An implementation of the parallelized General Triplet Track Fit for GPUs

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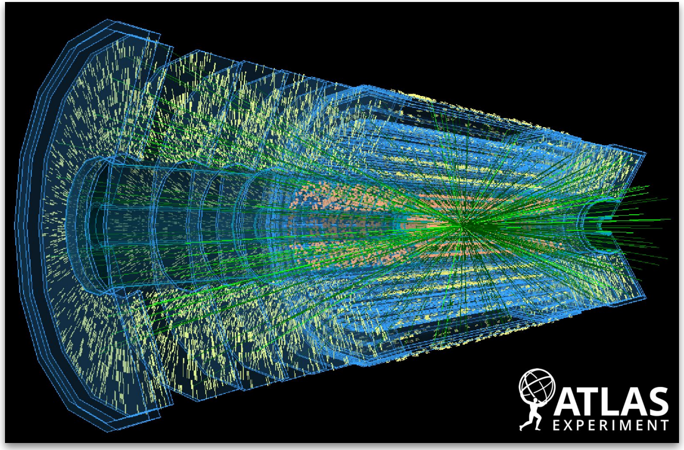
Challenges of track reconstruction

- Particle Physics experiments constantly need to deal with increasing particle rates - e.g. with High Luminosity upgrade of Large Hadron Collider.
- Number of hits in tracking detectors increase with the number of particles.
- Track reconstruction algorithms face an even steeper challenge due to rapidly increasing number of combinations between hits.
- Opportunity for **innovative algorithms** and **hardware acceleration**.



A simulated High Luminosity LHC event in the Inner Tracker (ITk) of the ATLAS detector with a pile-up of 140 simultaneous proton-proton collisions per bunch crossing. Source: ATLAS-PHOTO-2023-042

General Triplet Track Fit

- Generalized from multiple Coulomb scattering (MS)-only triplet fit [1] to include hit uncertainties [2].
- Fit quality for a track:

$$\chi^2(p, \vec{\boldsymbol{\delta}}) = \sum_{scatt.\ k=0}^{n_{scatt.}-1} \left(\frac{\Delta\Phi^2_{\mathrm{MS},k}}{\sigma^2_{\Phi_{\mathrm{MS},k}}} + \frac{\Delta\Theta^2_{\mathrm{MS},k}}{\sigma^2_{\Theta_{\mathrm{MS},k}}} \right) + \sum_{hit\ j=0}^{n_{hit}-1} \vec{\delta}_j^{\top} V_j^{-1} \vec{\delta}_j$$

$$\vec{\delta}_j = \vec{r}_{fit,j} - \vec{r}_{meas,j}$$

- The fit of the total momentum p & hit position shifts $\vec{\delta}_j$ is done in two steps: **local processing of hit triplets** & a **global track fit**.

[1]: Berger, Kozlinskiy, Kiehn, & Schöning, *A new three-dimensional track fit with multiple scattering* 2017
[2]: Schöning, *A General Track Fit Based on Triplets* 2024

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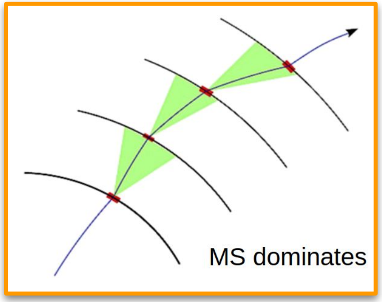

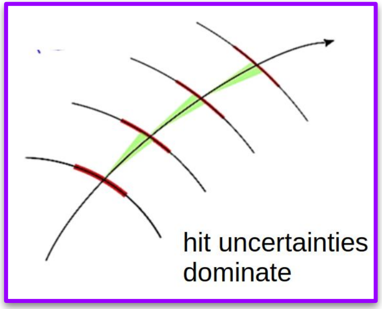

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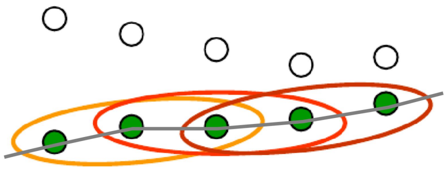
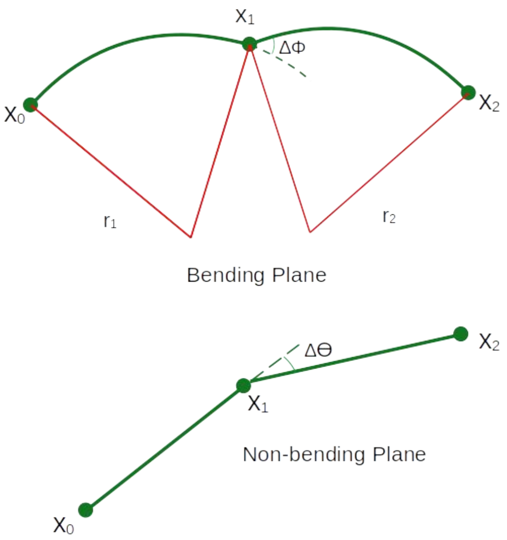
Hit triplet representation

- A track can be broken up into several triplets of hits, two hits shared between consecutive triplets
  - Exposes a new level of **parallelizability**.
- Motivating the triplet representation:
  - At least three hits required to measure track parameters.
  - Individual triplets can be fitted for momentum and hit positions.
  - Kinematics overconstrained ➔ filter bad hit combinations with fit quality.
  - Can be used to seed conventional track finding algorithms.



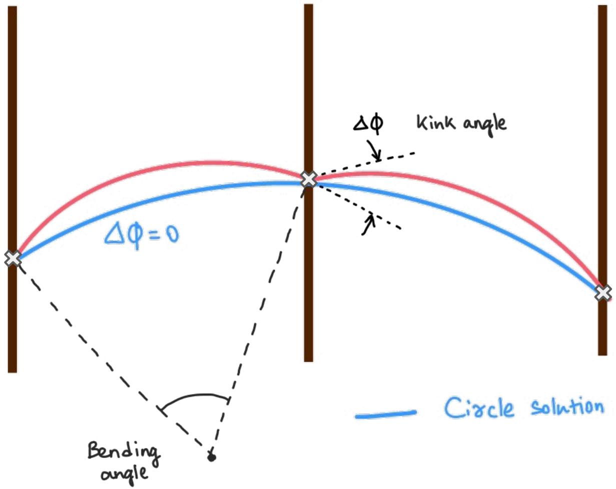


A triplet of hits projected onto bending and non-bending planes, showing the triplet kink angles ($\Delta\Phi$, $\Delta\Theta$).

Local processing of triplets

- Triplet kink angles can be obtained from bending angles and hit positions.
- Bending angles are given by transcendental equations ➔ no algebraic solution…
- Using a reference solution - connecting three hits with a circle in transverse plane - linearized expressions of the kink angle as a function of track curvature is obtained.

Bending plane



Linearized kink angle in bending plane:

$$\Delta\Phi = \Phi_0 + \rho_\Phi c_{3D} \qquad c_{3D} = \frac{|q|B}{p}$$

Kink angle for vanishing curvature ($c_{3D} = 0$)

Dependance of kink angle on curvature

…similarly in the non-bending plane

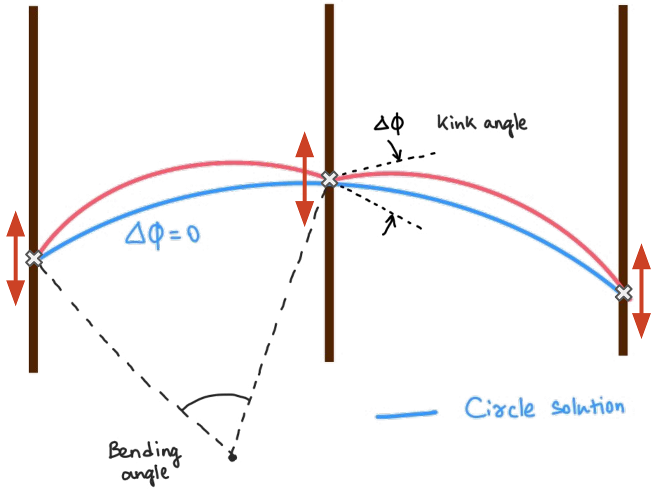
Local processing of triplets

Contributions to kink angles from position uncertainties ($\Delta\Theta_{hit}$, $\Delta\Phi_{hit}$) taken into account using directional derivatives with respect to hit positions.

$$\Delta\Theta_{\mathrm{MS}} = \Delta\Theta - \Delta\Theta_{\mathrm{hit}};\ \Delta\Phi_{\mathrm{MS}} = \Delta\Phi - \Delta\Phi_{\mathrm{hit}}$$

with apparent kinks $\Delta\Theta,\ \Delta\Phi$

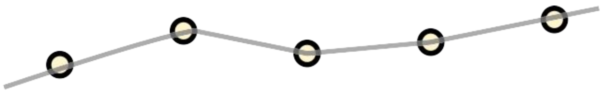
Bending plane



…similarly in the non-bending plane

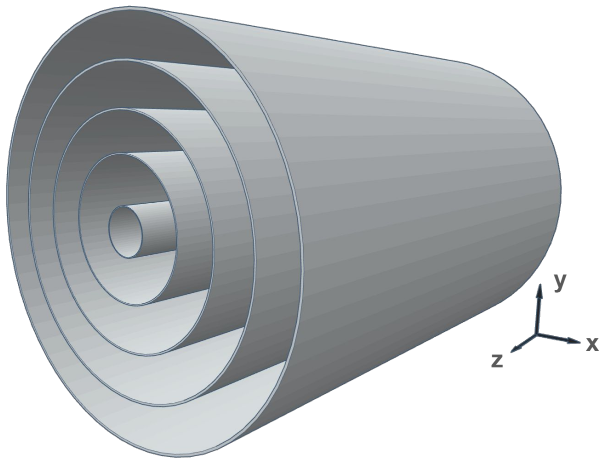
Global track fit

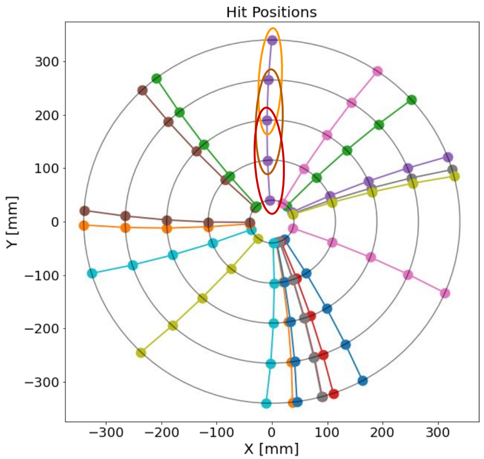
- Linearized triplet kink angles & contribution of position uncertainties are plugged back into track χ^2.
- Minimising χ^2 results in a system of linear equations ➔ Requires a matrix inversion.
  - Diagonal in case of dominating MS uncertainties.
- Global fit is detector independent since local processing of triplets absorb all detector dependencies (geometry, magnetic field).



Standalone testing - simulation of triplets

- Simplified simulation of tracks developed for standalone testing.
    - Offers full control.
- Cylindrical barrel tracking detector with 5 equally-spaced layers, axis parallel to magnetic field (uniform).
- Multiple Coulomb scattering at the layers and finite resolution of measurements simulated.

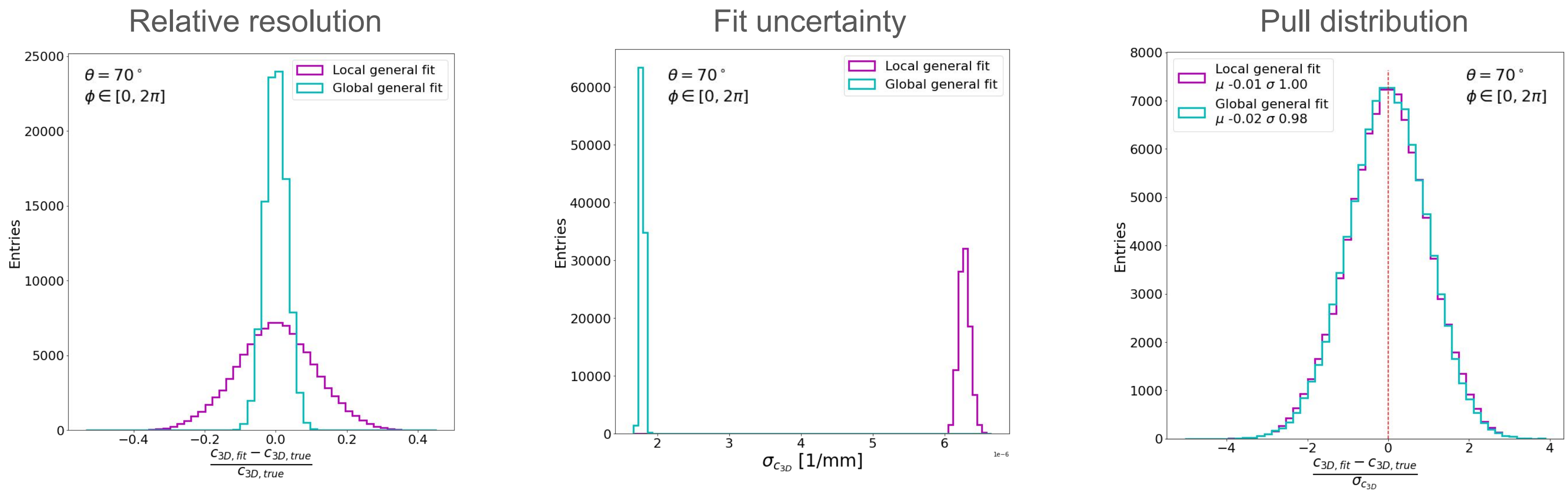




Standalone testing - implementation on CPU

$$c_{3D} = \frac{|q|B}{p}$$

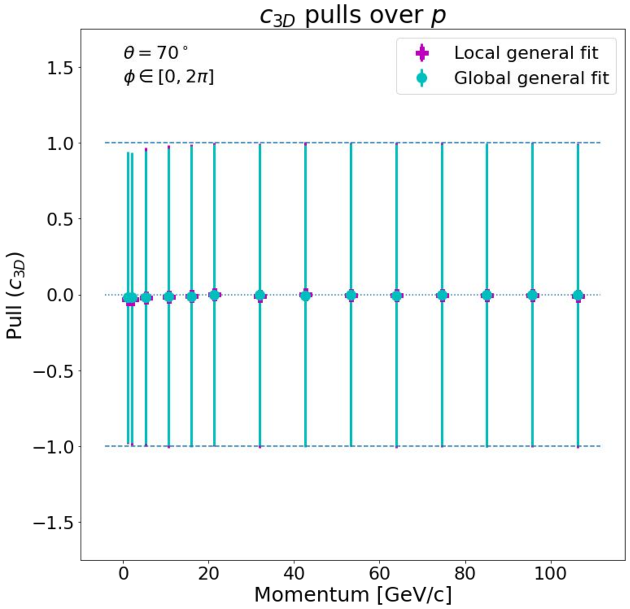
p_T = 10 GeV



Local fit = fit of an individual triplet (1 per track)

Implementation of the fit is working and uncertainties are correctly handled.

Standalone testing - implementation on CPU



The means and standard deviations of curvature pull distributions are shown at several momenta points.

➔ Correct estimation of curvatures and uncertainties over a large range of track momenta.

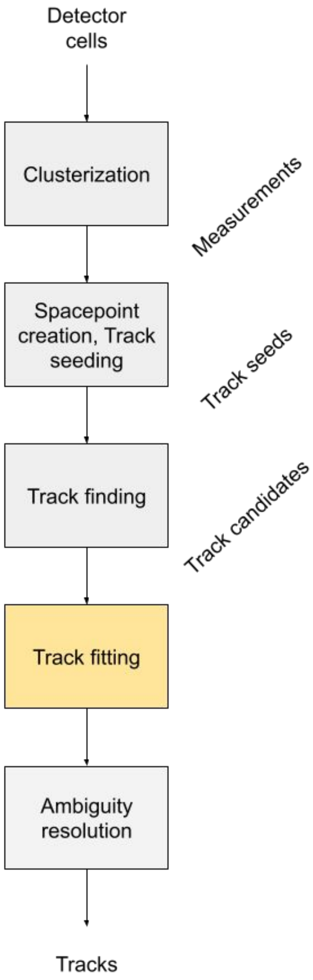
Track reconstruction on parallel architectures

- A Common Tracking Software (ACTS) [1, 2] is an experiment-independent toolkit for track reconstruction.
- `traccc` [3, 4] is a demonstrator for track reconstruction on Graphics Processing Units (GPUs), that grew out of ACTS:
    - Provides a chain of algorithms going from hits to fitted tracks, which can be run on CPUs or GPUs.
- We decided to use `traccc` since it provides a comparatively easy route to test the parallelizability and physics performance of the algorithm in a realistic detector as part of a complete tracking chain.

Talk by Beomki Yeo

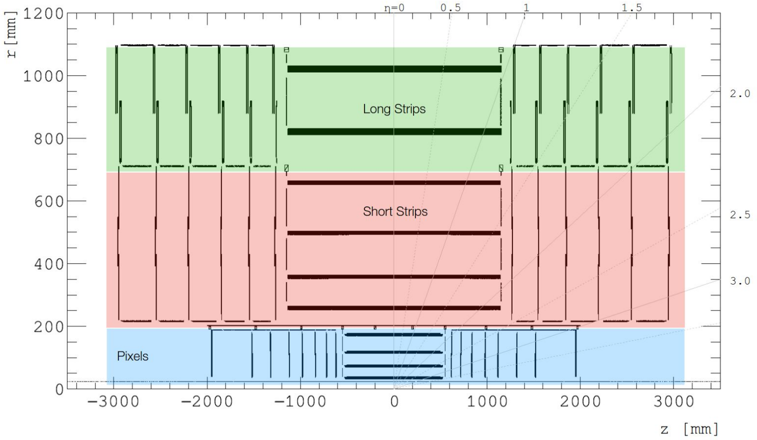
Challenges, experiences and current status

- Development of a completely new algorithm in `traccc` comes with a few challenges:
  - `traccc` makes extensive use of compile-time polymorphism (templates) to prevent dynamic memory allocations ➔ different programming paradigm.
  - Track fitting requires accessing the geometry, magnetic field etc. - handled and abstracted away through dedicated libraries [1, 2, 3].
  - Additionally adapting to Event Data Models (EDM), setting up interfaces.
  - Being a fast evolving project adds another layer of complexity.
- Many of these were overcome and an implementation of the General Triplet Track Fit as a track fitting algorithm in `traccc` has been done
  - Many thanks here to `traccc`/ACTS developers for their help!



`traccc` implementation

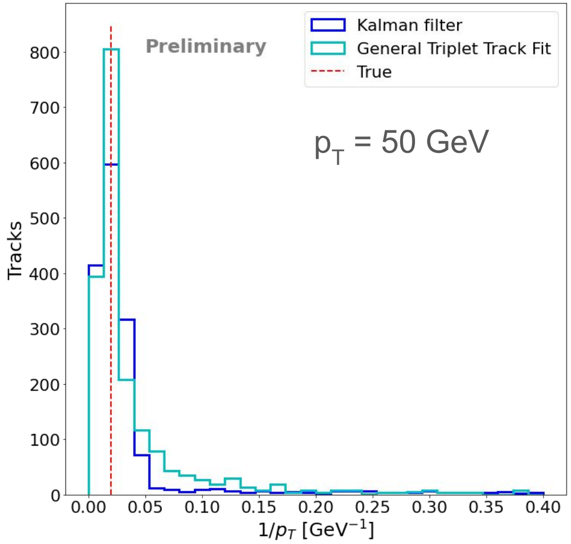
- Using the **O**pen**D**ata**D**etector [1]: a template High Luminosity-Large Hadron Collider type particle detector with (among other things) pixel and strip tracking systems.
- Homogeneous magnetic field along z-axis.
- Assumptions made:
    - Constant material budget per layer.



Geant4 hit locations in the ODD.
source: Salzburger, A. CHEP 2023

Preliminary verification of fitting performance

The `traccc` implementation of the fit is tested using muon tracks (uniformly distributed in η & ϕ).



Distribution of fitted track momenta from General Triplet Track Fit are comparable to Kalman filter. Dissimilarities are expected to be the result of detector material assumption.

Summary

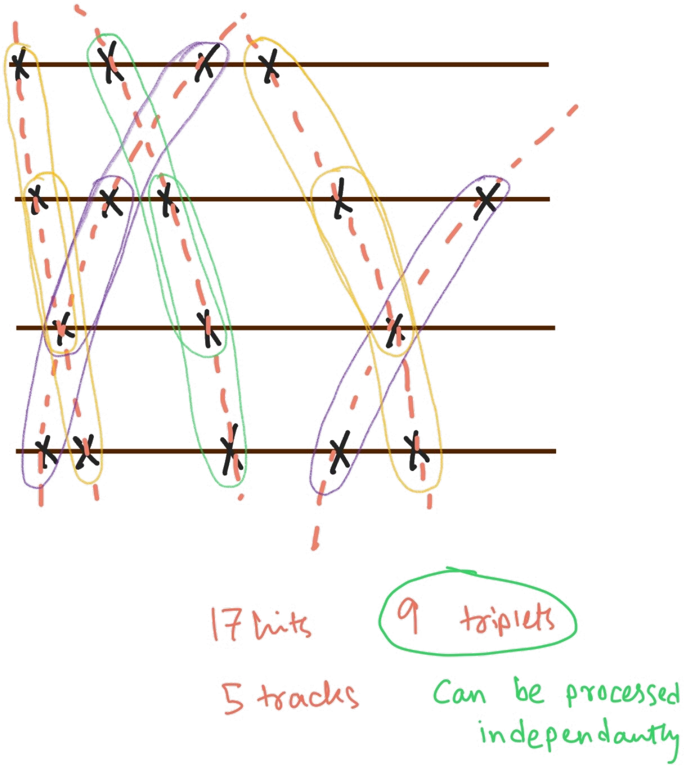
- General Triplet Track Fit introduced as a parallelizable algorithm for track fitting.
- Standalone testing showed correct estimation of curvatures and uncertainties.
- An implementation of the fit has been done in `traccc` - the GPU tracking demonstrator of ACTS, and the experience was reported.
- Preliminary fitting performance of the algorithm in `traccc` with the OpenDataDetector was presented.

Outlook

- Running the fit on GPUs in `traccc` taking advantage of its parallelizability.
- Optimization studies of the algorithm implementation.

Backup

Parallelization from Triplets



MS-only fit implemented on GPUs for Mu3e [1]

[1]: Mu3e Collaboration, *Technical design of the phase-I Mu3e experiment* 2021

Global fit χ^2

$$\chi^2(c_{3D};\vec{\boldsymbol{\delta}}) = \sum_{\text{triplet } j}^{n_{\text{hit}}-2} \frac{1}{\sigma^2_{\text{MS},j}} \left(\tilde{\Theta}_j + \rho_\Theta\, c_{3D} - \Delta\Theta_{\text{hit},j}(\vec{\boldsymbol{\delta}})\right)^2 + \sum_{\text{triplet } j}^{n_{\text{hit}}-2} \frac{\sin^2\hat{\vartheta}_j}{\sigma^2_{\text{MS},j}} \left(\tilde{\Phi}_j + \rho_\Phi\, c_{3D} - \Delta\Phi_{\text{hit},j}(\vec{\boldsymbol{\delta}})\right)^2 + \sum_{\text{hit } k=0}^{n_{\text{hit}}-1} \frac{{\delta_{k_i}}^2}{{\sigma_{k_i}}^2}$$

Global fit χ^2 :

Minimization involves a matrix inversion.

$$\chi^2(c_{3D};\vec{\boldsymbol{\delta}}) = \left(\boldsymbol{\Psi} + \boldsymbol{\rho}\, c_{3D} - \vec{\boldsymbol{H}}\vec{\boldsymbol{\delta}}\right)^T \boldsymbol{D}_{\text{MS}} \left(\boldsymbol{\Psi} + \boldsymbol{\rho}\, c_{3D} - \vec{\boldsymbol{H}}\vec{\boldsymbol{\delta}}\right) + \vec{\boldsymbol{\delta}}^T\, \vec{\vec{\boldsymbol{D}}}_{\text{hit}}\, \vec{\boldsymbol{\delta}}$$

Triplet parameters from linearization

Hit position derivatives of kink angles

Multiple scattering uncertainty matrix

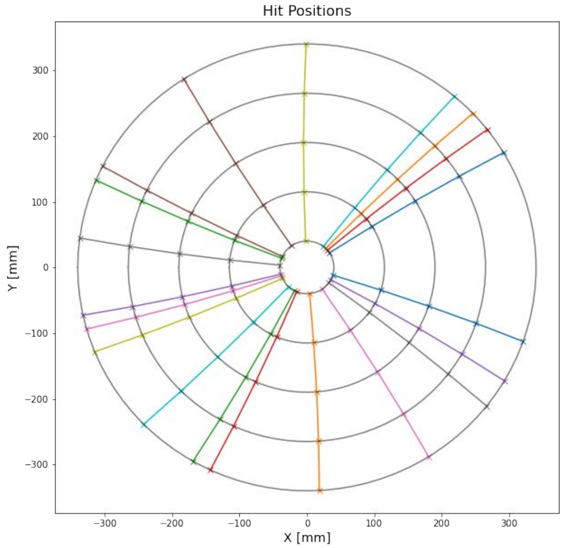
Hit position uncertainty matrix

Hit position shifts

More details on toy simulation

- Particles propagated through uniform, solenoidal B-field.
- Detector with 5 equally spaced barrel layers (axis parallel to B-field).

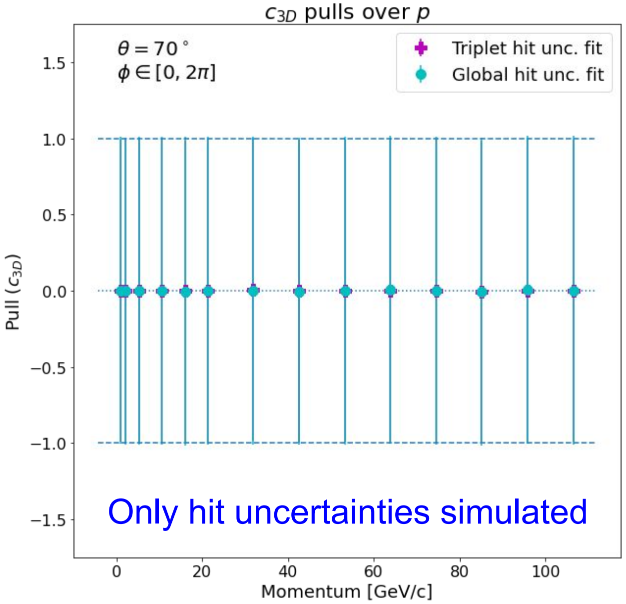
Generated particles	Position	(0,0,0) - beamspot of size 45 mm along z
	Azimuthal angle (Φ)	Uniform in [0, 2π]
	Polar angle (Θ)	70° (η= 0.36)
	Charge	+1
Simulated uncertainties	Multiple scattering	x/X_0 = 2% (per layer)
	Hit uncertainties	12 µm in sensor transverse directions

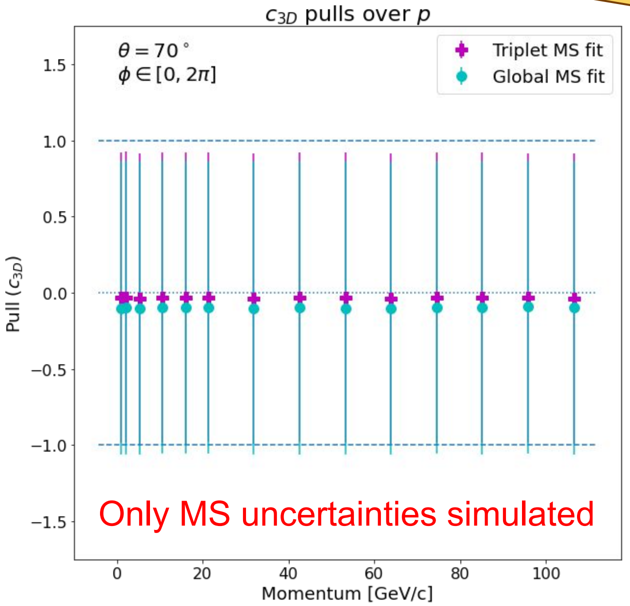


In bending plane

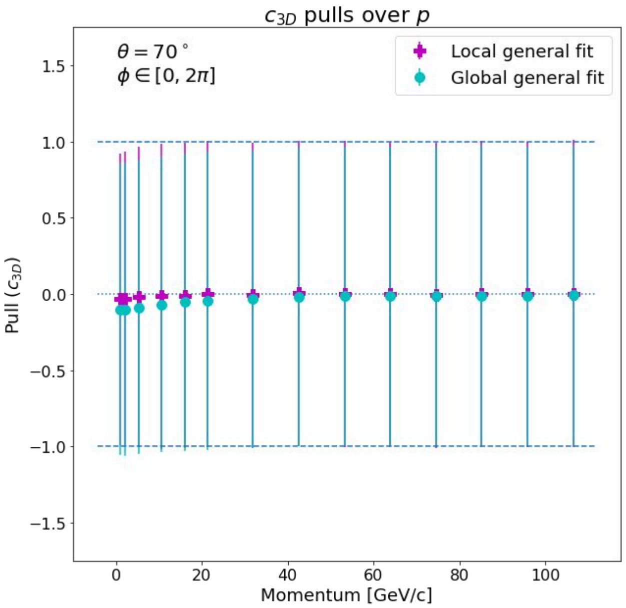
Curvature pulls with either one of uncertainties

The General Triplet Track Fit can be configured to only consider one of the uncertainties when those are dominating.





Standalone testing - curvature pull biases

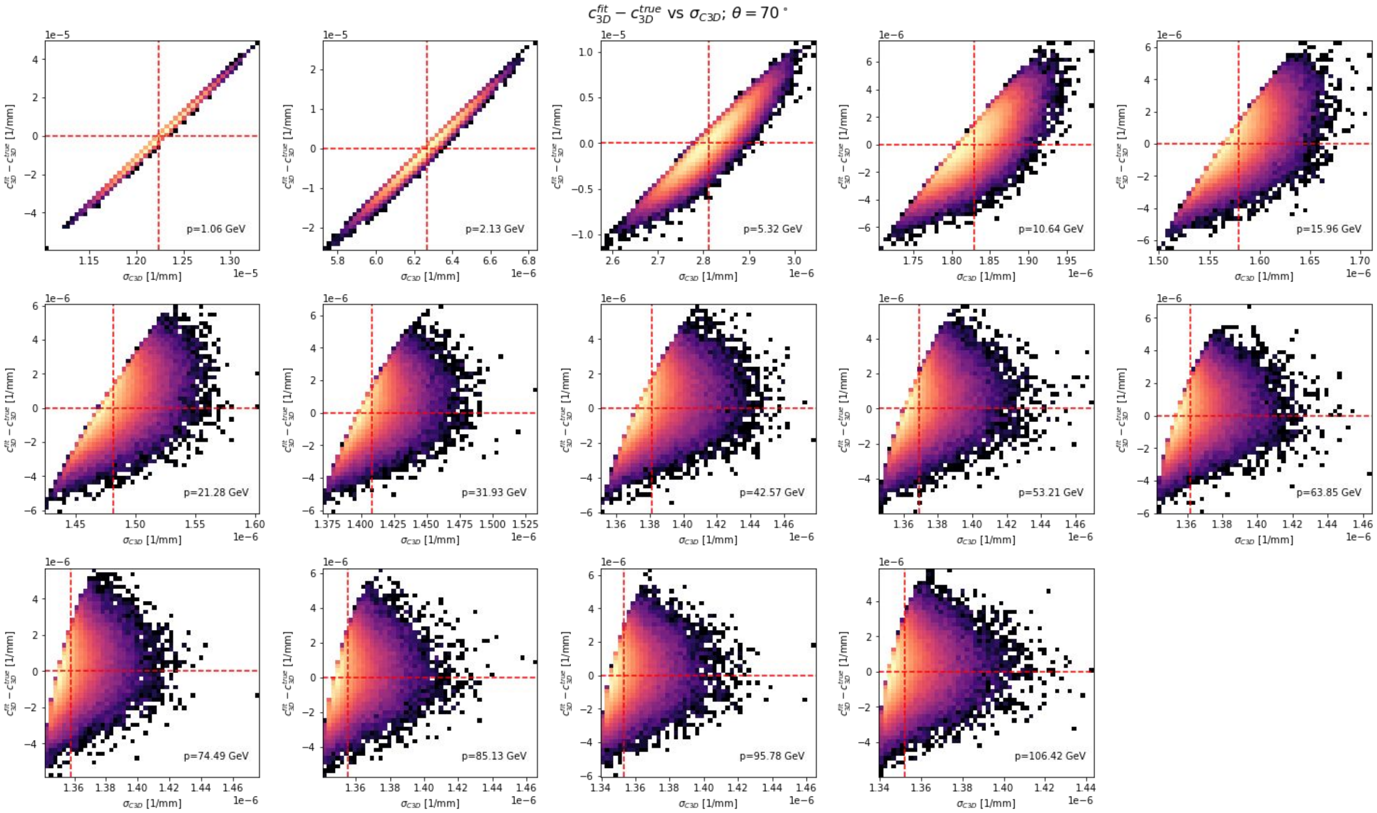


Correct estimation of curvatures and uncertainties over a large momentum range.

Dominating MS effects at low momenta → correlation between curvatures and uncertainties → bias in pull distributions.

➤ Well understood and is corrected using the best estimate of the MS uncertainties - using the curvature from the global fit.

Curvature pull biases



c_{3D} and $\sigma(c_{3D})$ strongly correlated at low momenta (dominating MS uncertainties)

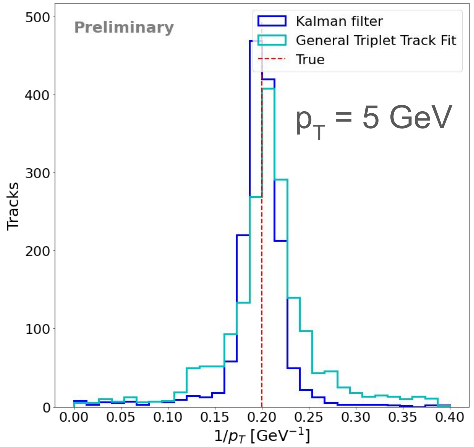
Two effects:

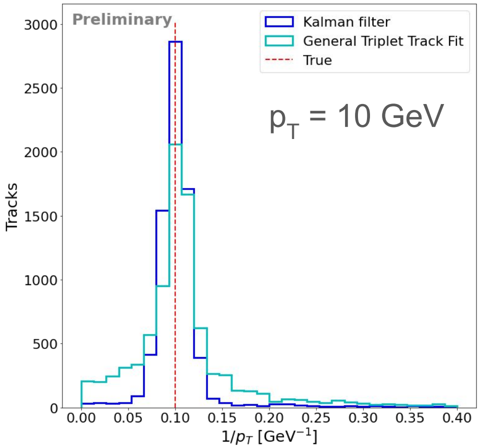
- pulls for a single triplet have $\mu < 0$ (negative residuals divided by smaller σ-s)

- statistical effect: combining triplets make the biases larger

Preliminary verification of fitting performance

The track fit is tested using muon tracks (uniformly distributed in η & ϕ) of a few different transverse momenta.





Distributions of fitted momenta from GTTF are comparable to Kalman filter. Dissimilarities are expected to be the result of detector material assumption.