An implementation of Neural Simulation-Based Inference for Parameter Estimation in ATLAS

CHEP

23 October 2024

Aishik Ghosh, on behalf of the ATLAS Collaboration

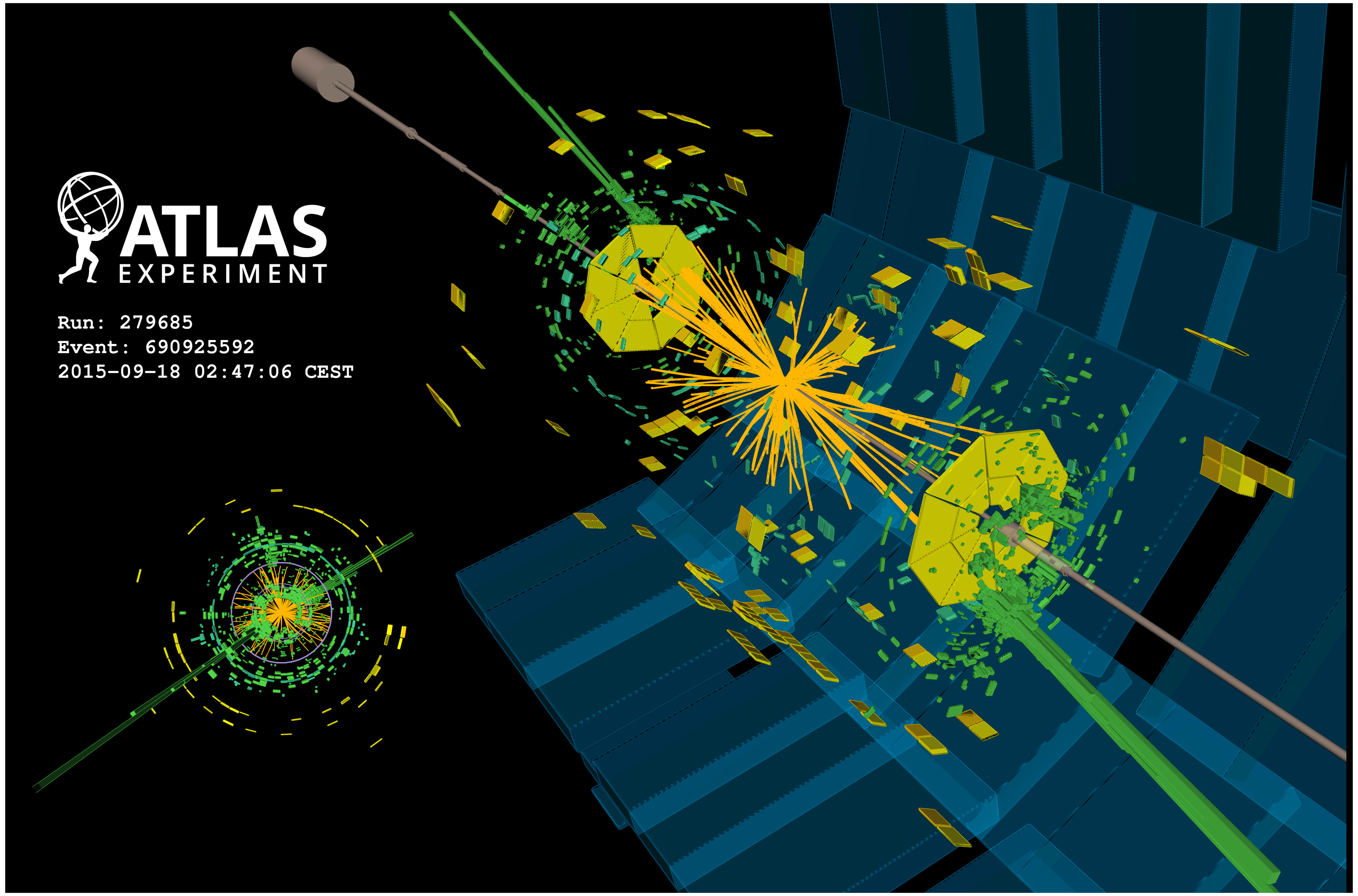
The motivation for high dimensional statistical inference

(Rather than using a low-dimensional histogram for statistical analysis)

Typical LHC Workflow

- Detector has O(100 million) sensors
- Can't build 100M dimensional histogram

- Reconstruction pipeline, event selection
- Design sensitive one-dimensional observable



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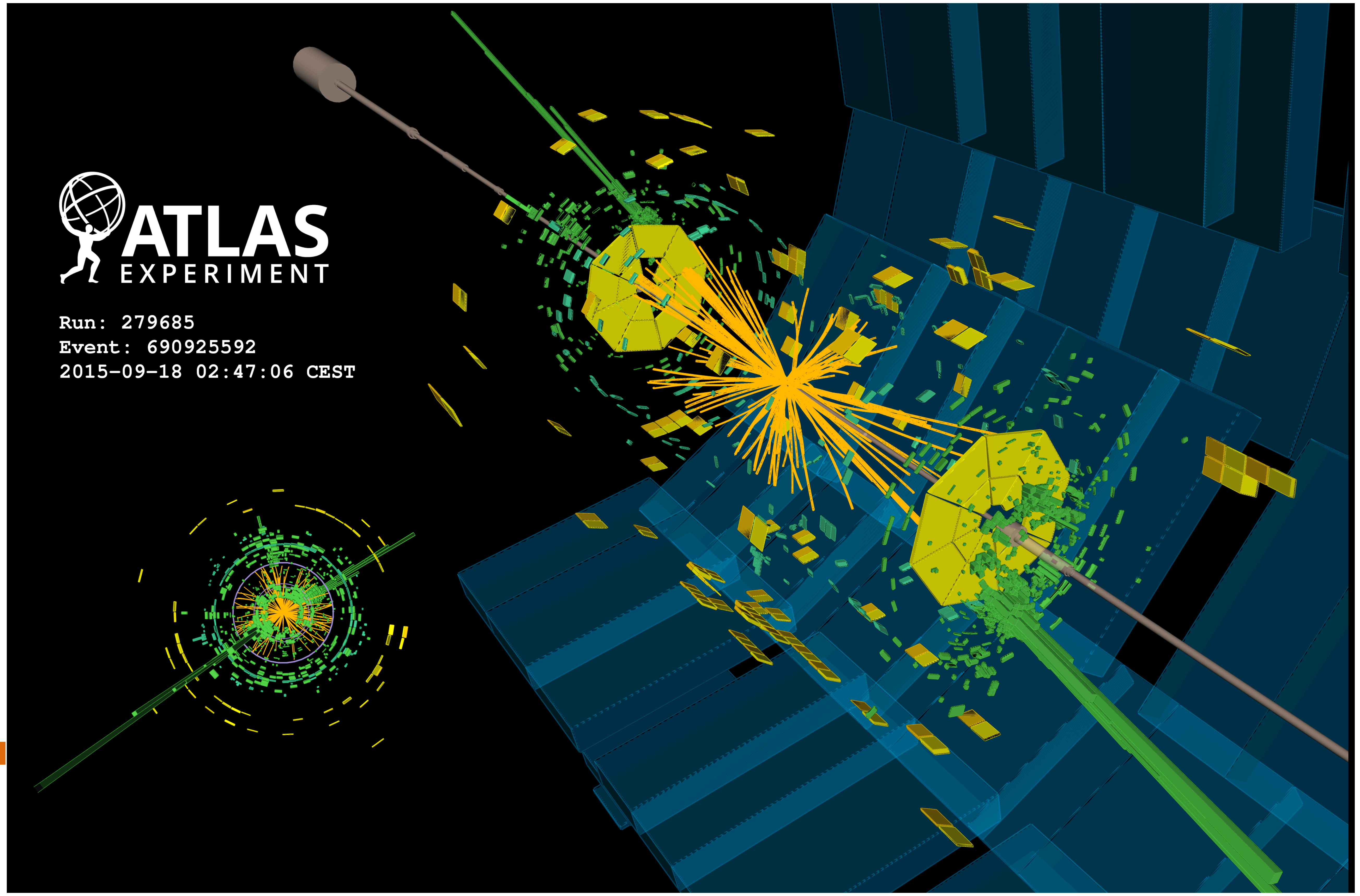
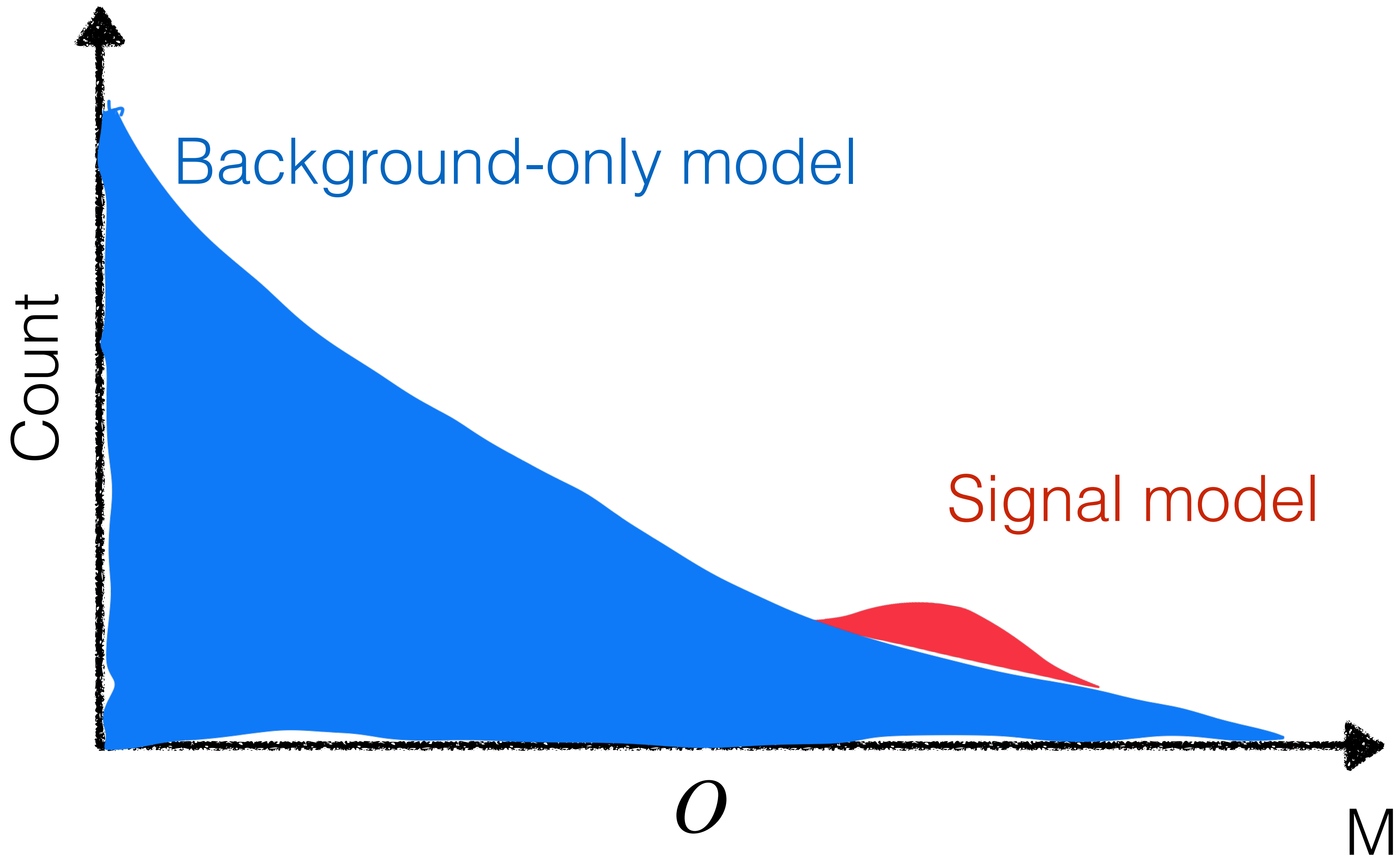
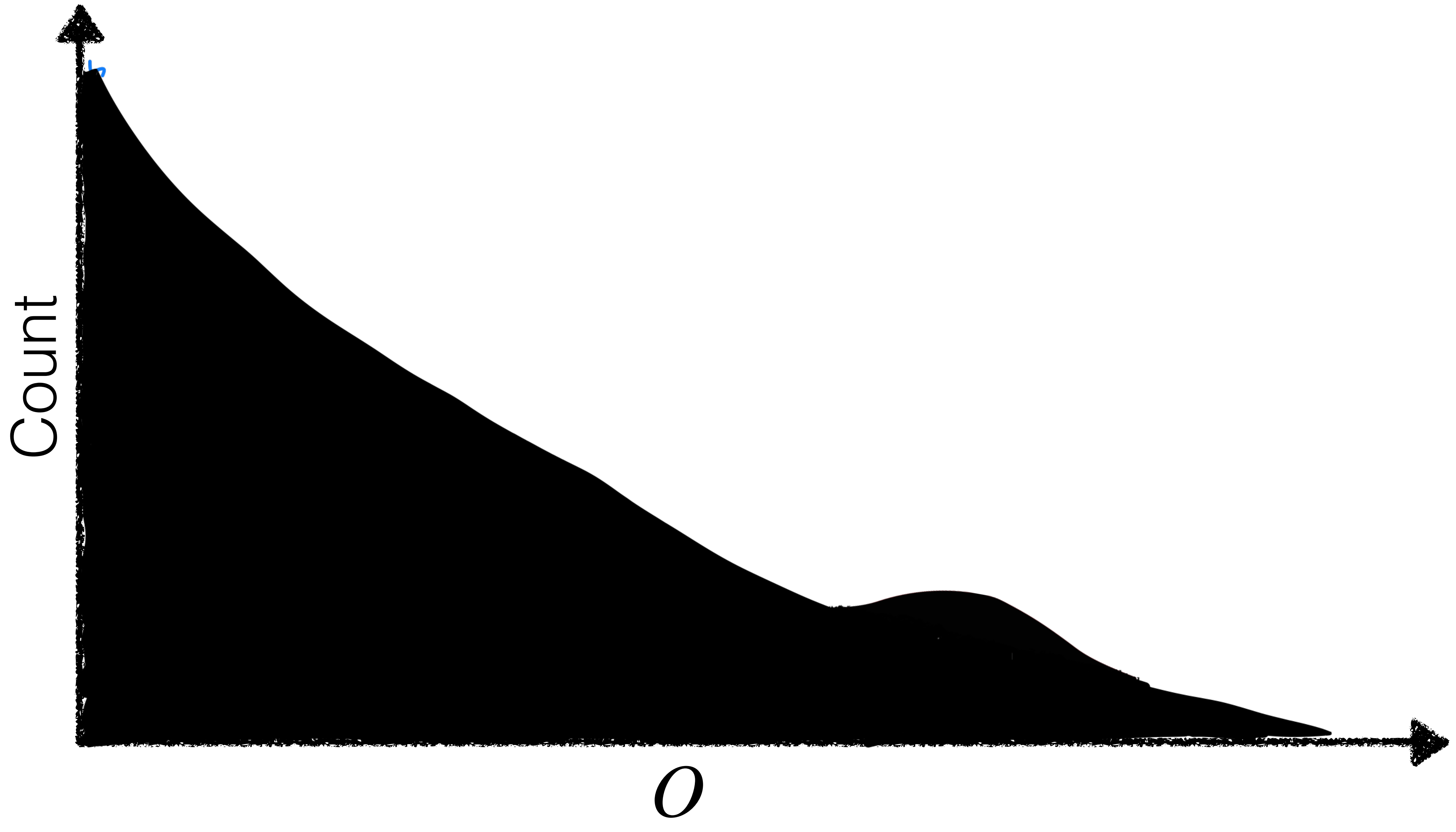


Probability Density Estimation: What we're used to doing..

Theory Predictions



Data



Measure signal strength μ

With histograms we can ask "Given the data, what is the likelihood of $\mu = 1$ hypothesis vs $\mu = 2$ hypothesis?"

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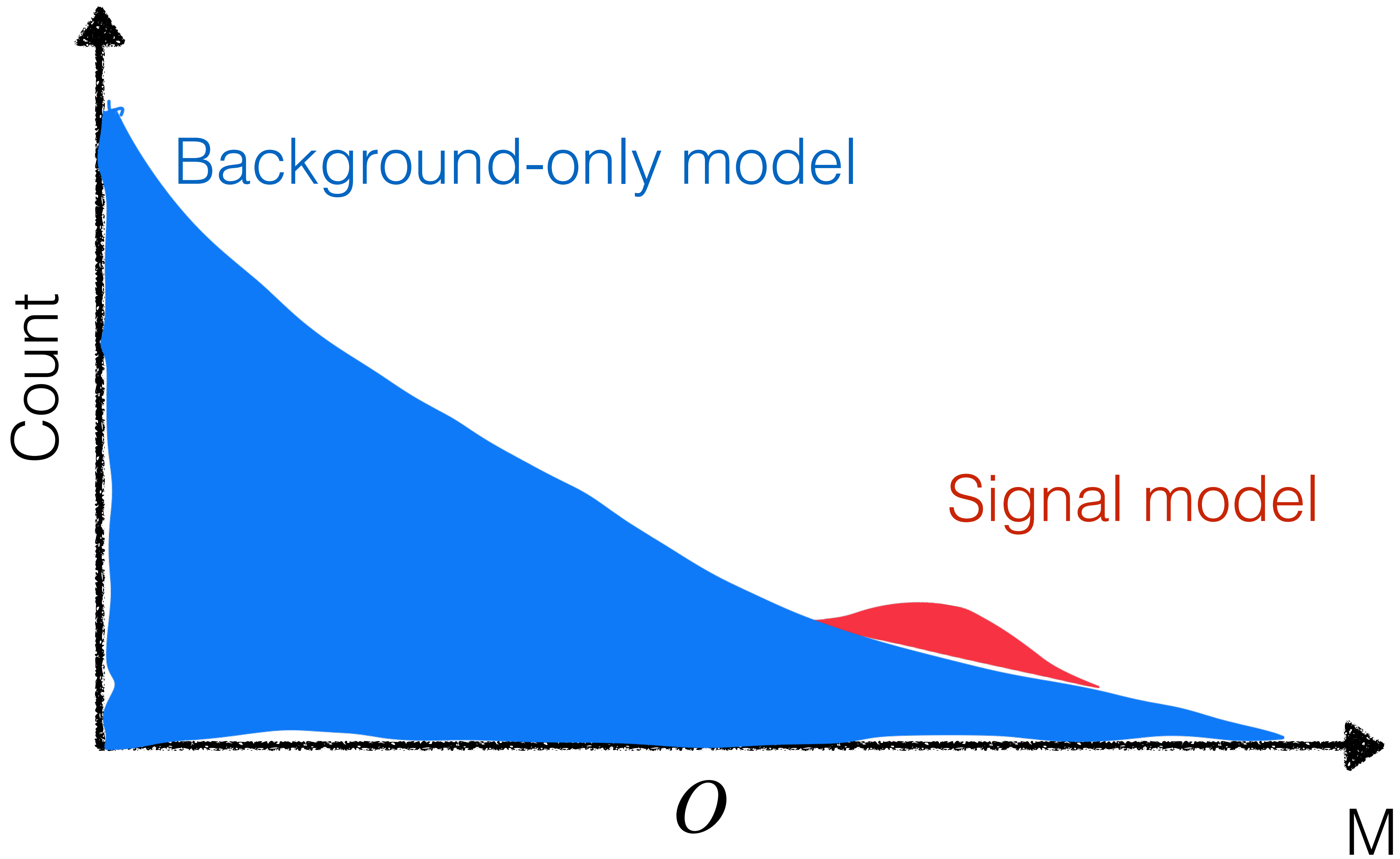

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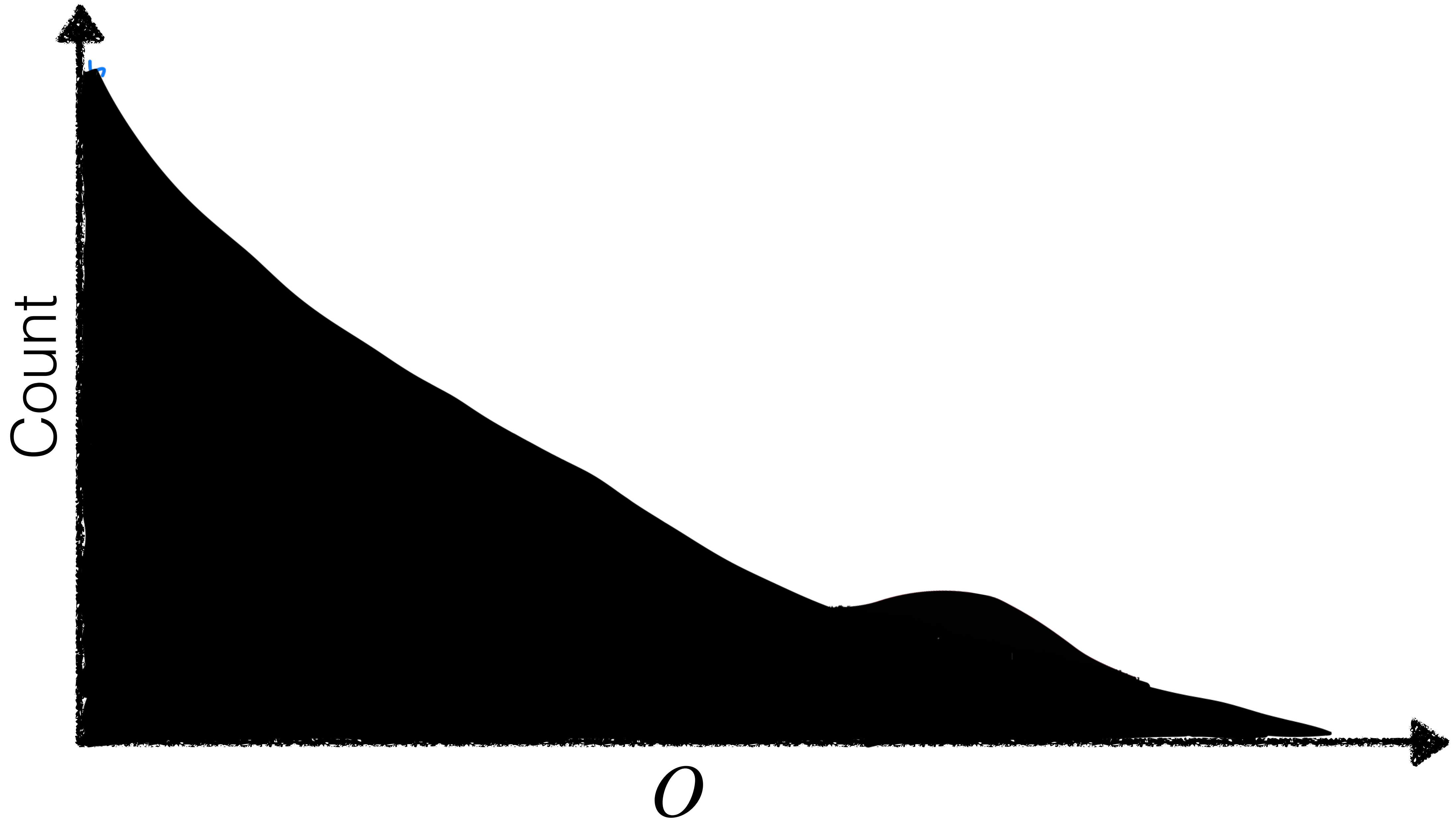

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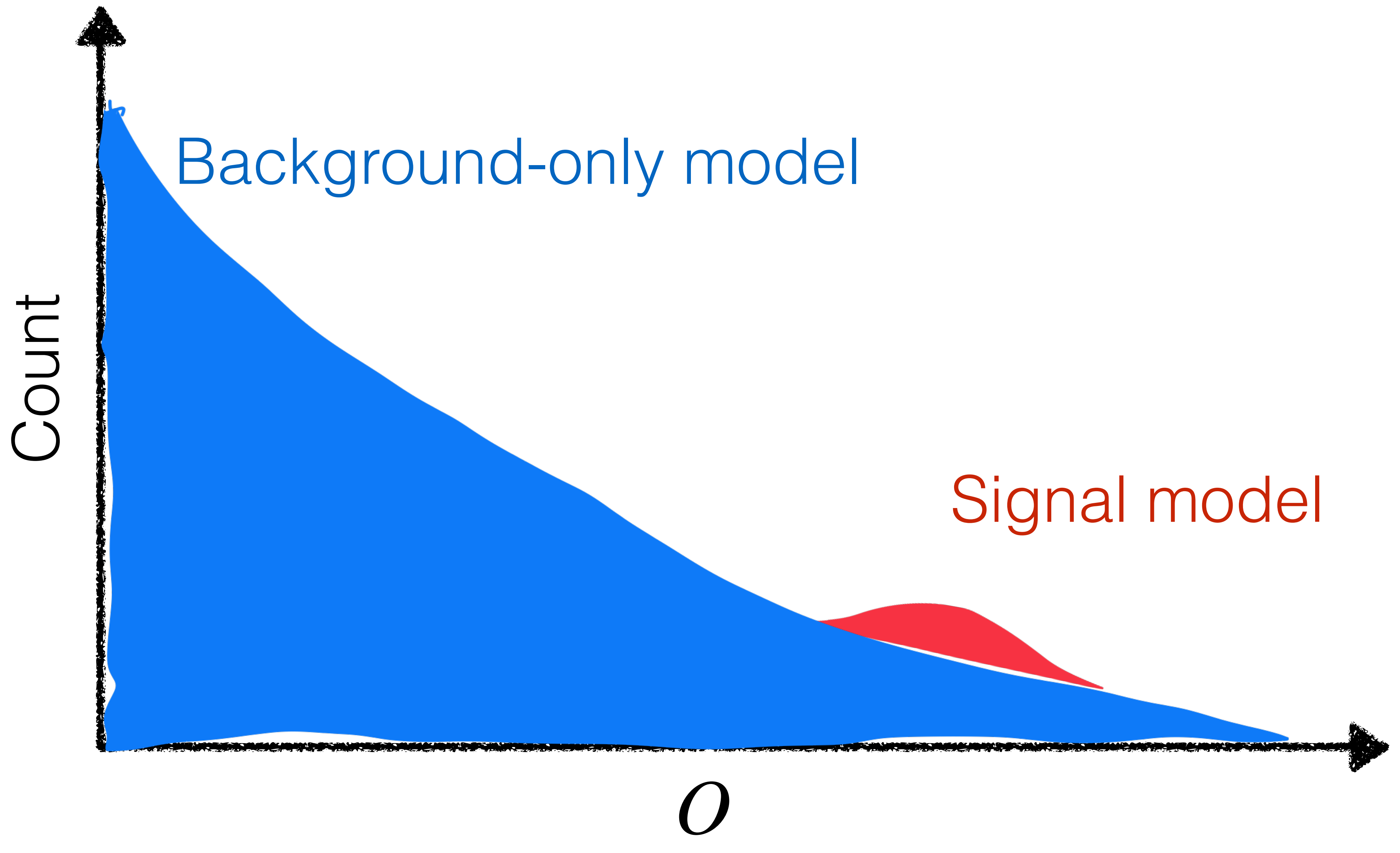
New challenge: Non-linear changes in kinematics (w.r.t. parameter of interest)



A histogram of any single observable is no longer optimal (see Ghosh et al: hal-02971995(p172)), but neural networks estimate high-dimensional likelihood ratios (see Cranmer et al: arXiv:1506.02169) !

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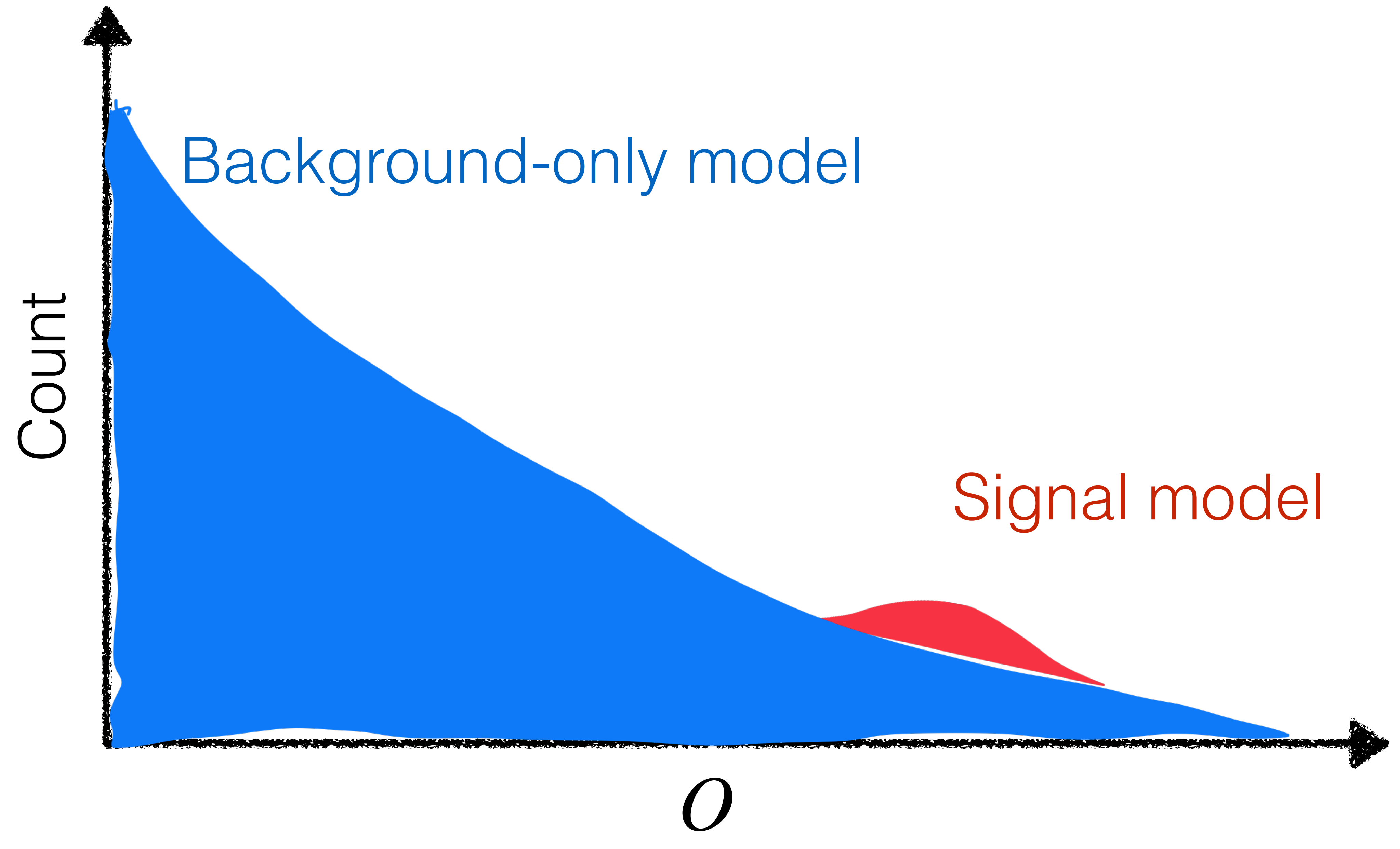
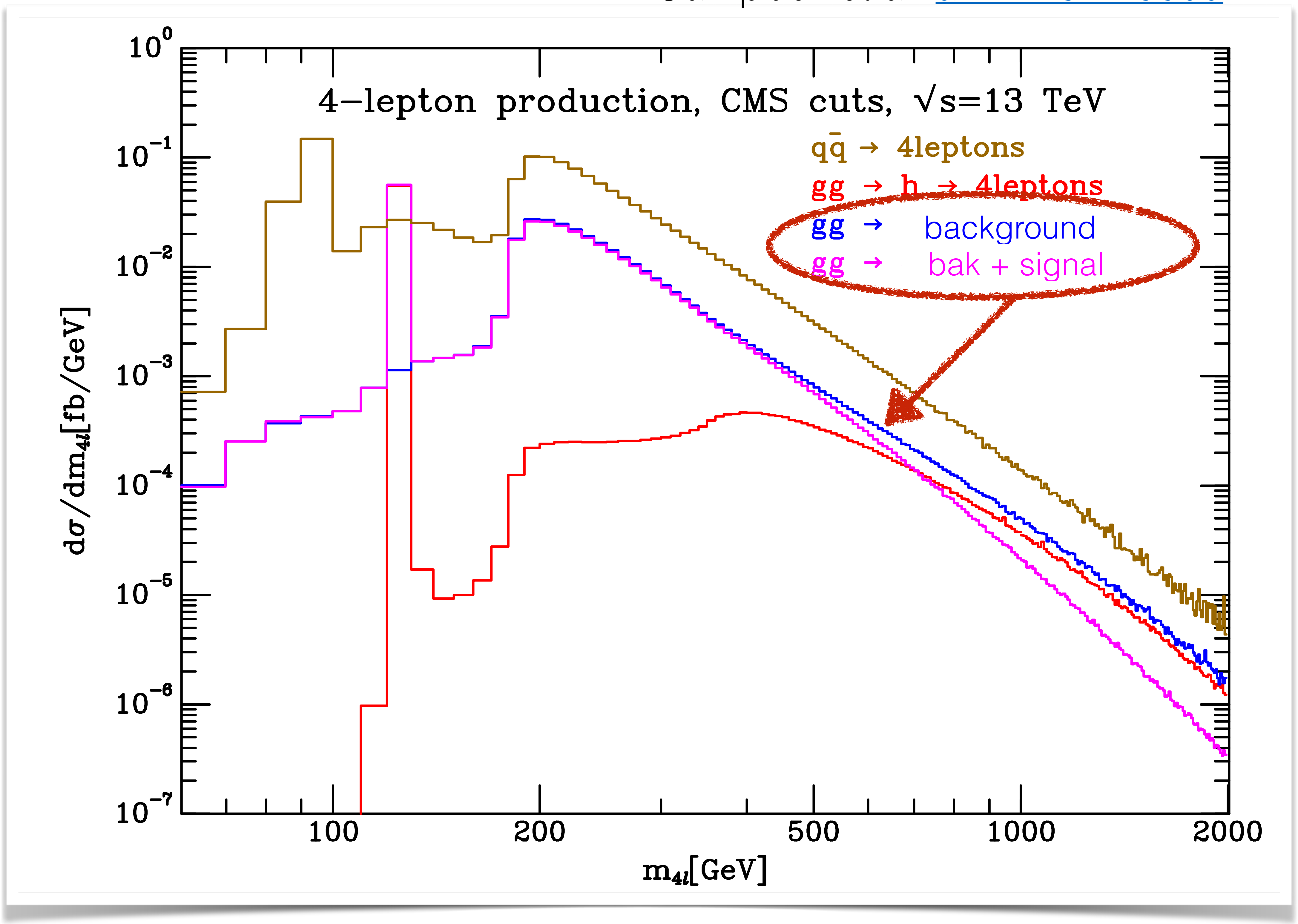
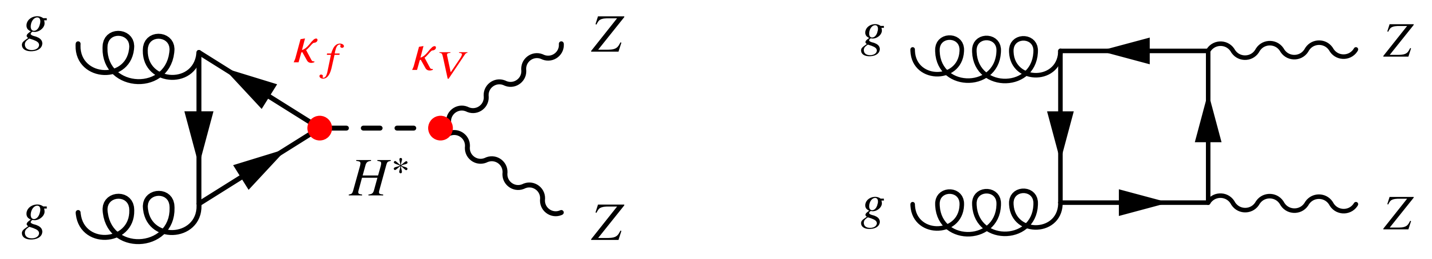


Campbell et al: [arXiv:1311.3589]



Quantum interference:



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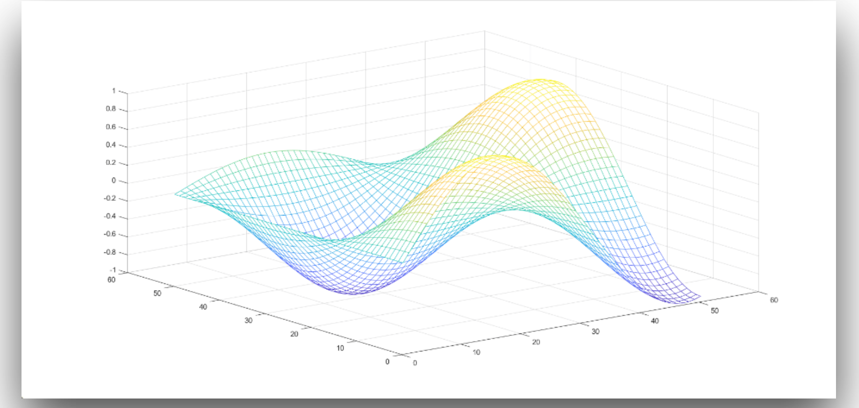
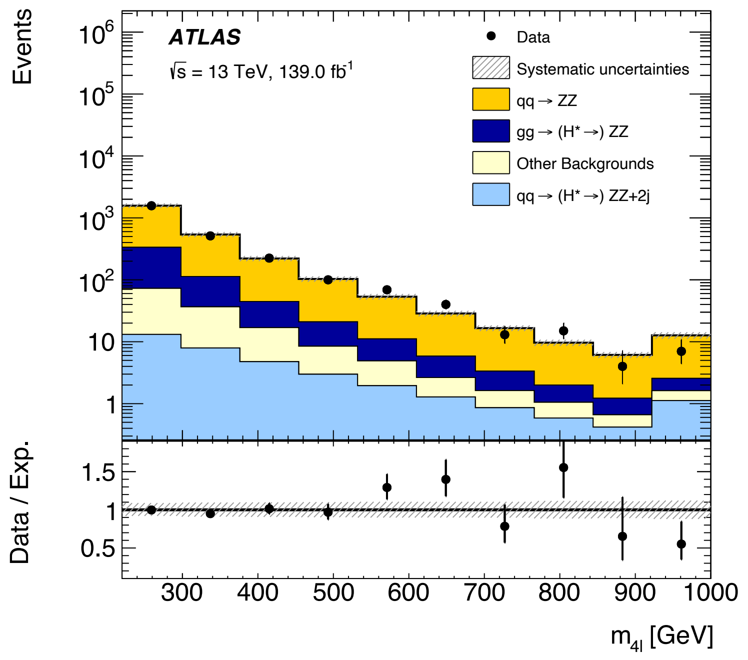
"Neural Simulation-Based Inference"

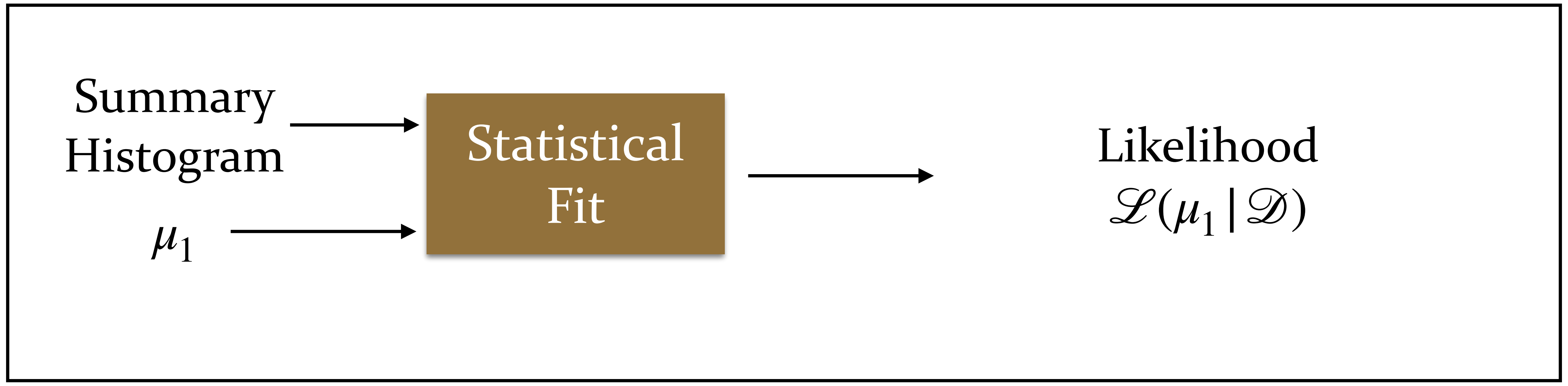
Traditional framework:



High-dim data

Summarisation to histogram





μ is now arbitrary parameter of interest(s)

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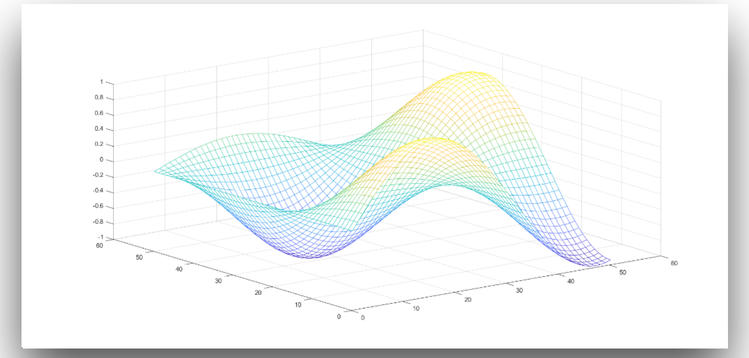

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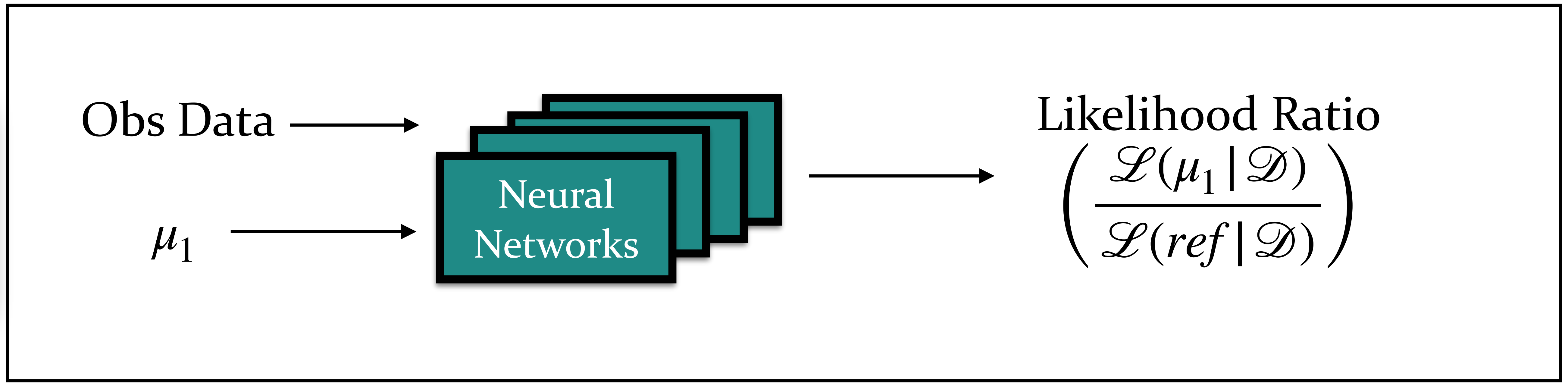

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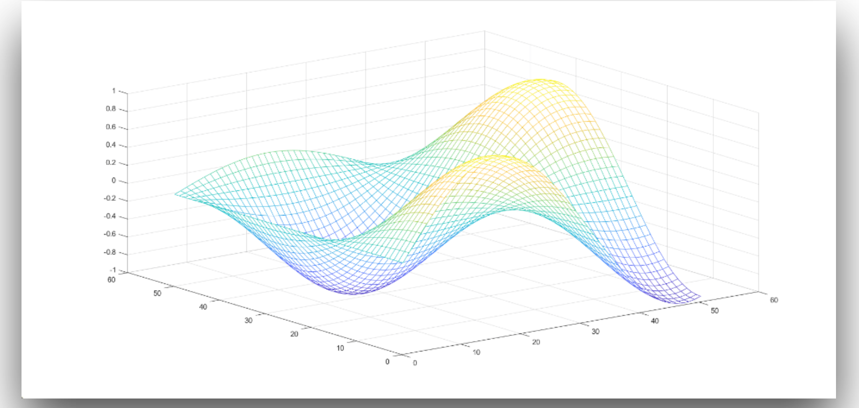

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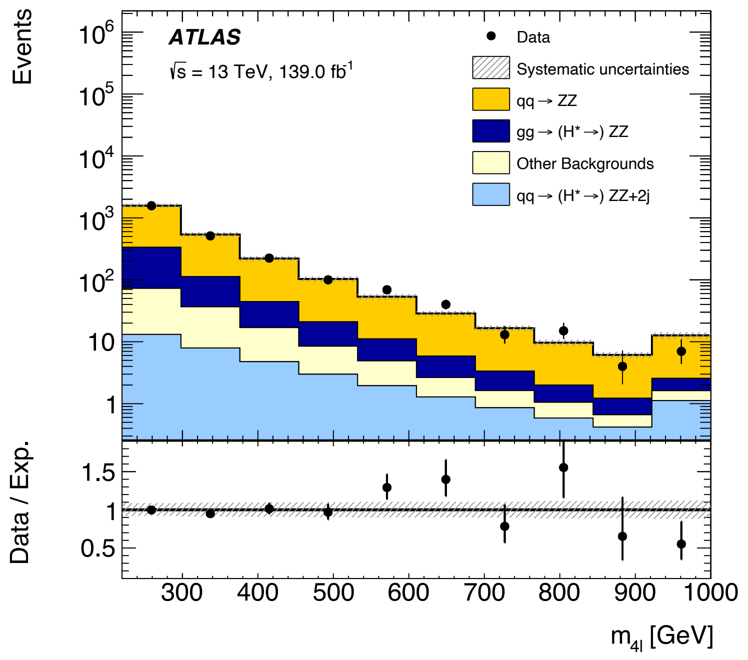

Summary Histogram → Statistical Fit → Likelihood $\mathscr{L}(\mu_1 | \mathscr{D})$

μ_1 → Statistical Fit

Hypothesis μ_1

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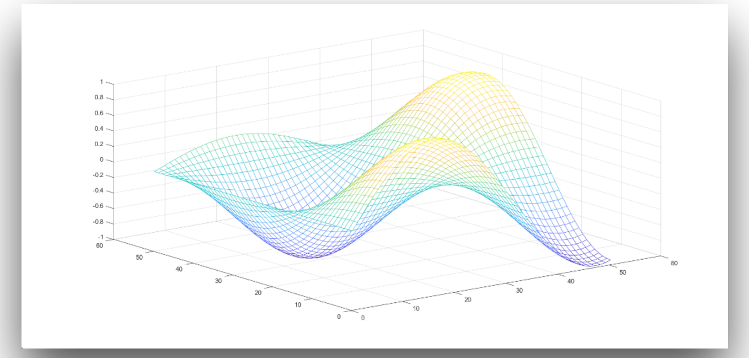

High-dim data

Obs Data → Neural Networks → Likelihood Ratio $\left(\frac{\mathscr{L}(\mu_1 | \mathscr{D})}{\mathscr{L}(ref | \mathscr{D})}\right)$

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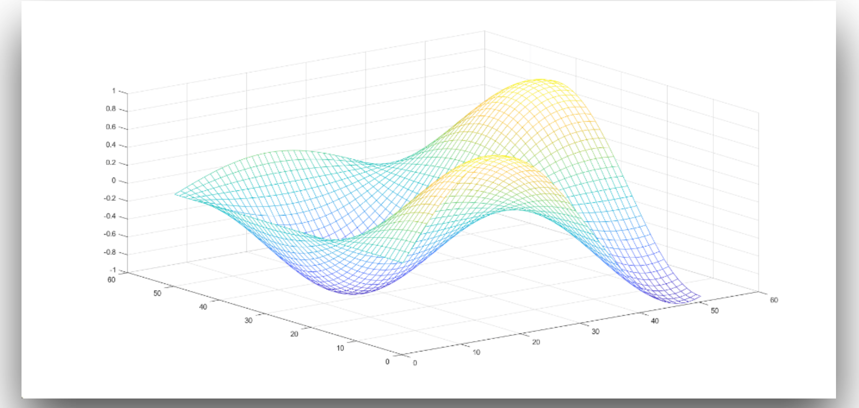

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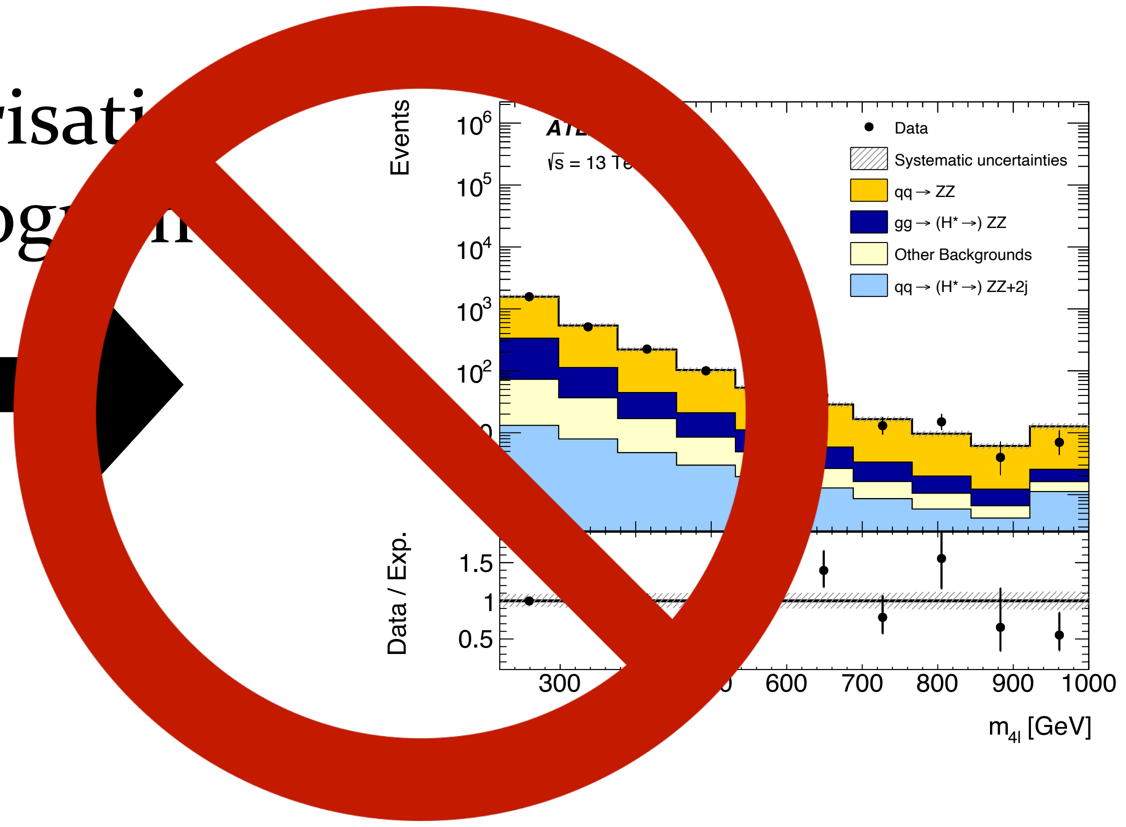

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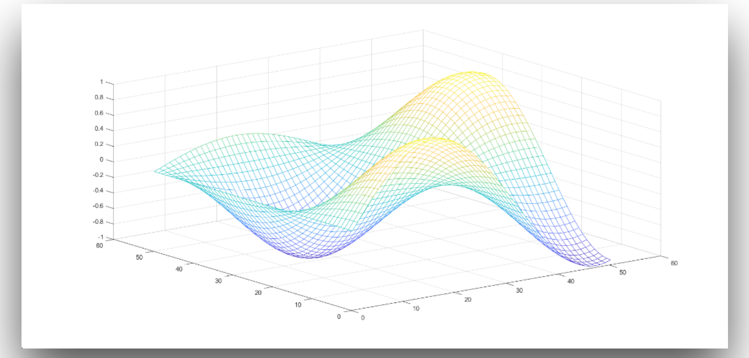

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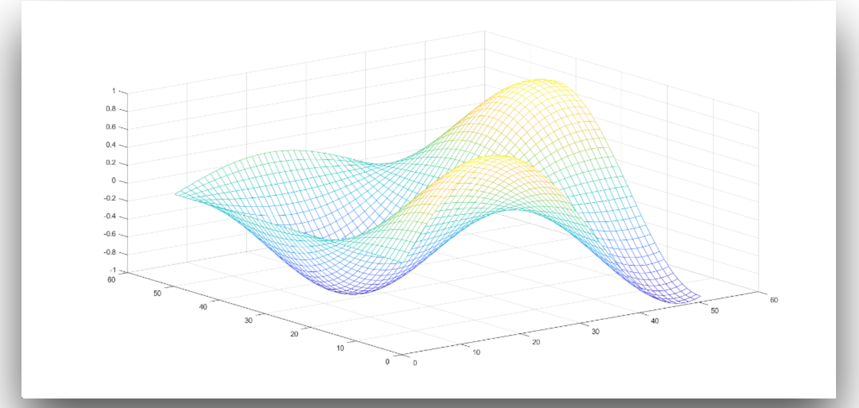

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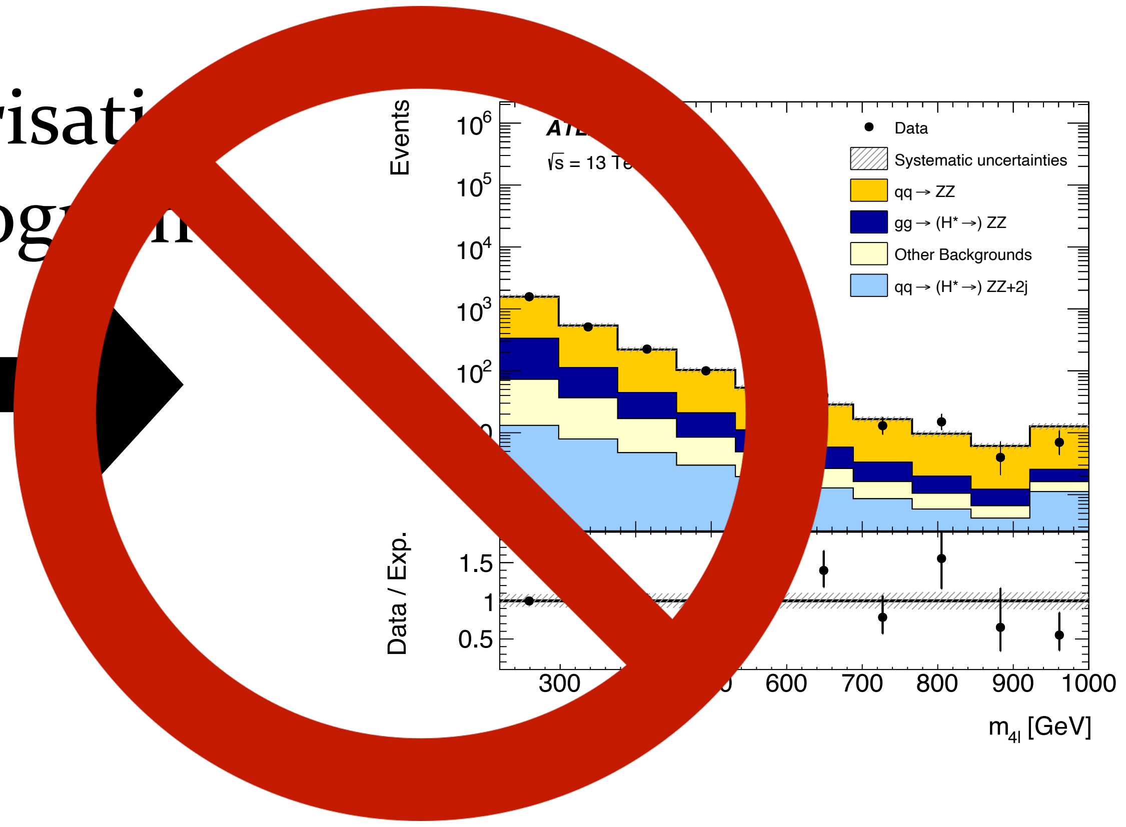

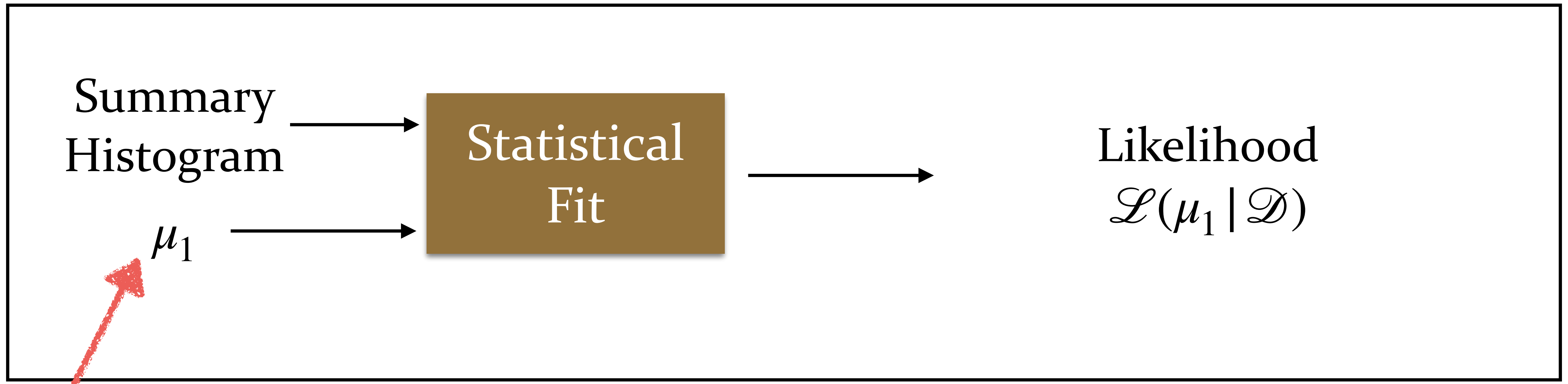

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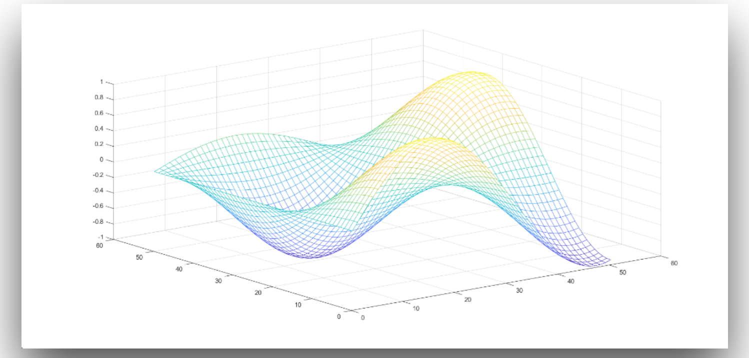

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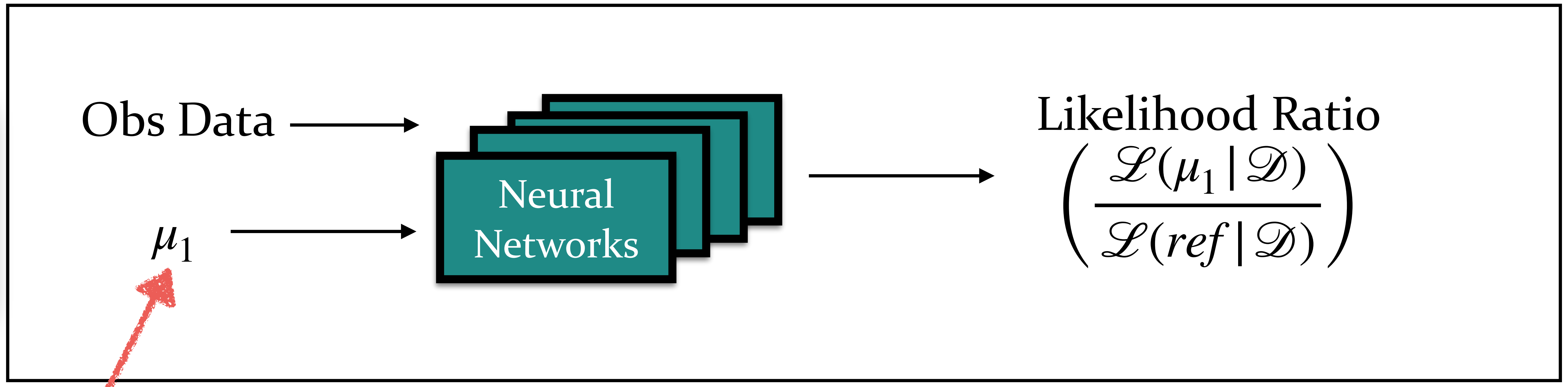

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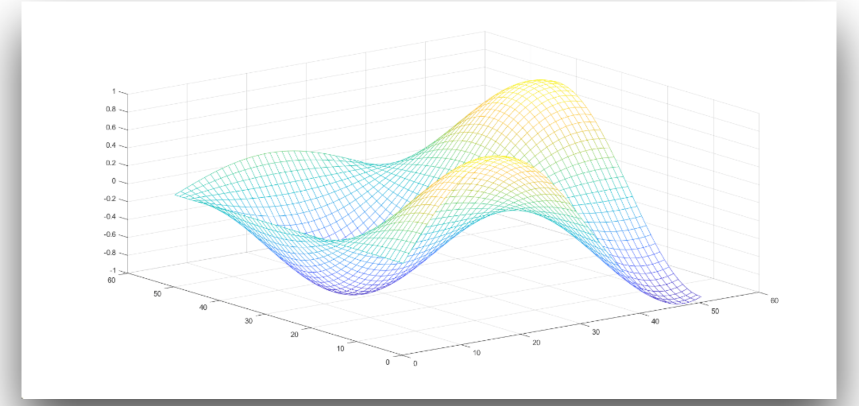
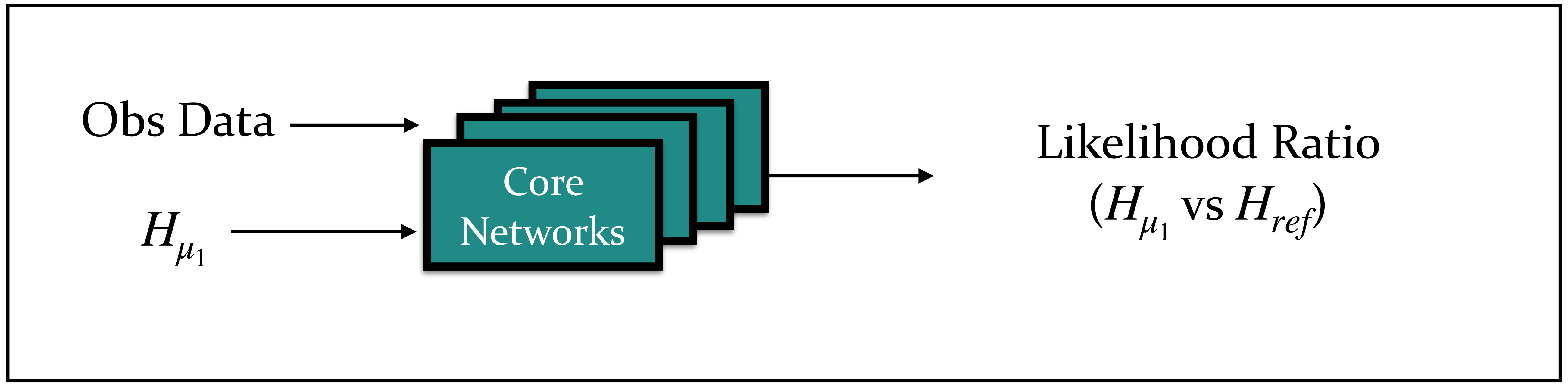
Open problems to extend to full ATLAS analysis:

- Robustness: Design and validation
- Systematic Uncertainties: Incorporate them in likelihood (ratio) model
- Neyman Construction: Throwing toys in a per-event analysis

Big picture of full solution developed in ATLAS



$O(16)$ observables



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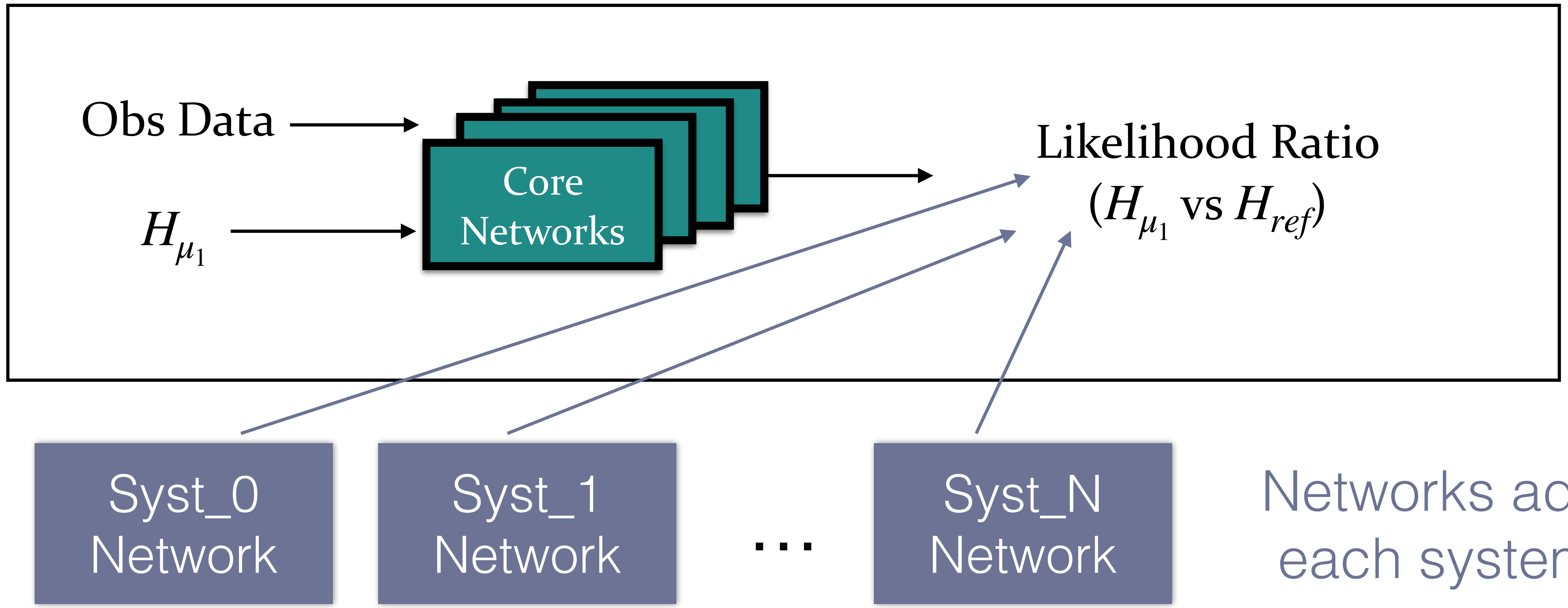

Networks adjust likelihood for each systematic uncertainty

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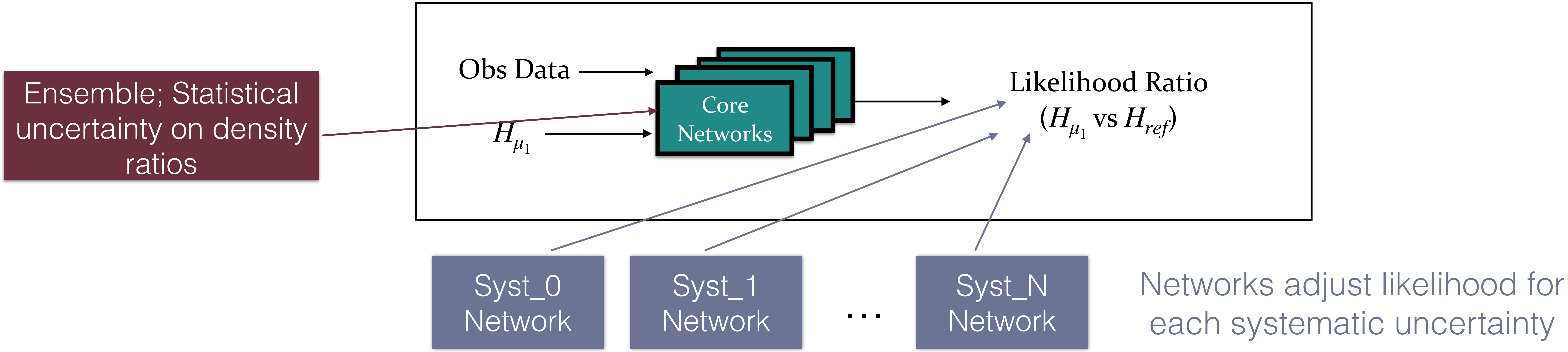

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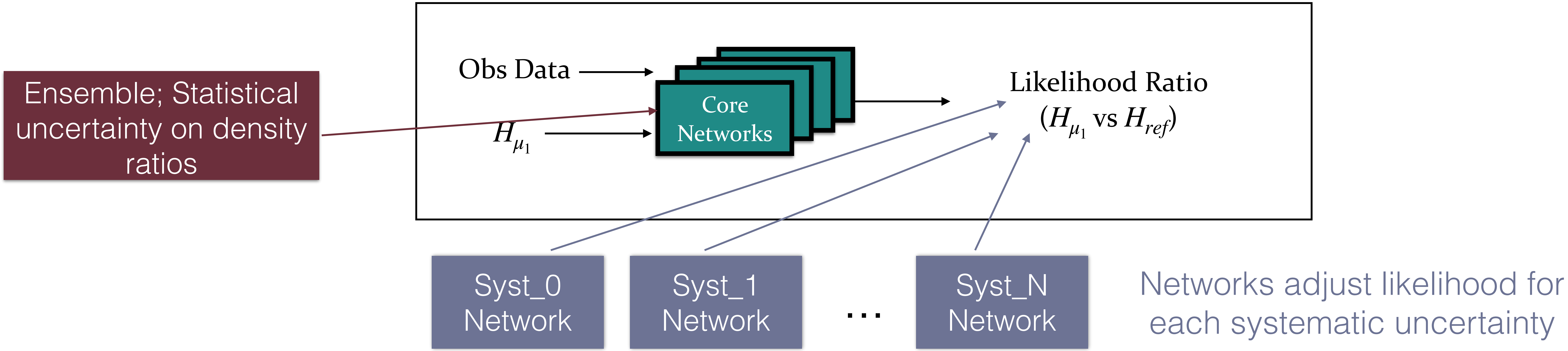

- ✦ Train $O(10^3)$ networks on TensorFlow
- ✦ **Computing resources provided by Google, SMU,** other HPC clusters
- ✦ Fits with JAX

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Search-Oriented Mixture Model

x_i is one individual event

General Formula

$$p(x_i|\mu) = \frac{1}{\nu(\mu)} \sum_{j}^{C} f_j(\mu) \cdot \nu_j \; p_j(x_i)$$

j runs over different physics process
(Eg. $gg \to H^* \to 4l$, $gg \to ZZ \to 4l$)

Example use case

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Comes from theory model chosen to interpret data

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Estimated using an ensemble of networks

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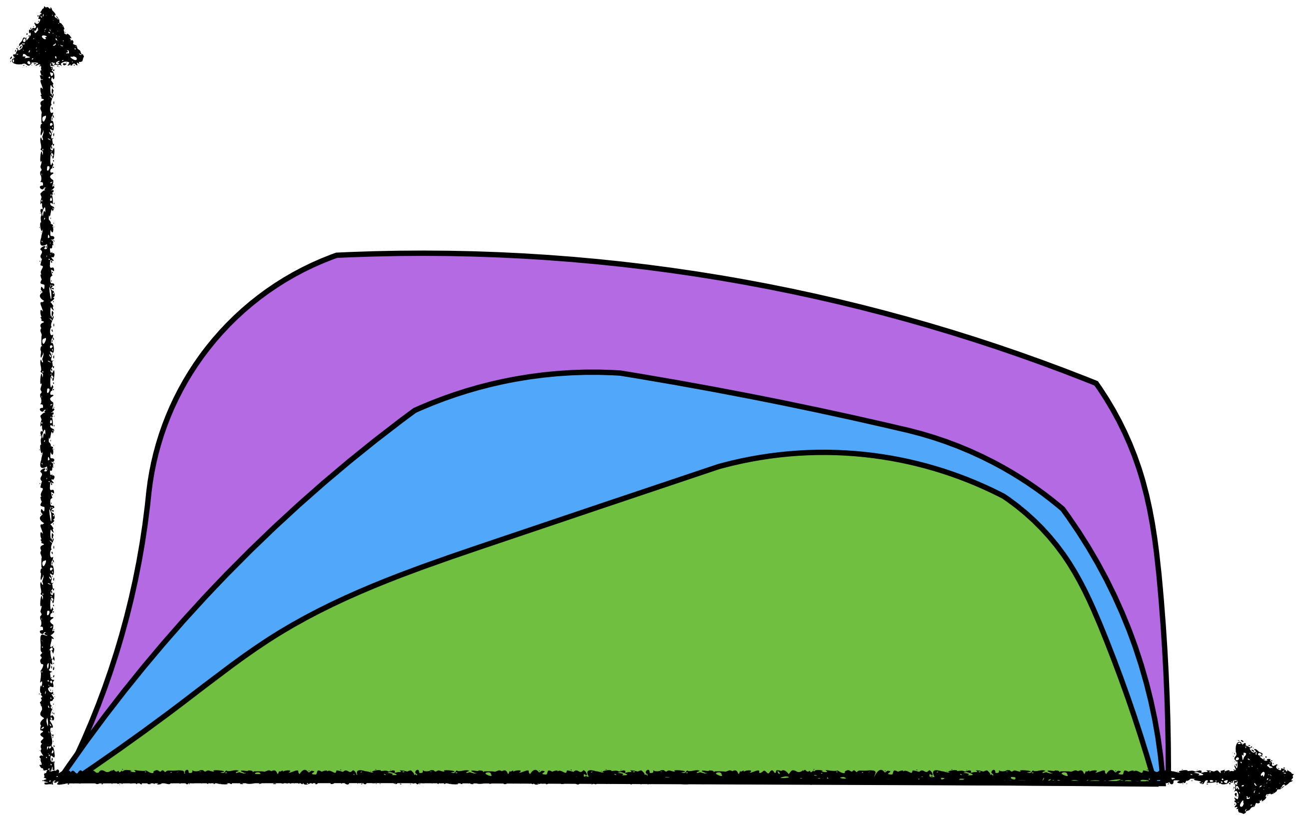
Robust, parameterised classifier without parameterising

H_{ref} : Reference hypothesis

H_μ

Hypothesis



VS

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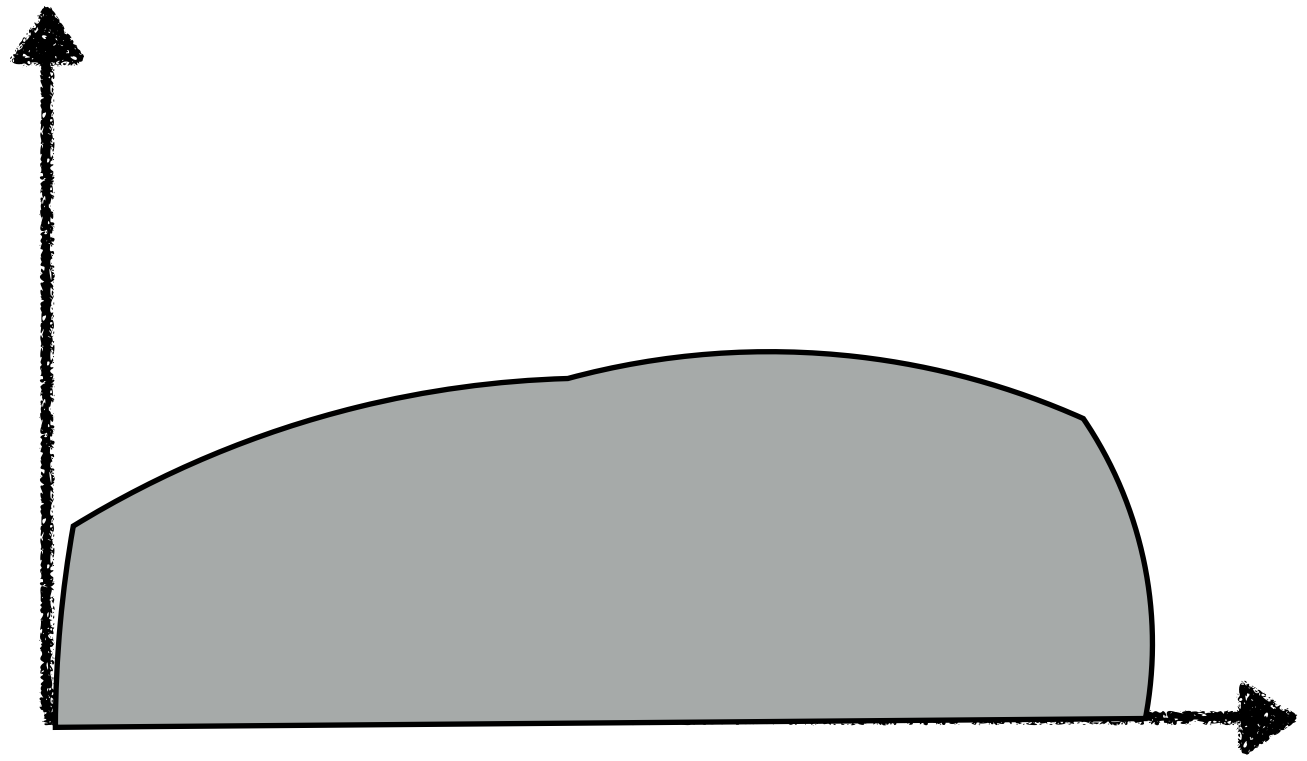

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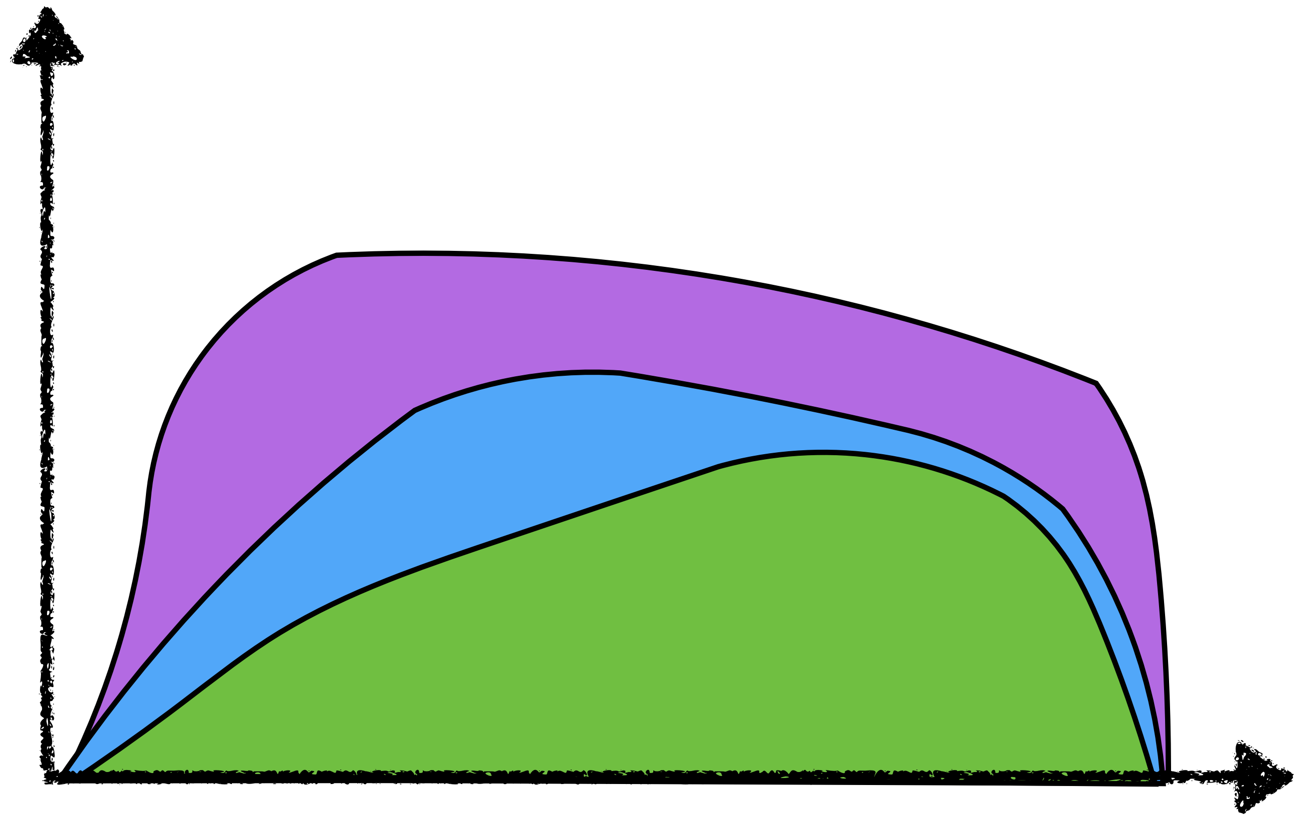

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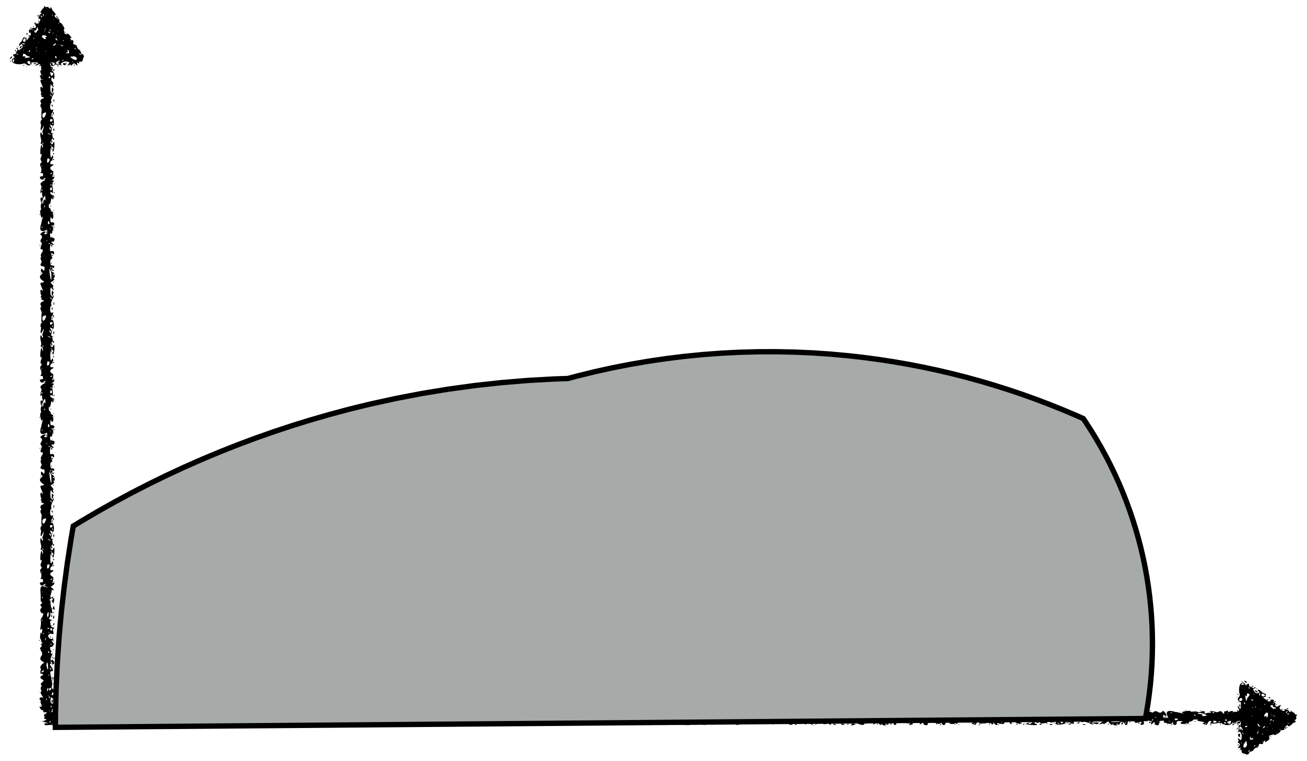

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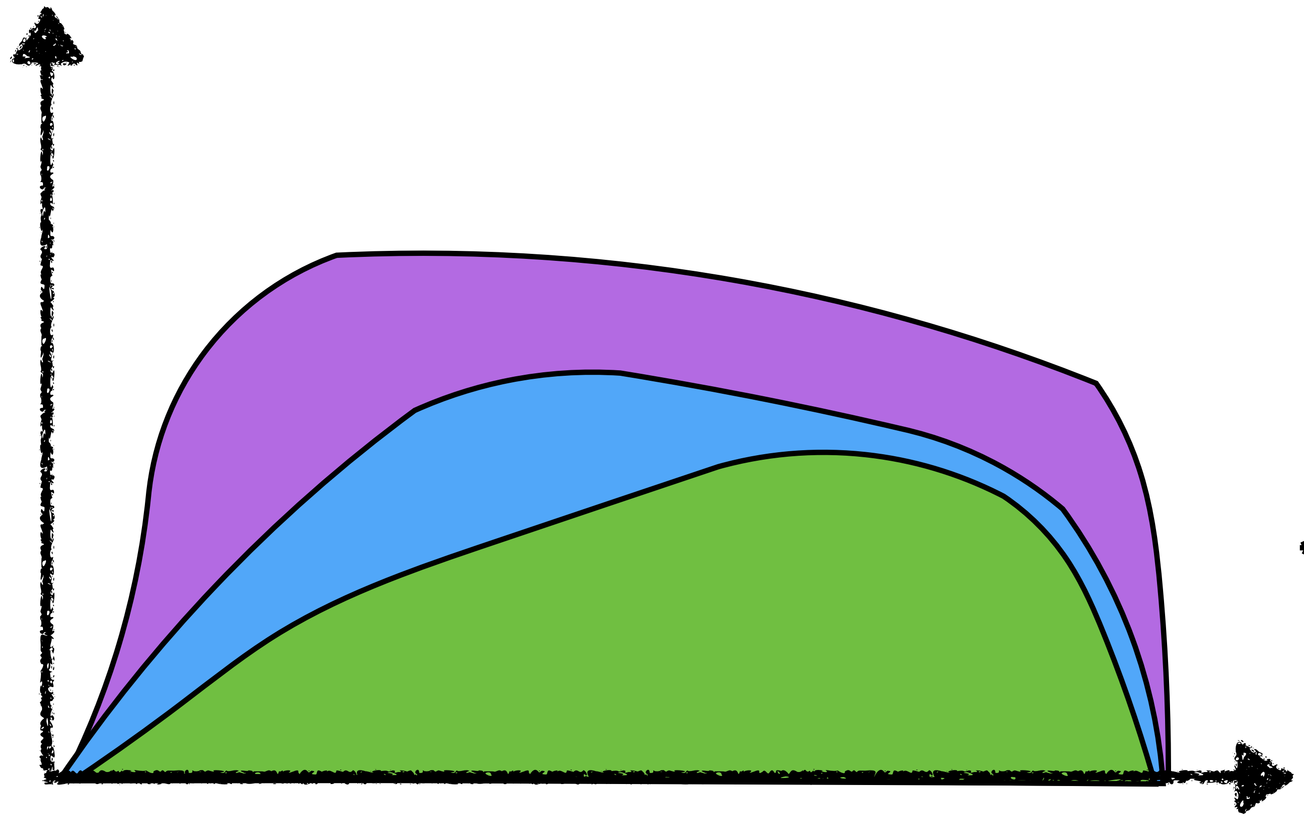

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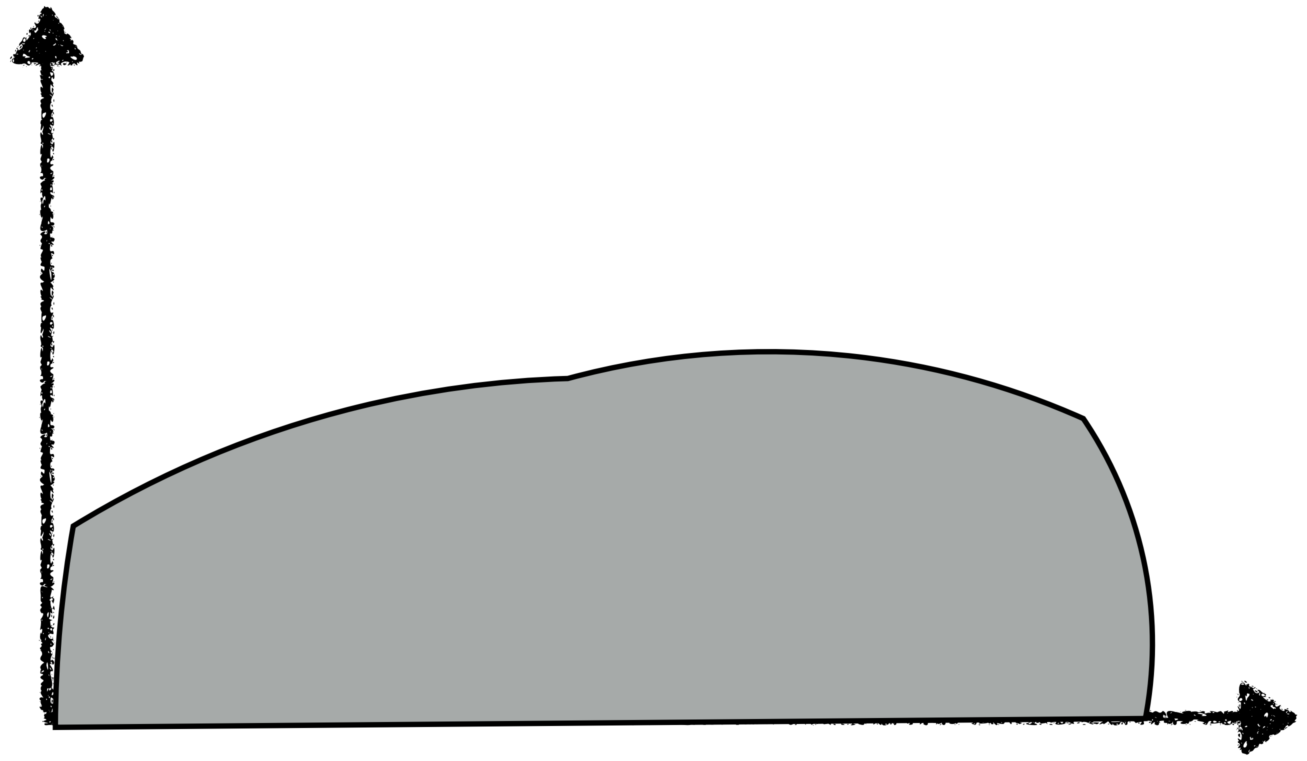

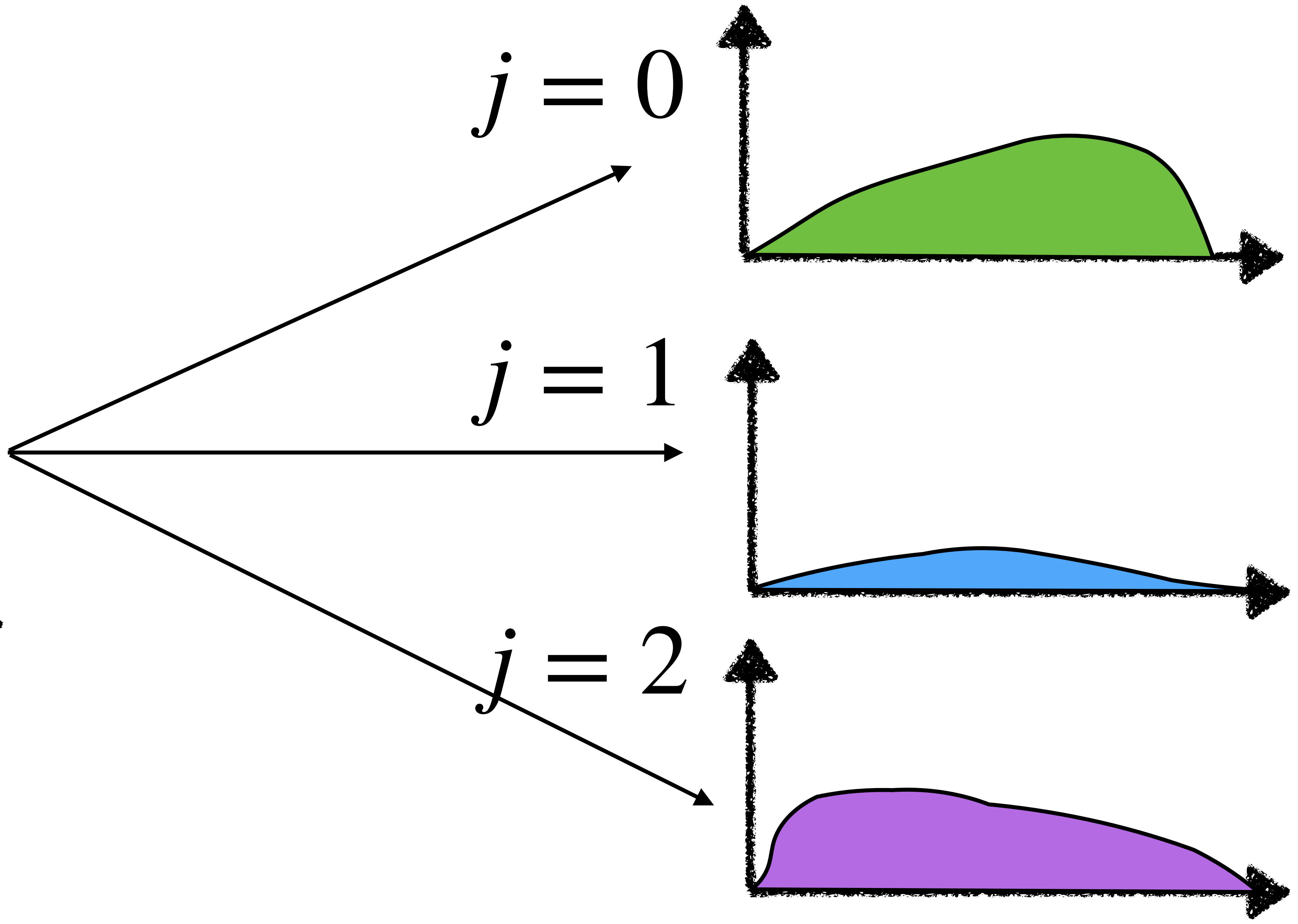

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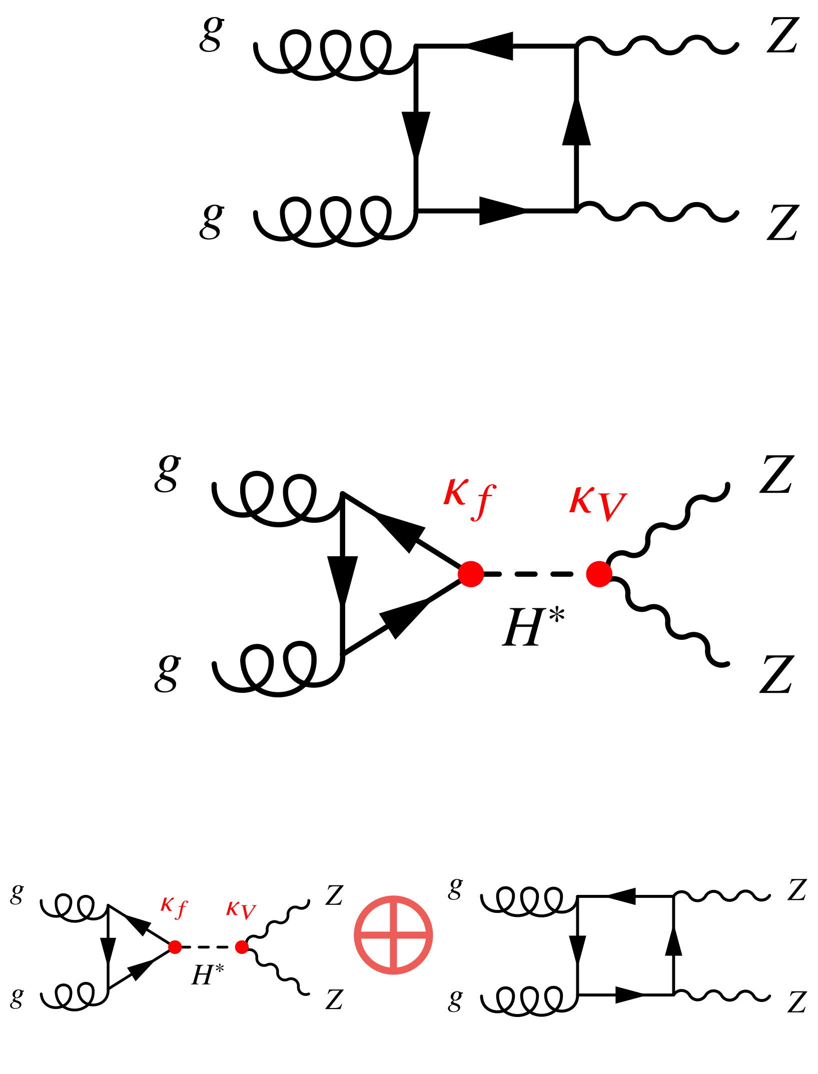
$f_1(\mu) = \mu$

$f_2(\mu) = \sqrt{\mu}$



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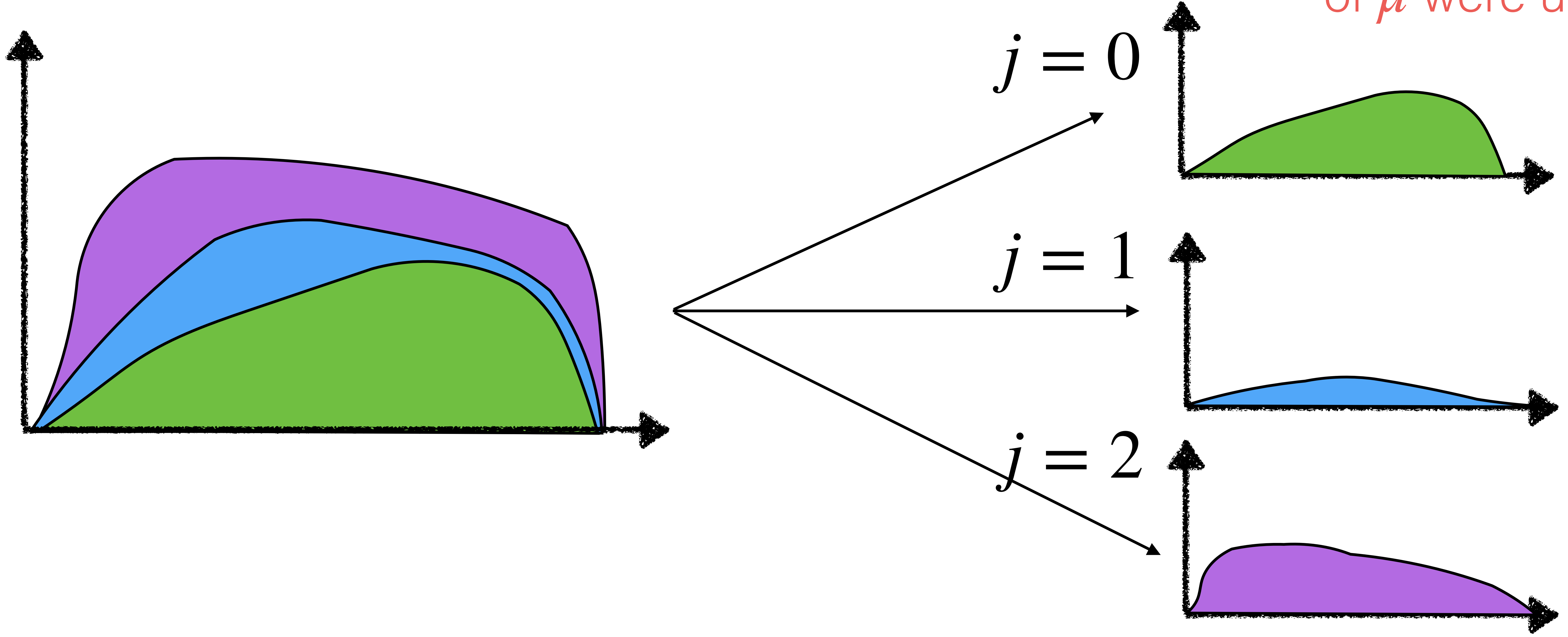
$f_i(\mu)$ will depend on morphing bases points (which values of μ were used to simulate samples)

H_μ

Hypothesis



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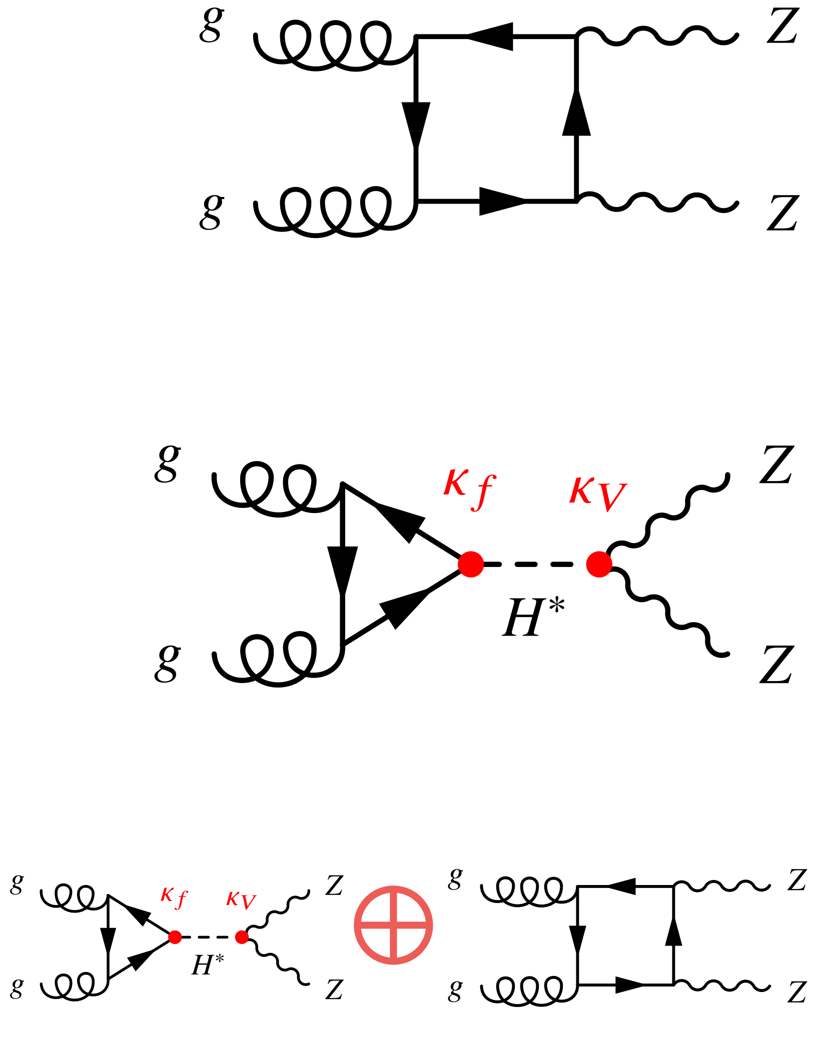
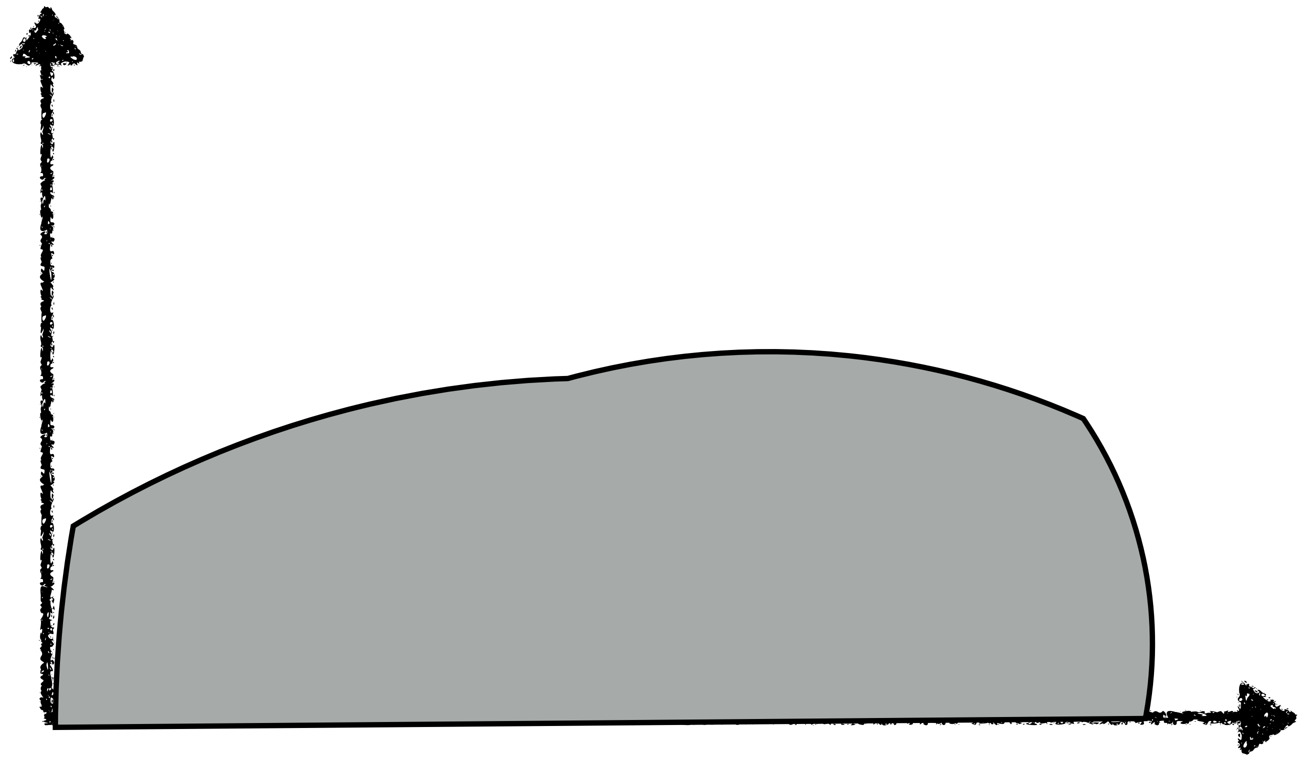
$f_2(\mu) = \sqrt{\mu}$



VS

H_{ref}

Reference hypothesis



$$\frac{p(x_i|\mu)}{p_{\mathrm{ref}}(x_i)} = \frac{1}{\nu(\mu)} \sum_j^C f_j(\mu) \cdot \nu_j \frac{p_j(x_i)}{p_{\mathrm{ref}}(x_i)}$$

A separate classifier per physics process j

(Eg. $gg \rightarrow H^* \rightarrow 4l$, $gg \rightarrow ZZ \rightarrow 4l$)

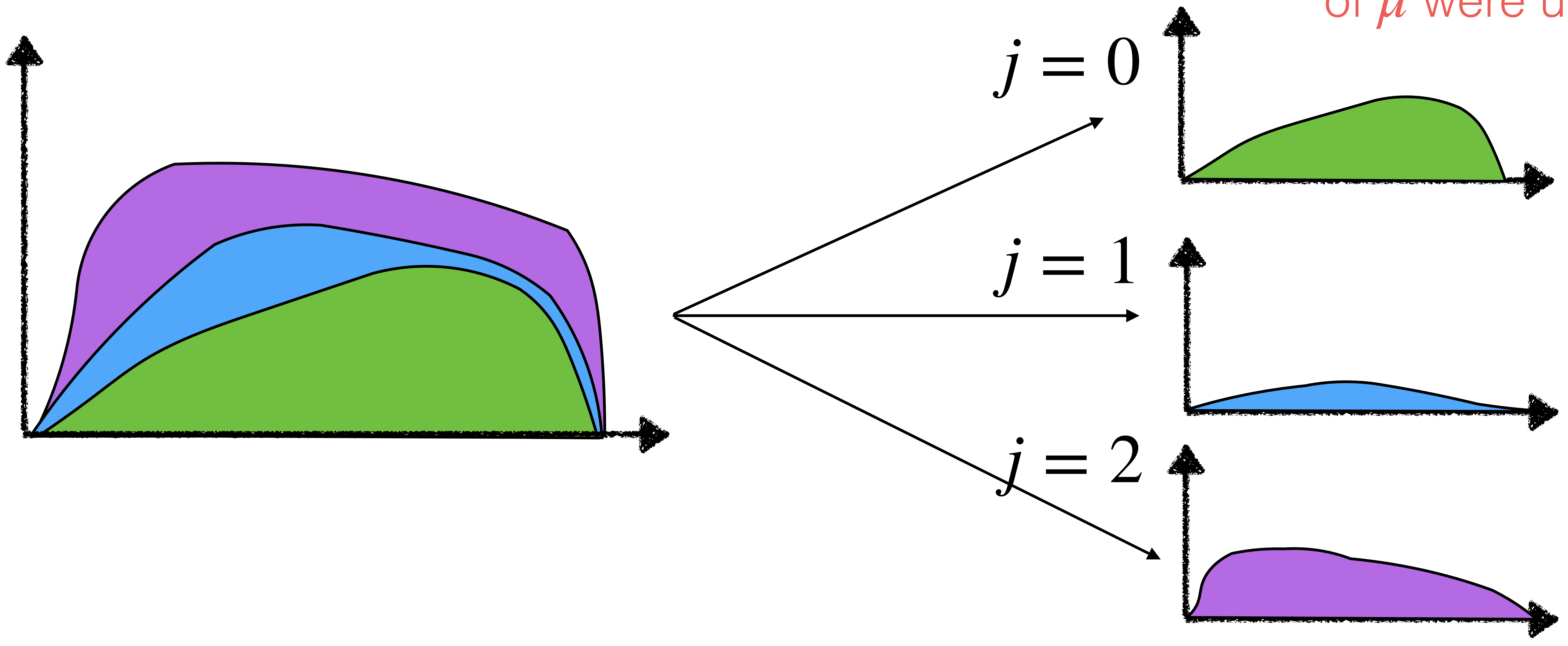
Robust, parameterised classifier without parameterising

H_{ref} : Reference hypothesis

$f_i(\mu)$ will depend on morphing bases points (which values of μ were used to simulate samples)

H_μ

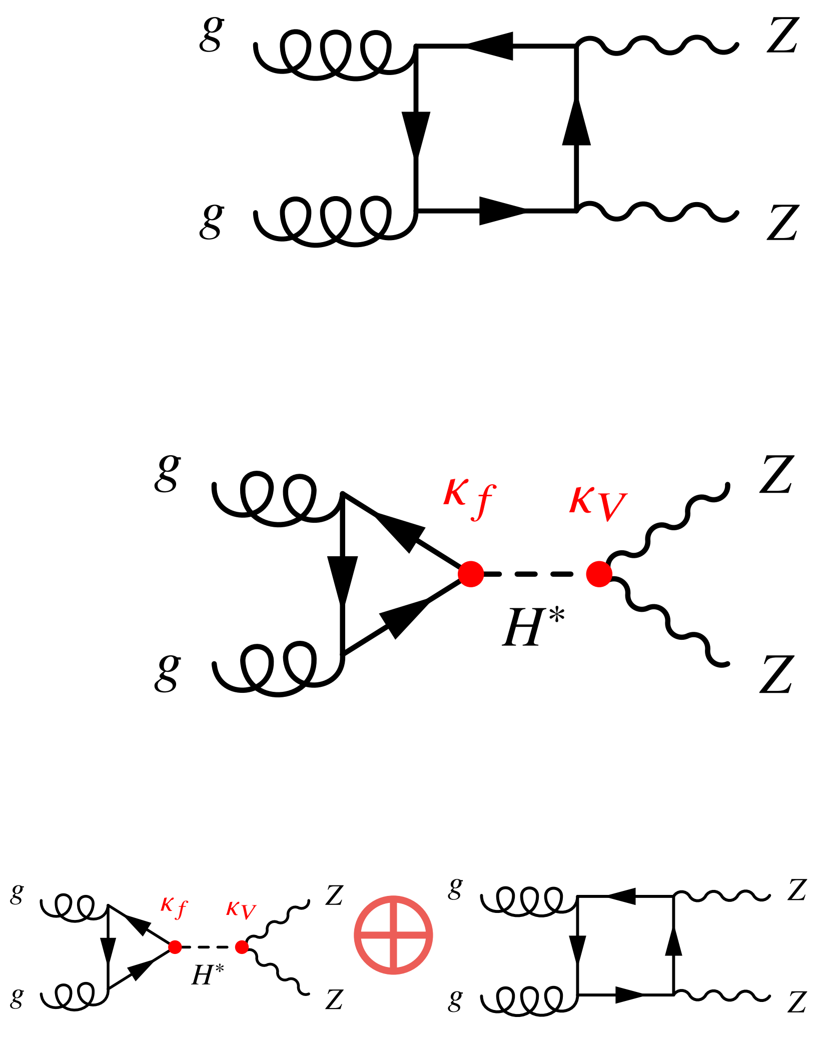
Hypothesis



$f_0(\mu) = 1$

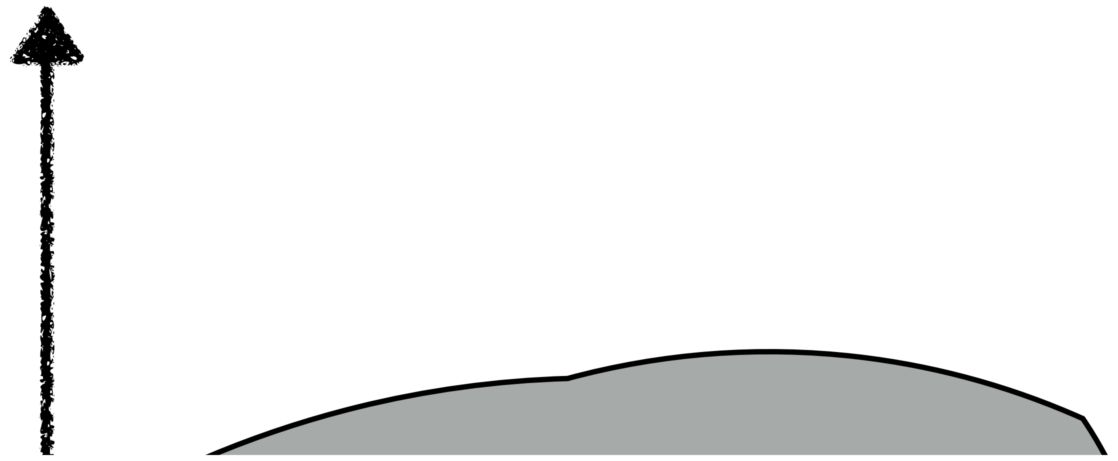
$f_1(\mu) = \mu$

$f_2(\mu) = \sqrt{\mu}$



vs

H_{ref}



$$\frac{p(x_i|\mu)}{p_{\text{ref}}(x_i)} = \frac{1}{\nu(\mu)} \sum_j^C f_j(\mu) \cdot \nu_j \frac{p_j(x_i)}{p_{\text{ref}}(x_i)}$$

Analytically parameterised in μ, allows to get LR for any hypothesis μ without training parameterised networks !

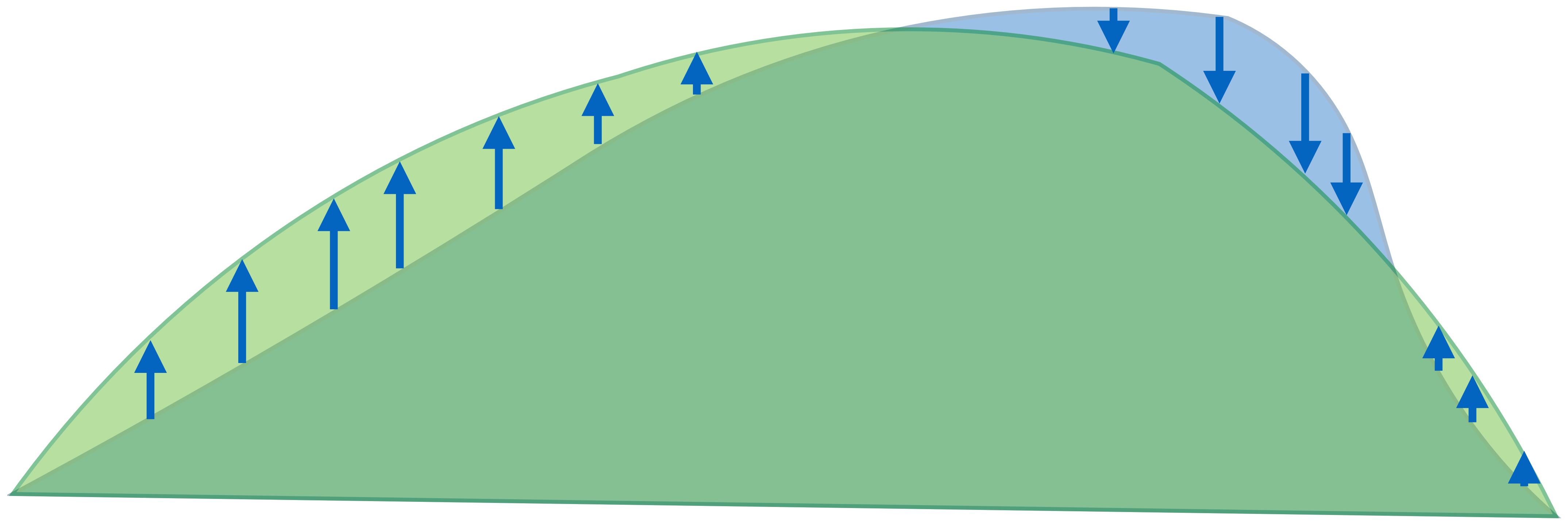
A separate classifier per physics process j (Eg. $gg \to H^* \to 4l$, $gg \to ZZ \to 4l$)

Open problems to extend to full ATLAS analysis:

- Robustness: Design and validation
- Systematic Uncertainties: Incorporate them in likelihood (ratio) model
- Neyman Construction: Throwing toys in a per-event analysis

Validate quality of LR estimation with re-weighting task

Reweighting: Calculate weights w_i for events x_i in blue sample to match green sample

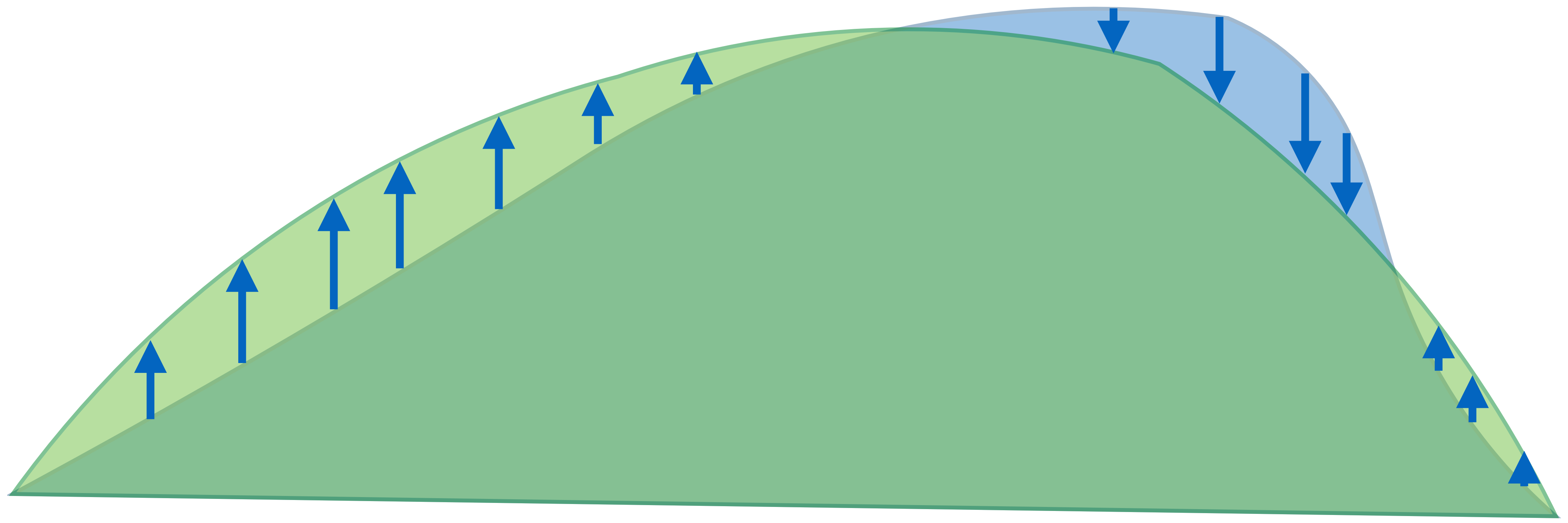


Validate quality of LR estimation with re-weighting task

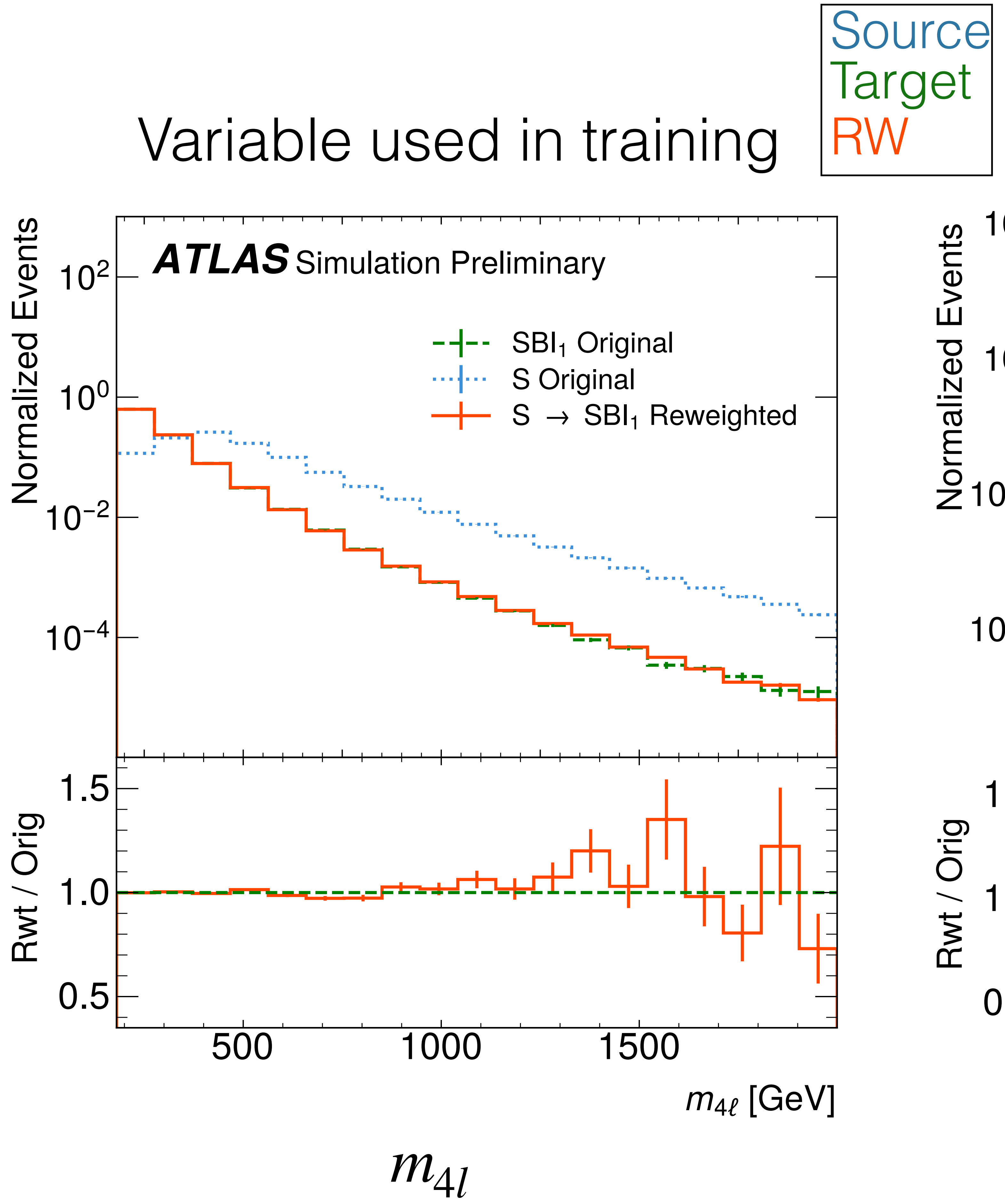
Reweighting: Calculate weights w_i for events x_i in blue sample to match green sample

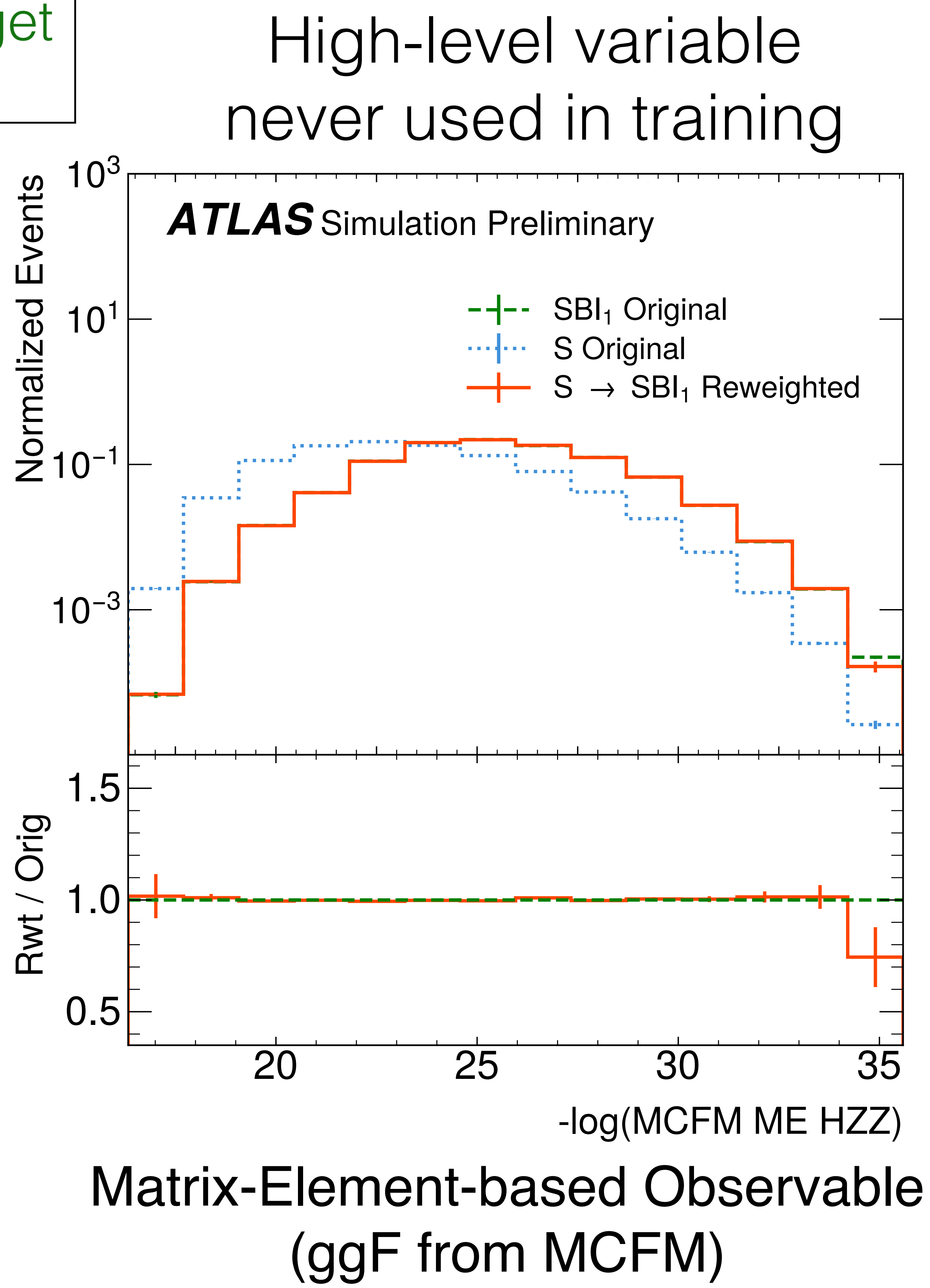
$$w_i = r(x_i, \mu_0, \mu_1) = \frac{p(x_i | \mu_0)}{p(x_i | \mu_1)}$$

Already estimated using an ensemble of networks



Re-weight closures





High-Dim Classifier Test:

Train independent classifier on RW vs Target, AUC=0.5 ⇒ LRs well estimated

See additional closure tests

Open problems to extend to full ATLAS analysis:

✓ Robustness: Design and validation

‣ Systematic Uncertainties: Incorporate them in likelihood (ratio) model

- Neyman Construction: Throwing toys in a per-event analysis

Systematic uncertainties

Experimental uncertainties:

Eg. Inaccuracies in the calibration of our detector

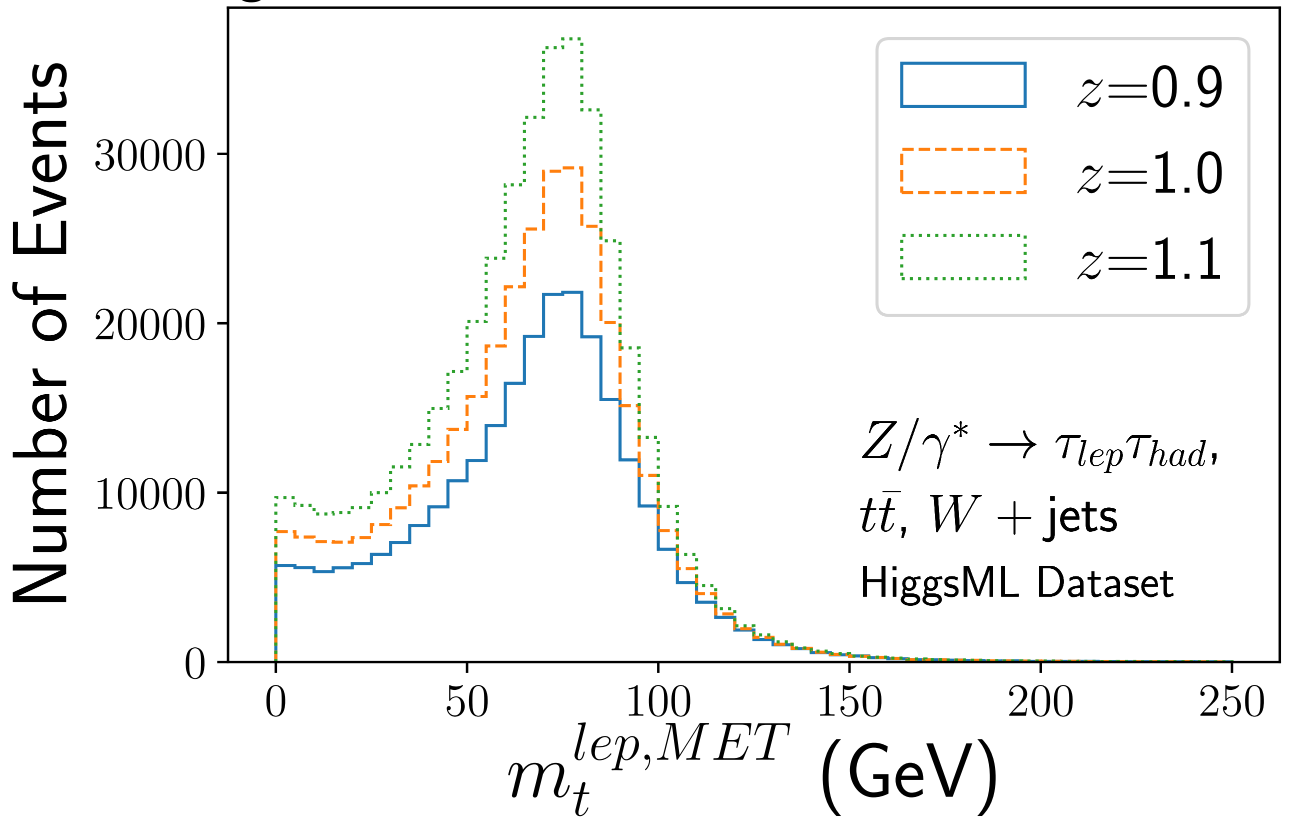


Image: arXiv:2105.08742

Theory uncertainties:

Eg. Inability to compute QFT to infinite order

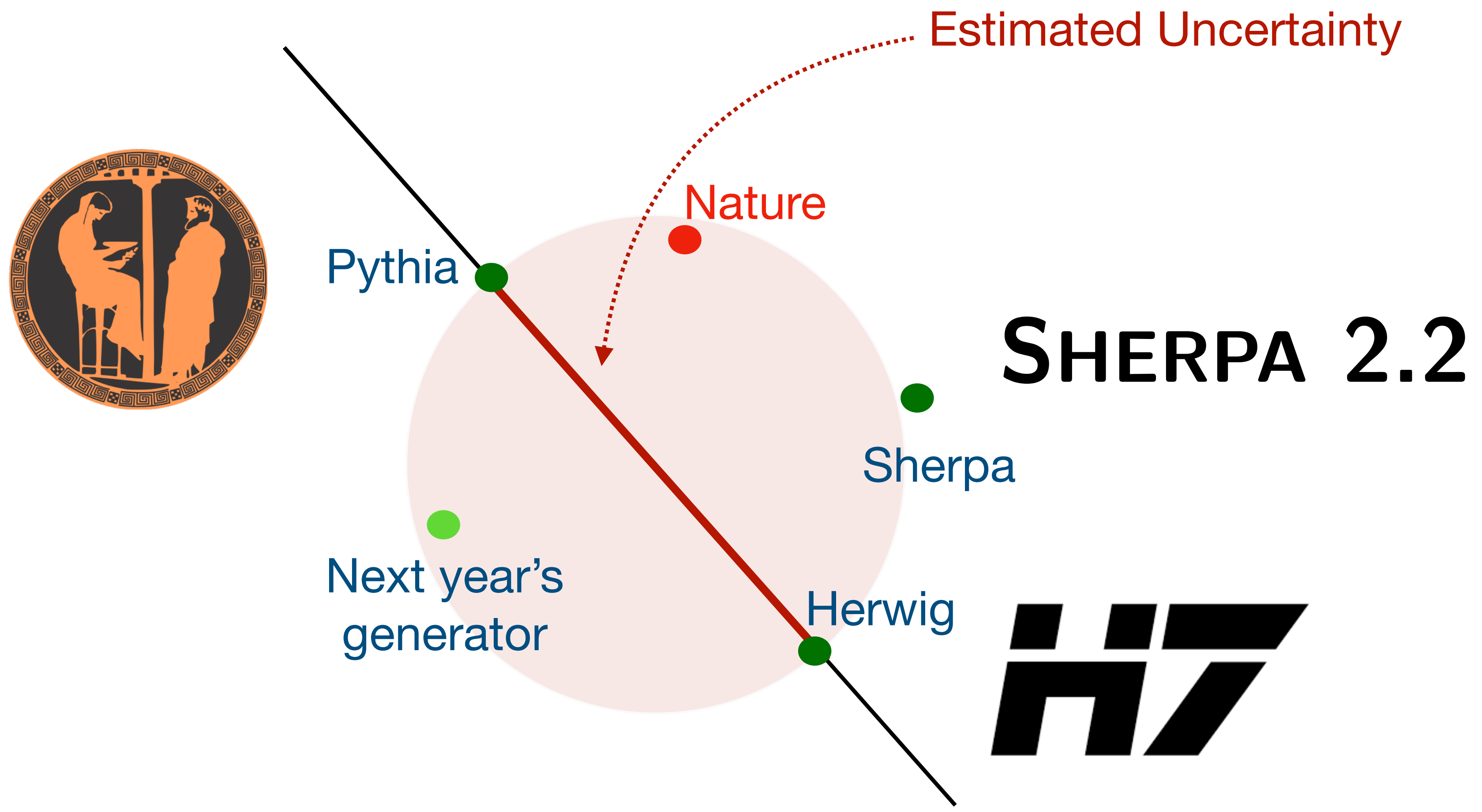


Image: arXiv:2109.08159

Systematic uncertainties

Experimental uncertainties:

Eg. Inaccuracies in the calibration of our detector

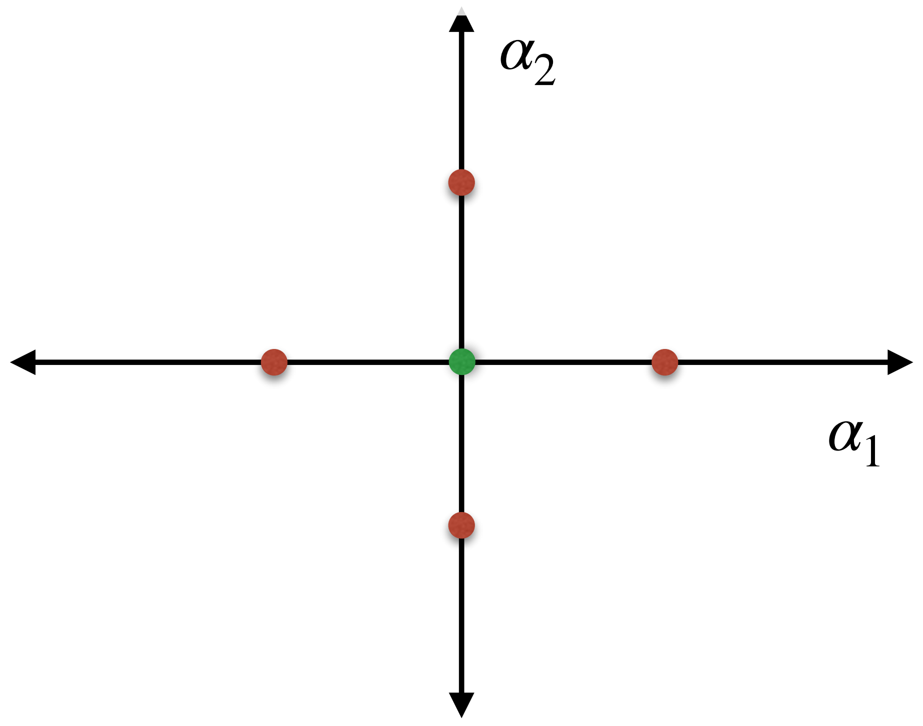
Image: arXiv:2105.08742

Theory uncertainties:

Eg. Inability to compute QFT to infinite order

Image: arXiv:2109.08159

- We only have simulations at 3 variations of each nuisance parameter α_k



Known interpolation strategies

See formula used

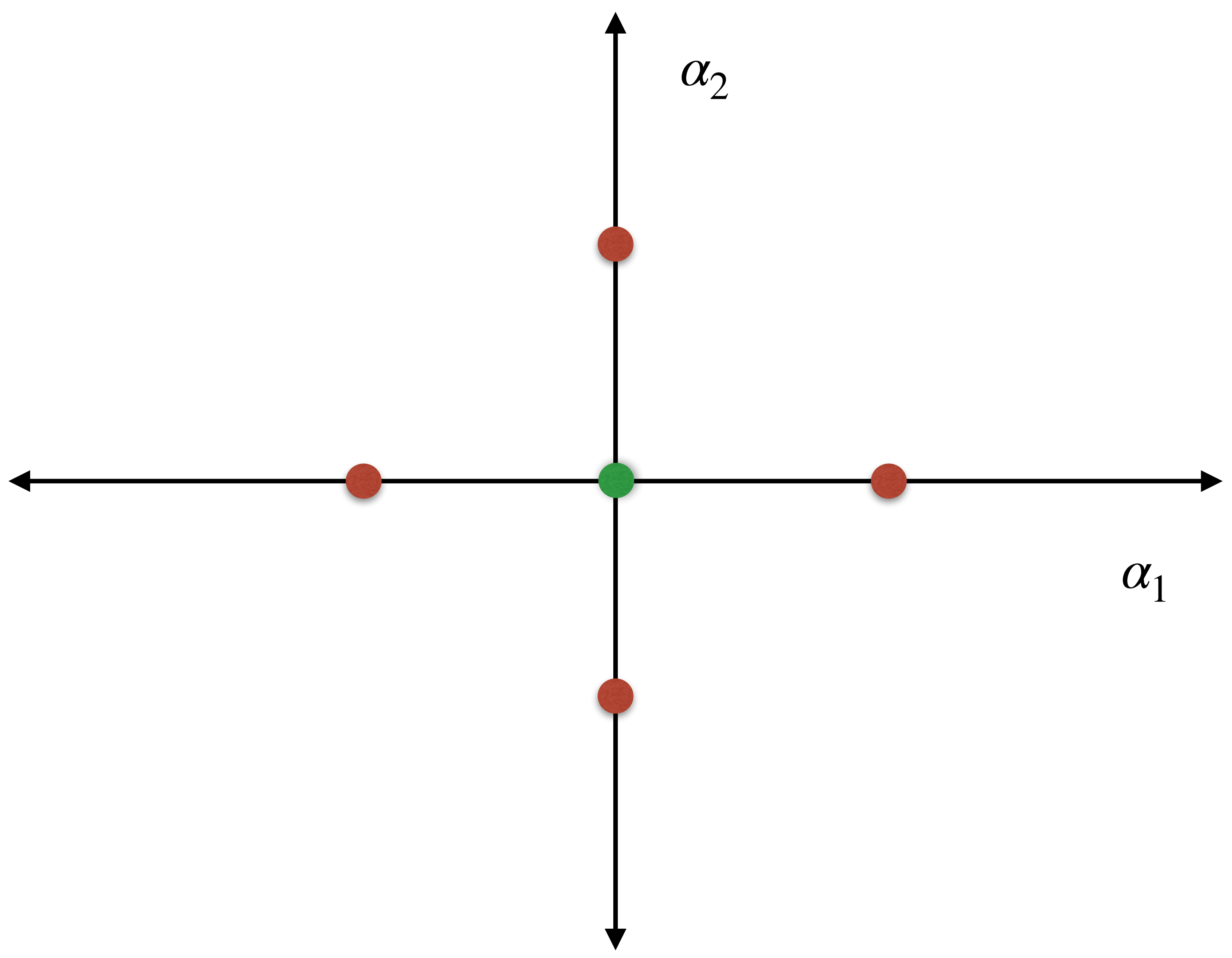
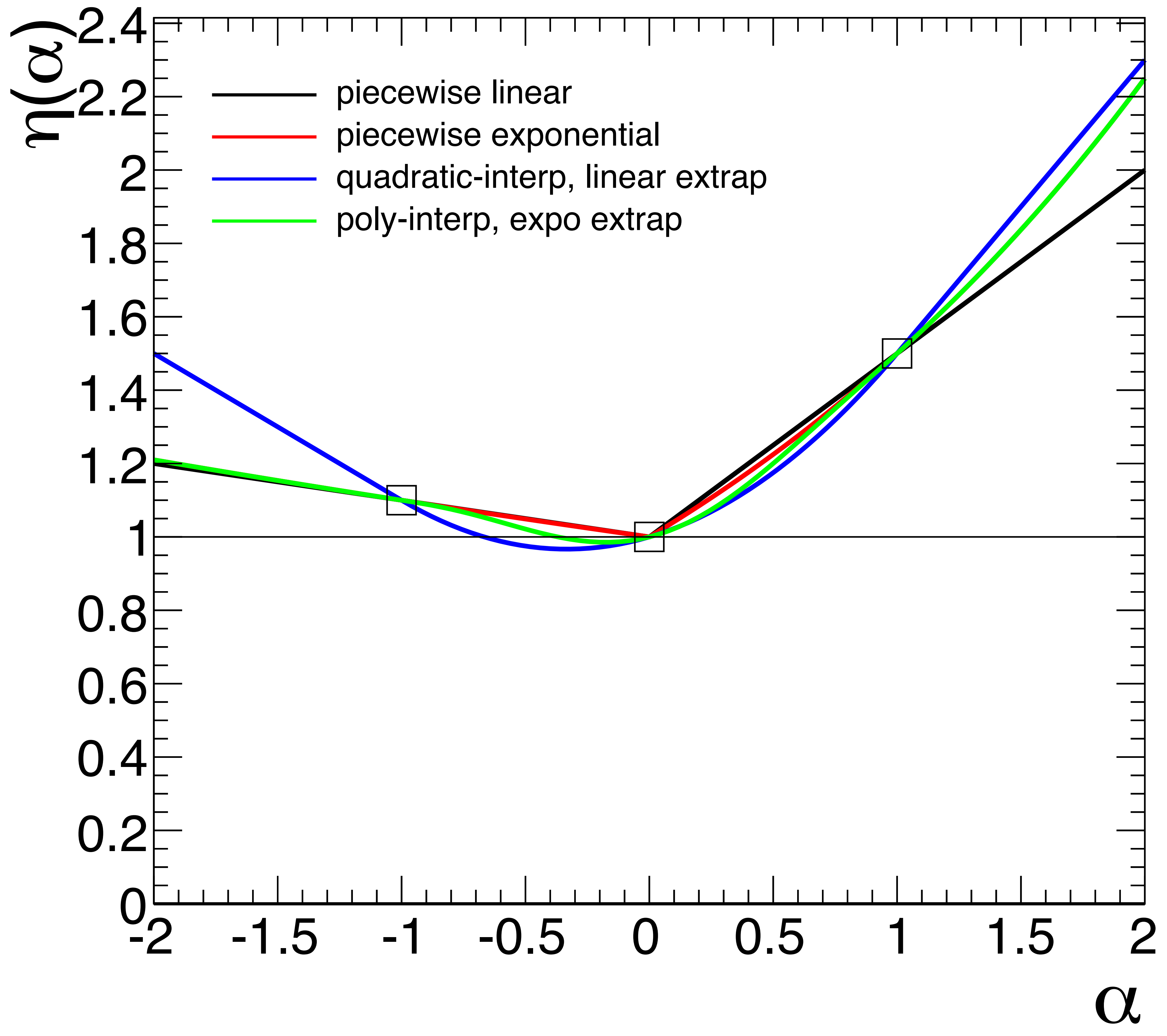


Image: arXiv:1503.07622

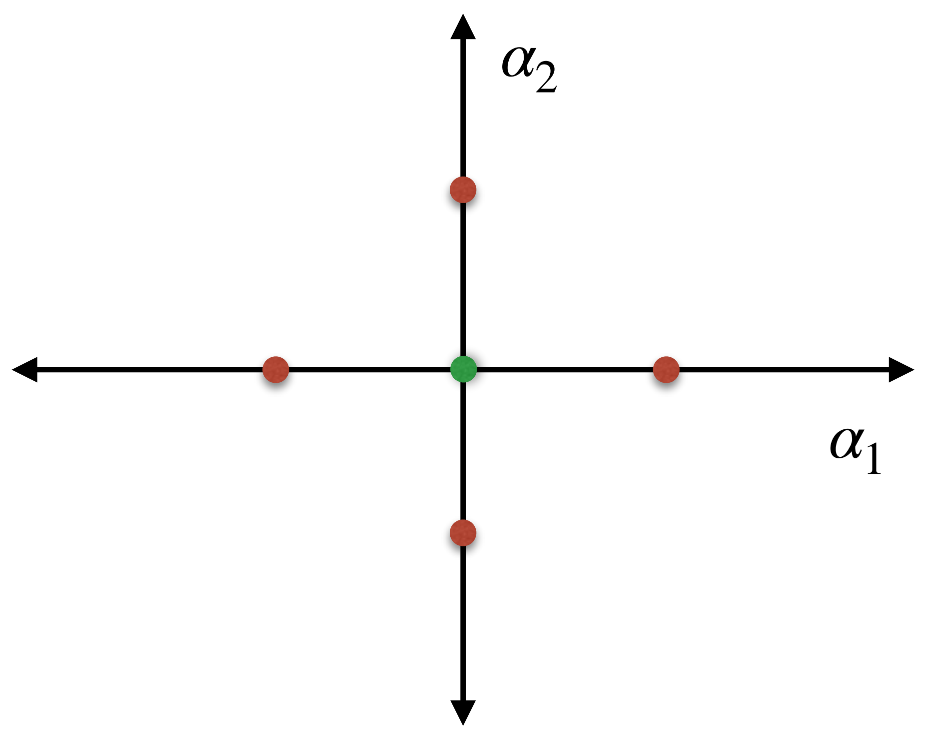


⇒ Combine these traditional interpolation with neural network estimation of per-event likelihood ratios

Probability density ratio including nuisance parameters (α)

x_i is one individual event

$$\frac{p(x_i \mid \mu, \alpha)}{p_{ref}(x_i)} =$$

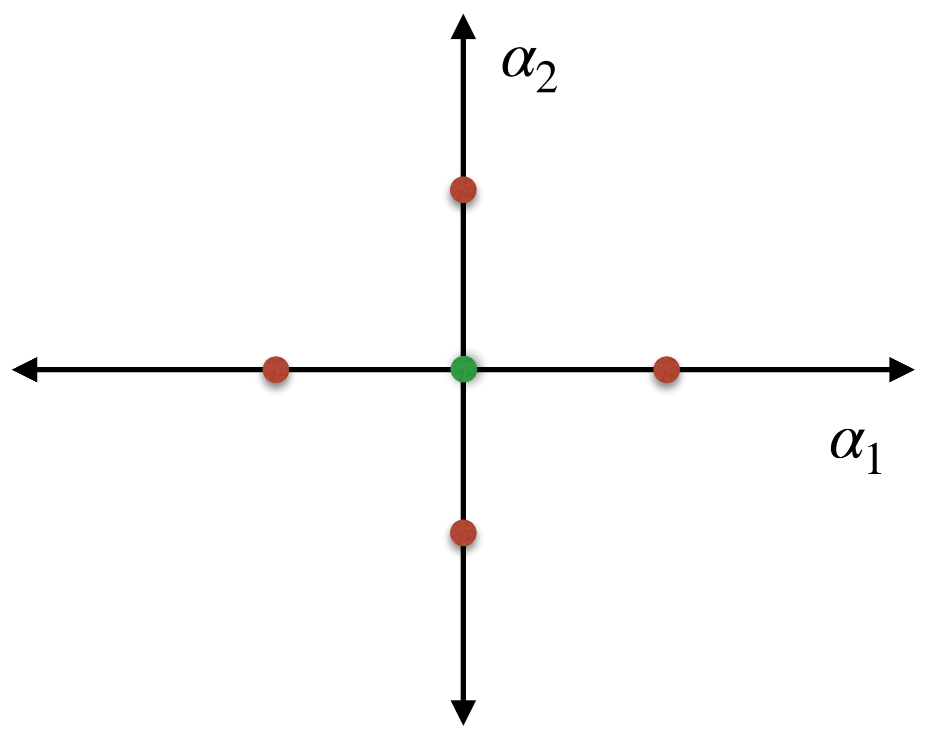


See details of vertical interpolation for $G_j(\alpha_k)$, $g_j(x_i, \alpha_k)$

Probability density ratio including nuisance parameters (α)

x_i is one individual event

$$\frac{p(x_i\,|\,\mu,\alpha)}{p_{ref}(x_i)} = \frac{1}{\nu(\mu,\alpha)} \sum_j^C f_j(\mu)\cdot\nu_j\cdot\frac{p_j(x_i)}{p_{ref}(x_i)}\cdot\prod_k^{N_{syst}} G_j(\alpha_k)\cdot g_j(x_i,\alpha_k)$$



$$g_j(x_i,\alpha_k) = \frac{p_j(x_i,\alpha_k)}{p_j(x_i)}$$

See details of vertical interpolation for $G_j(\alpha_k)$, $g_j(x_i,\alpha_k)$

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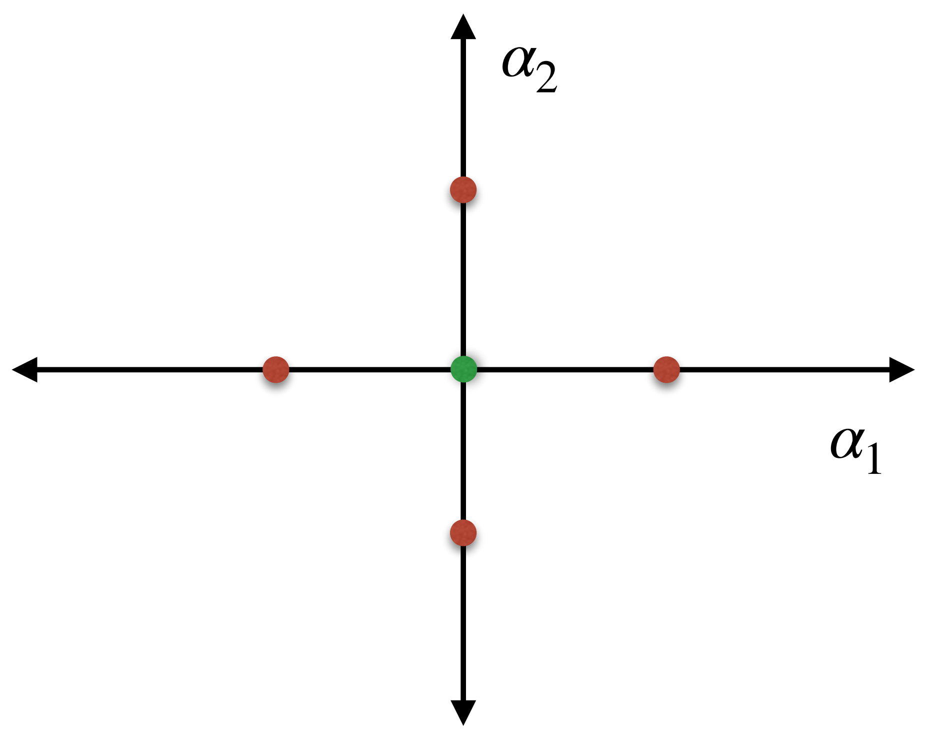
We have this already



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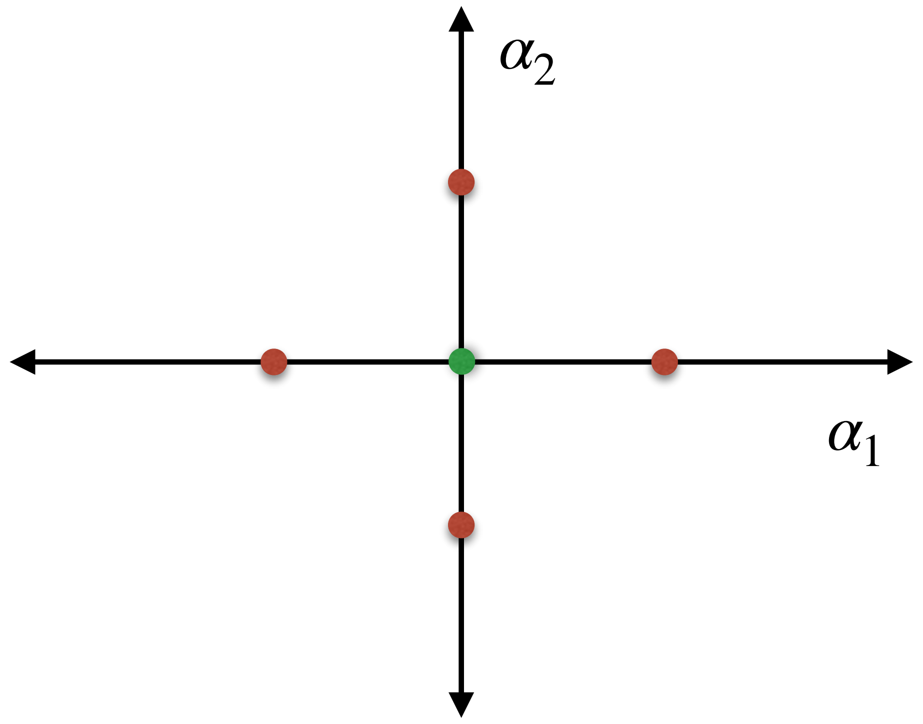
We have this already

Estimate from simulations and existing interpolation methods



$$g_j(x_i, \alpha_k) = \frac{p_j(x_i, \alpha_k)}{p_j(x_i)}$$

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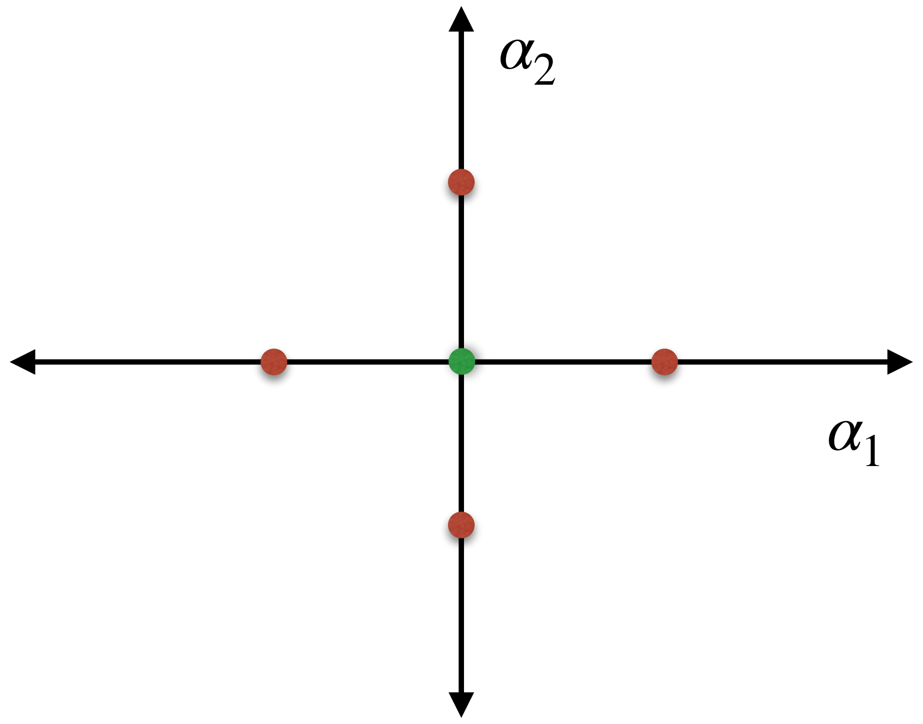
We have this already

Per-event terms estimated using another ensemble of networks and interpolation methods

$$g_j(x_i,\alpha_k) = \frac{p_j(x_i,\alpha_k)}{p_j(x_i)}$$

Estimate from simulations and existing interpolation methods



See details of vertical interpolation for $G_j(\alpha_k)$, $g_j(x_i,\alpha_k)$

Final test statistic

x_i is one individual event

$$\frac{L_{\text{full}}(\mu, \alpha|\mathcal{D})}{L_{\text{ref}}(\mathcal{D})} = \text{Pois}(N_{\text{data}}|\nu(\mu, \alpha)) \prod_{i}^{N_{\text{data}}} \frac{p(x_i|\mu, \alpha)}{p_{\text{ref}}(x_i)} \prod_{k} \text{Gaus}(a_k|\alpha_k, \delta_k)$$

Final test statistic

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From previous slide

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From previous slide

Prod over events

Final test statistic

x_i is one individual event

$$\frac{L_{\text{full}}(\mu, \alpha | \mathcal{D})}{L_{\text{ref}}(\mathcal{D})} = \text{Pois}(N_{\text{data}} | \nu(\mu, \alpha)) \prod_i^{N_{\text{data}}} \frac{p(x_i | \mu, \alpha)}{p_{\text{ref}}(x_i)} \prod_k \text{Gaus}(a_k | \alpha_k, \delta_k)$$

From previous slide

Rate term

Prod over events

Final test statistic

x_i is one individual event

$$\frac{L_{\text{full}}(\mu,\alpha|\mathcal{D})}{L_{\text{ref}}(\mathcal{D})} = \text{Pois}(N_{\text{data}}|\nu(\mu,\alpha)) \prod_{i}^{N_{\text{data}}} \frac{p(x_i|\mu,\alpha)}{p_{\text{ref}}(x_i)} \prod_{k} \text{Gaus}(a_k|\alpha_k,\delta_k)$$

From previous slide

Rate term

Prod over events

Constrain term

Final test statistic

x_i is one individual event

$$\frac{L_{\text{full}}(\mu,\alpha|\mathcal{D})}{L_{\text{ref}}(\mathcal{D})} = \text{Pois}(N_{\text{data}}|\nu(\mu,\alpha)) \prod_{i}^{N_{\text{data}}} \frac{p(x_i|\mu,\alpha)}{p_{\text{ref}}(x_i)} \prod_k \text{Gaus}(a_k|\alpha_k,\delta_k)$$

From previous slide

Rate term

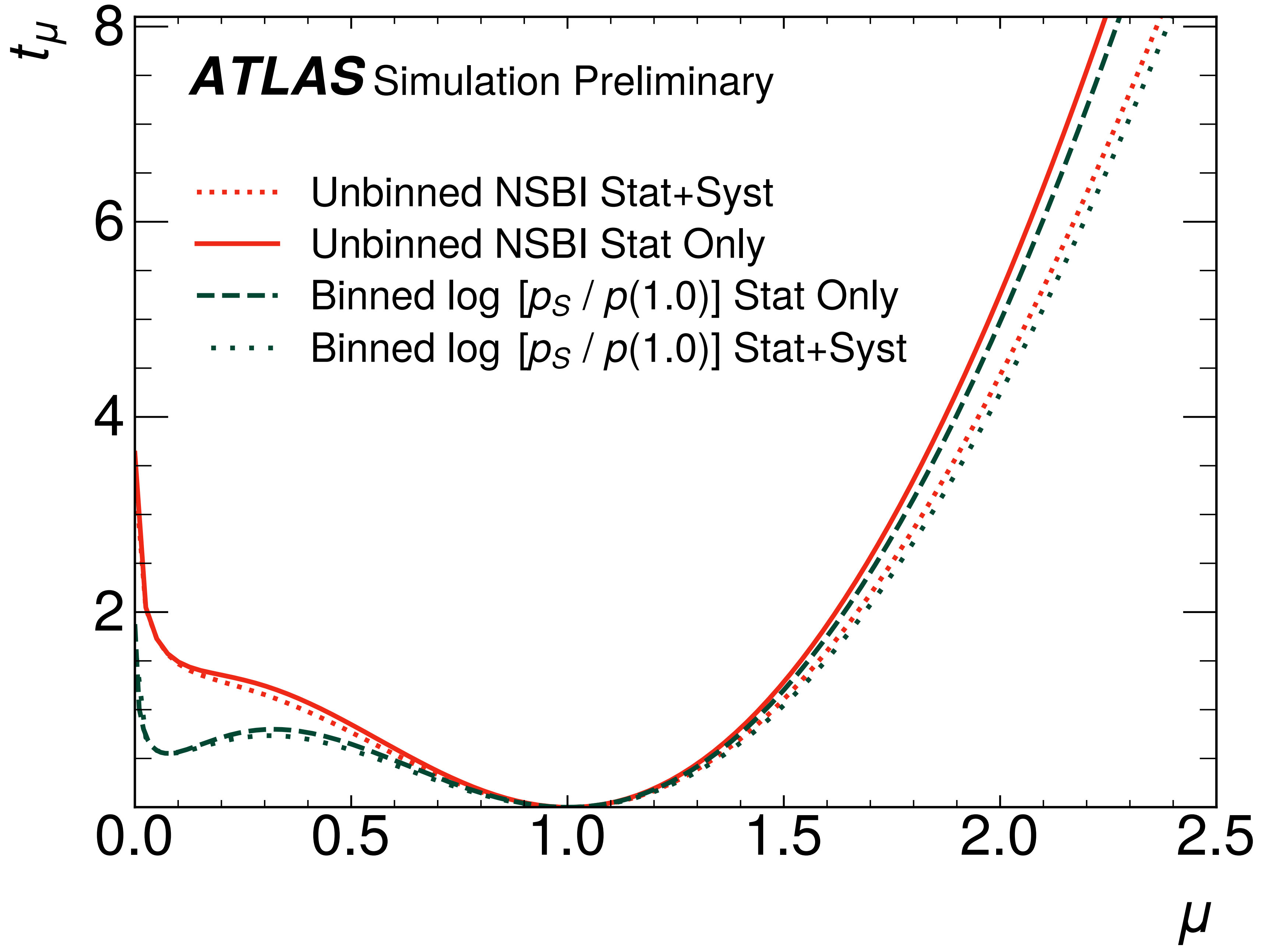
Prod over events

Constrain term

Profiling:

$$t_\mu = -2\ln\left(\frac{L_{\text{full}}(\mu,\hat{\hat{\alpha}})/\cancel{L_{\text{ref}}}}{L_{\text{full}}(\hat{\mu},\hat{\alpha})/\cancel{L_{\text{ref}}}}\right)$$

This is why we define p_{ref} to be independent of μ



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From previous slide

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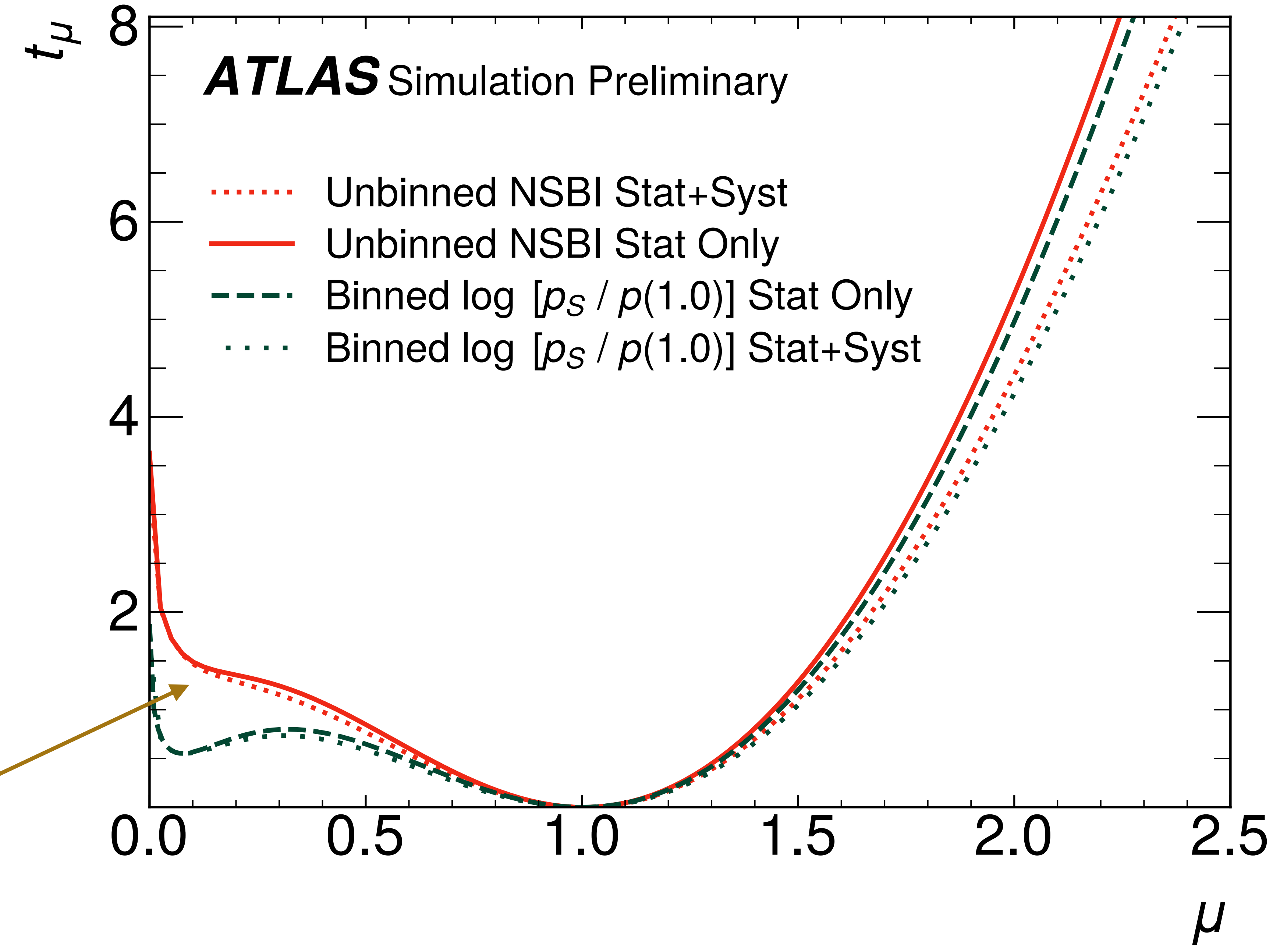


Non-parabolic shape due to non-linear effects from quantum interference

Reference Sample

A combination of signal samples, to ensure there's non-vanishing support entire region of analysis
Does not have to be physical!

$$p_{\text{ref}}(x_i) = \frac{1}{\sum_k \nu_k} \sum_{k}^{C_{\text{signals}}} \nu_k \cdot p_k(x_i)$$

⇒ In our dataset, $p_{ref}(\cdot) = p_S(\cdot)$

Choice of $p_{ref}(\cdot)$ can be made purely on numerical stability of training, as it drops out in profile step

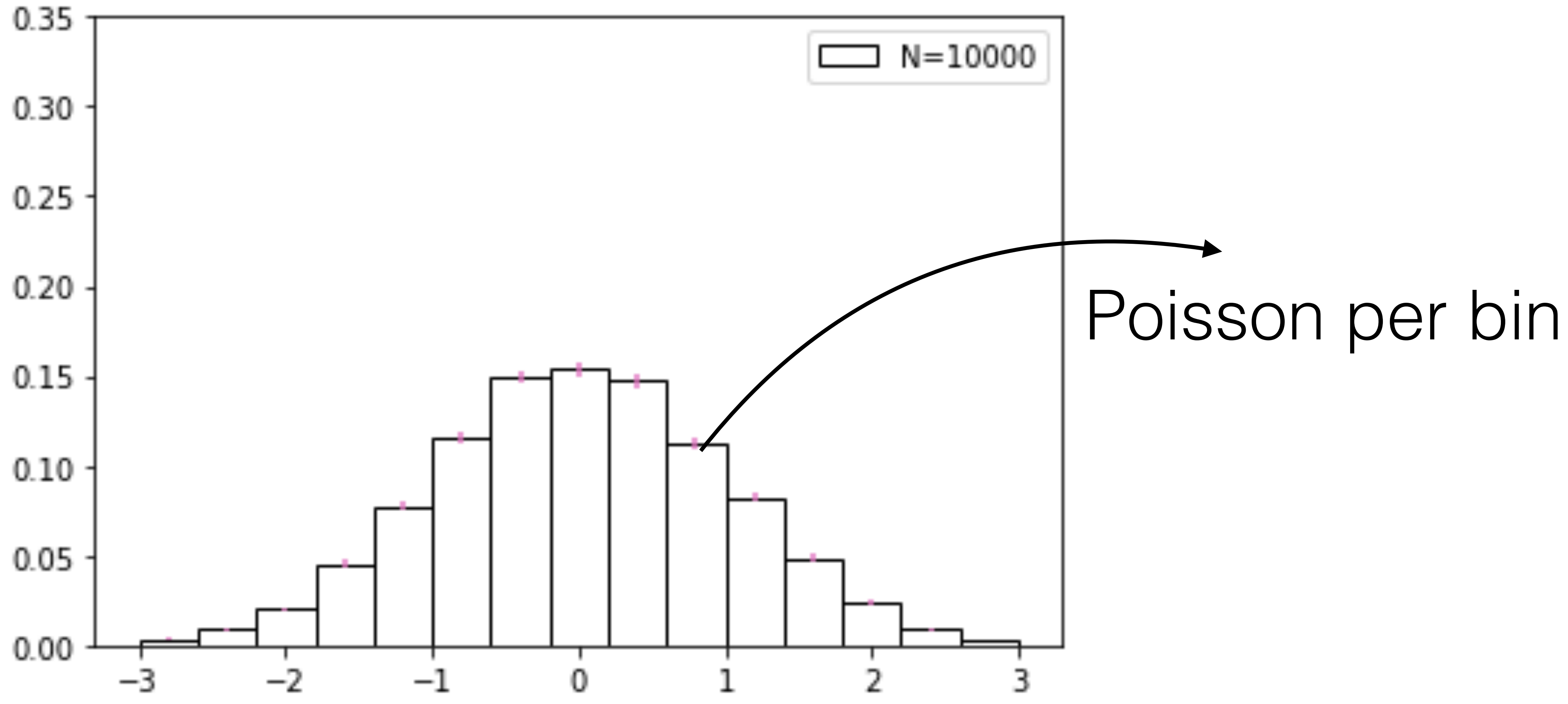
$$t_\mu = -2 \ln \left(\frac{L_{\text{full}}(\mu, \widehat{\widehat{\alpha}}) / \cancel{L_{\text{ref}}}}{L_{\text{full}}(\widehat{\mu}, \widehat{\alpha}) / \cancel{L_{\text{ref}}}} \right)$$

Open problems to extend to full ATLAS analysis:

- ✓ Robustness: Design and validation
- ✓ Systematic Uncertainties: Incorporate them in likelihood (ratio) model
- ‣ Neyman Construction: Throwing toys in a per-event analysis

Throwing event-level toys

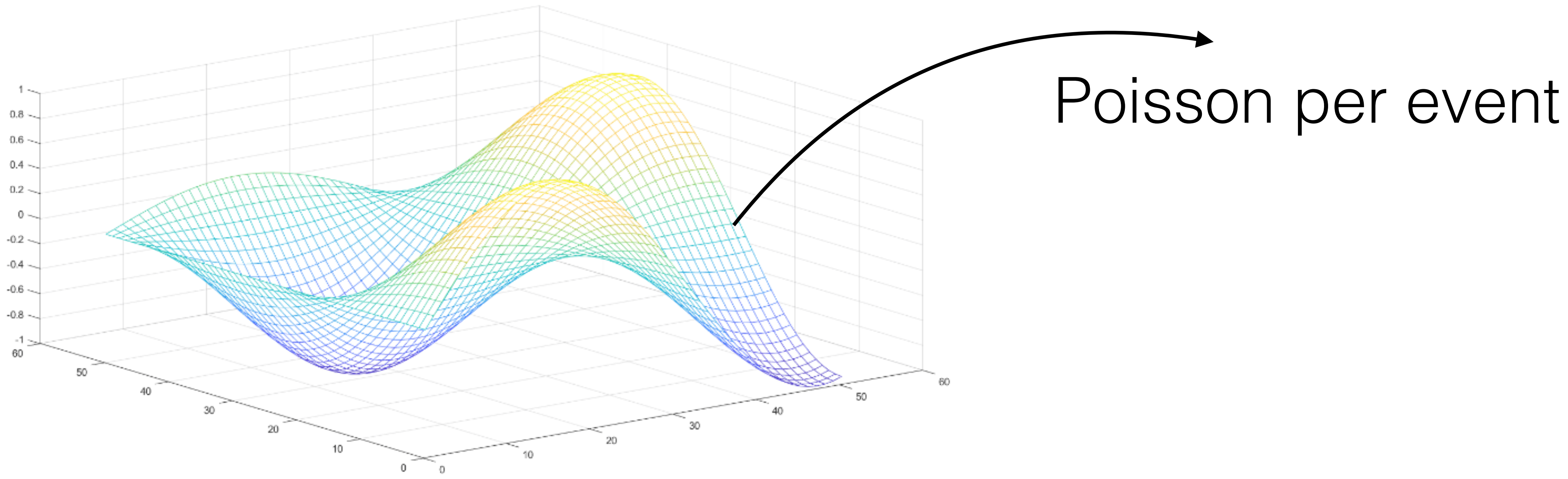
Traditionally:



Asimov Histogram

$$N_i^{toy} = Poisson(N_i^{Asimov})$$

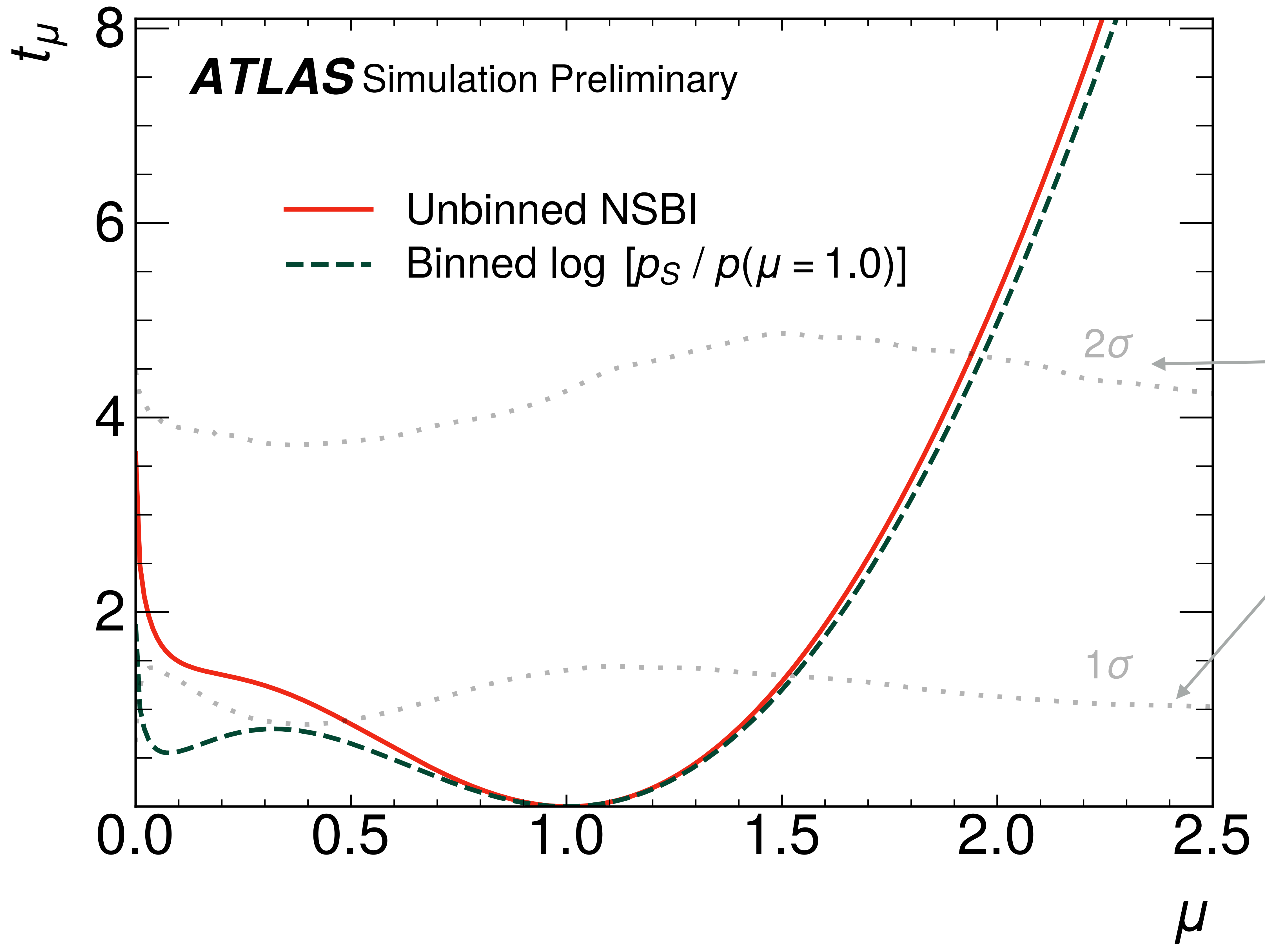
NSBI:



$$w_i^{toy} = Poisson(w_i^{Asimov})$$

('Unweighted' events, i.e. integer weights)

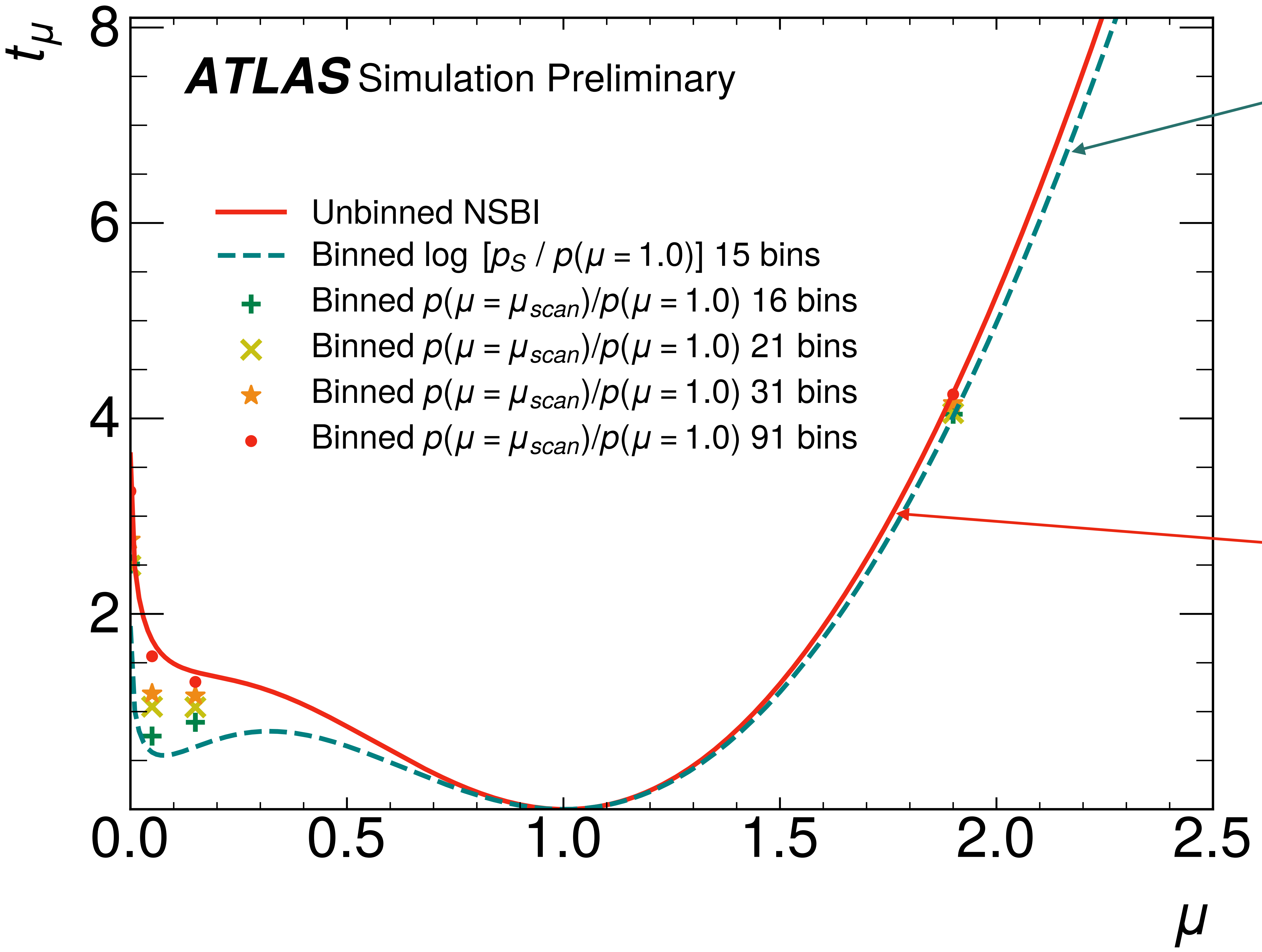
Confidence belts



Similar to structure seen in histogram analysis

Why does NSBI work better than traditional analyses?

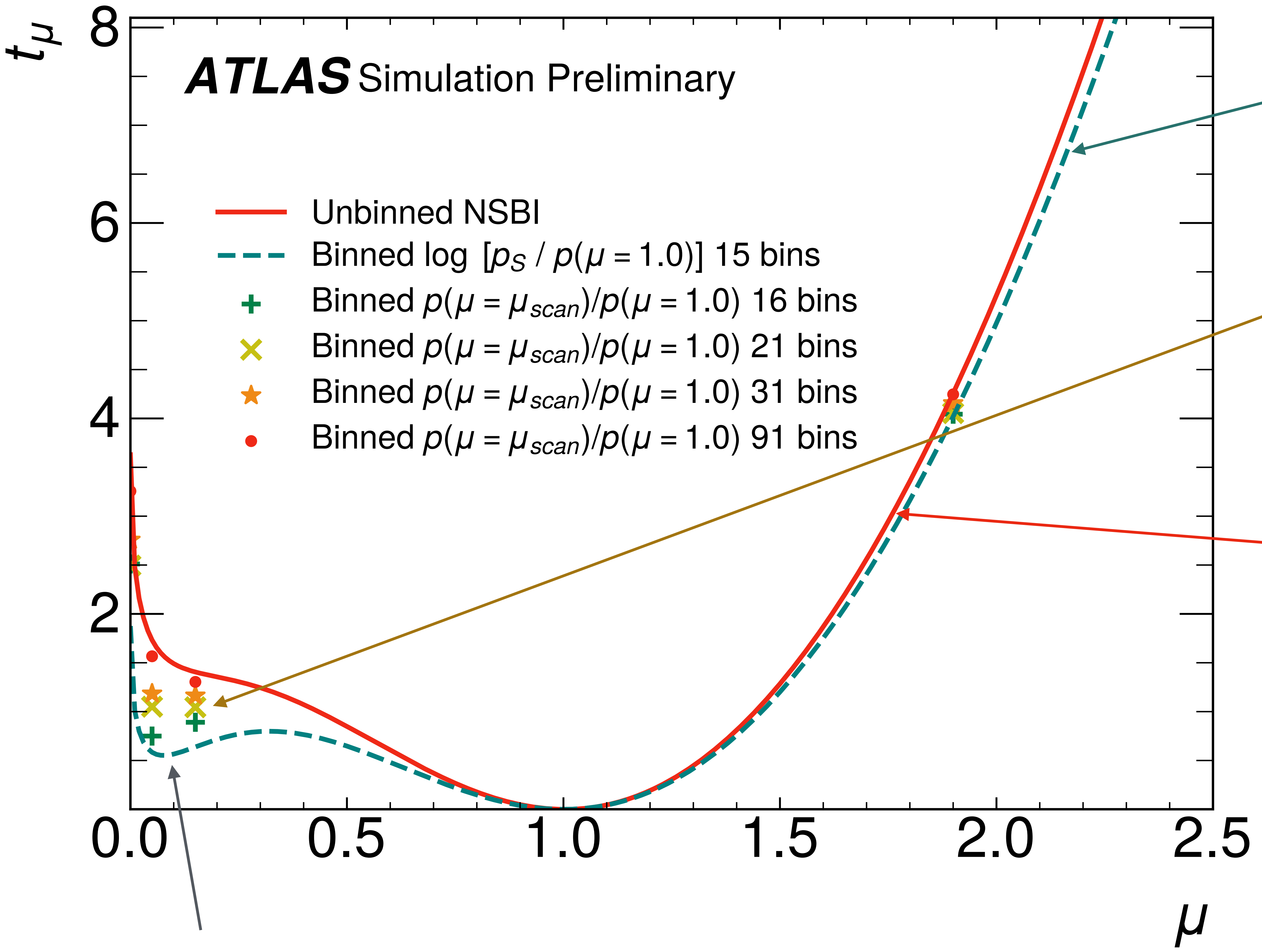
Why does it work better than traditional analyses?



$O_{fixed} = \log \frac{p_S(x_i)}{p_{SBI}(x_i)}$: Similar to histogram analysis

NSBI: Parameterised, unbinned

Why does it work better than traditional analyses?



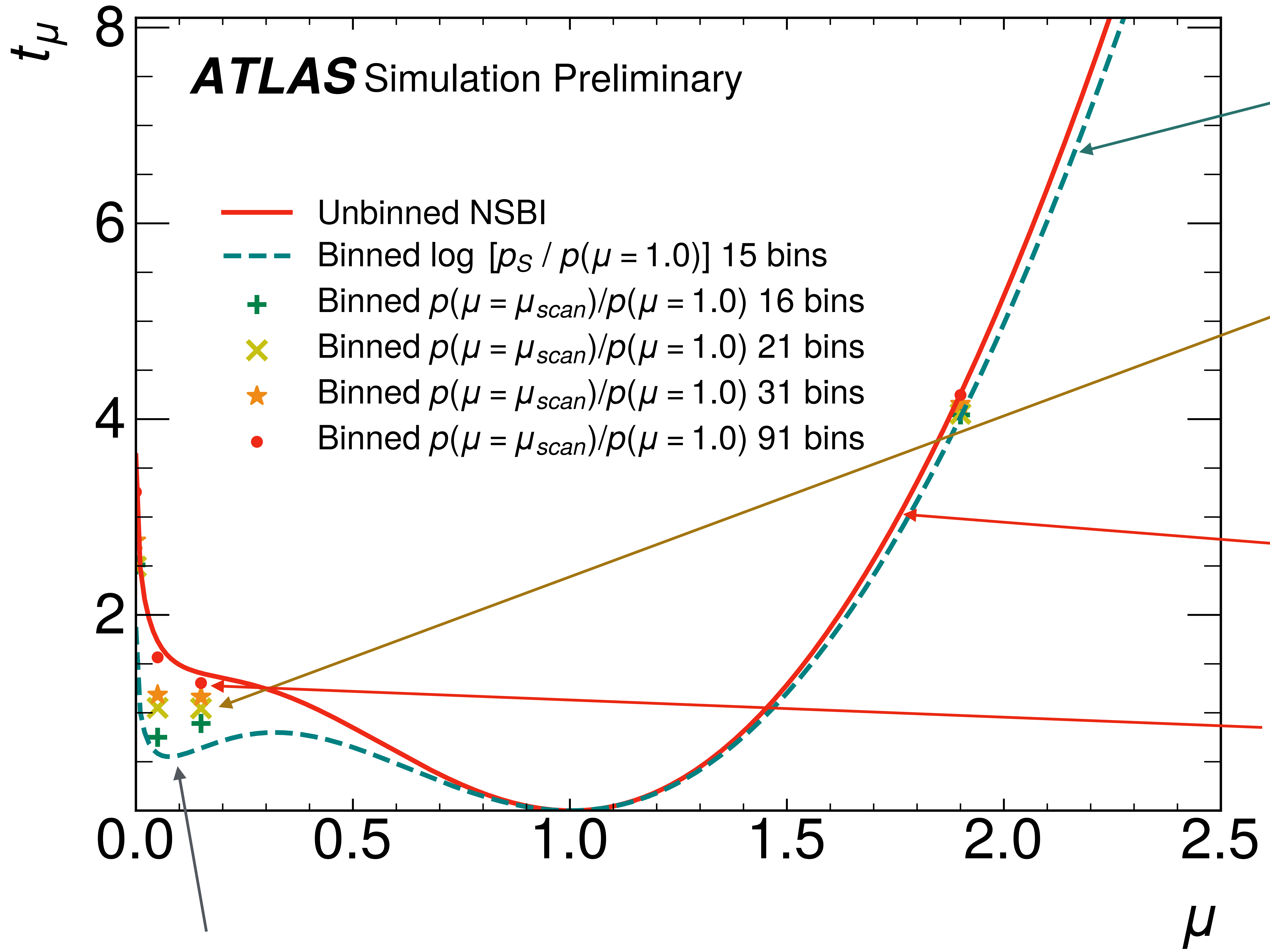
$O_{\text{fixed}} = \log \frac{p_S(x_i)}{p_{SBI}(x_i)}$: Similar to histogram analysis

$O_\mu = \frac{p(x_i \mid \mu)}{p(x_i \mid \mu = 1)}$: Parameterised observable, histogram fit

NSBI: Parameterised, unbinned

Significant improvement in QI impacted region

Why does it work better than traditional analyses?



$O_{\text{fixed}} = \log \frac{p_S(x_i)}{p_{SBI}(x_i)}$: Similar to histogram analysis

$O_\mu = \frac{p(x_i|\mu)}{p(x_i|\mu = 1)}$: Parameterised observable, histogram fit

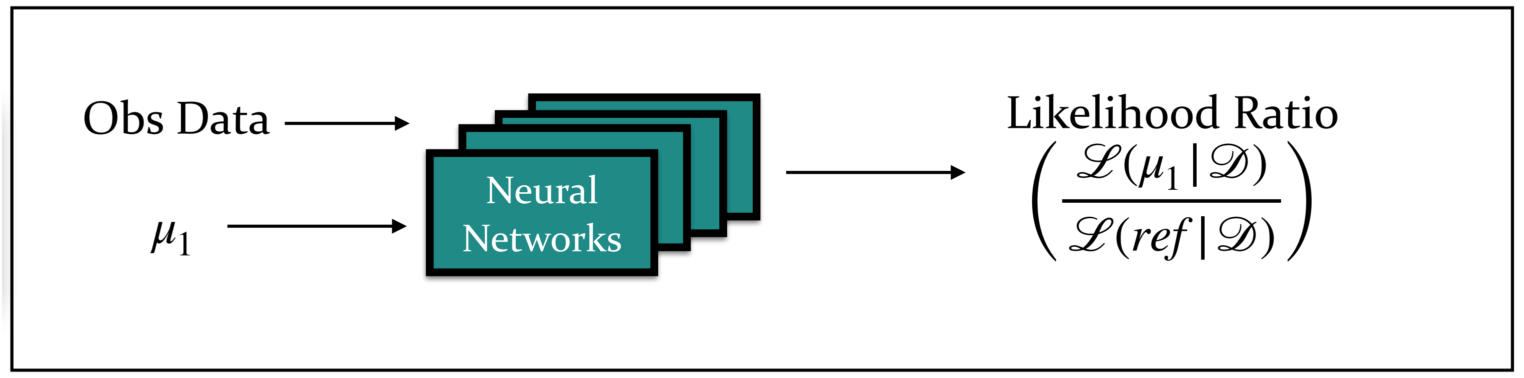
NSBI: Parameterised, unbinned

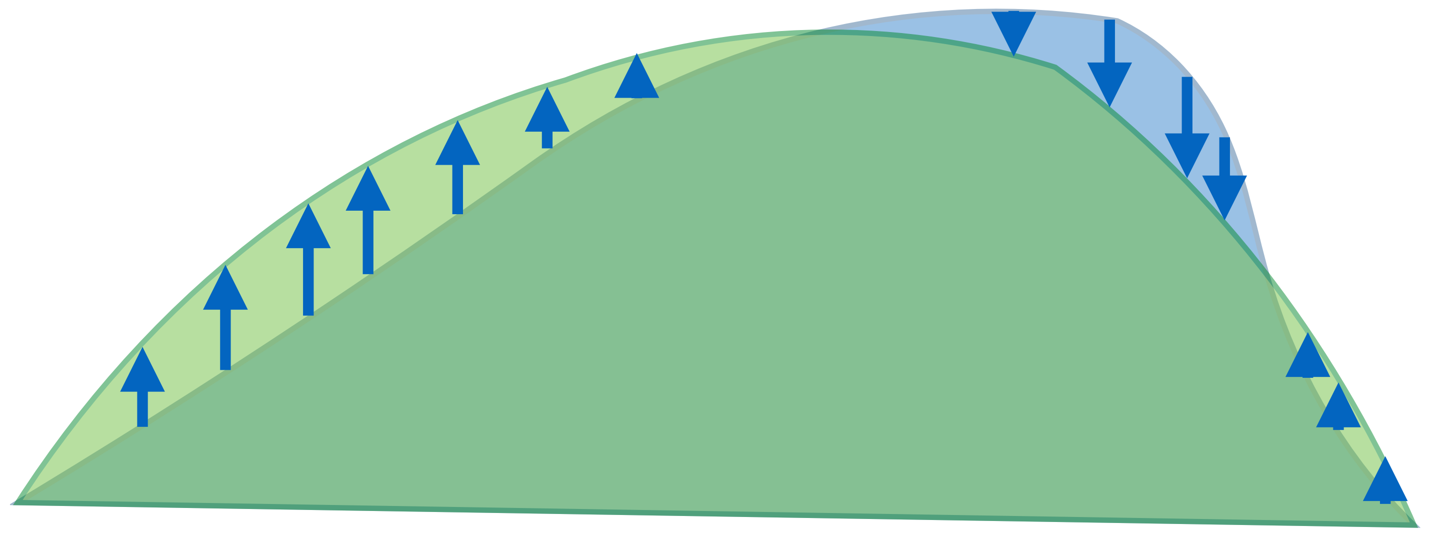
O_μ approaches NSBI as nBins $\to \infty$

Significant improvement in QI impacted region

Conclusion

- Developed a complete statistical framework for high-dimensional statistical inference
  - Builds upon traditional methodology in ATLAS
  - Developed diagnostic tools for validation
- Such methods are crucial for analyses where kinematic distributions change non-linearly with the parameter of interest, eg. EFT studies
- Weaknesses: Same as traditional analyses, requires well trained networks





Conclusion

- Developed a complete statistical framework for high-dimensional statistical inference
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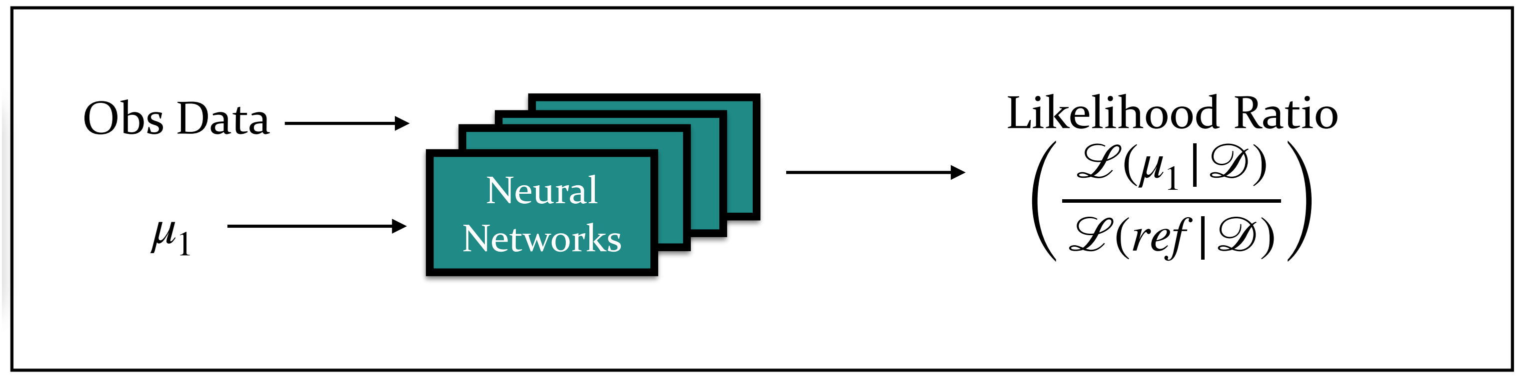

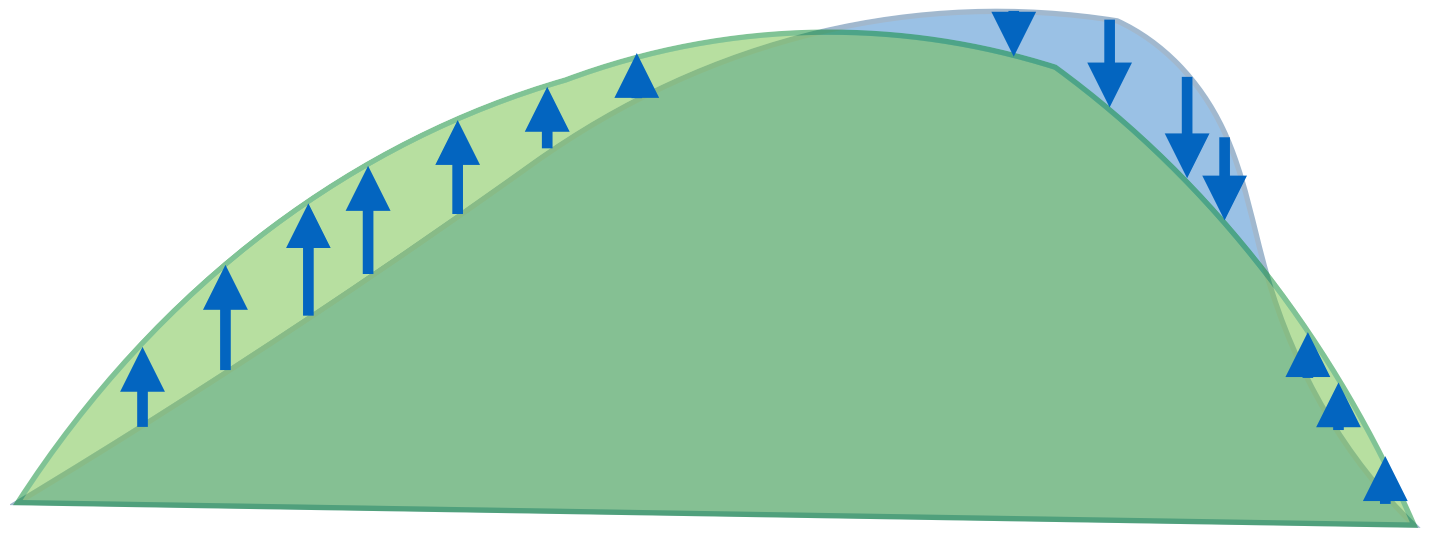

Thanks !

Backup

Choice of observable

Choice of observable

$$\mathscr{L}(\mu \,|\, \mathscr{D}) = p(\mathscr{D} \,|\, \mu)$$

Neyman–Pearson lemma: Likelihood ratio is the most powerful test statistic

We want to compare likelihoods: $$\frac{p(\mathscr{D} \,|\, \mu)}{p(\mathscr{D} \,|\, \mu_0)}$$

Choice of observable

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We want to compare likelihoods:

$$\frac{p(\mathscr{D} \,|\, \mu)}{p(\mathscr{D} \,|\, \mu_0)}$$

A neural network classifier trained on S vs B, estimates the decision function*:

$$s(x_i) = \frac{p(x_i \,|\, S)}{p(x_i \,|\, S) + p(x_i \,|\, B)}$$

* Equal class weights

Choice of observable

$$\mathscr{L}(\mu \,|\, \mathscr{D}) = p(\mathscr{D} \,|\, \mu)$$

Neyman–Pearson lemma: **Likelihood ratio is the most powerful test statistic**

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A neural network classifier trained on S vs B, estimates the decision function*: $$s(x_i) = \frac{p(x_i \,|\, S)}{p(x_i \,|\, S) + p(x_i \,|\, B)}$$

Which contains all the information required for the likelihood ratio:

$$\frac{p(x_i \,|\, \mu)}{p(x_i \,|\, \mu = 0)} = \frac{\mu \cdot \sigma_S \cdot p(x_i \,|\, S) + \sigma_B \cdot p(x_i \,|\, B)}{\sigma_B \cdot p(x_i \,|\, B)} = \mu \cdot \frac{\sigma_S}{\sigma_B} \cdot \frac{s(x_i)}{1 - s(x_i)} + 1.$$

Same observable s is optimal to test all μ hypotheses!

No need to develop separate analysis per hypothesis μ

* Equal class weights

What breaks down?

$$P(X) = |M_s(X) + M_b(X)|^2 = \underbrace{|M_s(X)|^2}_{P_s(X)} + \underbrace{|M_b(X)|^2}_{P_b(X)} + \underbrace{2\,\mathrm{Re}(\overline{M_s(X)}M_b(X))}_{P_i(X)}$$

$$N_{exp} = \mu \cdot S + B + \sqrt{\mu} \cdot I$$

A neural network classifier trained on S vs B, estimates the decision function: $s(x_i) = \frac{p(x_i|S)}{p(x_i|S) + p(x_i|B)}$

Which contains all the information required for the likelihood ratio:

$$\frac{p(x_i|\mu)}{p(x_i|\mu=0)} = \frac{\mu \cdot \sigma_S \cdot p(x_i|S) + \sigma_B \cdot p(x_i|B)}{\sigma_B \cdot p(x_i|B)} = \mu \cdot \frac{\sigma_S}{\sigma_B} \cdot \frac{s(x_i)}{1 - s(x_i)} + 1.$$

Same observable s is optimal to test all μ hypotheses!
No need to develop separate analysis per hypothesis μ

8

No longer in this convenient spacial case: The same observable **no longer optimal due to non-linear effects** coming from quantum interference

Also **does not generalise to an arbitrary theory parameter** θ, (eg. Effective Field Theory parameters)

Can we modify the LHC analysis methodology to **design near-optimal analyse for the general case?**

Cranmer et al

Estimating high-dimensional density ratios

$$\mathscr{L}(\mu \,|\, \mathscr{D}) = p(\mathscr{D} \,|\, \mu)$$

Neyman–Pearson lemma: **Likelihood ratio is the most powerful test statistic**

We want to compare likelihoods:

$$\frac{p(\mathscr{D} \,|\, \mu)}{p(\mathscr{D} \,|\, ref)}$$

A neural network classifier trained on simulated samples from θ_1 vs simulated samples from ref, estimates the decision function:

$$s(x_i) = \frac{p(x_i \,|\, \mu_1)}{p(x_i \,|\, \mu_1) + p(x_i \,|\, ref)}$$

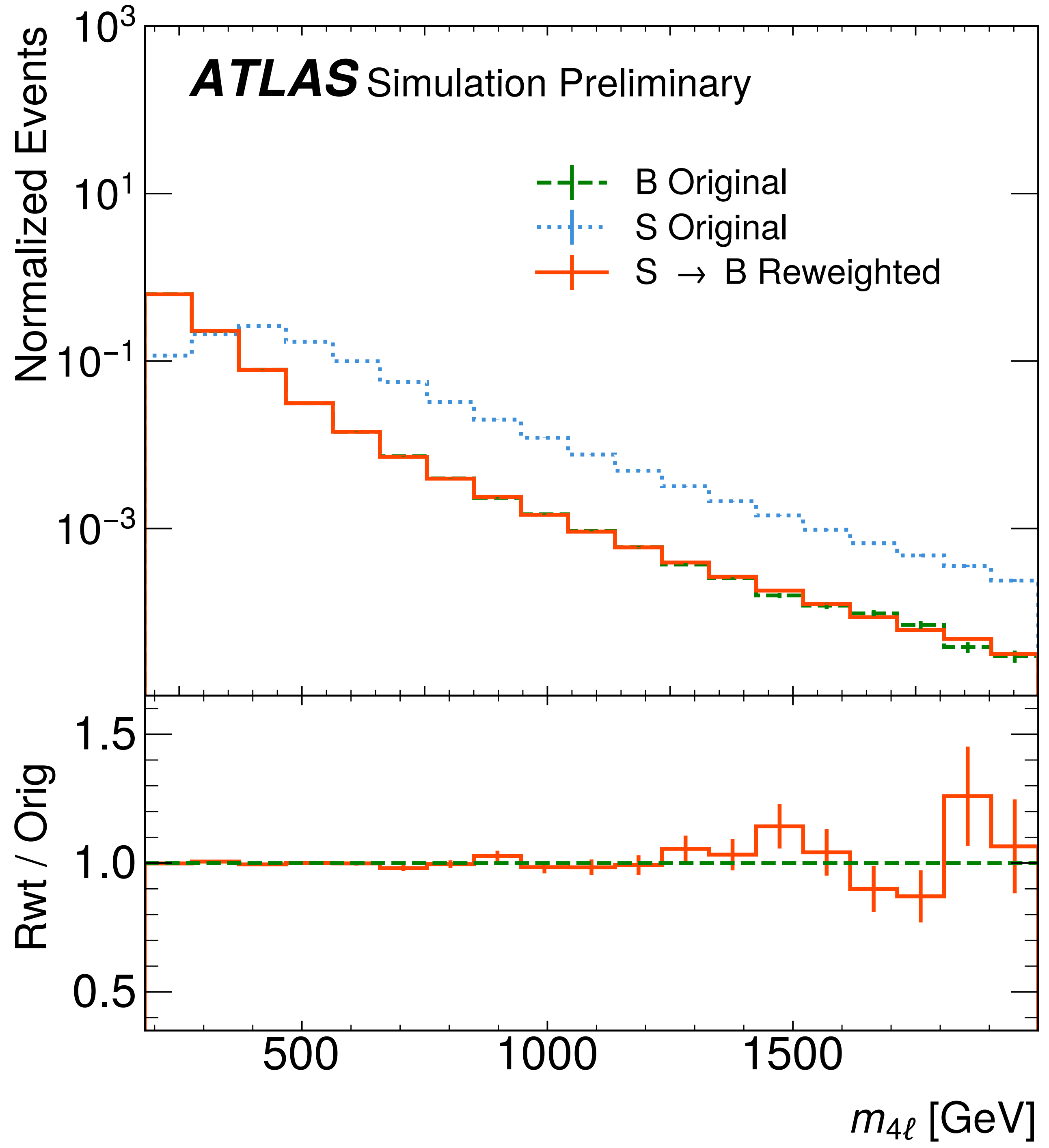
Which contains all the information required for the likelihood ratio:

$$\frac{p(x_i \,|\, \mu_1)}{p(x_i \,|\, ref)} = \frac{s(x_i)}{1 - s(x_i)}$$

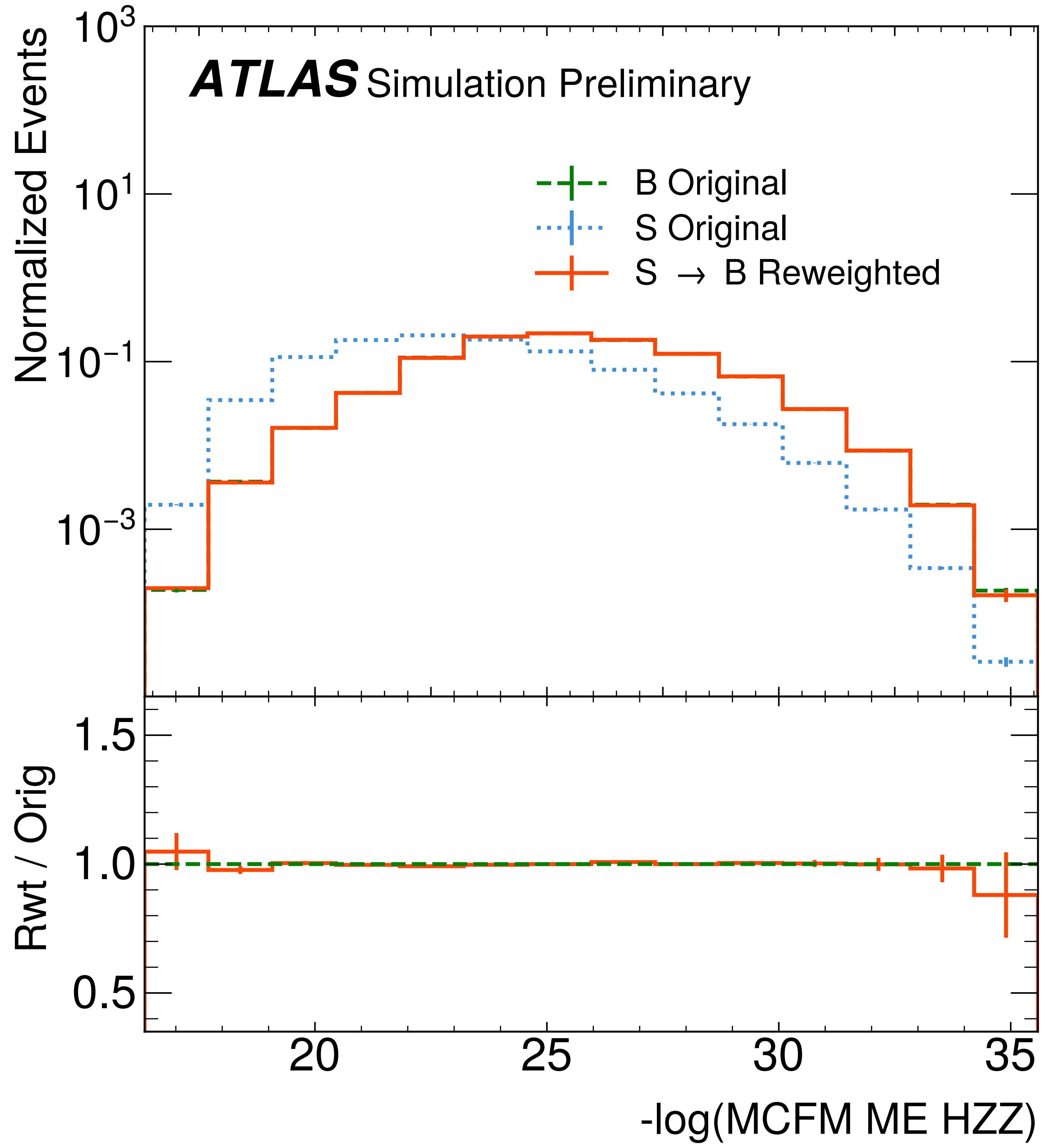
- ✴ Optimal statistic to test each value of μ
- ✴ We get the LR *per event* (unbinned)

Re-weight closures for B

Source
Target
RW



m_{4l}



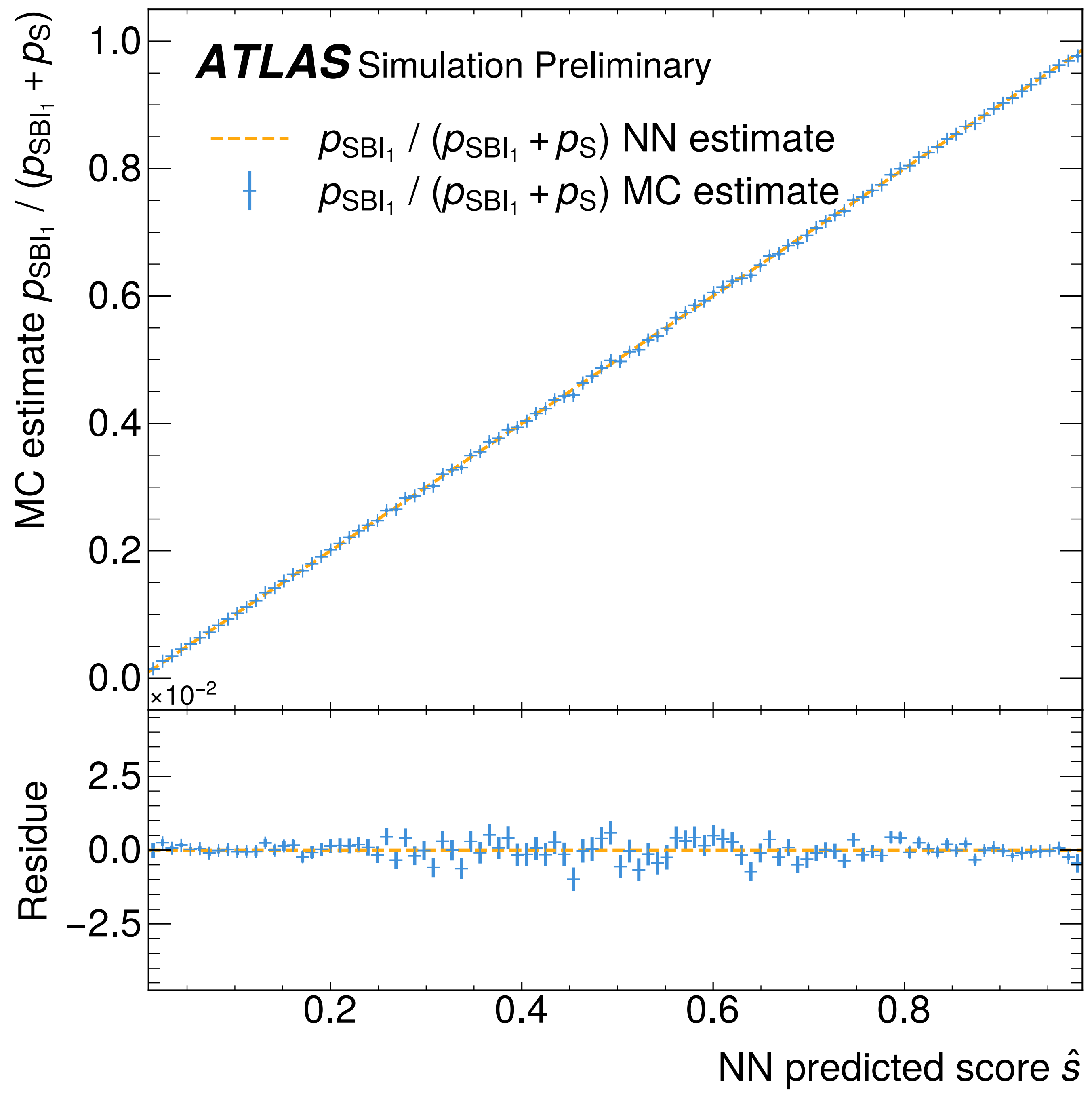
Matrix-Element-based Observable
(ggF from MCFM)

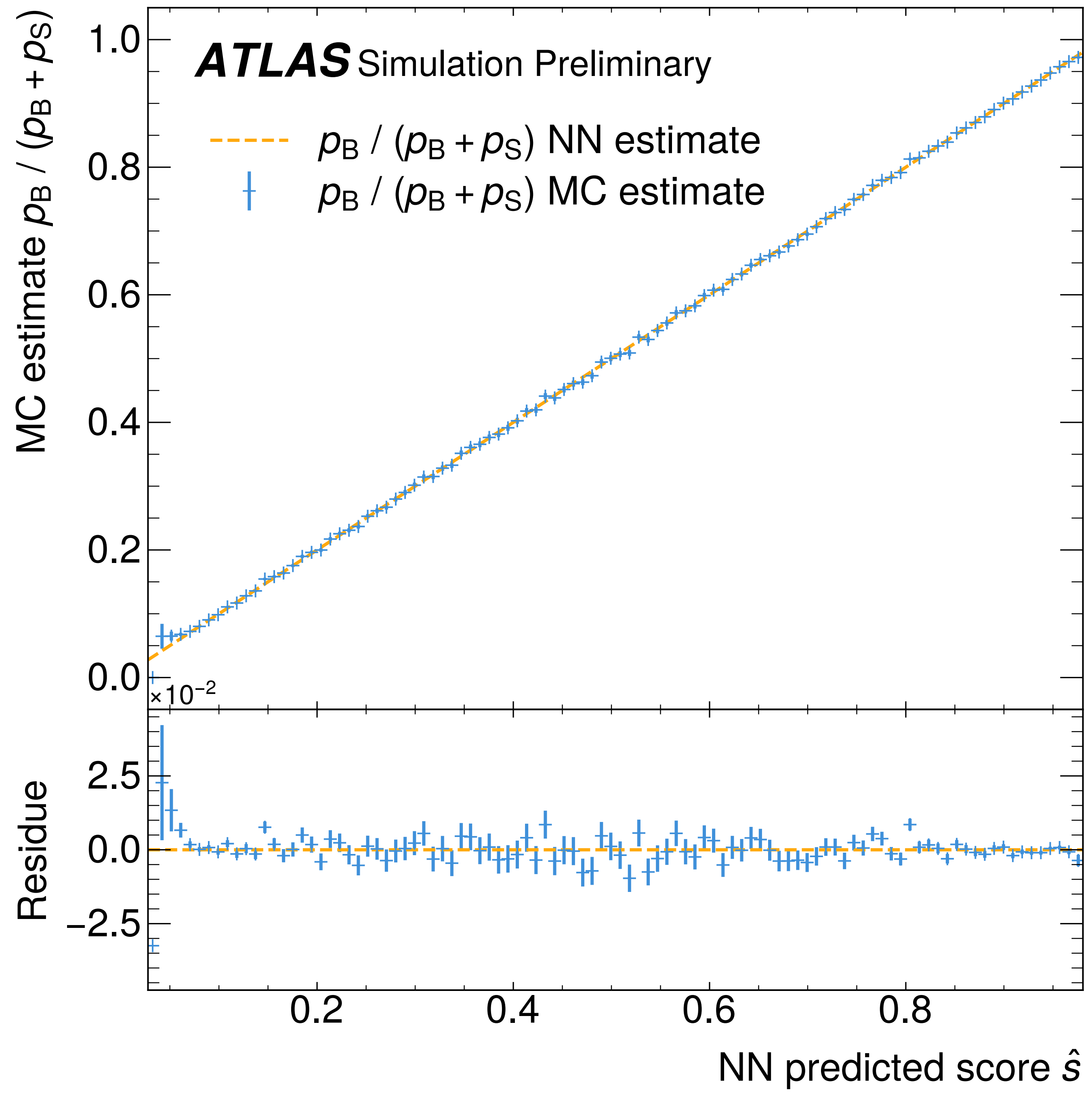
Calibration Curves

$$\frac{P_{SBI}}{P_{SBI} + P_{ref}}$$

$$\frac{P_B}{P_B + P_{ref}}$$

Binned estimate





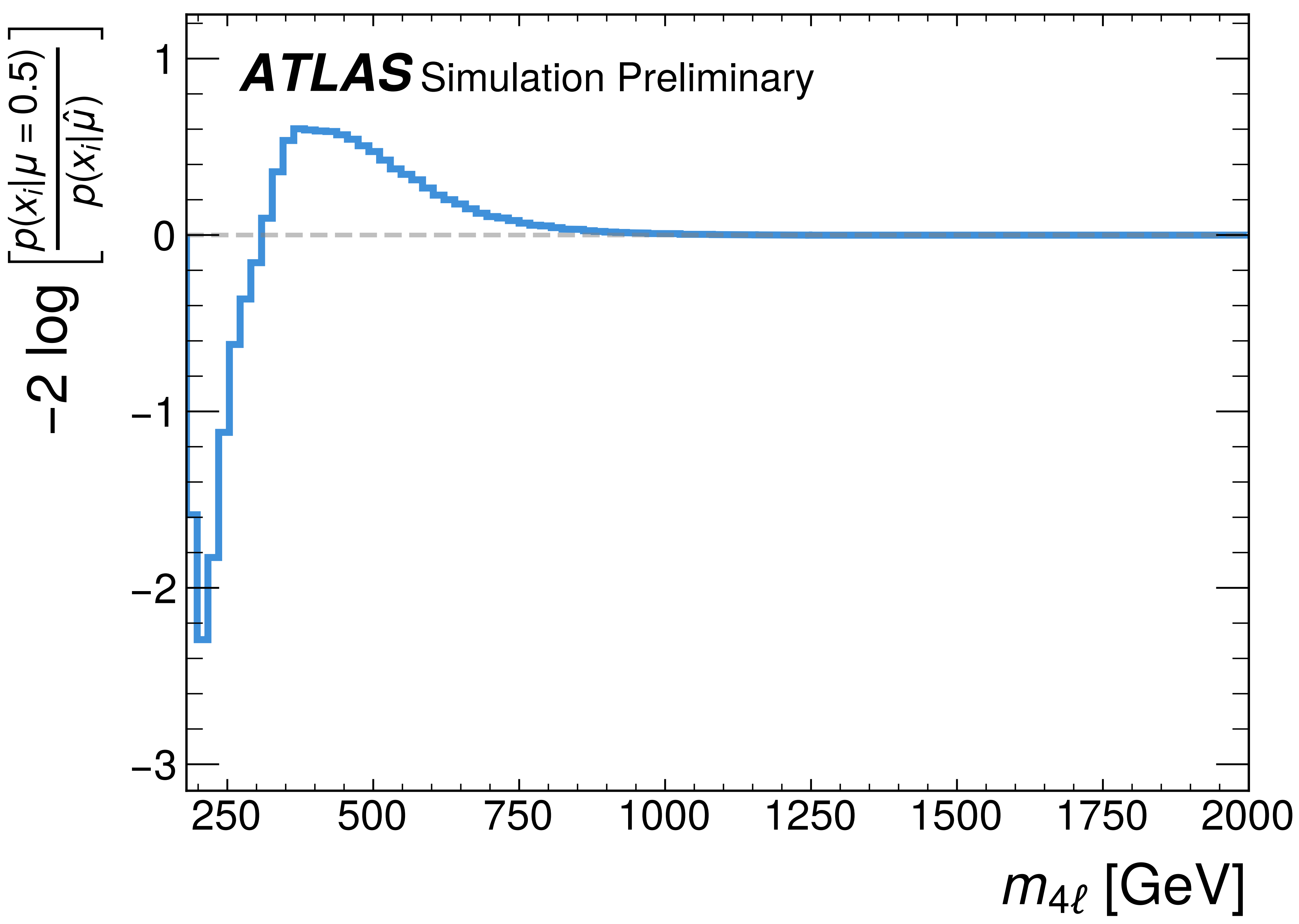
Ensemble prediction

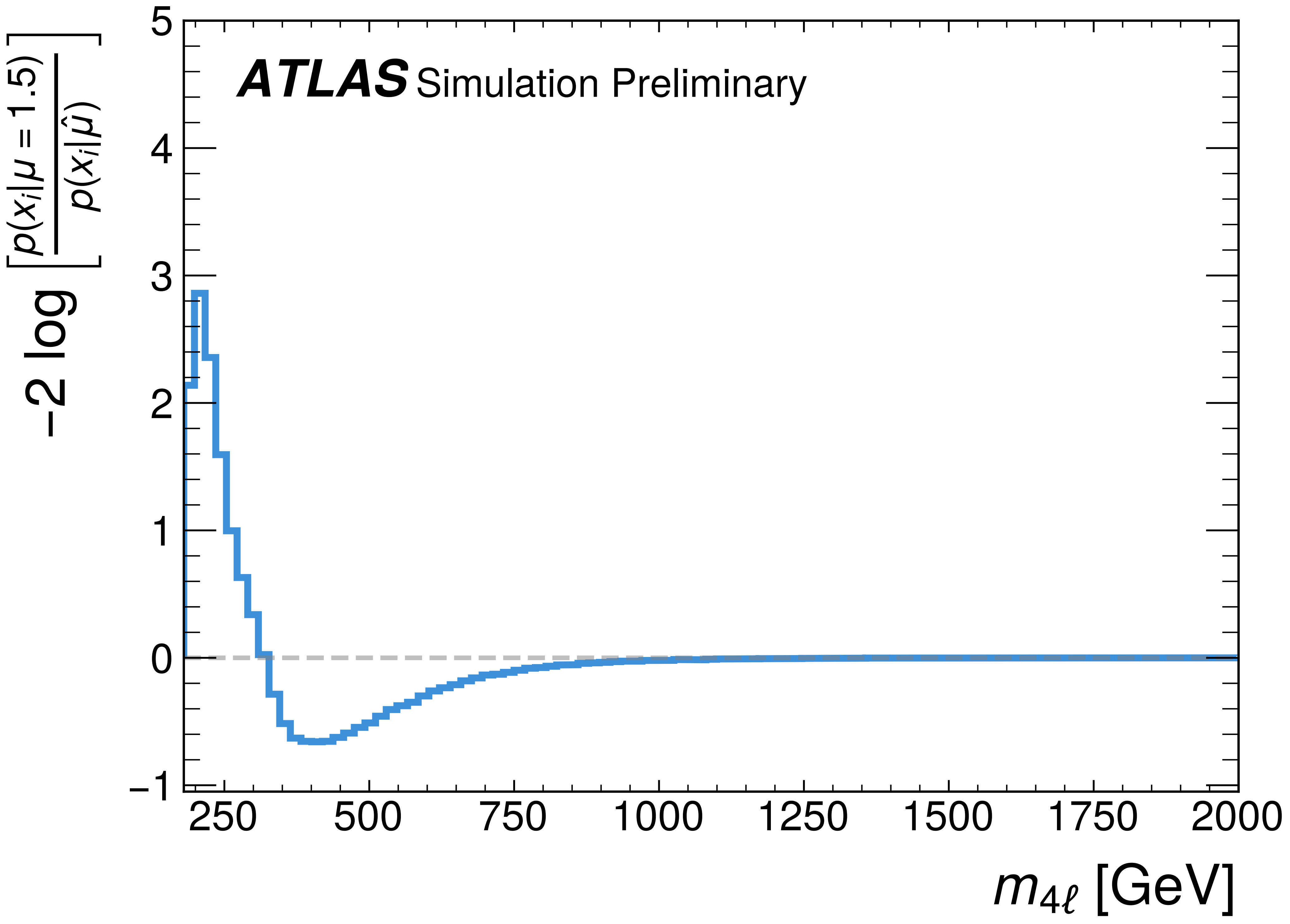
Ensemble prediction

Interpretability:
Which phase space favours one hypothesis over another?

$$-2 \cdot log \frac{P(x_i | \mu = 0.5)}{P(x_i | \mu = 1)}$$

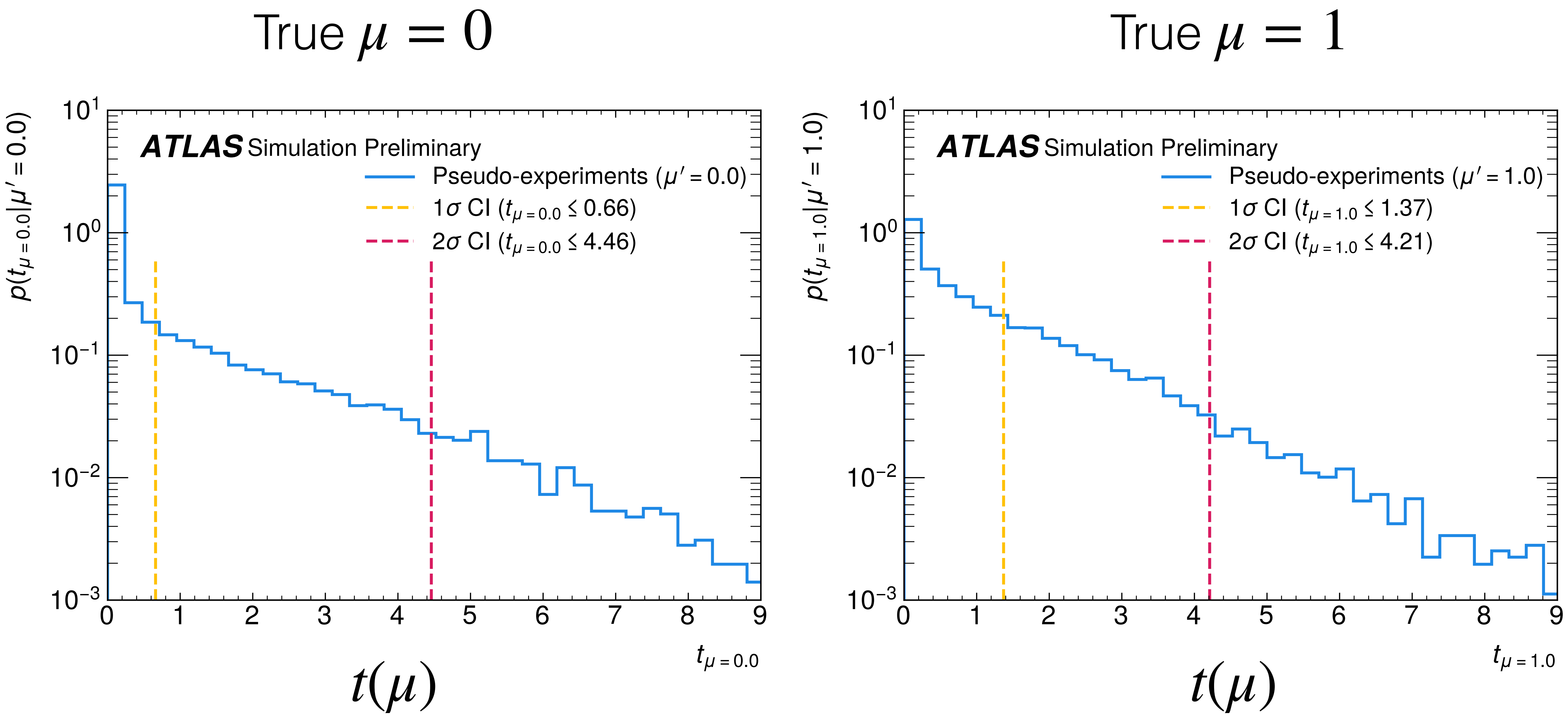
$$-2 \cdot log \frac{P(x_i | \mu = 1.5)}{P(x_i | \mu = 1)}$$





Neyman Construction

- To build confidence intervals, we need to 'invert the hypothesis test'
- Generate pseudo-experiments ('toys') and determine 1σ & 2σ CI as a function of parameter of interest



Negative Weighted Events

1. Start from a positive weighted reference sample instead
2. Re-weight to intended parameter point
3. Throw toys from this sample

$$w_i^{\text{rwt-ref}} \rightarrow w_i^{\text{Asimov}}(\mu) = \frac{\nu(\mu)}{\nu_{\text{rwt-ref}}} \cdot \frac{p(x_i\,|\,\mu)}{P_{\text{rwt-ref}}(x_i)} \cdot w_i^{\text{rwt-ref}}$$

Estimating the variance on mean: Bootstrapping

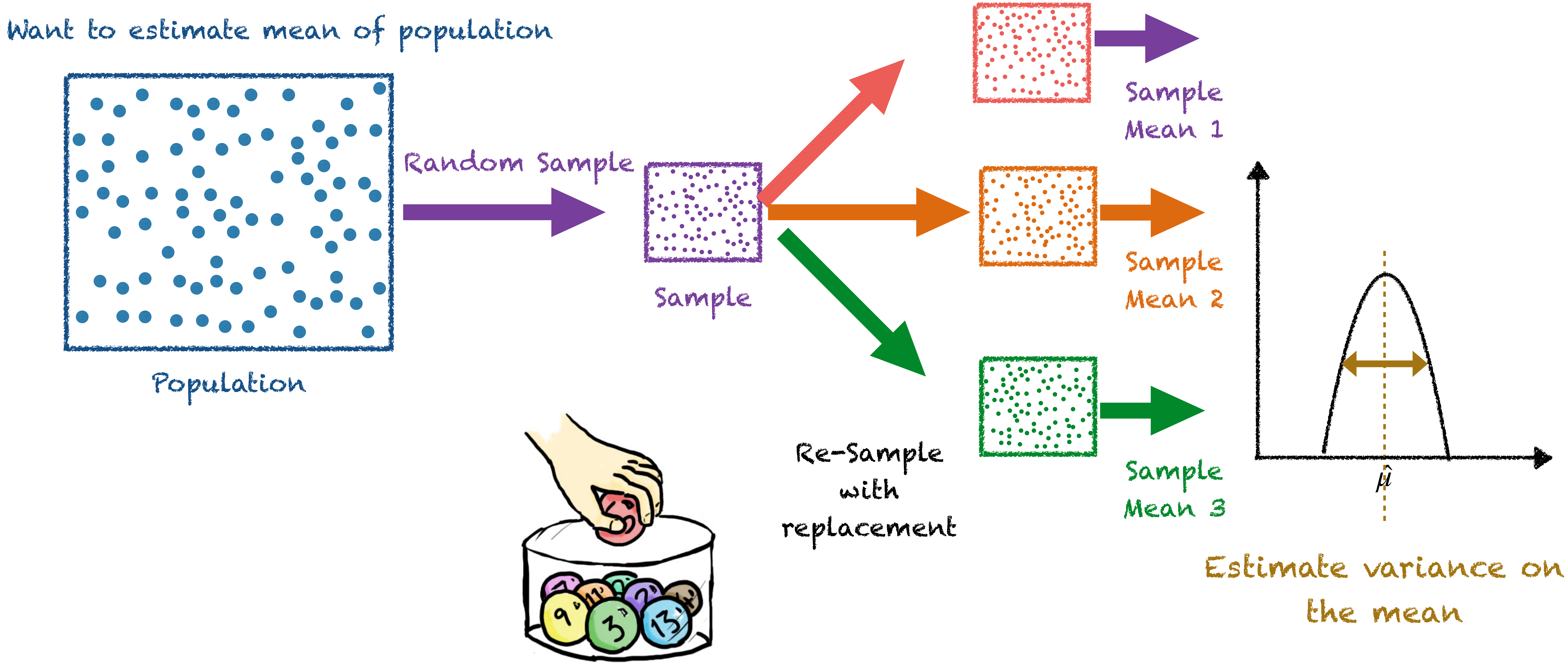


Image: Source

Quantifying uncertainty on estimated density ratio

$$w_i \rightarrow w_i \cdot Pois(1)$$

- Train an ensemble of networks, each on a Poisson fluctuated version of the training dataset

- Ensemble average used as final prediction, estimate the variance on mean from bootstrapped ensembles

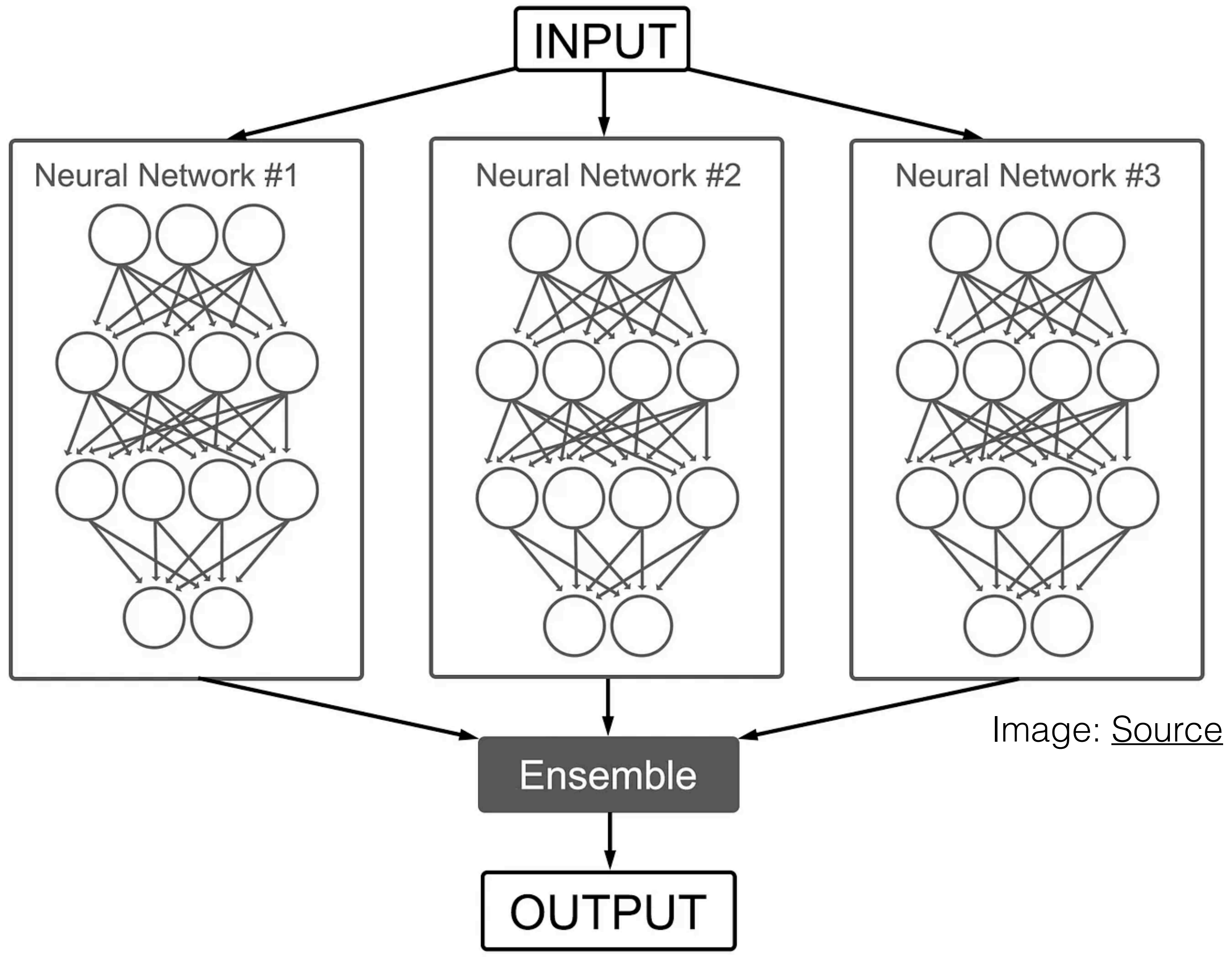


Image: Source

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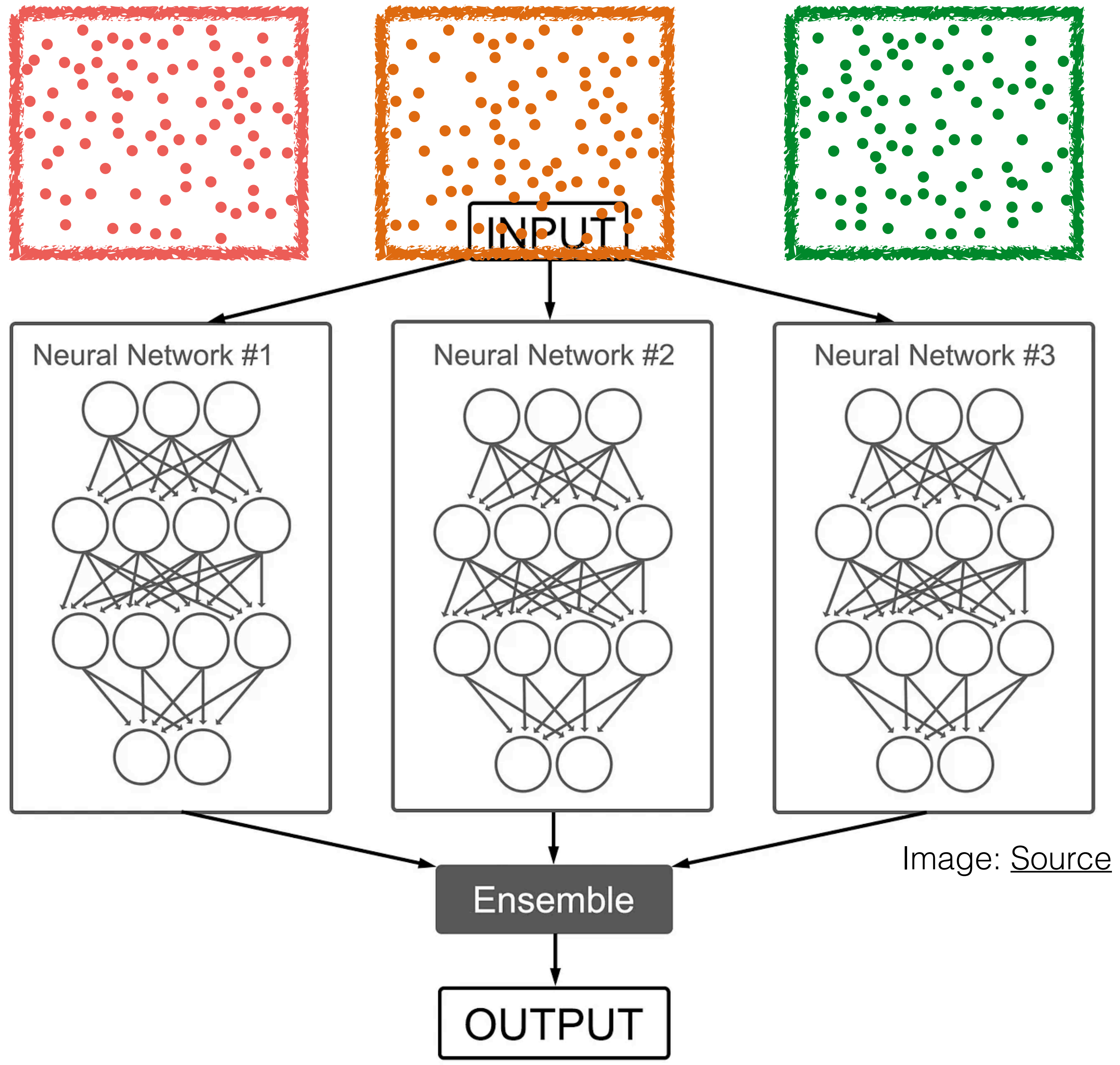

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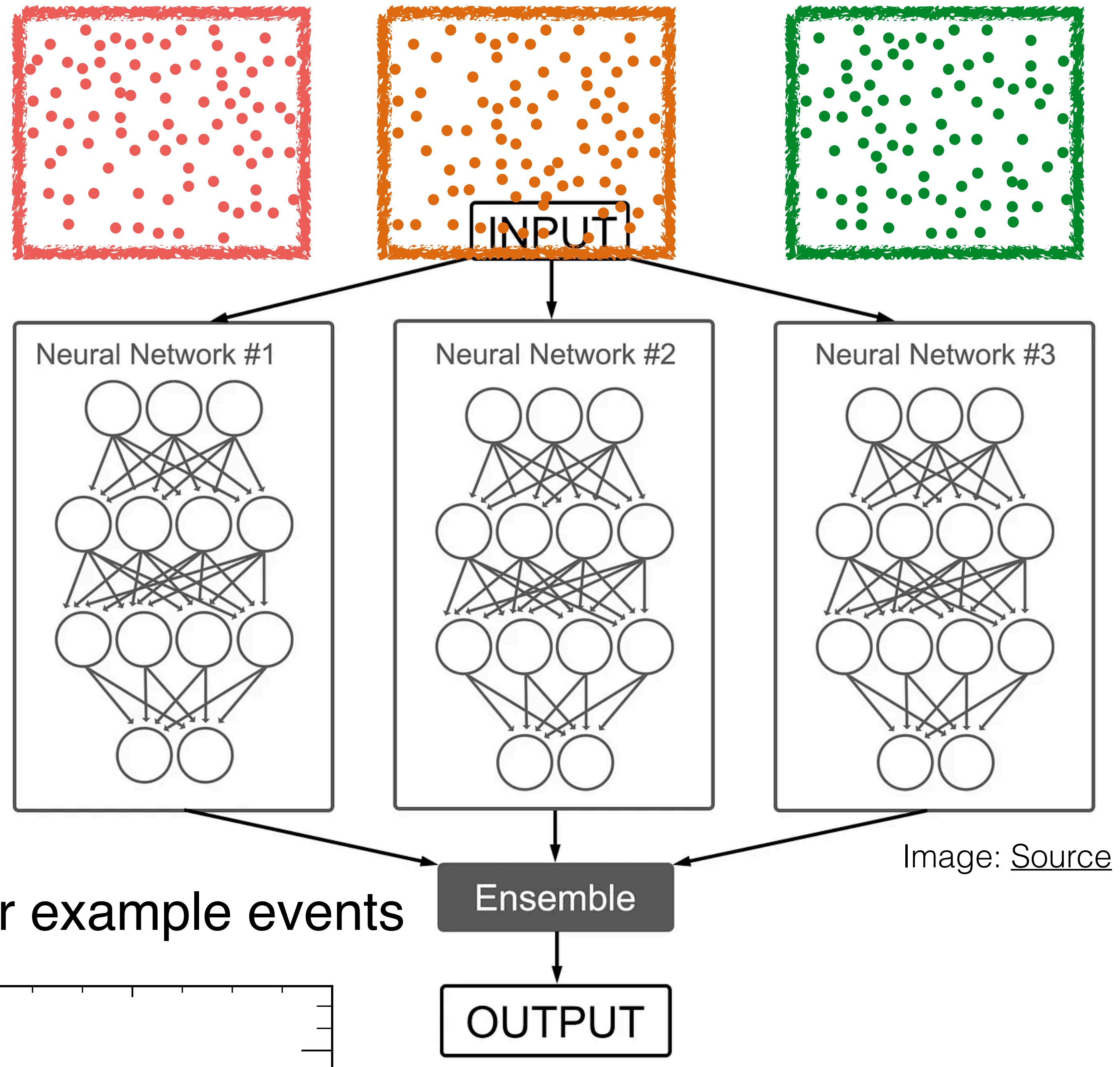
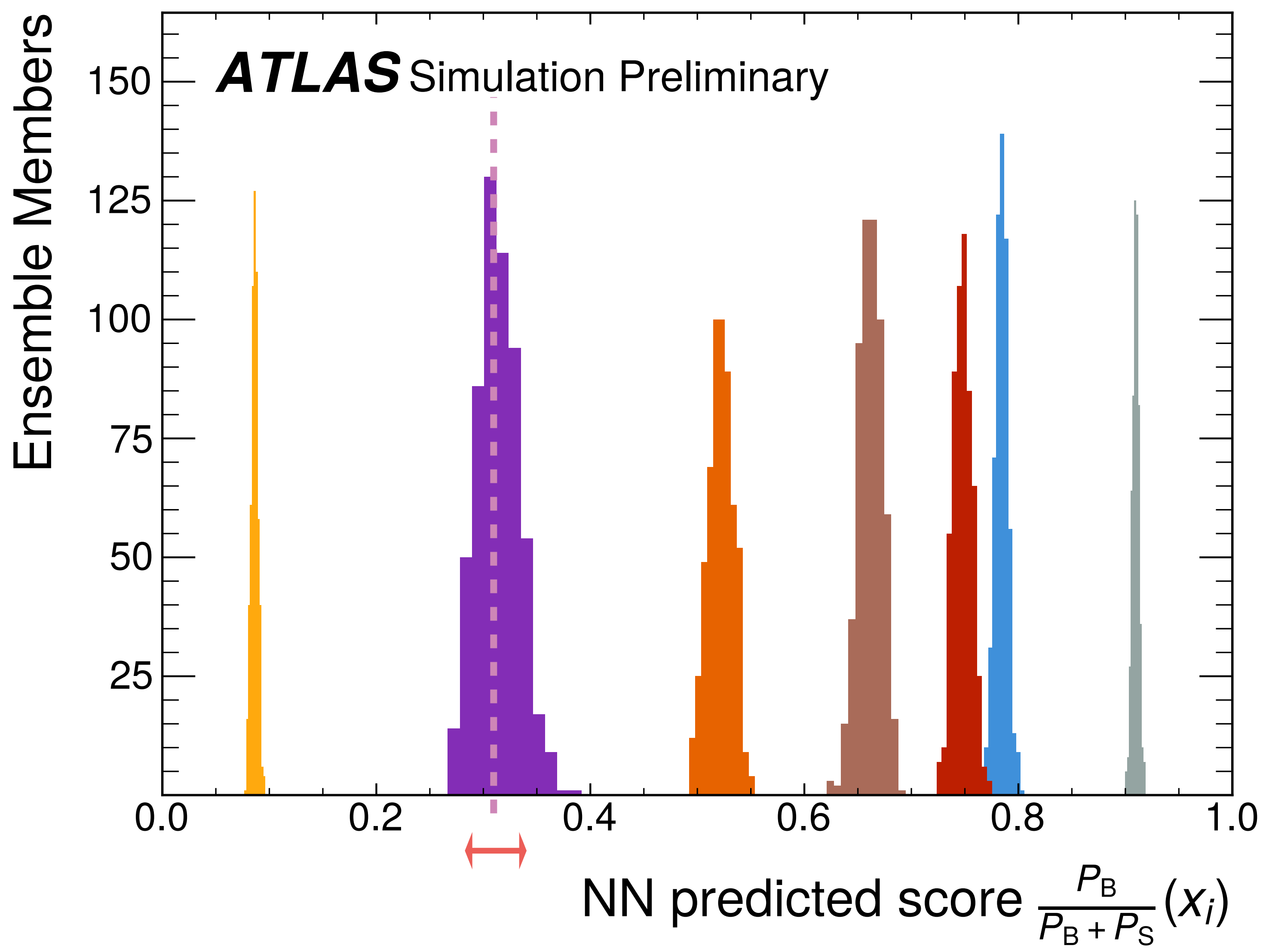


Image: Source

Distribution of NN predictions for example events



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- Propagate with spurious signal method

$$f_j(\mu) \rightarrow f_j(\mu + \alpha \cdot \Delta\hat{\mu}(\mu))$$

Constraint term: $Gauss(0,1)$

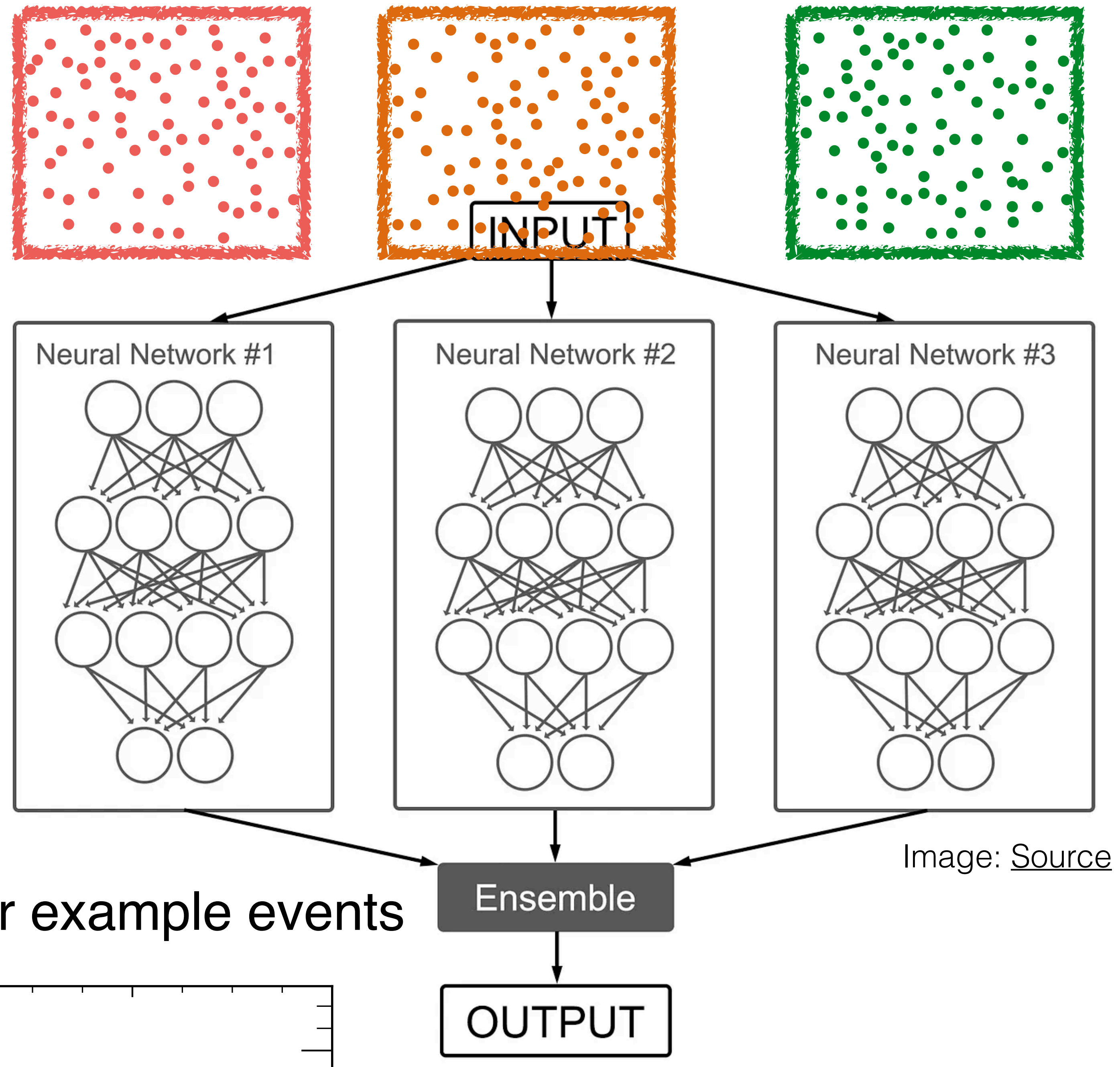

Image: Source

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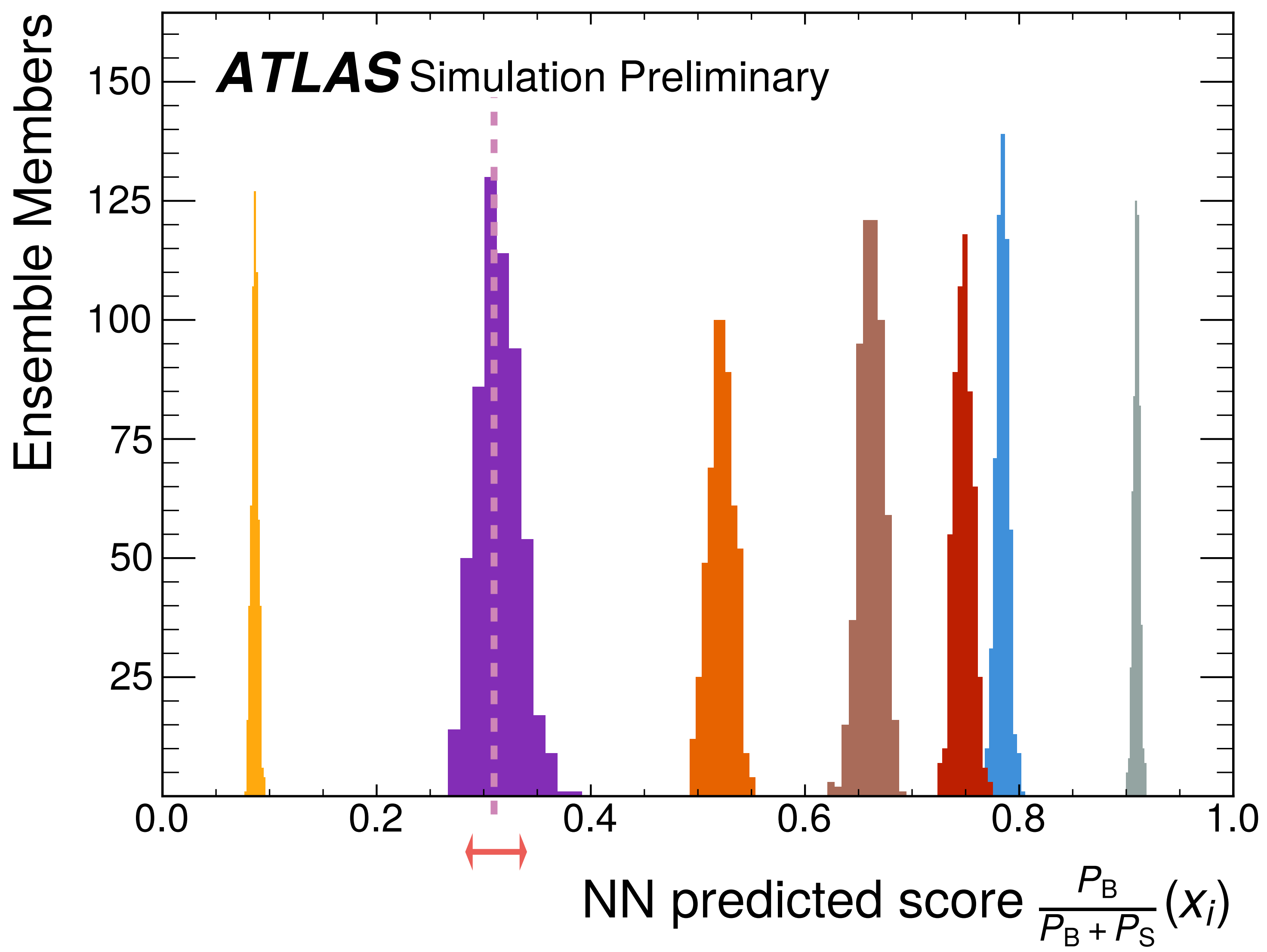

Simulated Samples

- PoI: Signal strength μ
- Simplified, unphysical dataset:
  - Processes: S: $gg \rightarrow H^* \rightarrow 4l$ & B: $gg \rightarrow ZZ \rightarrow 4l$, SBI: full process
  - No VBF processes or qqZZ background
  - Two systematics: ggF NLO K-factor uncertainty (shape + norm) & luminosity uncertainty (norm only)

Input variables

Variable	**Definition**
Production Kinematics	
$m_{4\ell}$	Four-lepton invariant mass
$p_T^{4\ell}$	Four-lepton transverse momentum
$\eta^{4\ell}$	Four-lepton pseudo-rapidity
Decay Kinematics	
m_{Z1}	Z_1 mass
m_{Z2}	Z_2 mass
$\cos\theta^*$	Higgs decay angle
$\cos\theta_1$	Z_1 decay angle
$\cos\theta_2$	Z_2 decay angle
ϕ	Angle between Z_1, Z_2 decay planes
ϕ_1	Z_1 decay plane angle

Combination with histogram analyses

$$\frac{L_{\text{comb}}(\mu, \alpha)}{L_{\text{ref}}} = \frac{L_{\text{full}}(\mu, \alpha)}{L_{\text{ref}}} \; L_{\text{hist}}(\mu, \alpha)$$

Calculating pulls and impacts in JAX

$$\lambda(\mu, \alpha) = -2\ln(L_{full}(\mu, \alpha)/L_{ref})$$

Hessian:

$$C_{nm} = \left[\frac{1}{2}\frac{\partial^2 \lambda}{\partial \alpha_n \partial \alpha_m}(\widehat{\mu}, \widehat{\alpha})\right]^{-1}$$

Pulls:

$$\frac{\widehat{\alpha}_k - \alpha_k^0}{\sqrt{C_{kk}}}.$$

Post-fit Impact:

$$\Gamma_k = \frac{\partial \widehat{\mu}}{\partial \alpha_k} \times \sqrt{C_{kk}}$$

$$= -\left[\frac{\partial^2 \lambda}{\partial^2 \mu}(\widehat{\mu}, \widehat{\alpha})\right]^{-1} \frac{\partial^2 \lambda}{\partial \mu \partial \alpha_k}(\widehat{\mu}, \widehat{\alpha}) \times \sqrt{C_{kk}},$$

Vertical interpolation

$$G_j(\alpha_k) = \begin{cases} \left(\frac{\nu_j(\alpha_k^+)}{\nu_j(\alpha_k^0)}\right)^{\alpha_k} & \alpha_k > 1 \\ 1 + \sum_{n=1}^{6} c_n \alpha_k^n & -1 \le \alpha_k \le 1 \\ \left(\frac{\nu_j(\alpha_k^-)}{\nu_j(\alpha_k^0)}\right)^{-\alpha_k} & \alpha_k < -1 \end{cases}$$

$$g_j(x_i, \alpha_k) = \begin{cases} \left(g_j(x_i, \alpha_k^+)\right)^{\alpha_k} & \alpha_k > 1 \\ 1 + \sum_{n=1}^{6} c_n \alpha_k^n & -1 \le \alpha_k \le 1 \\ \left(g_j(x_i, \alpha_k^-)\right)^{-\alpha_k} & \alpha_k < -1 \end{cases}$$

With some continuity requirements