EFT in Supersymmetry

Hilbert series in susy

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Outline

- Introduction: Why EFT?
- Hilbert Series
- Supersymmetry
- EOM & IBP redundancies
- Conclusions

Introduction

- EFTs are everywhere:
  - Fermi Theory for β-decay
  - Nucleon-nucleon interactions
  - Phonons in CM
  - ……
- They include the relevant degrees of freedom for a particular range of energies.

- In the case of the SM the process is as follows:
  - Include all operators consistent with the symmetries of the SM to a given mass dimensions (5,6,7,8….)
  - Remove those operators which are redundant under EOM and IBP
  - The task is gigantic since in principle there are a huge amounts of operators.
  - Is there a way to make sure how many there are?

1

$$-\frac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \frac{1}{4}g_s^2 f^{abc} f^{ade} g^b_\mu g^c_\nu g^d_\mu g^e_\nu +$$

$$\frac{1}{2}ig_s^2(\bar{q}^\sigma_i \gamma^\mu q^\sigma_j)g^a_\mu + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu \bar{G}^a G^b g^c_\mu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu -$$

2

$$M^2 W^+_\mu W^-_\mu - \frac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \frac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \frac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - \frac{1}{2}\partial_\mu H \partial_\mu H -$$

$$\frac{1}{2}m_h^2 H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - M^2\phi^+\phi^- - \frac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \frac{1}{2c_w^2}M\phi^0\phi^0 - \beta_h[\frac{2M^2}{g^2} +$$

$$\frac{2M}{g}H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)] + \frac{2M^4}{g^2}\alpha_h - igc_w[\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu -$$

$$W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu -$$

$$W^-_\nu \partial_\nu W^+_\mu)] - igs_w[\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu -$$

$$W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)] - \frac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu +$$

$$\frac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) +$$

$$g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w[A_\mu Z^0_\nu(W^+_\mu W^-_\nu -$$

$$W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu] - g\alpha[H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-] -$$

$$\frac{1}{8}g^2\alpha_h[H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2 H^2] -$$

$$gMW^+_\mu W^-_\mu H - \frac{1}{2}g\frac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \frac{1}{2}ig[W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) -$$

$$W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)] + \frac{1}{2}g[W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) - W^-_\mu(H\partial_\mu\phi^+ -$$

$$\phi^+\partial_\mu H)] + \frac{1}{2}g\frac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) - ig\frac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) +$$

$$igs_w MA_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) - ig\frac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) +$$

$$igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \frac{1}{4}g^2 W^+_\mu W^-_\mu[H^2 + (\phi^0)^2 + 2\phi^+\phi^-] -$$

$$\frac{1}{4}g^2\frac{1}{c_w^2}Z^0_\mu Z^0_\mu[H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-] - \frac{1}{2}g^2\frac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- +$$

$$W^-_\mu\phi^+) - \frac{1}{2}ig^2\frac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \frac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- +$$

$$W^-_\mu\phi^+) + \frac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\frac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- -$$

$$g^1 s_w^2 A_\mu A_\mu\phi^+\phi^- - \bar{e}^\lambda(\gamma\partial + m_e^\lambda)e^\lambda - \bar{\nu}^\lambda\gamma\partial\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + m_u^\lambda)u^\lambda_j -$$

3

$$\bar{d}^\lambda_j(\gamma\partial + m_d^\lambda)d^\lambda_j + igs_w A_\mu[-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \frac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \frac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)] +$$

$$\frac{ig}{4c_w}Z^0_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{u}^\lambda_j\gamma^\mu(\frac{4}{3}s_w^2 -$$

$$1 - \gamma^5)u^\lambda_j) + (\bar{d}^\lambda_j\gamma^\mu(1 - \frac{8}{3}s_w^2 - \gamma^5)d^\lambda_j)] + \frac{ig}{2\sqrt{2}}W^+_\mu[(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)e^\lambda) +$$

$$(\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)] + \frac{ig}{2\sqrt{2}}W^-_\mu[(\bar{e}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\lambda\kappa}\gamma^\mu(1 +$$

$$\gamma^5)u^\lambda_j)] + \frac{ig}{2\sqrt{2}}\frac{m_e^\lambda}{M}[-\phi^+(\bar{\nu}^\lambda(1-\gamma^5)e^\lambda) + \phi^-(\bar{e}^\lambda(1+\gamma^5)\nu^\lambda)] -$$

4

$$\frac{g}{2}\frac{m_e^\lambda}{M}[H(\bar{e}^\lambda e^\lambda) + i\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda)] + \frac{ig}{2M\sqrt{2}}\phi^+[-m_d^\kappa(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) +$$

$$m_u^\lambda(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j] + \frac{ig}{2M\sqrt{2}}\phi^-[m_d^\lambda(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m_u^\kappa(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1 -$$

$$\gamma^5)u^\kappa_j] - \frac{g}{2}\frac{m_u^\lambda}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \frac{g}{2}\frac{m_d^\lambda}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \frac{ig}{2}\frac{m_u^\lambda}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) -$$

$$\frac{ig}{2}\frac{m_d^\lambda}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 -$$

5

$$\frac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- -$$

$$\partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y -$$

$$\partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ -$$

$$\partial_\mu\bar{X}^- X^-) - \frac{1}{2}gM[\bar{X}^+ X^+ H + \bar{X}^- X^- H + \frac{1}{c_w^2}\bar{X}^0 X^0 H] +$$

$$\frac{1-2c_w^2}{2c_w}igM[\bar{X}^+ X^0\phi^+ - \bar{X}^- X^0\phi^-] + \frac{1}{2c_w}igM[\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-] +$$

$$igMs_w[\bar{X}^0 X^-\phi^+ - \bar{X}^0 X^+\phi^-] + \frac{1}{2}igM[\bar{X}^+ X^+\phi^0 - \bar{X}^- X^-\phi^0]$$

Hilbert Series

- Hilbert series provides with a way to count the number of operators for a given mass dimension.
- One just need to specify the field content, representation and quantum numbers.
- It works for different number of space time dimensions and also for different spacetime symmetries.

$$\mathcal{H} = \int d\mu \frac{1}{P} PE[\sum_i \phi_i \chi_{R,i}] + \Delta H$$

- $d\mu$: project out Lorentz scalars
- $\frac{1}{P}$: remove IBP
- $PE[\sum_i \phi_i \chi_{R,i}]$: remove EOM

$$\mathscr{H} = \int d\mu_{\mathscr{G}} \frac{1}{P} PE[\sum_{i=1}^{N} \phi_i \chi_{R,i}]$$

Hilbert series:
An algebraic function;
Encoding information of polynomials

Characters:
Representation of each field

Haar measure:
$d\mu_{\mathscr{G}} = d\mu_{SU(2)_L \times SU(2)_R} d\mu_{gauge}$
Group invariant volume measure

Plethystic exponential:
(Anti-)symmetrizing the arguments;
Fermi/Bose statistics

Spurions:
Both fields and derivatives;
Not real fields;
Treated as complex variables

Example: $Q = \{3, 2, \frac{1}{6}\},\ L = \{1, 2, -\frac{1}{2}\}$

$$\mathscr{I}(Q, L; x, y, u, z_1, z_2) = 3Q(x + \frac{1}{x})(y + \frac{1}{y})(z_1 + \frac{z_2}{z_1} + \frac{1}{z_2})u^{1/6} + 3L(x + \frac{1}{x})(y + \frac{1}{y})u^{-1/2}$$

x, y are group parameters of $SU(2)_L, SU(2)_R$, and u is the group parameter of $U(1)$

$$\mathscr{H} = \int d\mu\, PE[\mathscr{I}(Q, L; x, y, u, z_1, z_2)] = 1 + \boxed{57LQ^3} + 4818L^2Q^6 + \cdots$$

Counting operators ⟺ Finding the coefficients

- One has to correct the counting by redundancies due to EOM and IBP.

EOM: $\partial^2\phi \sim m^2\phi,\ i\gamma^\mu\partial_\mu\psi \sim m\psi^\dagger, \cdots$

IBP: $\mathcal{O}_1 \sim \mathcal{O}_2 + \partial\mathcal{O}_3$

Short conformal characters:

$$\bar{\chi}_{(0,0)} = P(\alpha,\beta,D)(1-D^2)$$

$$P(\alpha,\beta,D) = \left((1-D\alpha\beta)(1-\frac{D}{\alpha\beta})(1-\frac{D\alpha}{\beta})(1-\frac{D\beta}{\alpha})\right)^{-1}$$

$$\bar{\chi}_{(\frac{1}{2},0)} = P(\alpha,\beta,D)((\alpha+\frac{1}{\alpha})-D(\beta+\frac{1}{\beta})),\quad \bar{\chi}_{(0,\frac{1}{2})} = P(\alpha,\beta,D)((\beta+\frac{1}{\beta})-D(\alpha+\frac{1}{\alpha}))$$

$$\mathcal{H} = \int d\mu \frac{1}{P} PE[\sum_i \phi_i \chi_{R,i}] + \Delta H$$

Operators less than dimension 5.

- $d\mu$: project out Lorentz scalars
- $\frac{1}{P}$: remove IBP
- Short conformal characters: remove EOM

Supersymmetry:

	Non-supersymmetric theory	Supersymmetry
Fields	Scalar field ϕ, fermion field ψ, etc.	Chiral superfield Φ $\partial_{\dot{\alpha}}\Phi = 0,\ \partial_{\alpha}\Phi^{\dagger} = 0$
Derivatives	Partial derivative ∂_μ $\{\partial_\alpha, \partial_{\dot{\alpha}}\} = 2i\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_\mu$	Super derivatives $\partial_\alpha, \partial_{\dot{\alpha}}$ $D_\alpha \equiv \partial_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\sigma^{\mu}_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu;$ $\overline{D_{\dot{\alpha}}} \equiv \partial_{\dot{\alpha}} = -\frac{\partial}{\partial\theta^{\dot{\alpha}}} + i\theta^{\alpha}\sigma^{\mu}_{\alpha\dot{\alpha}}\partial_\mu$
Lagrangian	$S = \int d^4x\mathscr{L}(x)$ Lorentz scalar Singlet	$\mathscr{L} = \int d^4\theta K(\Phi, \Phi^{\dagger}) + \int d^2\theta W(\Phi) + h.c.$ Kahler potential — Superpotential

Which one to choose,

D-term or F-term?

$$\int d^4x W(\Phi_i, \partial^2_{\dot{\alpha}}S_i)_{\mathscr{F}}$$

$$= \int d^4x\{\partial^2_{\dot{\alpha}}[S_k h(\Phi_i, \partial^2_{\dot{\alpha}}S_i)]\}_{\mathscr{F}} \sim \int d^4x[S_k h(\Phi_i, \partial^2_{\dot{\alpha}}S_i)]_{\mathscr{D}},$$

EOM in Supersymmetry

For a chiral superfield Φ, equation of motion is given by $\partial_\alpha^2 \Phi \sim m\Phi^\dagger$. We can verify this relation by expanding Φ in components, i.e. $\Phi = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y)$, where $y^\mu = x^\mu + i\theta\sigma^m\bar{\theta}$, and we get $\partial_\alpha^2\phi = m\phi^*$, $i\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu\psi^\alpha = m\psi^\dagger_{\dot{\alpha}}$.

$$\begin{pmatrix} \Phi \\ \partial_\alpha\Phi \\ \partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \partial_\beta\partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \partial_{\dot{\beta}}\partial_\beta\partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \cdots \end{pmatrix} = \begin{pmatrix} \Phi \\ \partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \partial_{\dot{\beta}}\partial_\beta\partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \partial_{\dot{\gamma}}\partial_\gamma\partial_{\dot{\beta}}\partial_\beta\partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \cdots \end{pmatrix} + \begin{pmatrix} \partial_\alpha\Phi \\ \partial_\beta\partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \partial_\gamma\partial_{\dot{\beta}}\partial_\beta\partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \partial_\tau\partial_{\dot{\gamma}}\partial_\gamma\partial_{\dot{\beta}}\partial_\beta\partial_{\dot{\alpha}}\partial_\alpha\Phi \\ \cdots \end{pmatrix}$$

Indices are chosen to be symmetric combinations

Free of EOM

Bosonic Part ⟷ (SUSY) Fermionic Part

$\bar{\chi}_{(0,0)} = P(\alpha,\beta,D)(1-D^2)$

$\bar{\chi}_{(\frac{1}{2},0)} = P(\alpha,\beta,D)((\alpha+\frac{1}{\alpha}) - D(\beta+\frac{1}{\beta}))$

$$PE[\sum_i \phi_i\chi_{R,i}] = PE[\Phi\bar{\chi}_{(0,0)} + P\Phi\bar{\chi}_{(\frac{1}{2},0)}]$$

P: spurion of super derivative

IBP in Supersymmetry
—2 Independent IBP Relations

3 different derivatives $\partial_\mu,\ \partial_\alpha,\ \partial_{\dot\alpha}$ ⇒ ***3 IBP relations***

$$K \sim K' + \partial_\alpha X^\alpha$$
$$K \sim K' + \partial_{\dot\alpha} X^{\dot\alpha}$$
$$K \sim K' + \partial_\mu X^\mu$$

Only 2 of them are independent! $K \sim K' + \partial_\alpha(\partial_{\dot\alpha} X^{\alpha\dot\alpha}) + \partial_{\dot\alpha}(\partial_\alpha X^{\alpha\dot\alpha})$

Still, we have 2 relations and the previous 1/P factor doesn't work here. (P is not a rep here)

It is tempting to simply subtract the number of $X^\alpha, X^{\dot\alpha}$ to get the number of independent operators, because it seems like that one operator with one fewer derivative provides one IBP relation, and if we get rid of all these operators, our result is free of IBP. However it's incorrect because these IBP relations can be linearly dependent!

—Correction Space

Starting with a space $\mathcal{O}$, we define the **zeroth order equivalence relations** on $\mathcal{O}$ as follows:

$$o_1 \sim o_2 + \sum \mathcal{I}_i s_i, \quad o_i \in \mathcal{O}, s_i \in S_i^0 .$$

We call $\mathcal{S}_j^1$ the **first order correction space** if all elements in $\mathcal{S}_j^1$ satisfy the following conditions:

$$\mathcal{T}_{ij}^1 s_j \neq 0, \; and \;\; \mathcal{I}_i \mathcal{T}_{ij}^1 s_j = 0, \; (no \; sums \; over \; i), \forall s_j \in \mathcal{S}_j^1.$$

A **space** $\mathcal{S}_j^n$ is called the **nth-order correction** to $\mathcal{O}$ if there exist **maps**:

$\mathcal{T}_{ij}^n : S_j^n \to S_i^{(n-1)}$, such that:

$$\mathcal{T}_{ij}^n s_j \neq 0, \; and \;\; \mathcal{T}_{ki}^{n-1} \mathcal{T}_{ij}^n s_j = 0, \forall s_j \in S_j^n, \forall k,$$

and is denoted as $\mathcal{S}_j^n(\{\mathcal{S}_i^{n-1}\} \to \{\mathcal{S}_i^{n-2}\}), n \geq 2.$

$$\# \; of \; independent \; operators =$$

$$\#\{\mathcal{O}\} - \# \sum \{\mathcal{S}_i^0\} + \# \sum \{\mathcal{S}_i^1\} - \# \sum \{\mathcal{S}_i^2\} + \cdots$$

—SMEFT Example Revisit

e.g. $\mathcal{O}_i \sim \mathcal{O}_j + \sum_n \partial_\mu \mathcal{O}_n^\mu, \quad \mathcal{O}_i, \mathcal{O}_j \in \{X\}, \mathcal{O}_n^\mu \in \{X^\mu\}.$

Operator Space   IBP Space

If we identify $\mathcal{T}^1_{11} \equiv \partial_\mu$ and $\mathcal{I}_1 \equiv \partial_\nu$.

Terminates with four total antisymmetric indices in four dimensions.

$\mathcal{I}_1 \mathcal{T}^1_{11} s = \partial_\mu \partial_\nu X^{[\mu\nu]} = 0$ $X^{[\mu\nu]}$ is the first order correction space!

$\mathcal{T}^1_{11} \mathcal{T}^2_{11} X^{[\mu\nu\rho]} = \partial_\mu \partial_\nu X^{[\mu\nu\rho]} = 0$ $X^{[\mu\nu\rho]}$ is the second order correction space!

$\mathcal{T}^2_{11} \mathcal{T}^3_{11} X^{[\mu\nu\rho\sigma]} = \partial_\mu \partial_\nu X^{[\mu\nu\rho\sigma]} = 0$ $X^{[\mu\nu\rho\sigma]}$ is the third order correction space!

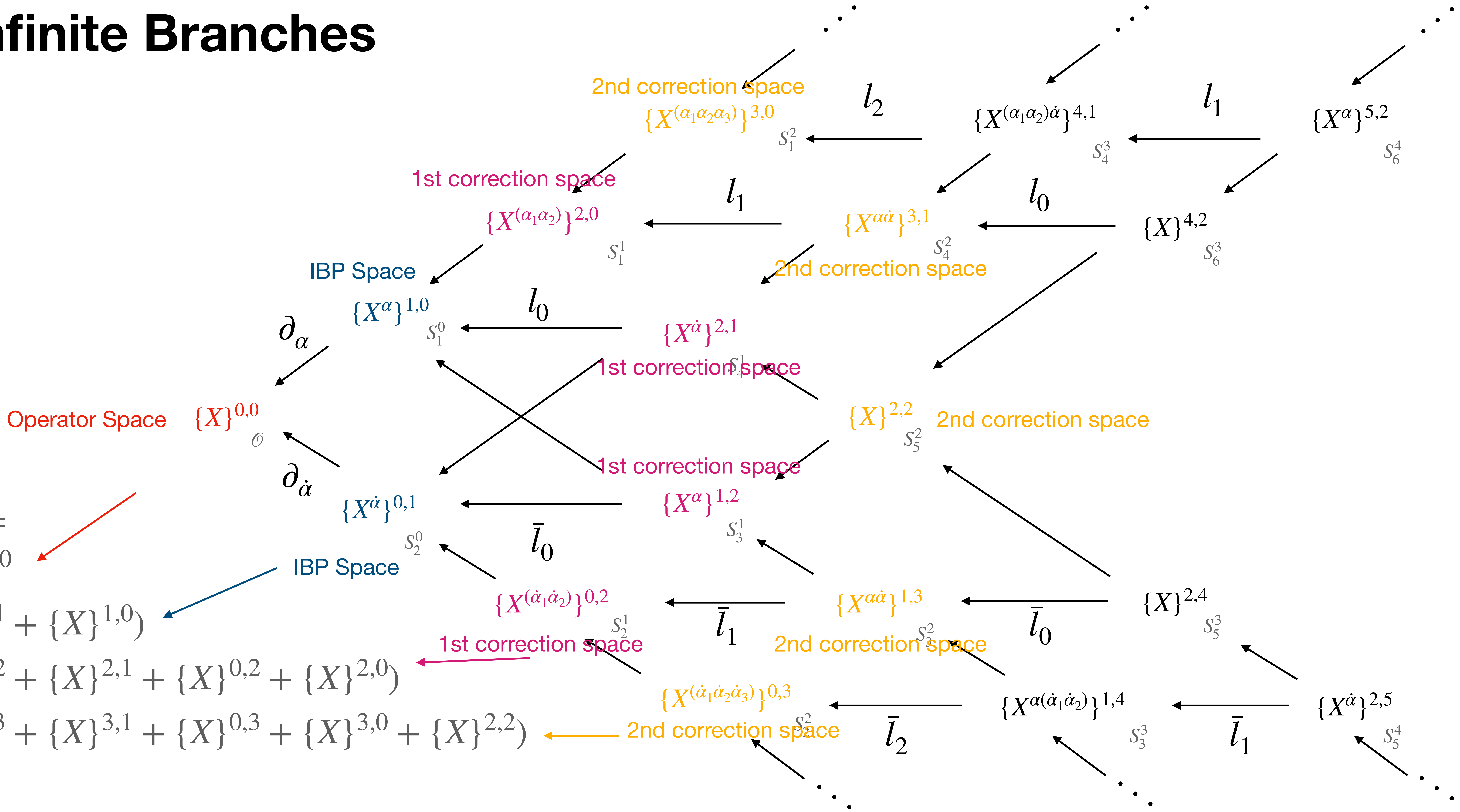
Diagram: $\{X\} \xleftarrow{\partial_\mu} \{X^\mu\} \xleftarrow{\partial_\nu} \{X^{[\mu\nu]}\} \xleftarrow{\partial_\rho} \{X^{[\mu\nu\rho]}\} \xleftarrow{\partial_\sigma} \{X^{[\mu\nu\rho\sigma]}\}$

Order: $1 \quad D \quad D^2 \quad D^3 \quad D^4$

$$\sum D^n \chi_{X^{[\mu_1\mu_2\cdots\mu_n]}} = 1 - D(\alpha + \frac{1}{\alpha})(\beta + \frac{1}{\beta}) + D^2[(1 + \alpha^2 + \frac{1}{\alpha^2}) + (1 + \beta^2 + \frac{1}{\beta^2})] - D^3(\alpha + \frac{1}{\alpha})(\beta + \frac{1}{\beta}) + D^4 = \frac{1}{P}$$

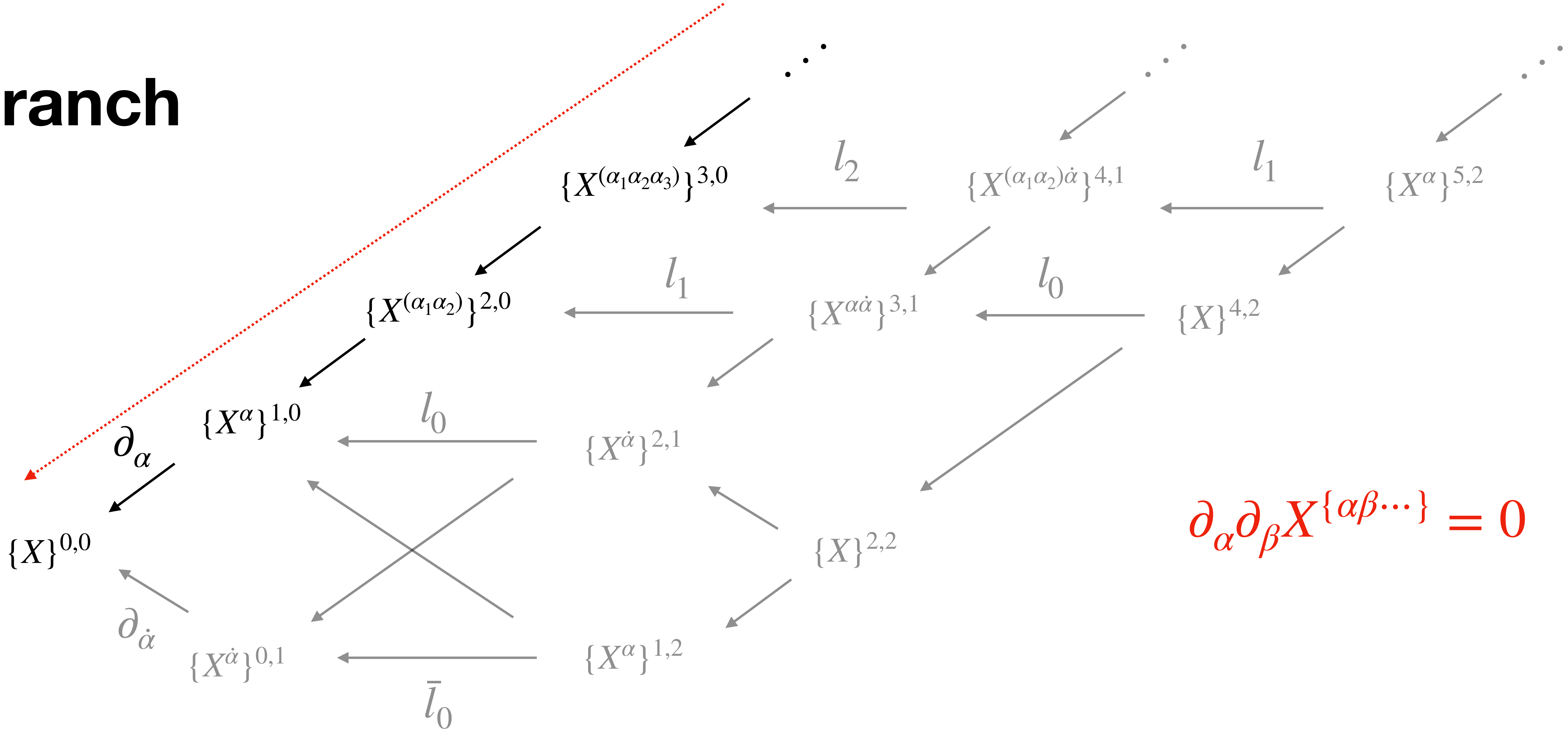
Operator Space - IBP Space + First order correction space - Second order correction space + Third order correction space

—6 Infinite Branches



$$number =$$
$$= \#\{X\}^{0,0}$$
$$-\#(\{X\}^{0,1} + \{X\}^{1,0})$$
$$+\#(\{X\}^{1,2} + \{X\}^{2,1} + \{X\}^{0,2} + \{X\}^{2,0})$$
$$-\#(\{X\}^{1,3} + \{X\}^{3,1} + \{X\}^{0,3} + \{X\}^{3,0} + \{X\}^{2,2})$$
$$\cdots$$

—First Branch



$$\partial_\alpha \partial_\beta X^{\{\alpha\beta\cdots\}} = 0$$

It doesn't terminate because one can form infinite numbers of indices to be fully symmetric, in contrast to the antisymmetric case.

$$\partial_\mu \partial_\nu X^{[\mu\nu\cdots]} = 0$$

SMEFT Diagram

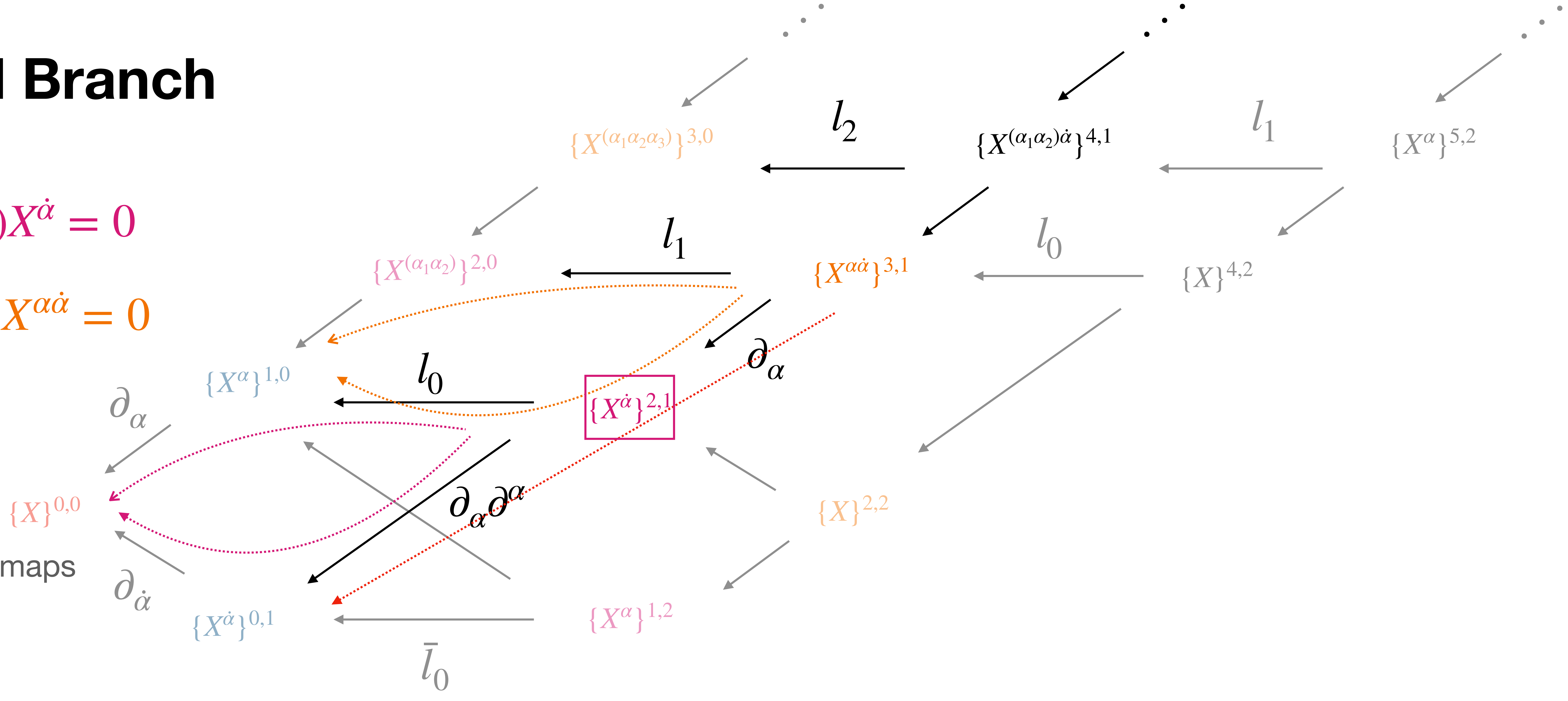
$$\{X\} \xleftarrow{\partial_\mu} \{X^\mu\} \xleftarrow{\partial_\nu} \{X^{[\mu\nu]}\} \xleftarrow{\partial_\rho} \{X^{[\mu\nu\rho]}\} \xleftarrow{\partial_\sigma} \{X^{[\mu\nu\rho\sigma]}\}$$

—Second Branch

$$(\partial_\alpha l_0 + \partial_{\dot\alpha}\partial_\alpha^2)X^{\dot\alpha} = 0$$

$$(l_0\partial_\alpha + \partial_\alpha l_1)X^{\alpha\dot\alpha} = 0$$

⋮



Fully determine the maps

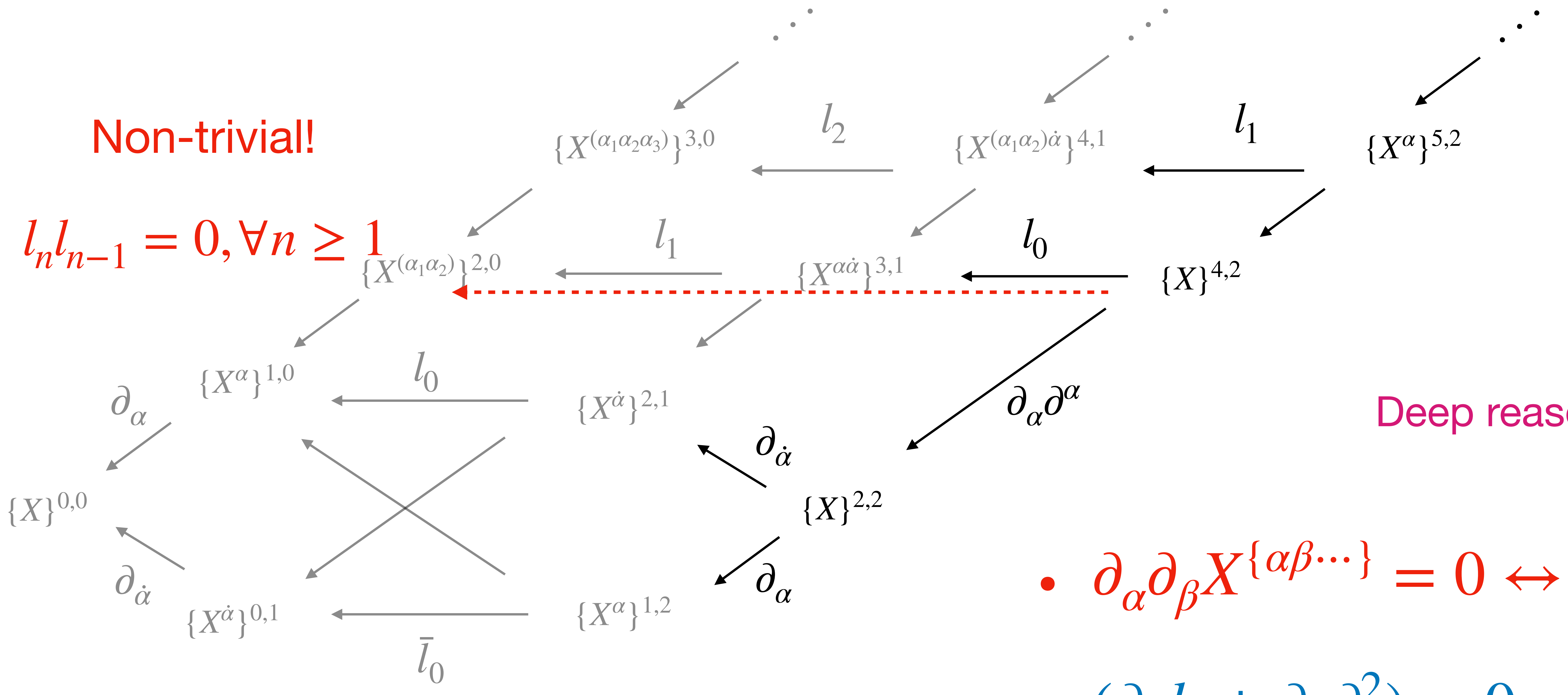
$$(l_n)_{\dot\alpha}^{(\alpha_1\alpha_2\cdots\alpha_n)(\tau\beta_1\beta_2\cdots\beta_n)}X^{\dot\alpha}_{(\alpha_1\alpha_2\cdots\alpha_n)} = (a_n\partial_{\dot\alpha}\partial_Z + b_n\partial_Z\partial_{\dot\alpha})\epsilon^{(Z\alpha_1\alpha_2\cdots\alpha_n)(\tau\beta_1\beta_2\cdots\beta_n)}X^{\dot\alpha}_{(\alpha_1\alpha_2\cdots\alpha_n)}$$

$$a_n = (-1)^n\frac{2}{(n+1)!},\quad b_n = (-1)^n\frac{2}{(n+2)n!}$$

—Third Branch

Non-trivial!

$l_n l_{n-1} = 0, \forall n \geq 1$



Deep reason?

- $\partial_\alpha \partial_\beta X^{\{\alpha\beta\cdots\}} = 0 \leftrightarrow \{\partial_\alpha, \partial_\beta\} = 0$
- $(\partial_\alpha l_0 + \partial_{\dot{\alpha}} \partial_\alpha^2) = 0 \leftrightarrow [\{\partial_\alpha, \partial_{\dot{\alpha}}\}, \partial_\beta] = 0$
- $l_n l_{n-1} = 0 \leftrightarrow [\{\partial_\alpha, \partial_{\dot{\alpha}}\}, \{\partial_\beta, \partial_{\dot{\beta}}\}] = 0$

Any additional branches/corrections?

- $p = q$
- All three composite maps must vanish

$$l_0\{X\}^{2,4} = l_0\mathcal{G}_1\{X\}^{p,p}$$

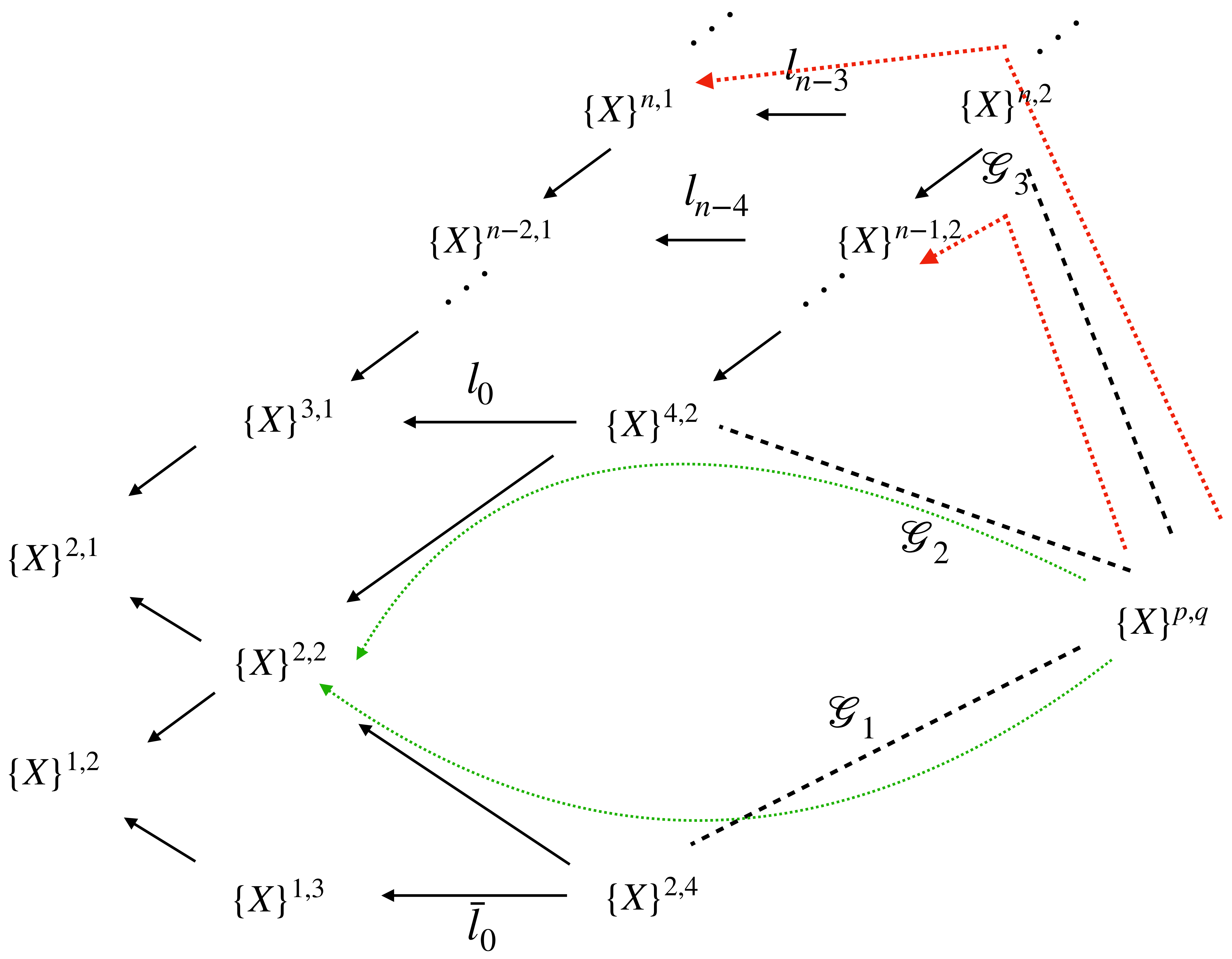
$$\bar{l}_0\{X\}^{4,2} = \bar{l}_0\mathcal{G}_2\{X\}^{p,p}$$

$$(\partial_\alpha^2\{X\}^{4,2} + \partial_{\dot{\alpha}}^2\{X\}^{2,4}) = (\partial_\alpha^2\mathcal{G}_1 + \partial_{\dot{\alpha}}^2\mathcal{G}_2)\{X\}^{p,p}$$

- $p \neq q$
- Suppose $p \geq 4$, the following two conditions must be satisfied

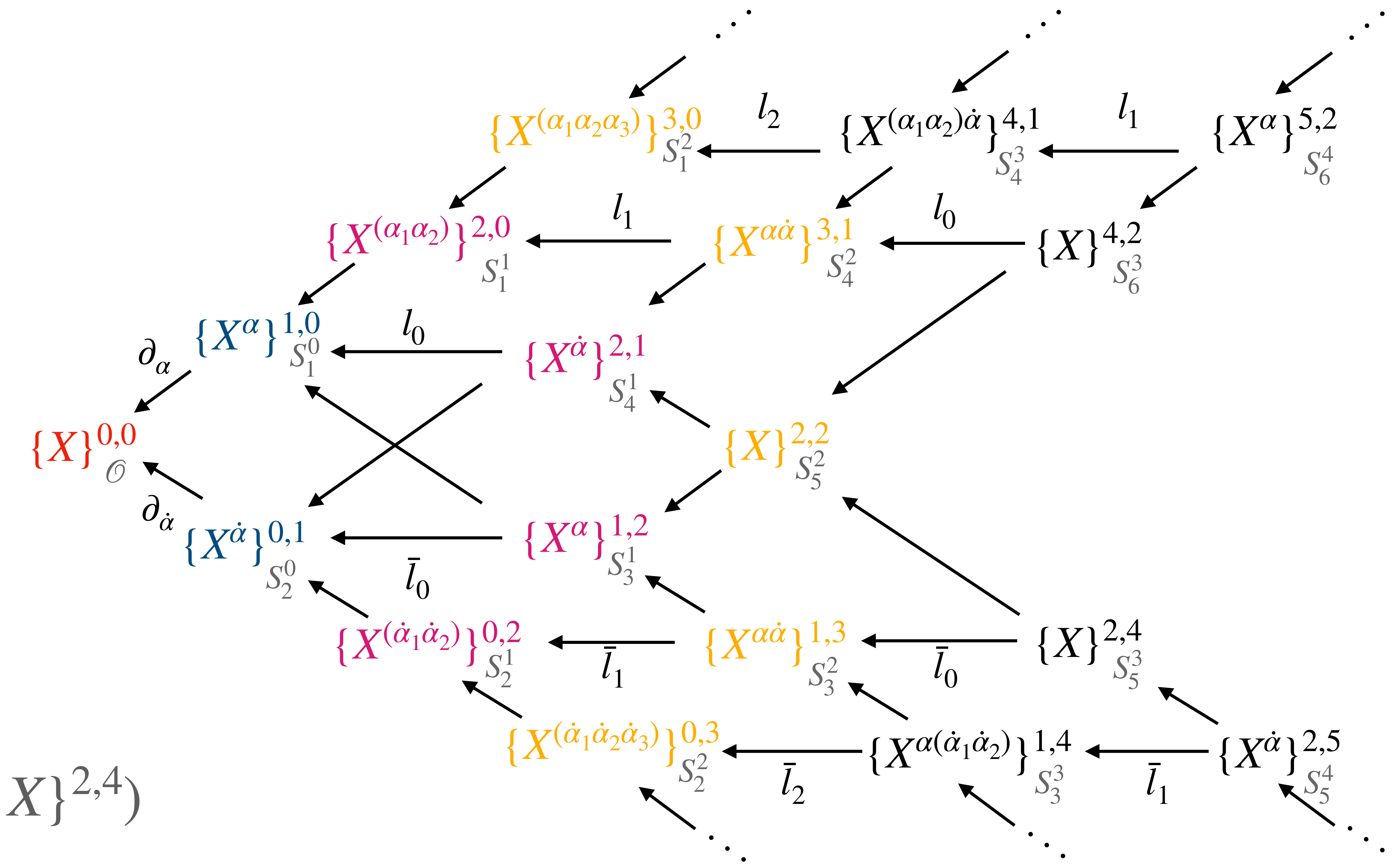
$$\partial_\alpha\{X\}^{n,2} = \partial_\alpha\mathcal{G}_3\{X\}^{p,q} = 0$$

$$l_{n-3}\{X\}^{n,2} = l_{n-3}\mathcal{G}_3\{X\}^{p,q} = 0$$



—Summation

$$number =$$
$$= \#\{X\}^{0,0}$$
$$-\#(\{X\}^{0,1} + \{X\}^{1,0})$$
$$+\#(\{X\}^{1,2} + \{X\}^{2,1} + \{X\}^{0,2} + \{X\}^{2,0})$$
$$-\#(\{X\}^{1,3} + \{X\}^{3,1} + \{X\}^{0,3} + \{X\}^{3,0} + \{X\}^{2,2})$$
$$+\#(\{X\}^{1,4} + \{X\}^{4,1} + \{X\}^{0,4} + \{X\}^{4,0} + \{X\}^{4,2} + \{X\}^{2,4})$$
$$\cdots$$



$$\sum P^p Q^q \chi_{X^{p,q}}$$

P,Q represent two super derivatives

$$= 1$$
$$-(Px + Qy)$$
$$+(PQ^2x + P^2Qy + P^2(x^2-1) + Q^2(y^2-1))$$
$$-(PQ^3xy + P^3Qxy + P^3(x^3-2x) + Q^3(y^3-2y) + P^2Q^2)$$
$$\cdots$$

This becomes the 1/P factor in supersymmetry, and when we put this into Hilbert series, it will automatically remove all IBP redundancies.

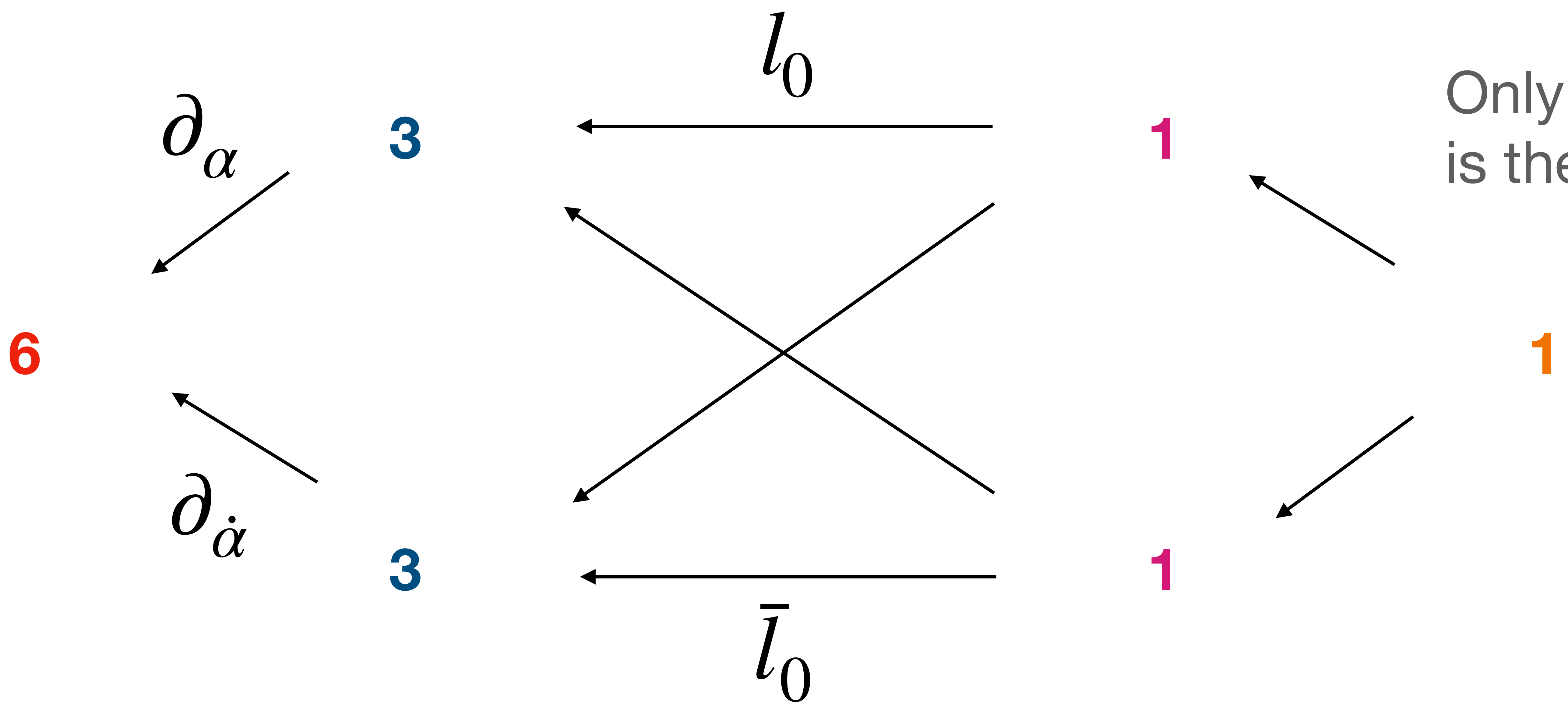
$$\mathcal{H} = \int d\mu \frac{1}{P_{new}} PE[\sum_i \phi_i \chi_{R,i}] + \Delta H$$

In practical use, we will truncate this infinite series.

—Examples

$$
\begin{aligned}
&\partial_\alpha[\Phi(\partial^\alpha\Phi)(\partial_{\dot\alpha}\Phi^\dagger)(\partial^{\dot\alpha}\Phi^\dagger)] \sim (\partial_\alpha\Phi)(\partial^\alpha\Phi)(\partial_{\dot\alpha}\Phi^\dagger)(\partial^{\dot\alpha}\Phi^\dagger) - 2\Phi(\partial_\alpha\Phi)(\partial^\alpha\partial^{\dot\alpha}\Phi^\dagger)(\partial_{\dot\alpha}\Phi^\dagger),\\
&\partial_\alpha[\Phi^2(\partial_{\dot\alpha}\Phi^\dagger)(\partial^\alpha\partial^{\dot\alpha}\Phi^\dagger)] \sim 2\Phi(\partial_\alpha\Phi)(\partial^\alpha\partial^{\dot\alpha}\Phi^\dagger)(\partial_{\dot\alpha}\Phi^\dagger) + \Phi^2(\partial_\alpha\partial_{\dot\alpha}\Phi^\dagger)(\partial^\alpha\partial^{\dot\alpha}\Phi^\dagger),\\
&\partial_\alpha[\Phi(\partial^{\dot\alpha}\partial^\alpha\Phi)\Phi^\dagger(\partial_{\dot\alpha}\Phi^\dagger)] \sim (\partial^{\dot\alpha}\partial^\alpha\Phi)(\partial_\alpha\Phi)\Phi^\dagger(\partial_{\dot\alpha}\Phi^\dagger) + \Phi(\partial^{\dot\alpha}\partial^\alpha\Phi)\Phi^\dagger(\partial_\alpha\partial_{\dot\alpha}\Phi^\dagger),\\
&\partial_{\dot\alpha}[(\partial_\alpha\Phi)(\partial^\alpha\Phi)\Phi^\dagger(\partial^{\dot\alpha}\Phi^\dagger)] \sim (\partial_\alpha\Phi)(\partial^\alpha\Phi)(\partial_{\dot\alpha}\Phi^\dagger)(\partial^{\dot\alpha}\Phi^\dagger) + 2(\partial_{\dot\alpha}\partial_\alpha\Phi)(\partial^\alpha\Phi)\Phi^\dagger(\partial^{\dot\alpha}\Phi^\dagger),\\
&\partial_{\dot\alpha}[(\partial_\alpha\Phi)(\partial^{\dot\alpha}\partial^\alpha\Phi)\Phi^{\dagger 2}] \sim (\partial_{\dot\alpha}\partial_\alpha\Phi)(\partial^{\dot\alpha}\partial^\alpha\Phi)\Phi^{\dagger 2} - 2(\partial_\alpha\Phi)(\partial^{\dot\alpha}\partial^\alpha\Phi)\Phi^\dagger(\partial_{\dot\alpha}\Phi^\dagger),\\
&\partial_{\dot\alpha}[\Phi(\partial_\alpha\Phi)\Phi^\dagger(\partial^\alpha\partial^{\dot\alpha}\Phi^\dagger)] \sim \Phi(\partial_{\dot\alpha}\partial_\alpha\Phi)\Phi^\dagger(\partial^\alpha\partial^{\dot\alpha}\Phi^\dagger) - \Phi(\partial_\alpha\Phi)(\partial^\alpha\partial^{\dot\alpha}\Phi^\dagger)(\partial_{\dot\alpha}\Phi^\dagger)\,.
\end{aligned}
$$

$\mathcal{O}(\partial_\alpha{}^2\partial_{\dot\alpha}{}^2\Phi^2\Phi^{\dagger 2})$

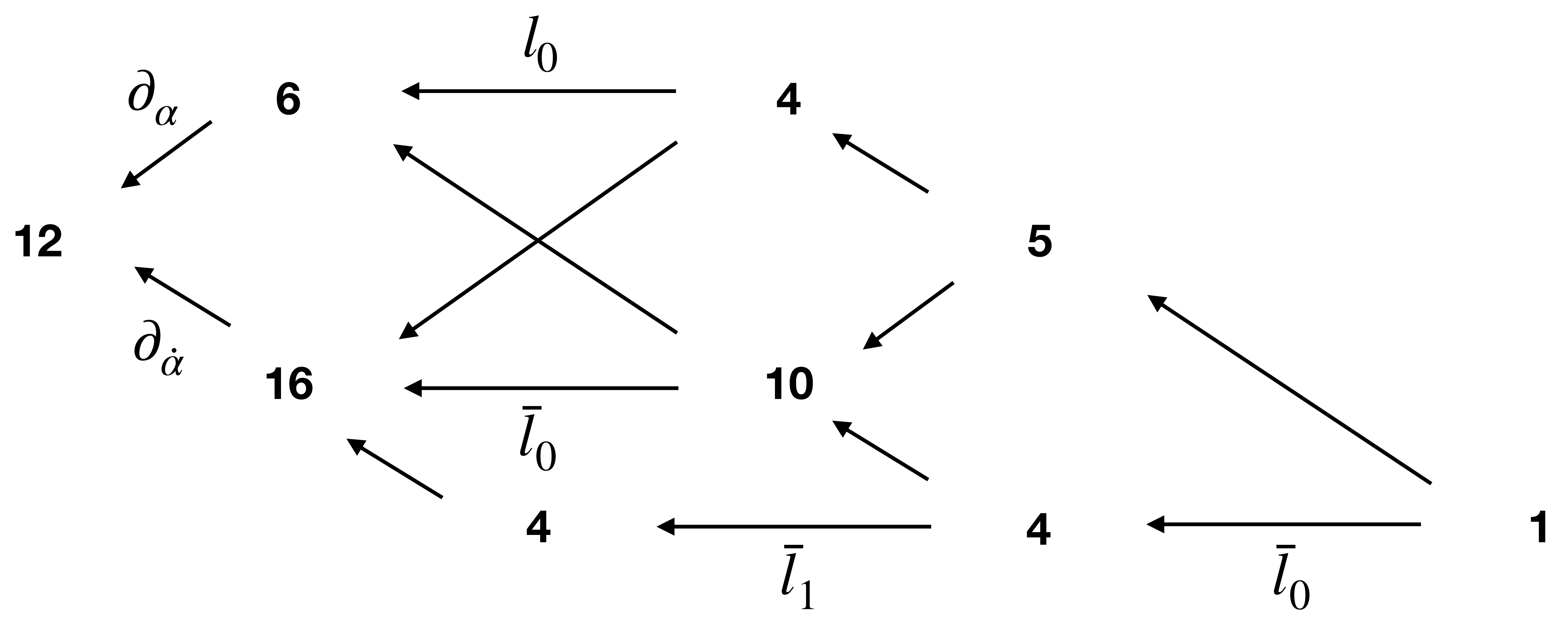


Only 5 of these are independent! The independent number is therefore 6-5=1, which is the same as 6-3-3+1+1-1=1.

We don't need to find all relations, not even the explicit form of operators in IBP spaces. What we do is using Hilbert series to count the number of operators in each correction space, and calculate the summation.

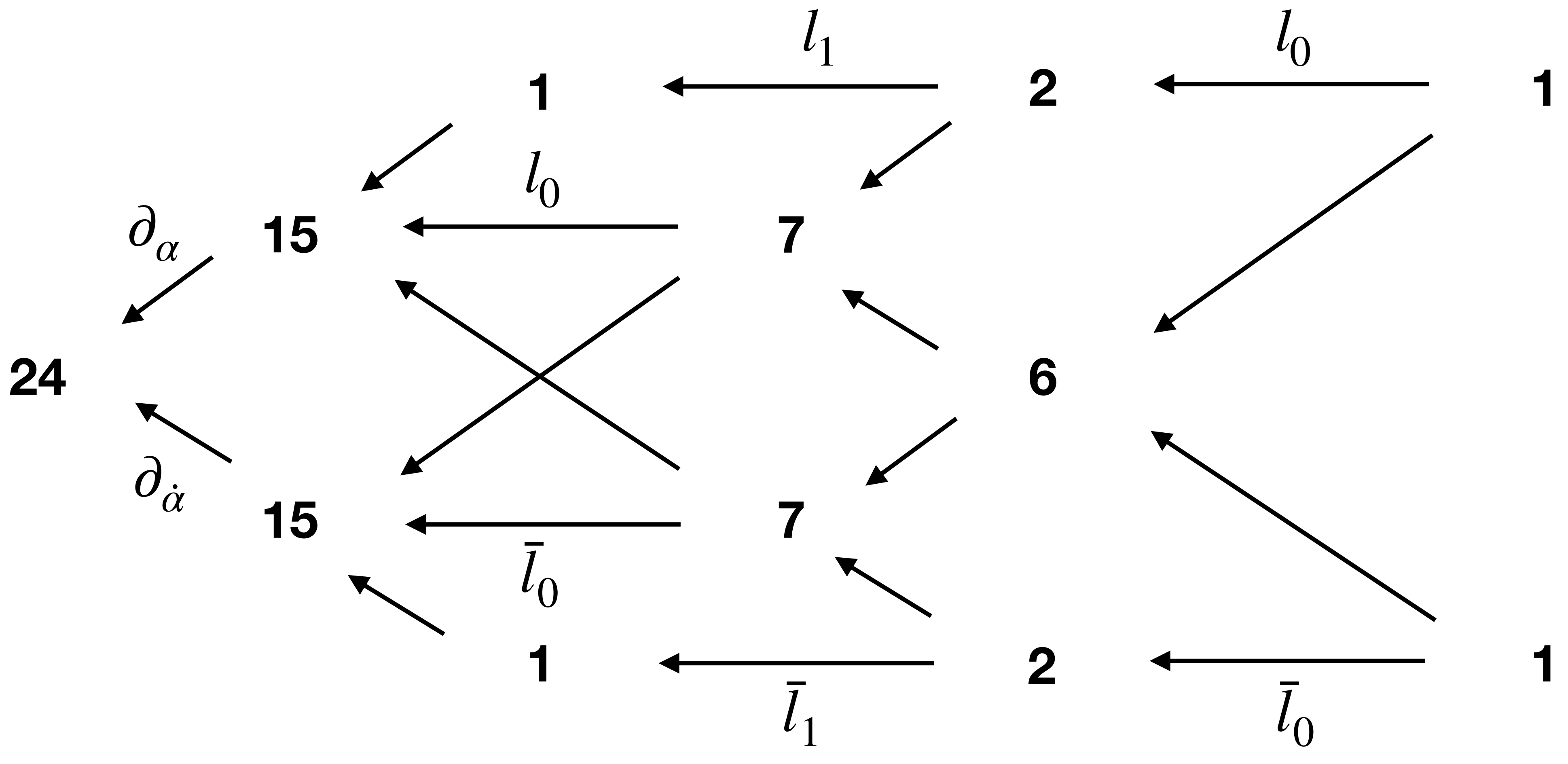
—More flavors and Derivatives

$\mathcal{O}(\partial_\alpha{}^4 \partial_{\dot\alpha}{}^4 \Phi_1 \Phi_2 \Phi_1^\dagger)$



12-6-16+4+10+4-5-4+1=0

$\mathcal{O}(\partial_\alpha{}^4 \partial_{\dot\alpha}{}^4 \Phi^2 \Phi^{\dagger 2})$



24-15-15+1+1+7+7-2-2-6+1+1=2

Schouten identity makes it even more difficult

Vector superfields

- So far we have just considered chiral superfields
- The formalism can also be applied for vector interactions.
- Lets remind how gauge abelian terms are included in supersymmetry:

$$\Phi_l \to \Phi'_l = e^{-it_l\Lambda}\Phi_l;\ \Phi_l^\dagger \to \Phi_l'^\dagger = e^{it_l\Lambda^\dagger}\Phi_l^\dagger$$

V a real superfield
V=V*

$$\Phi^\dagger e^{tV}\Phi \to \Phi'^\dagger e^{tV'}\Phi' = \Phi^\dagger e^{tV}\Phi$$

$$W_\alpha \equiv -\frac{1}{4}\overline{D}^2 D_\alpha V,\quad \overline{W}_{\dot\alpha} \equiv -\frac{1}{4}D^2\overline{D}_{\dot\alpha}V$$

- One subtlety is that we have to include the Haar measure for the $U(1)_R$.

- The Hilbert series works in the same way as in SMEFT except we have to add the Haar measure for two U(1) groups.

Same P factor as before!

$$\mathcal{H}(P,Q,\Phi,\Phi^\dagger,W_\alpha,\overline{W}^{\dot\alpha},e^V)$$
$$=\int d\mu_{Lorentz}d\mu_{gauge}d\mu_{U_R(1)}P^{-1}(P,Q,\alpha,\beta,z)PE[\mathcal{I}(\Phi,\Phi^\dagger,W_\alpha,\overline{W}^{\dot\alpha},e^V)]$$

$$d\mu_{Lorentz}=\frac{1}{(2\pi i)^2}\oint_{|\alpha|=1}\frac{d\alpha}{\alpha}(1-\alpha^2)\oint_{|\beta|=1}\frac{d\beta}{\beta}(1-\beta^2),$$
$$d\mu_{gauge}=\frac{1}{(2\pi i)^2}\oint_{|g_1|=1}\frac{dg_1}{g_1}\oint_{|g_2|=1}\frac{dg_2}{g_2},$$
$$d\mu_{U_R(1)}=\frac{1}{2\pi i}\oint_{|z|=1}\frac{dz}{z},$$

$$\mathcal{I}(\Phi,\Phi^\dagger,W_\alpha,\overline{W}^{\dot\alpha},e^V)_{\rm bos}=\Phi g_1^{-1}\tilde\chi_{(0,0)}+\Phi^\dagger g_2\tilde\chi_{(0,0)}+ P(DW_\alpha)\tilde\chi_{(1,0)}+Q(\overline{D}\overline{W}^{\dot\alpha})\tilde\chi_{(0,1)}+e^Vg_1g_2^{-1}$$

$$\mathcal{I}(\Phi,\Phi^\dagger,W_\alpha,\overline{W}^{\dot\alpha},e^V)_{\rm ferm}=P(D\Phi)g_1^{-1}z^{-1}\tilde\chi_{(\frac{1}{2},0)}+Q(\overline{D}\Phi^\dagger)g_2z\tilde\chi_{(0,\frac{1}{2})}+W_\alpha z\tilde\chi_{(\frac{1}{2},0)}+\overline{W}^{\dot\alpha}z^{-1}\tilde\chi_{(0,\frac{1}{2})}$$

- For a non-abelian case (e.g. SU(2) things work in the same way:

Same P factor as before!

$$\mathcal{H}(P,Q,\Phi,\Phi^\dagger,W_\alpha,\overline{W}^{\dot\alpha},e^V) = \int d\mu_{Lorentz} d\mu_{gauge} d\mu_{U_R(1)} P^{-1}(P,Q,\alpha,\beta,z) PE[\mathcal{I}(\Phi,\Phi^\dagger,W_\alpha,\overline{W}^{\dot\alpha},e^V)]$$

$$d\mu_{gauge} = \frac{1}{(2\pi i)^2} \oint_{|g_1|=1} \frac{dg_1}{g_1}(1-g_1^2) \oint_{|g_2|=1} \frac{dg_2}{g_2}(1-g_2^2)$$

$$\mathcal{I}(\Phi,\Phi^\dagger,W_\alpha,\overline{W}^{\dot\alpha},e^V)_{\mathrm{bos}} = \Phi(g_1+\frac{1}{g_1})\tilde\chi_{(0,0)} + \Phi^\dagger(g_2+\frac{1}{g_2})\tilde\chi_{(0,0)} + P(DW_\alpha)\tilde\chi_{(1,0)} + Q\overline{D}\overline{W}^{\dot\alpha}\tilde\chi_{(0,1)} + e^V(g_1+\frac{1}{g_1})(g_2+\frac{1}{g_2})$$

$$\mathcal{I}(\Phi,\Phi^\dagger,W_\alpha,\overline{W}^{\dot\alpha},e^V)_{\mathrm{ferm}} = P(D\Phi)(g_1+\frac{1}{g_1})z^{-1}\tilde\chi_{(\frac{1}{2},0)} + Q(\overline{D}\Phi^\dagger)(g_2+\frac{1}{g_2})z\tilde\chi_{(0,\frac{1}{2})} + W_\alpha z\tilde\chi_{(\frac{1}{2},0)} + \overline{W}^{\dot\alpha}z^{-1}\tilde\chi_{(0,\frac{1}{2})}$$

Conclusions

- Hilbert series are an useful tool to calculate the number of independent operators (IBP & EOM free) in an EFT.
- In this talk I have applied the formalism to an N=1 SUSY theory with chiral multiplets.
- EOMs redundancies can be treated similarly to non-supersymmetric theories.
- IBP generate a richer structure due to the existence of three derivatives and not just one.
- The techniques also works in super gauge theories.

- Future directions that we are exploring are the following:
  - Connection to the superconformal group
  - Identifying the structure of the operators using Young Tableaux techniques.
  - Studying the role of super amplitudes in this approach.

Thank you!

多謝

谢谢