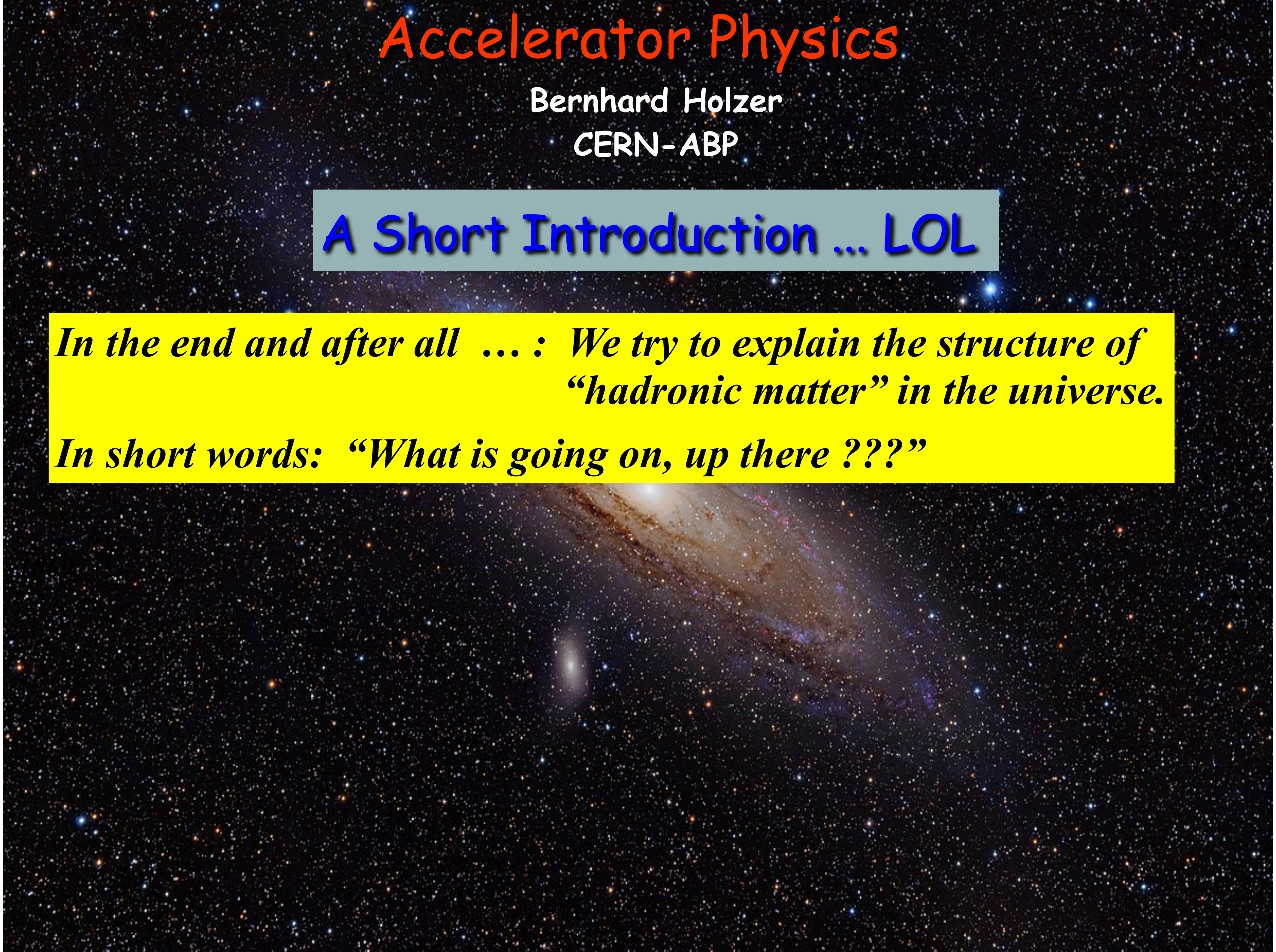
Accelerator Physics

Bernhard Holzer
CERN-ABP

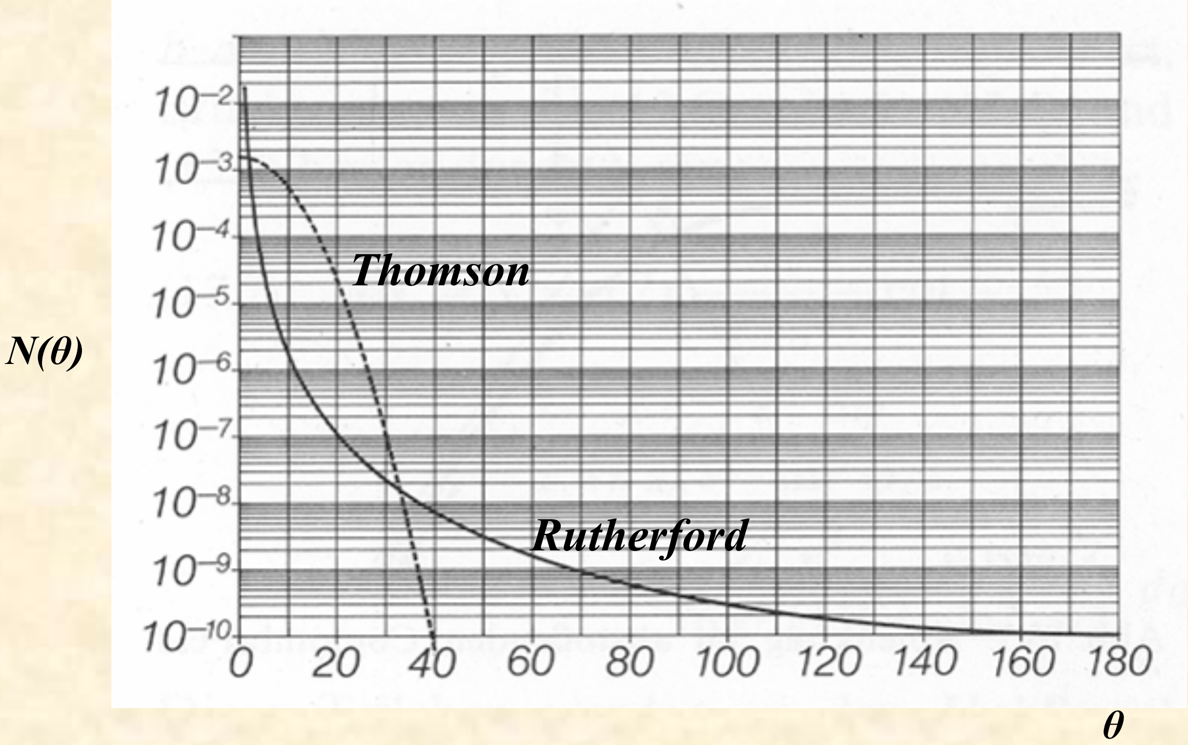
A Short Introduction … LOL

In the end and after all … : We try to explain the structure of "hadronic matter" in the universe.

In short words: "What is going on, up there ???"



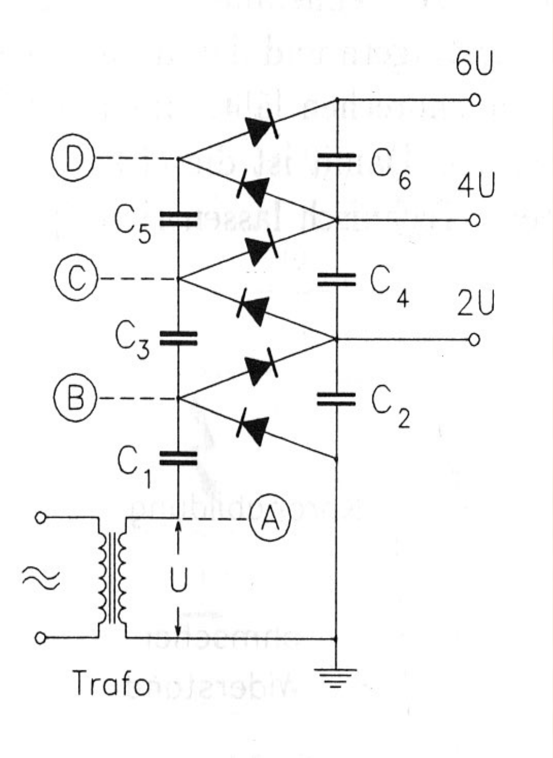
$$N(\theta) = \frac{N_i ntZ^2 e^4}{(8\pi\varepsilon_0)^2 r^2 K^2} * \frac{1}{\sin^4(\theta/2)}$$



Rutherford Scattering, 1911

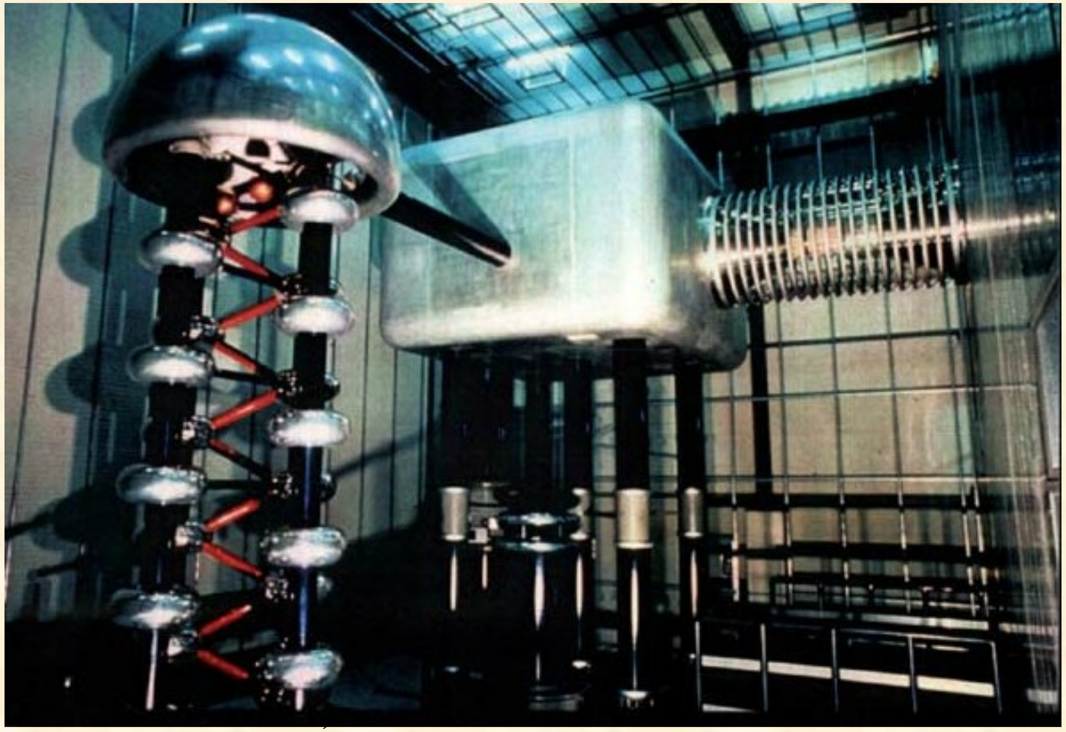
Using radioactive particle sources:
α-particles of some MeV energy

1.) Electrostatic Machines:
The Cockcroft-Walton Generator



1928: Encouraged by Rutherford Cockcroft and Walton start the design & construction of a high voltage generator to accelerate a proton beam

1932: First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV



Particle source:	Hydrogen discharge tube on 400 kV level
Accelerator:	evacuated glas tube
Target:	Li-Foil on earth potential

Technically: rectifier circuit, built of capacitors and diodes (Greinacher)

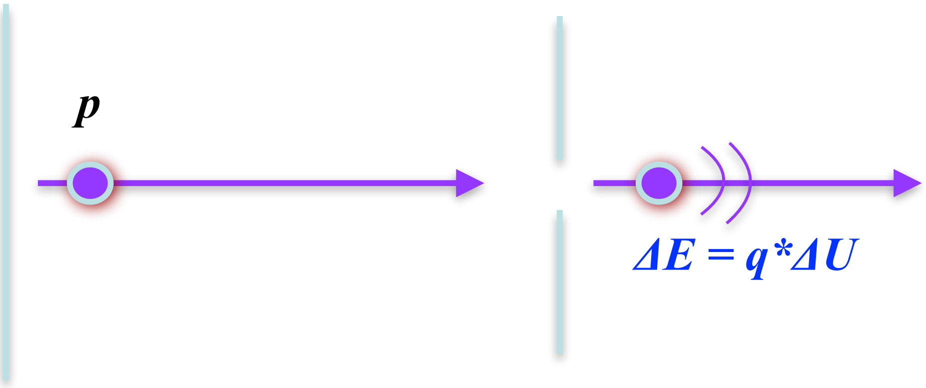
Problem:
DC Voltage can only be used once

2.) Electrostatic Machines:
van de Graaff Accelerator (1930 ...)

creating high voltages by mechanical transport of charges

** Terminal Potential: $U \approx 12 \ldots 28$ MV*
using high pressure gas to suppress discharge (SF_6)

Das Prinzip:

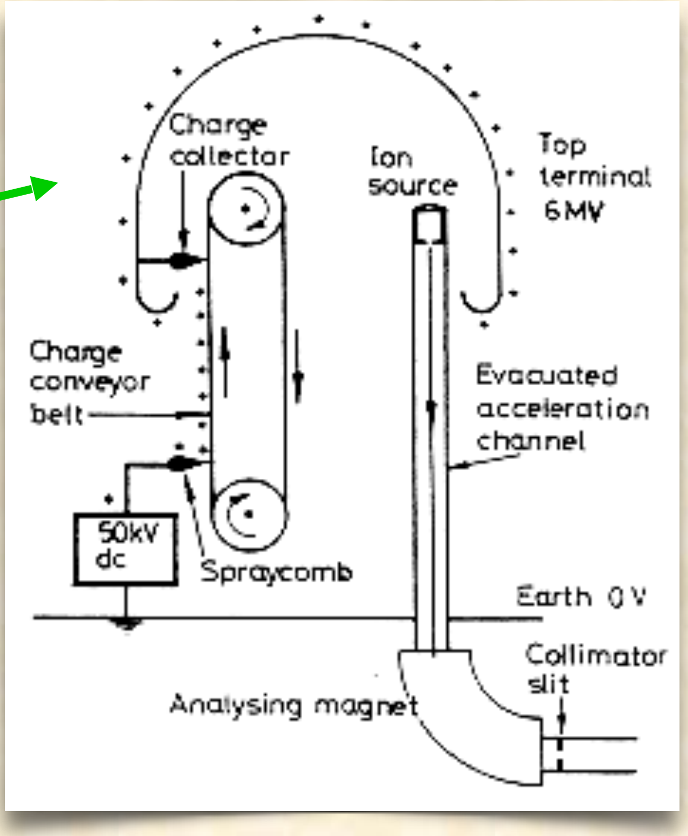


p

$\Delta E = q * \Delta U$

U = 12 MV

U = 0



*Energie=Ladung * Spannung (Differenz)*

Problems: * *Particle energy limited by high voltage discharges*
* *high voltage can only be applied once per particle ...*
... or twice ?

The „Tandem principle“: *Apply the accelerating voltage twice ...*

... by working with negative ions (e.g. H -) and

stripping the electrons in the centre of the structure

Example for such a „steam engine“: 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

Gretchen Frage (J.W. Goethe, Faust)

Fallen die Dinger eigentlich runter ?

$$l_{vdG} = 30m$$

$$v \approx 10\,\%\,c \approx 3 * 10^7 \; m/s$$

$$\Delta t = 1\mu s$$

Free Fall in Vacuum:

$$s = \frac{1}{2} g t^2$$

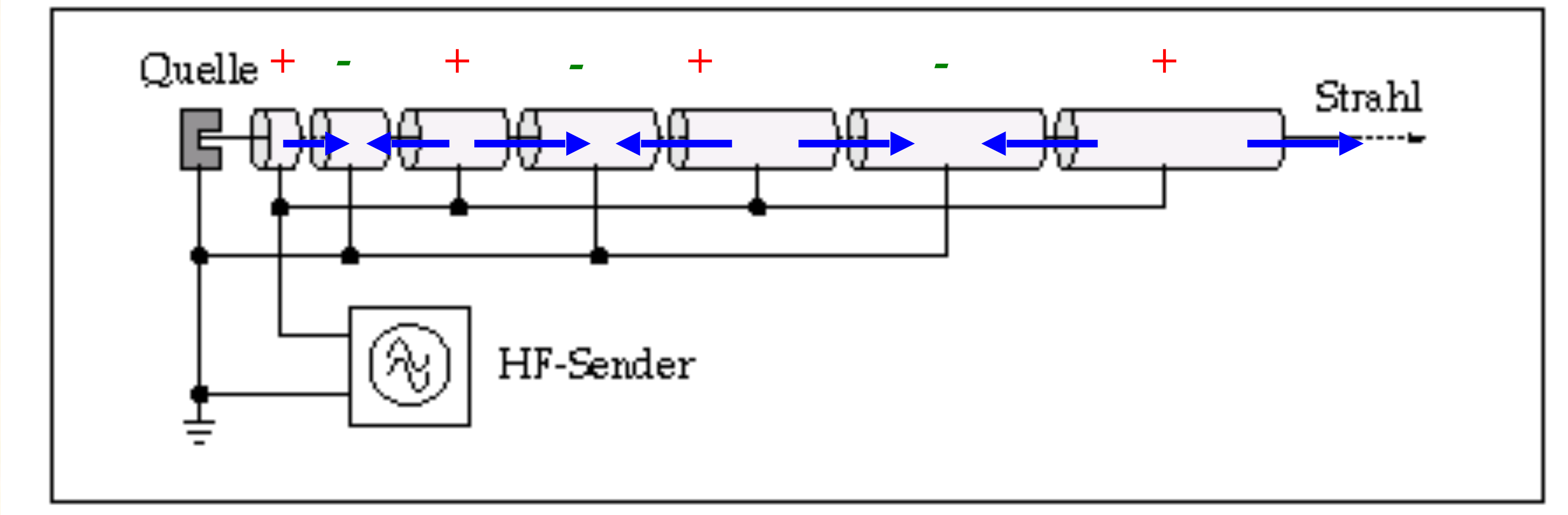
$$s = \frac{1}{2} \cdot 10 \frac{m}{s^2} \cdot (1\mu s)^2$$

$$s = 5 \cdot 10^{-12} m = 5pm$$

3.) The first RF-Accelerator: „Linac"

1928, Wideroe: ***how can the acceleration voltage be applied several times to the particle beam***

schematic Layout:



Energy gained after n acceleration gaps

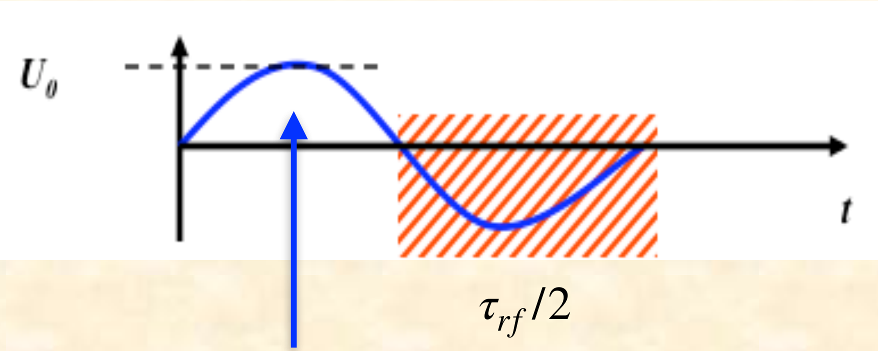
$$E_n = \boxed{n} \cdot q \cdot U_0 \cdot \boxed{\sin \psi_s}$$

n *number of gaps between the drift tubes*
q *charge of the particle*
U_0 *Peak voltage of the RF System*
Ψ_S *synchronous phase of the particle*

**** acceleration of the proton in the first gap***
**** voltage has to be „flipped" to get the right sign in the second gap → RF voltage***
→ shield the particle in drift tubes during the negative half wave of the RF voltage

Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



idealer Zeitpunkt
90 grad —> sin(90⁰)=1

Time span of the negative half wave: $\tau_{rf}/2$

Length of the Drift Tube: $l_n = v_n \cdot \frac{\tau_{rf}}{2}$

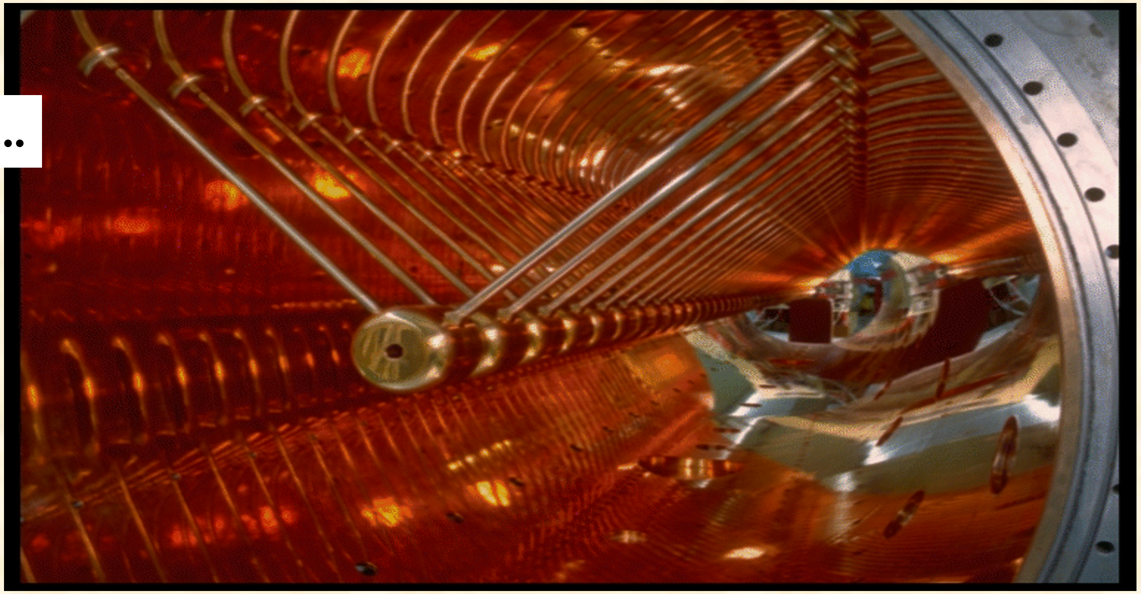
Kinetic Energy of the Particles $E_n = \frac{1}{2}mv^2 \longrightarrow v_n = \sqrt{2E_n/m}$

mit der kin. Energie $E_n = n \cdot q \cdot U_0 \cdot sin\ \psi_s$

ergibt das
$$v_n = \sqrt{\frac{2 \cdot n \cdot q \cdot U_0 \cdot sin(\psi_s)}{m}}$$

Bauplan fuer einen Wideroe Beschleuniger: $l_n = v_n \cdot \frac{\tau_{rf}}{2} = \frac{1}{f_{rf}} \cdot \sqrt{\frac{n \cdot q \cdot U_0 \cdot sin\psi_s}{2m}}$

Und so sieht das innen drinnen aus:



Achtung !!!   valid for non relativistic particles ...

Energy: ≈ 20 MeV per Nucleon

$\beta = v/c \approx$ ***0.04 … 0.6, Particles: Protons/Ions***

Zahlenbeispiel:

total energy $E_{total} = E_{kin} + m_0c^2$

kinetic energy $E_{kin} = E_{total} - m_0c^2$

Ruhe-Energie $E_0 = m_0c^2$

Linac III:

$E_{total} = 988\ MeV$

$m_0c^2 = 938\ MeV$

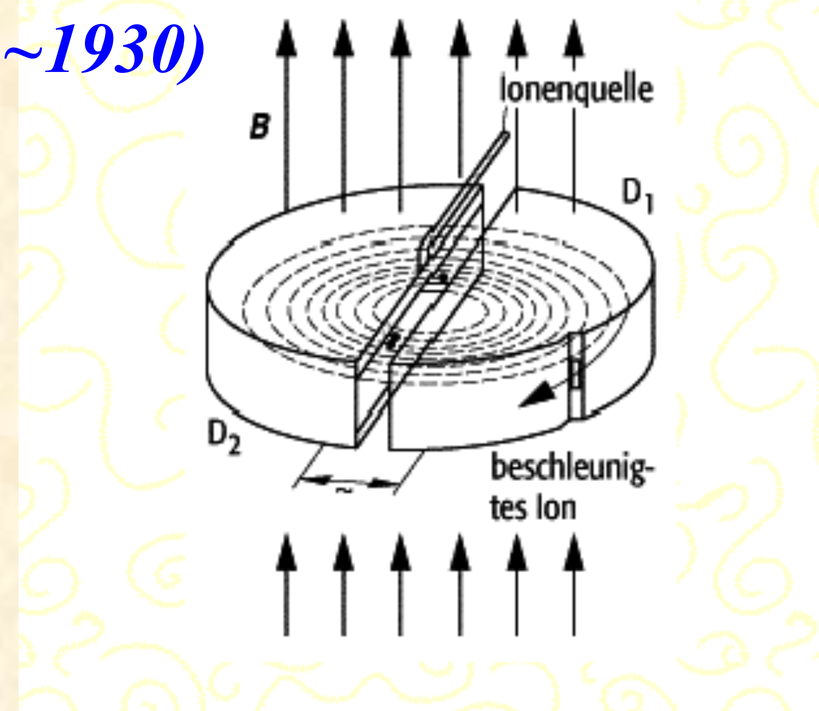
$E_{kin} = 50\ MeV$

man erinnert sich: $m \to \gamma \cdot m_0$

$$\gamma = \frac{E_{ges}}{E_0} = \frac{988}{938} = 1.05$$

—> im klassischen Bereich

3.) The Cyclotron: (Livingston / Lawrence ~1930)



Problem:
Linacs werden bei v=c sehr schnell sehr lanngggg.

—> Man erhaelt ne kompakte (d.h. billigere) Maschine, wenn man den Orbit der Teilchen aufwickelt.

Idea: Apply a magnetic field: $B = const$

Lorentzforce

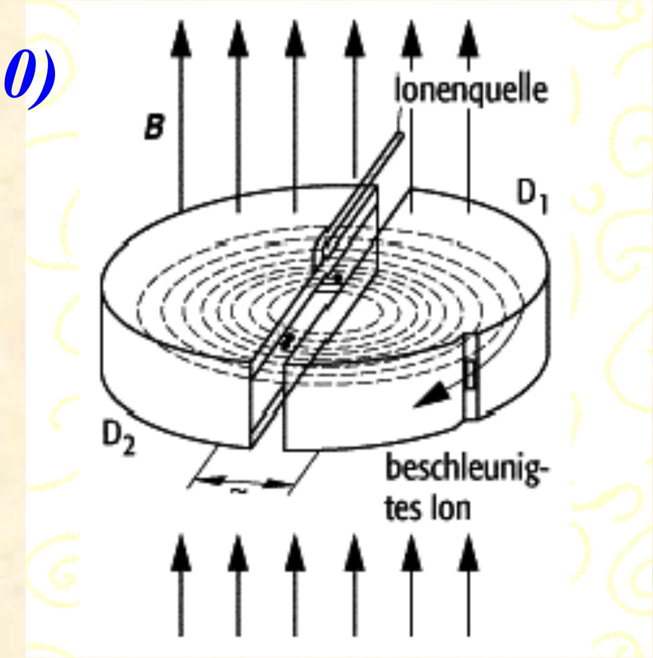
$$F = q \cdot v \cdot B$$

geladene Teilchen in Bewegung werden im Magnetfeld abgelenkt.

Kreisbahn-Bedingung:
Zentrifugalkraft wird durch die entgegengesetzte Lorentz-Kraft aufgehoben.

$$\left.\begin{array}{l} F_{Lorentz} = F_{zentrifugal} \\ q \cdot v \cdot B = \frac{mv^2}{r} \end{array}\right\} \quad B \cdot R = \frac{mv}{q} \longrightarrow \boxed{B \cdot R = \frac{p}{q}}$$

3.) The Cyclotron: (Livingston / Lawrence ~1930)



increasing radius for increasing momentum ➔ Spiral Trajectory

revolution frequency

$$\omega_{revol} = \frac{v}{r} = \frac{q}{m} \cdot B = \quad const!!!$$

Die Umlaufs-frequenz im Cyclotron ist konstant.
Wir lassen eine gleich-grosse konstante RF frequenz
auf die Teilchen los und die Kiste funktioniert.

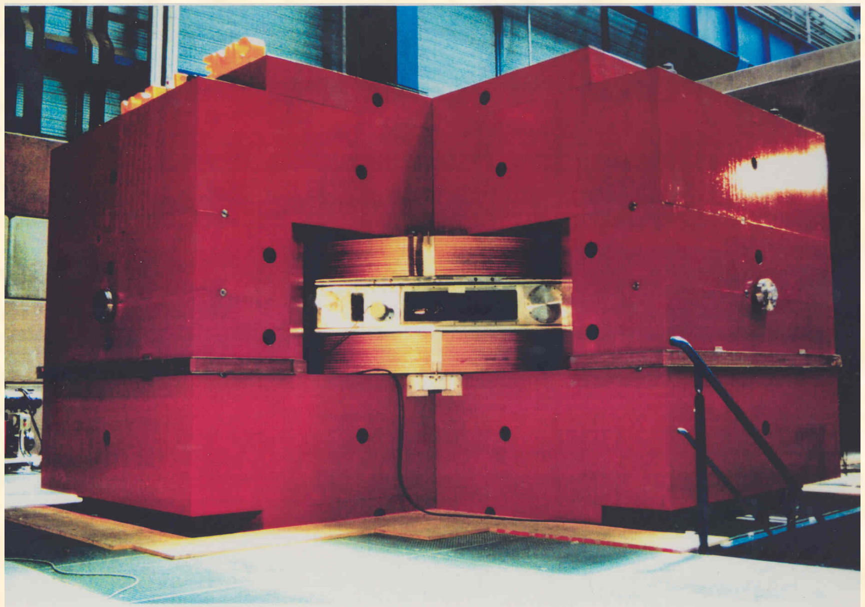
$\omega_{rf} = \omega_{revolution}$ **oder** $\omega_{rf} = h \cdot \omega_{revolution}$

Problem: Albert !!!

$$m \rightarrow \gamma \cdot m_0$$

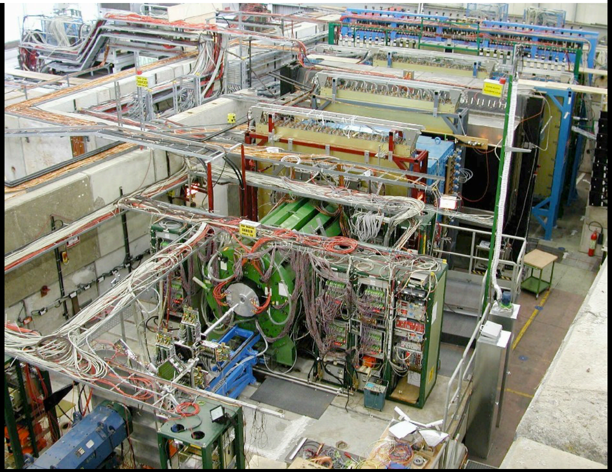
$$\omega_{revol} = \frac{q}{\boxed{\gamma\ m}} \neq \ const$$

Synchro-Cyclotron
Korrektur der RF
Frequenz

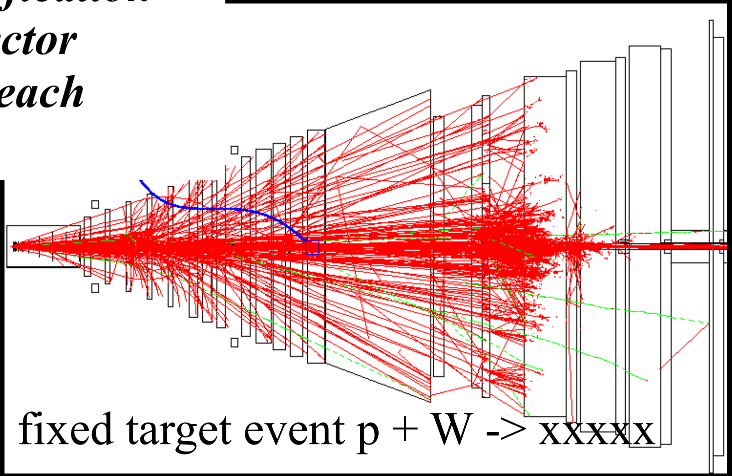


Particle Colliders:

Fixed target experiments:

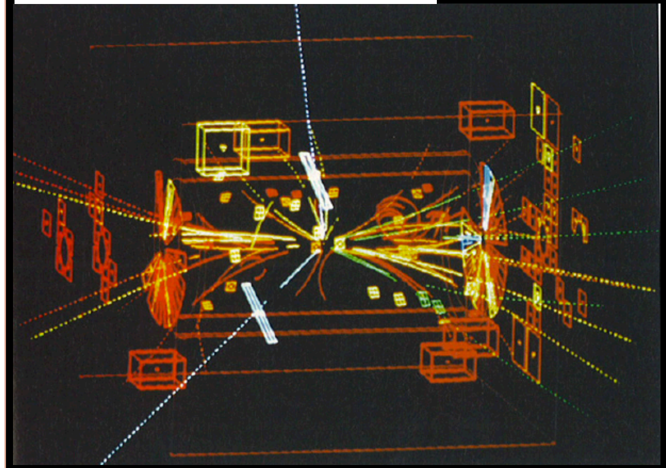


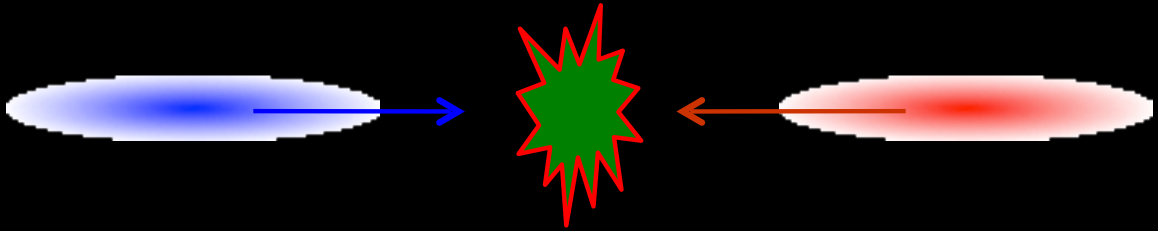
high event rate
easy track identification
asymmetric detector
limited energy reach



fixed target event p + W -> xxxxx

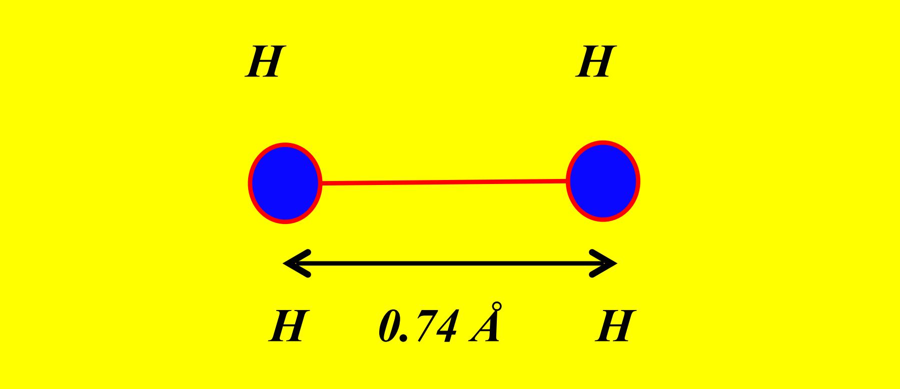
Particle Collider:





low event rate (luminosity)
challenging track identification
symmetric detector
$E_{lab} = E_{cm}$

Particle Density in matter



Atomic Distance in Hydrogen Molecule

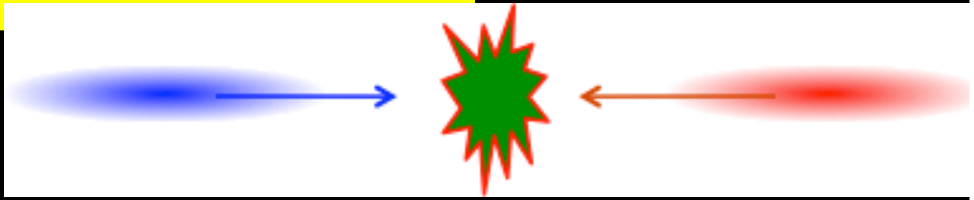
$R_B \approx 0.5$ Å

in solids / fluids $\lambda \approx 1 \ldots 3$ Å $(1 \ldots 3 * 10^{-10}$ m$)$

in gases $\lambda \approx 35$ Å $= 3.5$ nm

Particle Distance in Accelerators: $\lambda \approx 600$ nm (Arc) … 300nm (IP LEP)

$= 3000$ Å



II.) A Bit of Theory

The big storage rings: „Synchrotrons“

1.) Introduction and Basic Ideas

„ … in the end and after all it should be a kind of circular machine“

→ ***need transverse deflecting force***

Lorentz force

$$\vec{F} = q * (\cancel{\vec{E}} + \vec{v} \times \vec{B})$$

typical velocity in high energy machines:

$$v \approx c \approx 3 * 10^{8}\ {}^{m}\!/_{s}$$

Example:

$$B = 1\,T \quad \rightarrow \quad F = q * 3 * 10^{8} \frac{m}{s} * 1 \frac{Vs}{m^{2}}$$

$$F = q * \underbrace{300 \frac{MV}{m}}_{E}$$

equivalent electrical field:

Technical limit for electrical fields:

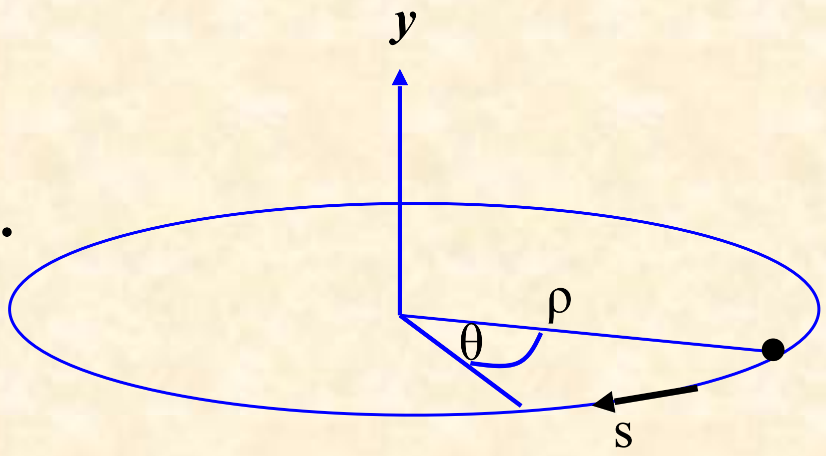
$$E \leq 1 \frac{MV}{m}$$

Ein Speicherring besteht aus
Magneten, Magneten und Magneten

***und ein wenig Vakuum-Kammern, Strahldiagnose,** und RF Systemen*

The ideal circular orbit

… das hatten wir schon.



circular coordinate system

condition for circular orbit:

Lorentz force $F_L = e\,v\,B$

centrifugal force $F_{centr} = \dfrac{\gamma\, m_0\, v^2}{\rho}$

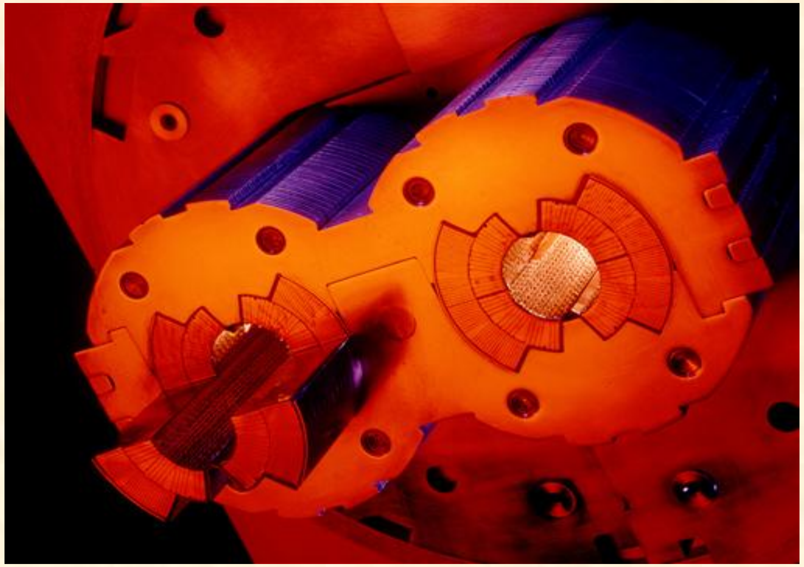
$$\frac{\gamma\, m_0\, v^{\cancel{2}}}{\rho} = e\,\cancel{v}\,B$$

$$\frac{p}{e} = B\,\rho$$

B ρ = "beam rigidity"

… und jetzt isses sogar relativistisch korrekt.

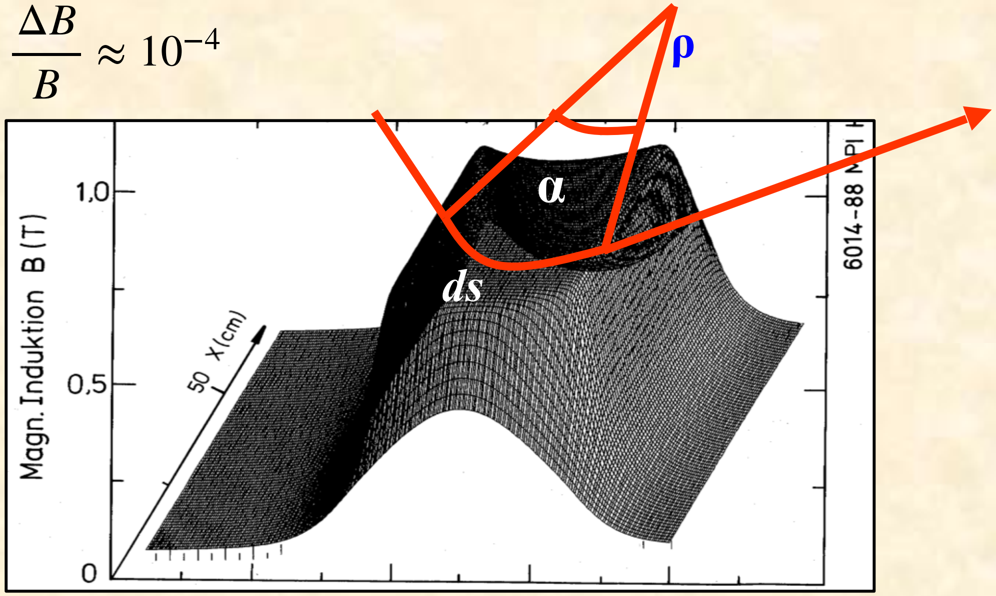
The Magnetic Guide Field



$B \approx 1 \dots 8\ T$

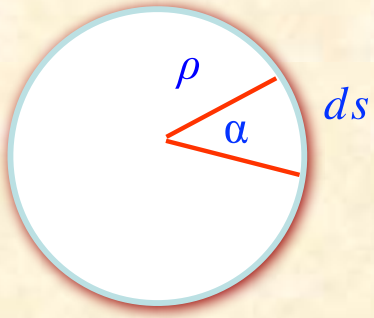
Dipole erzeugen ein konstantes (!) Magnetfeld

$$\frac{\Delta B}{B} \approx 10^{-4}$$



field map of a storage ring dipole magnet

„wieviele Dipole sollen's denn sein ???



Ablenkwinkel eines Dipols:

$$\alpha_{dipol} = \frac{ds}{\rho} = \frac{\int B\ ds}{B\ \rho} \approx \frac{B \cdot l_{dipol}}{B\ \rho}$$

Anzahl Dipol Magnete:

$$N_{dipole} = \frac{2\pi}{\alpha_{dipol}} = \ 1232\ !!!$$

Umfang des Speicherrings:

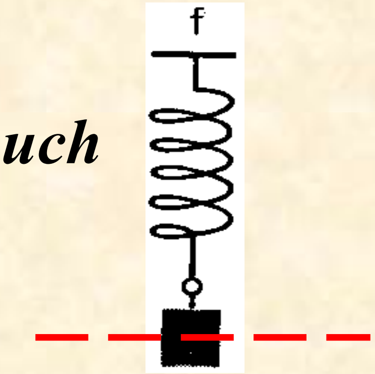
$$C_0 = 2\pi \cdot \rho$$

Ablenkradius:

$$\rho = 2.8\ km$$

2.) Focusing Forces: Hook's law

Federpendel im Physik Buch



there is a restoring force, proportional to the elongation x:

$$F = m * a = -\, const * x$$

$$F = m * \frac{d^2x}{dt^2} = -\, const * x$$

Hook's Federgesetz: $F = -\, k * x$

Integration liefert uns eine cos- artige Lösung $x(t) = A \cdot cos(\omega t)$

oder eine sinus artige $x(t) = B \cdot sin(\omega t)$

oder eine Kombination aus beiden $x_{allg}(t) = A \cdot cos(\omega t) + B \cdot sin(\omega t)$

Vorteil:

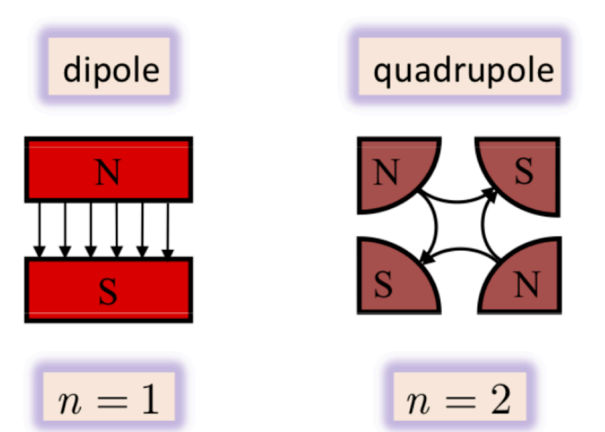
harmonische Schwingungen sind sehr (!!) stabil,
haben eine wohldefinierte Frequenz
sind in der Natur (i.e. Physik) weit verbreitet

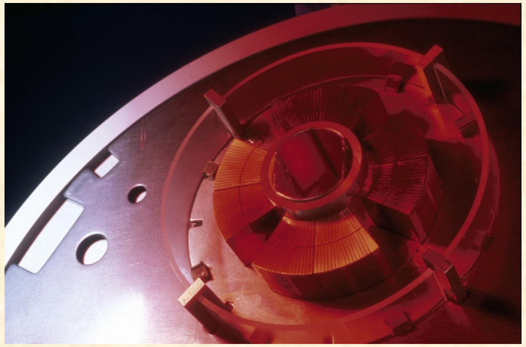
2.)Focusing Forces: Quadrupole Fields

Apply this concept to magnetic forces: we need a Lorentz force that rises as a function of the distance to ...

… the design orbit

$$F(x) = q * v * B(x)$$





Dipoles: Create a constant field $B_y = const$

Quadrupoles: Create a linear increasing magnetic field: $B_y(x) = g \cdot x, \quad B_x(y) = g \cdot y$

Focusing forces and particle trajectories:

normalise magnet fields to momentum
(remember: $B*\rho = p / q$)

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

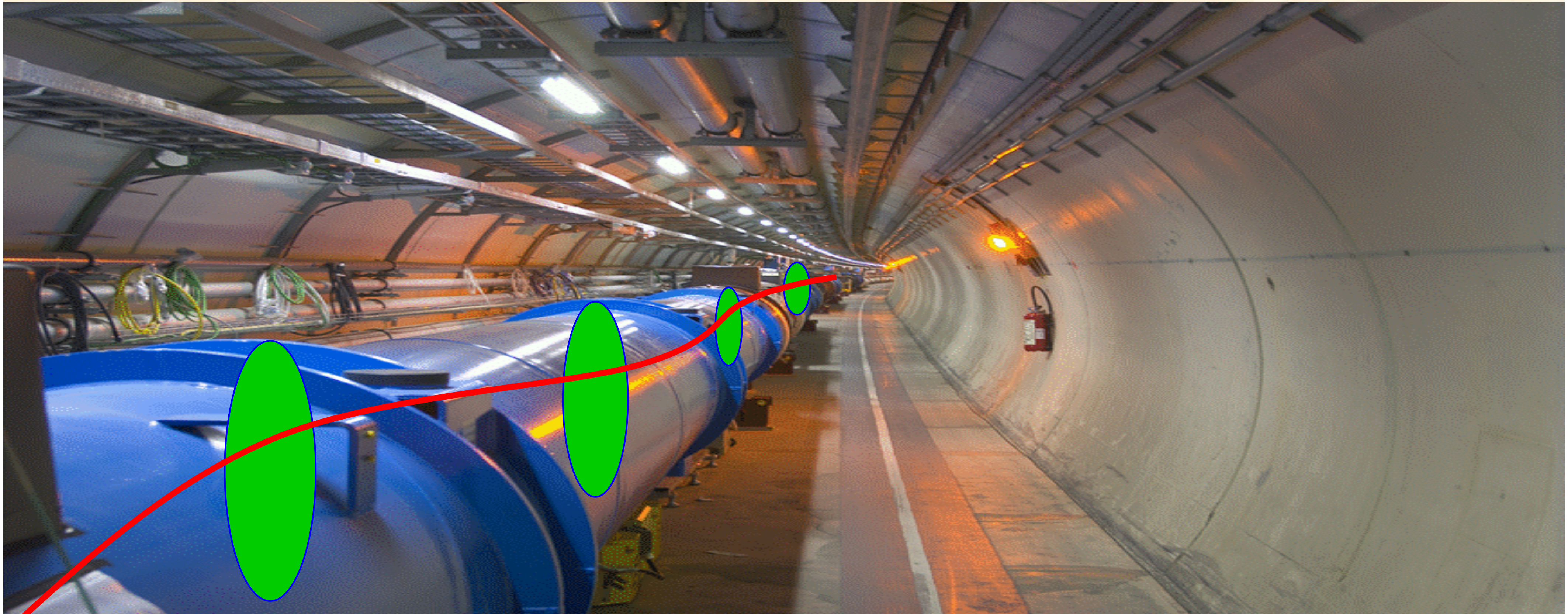
Quadrupole Magnet

$$k := \frac{g}{p/q}$$

Achtung:
um Energie unabhängige
Gleichungen zu erhalten teilen wir die
Felder durch "p"

„normalised bending strength"

„normalised gradient"



Soll das heissen, die Teilchen verhalten sie wie Omas Pendeluhr, während sie mit Lichtgeschwindigkeit um die Maschine rasen ???

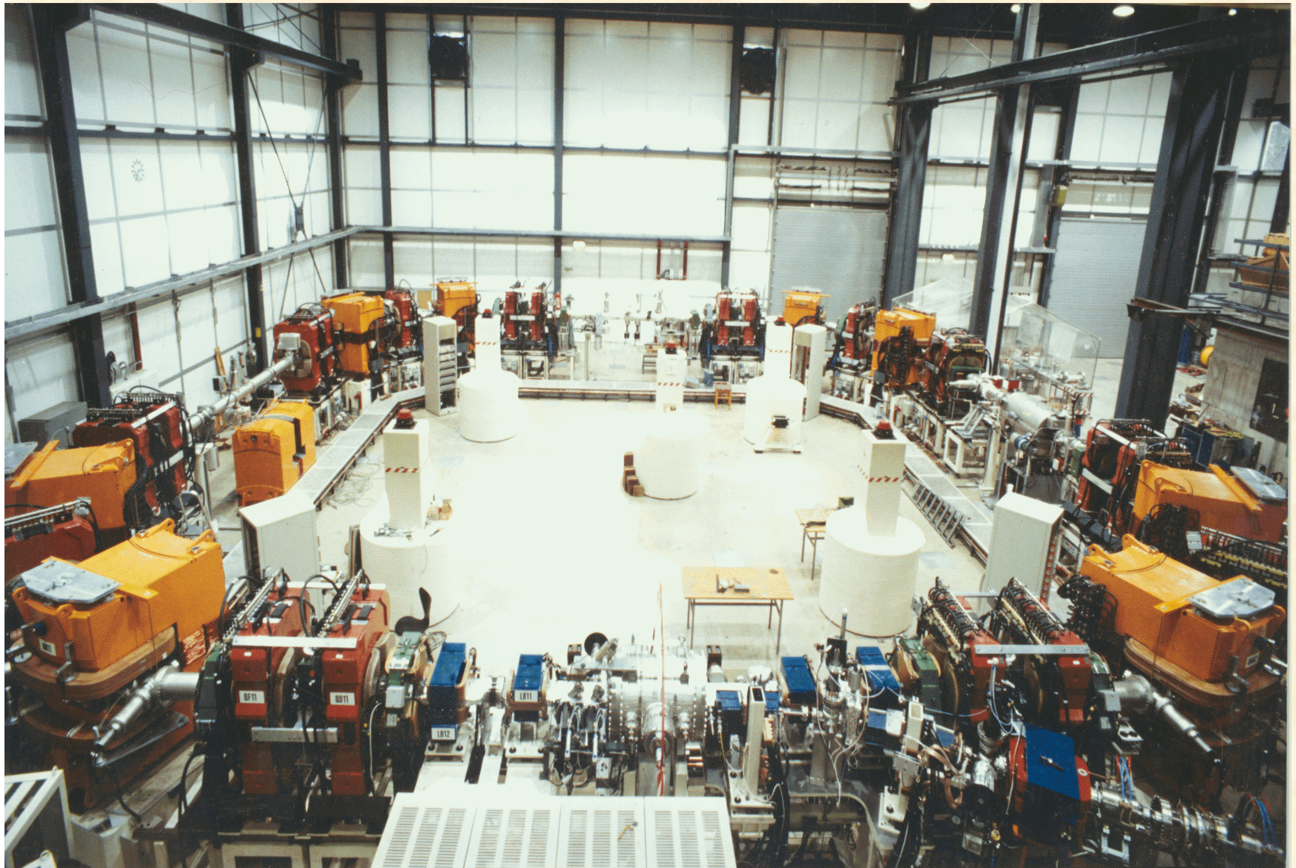
JA !

3.) The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \xcancel{\frac{1}{2!}\,m}\,x^{2} + \xcancel{\frac{1}{3!}\,n}\,x^{3} + \dots$$

only terms linear in x, y taken into account *dipole fields*

quadrupole fields



Separate Function Machines:

Split the magnets and optimise them according to their job:

bending, focusing etc

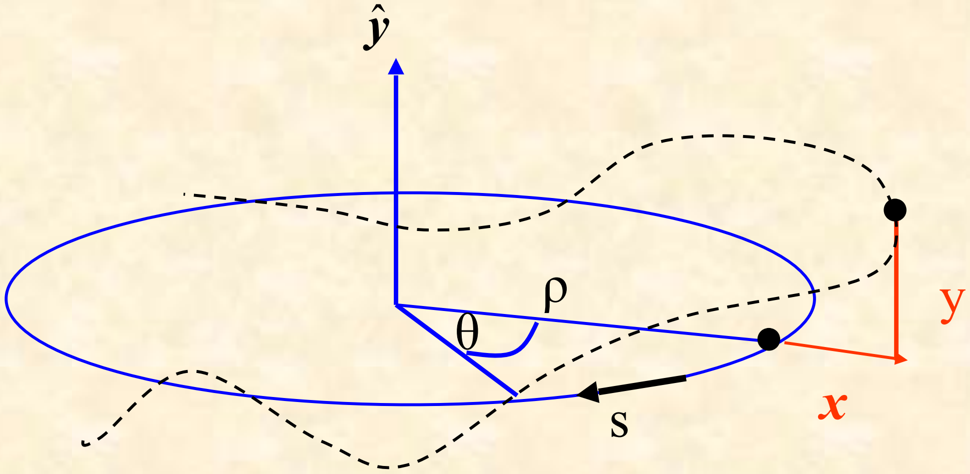
Example:

heavy ion storage ring TSR

* ***man sieht nur***

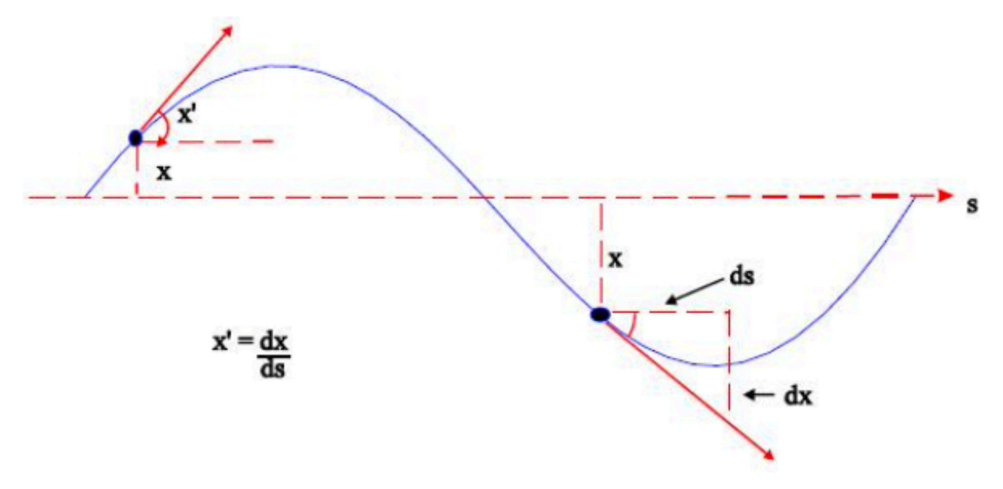
dipole und quads → ***linear***

The Equation of Motion:



* *Equation for the* ***horizontal motion:***

$$x'' + x \cdot k = 0$$



x =   particle amplitude

x' = dx/ds angle of particle trajectory

Hook's Gesetz fuer Speicherringe

… es gibt da nur ein kleines Problem:

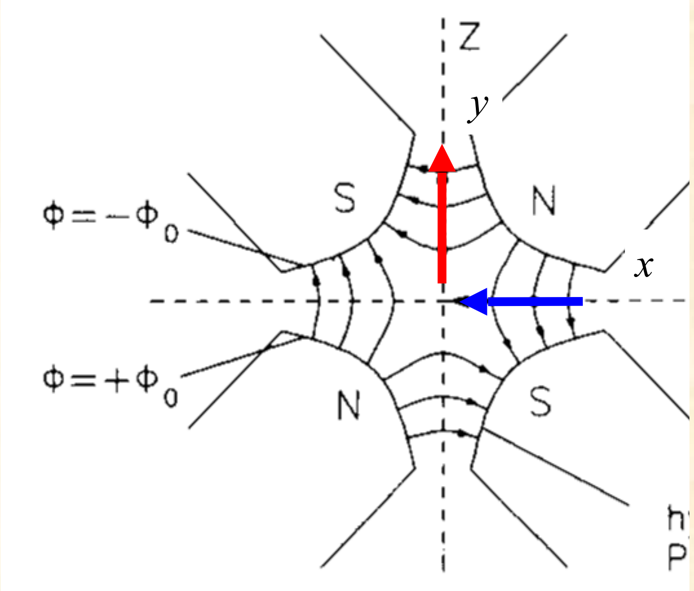
In der vertikalen Ebene drehen sich die Magnetfeld-Linien um

… und damit sieht die “rechte Hand Regel” etwas anders aus !!

- ***Equation for the vertical motion:***

$k \quad \leftrightarrow \quad -k$ ***quadrupole field changes sign***

$$\boxed{y'' - k \cdot y = 0}$$



… und Teilchen,
die in der horizontalen Ebene fokussiert werden,
werden im gleichen Atemzug in der vertikalen Ebene
aus der Maschine befördert.

!

4.) Solution of Trajectory Equations

Define … hor. plane: K= k

… vert. Plane: K = - k

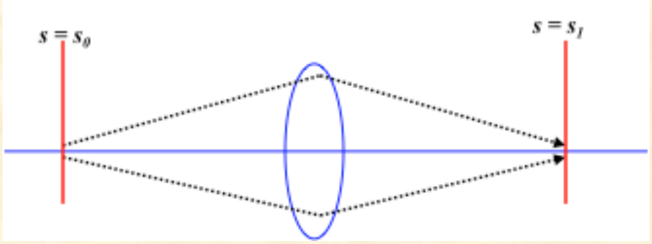
$$x'' + K\; x = 0$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: *Hor. Focusing Quadrupole K > 0:*

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x'_0 \cdot \cos(\sqrt{|K|}s)$$



… da ist wieder unsere Kuckucksuhr.

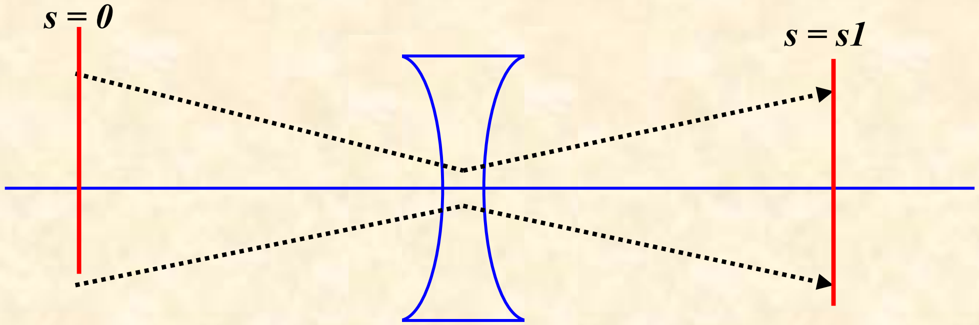
For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$M_{foc} = \begin{pmatrix} \cos\left(\sqrt{|K|}l\right) & \frac{1}{\sqrt{|K|}} \sin\left(\sqrt{|K|}l\right) \\ -\sqrt{|K|} \sin\left(\sqrt{|K|}l\right) & \cos\left(\sqrt{|K|}l\right) \end{pmatrix}$$

hor. defocusing quadrupole:

$$x'' - K\,x = 0$$



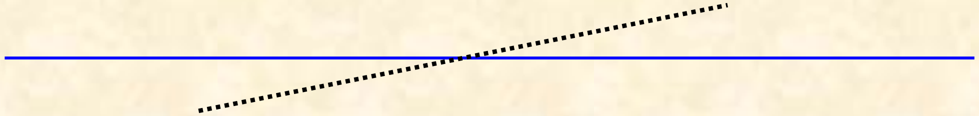
Ansatz: ***Remember from school***

$$x(s) = a_1 \cdot \cosh(\omega\, s) + a_2 \cdot \sinh(\omega\, s)$$

$$M_{defoc} = \begin{pmatrix} \cosh \sqrt{|K|}l & \frac{1}{\sqrt{|K|}} \sinh \sqrt{|K|}l \\ \sqrt{|K|} \sinh \sqrt{|K|}l & \cosh \sqrt{|K|}l \end{pmatrix}$$

drift space:

K = 0



$$x(s) = x'_0 \cdot s$$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

! with the assumptions made, the motion in the horizontal and vertical planes are independent **„ ... the particle motion in x & y is uncoupled“**

… zur Erinnerung:
hyperbolische Funktionen führen leicht zu Panik Attacken !

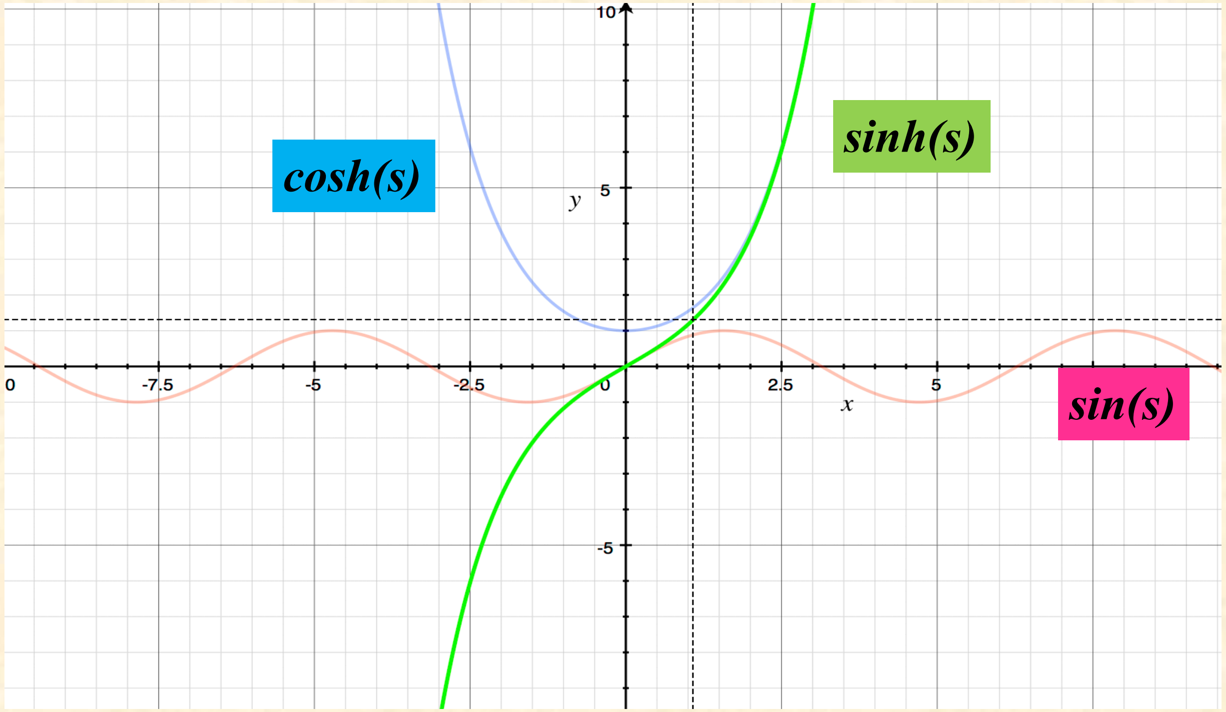
$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}l \\ \sqrt{|K|}\sinh\sqrt{|K|}l & \cosh\sqrt{|K|}l \end{pmatrix}$$

$f(s) = sin(s)$ $\qquad$ $f(s) = cos(s)$

$f(s) = sinh(s)$ $\qquad$ $f(s) = cosh(s)$

Ansatz für die Teilchenbewegung im defokusierenden Fall:

$$x(s) = a_1 \cdot \cosh(\omega\, s) + a_2 \cdot \sinh(\omega\, s)$$

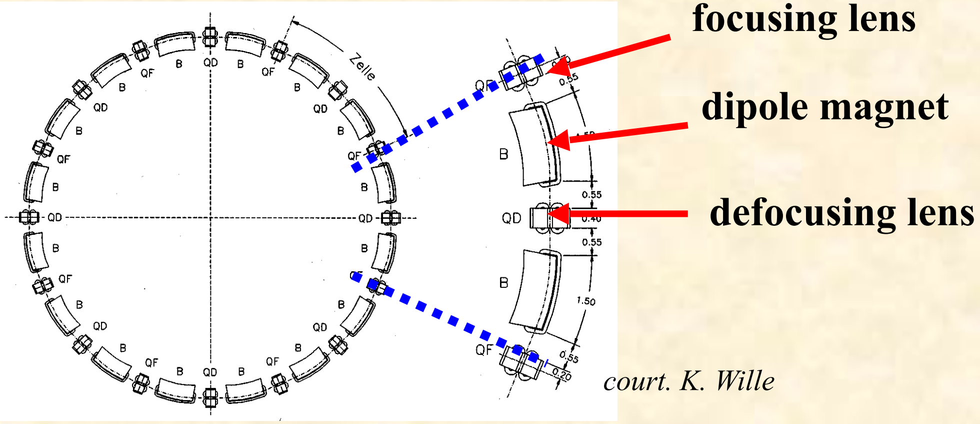


Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D*.....}$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s_2} = M(s_2, s_1) \cdot \begin{pmatrix} x \\ x' \end{pmatrix}_{s_1}$$



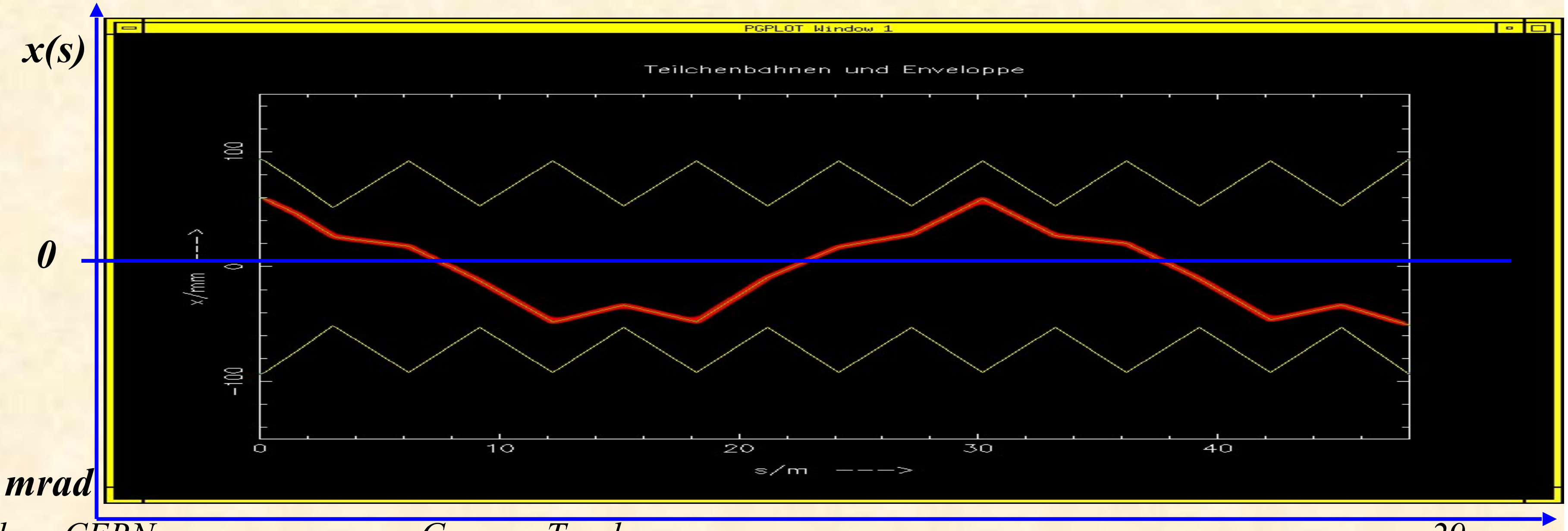
focusing lens

dipole magnet

defocusing lens

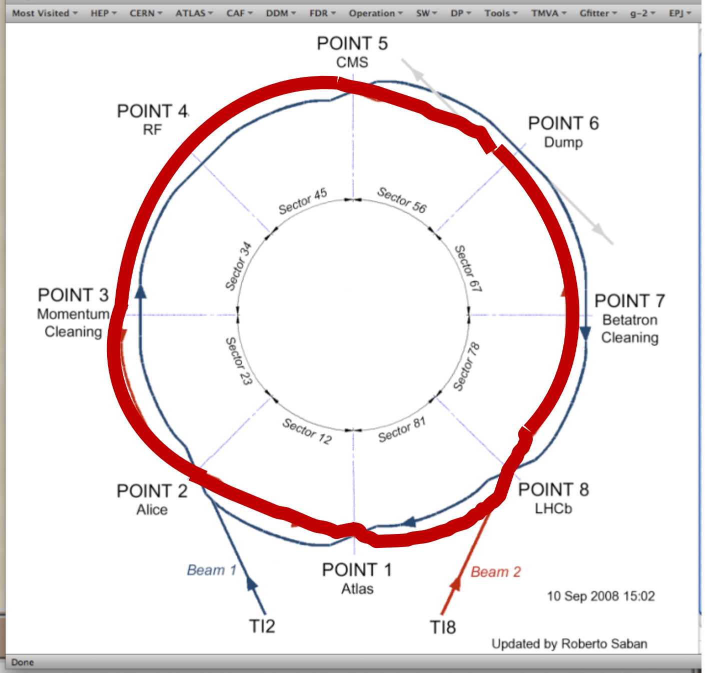
court. K. Wille

in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „



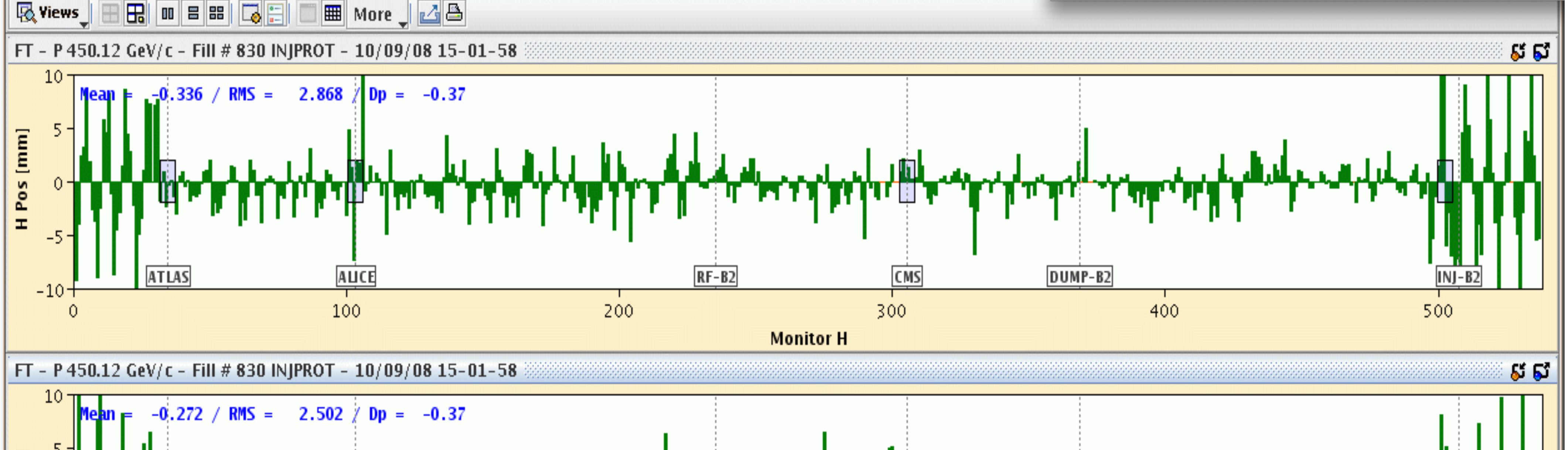
typical values in a strong foc. machine:
$x \approx$ mm, $x' \leq$ mrad

LHC Operation: Beam Commissioning



*The **transverse focusing fields** create a **harmonic oscillation of the particles** with a well defined "Eigenfrequency" which is called **tune***

First turn steering "by sector:"

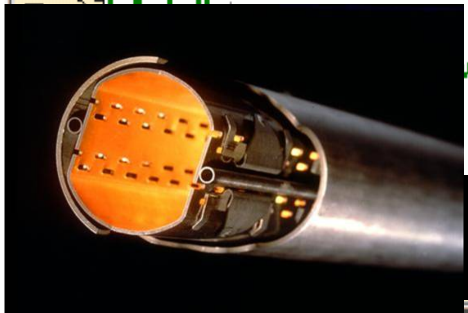

YASP DV LHCRING / INJ-TEST-NB / beam 2

FT - P 450.12 GeV/c - Fill # 830 INJPROT - 10/09/08 15-01-58

Mean = -0.336 / RMS = 2.868 / Dp = -0.37

FT - P 450.12 GeV/c - Fill # 830 INJPROT - 10/09/08 15-01-58

Mean = -0.272 / RMS = 2.502 / Dp = -0.37



aperture radius: r = 22 mm

"Once more unto the breach, dear friends, once more"
(W. Shakespeare, Henry 5)

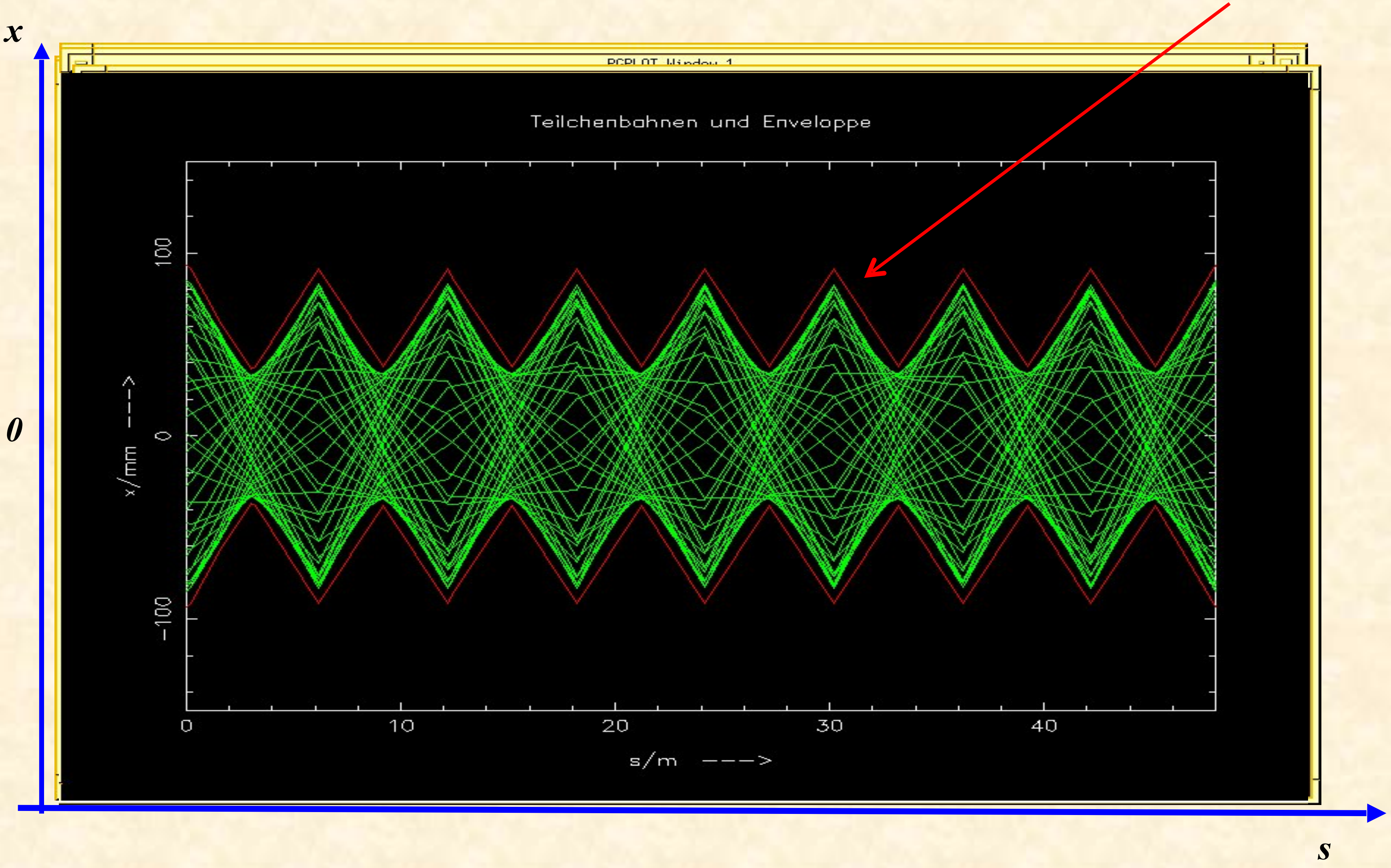
"Do they actually drop ?"

Answer: No

Question: ***what will happen, if the particle performs a second turn ?***

... or a third one or ... 10^{10} turns

$$\sigma = \sqrt{\varepsilon\beta}$$



Collisions

Die zwei wichtigsten Formeln fuer uns …

$$E = mc^2$$

die Energie unserer Strahlen kann in ***Masse neuer Teilchen*** *umgewandelt werden.*

$$\lambda = \frac{h}{p}$$

Teilchen verhalten sich wie Wellen *mit einer wohl definierten Wellenlaenge;* $h = 4.1 \cdot 10^{-21}\ MeV\ s$

Collisions

Lichtspektrum:

$\lambda = 400 \ldots 800\ nm$ *Lichtmikroskope haben damit eine Auflösung von etwas besser als μm*

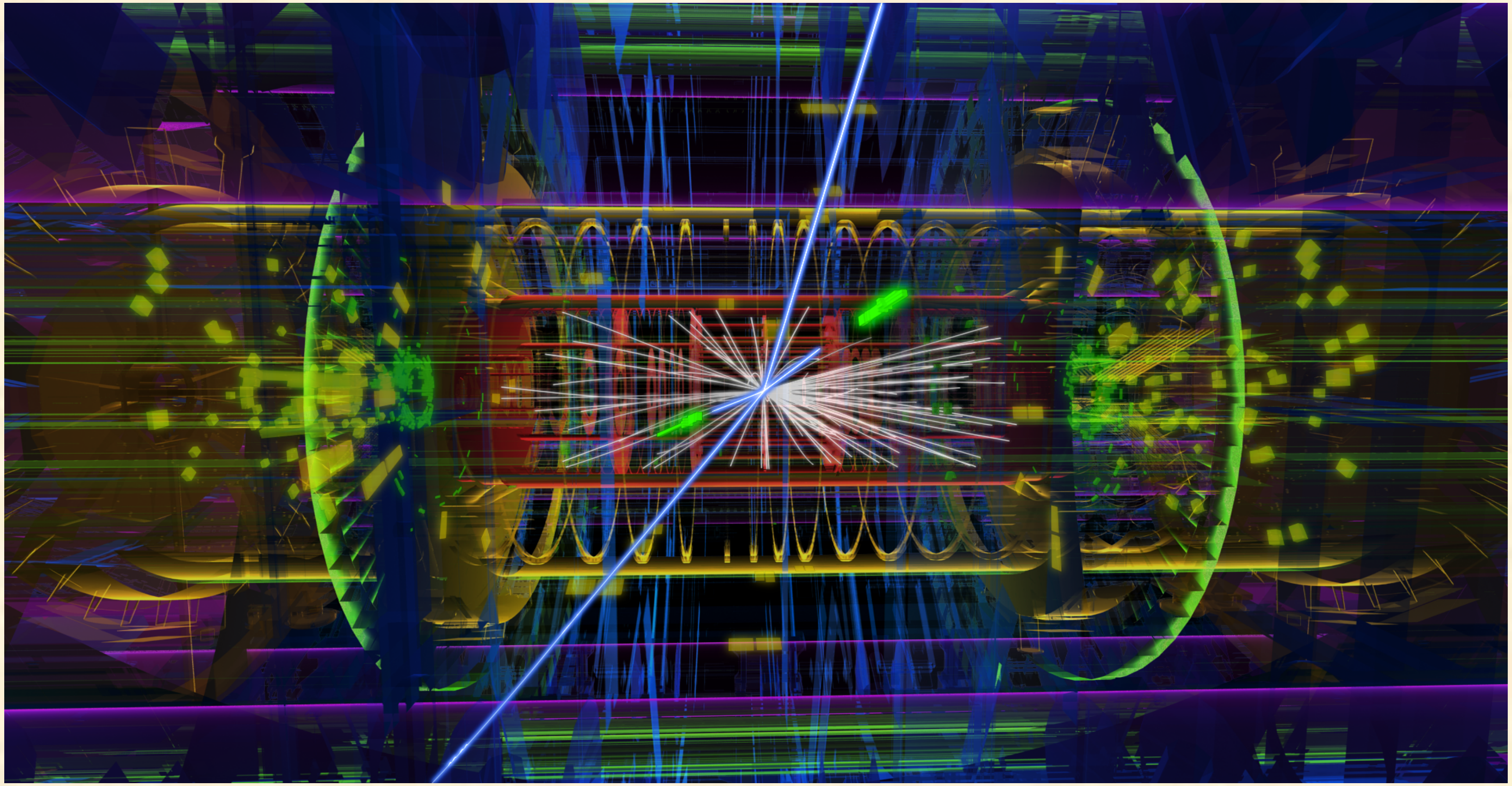
LHC:

$$E = p \cdot c \quad \rightarrow \quad p = \frac{E}{c} \qquad\qquad p = \frac{7 \cdot 10^{12}\, eV}{3 \cdot 10^{8}\, m/s}$$

$$\lambda = \frac{h}{p} \quad = 4.1 \cdot 10^{-21}\, MeV\, s \cdot \frac{3 * 10^{8} m/s}{7 \cdot 10^{12}\, eV}$$

$$\lambda \approx 2 \cdot 10^{-19}\, m$$

Collisions



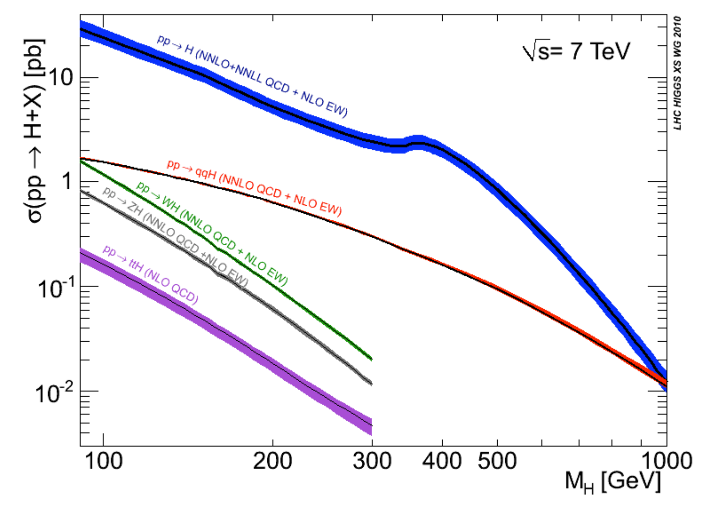
ATLAS event display: Higgs => two electrons & two muons

$$E = m_0c^2 = m_{e1} + m_{e2} + m_{\mu 1} + m_{\mu 2} = 125.4\ GeV$$

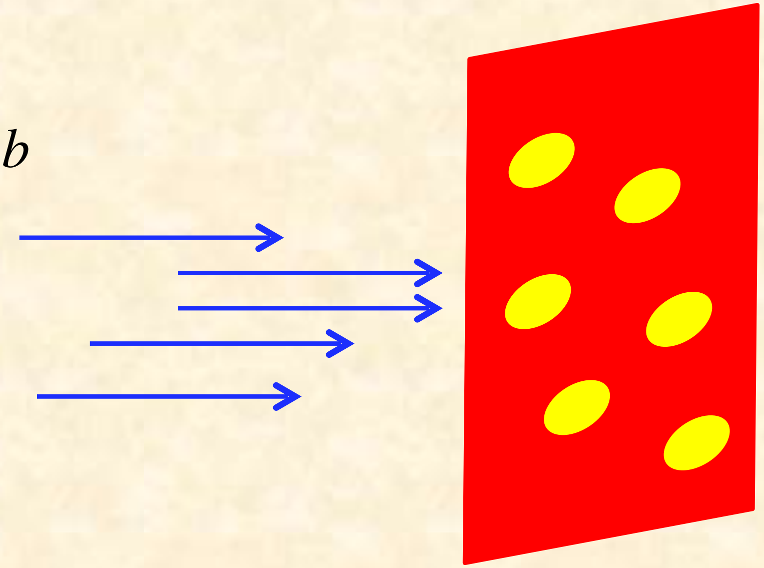
Problem: Our particles are VERY small !!

man trifft nicht so häufig.

Overall cross section of the Higgs:



$$\Sigma_{react} \approx 1pb$$



$$1b = 10^{-24} cm^2 = \frac{1}{mio} \cdot \frac{1}{mio} \cdot \frac{1}{mio} \cdot \frac{1}{10000} \quad mm^2 \longrightarrow \boxed{1pb = 10^{-12} b \approx ZERO}$$

The particles are "very small"

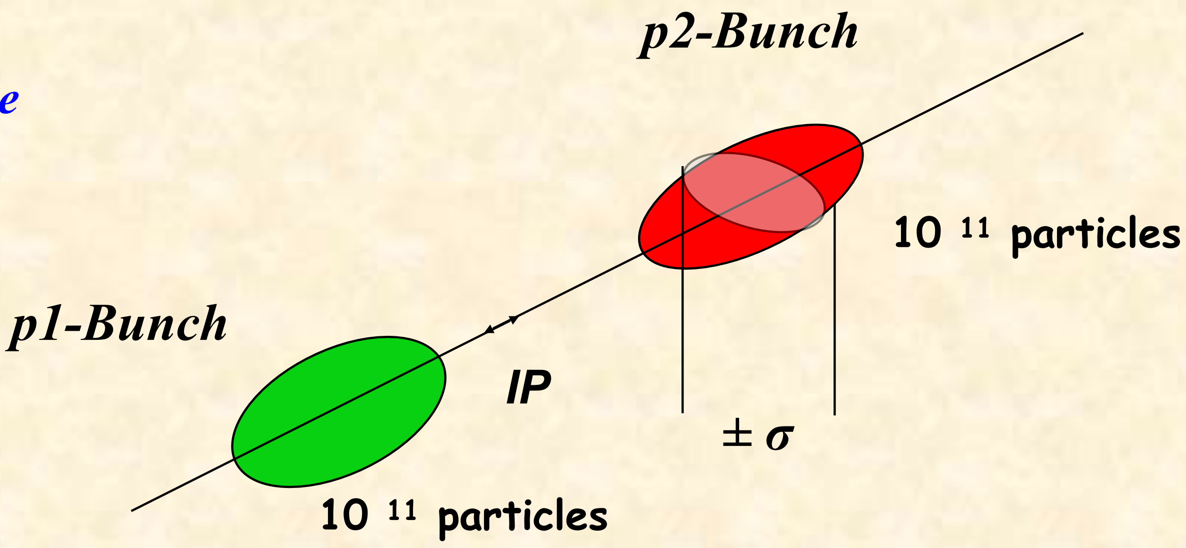
The only chance we have:
compress the transverse beam size … at the IP

LHC typical → 16 μm

5.) Luminosity

Ereignis Rate: "Physik" pro Sekunde

$$R = L \cdot \Sigma_{react}$$

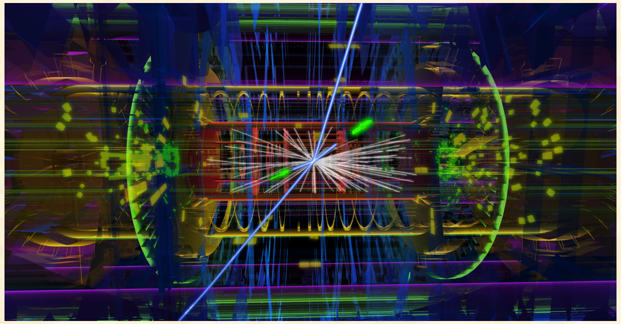


Example: Luminosity run at LHC

$\sigma_x = \sigma_y = 16 \mu m$	*Strahlgröße am IP*
$f_0 = 11.245\ kHz$	*Umlaufs-Frequenz*
$n_b = 2808$	*Zahl der Bunche*
$N_p = 1.2 \cdot 10^{11}$	*Teilchen in einem Bunch*
$I_p = 584\ mA$	*Strahlstrom*

$$L = 1.0 * 10^{34}\ \frac{1}{cm^2 s}$$

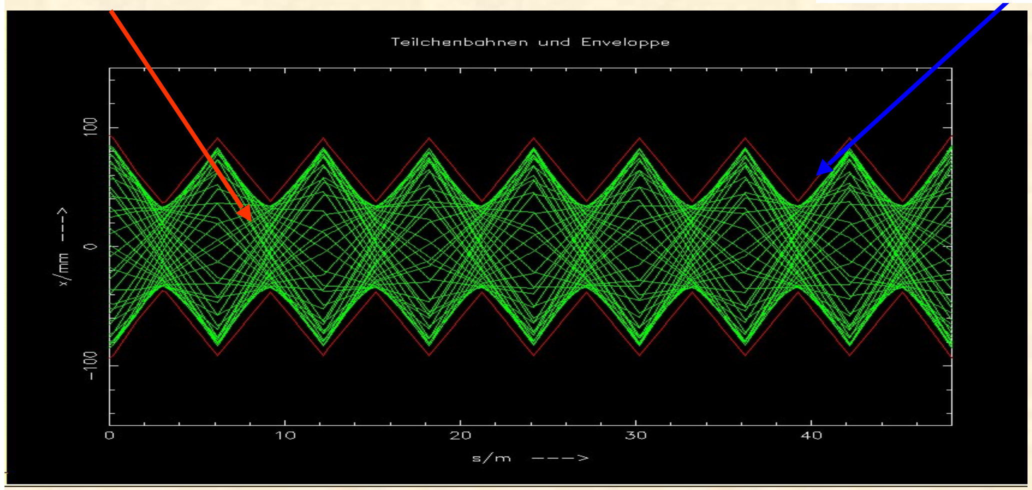
$$L = \frac{1}{4\pi} \cdot N_{p1} \cdot \frac{N_{p2}}{\sigma_x \sigma_y} \cdot (n_b \cdot f_0)$$



The Particle Ensemble:

$x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}\cdot\cos(\Psi(s)+\phi)$

$\hat{x}(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}$



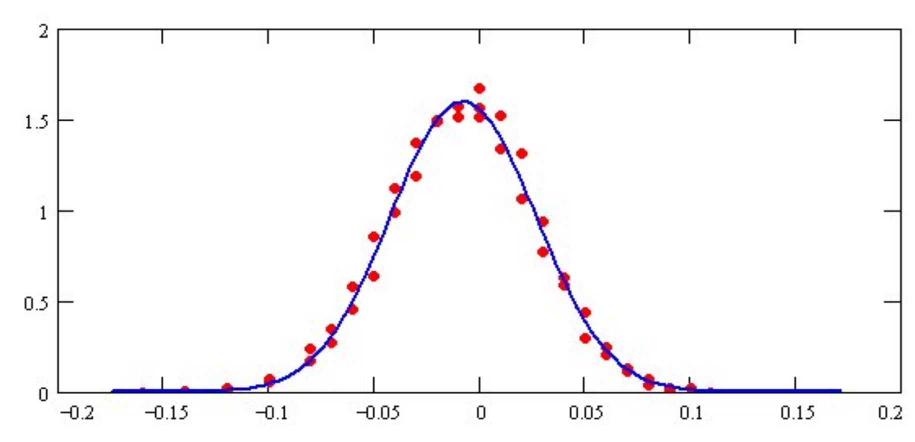
single particle trajectories, $N \approx 10^{11}$ per bunch

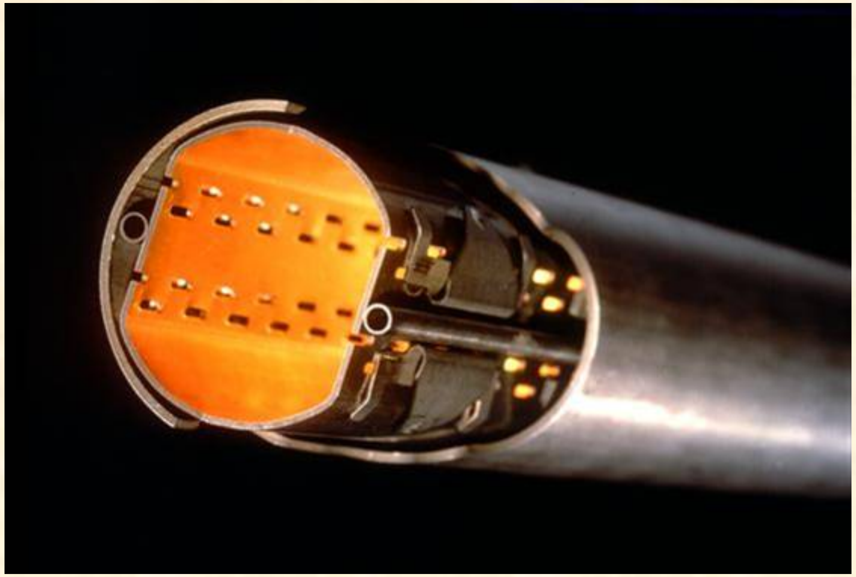
Gauß
Particle Distribution:

$$\rho(x) = \frac{N\cdot e}{\sqrt{2\pi}\,\sigma_x}\cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}$$

particle at distance 1 σ from centre
↔ 68.3 % of all beam particles

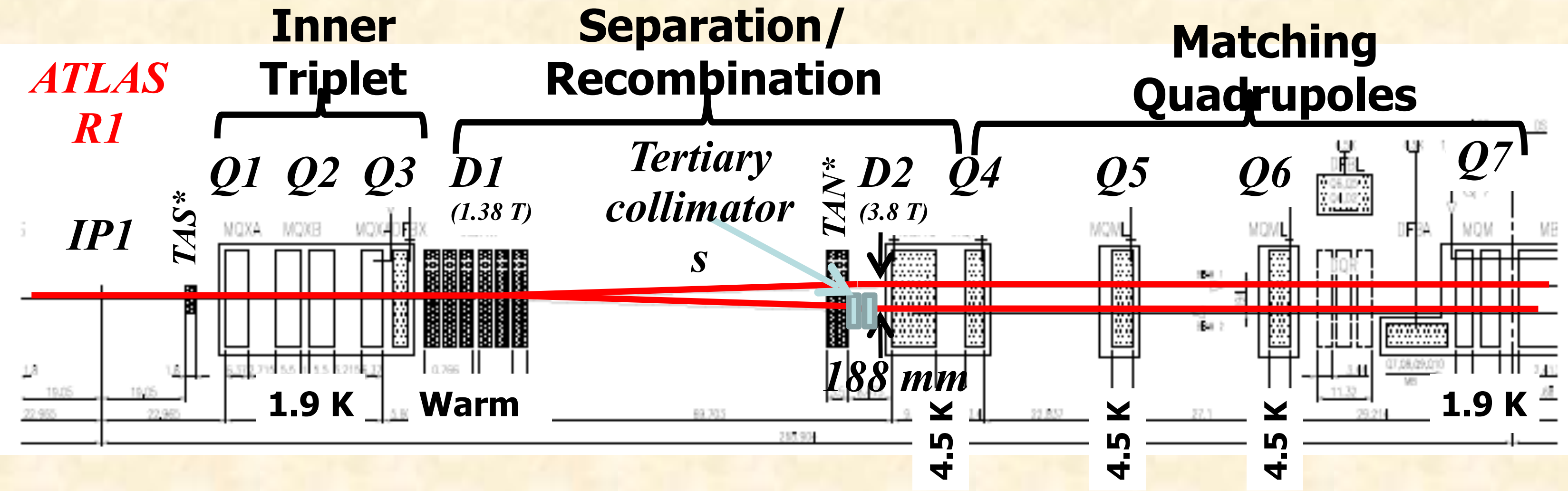
LHC: Strahlgroesse = σ ≃ 0.3 mm

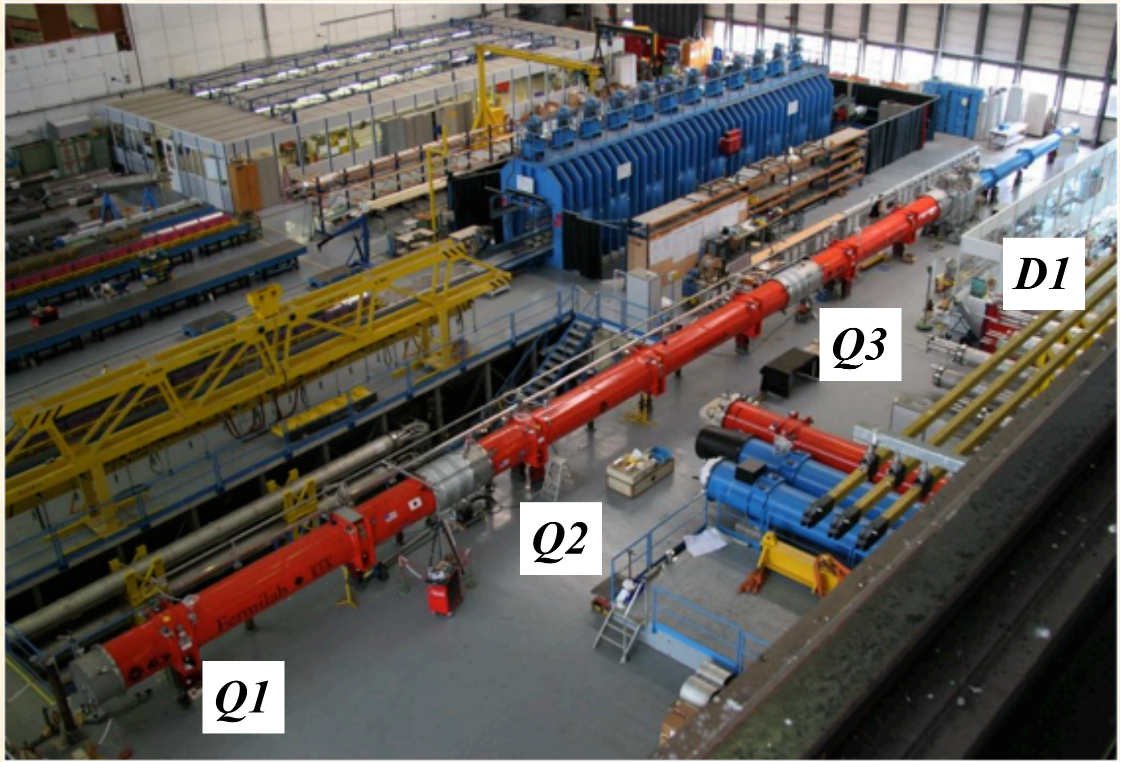




aperture requirements: $r_0 = 17 * \sigma$

The LHC Mini-Beta-Insertions





Extrem starke Fokussierung

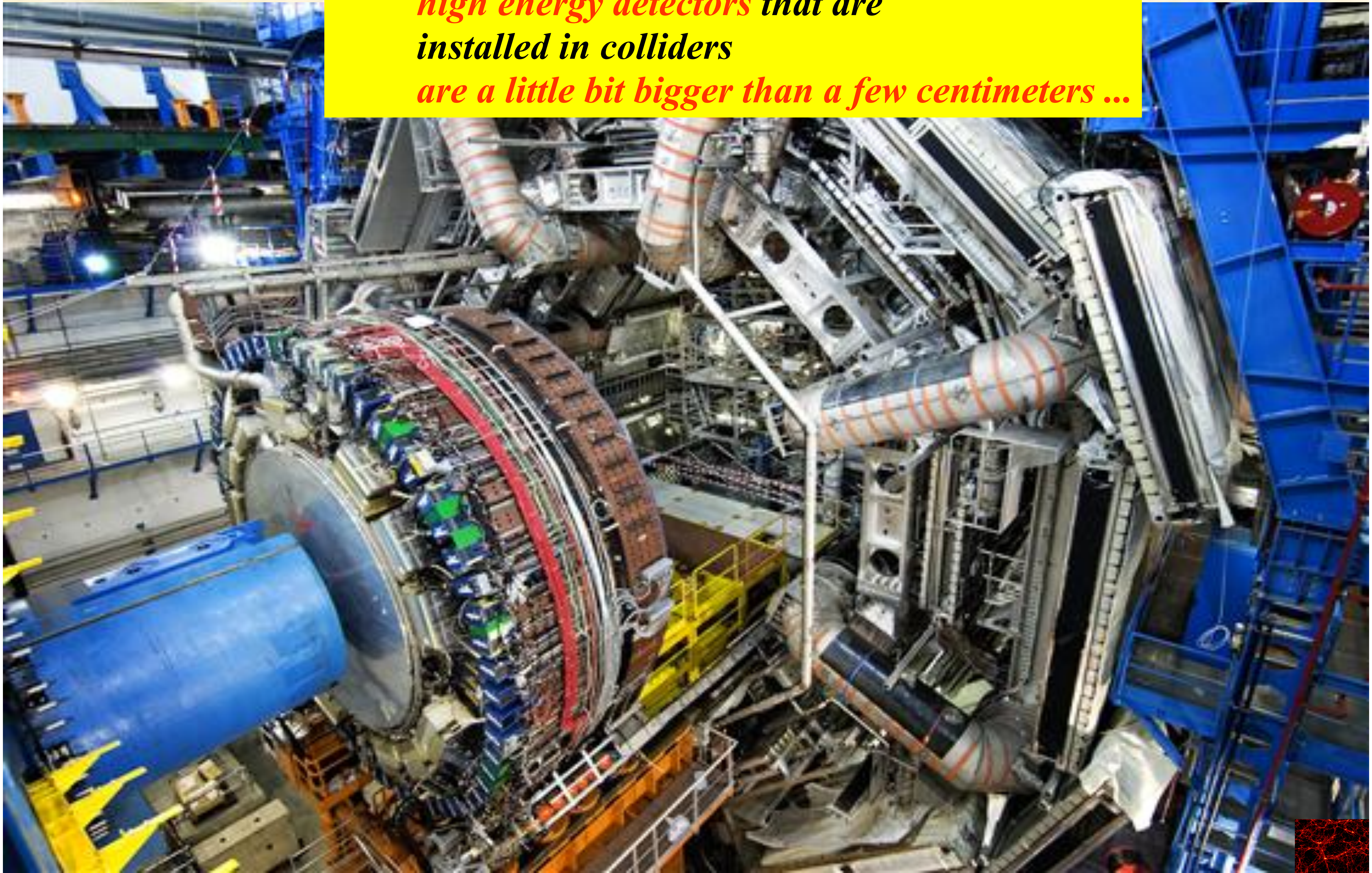
(in beiden Ebenen) für beide Strahlen, um

die Trajektorien der 10^{11} Teilchen auf

micro Meter zu komprimieren.

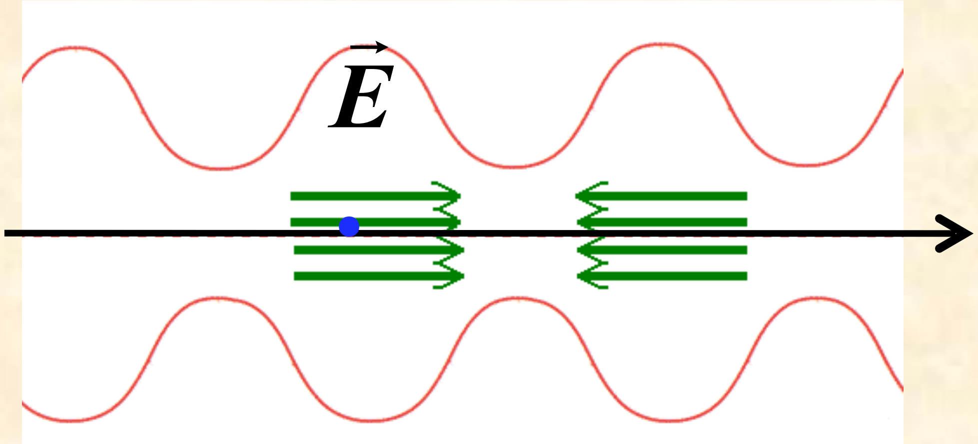
... clearly there is another problem !!!

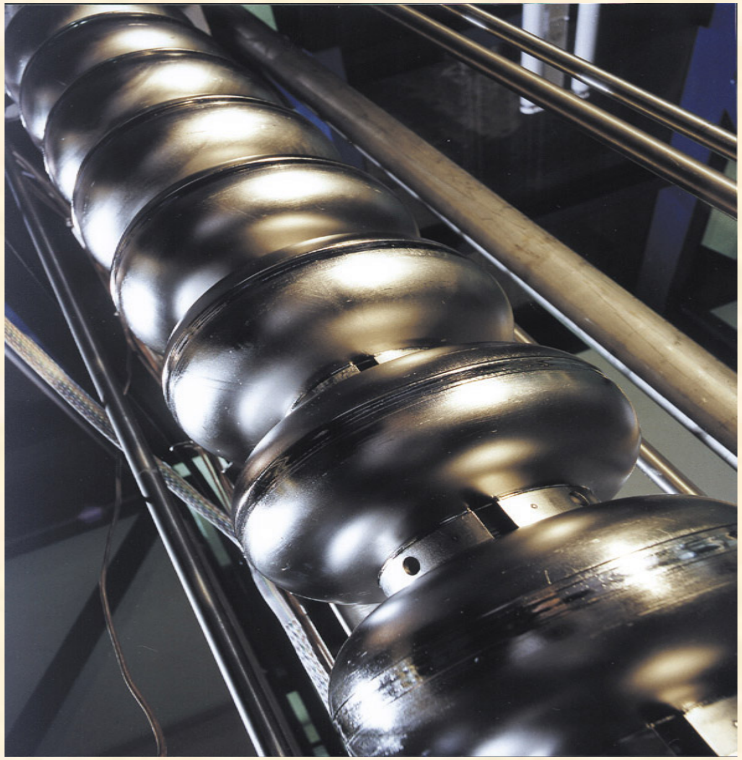
... unfortunately ... in general
high energy detectors that are
installed in colliders
are a little bit bigger than a few centimeters ...



The Acceleration

Install an RF accelerating structure in the ring:





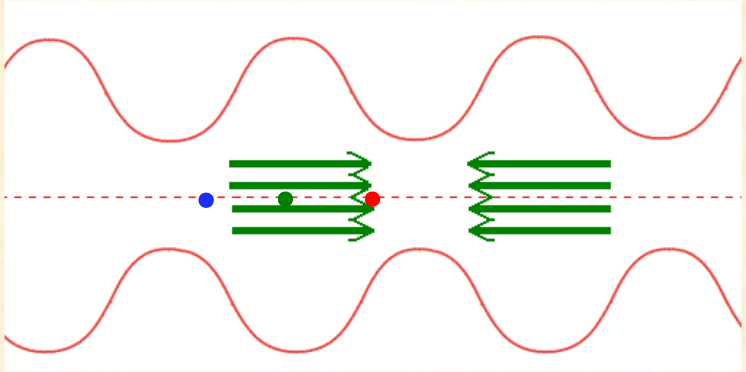
The Acceleration & "Phase Focusing"

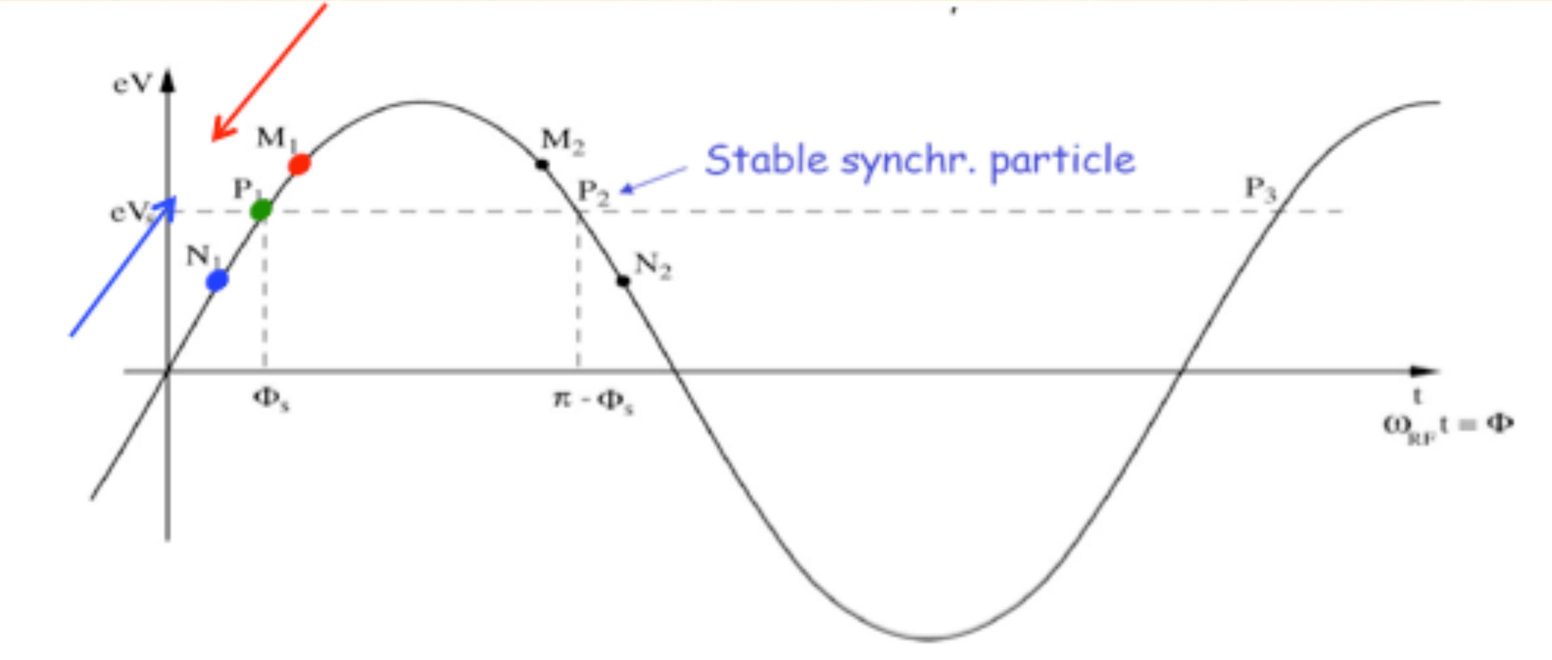
$\Delta p/p \neq 0$ below transition

ideal particle •

particle with $\Delta p/p > 0$ • *faster*

particle with $\Delta p/p < 0$ • *slower*





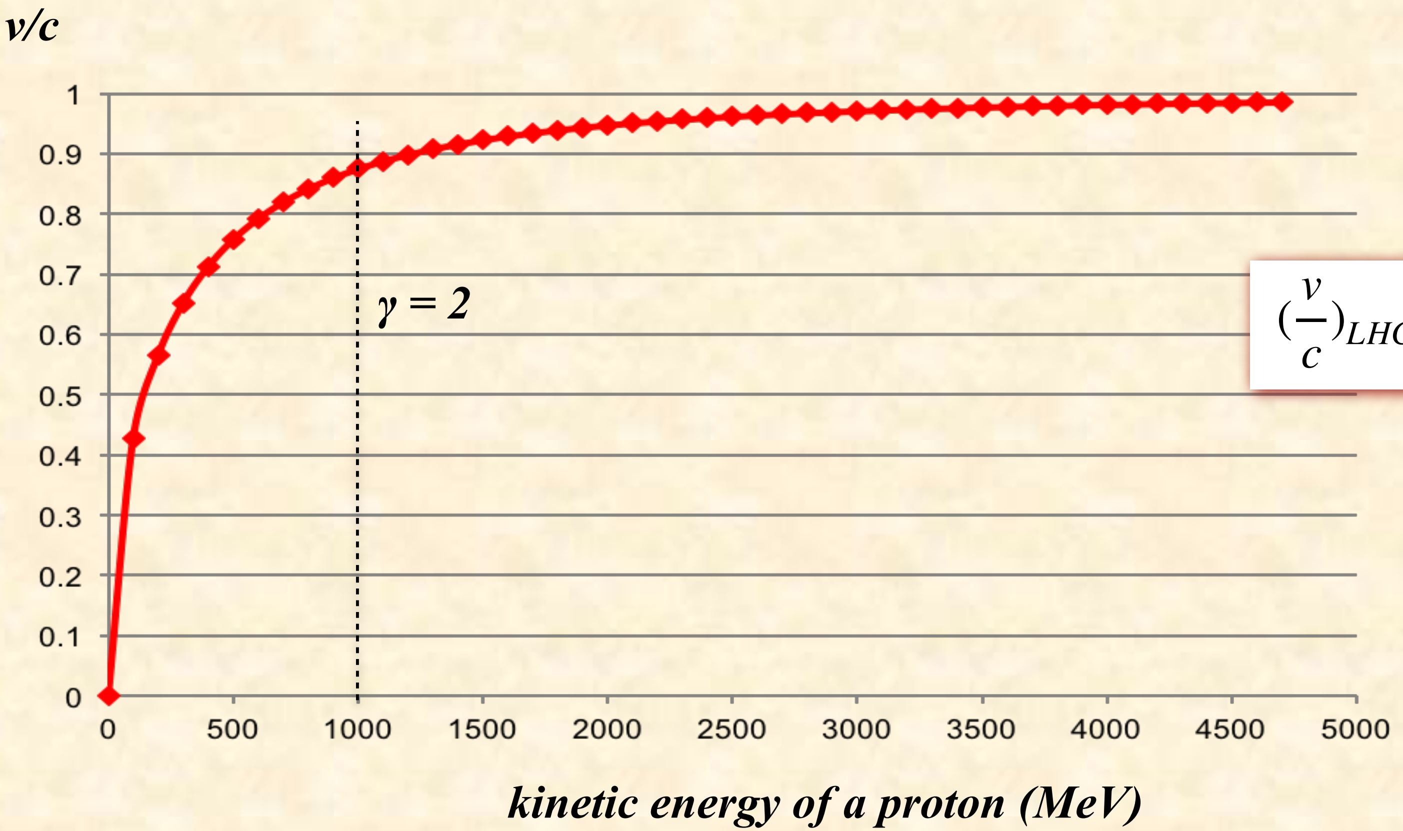
Focussing effect in the longitudinal direction keeping the particles close together … forming a "bunch"

... so sorry, here we need help from Albert:

was passiert, wenn wir die Teilchen immer "schneller" machen ?

$$\gamma = \frac{E_{total}}{m_0 c^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \quad \longrightarrow \quad \frac{v}{c} = \sqrt{1-\frac{mc^2}{E_{total}^2}}$$

die Teilchen werden irgendwann nicht mehr schneller !



$$\left(\frac{v}{c}\right)_{LHC} = 0.999999991$$

... some when the particles do not get faster anymore

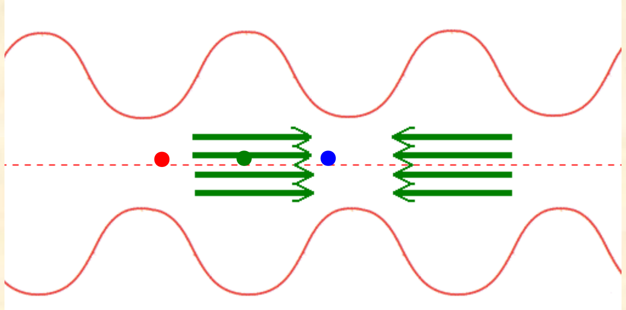
.... but heavier !

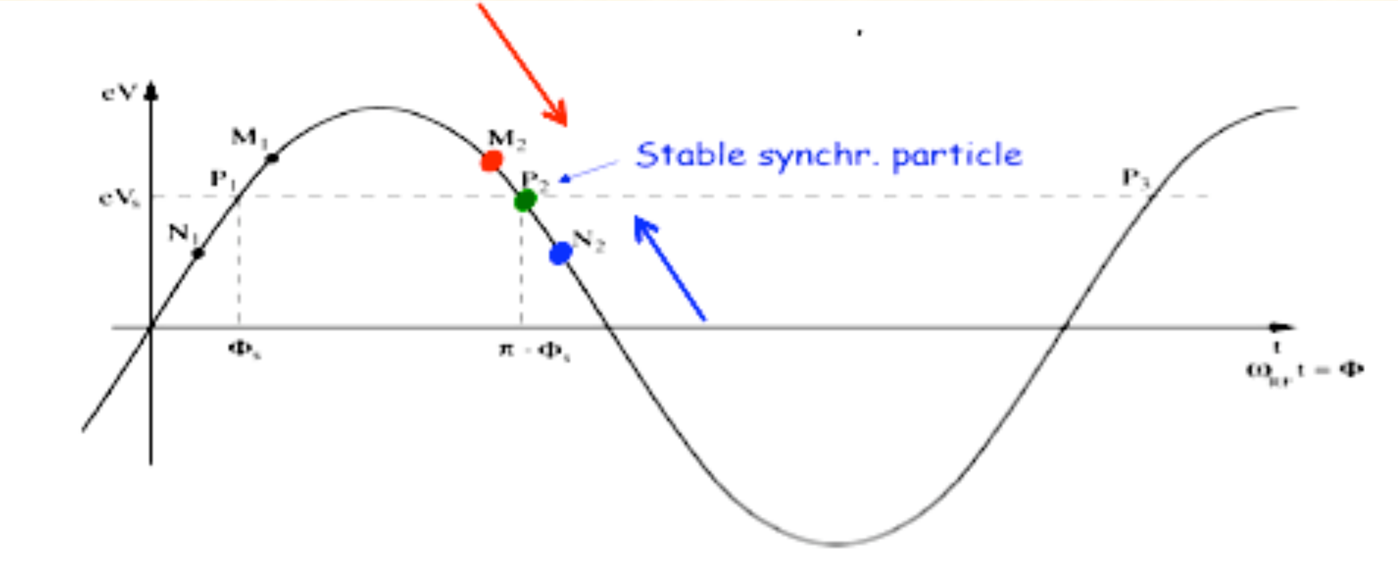
The Acceleration above transition

ideal particle ●

particle with $\Delta p/p > 0$ ● *heavier*

particle with $\Delta p/p < 0$ ● *lighter*



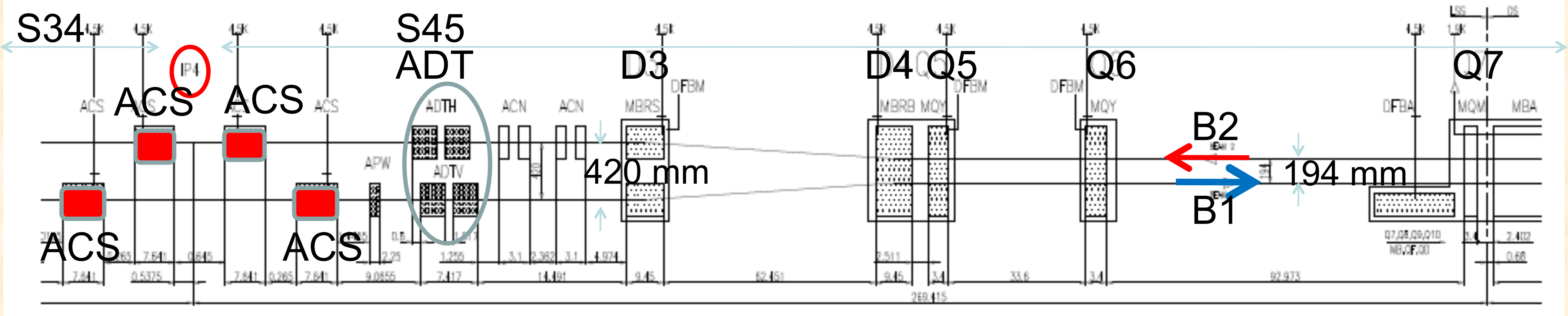


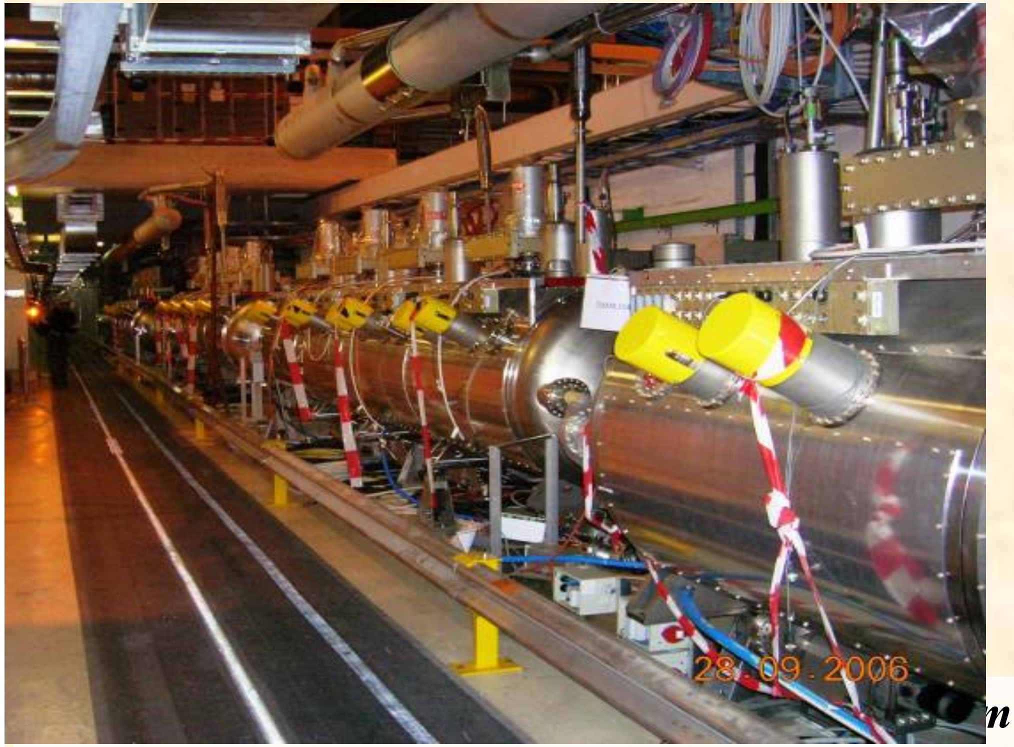
Focussing effect in the longitudinal direction keeping the particles close together ... forming a "bunch"

... and how do we accelerate now ???

with the dipole magnets !

The RF system: IR4





Nb on Cu cavities @4.5 K (=LEP2)
Beam pipe diam.=300mm

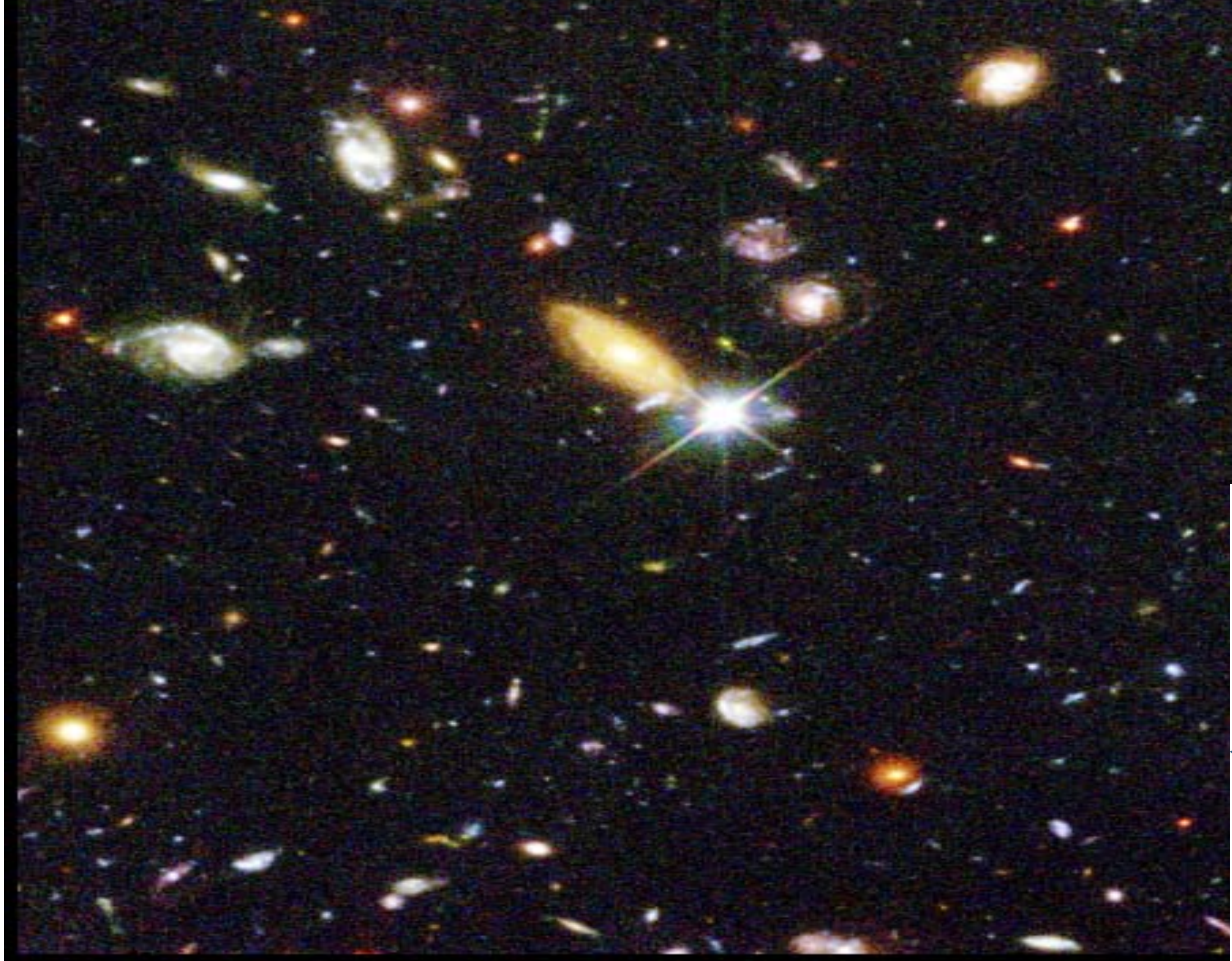
Bunch length (4σ)	ns	1.06
Energy spread (2σ)	10^{-3}	0.22
Synchr. rad. loss/turn	keV	7
Synchr. rad. power	kW	3.6
RF frequency	MHz	400
Harmonic number		35640
RF voltage/beam	MV	16
Energy gain/turn	keV	485
Synchrotron frequency	Hz	23.0

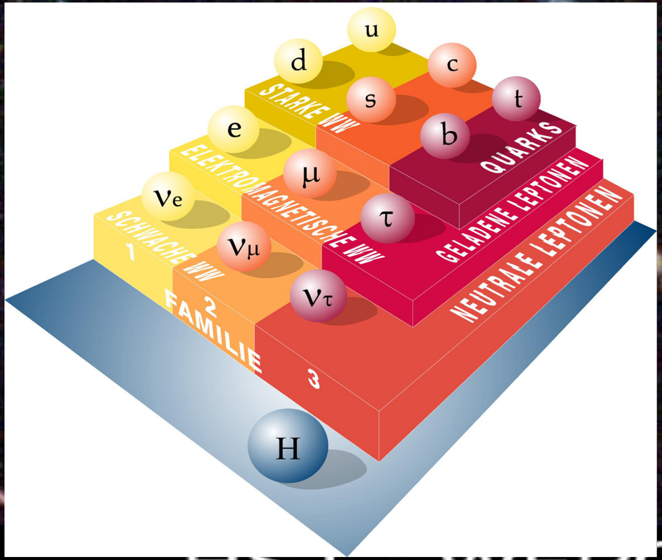
1.) Where are we ?

- * Standard Model of HEP
- * Higgs discovery

What‘s next ???

Dark Matter & Dark Energy
Physics beyond the Standard Model





Hubble Deep Field HST · WFPC2

PRC96-01a · ST ScI OPO · January 15, 1996 · R. Williams (ST ScI), NASA

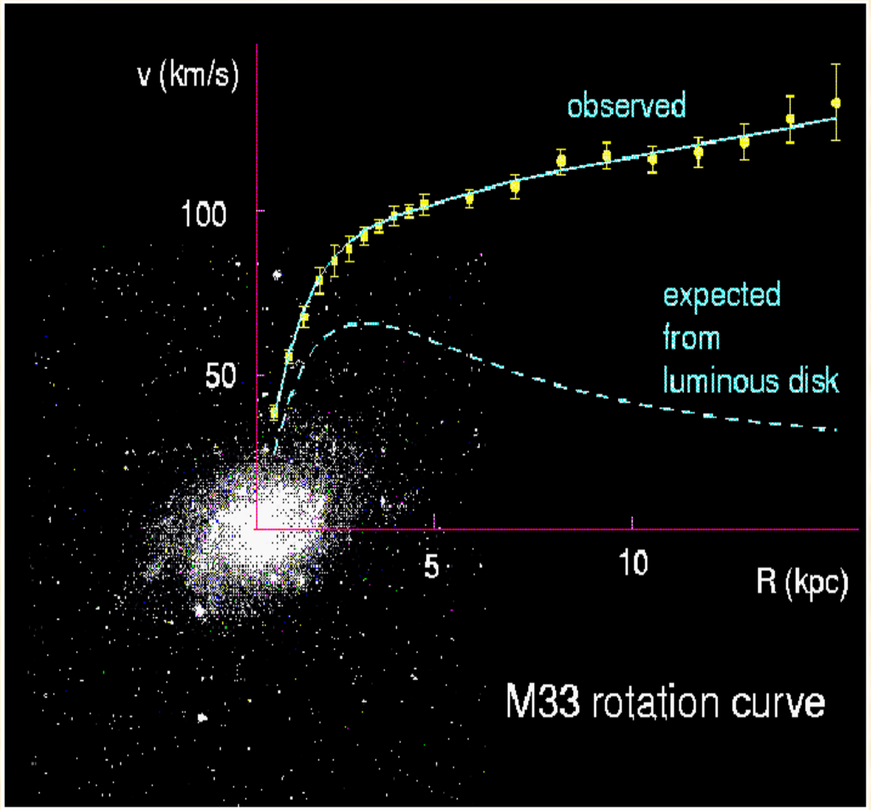
What‘s next ???
Dark Matter

The outer region of galaxies rotate faster than expected from visible matter

$$\frac{m \cdot v^2}{r} = \frac{m_1 \cdot M_2 \cdot G}{r^2}$$

$$v_{circ} = \sqrt{\frac{M_2(r) \cdot G}{r}}$$

Corbelli & Salucci (2000);
Bergstrom (2000)



Dark matter would explain this

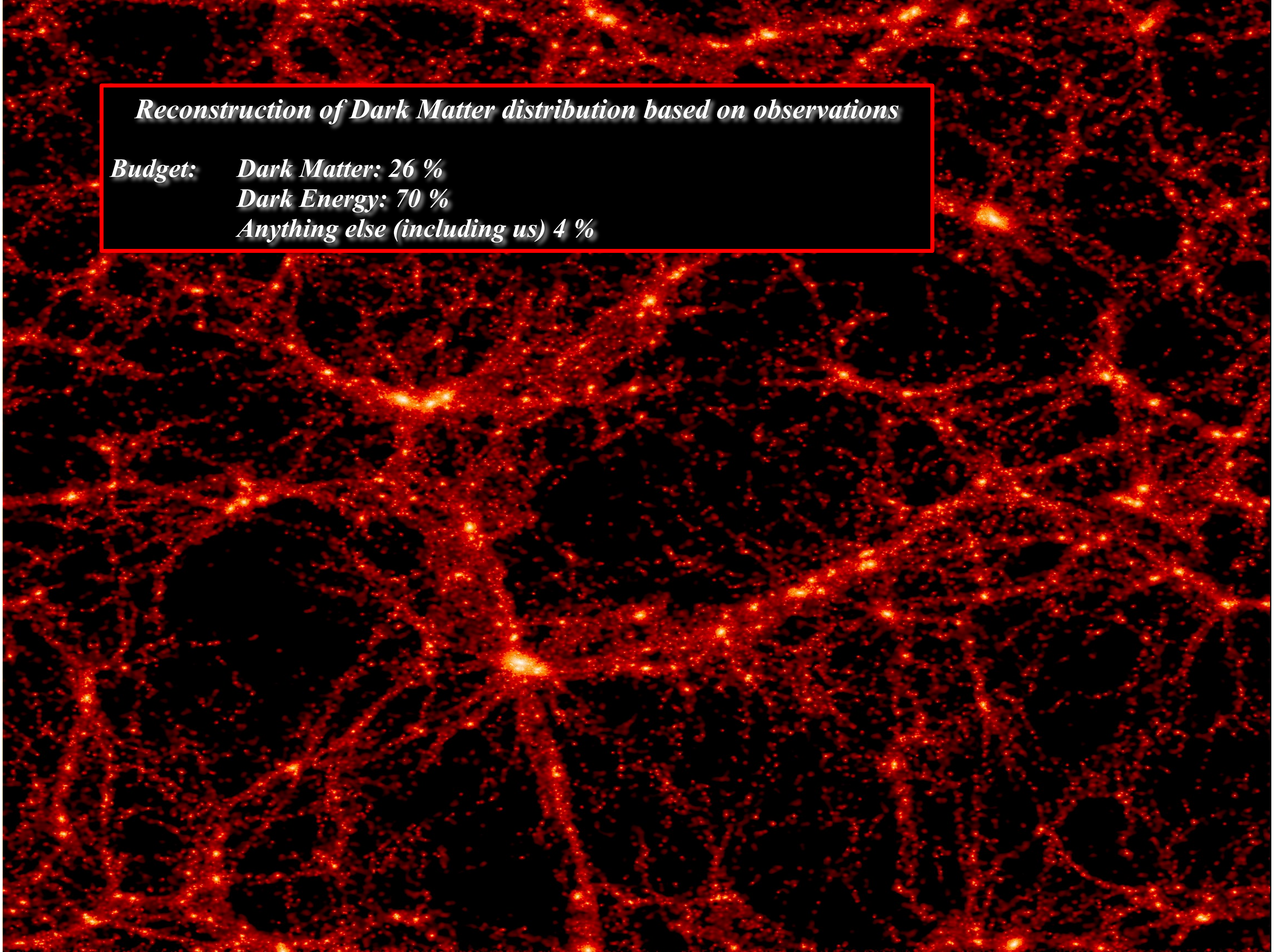
Other observations exist … (grav. lens effects)
but all through gravity

What is it?

(One explanation is super-symmetry)

Reconstruction of Dark Matter distribution based on observations

Budget: *Dark Matter: 26 %*
Dark Energy: 70 %
Anything else (including us) 4 %



Merci