SMEFT at future lepton colliders with machine learning

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Why SMEFT at future lepton colliders?

- **Build large colliders $\to$ go to high energy $\to$ discover new particles!**
- Higgs and nothing else?
- What's next?
  - Build an even larger collider (~ 100 TeV)?
  - No guaranteed discovery!

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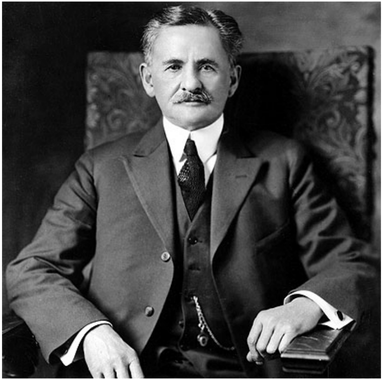
- **Build large colliders $\to$ do precision measurements $\to$ probe new physics!**
  - Higgs factory! (HL-LHC, or a future lepton collider)
  - Many other precision measurements! (Z, W, top, ...)
  - **S**tandard **M**odel **E**ffective **F**ield **T**heory (model independent approach)

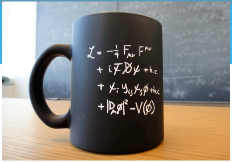
To summarize in one sentence...



"Our future discoveries must be looked for in the sixth place of decimals."

— Albert A. Michelson

The Standard Model Effective Field Theory

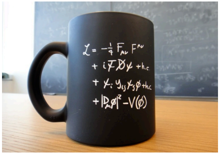


- $[\mathcal{L}_{\rm sm}] \leq 4$. Why?
  - ~~Bad things happen when we have non-renormalizable operators!~~
  - Everything is fine as long as we are happy with finite precision in perturbative calculation.
- **d=5:** $\frac{c}{\Lambda} LLHH \sim \frac{cv^2}{\Lambda}\nu\nu$, Majorana neutrino mass.
- Assuming Baryon and Lepton numbers are conserved,

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{c_i^{(6)}}{\Lambda^2}\mathcal{O}_i^{(6)} + \sum_j \frac{c_j^{(8)}}{\Lambda^4}\mathcal{O}_j^{(8)} + \cdots .$$

- If $\Lambda \gg v$, E, then **SM + dimension-6 operators** are sufficient to parameterize the physics around the electroweak scale.

The Standard Model Effective Field Theory



+

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r) \tau^I \varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r) \widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r) \tau^I \widetilde\varphi\, W^I_{\mu\nu}$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r) \widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) \varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r) \tau^I \varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) \varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p \gamma^\mu d_r)$

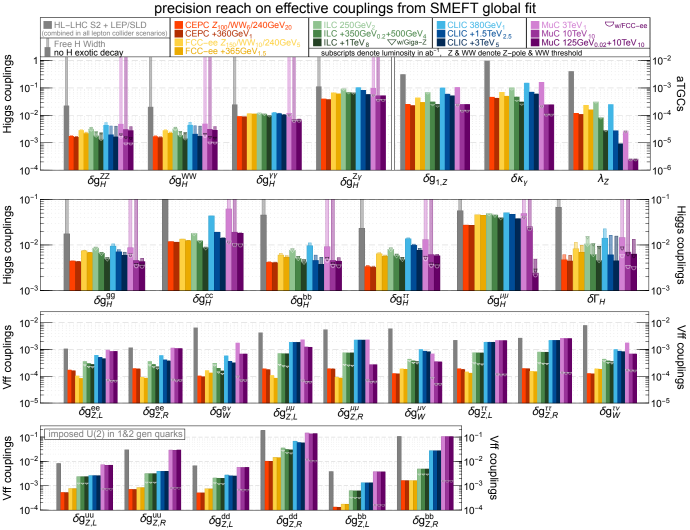
$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p \gamma_\mu l_r)(\bar l_s \gamma^\mu l_t)$	Q_{ee}	$(\bar e_p \gamma_\mu e_r)(\bar e_s \gamma^\mu e_t)$	Q_{le}	$(\bar l_p \gamma_\mu l_r)(\bar e_s \gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar q_s \gamma^\mu q_t)$	Q_{uu}	$(\bar u_p \gamma_\mu u_r)(\bar u_s \gamma^\mu u_t)$	Q_{lu}	$(\bar l_p \gamma_\mu l_r)(\bar u_s \gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p \gamma_\mu \tau^I q_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p \gamma_\mu d_r)(\bar d_s \gamma^\mu d_t)$	Q_{ld}	$(\bar l_p \gamma_\mu l_r)(\bar d_s \gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p \gamma_\mu l_r)(\bar q_s \gamma^\mu q_t)$	Q_{eu}	$(\bar e_p \gamma_\mu e_r)(\bar u_s \gamma^\mu u_t)$	Q_{qe}	$(\bar q_p \gamma_\mu q_r)(\bar e_s \gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p \gamma_\mu \tau^I l_r)(\bar q_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p \gamma_\mu e_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar u_s \gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p \gamma_\mu u_r)(\bar d_s \gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar u_s \gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p \gamma_\mu T^A u_r)(\bar d_s \gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p \gamma_\mu q_r)(\bar d_s \gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p \gamma_\mu T^A q_r)(\bar d_s \gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

- Write down all possible (non-redundant) dimension-6 operators ...
- 59 operators (76 parameters) for 1 generation, or 2499 parameters for 3 generations. [arXiv:1008.4884] Grzadkowski, Iskrzyński, Misiak, Rosiek, [arXiv:1312.2014] Alonso, Jenkins, Manohar, Trott.
- A **full global fit** with all measurements to all operator coefficients?
  - We usually only need to deal with a subset of them, *e.g.* $\sim$ 20-30 parameters for **Higgs and electroweak** measurements.
- Do a global fit and present the results with some fancy bar plots!

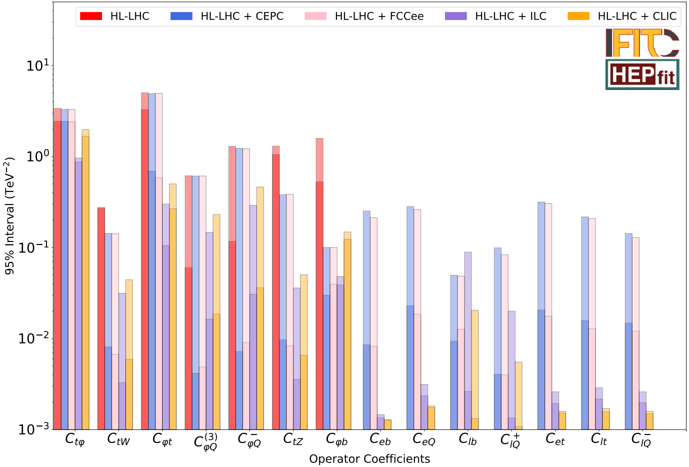
Higgs + EW, Results from the Snowmass 2021 (2022) study

[2206.08326] de Blas, Du, Grojean, JG, Miralles, Peskin, Tian, Vos, Vryonidou



Top operators with $e^+e^- \to t\bar{t}$

[2206.08326] de Blas, Du, Grojean, JG, Miralles, Peskin, Tian, Vos, Vryonidou

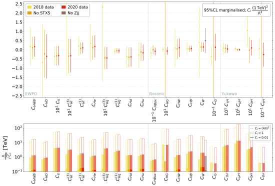


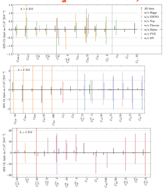
Many studies on SMEFT global fits!

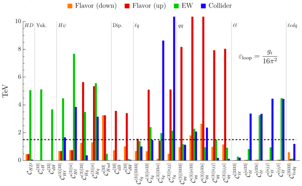
[2012.02779] Ellis, Madigan, Mimasu, Sanz, You

[2311.00020] Allwicher, Cornella, Isidori, Stefanek

[2311.04963] Bartocci, Biekötter, Hurth

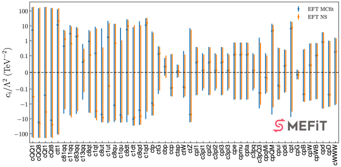


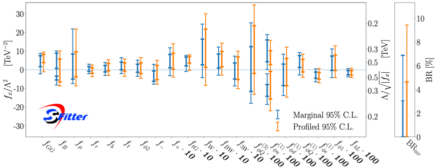




[2105.00006] The SMEFiT Collaboration

[2208.08454] Brivio, Bruggisser, Elmer, Geoffray, Luchmann, Plehn





Machine learning in SMEFT analyses

Machine learning is not physics!

past

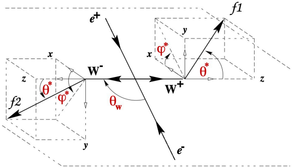
真香！(Delicious!)

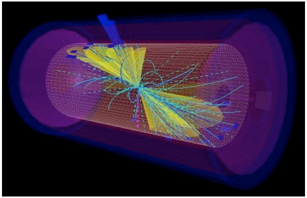
now

- [2401.02474] Shengdu Chai, JG, Lingfeng Li on $e^+e^- \to W^+W^-$.
- Many studies!
  - [1805.00013, 1805.00020] Brehmer, Cranmer, Louppe, Pavez,
    [2007.10356] Chen, Glioti, Panico, Wulzer ($pp \to ZW$),
    [2211.02058] Ambrosio, Hoeve, Madigan, Rojo, Sanz ($pp \to tt$, $pp \to hZ$),
    ......

Why Machine learning in SMEFT analyses?

- In many cases, the new physics contributions are sensitive to the differential distributions.
  - $e^+e^- \to W^+W^- \to 4f \;\Rightarrow$ 5 angles
  - $e^+e^- \to t\bar{t} \to bW^+\bar{b}W^- \to 6f$ $\Rightarrow$ 9 angles
  - How to extract information from the differential distribution?
  - If we have the full knowledge of $\frac{d\sigma}{d\Omega} \Rightarrow$ matrix-element method, optimal observables...

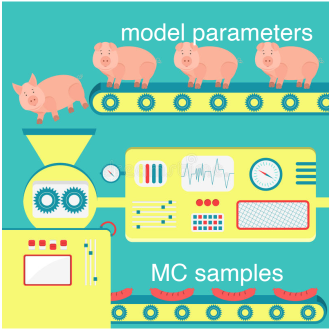




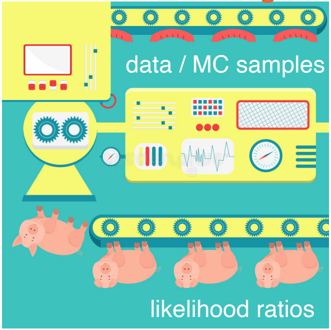
- The ideal $\frac{d\sigma}{d\Omega}$ we can calculate is not the $\frac{d\sigma}{d\Omega}$ that we actually measure!
  - detector acceptance, measurement uncertainties, ISR/beamstrahlung ...
  - In practice we only have **MC samples**, not analytic expressions, for $\frac{d\sigma}{d\Omega}$.

The “inverse problem”

Monte Carlo simulation



Machine Learning

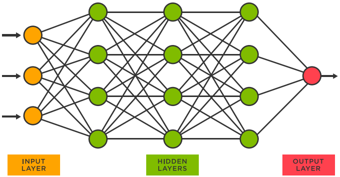


- **Forward:** From model parameters we can calculate the ideal $\frac{d\sigma}{d\Omega}$, simulate complicated effects and produce MC samples.
- **Inverse:** From data / MC samples, how do we know the model parameters?
- With **Neural Network** we can (in principle) reconstruct $\frac{d\sigma}{d\Omega}$ (or likelihood ratios) from MC samples.

A rough sketch

- We have a theory (SMEFT) that gives a differential cross section $\frac{d\sigma}{d\Omega}$ which is a function of the parameters of interest $\mathbf{c}$ (Wilson coefficients).
- For simplicity, let's ignore the total rate and focus on $\frac{1}{\sigma}\frac{d\sigma}{d\Omega} \equiv p(\mathbf{x}|\mathbf{c})$, *i.e.* it's a probability density function of the observables $\mathbf{x}$.
- Define the likelihood function $\mathcal{L}(\mathbf{c}|\mathbf{x}) \equiv p(\mathbf{x}|\mathbf{c})$. For a sample of N events, maximizing the total likelihood $\prod_{i=1}^{N} \mathcal{L}(\mathbf{c}|\mathbf{x}_i)$ (or the log likelihood) gives the best estimator for $\mathbf{c}$. (matrix-element method)
- For two model points c_0 and c_1, the likelihood ratio $r(\mathbf{x}|\mathbf{c}_0, \mathbf{c}_1) = \frac{p(\mathbf{x}|\mathbf{c}_0)}{p(\mathbf{x}|\mathbf{c}_1)}$ provides the optimal statistical test (Neyman–Pearson lemma).
  - We usually set c_1 to be SM.

A rough sketch

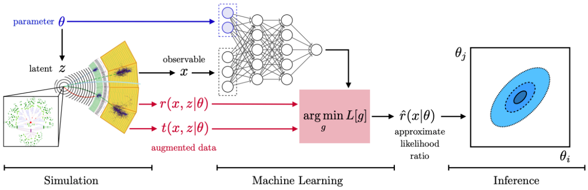


- We do not know $p(x|c)$ or $r(\mathbf{x}|\mathbf{c}_0, \mathbf{c}_1)$, but we can use neural network to construct an estimator $\hat{r}(\mathbf{x}|\mathbf{c}_0, \mathbf{c}_1)$ and a loss function(al) $L(\hat{r})$ which is minimized when $\hat{r} = r$.
- By minimizing $L(\hat{r})$ with respect to $\hat{r}$ we can find the true r in the ideal limit (large sample, perfect training).
- There are many ways to construct a loss function(al)....
- With additional assumptions on how $\frac{d\sigma}{d\Omega}$ depends on $\mathbf{c}$ (*i.e.*, a linear or a quadratic relation), we only need to train a finite number of times to obtain an estimator $\hat{r}(\mathbf{x}|\mathbf{c}_0, \mathbf{c}_1)$ for any c_0.

Particle physics structure

- One could make use of latent variable "z" (the parton level analytic result for $\frac{d\sigma}{d\Omega}$) to increase the performance of ML.

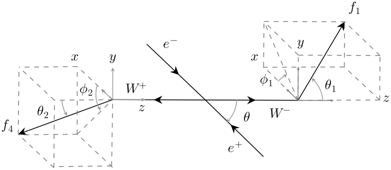
  [1805.00013, 1805.00020] Brehmer, Cranmer, Louppe, Pavez



- Assuming linear dependences $\frac{d\sigma}{d\Omega} = S_0 + \sum_i S_{1,i}\, c_i$, there is a method called SALLY (Score approximates likelihood locally).
  - In this case, for each parameter we only need to train once to obtain $\alpha_i \equiv \frac{S_{1,i}}{S_0}$. (It is basically the ML version of Optimal Observables.)
  - We can calculate the "ideal" $\alpha(z)$ which will help us train the actual $\alpha(x)$.

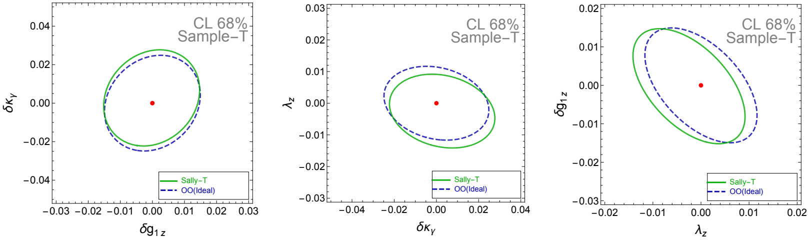
$$L[\hat{\alpha}(x)] = \sum_{x_i, z_i \sim \mathrm{SM}} \left|\alpha(z_i) - \hat{\alpha}(x_i)\right|^2 .$$

The ML analysis of $e^+e^- \to W^+W^-$



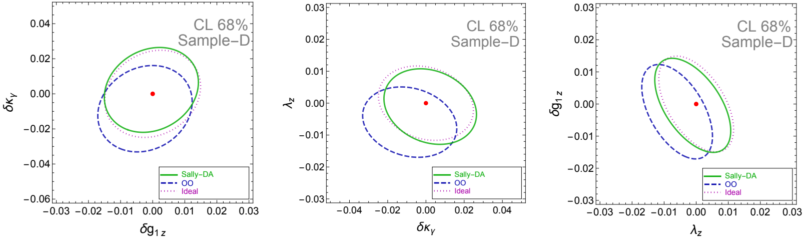
- $e^+e^- \to W^+W^-$, 240 GeV, unpolarized beams, semileptonic channel.
- Training sample: 2×10^6 events. Validation sample: 5×10^5 events.
  - This is much smaller than the actual data set ($\sim 10^8$ events) we will have!
- MadGraph/Pythia/Delphes, ILD-like detector card, IRS implemented.
- Background: $e^+e^- \to ZZ \to jj\ell^+\ell^-$ with a missing lepton.
- Inputs: particle 4 momenta + 5 reconstructed angles.
- Fully connected neural network (FCNN), 9 layers and 200 nodes each layer.

3-aTGC fit, truth-level sample



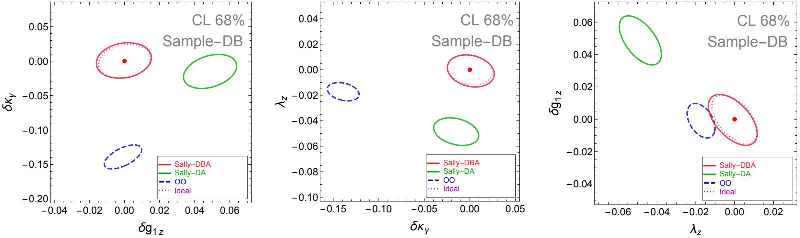
- The results are scaled to 10^4 events.
- At the truth level, Optimal Observables (OO) gives the ideal results by construction.
- Machine learning suffers from imperfect training and has no advantage.

3-aTGC fit, detector-level sample



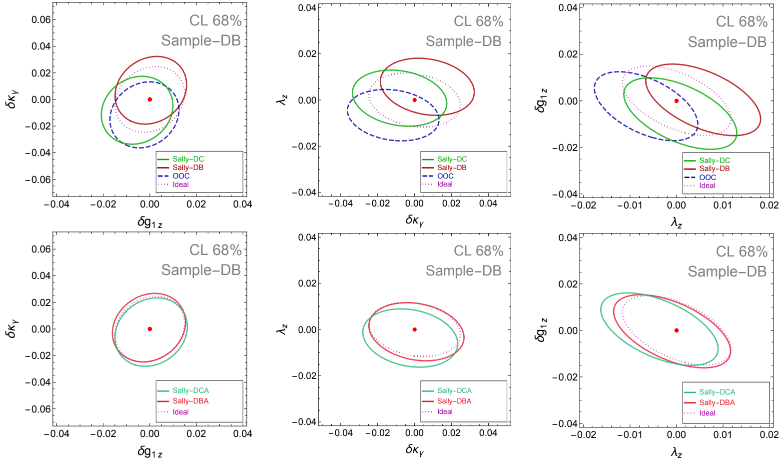
- Naively applying truth-level optimal observables to detector-level samples could lead to a large bias!
- ML model trained on detector-level samples (Sally-DA) automatically take care of the detector (and ISR) effects and are more robust.

3-aTGC fit, detector-level sample with background



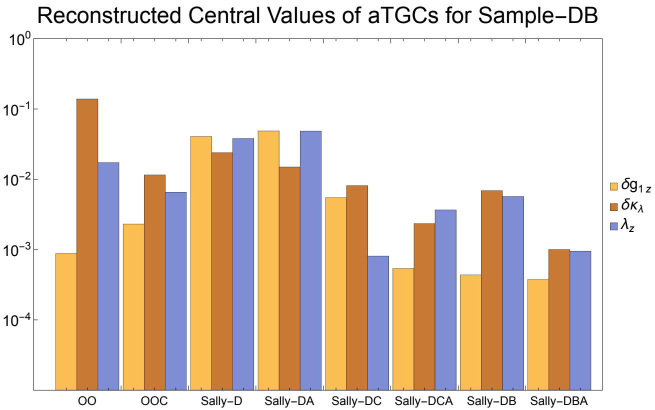
- 10% *ZZ* background. A large bias can be introduced if we failed to take account of it!
- SALLY-DBA: trained with both signal and backgrounds with the correct weighting to reconstruct the $\hat{\alpha}(x)$ for the combined differential cross section.

Comparisons between methods



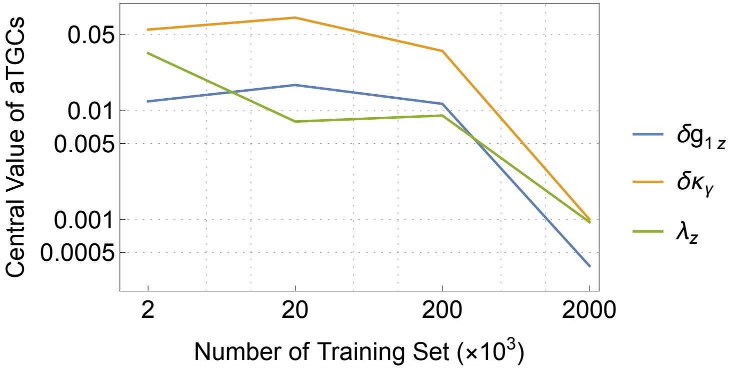
- OOC, Sally-DC(A): Optimal observables and Sally-D(A) combined with a classifier.
- A: averaged over 8 models to reduce the noises from the training phase.

Comparison of biases (reconstructed central values)



- For the detector-level sample with background, Sally-DBA has the least bias.

bias vs. training sample size

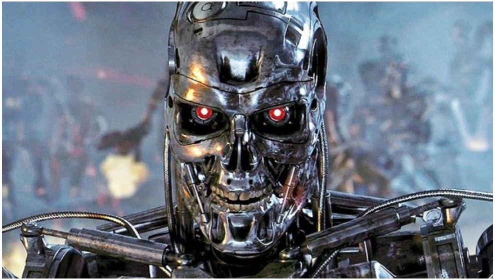


- The current bias is still unacceptable for future colliders with $\sim 10^8$ *WW* events.
- Hopefully with more computing resources in the future, the bias can be reduced to the desired level.

Conclusion

- **We have no idea what is the new physics beyond the Standard Model.**
- **One important direction to move forward is to do precision measurements of the Standard Model processes.**
  - HL-LHC is ok, but a future lepton collider is better!
  - SMEFT is a good theory framework (but is not everything).
- **Machine learning is** (likely to be) **the future!**
  - High precision $\Rightarrow$ high demand on reducing biases/systematics to the same level.
  - ML helps take care of the detector/ISR/background effects.

Conclusion



- **When will Machine take over?**
  - Before or after a future lepton collider is built?
- **Many more studies to do!**
  - Di-leptonic & fully hadronic channels.
  - Other processes, *e.g.* $e^+e^- \to t\bar{t}$ (current work with Yifan Fei, Tong Shen, Kerun Yu), ...
  - In reality, MC simulation does not perfectly describe data...

backup slides

SMEFT parameterization of $e^+e^- \to W^+W^-$

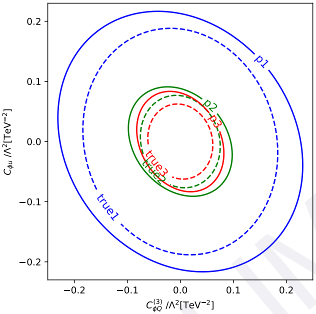
Tree-level dim-6 CP-even operators: 6 parameters (excluding modifications in m_W):

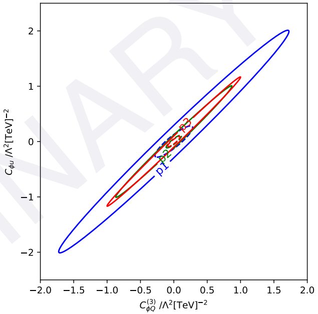
$$\delta g_{1Z}\,, \quad \delta\kappa_\gamma\,, \quad \lambda_Z\,, \quad \delta g_W^\ell\,, \quad \delta g_{Z,L}^e\,, \quad \delta g_{Z,R}^e\,. \tag{1}$$

$$\begin{aligned}\mathcal{L}_{\rm TGC} = \; & ie(W^+_{\mu\nu}W^{-\mu} - W^-_{\mu\nu}W^{+\mu})A^\nu + ie(1+\delta\kappa_\gamma)A^{\mu\nu}W^+_\mu W^-_\nu \\ & + igc_w\left[(1+\delta g_{1Z})(W^+_{\mu\nu}W^{-\mu} - W^-_{\mu\nu}W^{+\mu})Z^\nu + (1+\delta g_{1Z} - \frac{s_w^2}{c_w^2}\delta\kappa_\gamma)\,Z^{\mu\nu}W^+_\mu W^-_\nu\right] \\ & + \frac{ig\lambda_Z}{m_W^2}\left(s_w W^{+\nu}_\mu W^{-\rho}_\nu A^\mu_\rho + c_w W^{+\nu}_\mu W^{-\rho}_\nu Z^\mu_\rho\right)\,,\end{aligned} \tag{2}$$

$$\begin{aligned}\mathcal{L}_{Vff} = & -\frac{g}{\sqrt{2}}(1+\delta g_W^\ell)\left[W^+_\mu \bar{\nu}_L\gamma^\mu e_L + \text{h.c.}\right] \\ & -\frac{g}{c_w}Z_\mu\left[\bar{e}_L\gamma^\mu(-\frac{1}{2} + s_w^2 + \delta g_{Z,L}^e)e_L + \bar{e}_R\gamma^\mu(s_W^2 + \delta g_{Z,R}^e)e_R\right] + \ldots\,,\end{aligned} \tag{3}$$

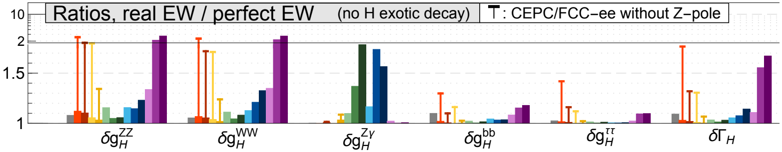
Machine Learning in $e^+e^- \to t\bar{t}$ (very preliminary results, Yifan Fei, JG, Tong Shen, Kerun Yu)



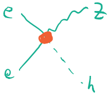


- $e^+e^- \to t\bar{t}$, 3 different channels (no background yet)
- **Left:** $\sqrt{s}=1\,\text{TeV}$, **Right:** $\sqrt{s}=360\,\text{GeV}$

Impacts of (lack of) the Z-pole run



- Without good *Z*-pole measurements, the *eeZh* contact interaction may have a significant impact on the Higgs coupling determination.
- Current (LEP) *Z*-pole measurements are not good enough for CEPC/FCC-ee Higgs measurements!
  - **A future Z-pole run is important!**



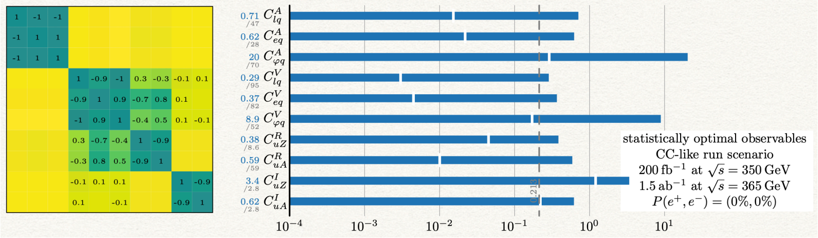
- Linear colliders suffer less from the lack of a Z-pole run. **(Win Win!)**

Probing Top operators with $e^- e^+ \to t\bar{t}$ [arXiv:1807.02121] Durieux, Perelló, Vos, Zhang

$$
\begin{aligned}
O^1_{\varphi q} &\equiv \tfrac{y_t^2}{2}\ \bar{q}\gamma^\mu q\ \ \varphi^\dagger i\overleftrightarrow{D}_\mu\varphi, & O_{uG} &\equiv y_t g_s\ \bar{q}T^A\sigma^{\mu\nu}u\ \epsilon\varphi^* G^A_{\mu\nu},\\
O^3_{\varphi q} &\equiv \tfrac{y_t^2}{2}\ \bar{q}\tau^I\gamma^\mu q\ \ \varphi^\dagger i\overleftrightarrow{D}^I_\mu\varphi, & O_{uW} &\equiv y_t g_W\ \bar{q}\tau^I\sigma^{\mu\nu}u\ \epsilon\varphi^* W^I_{\mu\nu},\\
O_{\varphi u} &\equiv \tfrac{y_t^2}{2}\ \bar{u}\gamma^\mu u\ \ \varphi^\dagger i\overleftrightarrow{D}_\mu\varphi, & O_{dW} &\equiv y_t g_W\ \bar{q}\tau^I\sigma^{\mu\nu}d\ \epsilon\varphi^* W^I_{\mu\nu},\\
O_{\varphi ud} &\equiv \tfrac{y_t^2}{2}\ \bar{u}\gamma^\mu d\ \ \varphi^T\epsilon\, iD_\mu\varphi, & O_{uB} &\equiv y_t g_Y\ \bar{q}\sigma^{\mu\nu}u\ \epsilon\varphi^* B_{\mu\nu},
\end{aligned}
$$

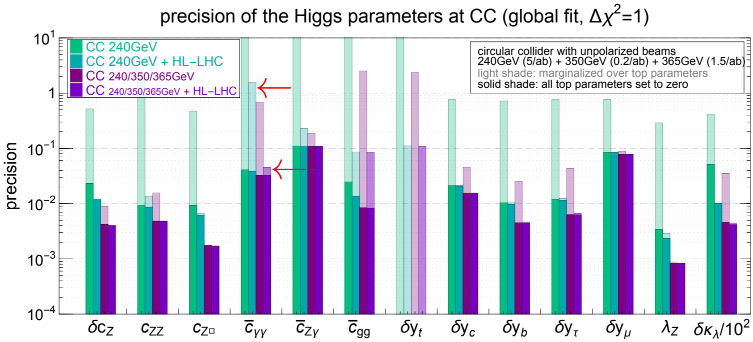
$$
\begin{aligned}
O^1_{lq} &\equiv \tfrac{1}{2}\ \bar{q}\gamma_\mu q\ \ \bar{l}\gamma^\mu l,\\
O^3_{lq} &\equiv \tfrac{1}{2}\ \bar{q}\tau^I\gamma_\mu q\ \ \bar{l}\tau^I\gamma^\mu l,\\
O_{lu} &\equiv \tfrac{1}{2}\ \bar{u}\gamma_\mu u\ \ \bar{l}\gamma^\mu l,\\
O_{eq} &\equiv \tfrac{1}{2}\ \bar{q}\gamma_\mu q\ \ \bar{e}\gamma^\mu e,\\
O_{eu} &\equiv \tfrac{1}{2}\ \bar{u}\gamma_\mu u\ \ \bar{e}\gamma^\mu e,
\end{aligned}
$$

- Also need to include **top dipole** interactions and *eett* contact interactions!
- Hard to resolve the top couplings from $4f$ interactions with just the 365 GeV run.
  - Can't really separate $e^+e^- \to Z/\gamma \to t\bar{t}$ from $e^+e^- \to Z' \to t\bar{t}$.
  - Is that a big deal?

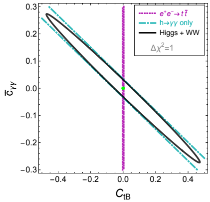


Top operators in loops (Higgs processes)

[1809.03520] G. Durieux, JG, E. Vryonidou, C. Zhang



- $O_{tB} = (\bar{Q}\sigma^{\mu\nu} t)\, \tilde{\varphi} B_{\mu\nu} + h.c.$ is not very well constrained at the LHC, and it generates dipole interactions that contributes to the $h\gamma\gamma$ vertex.
- Deviations in $h\gamma\gamma$ coupling $\Rightarrow$ run at ~ 365 GeV to confirm?



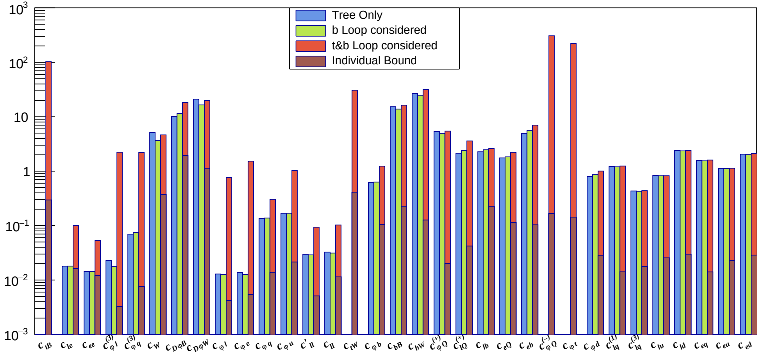
Top operators in loops (current EW processes)

[2205.05655] Y. Liu, Y. Wang, C. Zhang, L. Zhang, JG

	Experiment	Observables
Low Energy	CHARM/CDHS/ CCFR/NuTeV/ APV/QWEAK/ PVDIS	Effective Couplings
Z-pole	LEP/SLC	Total decay width Γ_Z Hadronic cross-section σ_{had} Ratio of decay width R_f Forward-Backward Asymmetry A_{FB}^f Polarized Asymmetry A_f
W-pole	LHC/Tevatron/ LEP/SLC	Total decay width Γ_W W branching ratios $Br(W \to l\nu_l)$ Mass of W Boson M_W
$ee \to qq$	LEP/TRISTAN	Hadronic cross-section σ_{had} Ratio of cross-section R_f Forward-Backward Asymmetry for b/c A_{FB}^f
$ee \to ll$	LEP	cross-section σ_f Forward-Backward Asymmetry A_{FB}^f Differential cross-section $\frac{d\sigma_f}{dcos\theta}$
$ee \to WW$	LEP	cross-section σ_{WW} Differential cross-section $\frac{d\sigma_{WW}}{dcos\theta}$

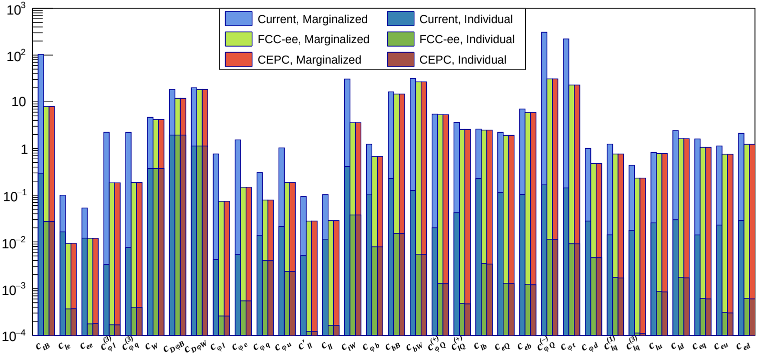
- Top operators (1-loop) + EW operators (tree, including bottom dipole operators)
- $e^+e^- \to f\bar{f}$ at different energies, $e^+e^- \to W^+W^-$.

Top operators in loops (current EW processes)



- Good sensitivities, but too many parameters for a global fit...

Top operators in loops (future EW processes)



- Good sensitivities, but too many parameters for a global fit...
- It shows the importance of directly measuring $e^+e^- \to t\bar{t}$.

Probing dimension-8 operators?

- The dimension-8 contribution has a large energy enhancement ($\sim E^4/\Lambda^4$)!
- It is difficult for LHC to probe these bounds.
  - Low statistics in the high energy bins.
  - Example: Vector boson scattering.
  - $\Lambda \lesssim \sqrt{s}$, the EFT expansion breaks down!
- Can we separate the dim-8 and dim-6 effects?
  - Precision measurements at several different $\sqrt{s}$?

    (A **very** high energy lepton collider?)
  - Or find some special process where dim-8 gives the leading new physics contribution?

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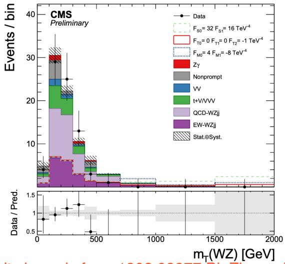
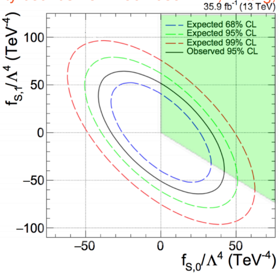


positivity bounds from 1902.08977 Bi, Zhang, Zhou



The diphoton channel [arXiv:2011.03055] Phys.Rev.Lett. 129, 011805, JG, Lian-Tao Wang, Cen Zhang

- $e^+e^- \to \gamma\gamma$ (or $\mu^+\mu^- \to \gamma\gamma$), SM, non-resonant.
- Leading order contribution: **dimension-8 contact interaction**.
  ($f^+f^- \to \bar{e}_L e_L$ or $e_R \bar{e}_R$)

$$\mathcal{A}(f^+f^-\gamma^+\gamma^-)_{\mathrm{SM+d8}} = 2e^2\frac{\langle 24\rangle^2}{\langle 13\rangle\langle 23\rangle} + \frac{a}{v^4}[13][23]\langle 24\rangle^2 \,.$$

- Can probe dim-8 operators (and their positivity bounds) at a **Higgs factory** (~ 240 GeV)!