From Hope to Heuristic: Realistic Runtime Estimates for Quantum Optimisation in NHEP

Maja Franz[1], Manuel Schönberger[1], Melvin Strobl[2], Eileen Kuehn[2], Achim Streit[2], Pía Zurita[3], Markus Diefenthaler[4], Wolfgang Mauerer[1,5]

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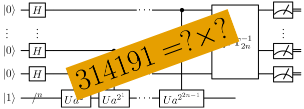
[1] Technical University of Applied Scienes Regensburg [2] Karlsruhe Institute of Technology [3] Complutense University of Madrid
[4] Jefferson Lab [5] Siemens AG, Corporate Research

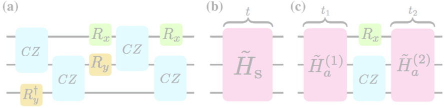
"Low-Level" Algorithms

- Grover's & Shor's algorithms
- Provable speedup / error correction required



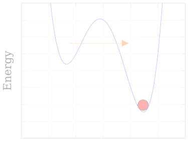
Quantum Simulation

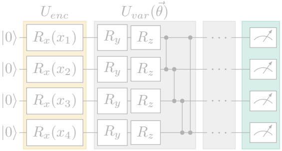
- Mimic system using simplified model
- Classically likely intractable



NISQ Algorithms

- Quantum annealing
- Variational algorithms: Hybrid quantum-classical
- Less resources / potential speedups





M. S. Lucas Lamata Adrian Parra-Rodriguez and E. Solano, *Digital-analog quantum simulations with superconducting circuits* (2018)

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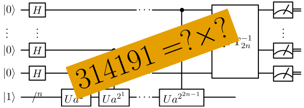

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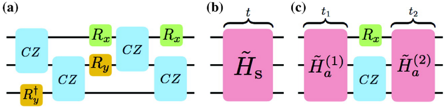

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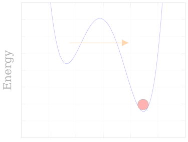

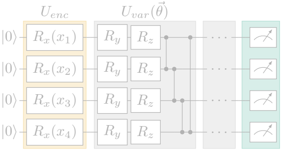

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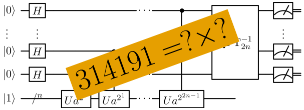

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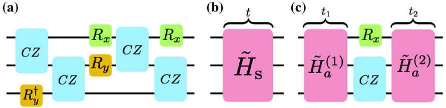

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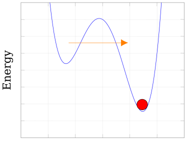

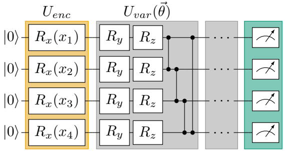

M. S. Lucas Lamata Adrian Parra-Rodriguez and E. Solano, *Digital-analog quantum simulations with superconducting circuits* (2018)

Ingredients

- Problem Hamiltonian H_P
  $\rightarrow$ (Eigen-)Ground state encodes solution
- Initial Hamiltonian H_0
  $\rightarrow$ Ground state easy to prepare
- Total anneal time T

$$H_0 \overset{\text{evolve}}{\rightsquigarrow} H_P$$

Adiabatic Theorem

The system remains in its groundstate if it evolves *slowly enough*.

Adiabatic Evolution

$$H(t) = (1 - s)H_0 + sH_P, \quad s = t/T$$

T. Albash and D. A. Lidar, *Adiabatic quantum computation* (2018)

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Quantum Annealing

- Theoretical ideal: Perfect isolation
- Reality: Environmental imperfections

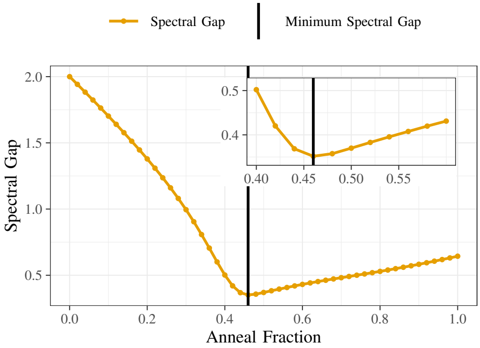
$\Rightarrow$ Combine adiabatic with stochastic processes

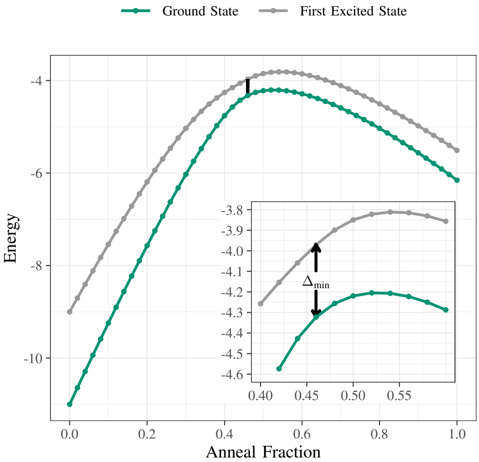
T. Albash and D. A. Lidar, *Adiabatic quantum computation* (2018)

How Slow is *Slowly Enough?*

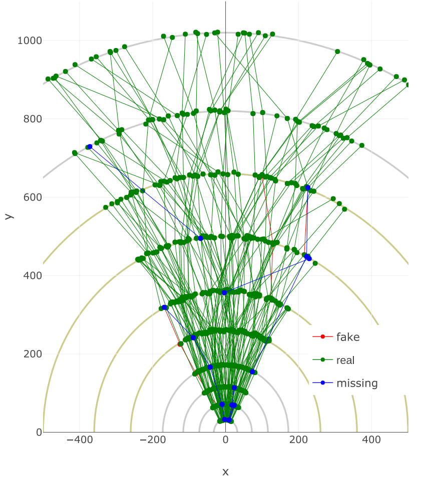
Runtime

$$T = \mathcal{O}\left(\frac{1}{\Delta_{\text{min}}^2}\right)$$





Application: Track Reconstruction



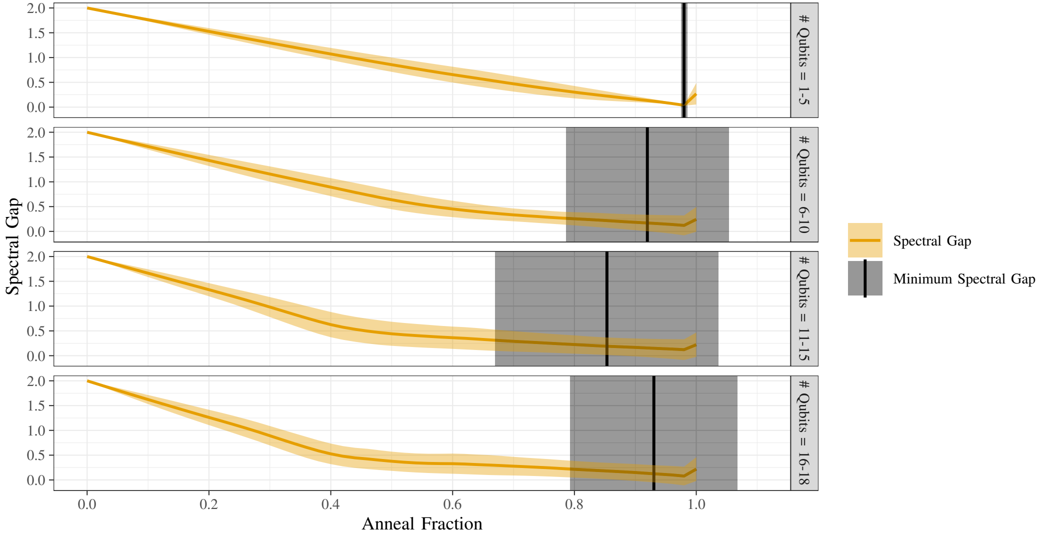
QUBO Formulation

$$O(\alpha, b, T) = \alpha \sum_{i=1}^{N} T_i + \sum_{i}^{N} \sum_{j<i}^{N} b_{ij} T_i T_j$$

- $T_i \in \{0, 1\}$: with variables (potential particle triplets)
- α, b_i: bias weight and coupling strengths
- Cast to Ising Hamiltonian via $T_i = \frac{\sigma_i^z + 1}{2}$

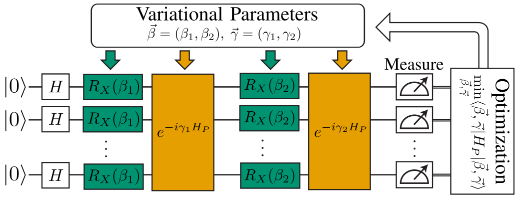
F. Bapst et al., *A Pattern Recognition Algorithm for Quantum Annealers* (2019)
T. Schwägerl et al., *Particle track reconstruction with noisy intermediate-scale quantum computers* (2023)
L. Linder, *GitHub: HEPQPR-Qallse* (2019)

Spectral Gaps in NHEP



Quantum Approximate Optimisation Algorithm (QAOA)

- Approximation of adiabatic evolution
- Instead of a continuous time evolution: Discretise in p timesteps (Trotterisation)
- $p \uparrow \Leftrightarrow$ Approximation quality $\uparrow$



E. Farhi, J. Goldstone, and S. Gutmann, *A Quantum Approximate Optimization Algorithm* (2014)
K. Wintersperger, H. Safi, and W. Mauerer, *QPU-System Co-Design for Quantum HPC Accelerators* (2022)

Discretised Hamiltonian

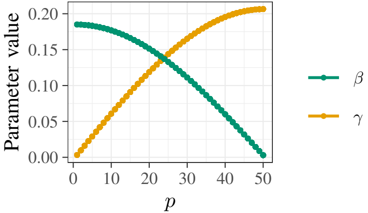
$$H_{\text{QAOA}}(t) = (1 - f(t))H_0 + f(t)H_P,$$

Discretised (non-linear) Anneal Schedule

$$f\left(t_i = \sum_{j=1}^{i}(|\gamma_j| + |\beta_j|) - \frac{1}{2}(|\gamma_i| + |\beta_i|)\right) = \frac{\gamma_i}{(|\gamma_i| + |\beta_i|)}$$

Ingredients

- Problem Hamiltonian H_P
- Initial Hamiltonian H_0
- p discrete timesteps t_i



L. Zhou et al., *Quantum Approximate Optimization Algorithm: Performance, Mechanism, and Implementation on Near-Term Devices* (2020)

Approximate Parameter Path

Instead of $2p$ parameters $(\vec{\beta}, \vec{\gamma}) \in \mathrm{R}^{2p}$,
$\rightarrow 2q$ parameters $(\vec{v}, \vec{u}) \in \mathrm{R}^{2q}$, with $q < p$.

Advantage

Reduced optimisation complexity

Parameter Transformation

$$\beta_i = \sum_{k=1}^{q} v_k \cos\left[\left(k - \frac{1}{2}\right)\left(i - \frac{1}{2}\right)\frac{\pi}{p}\right]$$

$$\gamma_i = \sum_{k=1}^{q} u_k \sin\left[\left(k - \frac{1}{2}\right)\left(i - \frac{1}{2}\right)\frac{\pi}{p}\right]$$

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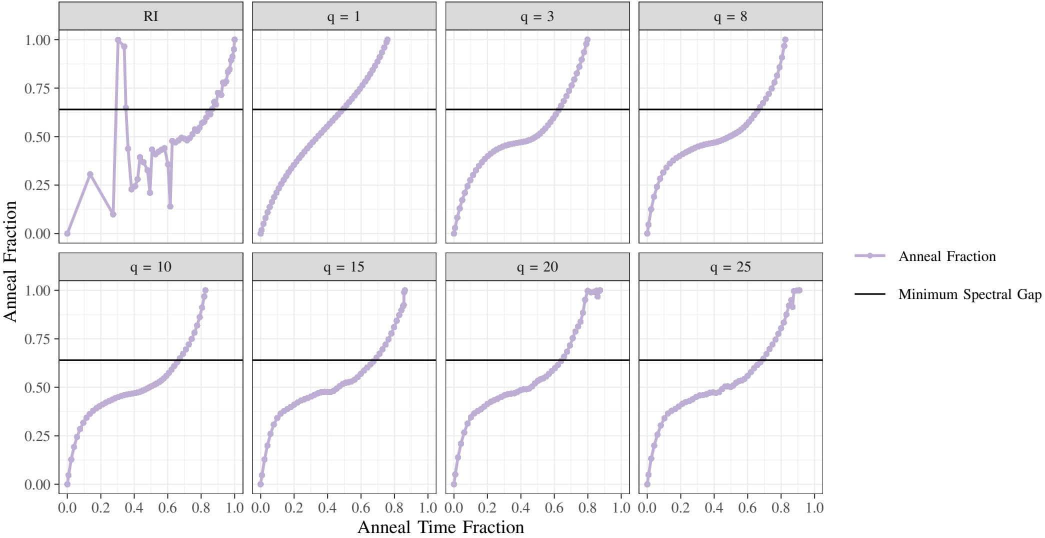
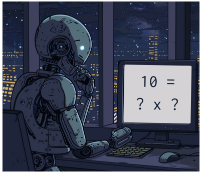
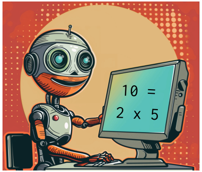

RI
q = 1
q = 3
q = 8
q = 10
q = 15
q = 20
q = 25
Anneal Fraction
0.00
0.25
0.50
0.75
1.00
0.0
0.2
0.4
0.6
0.8
1.0
Anneal Time Fraction
Anneal Fraction
Minimum Spectral Gap

Open Questions

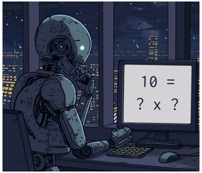
- Generalisation?
- Experimental Evaluation (D-Wave)?
- Optimal QAOA Parameters?
- Noise?
  → Co-Design promising



CPU

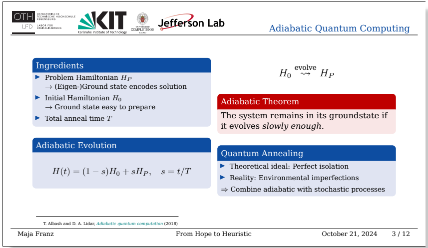


Error-corrected QPU



NISQ QPU

Thank You!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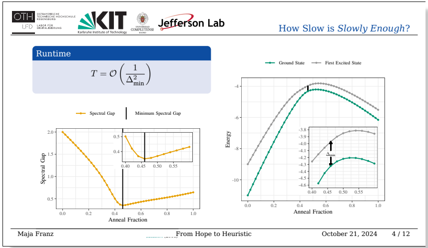
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Spectral Gap
Minimum Spectral Gap
Ground State
First Excited State
Energy
Anneal Fraction
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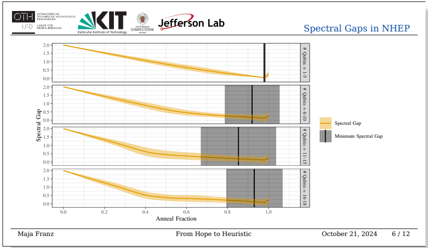
Spectral Gaps in NHEP
Spectral Gap
Anneal Fraction
Qubits = 1-5
Qubits = 6-10
Qubits = 11-15
Qubits = 16-18
Spectral Gap
Minimum Spectral Gap
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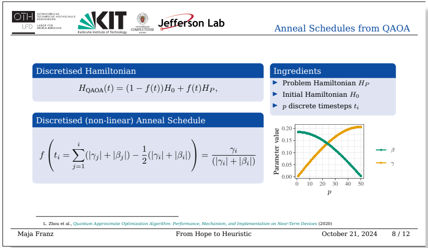
Anneal Schedules from QAOA
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Parameter value
p
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γ
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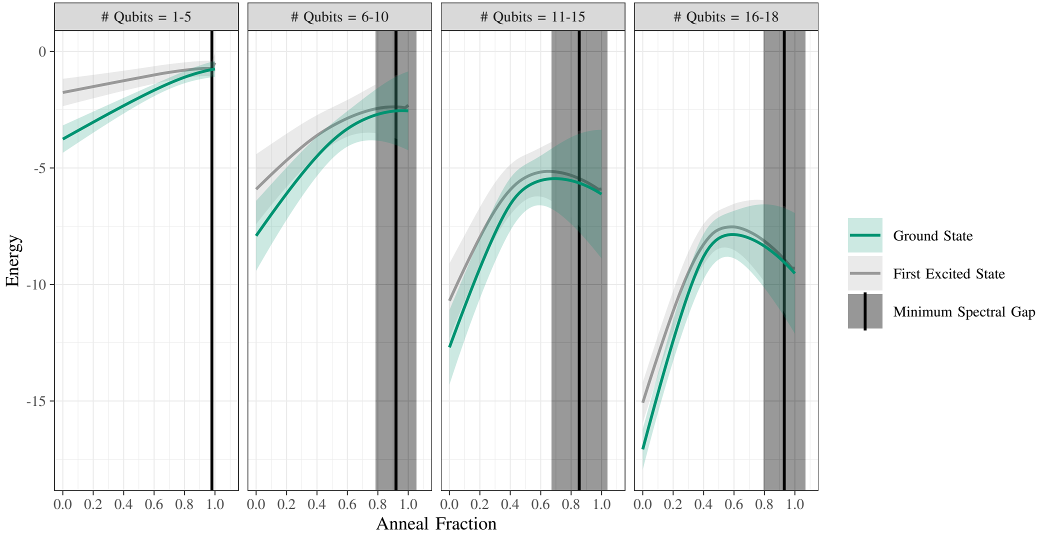
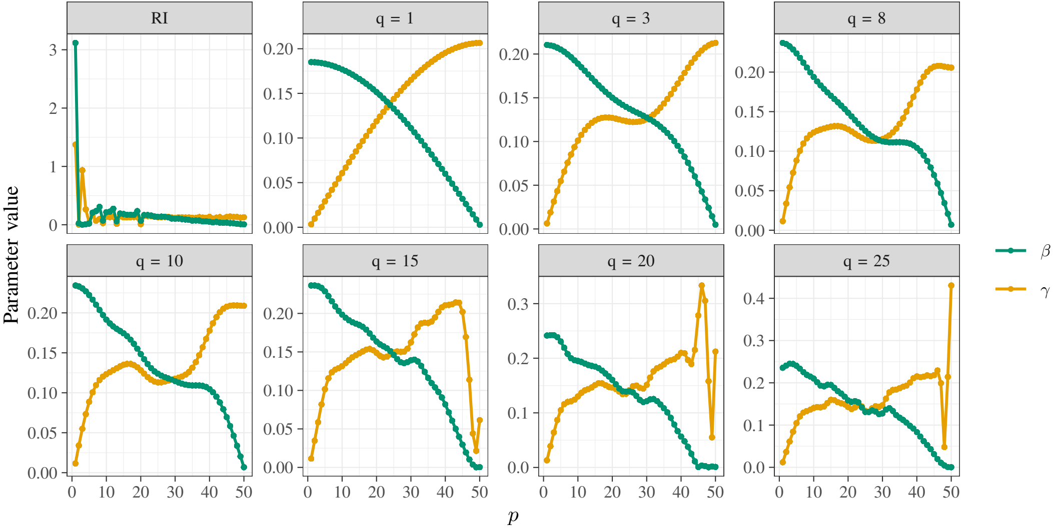
Backup Slides

Energy Spectra

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QAOA Solution Quality