Mini-workshop HKUST, Jan 16 – 17, 2024

Topological production of

Axion dark matter from cosmic string network: revisited

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Independent check from scratch by two students at KAIST,

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arXiv:2401.xxxx

Supported by KISTI super-computing center

Peccei-Quinn U(1)_PQ Phase Transition

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi - \frac{m_r^2}{2 f_a^2}\left(|\phi|^2 - \frac{f_a^2}{2}\right)^2 \qquad \phi = |\phi| e^{i\frac{a}{f_a}}$$

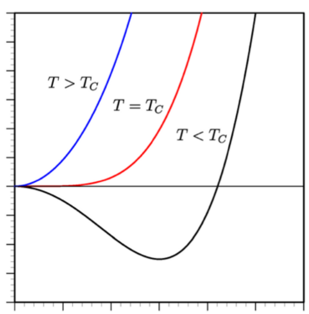
Post-inflationary scenario

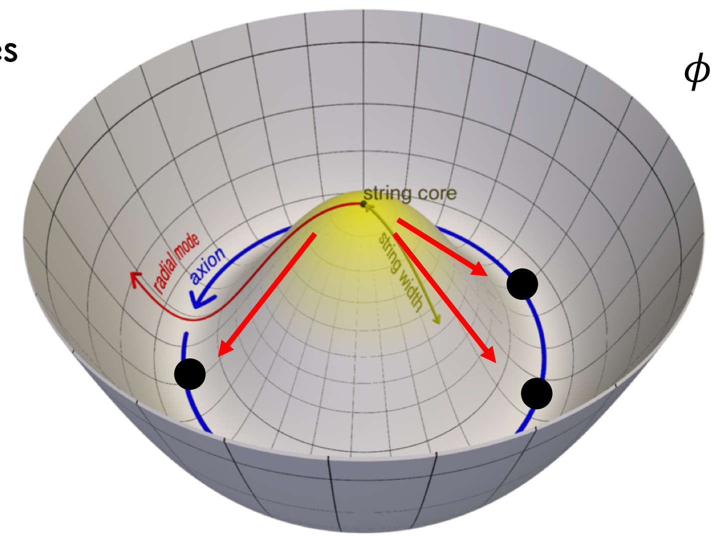
$H, T \gtrsim f_a$ Uncorrelated initial axion field on scales larger than the horizon, with neighboring Hubble patches coming into causal contact in subsequent evolution of the Univ.

During the phase transition phases are randomly distributed

$$\phi = \frac{r(x) + f_a}{\sqrt{2}} e^{i\frac{a}{f_a}}$$

As Universe cools down, it goes through the phase transition





Taken from B. Safdi

On the FRW metric

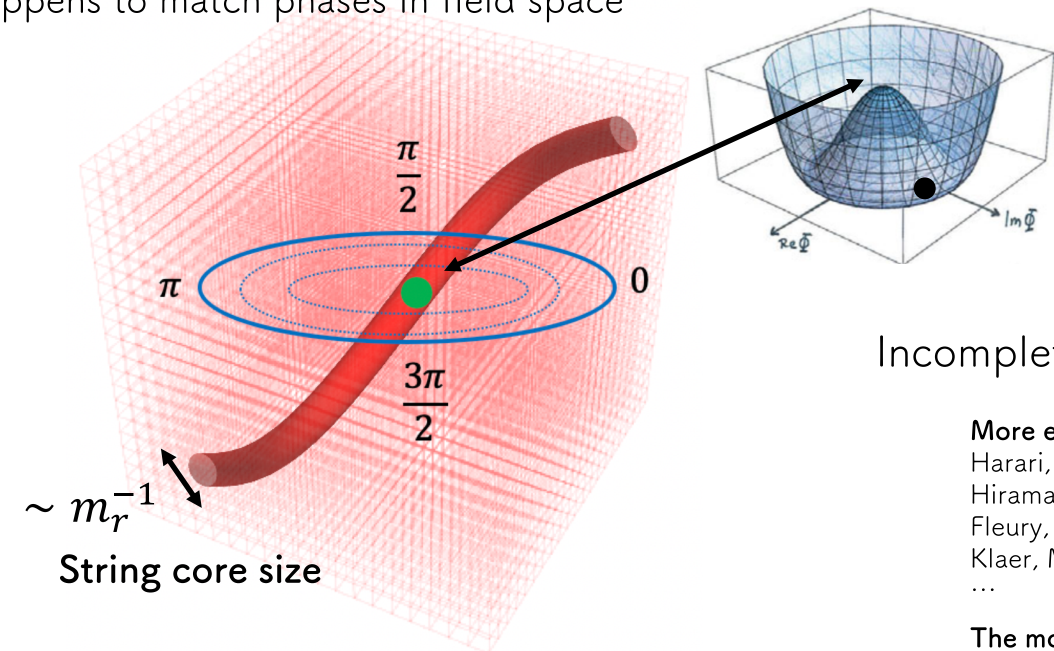
$$ds^2 = dt^2 - R^2(t) d\vec{x}^2$$

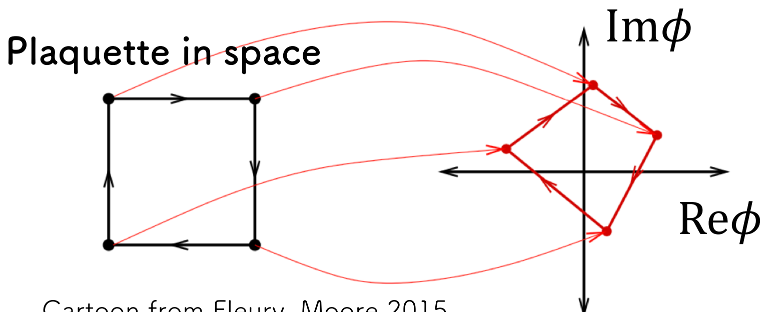
$$\ddot{\phi} + 3H\dot{\phi} - \frac{1}{R^2}\nabla^2\phi + \frac{m_r^2}{f_a^2}\left(|\phi|^2 - \frac{f_a^2}{2}\right)\phi = 0$$

has a solitonic string-like solution

Formation of topological cosmic string

Sometimes phases in real space
happens to match phases in field space



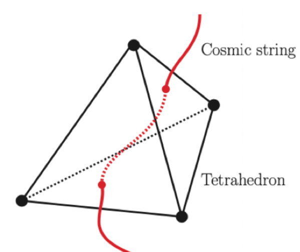


Cartoon from Fleury, Moore 2015

Incomplete list of references

More earlier literature missed here
Harari, Sikivie 1987 'Hagmann, Chang, Sikivie 1999
Hiramatsu, Kawasaki, Sekiguchi, Yamaguchi, Yokoyama 2011
Fleury, Moore 2015
Klaer, Moore 2017
…

The most recent update on global string …
Kawasaki, Sekiguchi, Yamaguchi, Yokoyama, 2018
Vaquero, Redondo, Stadler 18'
Gorghetto, Hardy, Villadoro 2018, 2020
Buschmann, Foster, Safdi 2019
Buschmann, Foster, Hook, Peterson, Willcox, Zhang, Safdi 2021
Kim, Park, SON in progress



New tetrahedron-based string identification

: ensures the connectedness of strings.
Add no extra CPU time

Kim, Park, **SON** 2401.xxx

Topological production of axion dark matter

$$\rho_{\rm tot} = \left\langle \left|\dot\phi\right|^2 + |\nabla\phi|^2 + V(\phi) \right\rangle$$

3D volume (spatial) average

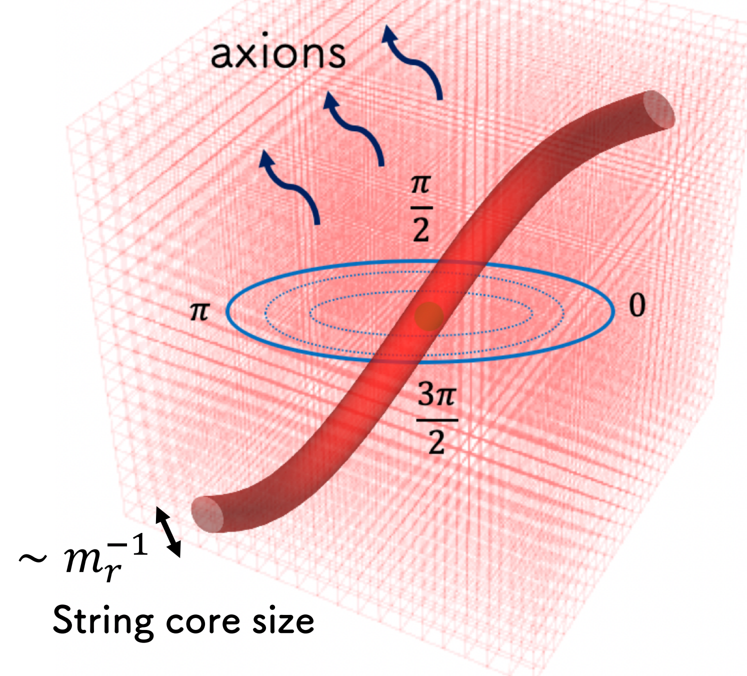
$$\phi = \frac{r(x) + f_a}{\sqrt{2}} e^{i\frac{a}{f_a}}$$

$$= \left\langle \frac{1}{2}\dot a^2 + \frac{1}{2}|\nabla a|^2 \right\rangle + \left\langle \frac{1}{2}\dot r^2 + \frac{1}{2}|\nabla r|^2 + V(r) \right\rangle$$

2× : stored in axions : stored in radial modes

$$+ \left\langle \left(\frac{r^2}{2f_a^2} + \frac{r}{f_a} \right) (\dot a^2 + |\nabla a|^2) \right\rangle$$

Axion radiation from strings



String energy density

$$\rho_s = \rho_{\rm tot} - \rho_a - \rho_r$$

Axions produced from strings at the time of QCD crossover is what we need

Non-linear evolution between two vastly separated two scales

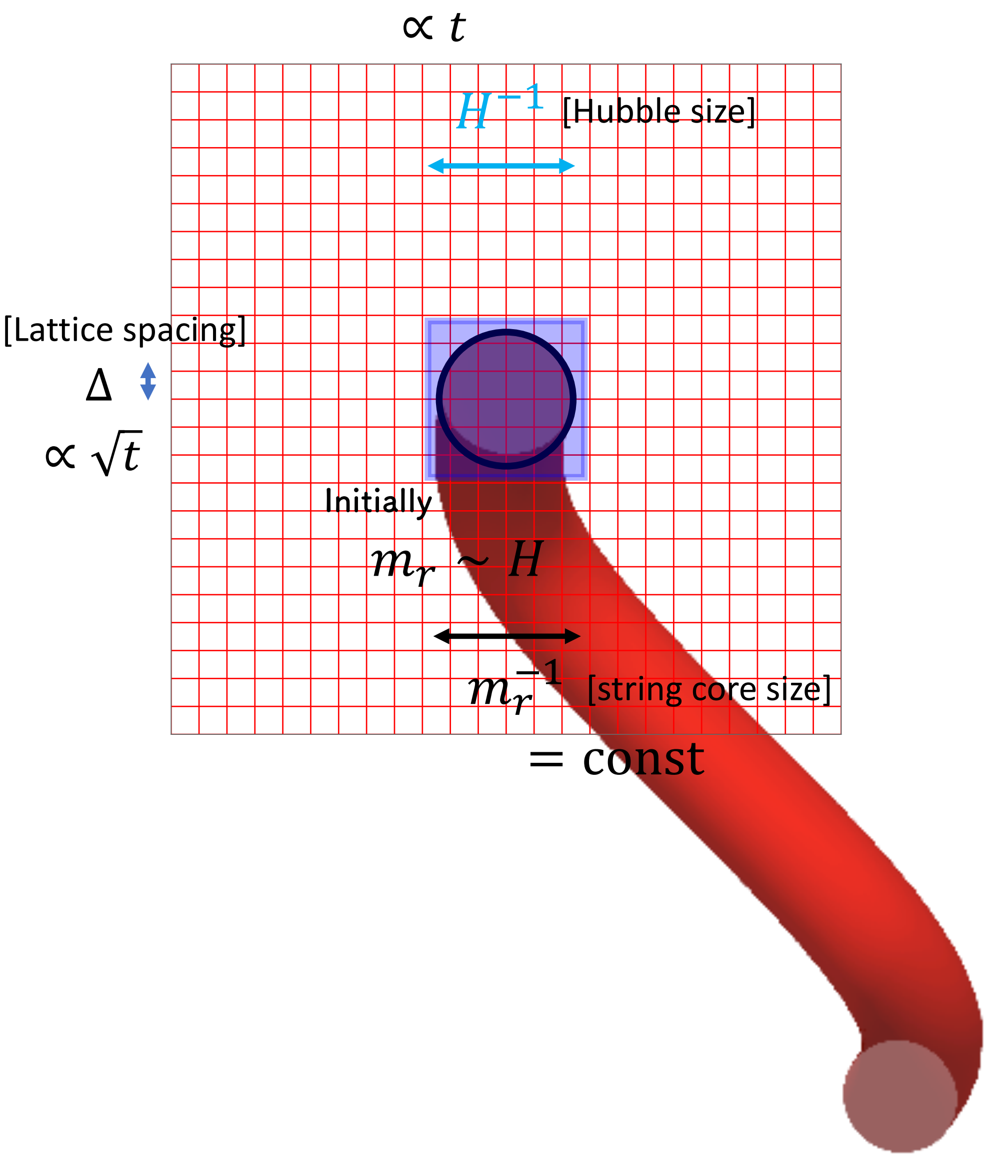
$$[m_r\,, H(t_{QCD})]$$

No EFT approach.
Only numerical lattice simulation

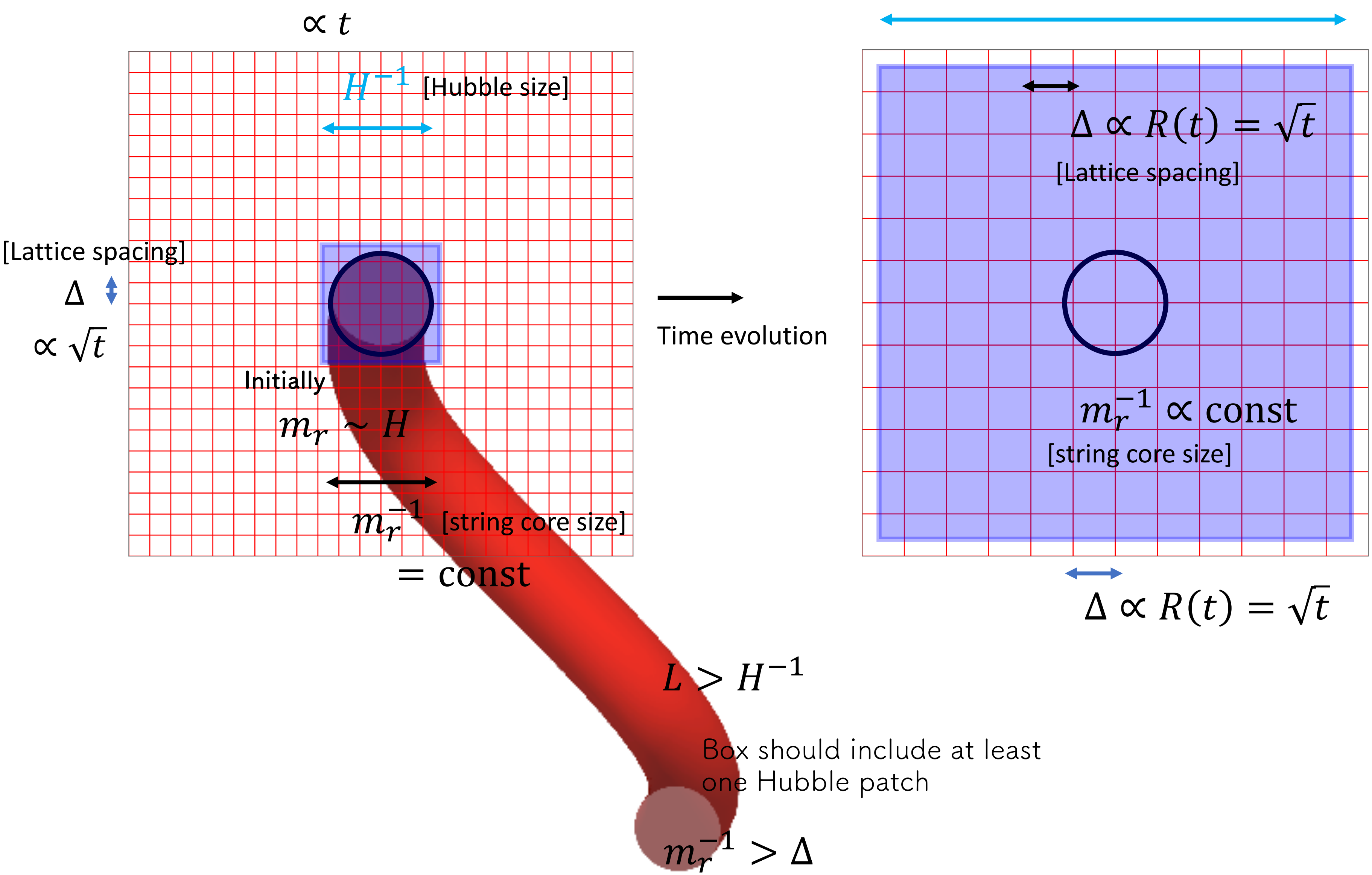
In Logarithmic time evolution in terms of Hubble e-folding

$$\log\frac{m_r}{H} = \log\frac{t}{t_0} \sim [1, 70]$$

Time coverage in expanding Universe



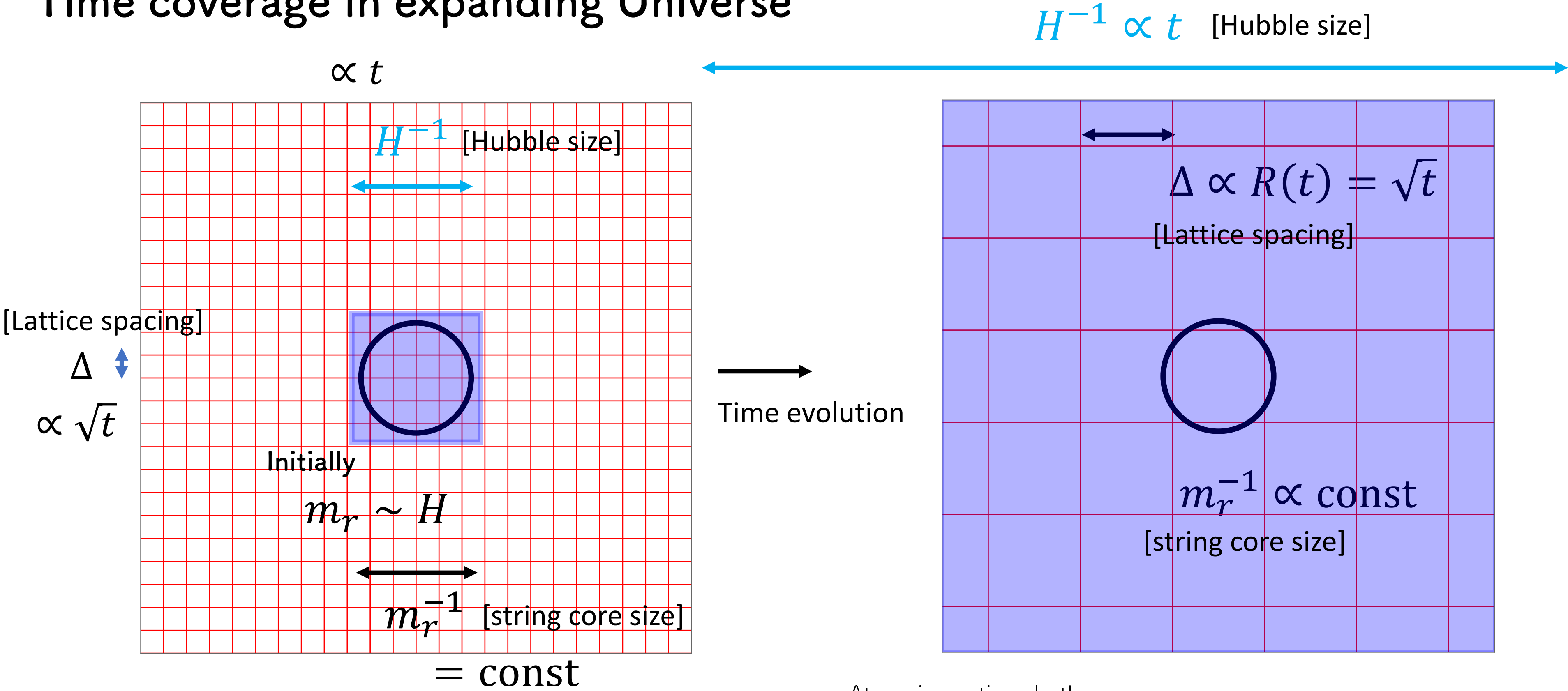
Time coverage in expanding Universe



Box should include at least one Hubble patch

Core should include at least one lattice spacing

Time coverage in expanding Universe



At maximum time, both criteria is violated

$$L > H^{-1}$$

Box should include at least one Hubble patch

$$m_r^{-1} > \Delta$$

Core should include at least one lattice spacing

$$\log \frac{m_r}{H} = \log \frac{t}{t_0} \lesssim \log \frac{L}{\Delta} = \log N \;\; \rightarrow \log \frac{N}{n_c n_H}$$

: maximum time evolution in terms of e-folding is limited by the lattice size N

$$\Delta x = \left. \frac{m_r^{-1}}{n_c R(t)} \right|_{t=(1/2H)_{\max}} = \frac{m_r^{-1}}{n_c} \left(\frac{2 n_c n_H}{N} \right)^{\frac{1}{2}}$$

Time coverage in expanding Universe

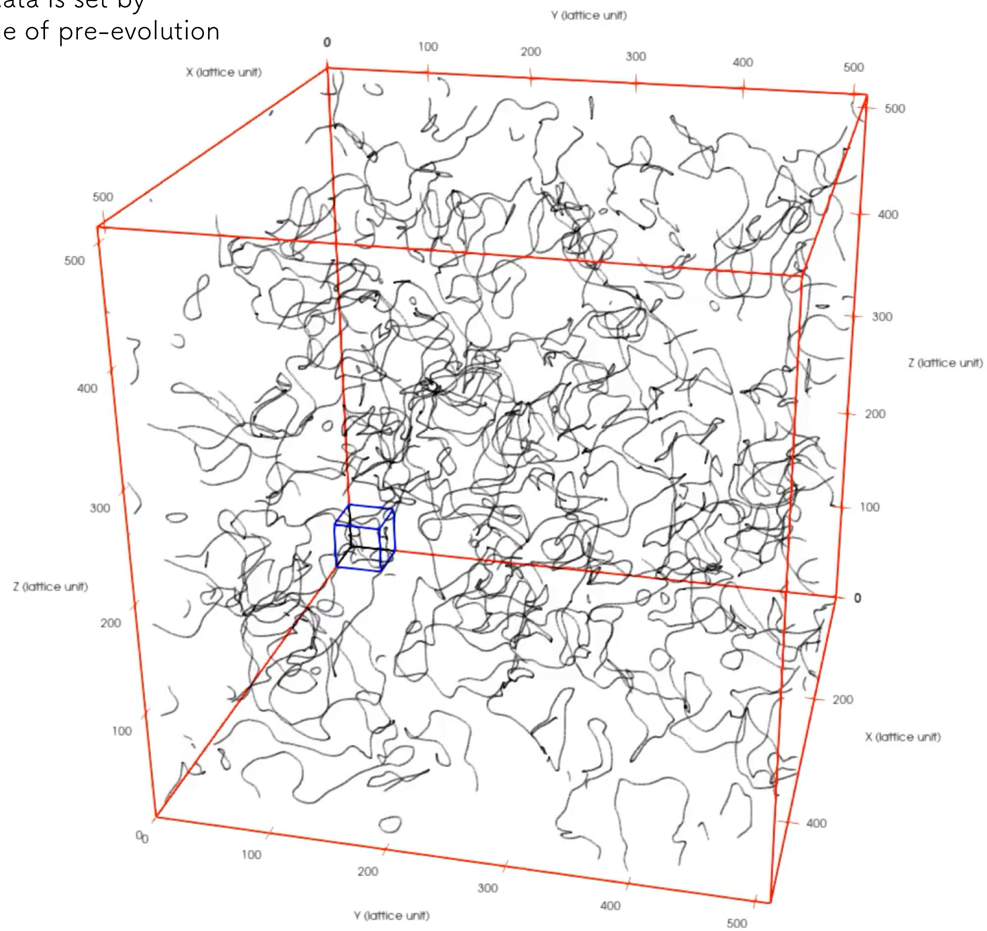
$N^3 = 512^3$

In comoving simulation box with constant lattice spacing

Δ = const.= L/512

log(m_r/H) = 2.000

Initial data is set by
outcome of pre-evolution



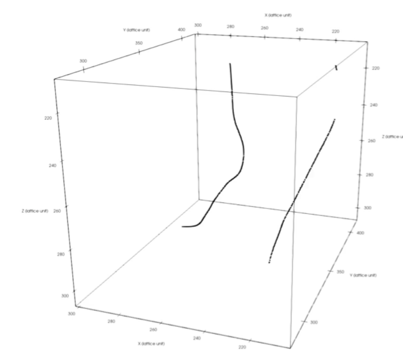
Hubble size

$$\frac{H^{-1}}{\Delta} \propto t^{1/2}$$

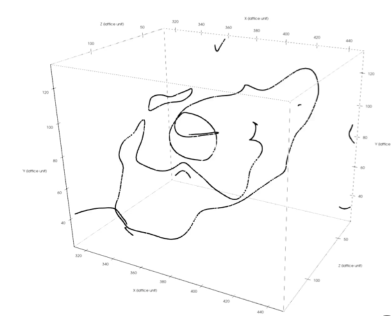
String core size

$$\frac{m_r^{-1}}{\Delta} \propto t^{-1/2}$$

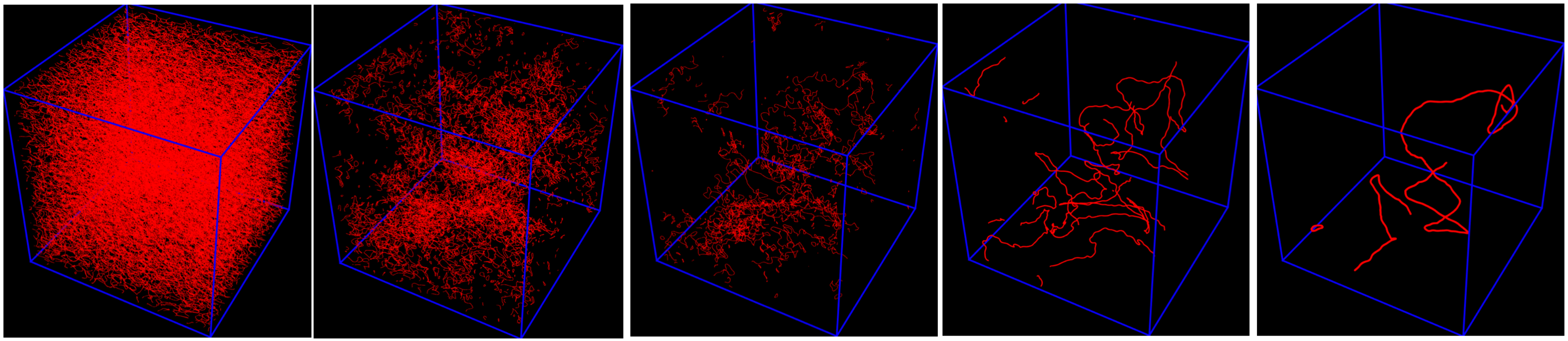
Inter-commutation



Decay of closed string



Toy simulation by **SON** et al

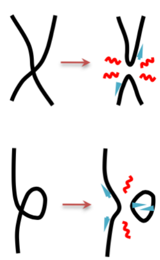


Competition :
1. more strings enter in Hubble box due to expansion
2. strings recombine and/or decay

Reaches attractor solution when above two balance

$\log \frac{m_r}{H} \sim [1, 8\sim9]$

$\boldsymbol{H, T \gtrsim f_a}$



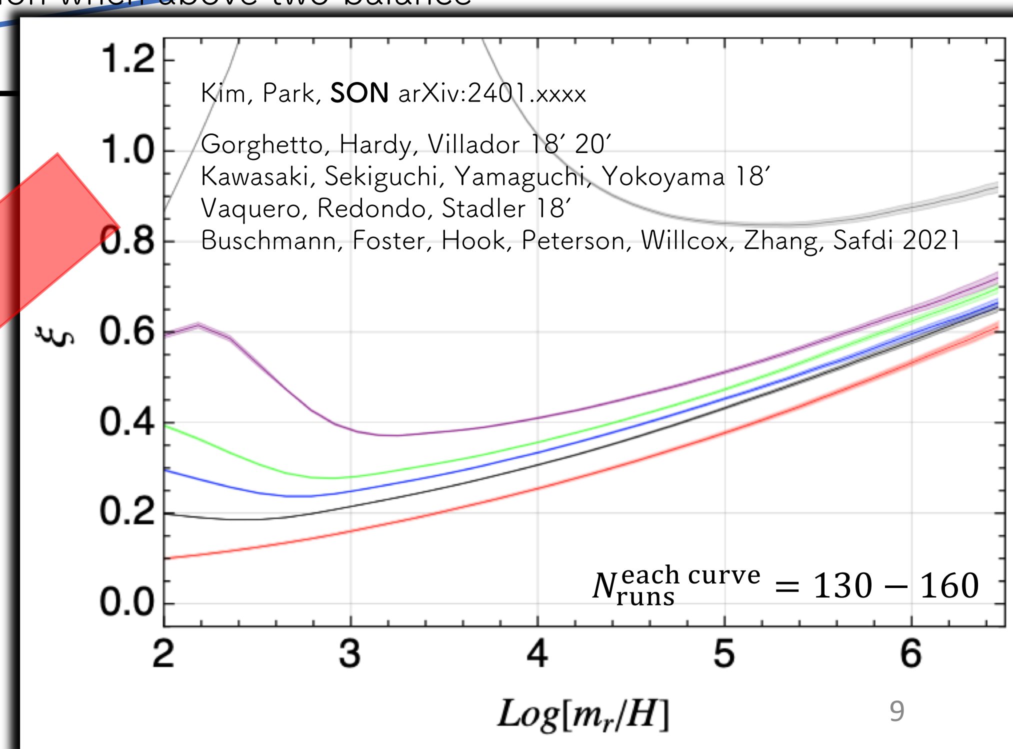
$\ell_{\text{tot}}(L)$ = total length stored inside box of L

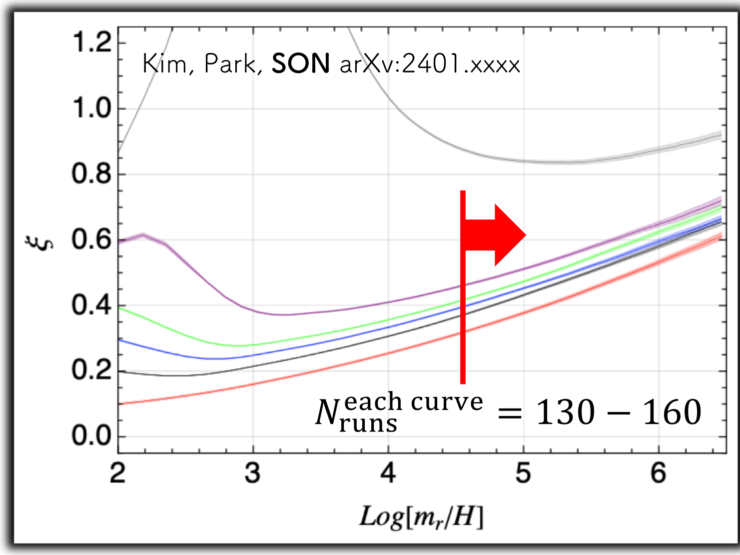
$$\xi \propto \frac{\ell_{\text{tot}}(L)}{\left(L^3/(H^{-1})^3\right)} \frac{1}{H^{-1}} = \frac{\ell_{\text{tot}}(L) t^2}{L^3}$$

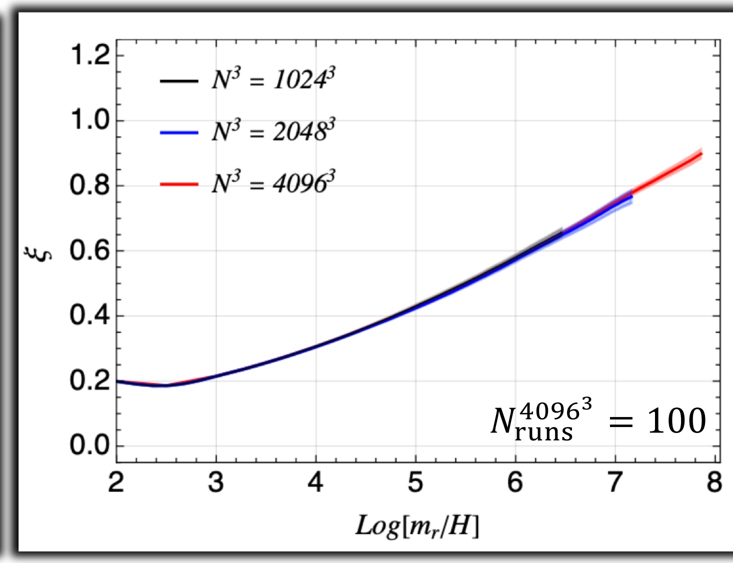
: total length per Hubble volume

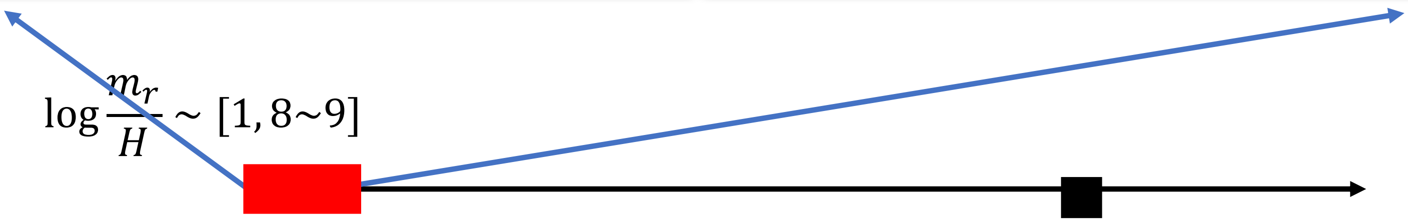
: number of strings per Hubble patch

#1









$\boldsymbol{H, T \gtrsim f_a}$

$\boldsymbol{H \sim m_a}$
$\boldsymbol{(T \sim \Lambda_{QCD})}$

$\ell_{\text{tot}}(L)$ = total length stored inside box of L

$$= \beta + \alpha \log\frac{m_r}{H}$$

$$\xi \propto \frac{\ell_{\text{tot}}(L)}{\left(L^3/(H^{-1})^3\right)}\frac{1}{H^{-1}} = \frac{\ell_{\text{tot}}(L)t^2}{L^3}$$

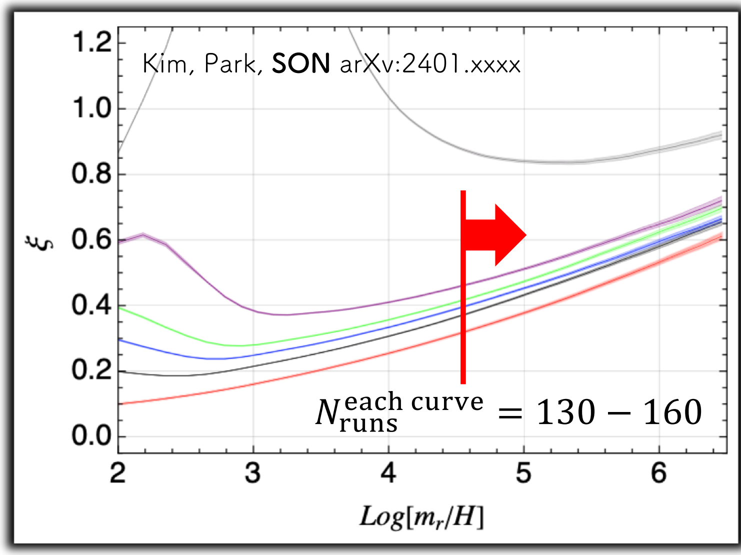
: total length per Hubble volume

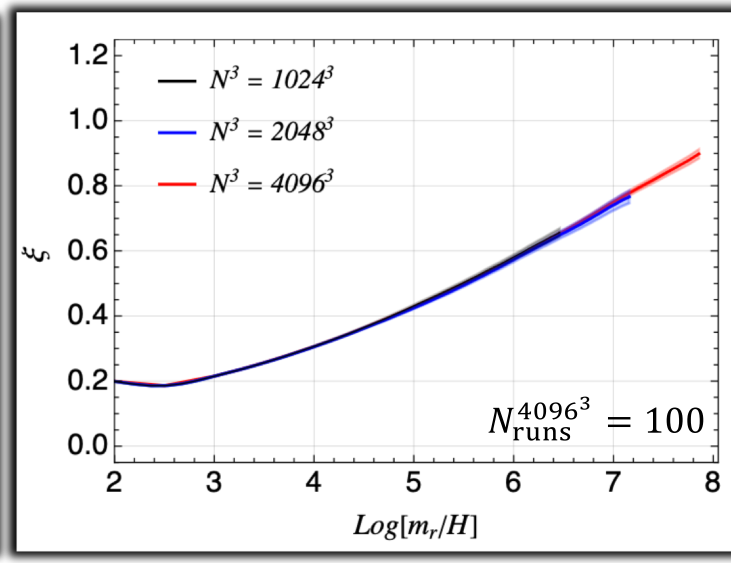
: number of strings per Hubble patch

With global Log-Hypothesis

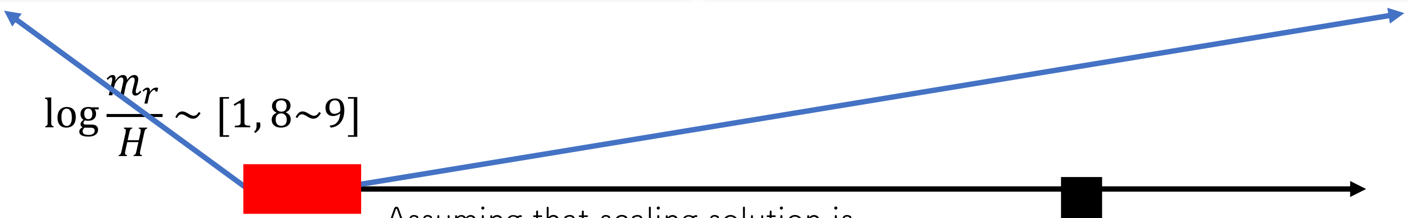
$$\xi_i = \cdots + \frac{d_{2i}}{\log^2\frac{m_r}{H}} + \frac{d_{1i}}{\log\frac{m_r}{H}} + \beta + \alpha\log\frac{m_r}{H}$$

Pre-evolution type	Fit with global log-scaling hypothesis	Interval for fit
Fat-string	$\xi \sim -0.81 + 0.21\log\frac{m_r}{H}$	$\log\frac{m_r}{H} = [5.0, -]$
Thermal	$\xi \sim -1.15 + 0.26\log\frac{m_r}{H}$	$\log\frac{m_r}{H} = [5.0, -]$





$\log \frac{m_r}{H} \sim [1, 8\sim9]$



$\boldsymbol{H, T \gtrsim f_a}$

Assuming that scaling solution is maintained at later times, extrapolate average number of strings per Hubble patch up to the relevant separation

$\boldsymbol{H \sim m_a}$
$\boldsymbol{(T \sim \Lambda_{QCD})}$

$\ell_{tot}(L)$ = total length stored inside box of L

$$= \beta + \alpha \log \frac{m_r}{H}$$

$$\xi \to \mathcal{O}(10) \ @ \log \frac{m_r}{H} = 70$$

$$\xi \propto \frac{\ell_{tot}(L)}{\left(L^3/(H^{-1})^3\right)} \frac{1}{H^{-1}} = \frac{\ell_{tot}(L) t^2}{L^3}$$

: total length per Hubble volume

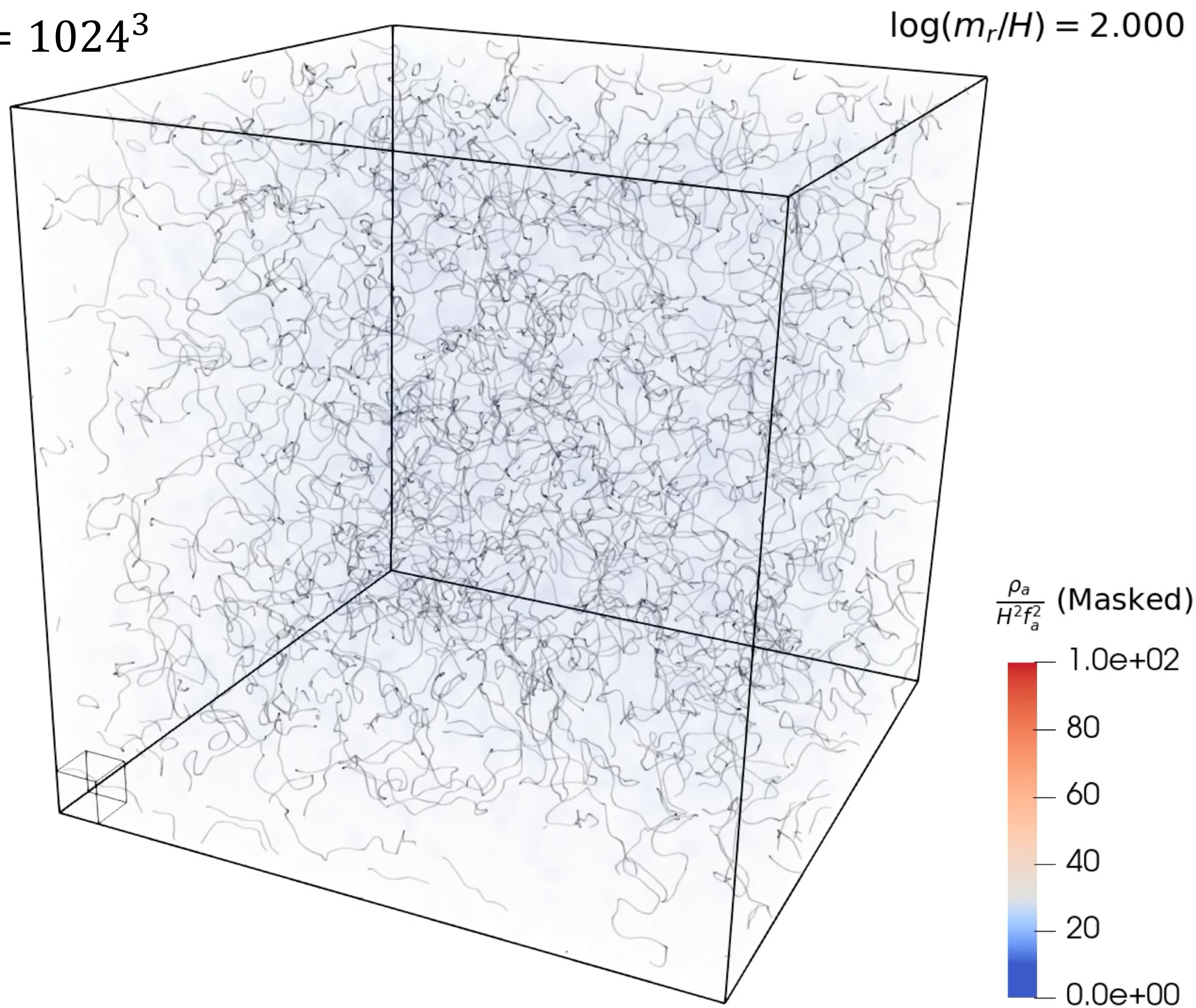
: number of strings per Hubble patch

$$\frac{\ell_{tot}(L)}{L^3} = \frac{1}{d_{str}^2} \to d_{str} = \frac{t}{\sqrt{\xi}} = \frac{1}{2H\sqrt{\xi}}$$

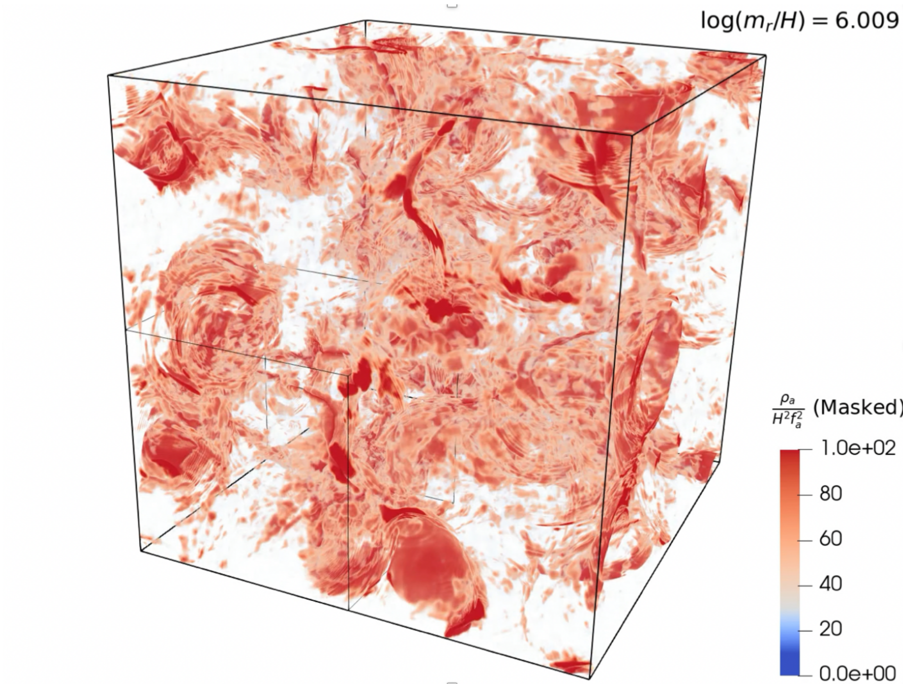
: inter-string distance

Axion spectrum, abundance

$N^3 = 1024^3$



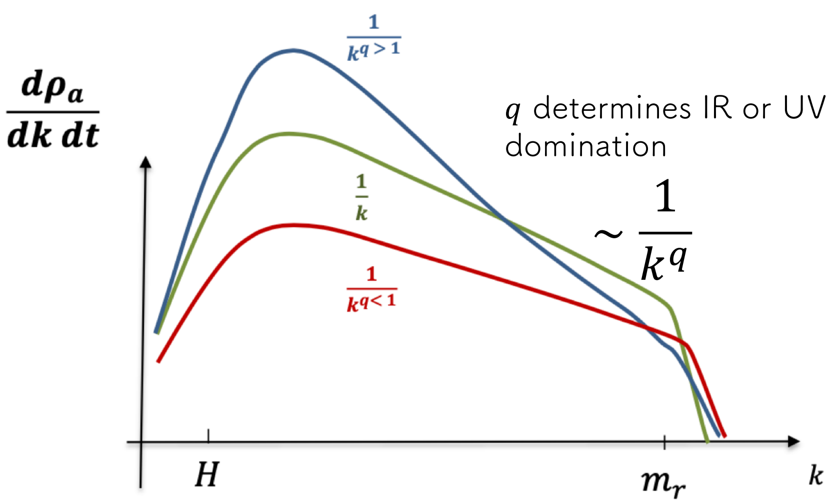
Axion spectrum, abundance



$$n_a = \int \frac{dk}{k} \frac{\partial \rho_a}{\partial k}$$

: We want to evaluate axion number density as a function of time

: should follow a **power law** due to absence of other non-trivial scales and sampling has to be done in scaling regime



From Gorghetto, Hardy, Villador 18'

$q > 1$ — **Sizeable**

Most energy emitted order of $O(H)$. n_a comes from ρ_s divided by typical momentum of order of $O(H)$

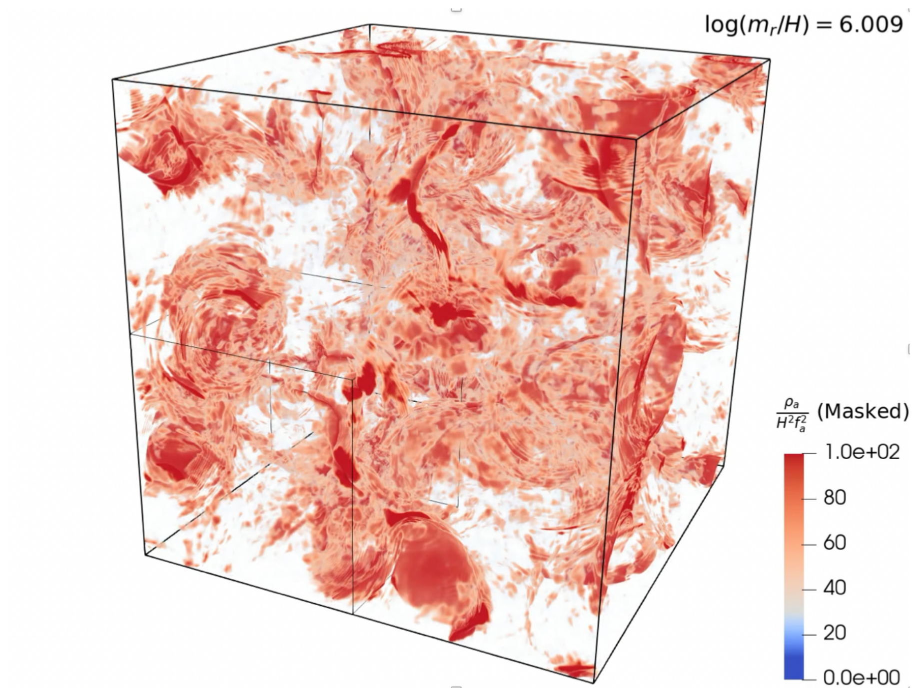
$q = 1$ — **Moderate**

Energy is equally distributed in logarithmic intervals of k. n_a is smaller by a factor of log compared to the above

$q < 1$ — **Suppressed**

UV dominated spectrum. n_a is power-suppressed

Axion spectrum, abundance



$$n_a = \int \frac{dk}{k} \frac{\partial \rho_a}{\partial k}$$

: We want to evaluate axion number density as a function of time

: should follow a **power law** due to absence of other non-trivial scales and sampling has to be done in scaling regime

or normalized transfer rate $\Gamma(t)$

: Instantaneous emission function

$$\frac{\partial \rho_a}{\partial k} = \int^t dt' \frac{\Gamma[t']}{H(t')} \left(\frac{R(t')}{R(t)}\right)^3 F\left[\frac{k'}{H(t')}, \frac{m_r}{H(t')}\right] \rightarrow F \sim \frac{1}{k^q}$$

: from fitting the data

$$\Gamma[t] \sim \frac{\xi \mu_{\mathrm{eff}}}{t^3} \sim 8\pi H^3 f_a^2 \, \xi \log \frac{m_r}{H}$$

Assuming that strings dominantly decay into axions at late times

at later times

$$\frac{1}{R^4(t)} \frac{\partial}{\partial t} (R^4(t) \rho_a(t)) = \Gamma_a + \cdots \sim \Gamma(t)$$

: transfer rate from strings to axions $\sim \int dk \frac{\partial \Gamma(t)}{\partial k}$

: **Instantaneous emission function**

$$\frac{\partial \rho_a}{\partial k} = \int^t dt' \frac{\Gamma[t']}{H(t')} \left(\frac{R(t')}{R(t)}\right)^3 F\left[\frac{k'}{H(t')}, \frac{m_r}{H(t')}\right]$$

$$F \sim \frac{1}{k^q}$$: from fitting the data

$$\Gamma[t] \sim \frac{\xi \mu_{\rm eff}}{t^3} \sim 8\pi H^3 f_a^2\, \xi \log\frac{m_r}{H}$$

Assuming that strings dominantly
decay into axions at late times

Assuming $q > 1$

$$\frac{\partial \rho_a}{\partial k} \sim \frac{1}{k} \qquad k \sim [H, \sqrt{m_r H}]$$

$$\frac{\partial \rho_a}{\partial k} \sim \frac{1}{k^q} \qquad k \sim [\sqrt{m_r H}, m_r]$$

A large range of possible number density

#2

$$\left.\frac{n_a^{q>1}}{n_a^{\rm mis}}\right|_{t_\ell} \propto \left(\xi_\star \log\frac{m_r}{H_\star}\right)^{\frac{1}{2}+\cdots}, \qquad \cdots$$

Gorghetto, Hardy, Villadoro 20'

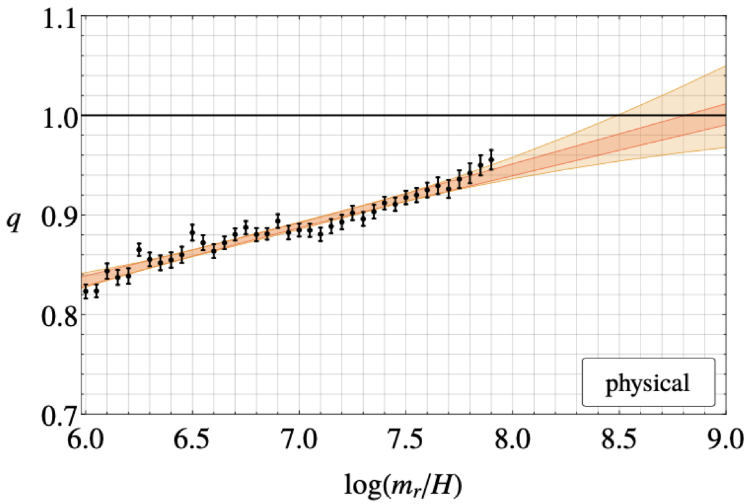
QCD axion relic abundance
from misalignment :

$$\Omega_a^{\rm mis} h^2 \sim 0.12 \left(\frac{f_a}{10^{12}\ {\rm GeV}}\right)^{7/6} \langle \theta_{a,i}^2 \rangle$$

$$\mathrm{m}_a \sim 10^{-5}\ \mathrm{eV}$$

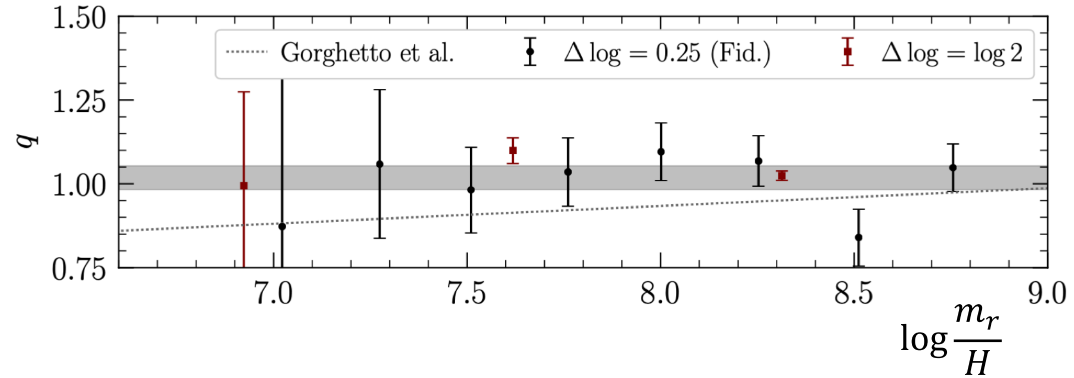
Logarithmic gross of q & IR dominance at late times vs No-log, which one?

Gorghetto, Hardy, Villadoro 2020, 4500^3



$$15H < k < {m_r}/_{4}$$

Buschmann, Foster, Hook, Peterson, Willcox, Zhang, Safdi 2021, $2048^3 + 2^4$ AMR (Adaptive Mesh Refinement)



$$50H < k < {m_r}/_{16}$$

Log hypothesis

$$q \sim q_0 + (0.053 \pm 0.005) \log \frac{m_r}{H}$$

: strong hint for IR domination at later times

VS

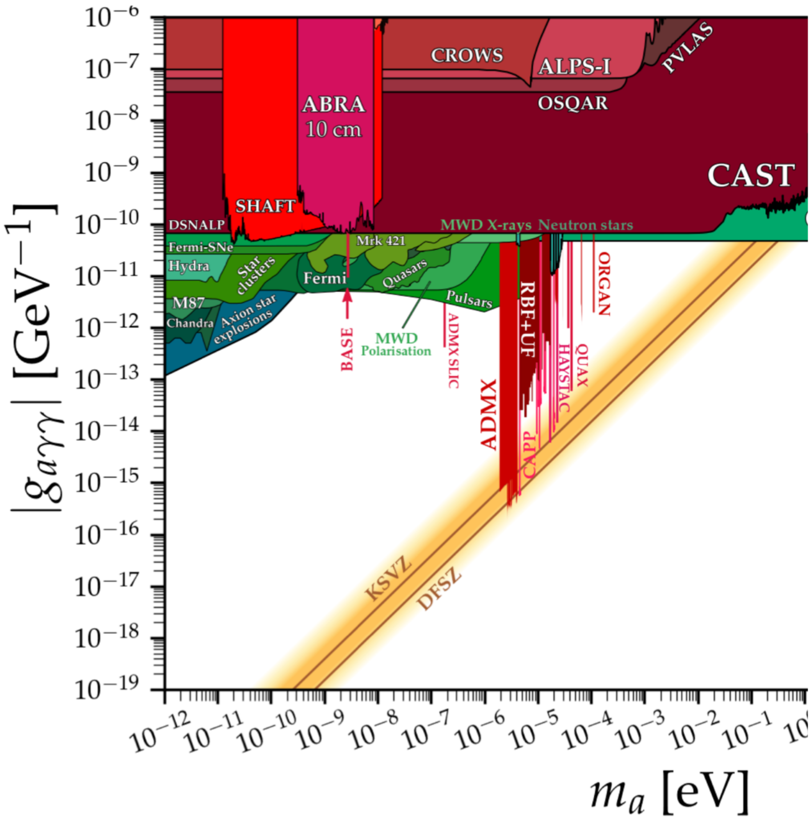
Log hypothesis

$$q \sim 1.36 \pm 0.69 + (-0.04 \pm 0.08) \log \frac{m_r}{H}$$

No Log hypothesis

$$q \sim 1.02 \pm 0.04$$

Implication on the QCD axion search



Abundance should not exceed the current observed dark matter value leads to

Buschmann, Foster, Hook, Peterson, Willcox, Zhang, Safdi 2021

$$m_a \sim [0.4 - 1.8] \times 10^{-4} \text{ eV}$$
$$(= 40 - 180\ \mu\text{eV})$$

Gorghetto, Hardy, Villadoro 2020

$$m_a \gtrsim 5 \times 10^{-4} \text{ eV}, \quad f_a \lesssim 10^{10} \text{ GeV}$$
$$(= 500\ \mu\text{eV})$$

$$\xi_* = 15, x_{IR} = 10, q > 2, \log_* = 64, N = 1$$

benchmark (at QCD PT)

our independent check

arXiv:2401.xxxx

Actual beginning of simulation

Beginning of physical string simulation

Generation of Random field configuration

In practice, we need pre-step. Justification is our belief that the scaling solution is insensitive to all details in early times

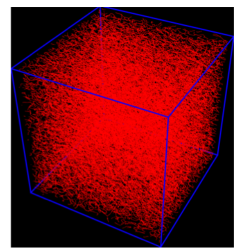
Pre-evolution for relaxation

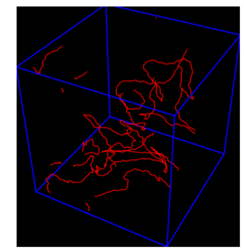
Role of pre-evolution is to relax noisy string network to a relatively clean level. It stops when this requirement is met

End result of pre-evolution becomes the initial data of the physical string evolution

Lots of high-freq. modes are relaxed during this step

Beginning of physical string evolution



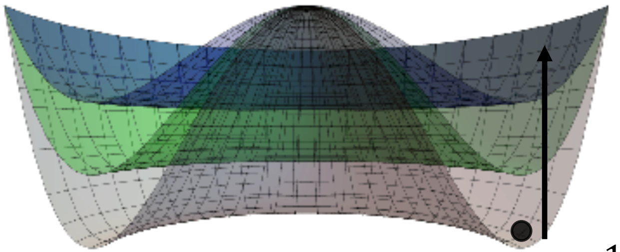


Dynamic time range of physical string evolution

$$\log \frac{m_r}{H} \sim [2, 8\sim 9]$$

1. Fat-string pre-evolution

Normal distribution as if it is symmetric

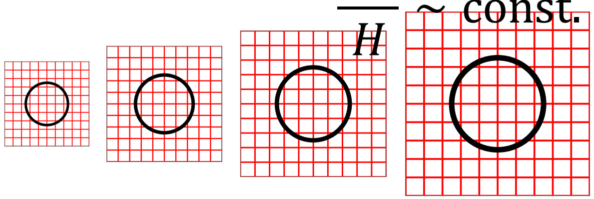


Gorghetto, Hardy, Villadoro 2020

$m_r^2 \propto \lambda f_a^2 \,, \;\; \lambda \propto \frac{1}{R^2}$

${m_r}^{-1} \propto R(t) \propto t$

$\frac{m_r}{H} \sim \text{const.}$



ξ_0

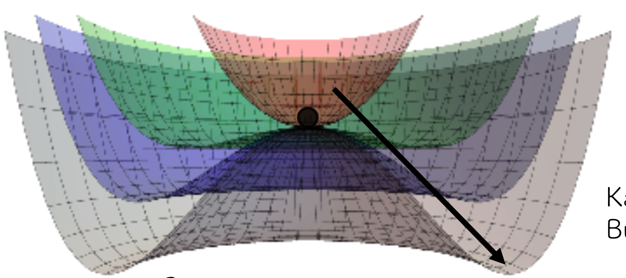
Generation of Random field configuration

$\tau_i = 0.1\,\tau_c$

ζ

2. Thermal pre-evolution

Gaussian random field config at finite temp.

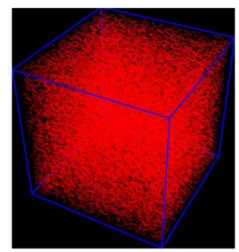


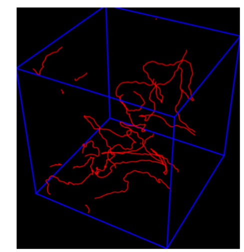
Kawasaki, Sekiguchi, Yamaguchi, Yokoyama 2018
Buschmann, Foster, Hook, Peterson, Willcox, Zhang, Safdi 2021

$$V = \lambda\left(|\phi|^2 - \frac{f_a^2}{2}\right)^2 + \frac{\lambda}{3}T^2|\phi|^2 \quad T^2 = 2\zeta H^2\,, \quad \left.\frac{m_r}{H}\right|_{T_c} = \frac{2}{3}\zeta$$

Beginning of physical string evolution

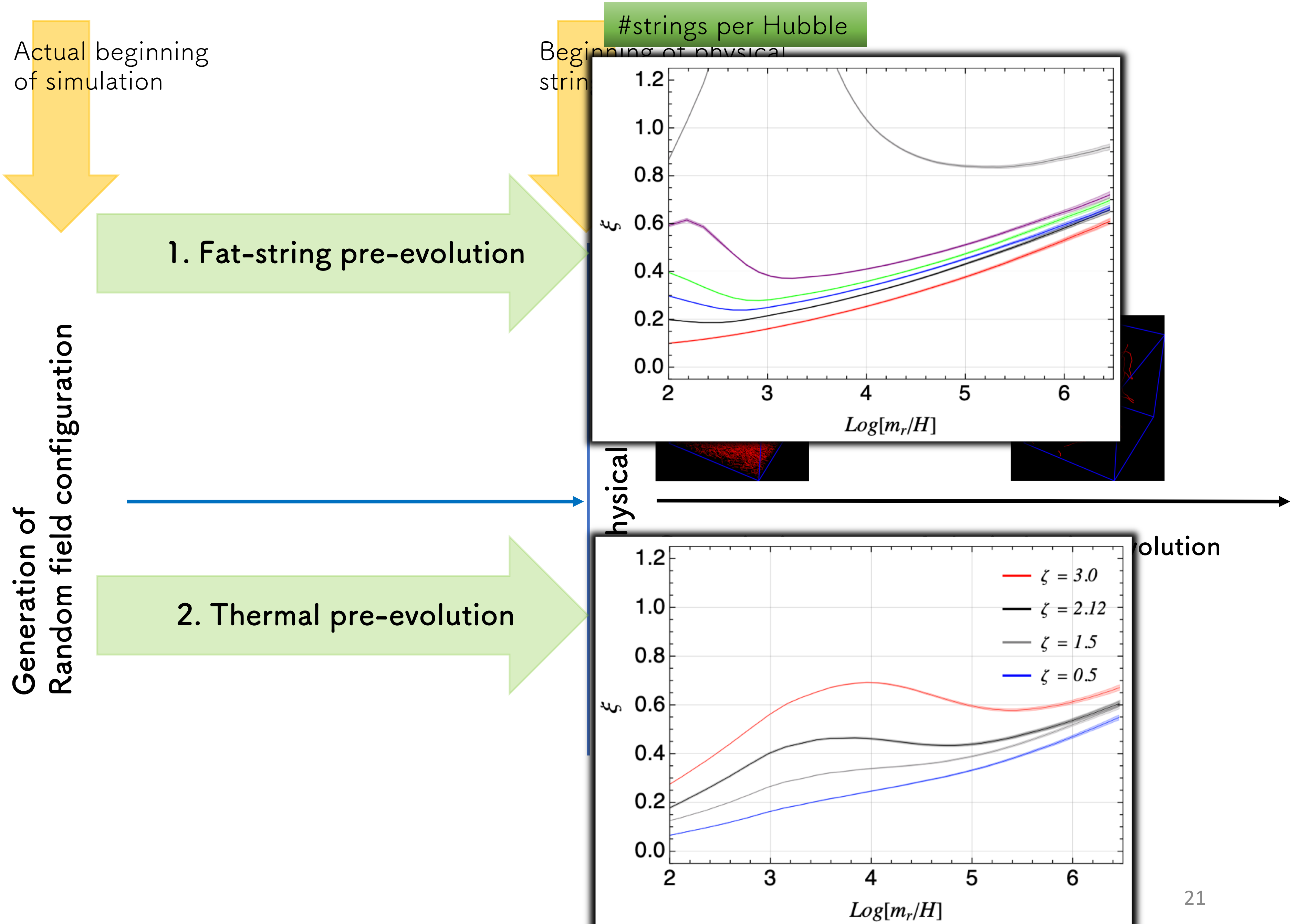
Role of pre-evolution is to relax noisy string network to a relatively clean level.

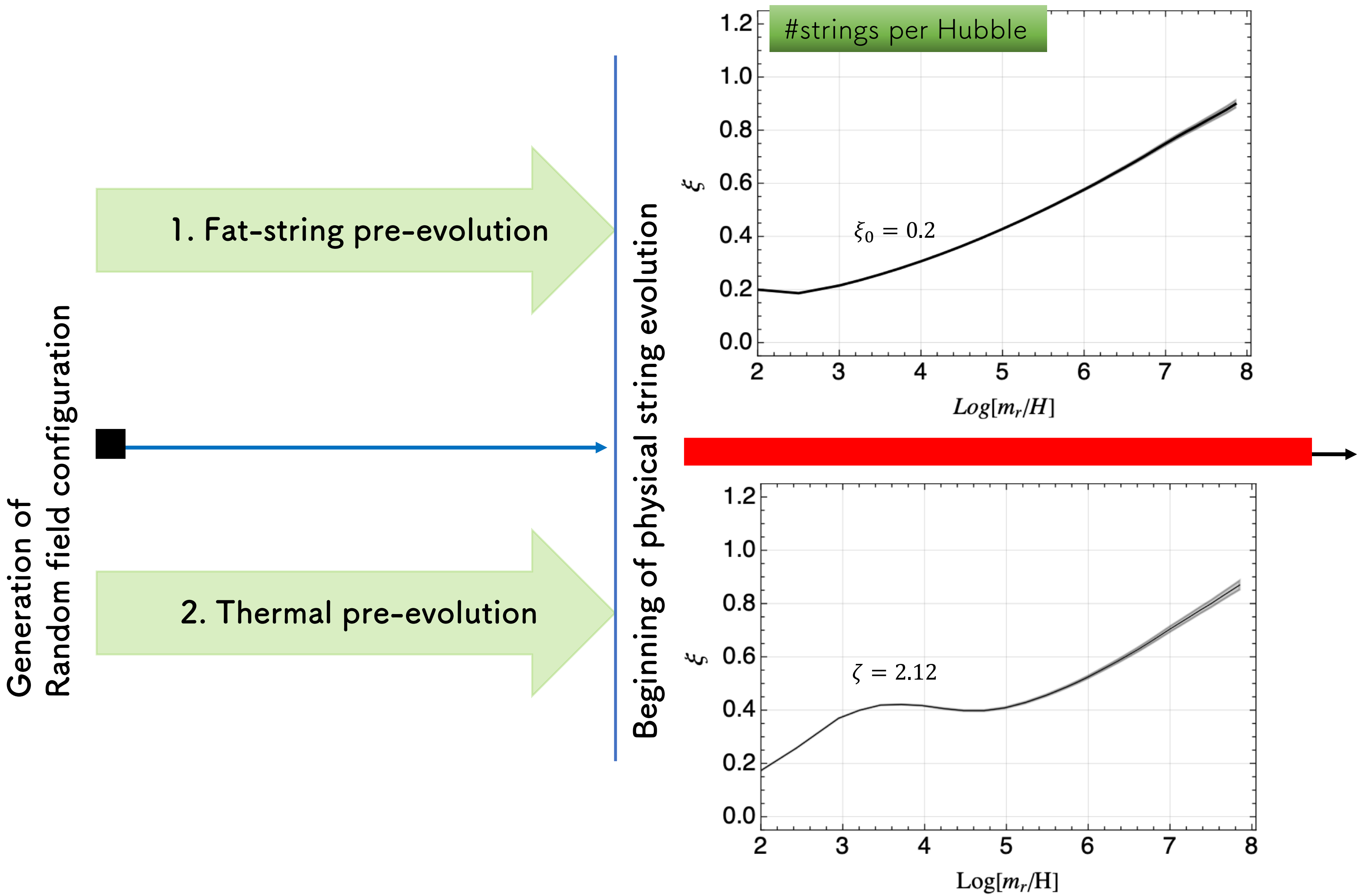




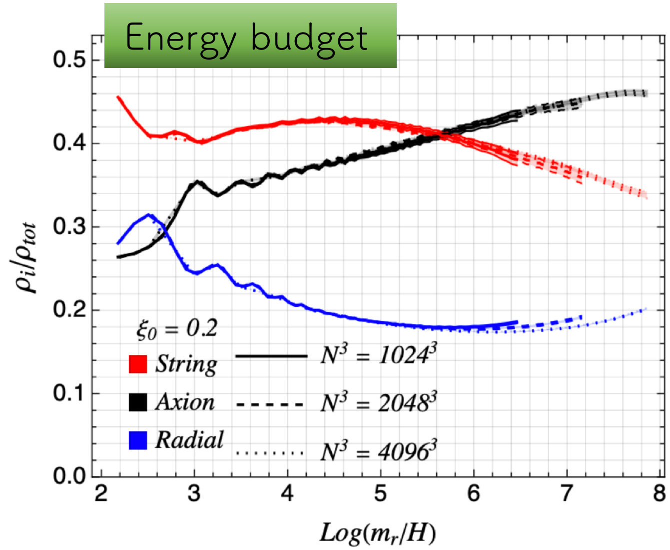
Dynamic time range of physical string evolution

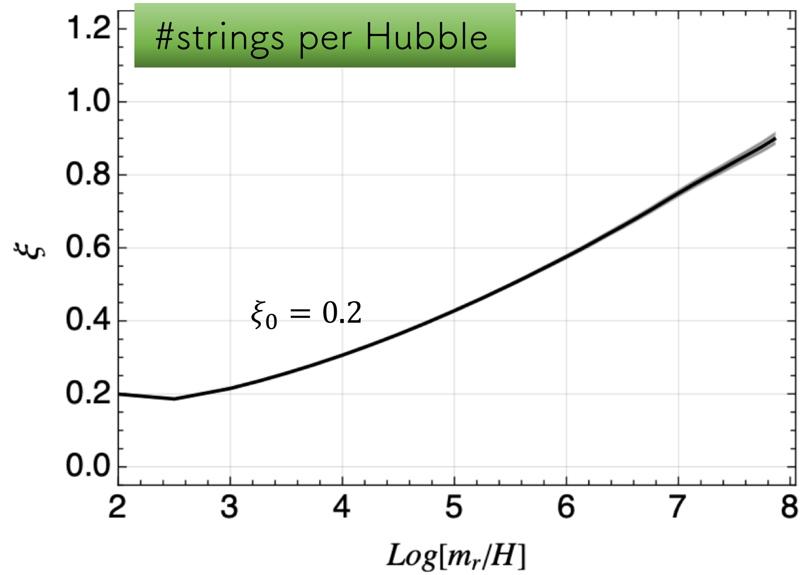
$$\log\frac{m_r}{H} \sim [2, 8\sim 9]$$


#strings per Hubble
Actual beginning of simulation
1. Fat-string pre-evolution
2. Thermal pre-evolution
Generation of Random field configuration
ξ
Log[m_r/H]
ζ = 3.0
ζ = 2.12
ζ = 1.5
ζ = 0.5

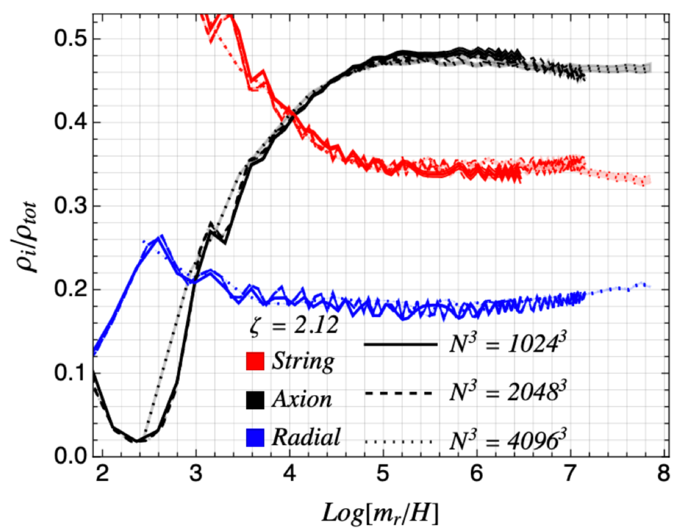

Generation of
Random field configuration
1. Fat-string pre-evolution
2. Thermal pre-evolution
Beginning of physical string evolution
#strings per Hubble
$\xi_0 = 0.2$
ξ
$Log[m_r/H]$
$\zeta = 2.12$
ξ
$\mathrm{Log}[m_r/\mathrm{H}]$
1.2
1.0
0.8
0.6
0.4
0.2
0.0
2
3
4
5
6
7
8

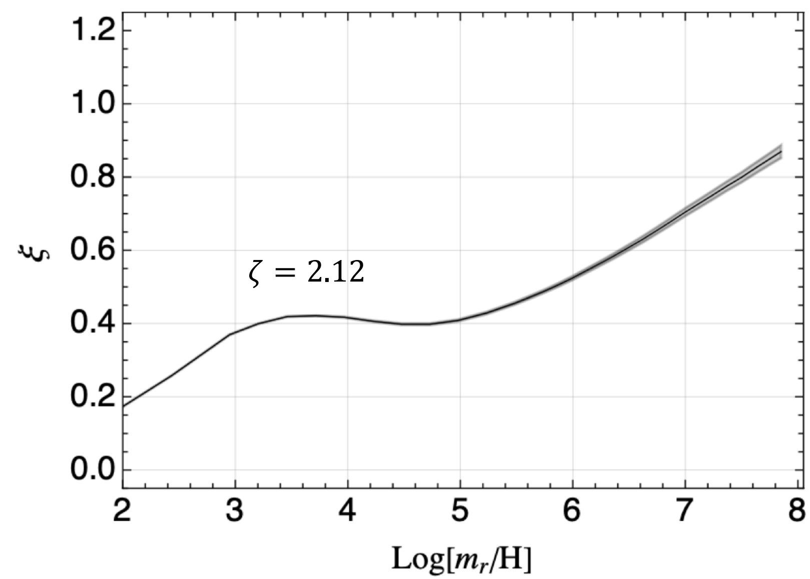
1. Fat-string pre-evolution





2. Thermal pre-evolution

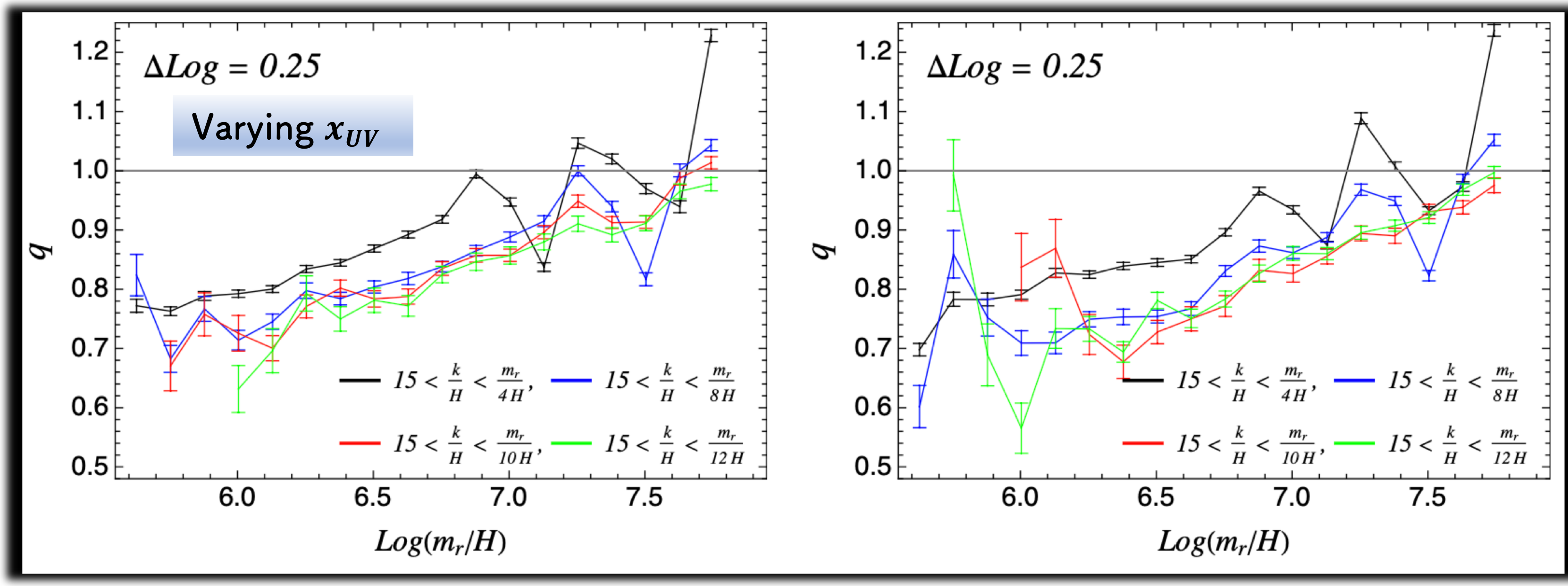




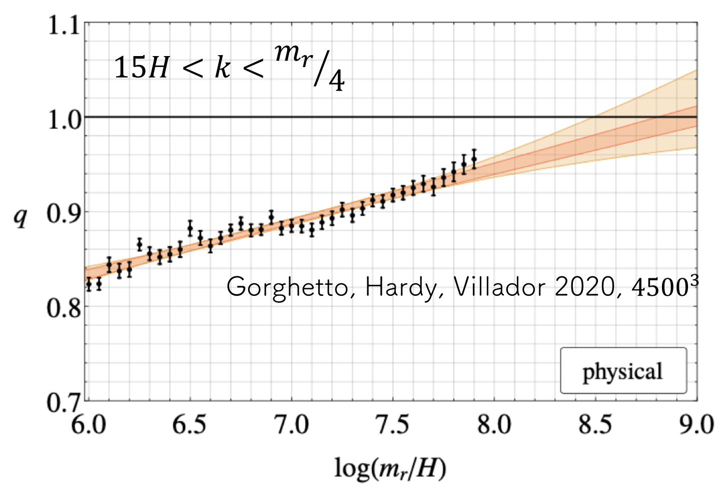
Logarithmic gross of q & IR-dominance

1. Fat-string pre-evolution type

2. Thermal pre-evolution type



✓ Looks consistent with logarithmic growth

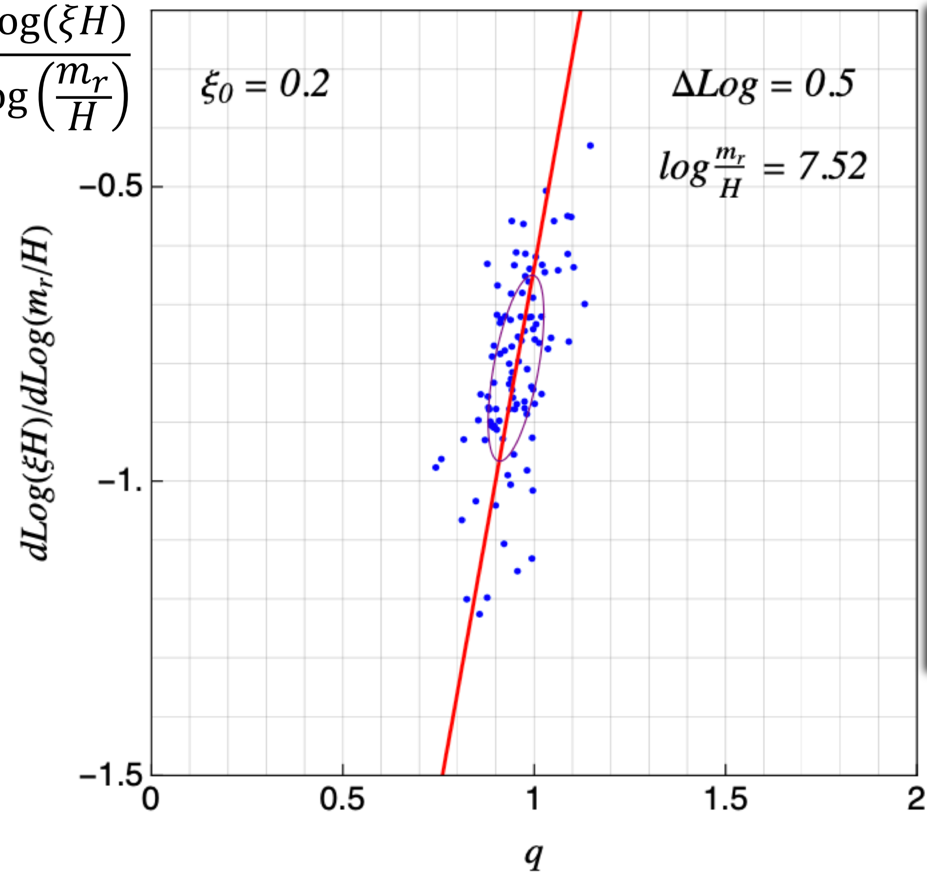


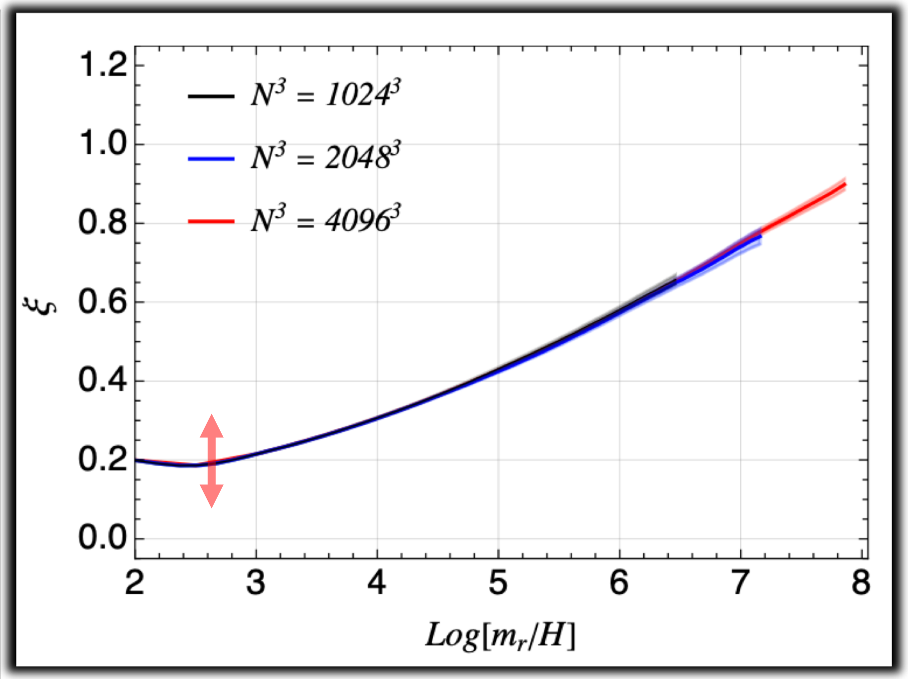
Justification for our benchmark

Correlation between strings and spectral index

Over 100 ensemble elements of one benchmark

$$\frac{d\mathrm{Log}(\xi H)}{d\mathrm{Log}\left(\frac{m_r}{H}\right)}$$





Summary

We independently support the logarithmic growth of ξ (strings per Hubble) and spectral index of axion spectrum. $N^3 = 8000^3$ should be possible for us (very near future).