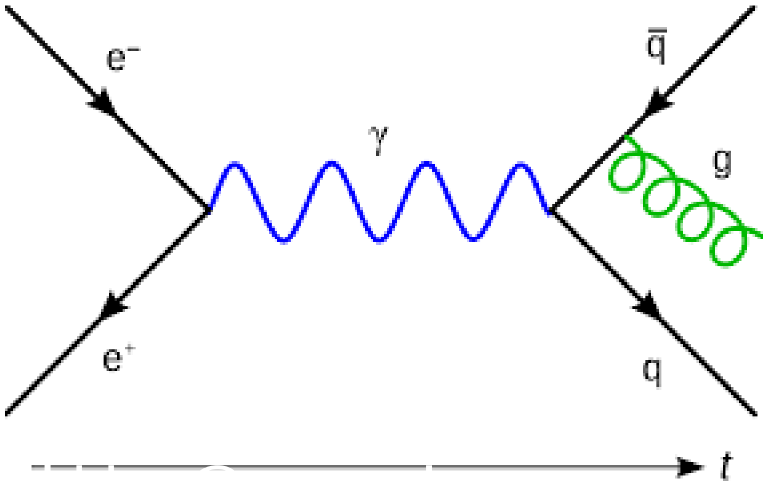
Quantum error mitigation for Fourier moments computation



Oriel Kiss, Michele Grossi and Alessandro Roggero

arXiv: 2401.13048

CHEP 2024- Krakow – 24.10.24

// What should we do with available quantum computers ?

1) Variational algorithms?

NISQ-friendly
BUT expensive classical-quantum feedback loop
+ vanishing gradients.

2) QSVT?

Can perform arbitrary Hamiltonian transformations
with Heisenberg scaling
BUT large depth overhead + non-local operations.

3) Fourier moments !

Incoherent QSVT
+ requires only a simple controlled time-evolution
+ no feedback loop
BUT larger sample overhead.

// Why do we care about Fourier moments ?

We can compute any Hamiltonian transformation incoherently! (Depth <-> Samples)

Example: ground state energy estimation

Spectral measure

$$C(x) = p(x) * \Theta(x)$$ Heaviside step function

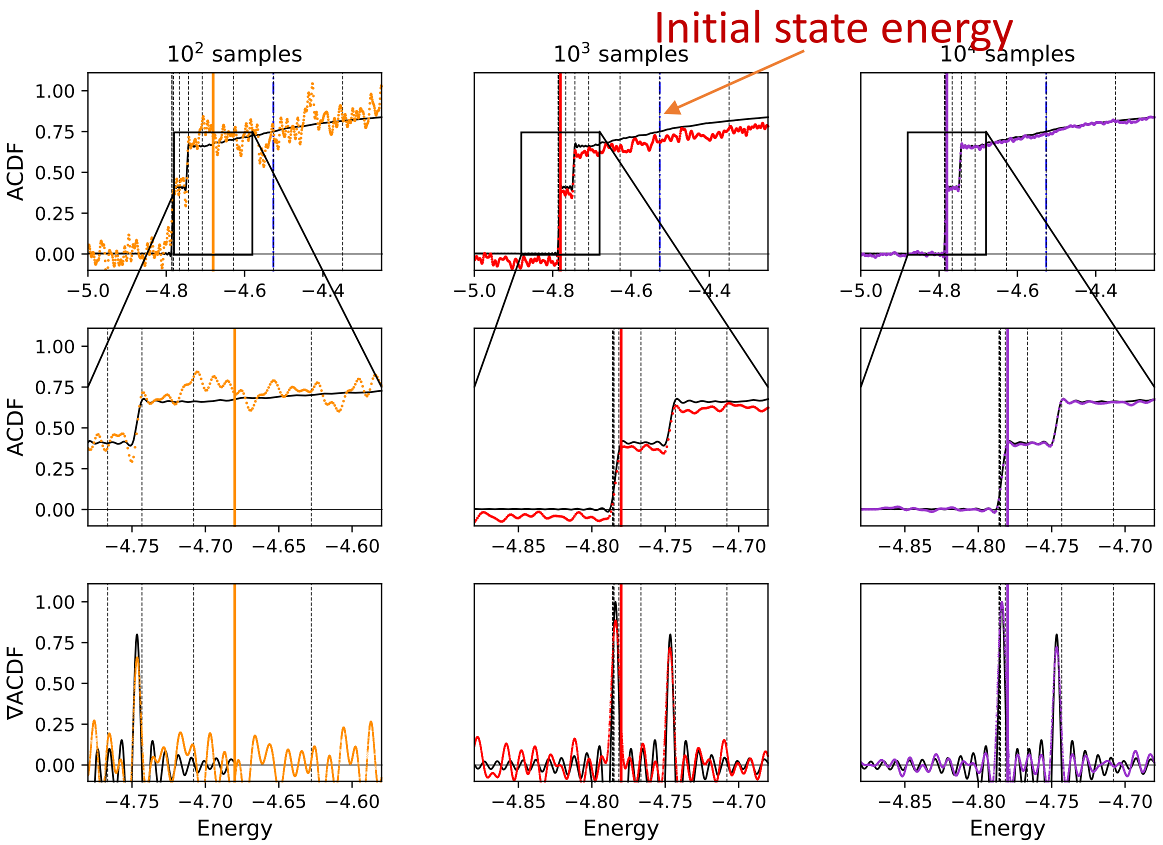
$$\tilde{C}(x) = \int_{-\pi/2}^{\pi/2} p(y)F(x-y)dy$$

$$= \sum_{|k|\leq D} F_k e^{ikx} \langle\Psi|e^{-i\tau k\mathcal{H}}|\Psi\rangle.$$

Fourier moments are computed on the QC with a Hadamard test.

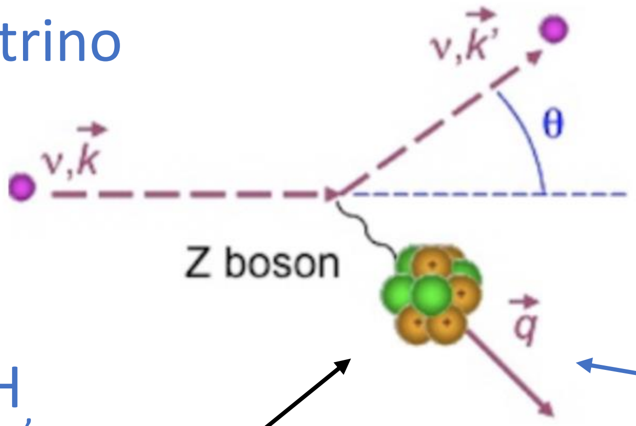
Lin and Tong PRX Quantum 3, 010318 (2022)
Wan et al. Phys. Rev. Lett. 129, 030503 (2022)

fully-connected 26 spins **Heisenberg model** with random couplings [Kiss et al. arXiv:2405.03754 (2024)]



// Scattering process

Incoming neutrino



momentum transfer described by O (external probing).

Nuclei described by H, in its ground state.

Effective field theory in 1st quantization

Two-dimensional Fermi-Hubbard (2x2)

$$H = 4.5 \cdot \mathbb{1} - 2\sum_{i=1}^{4} X_i$$

$$+ 1.75 \left(\sum_{i<j<k} Z_i Z_j Z_k + Z_1 Z_4 + Z_2 Z_3 \right)$$

$$\hat{O}(\vec{q}_k) = \sum_{f=\uparrow,\downarrow} \rho_f(\vec{q}_k) = \sum_{f=\uparrow,\downarrow} e_f \sum_i e^{i\vec{q}_k \cdot \vec{r}_i} n_{i,f},$$

Reciprocal vector

// Linear response function (~inclusive reaction cross-section)

$$S(\omega,\vec{q}) = \langle\Psi_0|\hat{O}(\vec{q})^\dagger\delta(\omega-(E_0-E_f))\hat{O}(\vec{q})|\Psi_0\rangle$$
$$= \sum_f \left|\langle\Psi_0|\hat{O}(\vec{q})|f\rangle\right|^2 \delta(\omega-(E_0-E_f)),$$

Challenging, since it requires the full spectrum.

Instead: expand in a suitable basis of polynomial (plane waves)

$$\Phi(\nu,\vec{q}) = \int d\omega K(\nu,\omega)S(\omega,\vec{q})$$
$$= \langle\Psi_0|\hat{O}(\vec{q})^\dagger K(\nu,(H-E_0))\hat{O}(\vec{q})|\Psi_0\rangle$$

$$\Phi_N^\chi(\nu) = \frac{1}{\chi\|H\|}\sum_{n=-N}^{N} g_n^\chi(\nu)\boxed{\langle\Psi_0|\,\hat{O}(\vec{q})^\dagger e^{-in\delta tH}\hat{O}(\vec{q})\,|\Psi_0\rangle}\,,$$

Fourier moments

Hamiltonian moments
Easy with quantum computers!

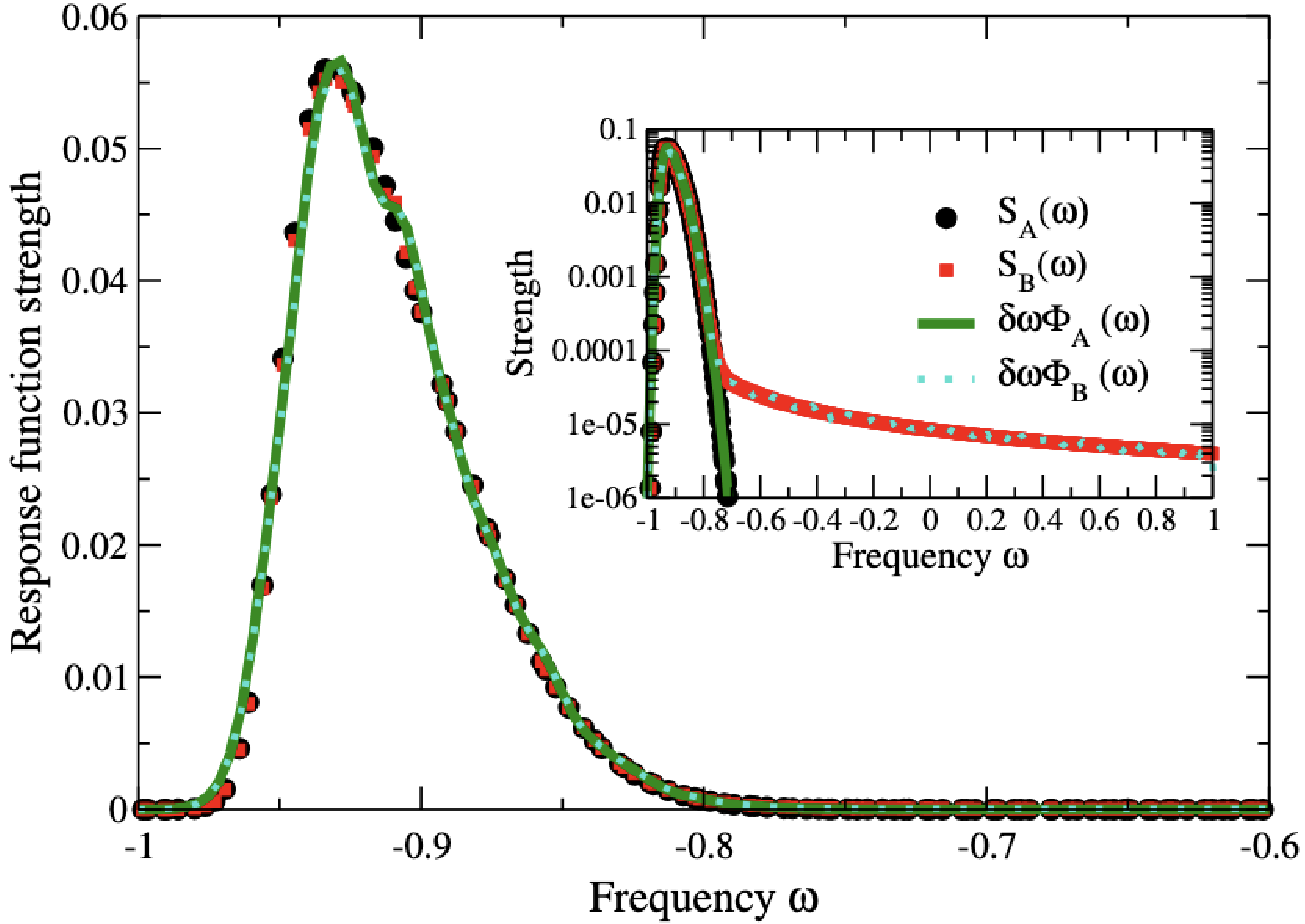
Roggero, Phys. Rev. A 102, 022409 (2020)

Hartse & Roggero, Eur. Phys. J. A **59**, 41 (2023)

// Artificial example

We obtain the resonance frequency of the physical system.

The integral transform is a good approximation.



Hartse & Roggero, Eur. Phys. J. A **59**, 41 (2023)

// Why error mitigation?

We have quantum computers, and many applications, but can not implement them!

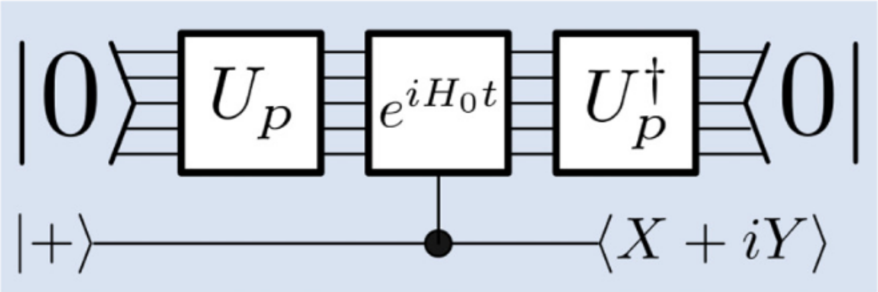
1. Noise prevents any chance of quantum advantage.
2. Waiting on quantum error correction could take a long time.
3. Instead, try to make the best out of the machines we have today and build a bridge towards fault tolerance.

This talk: look for synergies in quantum error mitigation protocols.

// Purified echo verification: two ingredients

Verification:

Unprepare the state and verify!



Verification passed → state contributes +/- 1 to the expectation value.

Otherwise → garbage

O'Brien, et al., PRX Quantum 2, 020317 (2021)

Purification of the ancilla:

$$\rho = \lambda|\psi\rangle\langle\psi| + (1-\lambda)\sum_{k=2}^{2^N} p_k|\psi_k\rangle\langle\psi_k|.$$

main component

Noisy components

Without noise: the ancilla is pure after post-selection

With noise: It is not. Extract the closest pure state from measurements.

Phys. Rev. A 105, 022427 (2022)

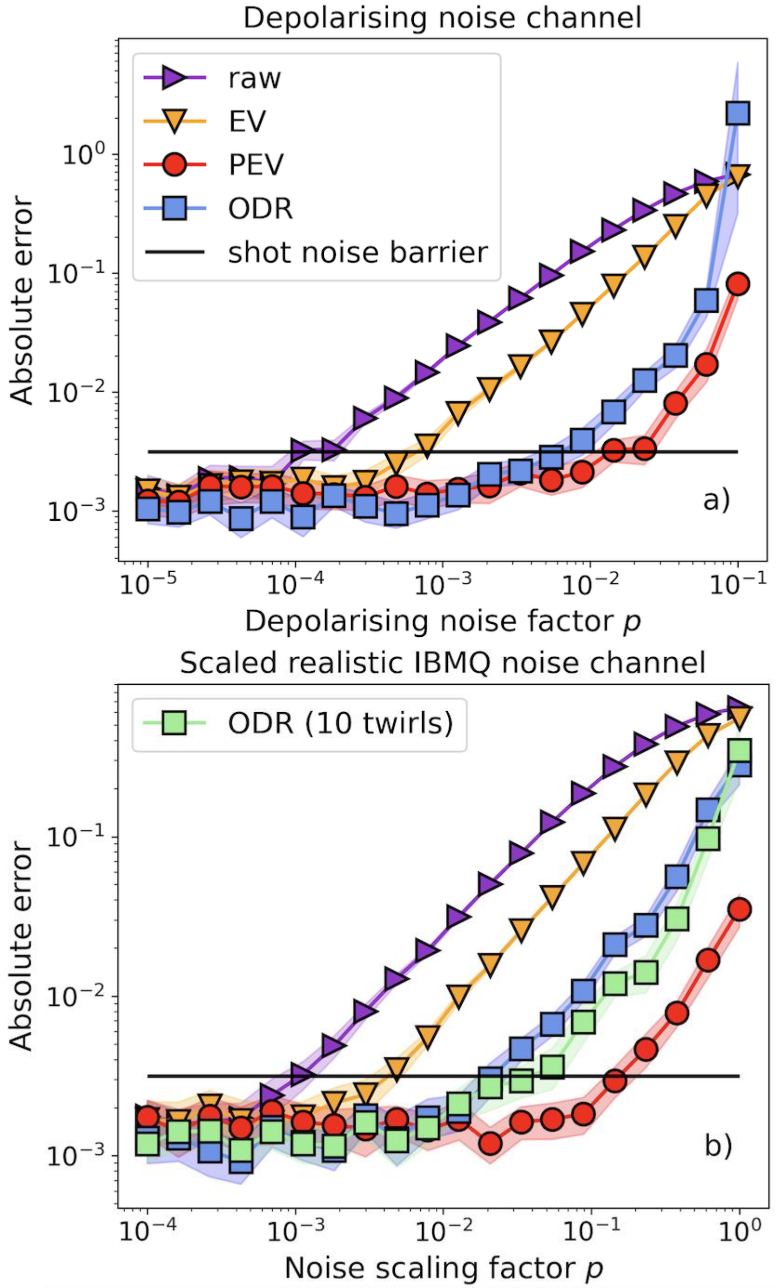
// PEV is resilient to noise models

1. We can show that PEV diminishes the error from a depolarising channel by a factor of 2^n.

2. O'Brien showed numerical evidence against **damping and dephasing** channels.

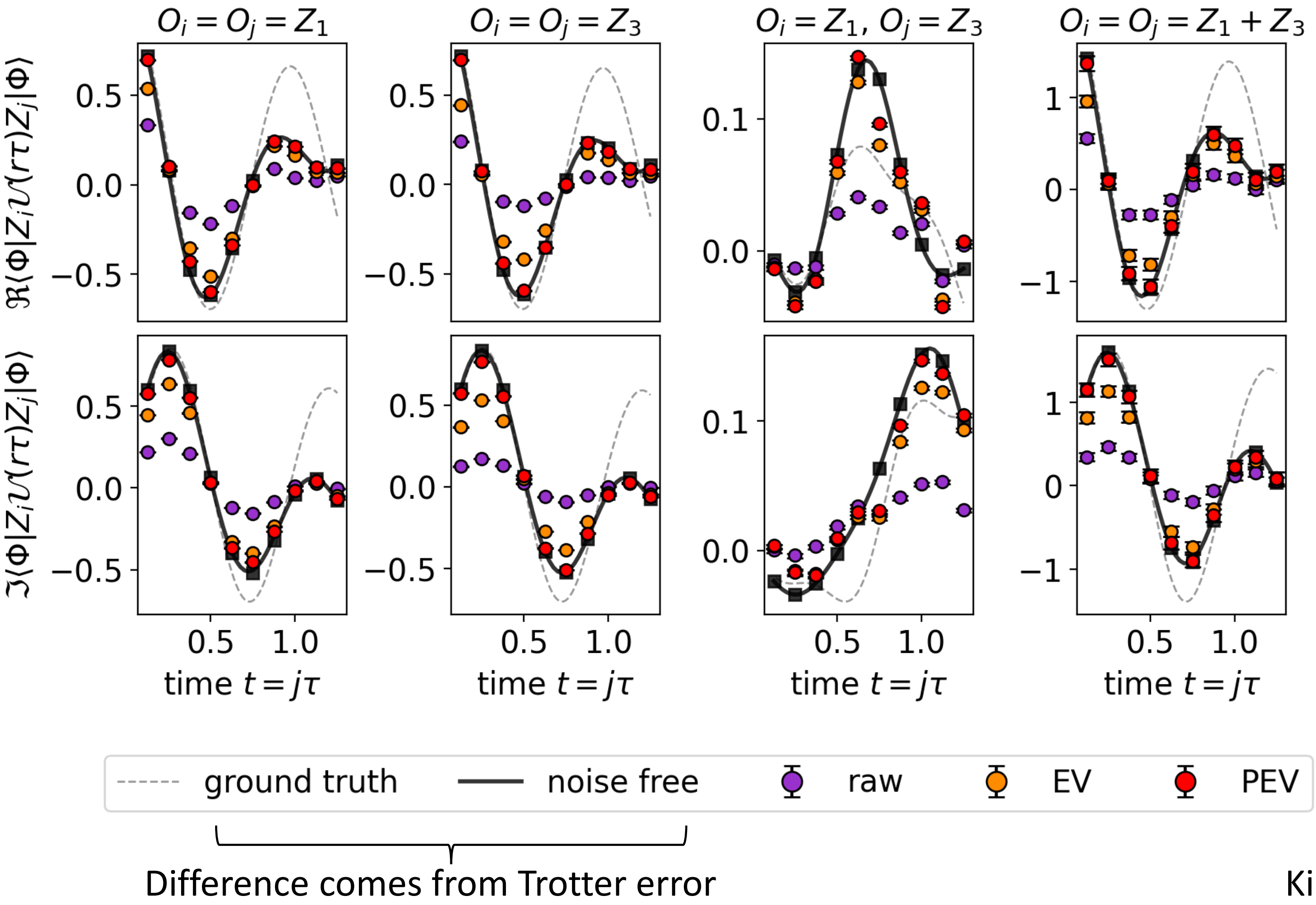
3. Here: depolarising and **scaled "realistic"** noise channel.

4. Cross the sampling noise threshold **100** times faster.



O'Brien et al. PRX Quantum 2, 020317 (2021)
Kiss et al, arXiv: 2401.13048 (2024)

// Results on superconducting quantum hardware



Single Trotter step

PEV effectively mitigates the noise on a real quantum computer.

Kiss et al, arXiv: 2401.13048 (2024)

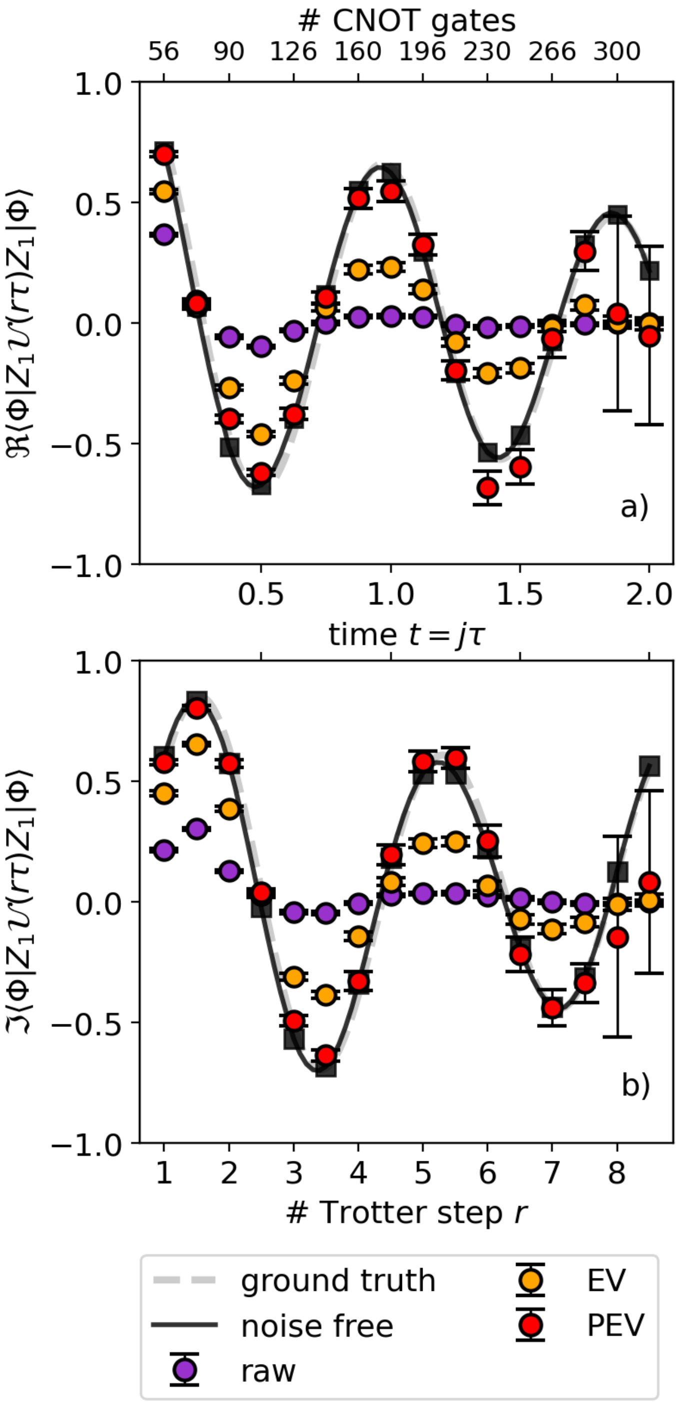
// Multiple Trotter steps

Trotter decomposition

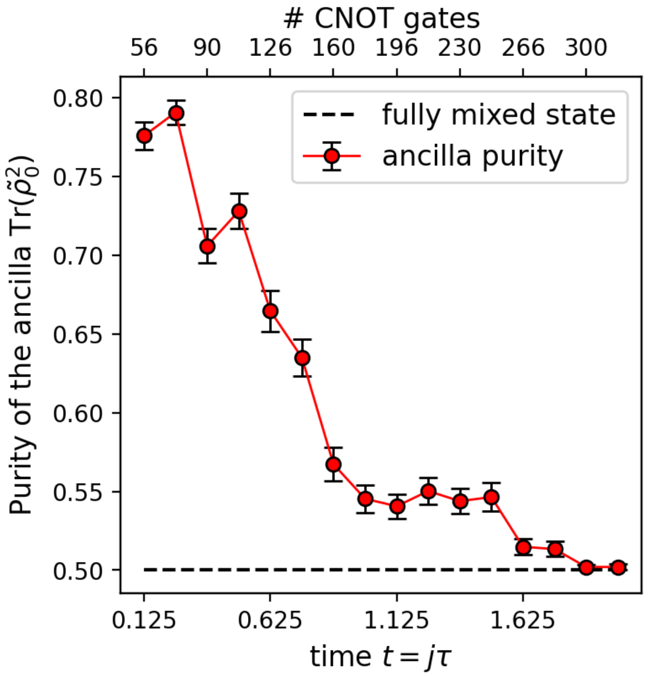
$$\mathcal{U}_1(t) = \overrightarrow{\prod_{\gamma}} e^{-itH_\gamma} = \mathcal{U}(t) + \mathcal{O}(t^2),$$

Decrease the error by breaking the unitary into smaller steps.

$$\mathcal{U}_{2j}\left(\frac{t}{r}\right)^r = \mathcal{U}(t) + \mathcal{O}\left(\frac{t^{2j+1}}{r^{2j}}\right)$$



Purity of the ancilla



Purity =1/2

Fully mixed state

Method breaks

// Conclusions

- Fourier moments are promising candidates for quantum utility on near-term quantum computers.

- They are versatile (arbitrary Hamiltonian transformation), and efficient to compute on quantum computers.

- Purified echo verification is a powerful technique to estimate Fourier moments.

arXiv: 2401.13048

// Thank you for your attention!
Questions ?

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// Noise renormalization

Urbanek et al, **Phys. Rev. Lett. 127, 270502 (2021)**

Assume depolarising channel:

$$\mathrm{Tr}\{\sigma\mathcal{N}(\rho)\} = (1-p)\,\mathrm{Tr}\{\sigma\rho\},$$

If we can estimate *p*,

$$1 = \langle\bar{0}|BO_l\mathcal{U}(j\tau)\mathcal{U}(-j\tau)O_l^\dagger B^\dagger|\bar{0}\rangle,$$

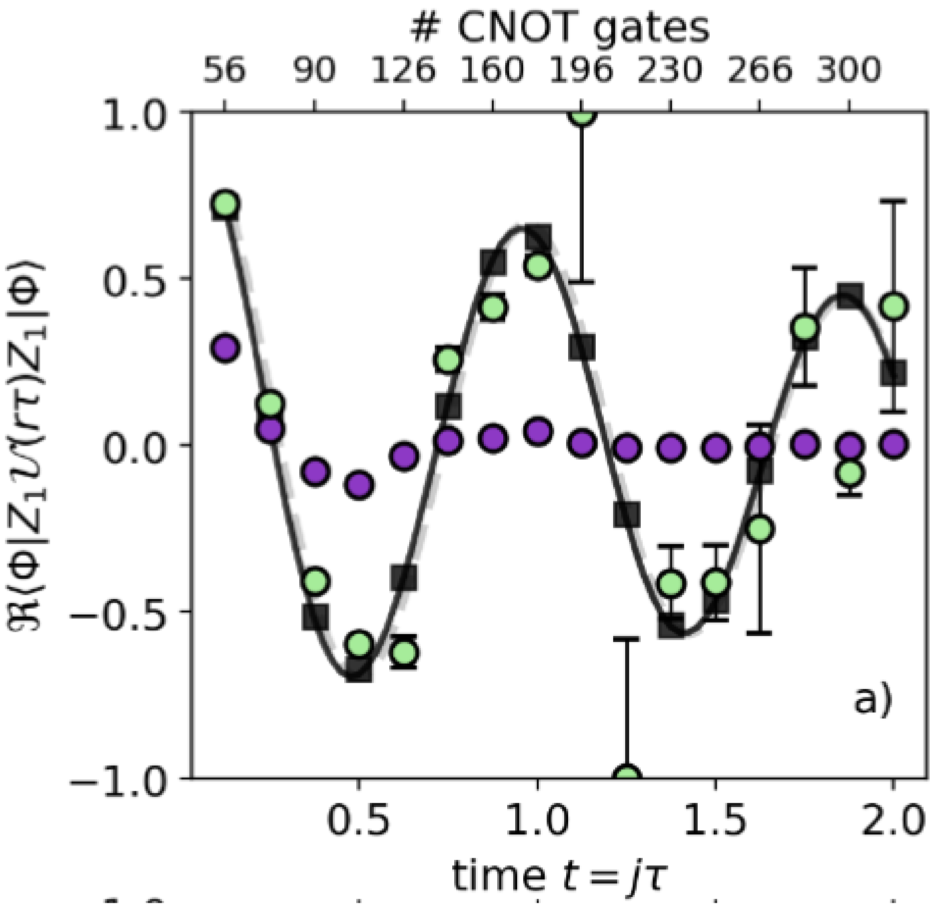
we can renormalize.

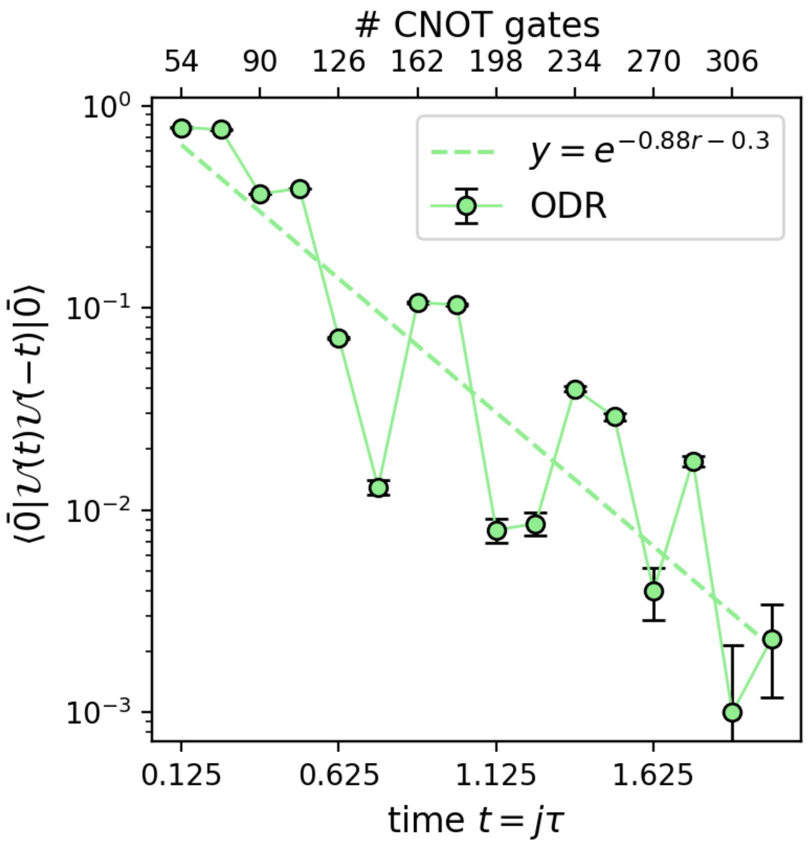
$$\langle\psi|\mathcal{U}(2j\tau)|\psi_k\rangle_{ODR} = \frac{\langle\psi|\mathcal{U}(2j\tau)|\psi\rangle}{\langle\Psi|\mathcal{U}(j\tau)\mathcal{U}(-j\tau)|\psi\rangle}.$$

But it only works with depolarising channels ?!

Yes, but you can use Pauli twirling to make the noise look more depolarising!

// Multiple Trotter steps





Less stable than PEV,
but easier to implement
and can estimate multiple observables at the same time

Exponential decay in depth …

// Kibble-Zurek Mechanism

Teplitskiy, Kiss et al. arXiv:2410.06250

Transverse Field Ising model

$$H(t) = -J(t)\sum_{i=1}^{N-1} X_i X_{i+1} - h(t)\sum_{i=1}^{N} Z_i$$

Defect-density

$$\hat{n} = \frac{1}{2N}\sum_{i=1}^{N-1}(\mathbb{1} - X_i X_{i+1}).$$

Schedules

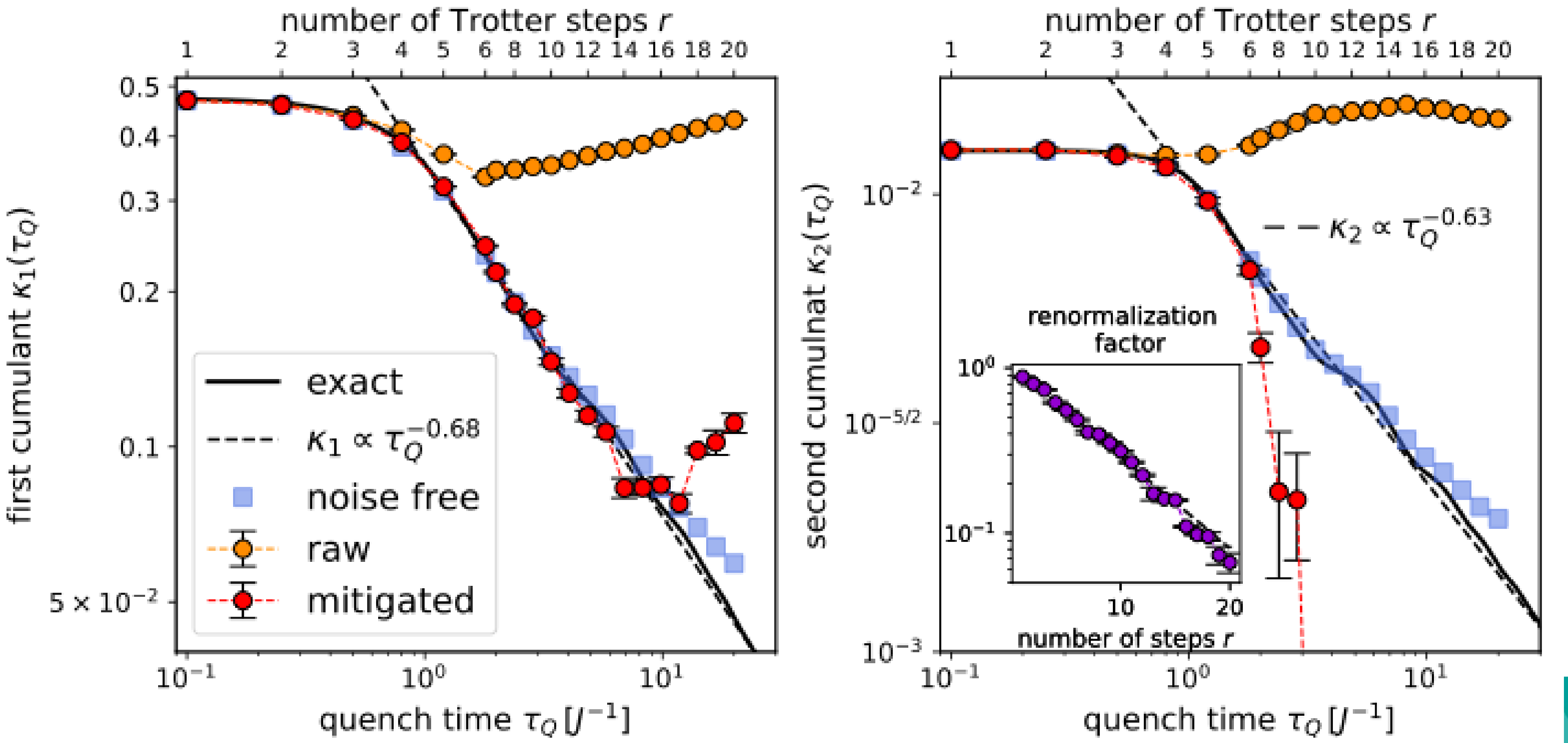
$$J(t) = J_0 t/\tau_Q \text{ and } h(t) = (1 - t/\tau_Q)h_0.$$

KZM

$$n \sim \frac{1}{\xi(\lambda(\hat{t}))^d} \propto \tau_Q^{-\frac{d\nu}{1+z\nu}},$$

We cross the QPT at $\tau_Q/2$.

Experience on the 20 Transmon qubits IQM devices



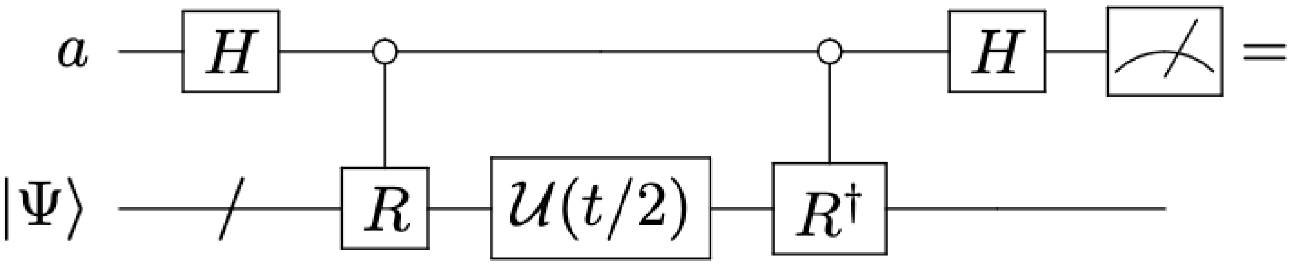
// Control Reversal Gates

Or how to avoid the control operation?

R anti commutes with H: {H,R} = 0
Use R to toggle the flow of time

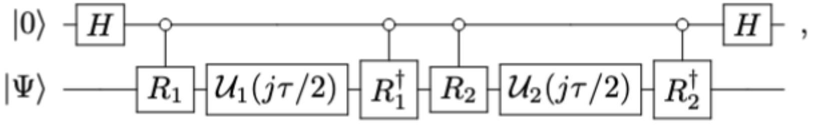
$$R\exp\{-iHt\}R^\dagger = R\sum_n \frac{(-itH)^n}{n!}R^\dagger = \sum_n \frac{(itH)^n}{n!}RR^\dagger = \exp\{itH\},$$

Hadamard test with control reversal gates



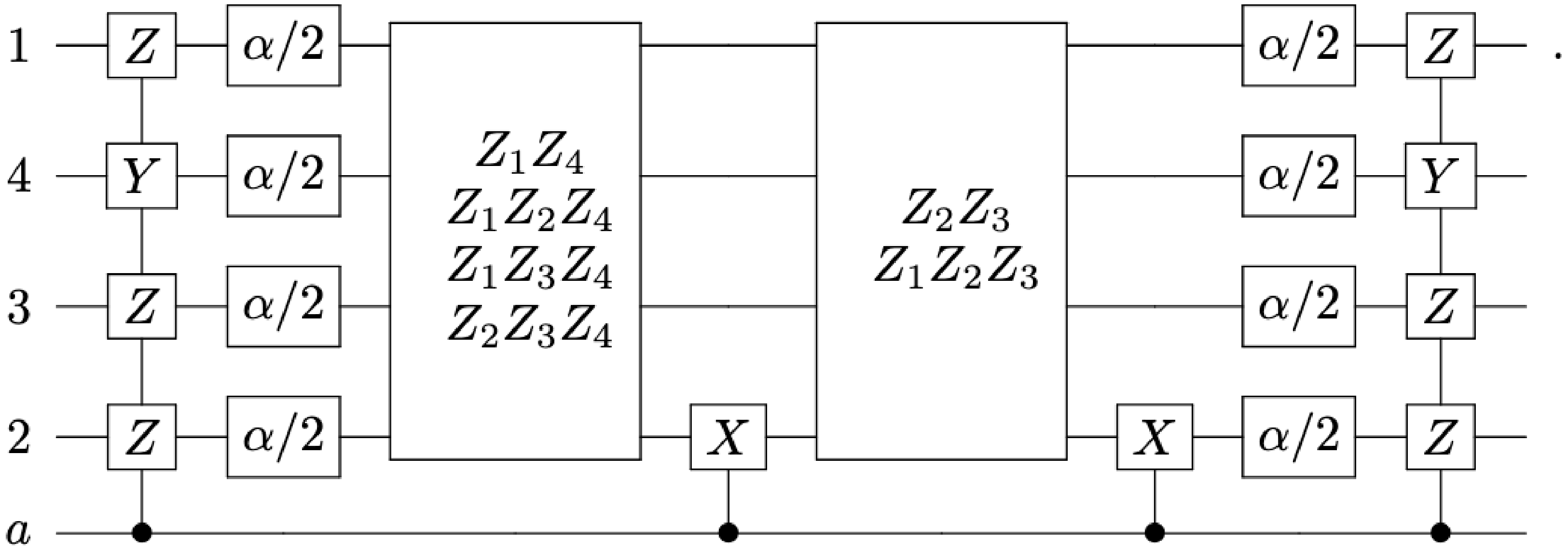
What if we can not find such a R?

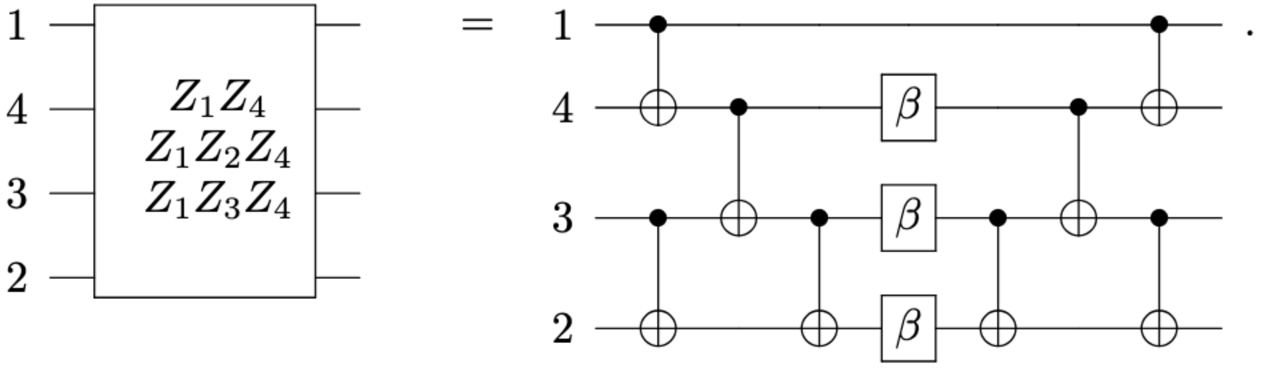
$$H = H_1 + H_2$$
$$\mathcal{U}(t) = \mathcal{U}_1(t)\mathcal{U}_2(t)$$

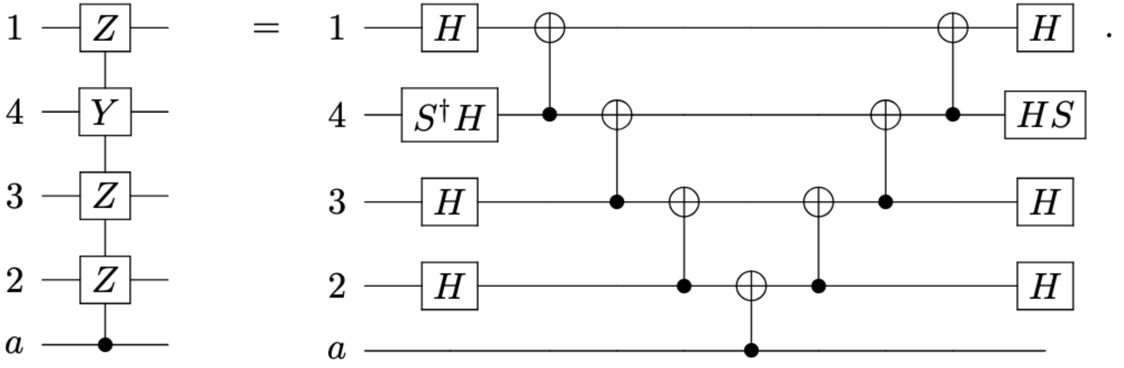


Always cheaper than direct implementation!

// How do you do it in practice?
-> Lots of manual optimization

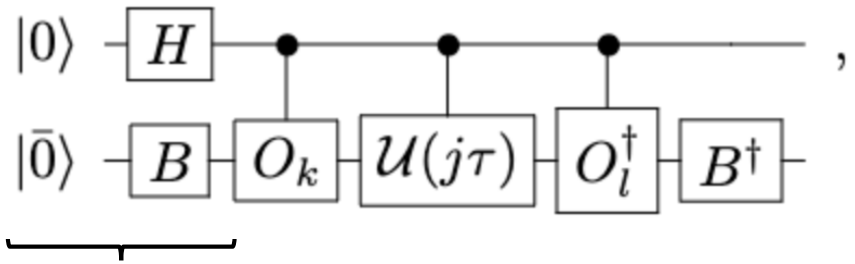






// Purified Echo Verification (more details)

Hadamard test (quantum phase estimation)



Initial state

$$|\Phi\rangle = \frac{1}{\sqrt{2}}\left(|\bar{0}\rangle \otimes |0\rangle + B^\dagger O_k^\dagger \mathcal{U}(j\tau) O_l B|\bar{0}\rangle \otimes |1\rangle\right)$$
$$\equiv \frac{1}{\sqrt{2}}\left(|\bar{0}\rangle \otimes |0\rangle + |\phi\rangle \otimes |1\rangle\right),$$

$$|\phi\rangle = \alpha|\bar{0}\rangle + \beta|\bar{0}^\perp\rangle.$$

We only care about $\boldsymbol{\alpha}$, so we can disregard any orthogonal states!

1. We measure the 3 single-qubit Pauli expectation (X,Y and Z) values of the ancilla.

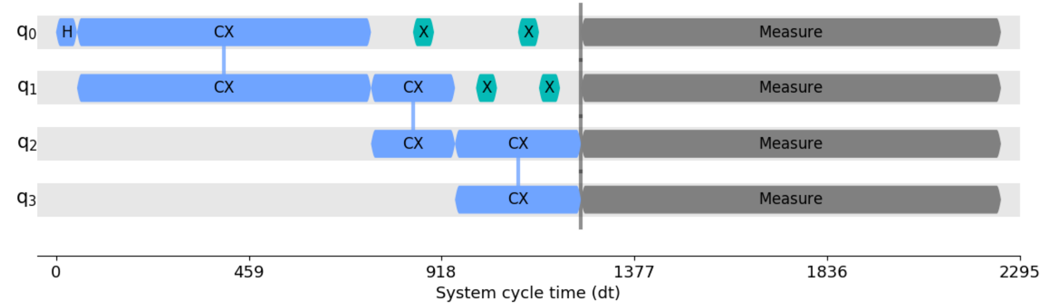
2. Construct the closest compatible pure state (**purification + tomography**).

$$\mathrm{Re}\{\alpha\} = \frac{\langle X_a\rangle_0}{1+\langle Z_a\rangle_0}, \quad \mathrm{Im}\{\alpha\} = \frac{\langle Y_a\rangle_0}{1+\langle Z_a\rangle_0}.$$

// What can we improve?
Turn coherent errors to incoherent ones!

Dynamical Decoupling

Decrease decoherence by taking advantage of **rapid, time-dependent control modulation** when the qubits are idling.

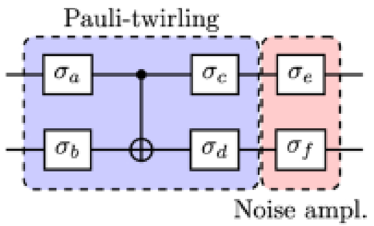


Which pulse sequences should we use? (XY8)

Randomized Compiling

Turn a noisy operator into a Pauli channel, via gate conjugation.

$$\mathcal{T}_W(\overline{M}) = \frac{1}{|W|} \sum_{w \in W} \overline{wMw^\dagger}\,.$$



(a) Pauli-twirling and noise amplification.

σ_a	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$	$\mathbb{1}$	σ_x	σ_x	σ_x	σ_x	σ_y	σ_y	σ_y	σ_y	σ_z	σ_z	σ_z	σ_z
σ_b	$\mathbb{1}$	σ_x	σ_y	σ_z	$\mathbb{1}$	σ_x	σ_y	σ_z	$\mathbb{1}$	σ_x	σ_y	σ_z	$\mathbb{1}$	σ_x	σ_y	σ_z
σ_c	$\mathbb{1}$	$\mathbb{1}$	σ_z	σ_z	σ_x	σ_x	σ_y	σ_y	σ_y	σ_y	σ_x	σ_x	σ_z	σ_z	$\mathbb{1}$	$\mathbb{1}$
σ_d	$\mathbb{1}$	σ_x	σ_y	σ_z	σ_x	$\mathbb{1}$	σ_z	σ_y	σ_x	$\mathbb{1}$	σ_z	σ_y	$\mathbb{1}$	σ_x	σ_y	σ_z

(b) Valid combinations for Pauli-twirling of the CX gate.

Fuchs, et al., *Eur. Phys. J. Plus* **135**, 353 (2020)