Monte Carlo challenges for Non Perturbative QED

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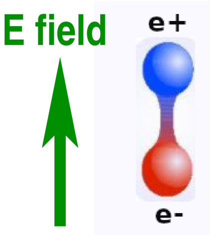
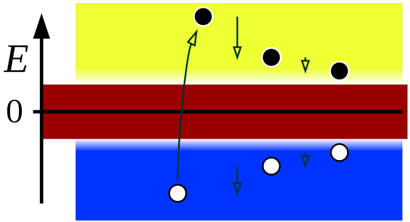
The strong field scale: Schwinger pair creation



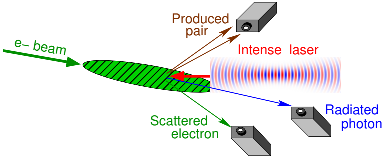


" Schwinger pairs created if virtual pairs separated by Compton wavelength $\lambda = \hbar/m_e c$ within the virtual pair lifetime $\Delta t = \hbar/m_e c^2$. "
" In strong external fields the normal vacuum is unstable and decays into a new vacuum that contains real particles. "

Greiner and Muller, QED of Strong Fields

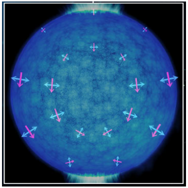
- The Schwinger critical field ($E_{\text{cr}} = m_e^2 c^3/e\hbar = 1.32 \times 10^{18}$ V/m)
- What novel experimental effects can we expect as $E \to E_{\text{cr}}$
- How do we calculate non perturbative cross-sections?
- What do experiments look like?

Where might we expect non perturbative effects?



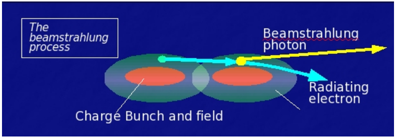
Electron/laser interactions

E_{cr} in the e-beam rest frame



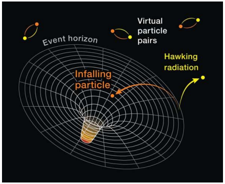
Magnetar B_{cr} near surface

Vacuum birefringence



q+q- particle collider

E_{cr} in each bunch's rest frame



Hawking radiation

G_{cr} equivalent critical gravitational field

Furry Picture: a non perturbative, semi classical QFT

[**Int J Mod Phys A33, 1830011 (2018)**]

Furry Pic Lagrangian, background field A^{ext}

$$\mathcal{L}^{\text{Int}}_{\text{QED}} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \tfrac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}(A\!\!\!/^{\text{ext}} + A\!\!\!/)\,\psi$$

$$\mathcal{L}^{\text{FP}}_{\text{QED}} = \bar{\psi}^{\text{FP}}(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} - \tfrac{1}{4}(F_{\mu\nu})^2 - e\bar{\psi}^{\text{FP}}A\!\!\!/\,\psi^{\text{FP}}$$

↪ **Bound Dirac equation**

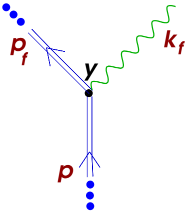
$$(i\partial\!\!\!/ - eA\!\!\!/^{\text{ext}} - m)\psi^{\text{FP}} = 0$$

↪ **Dressed wave functions**

$$\psi^{\text{FP}} = \mathbf{E}_p\, e^{-ip\cdot x}\, u_p$$

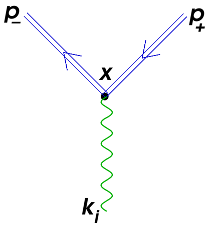
$$\mathbf{E}_p = \exp\left[-\frac{1}{2(k\cdot p)}\left(eA\!\!\!/^{\text{ext}}k\!\!\!/ + i2e(A^e\cdot p) - ie^2 A^{\text{ext}\,2}\right)\right]$$

Dressed Feynman vertex

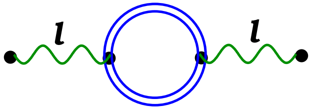


- New Feynman rules
- Exact wavefunctions only for some fields (plane wave, 1935)
- cross-sections are complicated & **still in progress**
- Effective, field dependent coupling constant, $f(\alpha, \chi)$
- χ is the field strength in rest frame of particle

Novel non perturbative processes

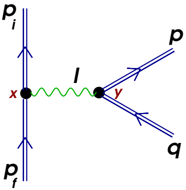


One photon pair prod^{n} (photon decay)

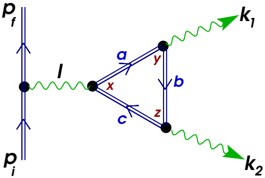


$$\text{propagator} = \frac{1}{l^2 + iM(l^2, \chi)}$$

Complex mass (resonant propagators)

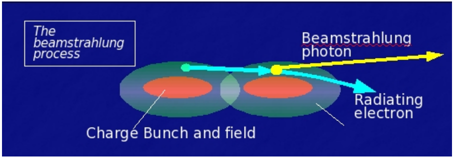


Trident process (resonant production)



Photon splitting (vacuum birefringence)

Strong fields at the collider Interaction Point



- SQED χ depends on collider bunch parameters and the pinch effect
- **All collider processes are SQFT processes**: backgrounds **and** signal
- Coherent e+e- pair production, depolarisation, WW pair production

  [A. Hartin, IJMPA **33**, 1830011 (2018)]

Machine	LEP2	SLC	ILC	CLIC
E (GeV)	94.5	46.6	500	1500
$N(\times 10^{10})$	334	4	2	0.37
σ_x, σ_y (μm)	190, 3	2.1, 0.9	0.49, 0.002	0.045, 0.001
σ_z (mm)	20	1.1	0.15	0.044
χ_{av}	0.00015	0.001	0.24	4.9

Field strength parameter, χ

Schwinger critical field, E_c

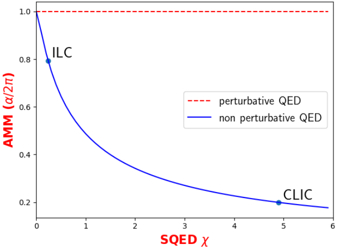
$$\chi = E_{rest}/E_{cr}, \quad E_c = 1.3 \times 10^{18} \text{ V/m}$$

precision spin physics/IP depolarisation needs:

e- anomalous magnetic moment

perturbative: $\frac{\Delta\mu}{\mu_0} = \frac{\alpha}{2\pi}$

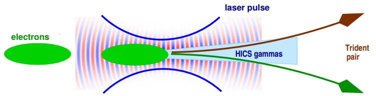
$$\frac{\Delta\mu}{\mu_0} = \frac{\alpha}{2\pi}\int_0^\infty \frac{2\pi\, dx}{(1+x)^3}\left(\frac{x}{\chi}\right)^{\frac{1}{3}} \mathrm{Gi}\left(\frac{x}{\chi}\right)^{\frac{1}{3}}$$

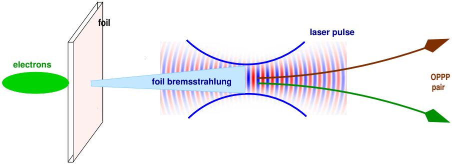


Experiment example: electron/laser interactions

[**Phys Rev D 99, 036008 (2019)**]

- Design experiment to test non perturbative phenomena, eg. mass shift, assisted Schwinger production
- Near head on collision between high energy electrons and focussed laser
- Field strength of laser relativistically boosted, parameters ξ, χ
- Several complementary experiments, spread of SQED parameters
- Different experimental configs allow several SQED processes
- Real experiment has gaussian pulse with varying intensity (ξ)
- Polarisation state may not be ”pure”
- Experimental effects need to be ”unpacked” - angular spread, gaussian pulse, crossing angle





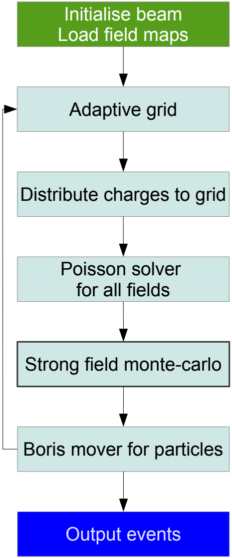
$$\chi = \xi \frac{k \cdot p}{m^2}$$ **recoil parameter**

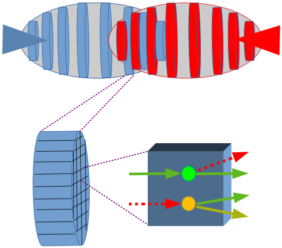
$$\xi (= a_0) = \frac{e|\vec{A}|}{m}$$ **intensity parameter**

Experiment	$\lambda(nm)$	E_{laser} (J)	focus (μm^2)	pulse (fs)	E_{e^-} (GeV)	ξ	χ
SLAC E144	527/1053	2	50	1880	46.6	0.66	2.7
LUXE Phase0	800	0.35	100	35	17.5	1.54	0.29
LUXE Phase1	800	7	100	35	17.5	6.9	1.29
FACET II	800	0.7	64	35	10	2.3	0.29
ELI-NP	1053	2.2	100	22	0.750	6.4	0.04
AWAKE	800	3	64	20	50	7.45	4.0

IPstrong - SQED monte carlo PIC simulation code

[**https://anthonyhartin.wixsite.com/physics/software**]

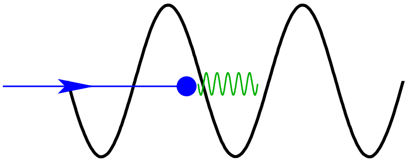




- Furry picture SQED interactions, via monte carlo, embedded in a 3D PIC electromagnetic solver
- Calculate SQED parameters (ξ, χ) in each voxel
- Monte carlo for each SQED process (rarest first)
- e-/laser, e+e- collisions, crystal lattices
- Internally generated or externally loaded bunches
- Fortran 2008 with openMPI (extendable to GPU)

High Intensity Compton Scattering - rest mass shift

Electron in background field needs extra energy just to exist



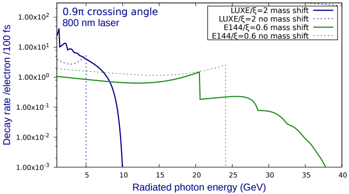
- An effective increase in rest mass (**Non perturbative effect!**)

$$p_{\rm i} + k\frac{\xi^3}{2\chi_{\rm i}} \rightarrow p_{\rm i}^2 = m^2(1+\xi^2)$$

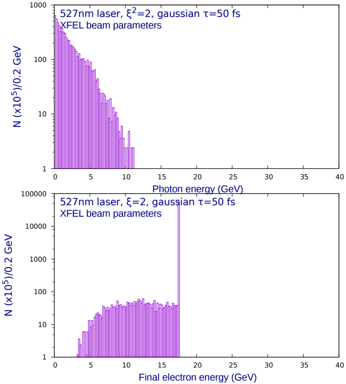
- Significant part of electron energy taken up by electron motion in the field/dispersive vacuum. Less energy available for radiated photon

- Manifests in Compton edge shift

HICS with mass shift for LUXE and E144

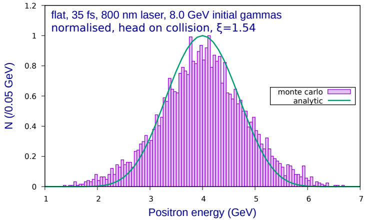


IPstrong monte-carlo, HICS for LUXE/ξ2



IPstrong validation, datasets and development

OPPP positron spectrum



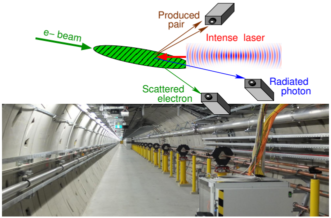
Features to be added

- Linear/elliptical polarised laser
- Initial and final state spin and polarisation
- Trident (with exact, non perturbative photon propagator)
- Milli-charged particles
- Other higher order processes

Monte carlo datasets

- initial and final states of beam as well as stdhep events
- BPPP: 5m and 12m foil to IP
- HICS + OPPP : gaussian pulse, 17.2^{o} crossing angle
- Ideal, flat laser pulses, head on collisions
- Latest version V1.2.02

/afs/desy.de/user/h/hartin/public/IPstrong

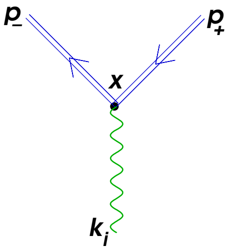


BACKUP

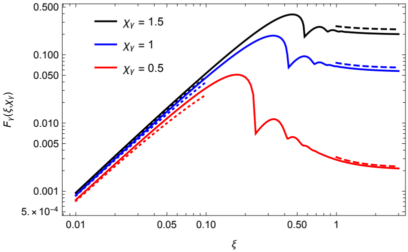
OPPP and Schwinger critical field measurement

[**Phys Rev D 99, 036008 (2019)**]

One photon pair production (OPPP)



OPPP rate, non perturbative regime



$$\Gamma_{\mathrm{OPPP}} = \frac{\alpha m^2}{2\omega_{\mathrm{i}}} \sum_{s>s_0}^{\infty} \int_1^{v_s} \frac{dv}{v\sqrt{v(v-1)}} \left[\mathrm{J}_s^2 + \frac{\xi^2}{2}(2v-1)\left(\mathrm{J}_{s+1}^2 + \mathrm{J}_{s-1}^2 - 2\,\mathrm{J}_s^2\right)\right] \propto \frac{\alpha m^2}{2\omega_{\mathrm{i}}} \frac{E}{E_c} \exp\left[-\frac{8 m E_c}{3\omega_i E}\right]$$

OPPP Rate at constant χ reaches non perturbative asymptote for $\xi \geq 1$

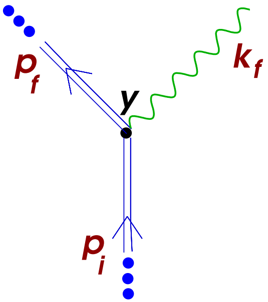
Note the similarity with the rate of Schwinger pair creation (from the vacuum):

$$\Gamma_{\mathrm{Schwinger}} = \frac{m^4}{(2\pi)^3} \frac{E^2}{E_c^2} \exp\left[-\pi \frac{E_c}{E}\right]$$

An experiment to measure the non perturbative OPPP process also informs us about Schwinger pair creation

Theoretical challenges

Theoretical challenges are simulation challenges as well!



Dressed vertices...

- The vertex is dressed with coupled spinors/momentum

$$\gamma^\mu_{\mathrm{y}} = \int \mathrm{d}^4y\, \bar{E}_f(y)\gamma^\mu E_i(y)\, e^{i[S_F(y)-S_I(y)]}$$

Smeared fermion wavepackets...

- We don't have "free" fermions, but "bound" fermions
- IN/OUT states require background field to vanish at $t = \pm\infty$ **[Meyer, J Math Phys 11 312 (1970)]**

Looong trace calculations...

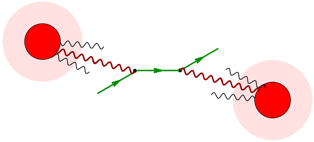
- Employ Fierz transformations for Volkov spinors **[Phys Rev D 94, 073002 (2016)]**

Integrated background contributions...

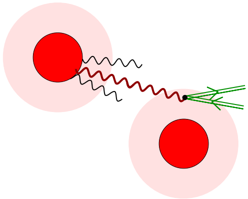
- Infinte summations over special functions and resonances. Analytic solutions help computation!

$$\sum_{r=-\infty}^{\infty} \frac{1}{r-a}\, \mathrm{J}_r(z_i)\, \mathrm{J}_{l-r}(z_f) = ?$$

SQED in ultra peripheral heavy ion collisions



heavy ion UPC: 2 equivalent photons



SQED heavy ion UPC: 1 equivalent photon + intense field

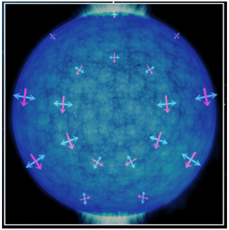
The "usual" EPA approach

- Approaching ions considered equivalent photons
- Search for low activity collisions, no QCD
- Gamma-gamma physics with Coulomb corrections
- Recent ATLAS Pb-Pb photon scattering search, hep-ex:1904.03536
- **CURIOUS:** unexplained resonances in heavy ion positron spectra at GSI, Darmstadt

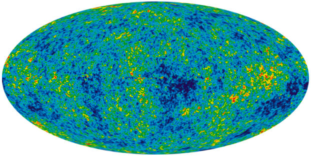
An SQED approach (new possible studies)

- Extremely strong fields operating over very short time scale, use SQED
- Equivalent gammas from one ion pass through the field of oncoming ion
- Ion field adjusts screening charge. photon has an effective mass
- SQED assisted Schwinger production
- SQED trident pair production, has SQED resonance in effective propagator (resonant positron spectrum)

Strong field Astrophysical/Cosmological arena



Magnetar Vacuum birefringence



CMB signatures

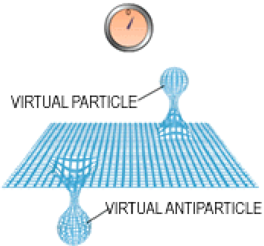
Vacuum birefringence

- Strong field effects in observation of polarised light from Magnetar
- Possible evidence of strong field vacuum birefringence (10^{13} G)
- Polarisation should be correlated with magnetic field relative to Earth
- Reported by arxiv:1610.08323

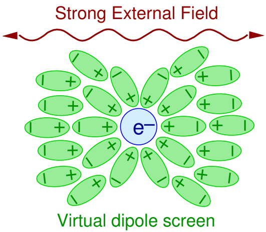
Cosmological Schwinger effect

- Hawking radiation is Schwinger pair creation in strong gravitational field
- Non perturbative QED $\leftrightarrow$ QED in curved space-times (Hollands, Wald arxiv:1401.2026)
- Strong field provided by gravity in early universe (Martin arxiv:0704.3540)
- Two point correlation function linked to CMB fluctuations

The polarisable quantum vacuum



Heisenberg uncertainty

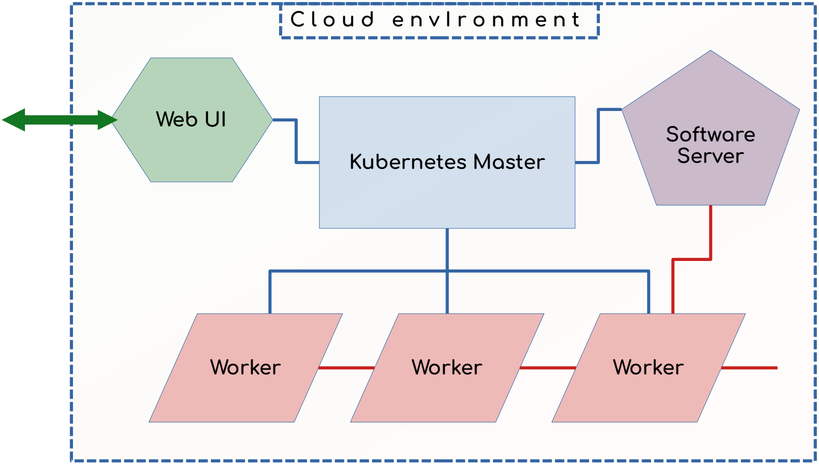


Bare/Dressed charge

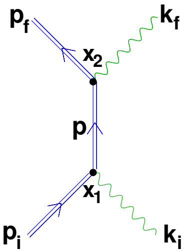
- Casimir force implies virtual particles have real physical effects
- Strong background field couples to charged virtual particles and polarises the vacuum
- The screening charge is rearranged, leading to possibly large effects even at modest field strengths
- At Schwinger critical field strength, vacuum decays into real pairs
- New phenomenology results - odd vertex diagrams, resonant propagators, different manifestations of IR divergences
- Polarisable vacuum applicable to strong gravity as well as strong EM fields
- Need to investigate experimental signatures within reach using today's and upcoming technology

IPstrong monte carlo as a service

- We want the non perturbative community to use customised software
- Exploit modern cloud computing resources
- Orchestrate services and resources with Kubernetes
- Create more workers to scale up service provision



1st challenge: Dressed vertices & higher order traces



SQED Feynman diagrams are easy...

- Double fermion lines are Volkov-type solutions
- Volkov E_p functions can be grouped around the vertex
- only need one new Feynman picture element - the dressed vertex

$$\gamma^{\mu}_{\mathsf{fpx}_2} = \int d^4x_2\, \bar{E}_{p_f}(x_2)\gamma^{\mu}E_{p_i}(x_2)\, e^{i(p_f+k_f-p)\cdot x_2}$$

... but the calculations, not

- 2nd order trace has 4 dressed vertices. How many terms?

$$\sum |M_{\mathsf{fi}}|^2 \propto \mathrm{Tr}\left[(\not{p}_{\mathsf{f}}+m)\gamma^{\mu}_{\mathsf{fpx}_2}(\not{p}+m)\gamma^{\nu}_{\mathsf{ipx}_2}(\not{p}_{\mathsf{i}}+m)\bar{\gamma}_{\mathsf{fpx}_1\,\nu}(\not{p}+m)\bar{\gamma}_{\mathsf{fpx}_1\,\mu}\right]$$

- 4 channels x 4 γ x (2x2) E_{p} x (4x2) spin sum = 512 terms
- Higher order terms become intractable
- Feyncalc strategy no good for strong field particle generator
- Need schema for Furry pic trace simplification for any order

Fierz transformation for Volkov spinors

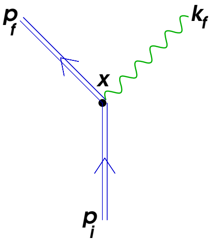
[Phys Rev D 94, 073002 (2016)]

New bound Dirac solutions. Canonical momentum Π_{px}. Define the Volkov spinor V_{px}

$$\Psi^{\text{V}}_{\text{prx}} = n_p \left[\not{\Pi}_{\text{px}} + m\right] \frac{\not{k}}{2k\cdot p}\, u_{\text{pr}}\, e^{-i\Delta_{\text{px}}} \equiv V_{\text{pxr}}\, e^{-i\Delta_{\text{px}}}$$

Extend Fierz transformations to Volkov spinors

$$\sum_{\text{rsr's'}} [\bar{V}_{\text{frx}}\, \Gamma^j_{\text{J}}\, V_{\text{isx}}][\bar{V}_{\text{is'x'}}\, \Gamma_{\text{J}\ j} V_{\text{fr'x'}}] = \sum_{\text{rsr's'K}} F_{\text{JK}} \left[\bar{V}_{\text{frx}}\, \gamma^\mu \Gamma^k_{\text{K}} \gamma_\mu\, V_{\text{fr'x'}}\right] \left[\bar{V}_{\text{is'x'}}\, \Gamma_{k\text{K}}\, V_{\text{isx}}\right]$$



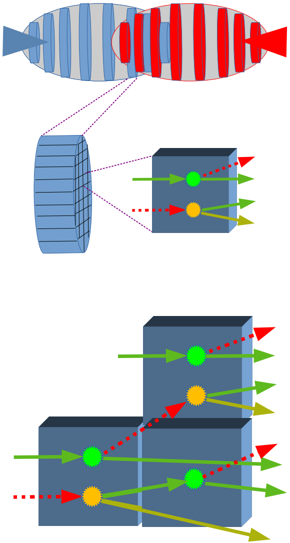
example: amplitude for HICS

$$M_{\text{fi}} = -ie \int d^4x\, \bar{\psi}^{\text{V}}_{\text{frx}}\, \not{A}_{\text{fx}}\, \psi^{\text{V}}_{\text{isx}}$$

squared amplitude splits into two traces

$$|M_{\text{fi}}|^2 \propto \sum_{\text{K}} F_{\text{SK}}\, \text{Tr}\left[\bar{V}_{\text{fx}}\, \gamma^\mu \Gamma^k_{\text{K}} \gamma_\mu\, V_{\text{fx}'}\right] \text{Tr}\left[\bar{V}_{\text{ix}'}\, \Gamma_{k\text{K}}\, V_{\text{ix}}\right]$$

2nd challenge: Charge bunch interactions



Discretise the interaction

- transform to head on collision
- Divide into overlapping slices
- Divide slices into mc voxels
- Calculate SQED parameters (ξ, χ) in each voxel
- Monte carlo for each SQED process (rarest first)

Macro vs Micro

- Real particles enter/leave voxel
- Higher order processes are tested **within** each voxel
- Distinguish between analytic rate within one voxel, and the effective global rate from sampling across whole bunch/pulse
- final particle ensemble built up over successive voxel monte carlo + time step through the whole collision (typically 5σ separation)