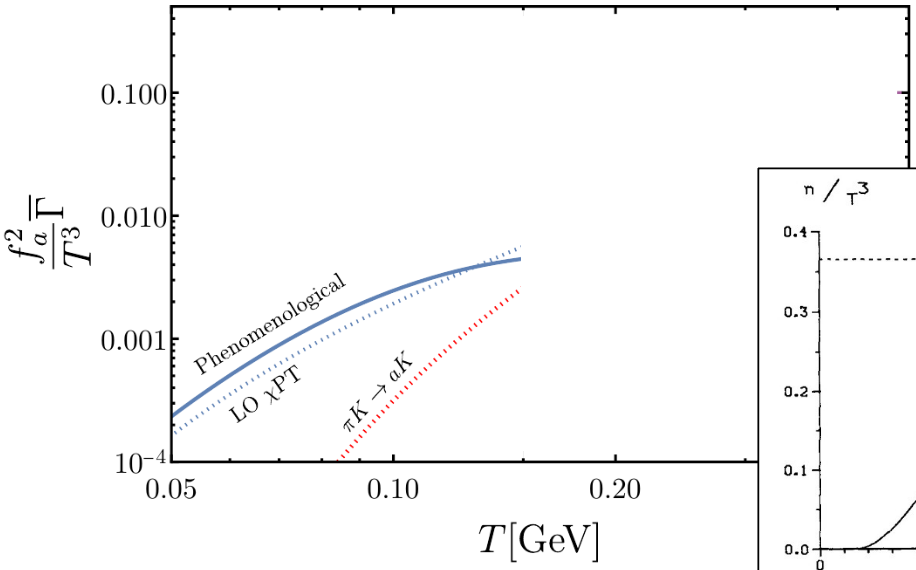
In reasonable agreement with:

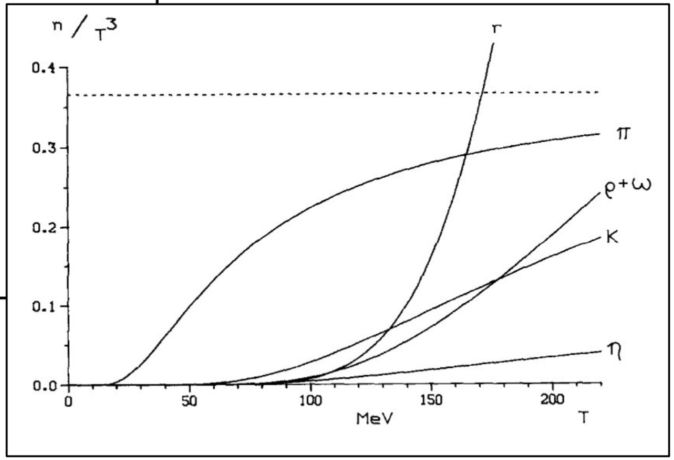
Di Luzio, Camalich, Martinelli, Oller, Piazza ‘22
(using NLO+unitarization)

1. The Thermalization Rate Γ



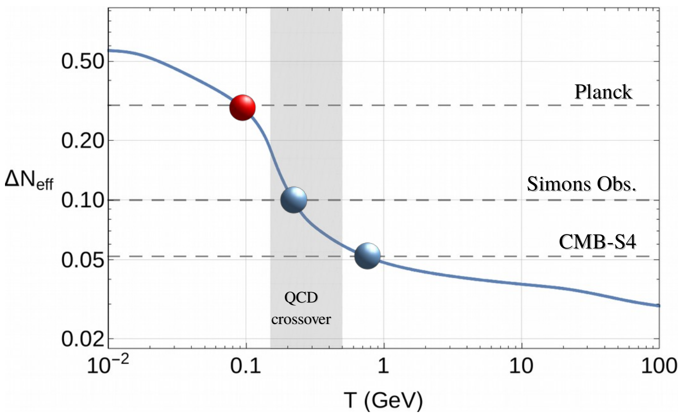
1. The Thermalization Rate Γ



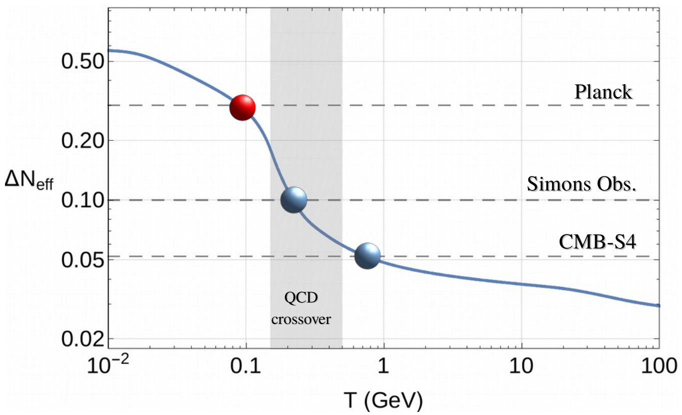


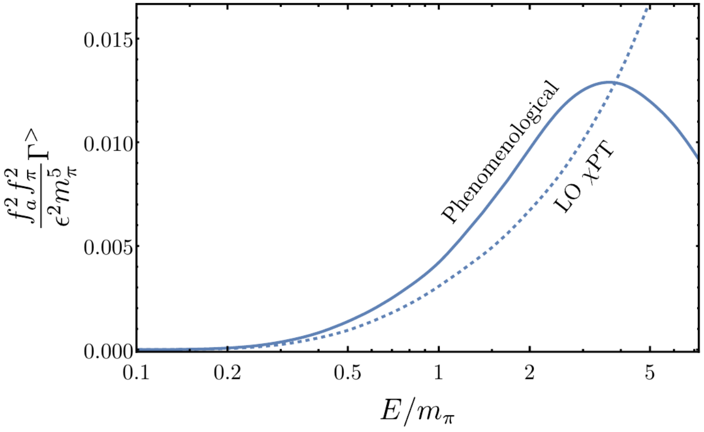
Gerber Leutwyler '89

2. Momentum Dependence

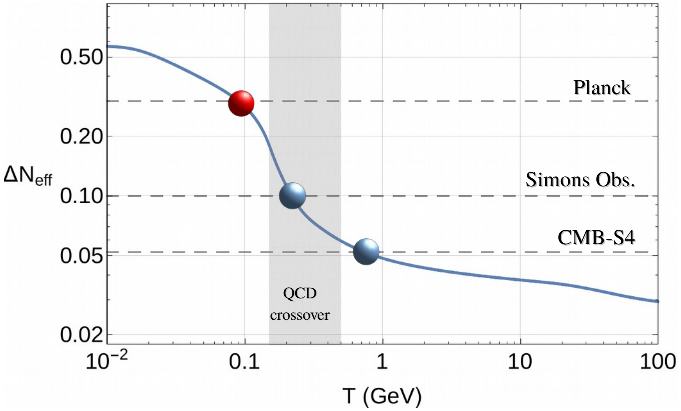


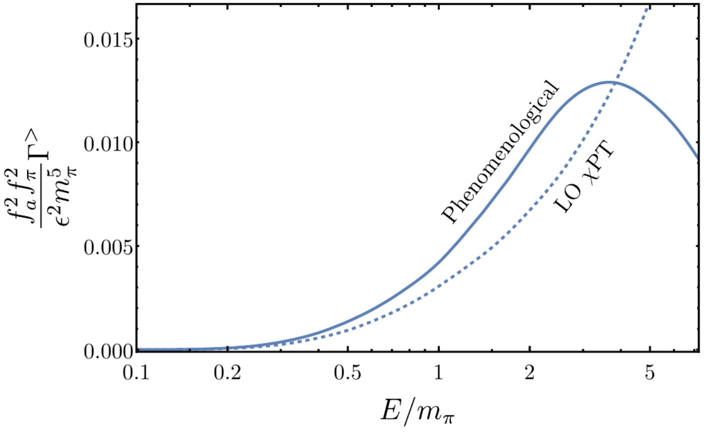
2. Momentum Dependence





2. Momentum Dependence

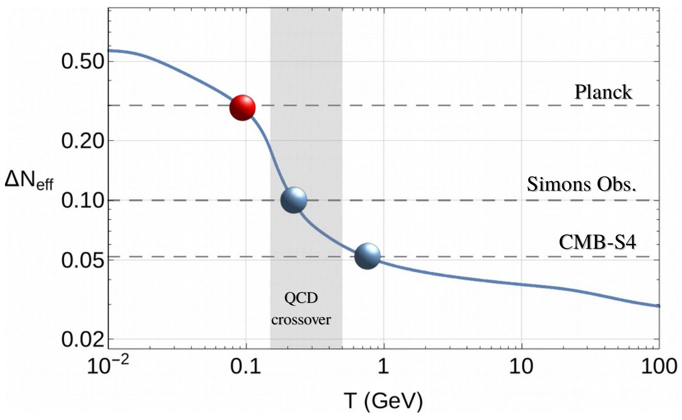


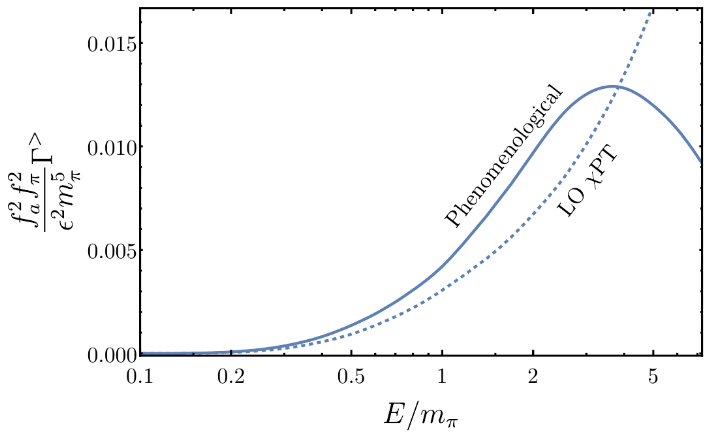


Boltzmann Eq.

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^< - f_{\mathbf{p}}\,\Gamma^>$$

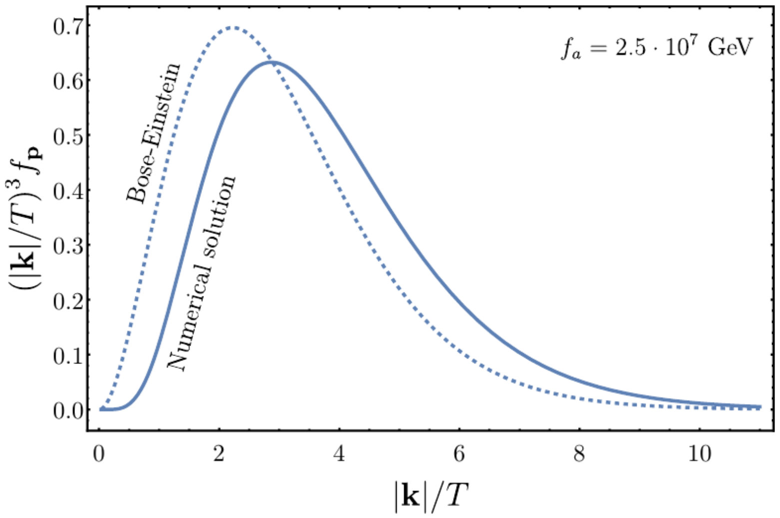
2. Momentum Dependence





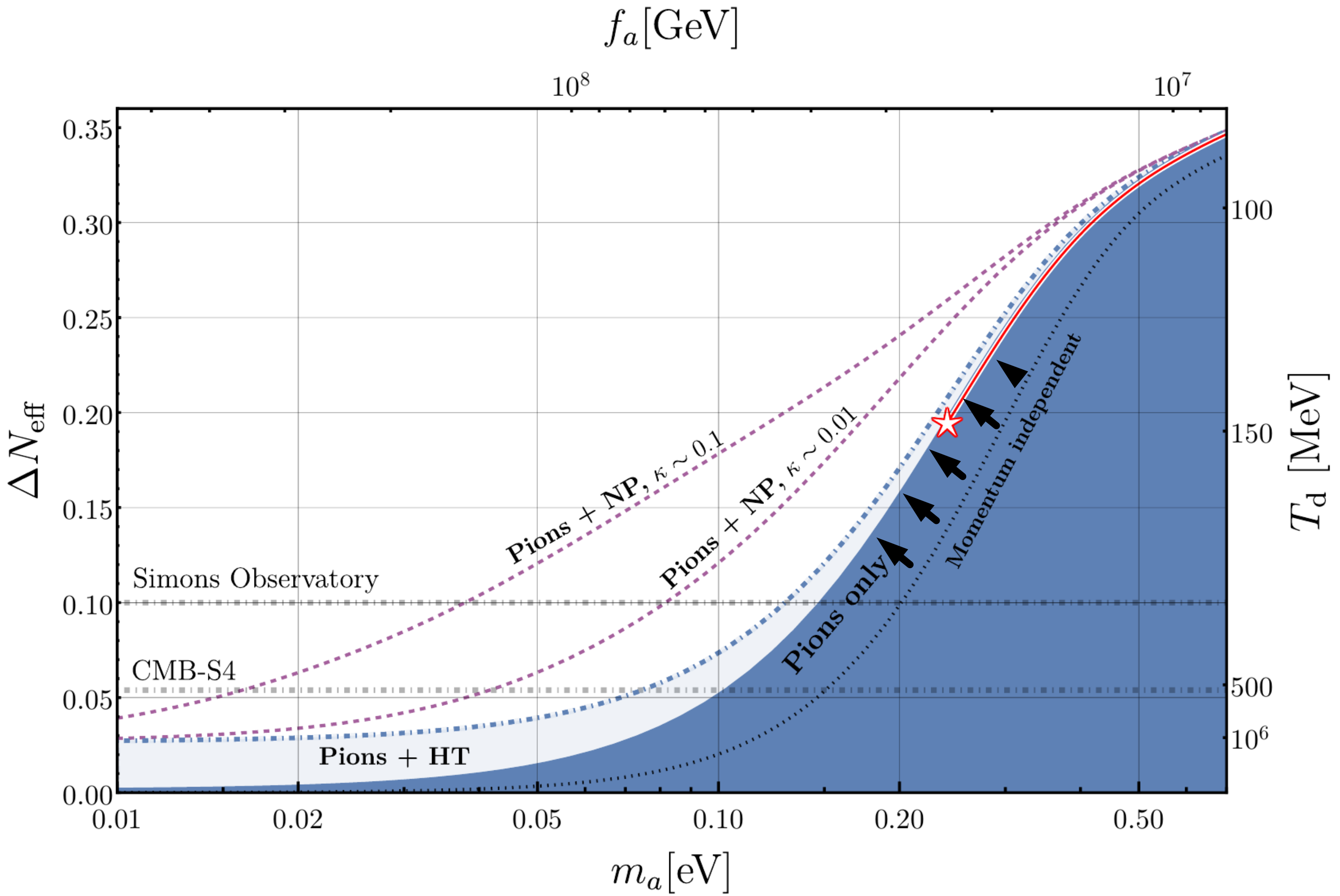
Boltzmann Eq.

$$\frac{df_{\mathbf{p}}}{dt} = (1 + f_{\mathbf{p}})\,\Gamma^< - f_{\mathbf{p}}\,\Gamma^>$$

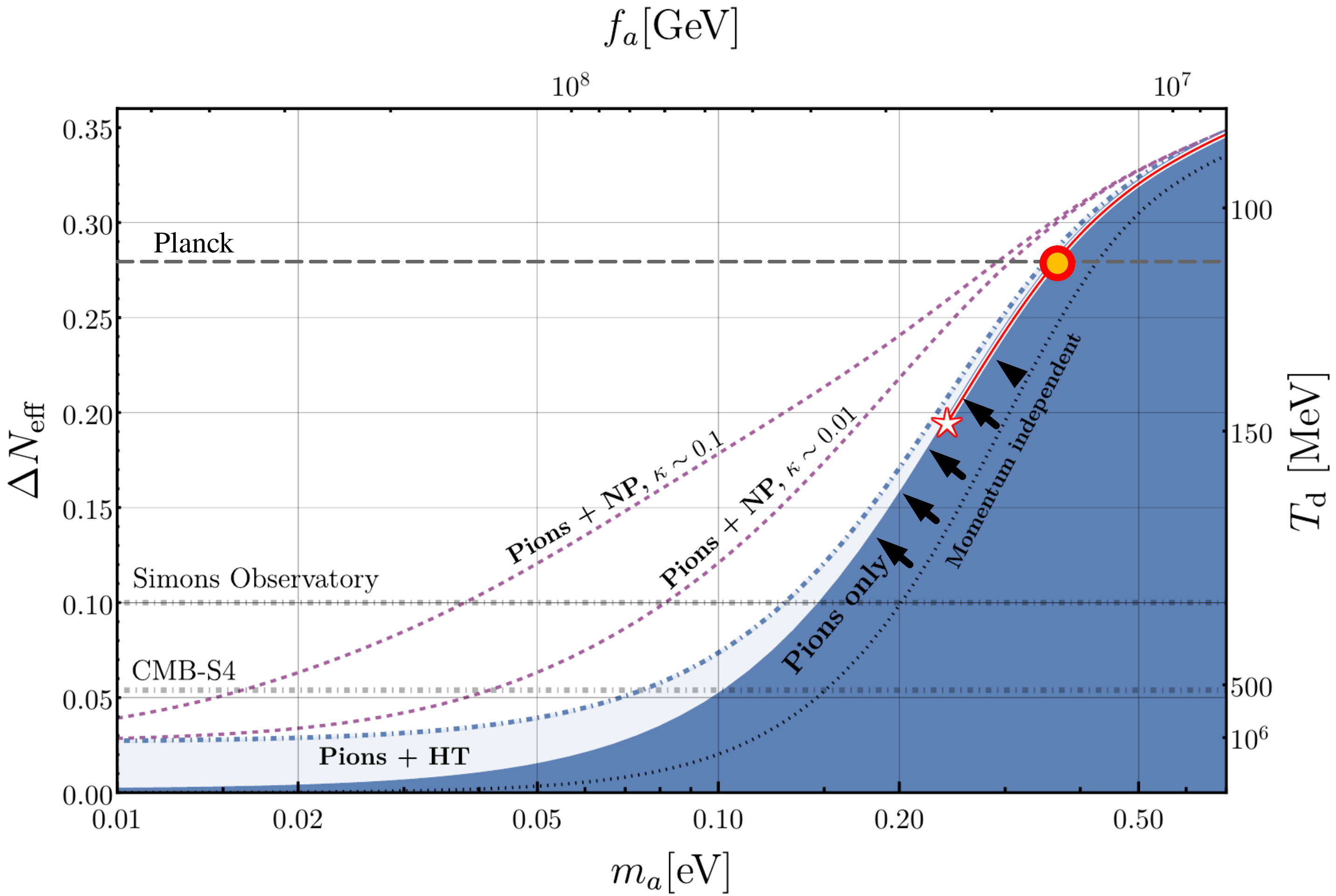


~ 40% enhancement

Future Reach

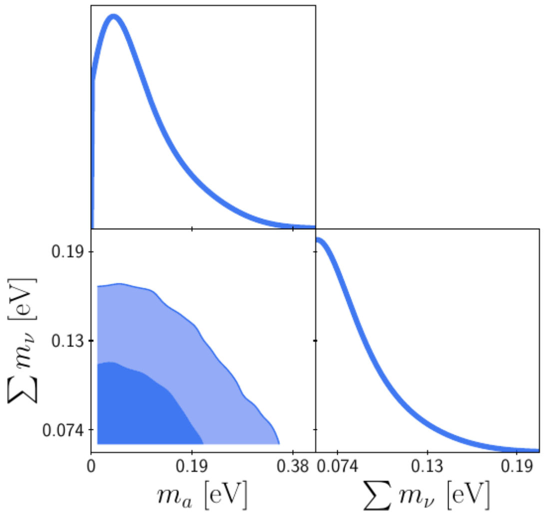


Future Reach



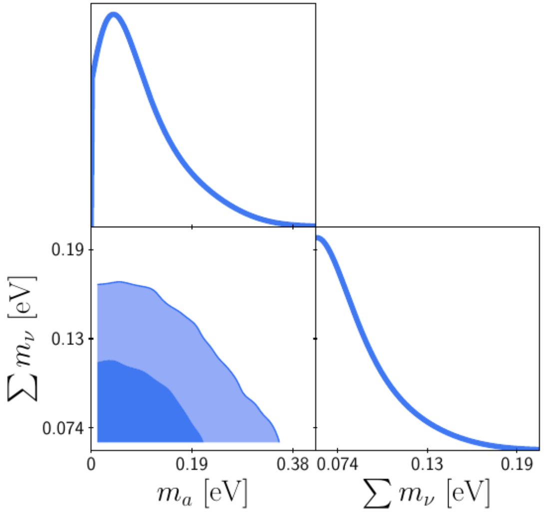
3. Cosmological Fit ($\Lambda_{\mathrm{CDM}} + \Sigma\, m_\nu + m_a$)

3. Cosmological Fit ($\Lambda_{CDM} + \Sigma\, m_\nu + m_a$)



$m_a \leq 0.24$ eV $\qquad f_a \geqslant 2.4 \cdot 10^7$ GeV

3. Cosmological Fit ($\Lambda_{CDM} + \Sigma m_\nu + m_a$)



$m_a \leq 0.24$ eV

$f_a \geqslant 2.4 \cdot 10^7$ GeV

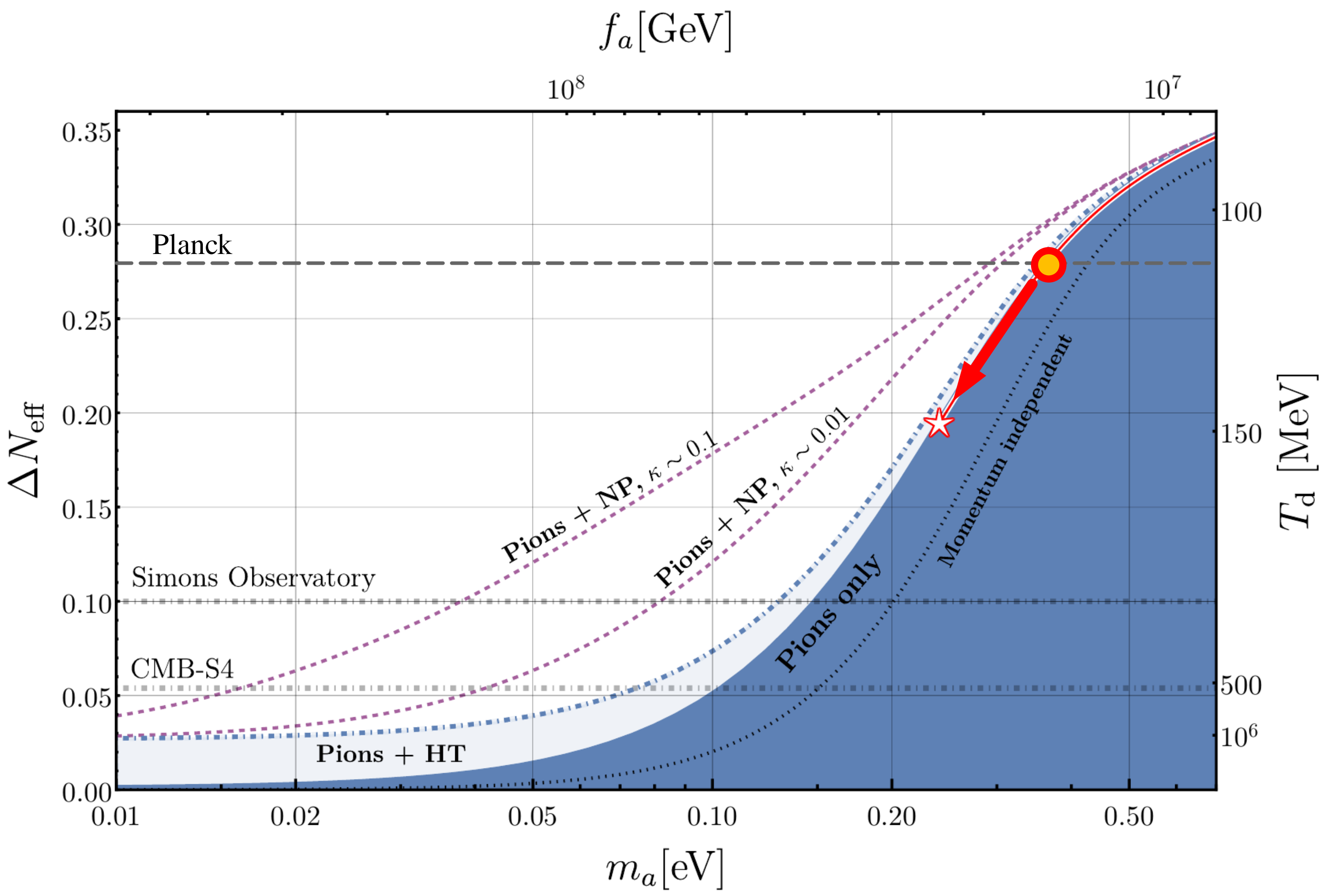
⇔

$\Delta N_{\text{eff}} \lesssim 0.19$

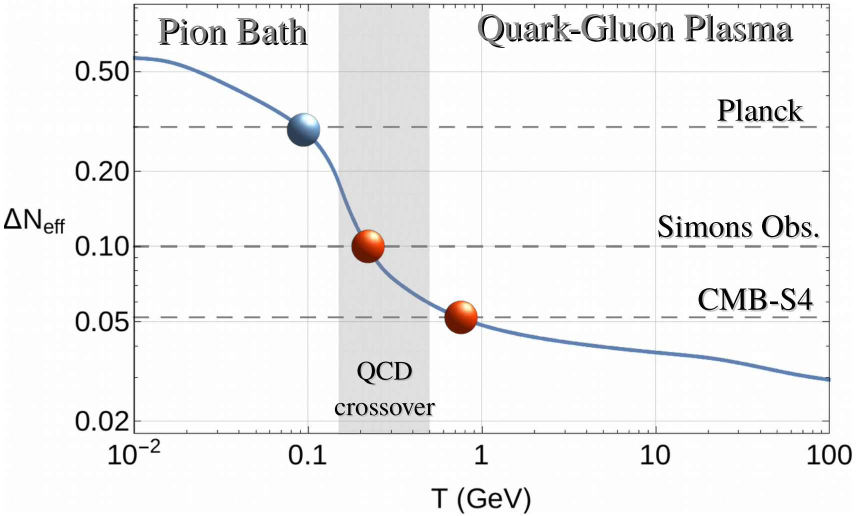
finite mass effect

~ 40% enhancement

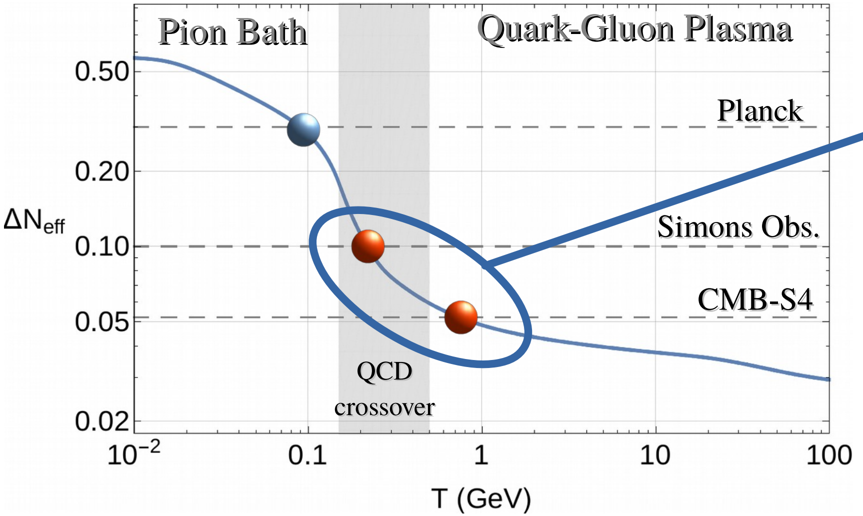
Future Reach



Future Reach



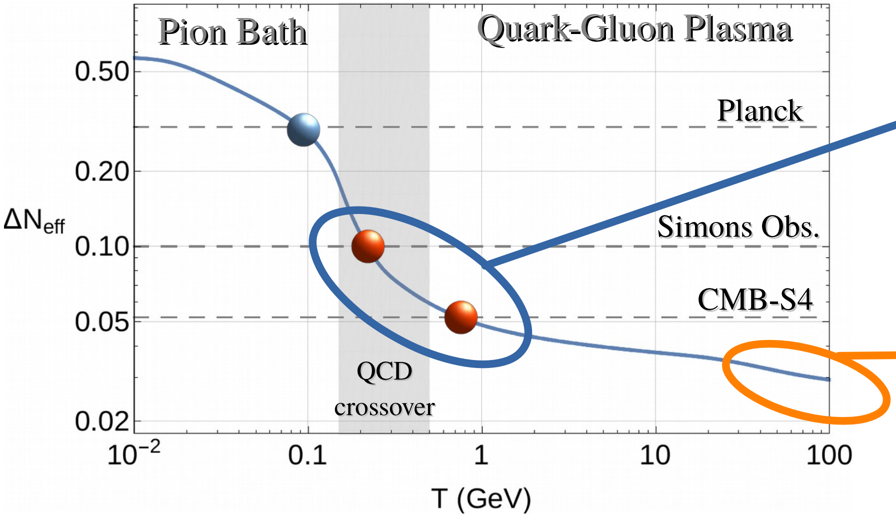
Future Reach



$$\Gamma_{\rm top}^{>} \equiv \int d^4x \, e^{ik^\mu x_\mu} \left\langle \frac{\alpha_s}{8\pi} G\tilde{G}(x^\mu) \, \frac{\alpha_s}{8\pi} G\tilde{G}(0) \right\rangle$$

Non-Perturbative

Future Reach



$$\Gamma^{>}_{\rm top} \equiv \int d^4x\, e^{ik^\mu x_\mu} \left\langle \frac{\alpha_s}{8\pi} G\tilde{G}(x^\mu)\, \frac{\alpha_s}{8\pi} G\tilde{G}(0) \right\rangle$$

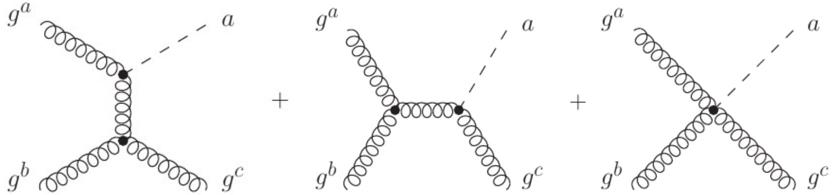
Non-Perturbative

$gg \leftrightarrow ga$

Perturbative ?

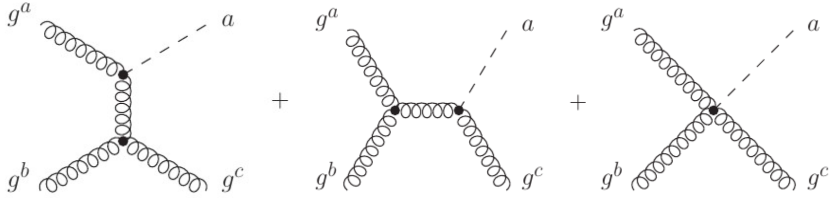
$$\overline{\Gamma}_{\rm pert} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

High Temperatures Regime



$$\overline{\Gamma}_{\rm pert} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

High Temperatures Regime



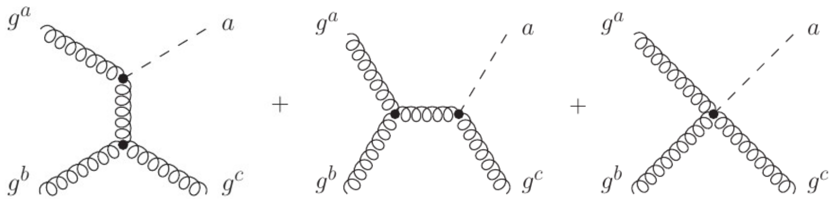
$$\overline{\Gamma}_{\rm pert} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

Masso, Rota, Zsembinszki '02
Graf, Steffen '10

$$F_3 = g_s^2 \log\left(\frac{3T^2}{2m_g^2}\right) = g_s^2 \log\left(\frac{3}{2g_s}\right)^2$$

for $g_s \ll 1$

High Temperatures Regime



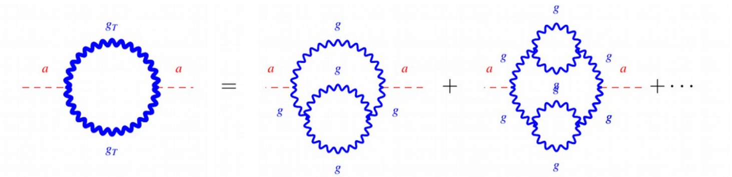
$$\overline{\Gamma}_{\rm pert} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

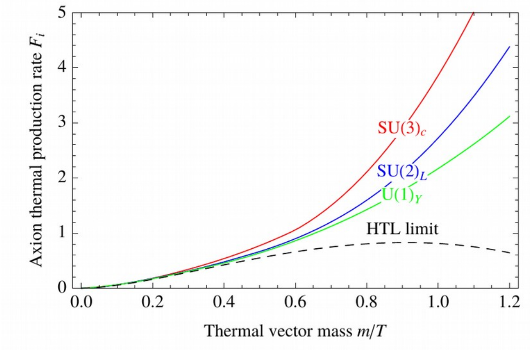
Masso, Rota, Zsembinszki ‘02
Graf, Steffen ‘10

$$F_3 = g_s^2 \log\left(\frac{3T^2}{2m_g^2}\right) = g_s^2 \log\left(\frac{3}{2g_s}\right)^2$$

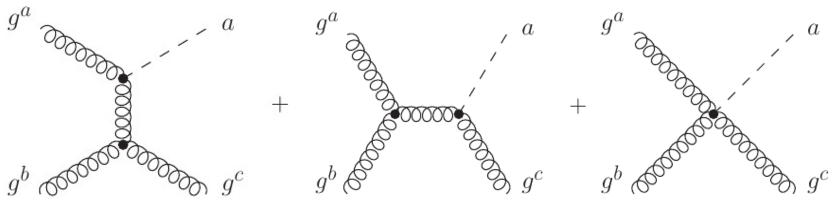
for $g_s \ll 1$

Salvio, Strumia, Xue ‘13





High Temperatures Regime



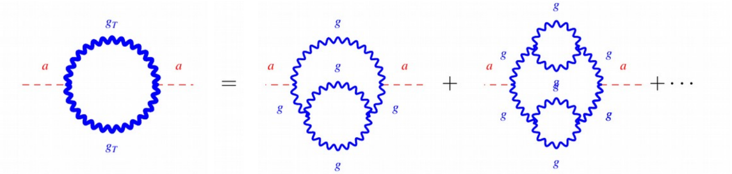
$$\overline{\Gamma}_{\rm pert} = \frac{\alpha_s^2 T^3}{4\pi^3 f_a^2} F_3$$

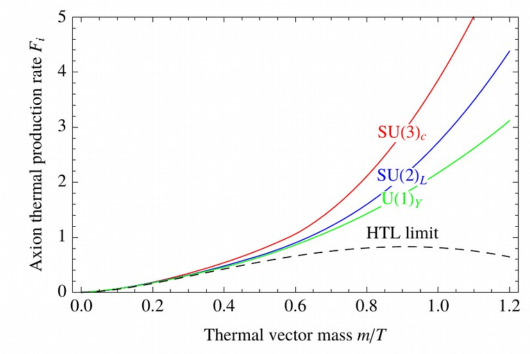
Masso, Rota, Zsembinszki ‘02
Graf, Steffen ‘10

$$F_3 = g_s^2 \log\left(\frac{3T^2}{2m_g^2}\right) = g_s^2 \log\left(\frac{3}{2g_s}\right)^2$$

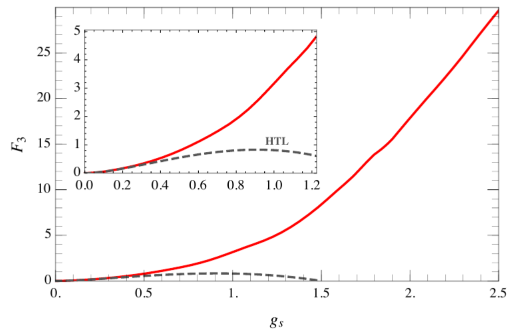
for $g_s \ll 1$

Salvio, Strumia, Xue ‘13

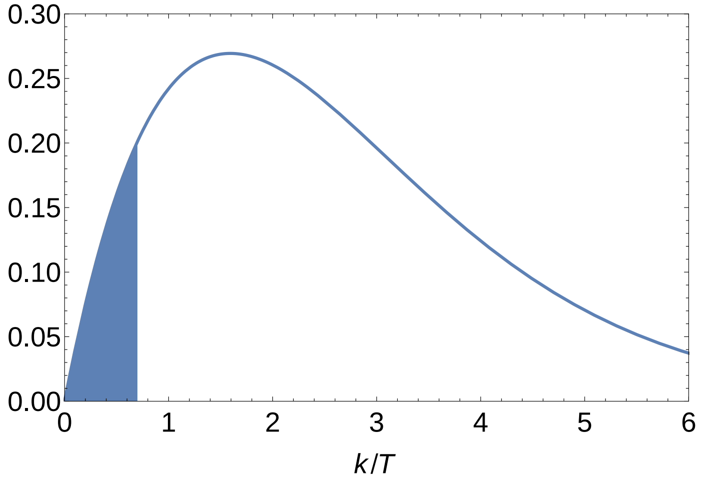




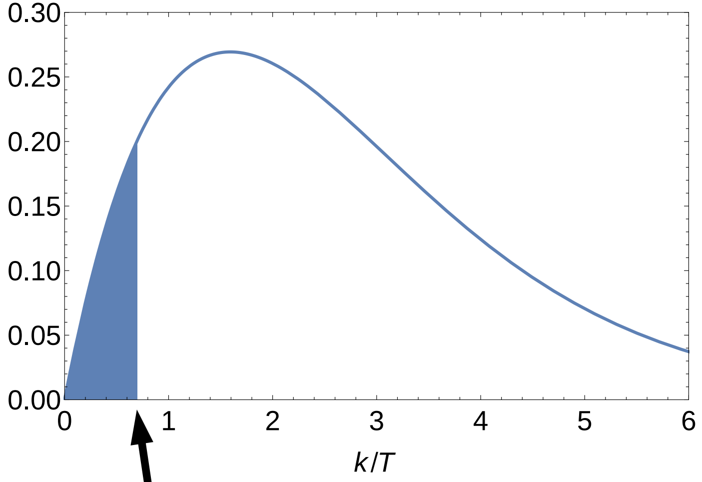
D’Eramo, Hajkarim, Yun ‘21



High Temperatures Regime

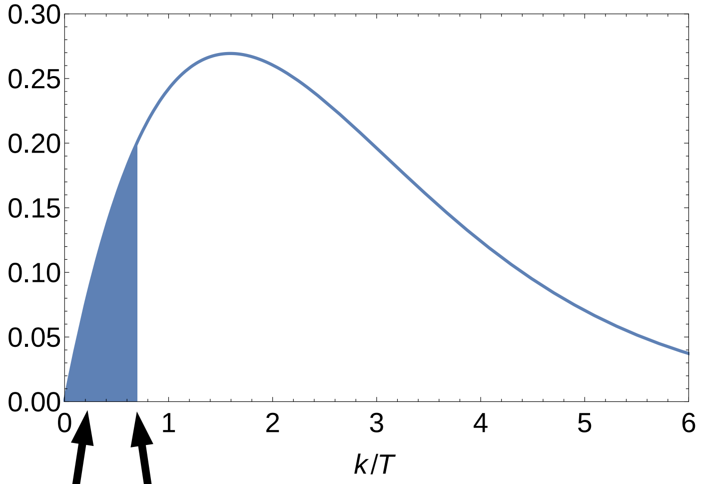


High Temperatures Regime



$k \sim m_e \sim g_s T$

High Temperatures Regime



$k \sim m_e \sim g_s T$

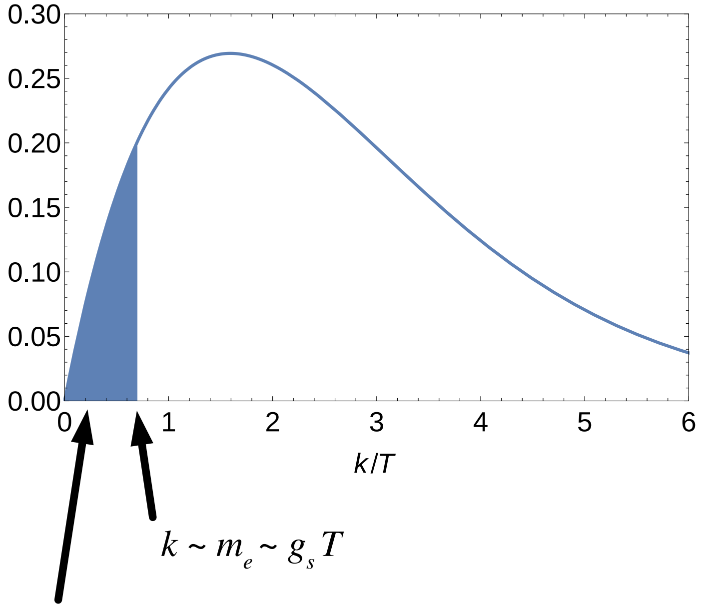
$k \sim m_m \sim g_s^2 T$

$\# \sim 1/g_s^2$

Linde '80

Gross, Pisarski, Yaffe '81

High Temperatures Regime



$k \sim m_e \sim g_s T$

$k \sim m_m \sim g_s^2 T$

$\# \sim 1/g_s^2$

Linde '80

Gross, Pisarski, Yaffe '81

@ $g_s \ll 1$:

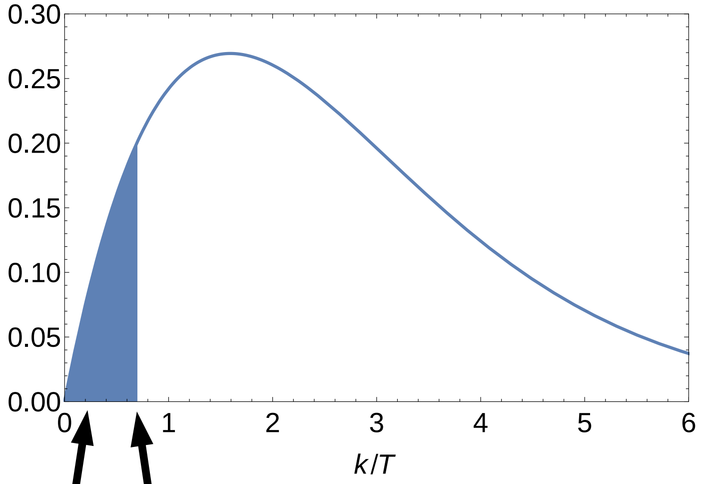
collective effects are phase-space suppressed $O(g_s^n)$

[e.g. for free energy $O(g_s^6)$]

large occupation numbers → dominated by semi-classic

[non-linear YM equations - e.g. strong sphalerons]

High Temperatures Regime



$k \sim m_e \sim g_s T$

$k \sim m_m \sim g_s^2 T$

$\# \sim 1/g_s^2$

Linde '80

Gross, Pisarski, Yaffe '81

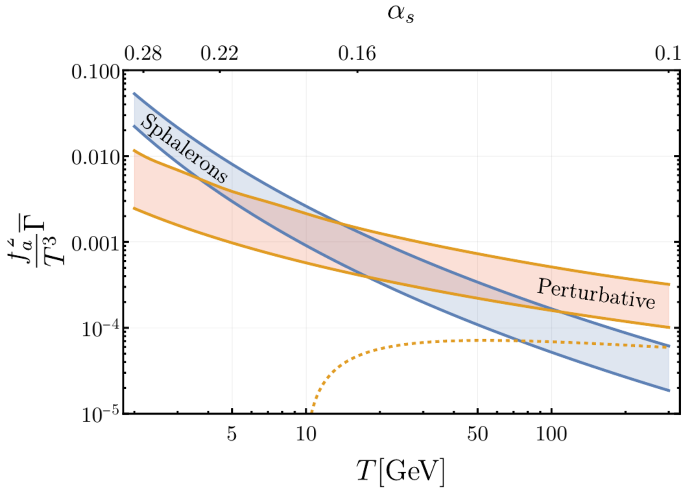
@ $g_s \ll 1$:

collective effects are phase-space suppressed $O(g_s^n)$

[e.g. for free energy $O(g_s^6)$]

large occupation numbers → dominated by semi-classic

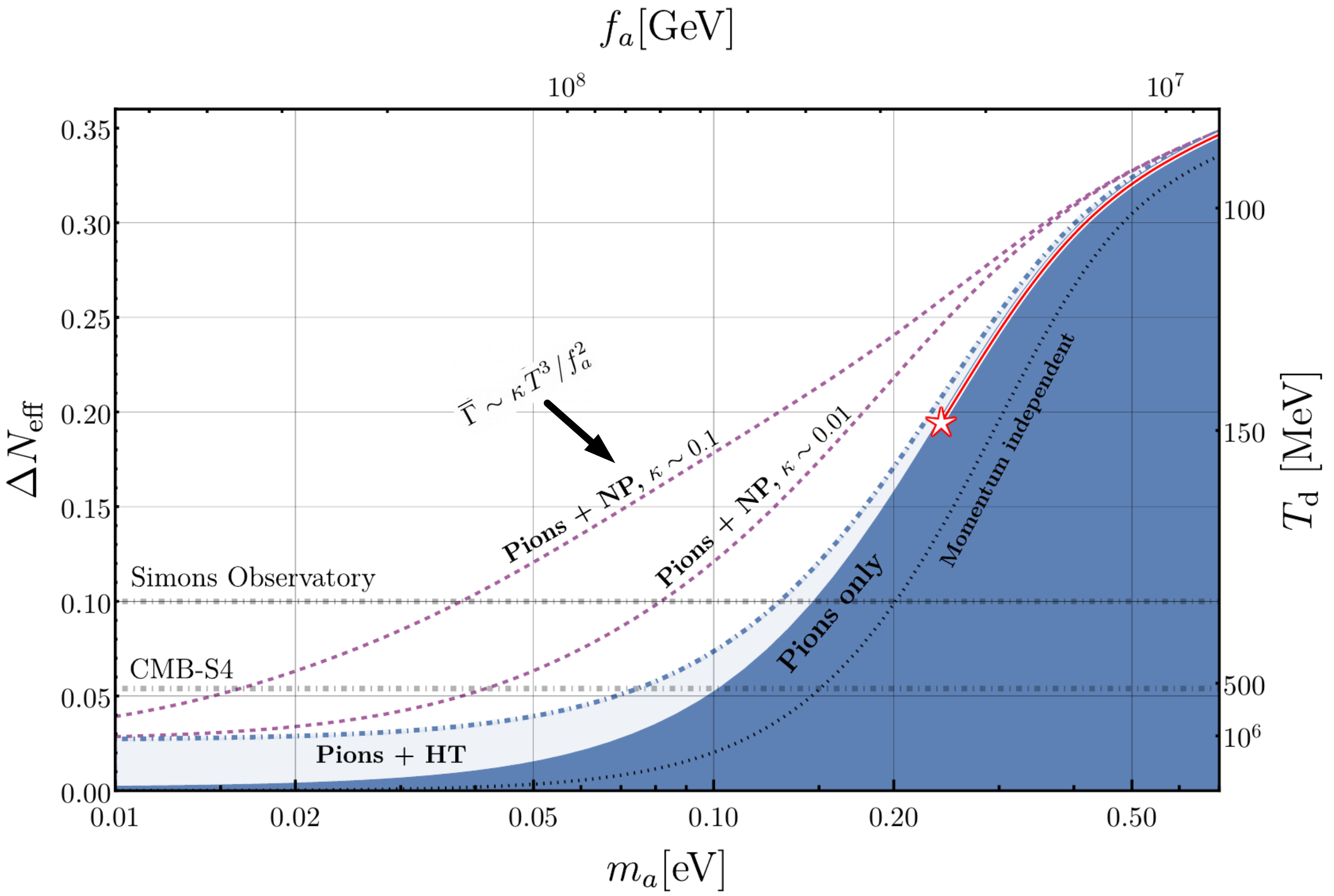
[non-linear YM equations - e.g. strong sphalerons]



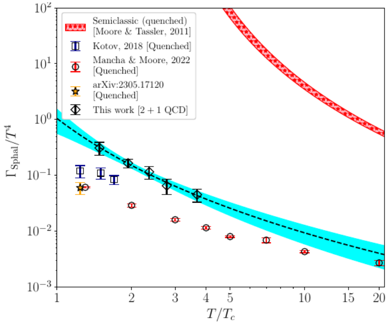
$\Gamma_{\text{sphal}} \simeq (N_c \alpha_s)^5 T^4$

Moore, Tassler '10

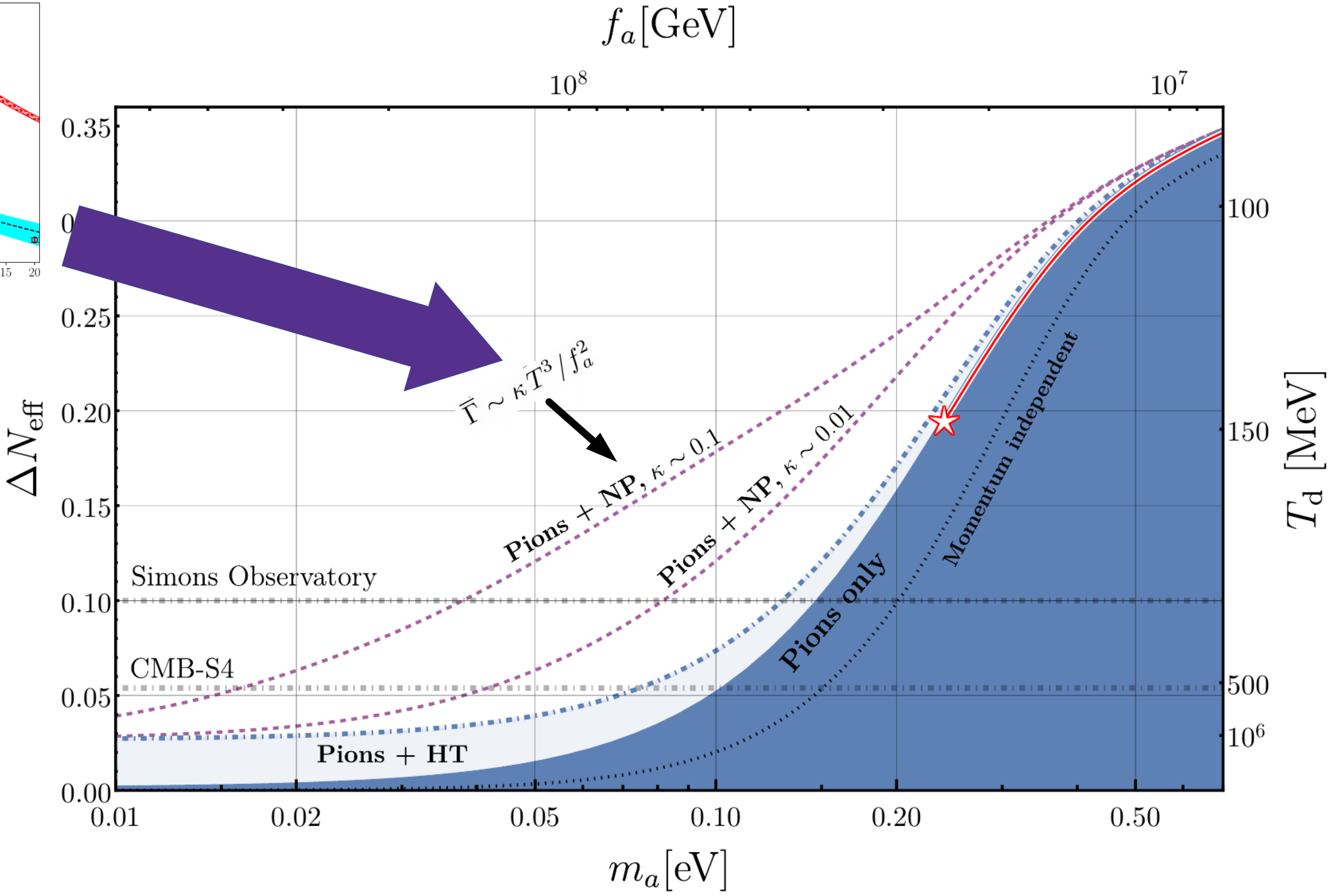
Future Reach



Future Reach



Bonanno et al.
2308.01287



Conclusions:

- More reliable upper bound on m_a (< 0.24 eV) from cosmology (for minimal KSVZ-like axions)
- Importance of momentum dependence on Boltzmann equation @ around QCD scale
- Doubts about reliability of perturbative rates above T_c
- Non-perturbative rates crucial for interpreting upcoming CMB experiments

  Promising preliminary results from Lattice QCD...

Thanks!

Back Up

1. The Thermalization Rate Γ

General form of low energy axion QCD Lagrangian:

$$\mathcal{L} = \bar{q}\left(i\partial\!\!\!/ + \frac{c_0}{2f_a}\partial\!\!\!/ a\,\gamma_5\right)q - \bar{q}_L M_a q_R + h.c.\,, \qquad M_a \equiv \begin{pmatrix} m_u & 0 \\ 0 & m_d \end{pmatrix} e^{i\frac{a}{2f_a}(1+c_3\sigma^3)}$$

$$\frac{\partial_\mu a}{2f_a} j_A^\mu \overset{\chi\mathrm{PT}}{=} \mathcal{O}(M_q)$$

$$\pi^0 = \cos(\theta_{a\pi})\,\pi^0_{\rm phys} + \sin(\theta_{a\pi})\,a_{\rm phys} \simeq \pi^0_{\rm phys} + \theta_{a\pi}a_{\rm phys}$$

<u>@ all orders in χPT</u>

$$\mathcal{M}_{a\pi^i\to\pi^j\pi^k} = \theta_{a\pi}\cdot\mathcal{M}_{\pi^0\pi^i\to\pi^j\pi^k} + \mathcal{O}\left(\frac{m_\pi^2}{s}\right)$$

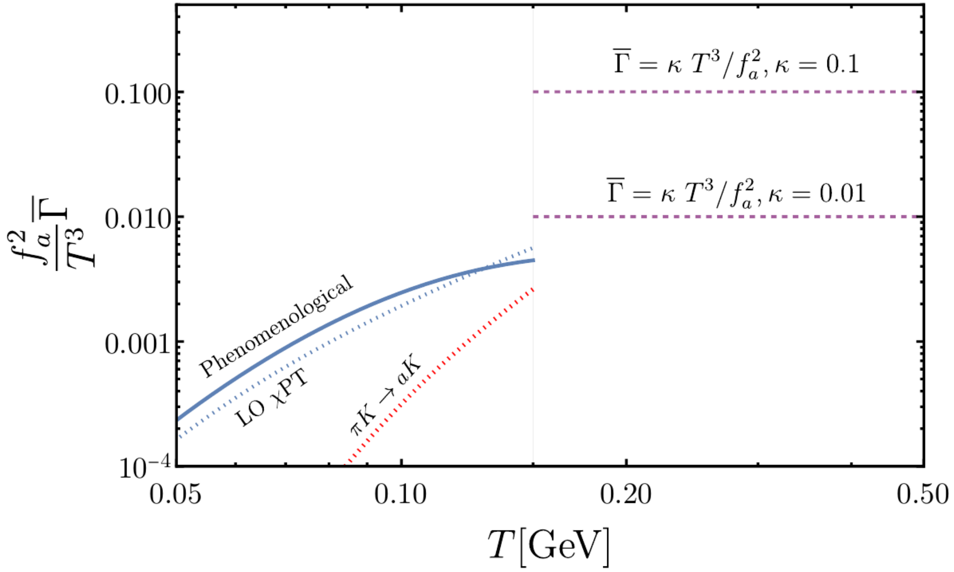
$\lesssim 10\%$

e.g. @ LO

$$|\mathcal{M}^{\rm LO}|^2 = \theta_{a\pi}^2\frac{s^2+t^2+u^2-3m_\pi^4}{f_\pi^4} \qquad |\mathcal{M}^{\rm LO}_{\pi\pi}|^2 = \frac{s^2+t^2+u^2-4m_\pi^4}{f_\pi^4}$$

Strong Sphaleron-like contribution to Axion rate

$$\overline{\Gamma}_{\rm sphal} = \frac{1}{n^{\rm eq}} \int^{|\mathbf{k}|<|\mathbf{k}_s|} \frac{d^3\mathbf{k}}{(2\pi)^3 2E} \frac{\Gamma_{\rm sphal}}{f_a^2} e^{-E/T} = \frac{(N_c\alpha_s)^5 T^3}{4\zeta_3 f_a^2} \left(1 - \left(1 + \frac{|\mathbf{k}_s|}{T}\right) e^{-|\mathbf{k}_s|/T}\right)$$



$$\Gamma^{>}_{\rm top}(E = |\mathbf{k}| < |\mathbf{k}_s|) \simeq \Gamma_{\rm sphal} \simeq (N_c\alpha_s)^5 T^4$$

$$|\mathbf{k}_s| \sim N_c\alpha_s T$$

The Thermal Width:

Challenge for Lattice QCD: <u>Compute Γ_k for $T > T_c$</u>

Existing Attempts (at k=0) e.g.

- Moore, Tassler '10 : Classical SU(N) simulations
- Kotov '18 : Quantum Euclidean (anal. cont.)
- Altenkort et al. '20 : Quantum Euclidean (anal. cont.)
- Mancha, Moore '22 : Quantum Euclidean (plus modeling)

$$\Gamma_{\rm sphal} = 2T \lim_{\omega\to 0} \frac{\rho(\omega)}{\omega}$$

$$G(\tau) = \int d^3x \, \langle q(\vec{0},0) q(\vec{x},\tau)\rangle = -\int_0^\infty \frac{d\omega}{\pi}\, \rho(\omega) \frac{\cosh[\omega(1/2T-\tau)]}{\sinh(\omega/2T)}$$

Important to exploit upcoming experiments!