Adaptive Hough Transform for Charged Particles Tracking at the LHC

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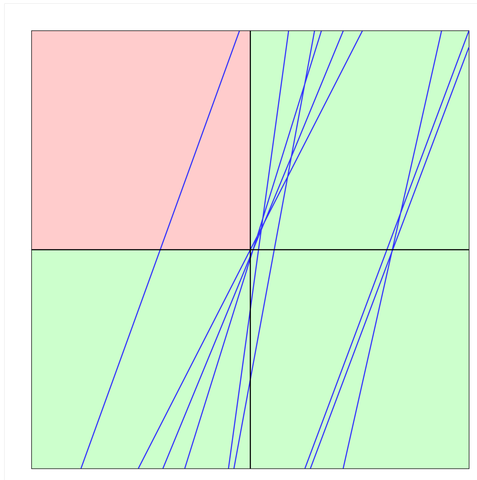
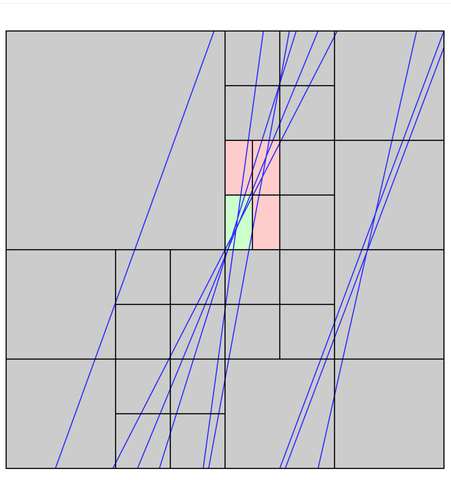
1. Adaptive Hough Transform (AHT)

- 'virtual' accumulator, memory is never allocated for the entire image
- Image space in searched in the recursive manner, starting from one cell for the entire parameter space and divided until desired precision is reached
- In first approximation, the memory size is 'size of cell description' × 'number of cells needed to divide image space down to desired size' e.g., when dividing into 4: 4+4+4+4+...+4 = 4 × N, N ≈ 13
- More accumulator dimensions only mildly add to the memory use
- Recursive implementation replaced with stack [pop cell → discard or divide → possibly produce solution]
- Transformation equation (φ and r – space point polar coordinates in xy plane):

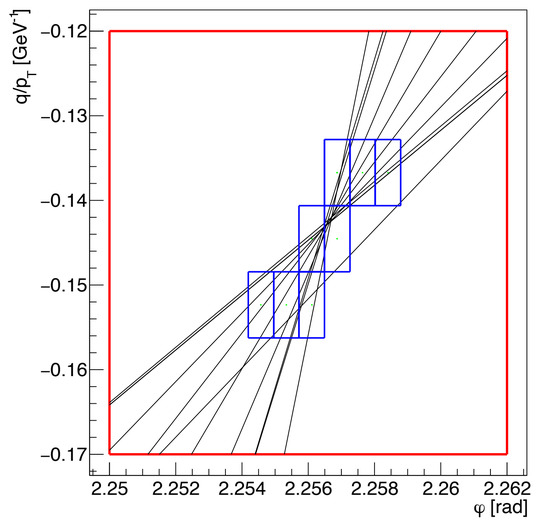
$$\frac{q}{p_T}(\varphi_0) = \frac{1}{rA}\varphi_0 - \frac{1}{rA}\varphi$$

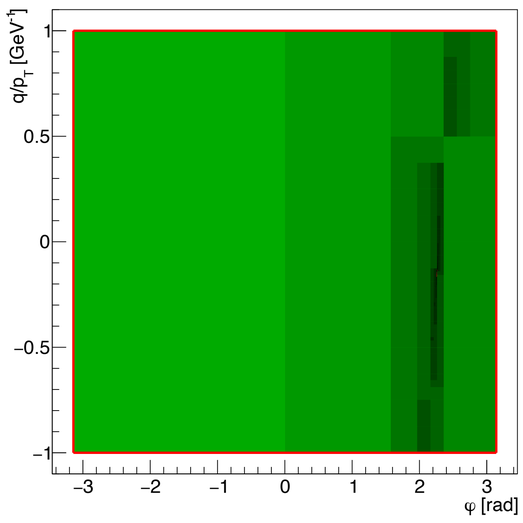




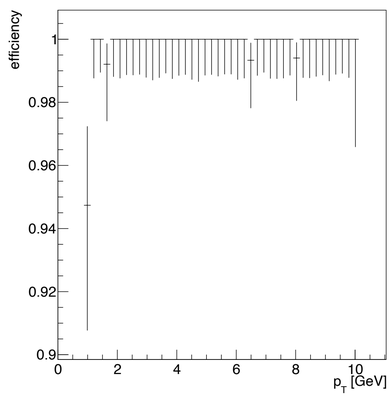
2. Validation with single muon tracking

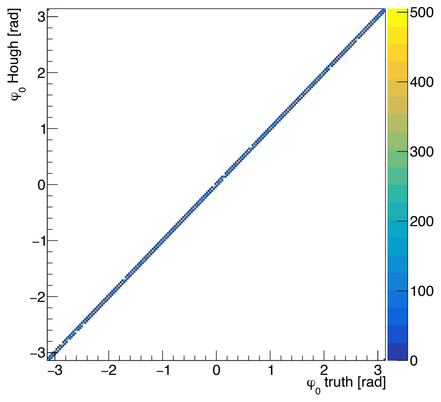
- Transverse momentum in the range from 1 GeV to 10 GeV
- Precision in φ_0 and q/p_T minimizes number of solutions, while maintaining 100% efficiency ($0.001\ rad$ and $0.01\ GeV^{-1}$, 13 and 8 divisions respectively)
- Cell counts threshold (to further divide accumulator section) – 6
- Very high tracking efficiency, precise estimate of φ_0 angle
- On average 10 solutions are found for a single truth particle → necessity to incorporate additional filtering phases
- Green picture below – the darker the color the deeper in the division level cell is

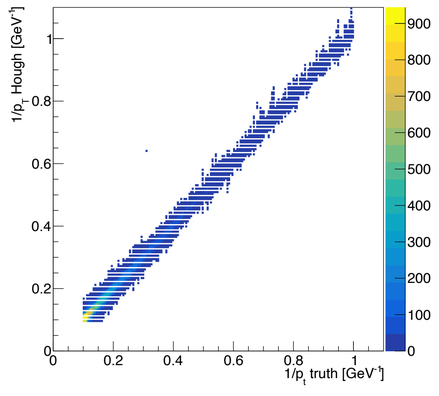




- Efficiency as a function of p_T, scatter plot for truth vs Hough φ_0 and

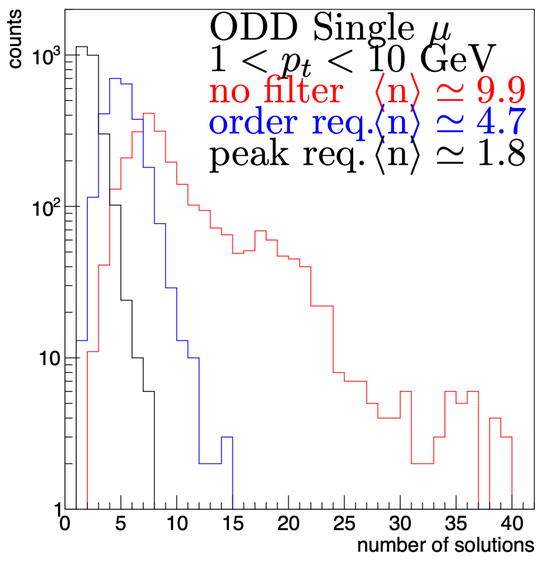


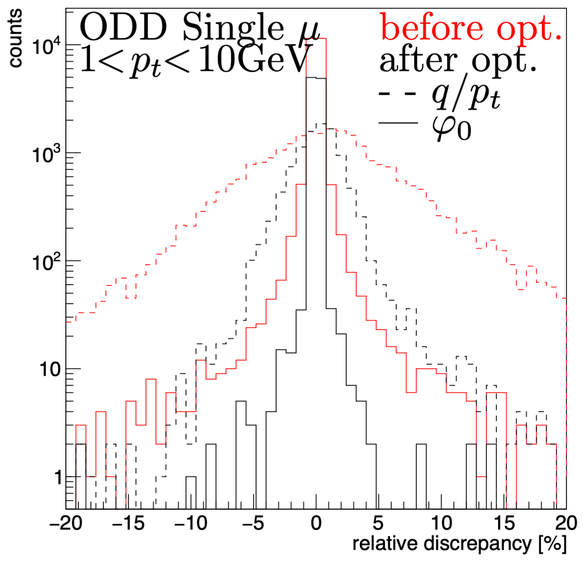




3. Filtering algorithms in Hough space

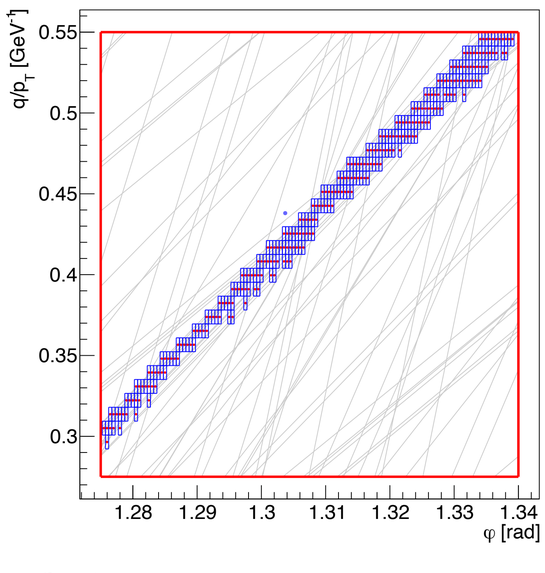
- Counting number of line order changes within the cell, no order change indicates that lines do not intersect → no solution within the cell
- Peak finding – for a solution candidate, calculate line crossing coordinates and reject points which are outside $\mu \pm n \cdot \sigma$ range, until none is outside that range, verify compliance with the count threshold
- Data partitioning - Divide space points into wedges (defined in terms of η and φ), allows for parallel tracking, implemented only for pile-up events

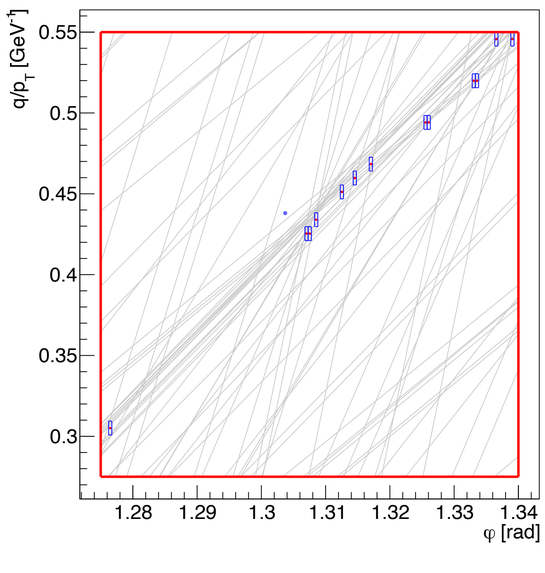


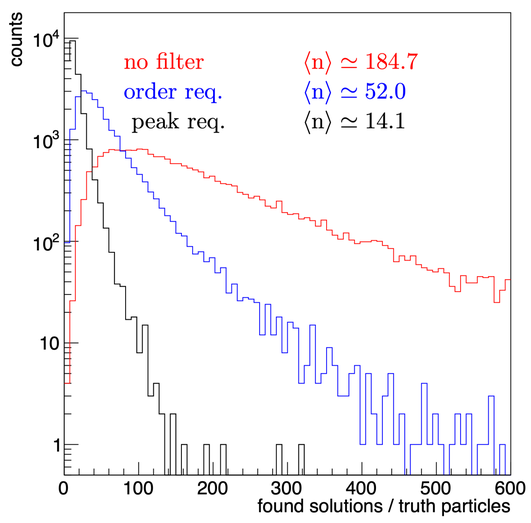


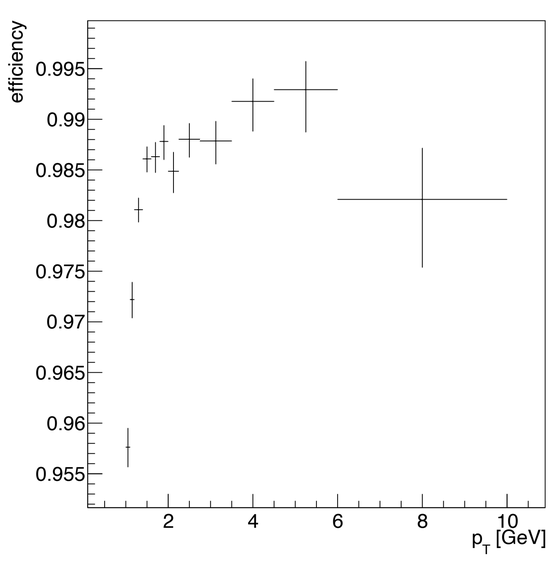
4. Application in high pile-up events

- On average 200 proton-proton interactions in HL-LHC
- η - φ space divided into 8 and 15 regions respectively, $|\eta| < 1.1$
- Share of fake solution reduced from 62.2% to 38.2%
- Maintained very high tracking efficiency for reduced total number of solutions









5. References:

- GitHub project – https:/github.com/tboldagh/helix-solver
- Exploration of adaptive Hough algorithm variants for charged particles tracks finding for the ATLAS experiment for the LHC Run 4 – https://cds.cern.ch/record/2912867

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