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FÜR PHYSIK



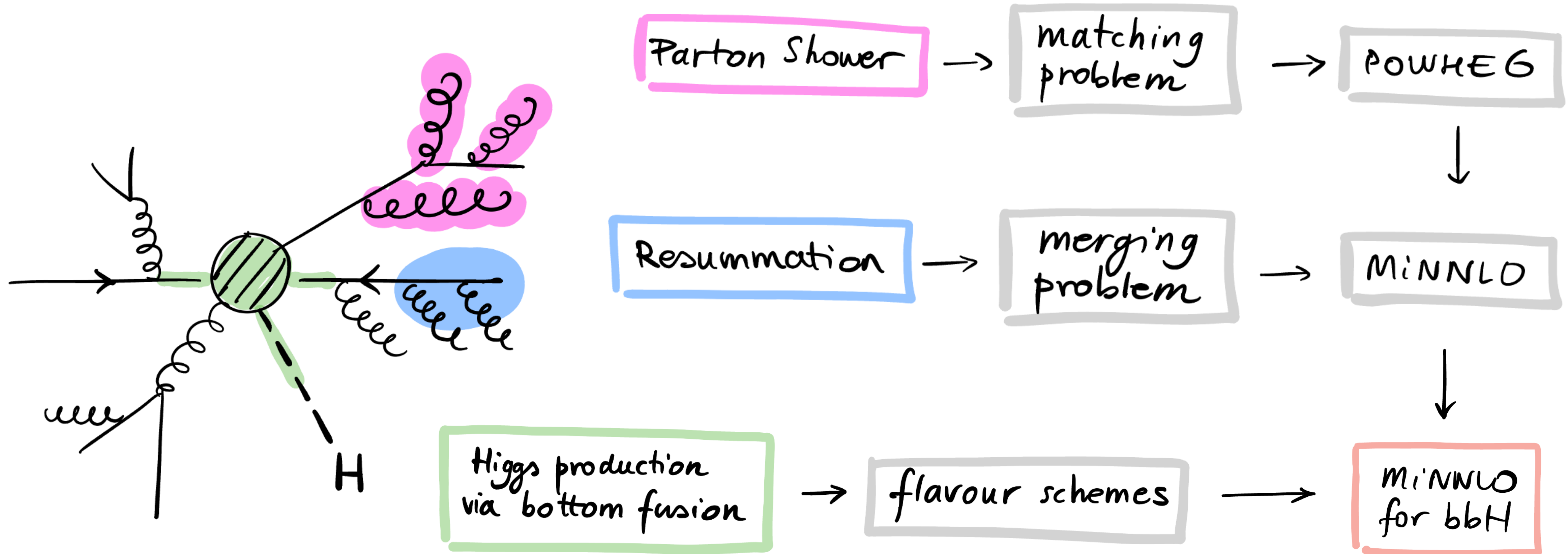
# Higgs production via bottom fusion in MiNNLOPS

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Max-Planck Institute for Physics

in collaboration with  
**A. Sankar, M. Wiesemann, G. Zanderighi**

**Milan Christmas Meeting**  
**Università degli Studi di Milano and INFN**  
**December 22nd, 2023**

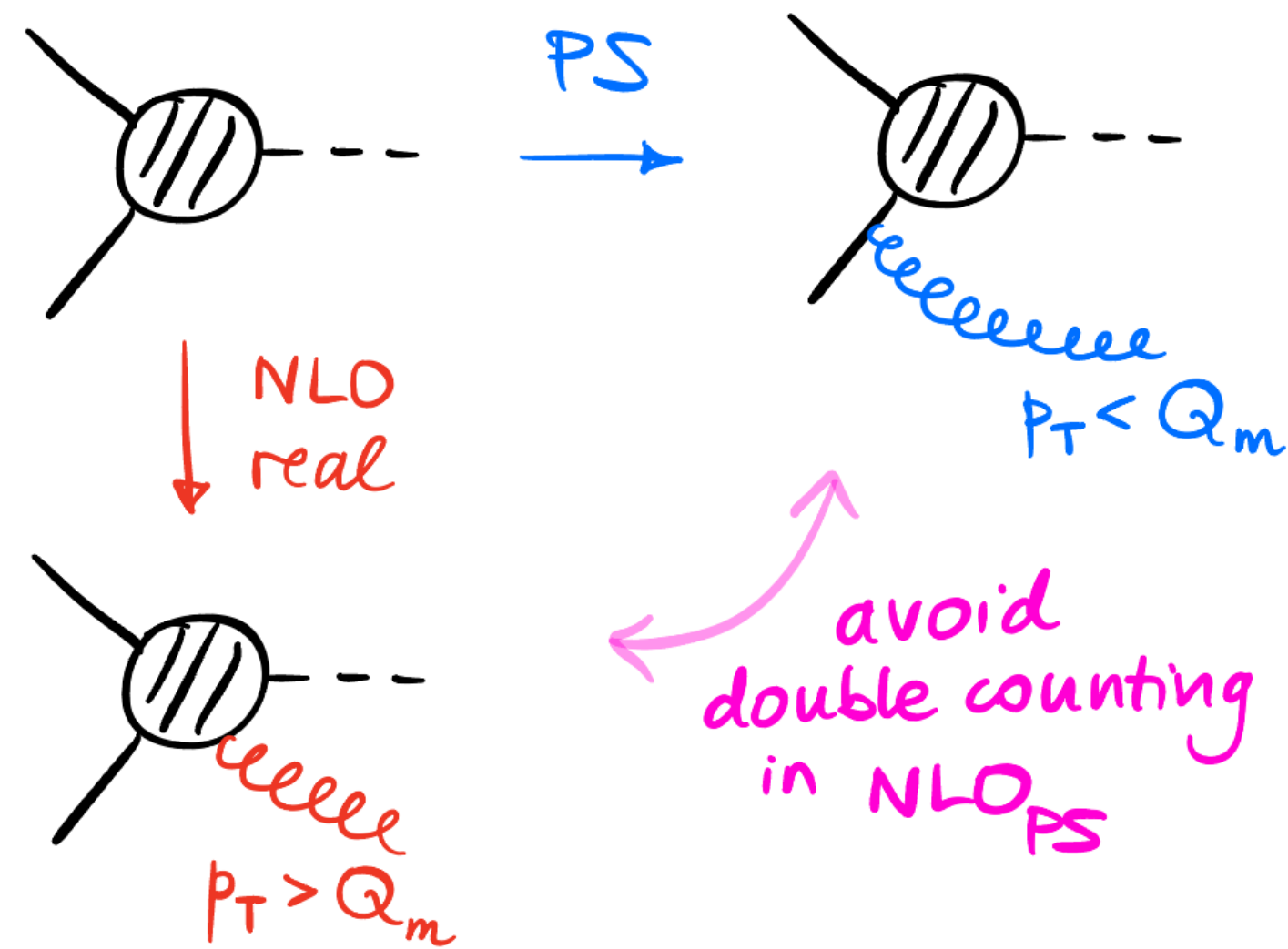
# Outline



# The method



# Matching problem



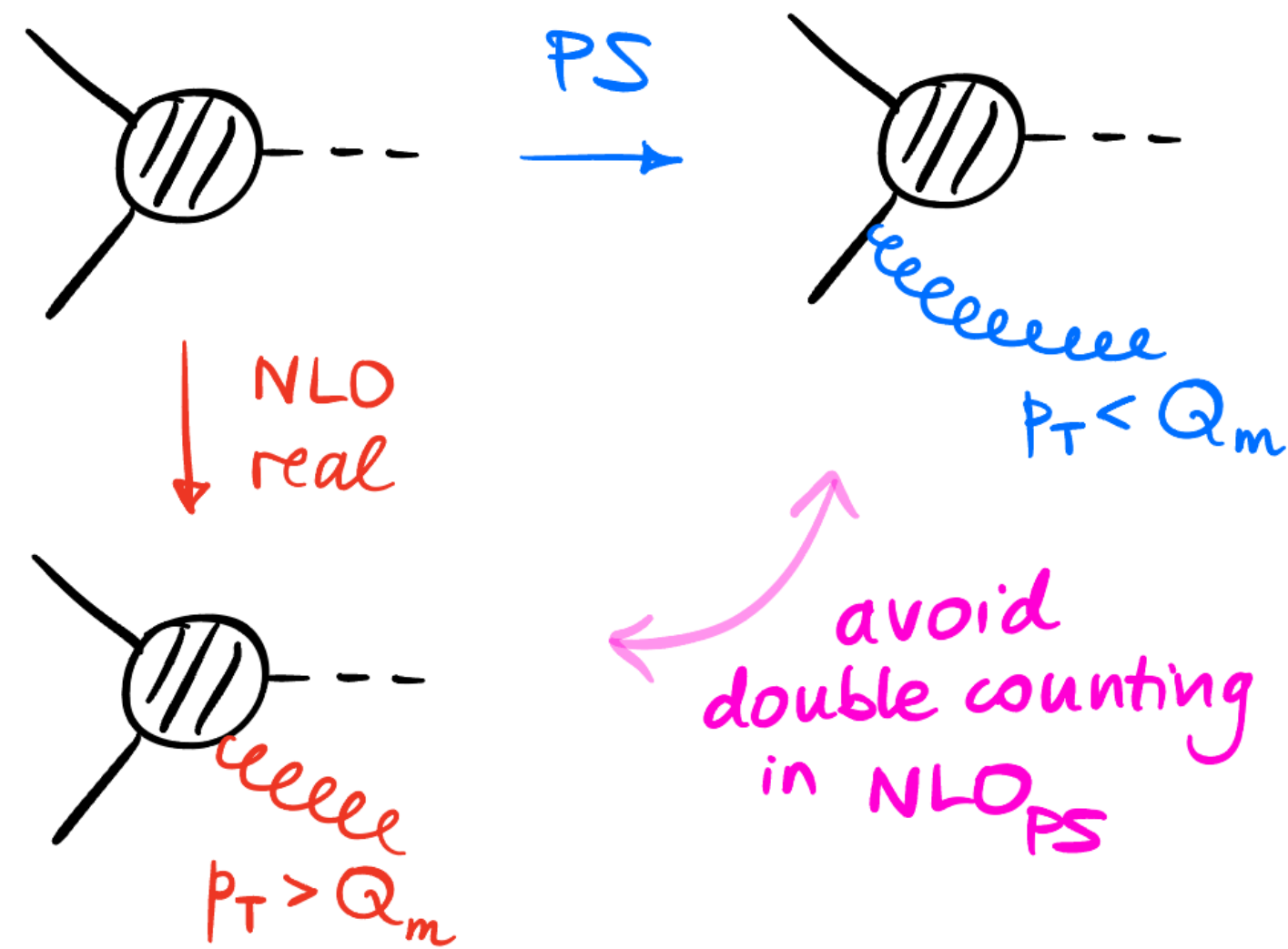
NLO predictions contain real corrections that also the Shower Monte Carlo produces.

**POWHEG solution:** Write a simplified Monte Carlo that generates just one emission (the hardest one) which alone gives the correct NLO result.

Nason [hep-ph/0409146]



# Matching problem



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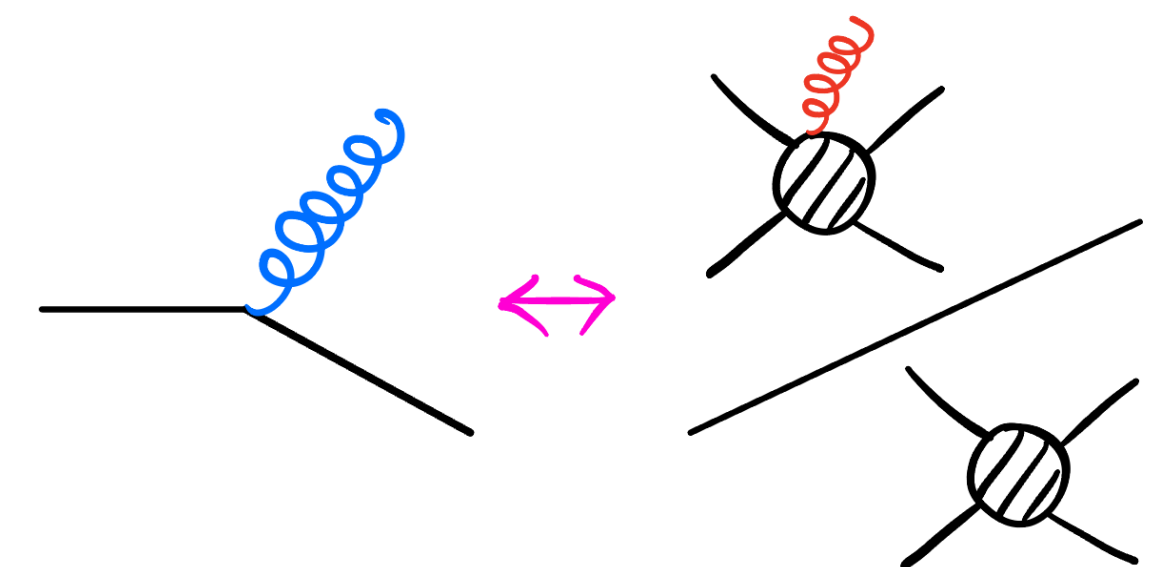
Nason [hep-ph/0409146]

## POWHEG master formula

$$d\sigma = d\Phi \bar{B}(\Phi) \left[ \Delta_{t_0}^{pwg} + d\phi_{rad} \Delta_t^{pwg} \frac{R(\Phi_n, \phi_{rad})}{B(\Phi)} \right]$$

with

$$\bar{B} = B + V + \int d\phi_{rad} R \quad \text{and} \quad \Delta_t^{pwg} = \exp \left[ - \int d\phi'_{rad} \frac{R(\Phi_n, \phi'_{rad})}{B(\Phi_n)} \Theta(t' - t) \right]$$





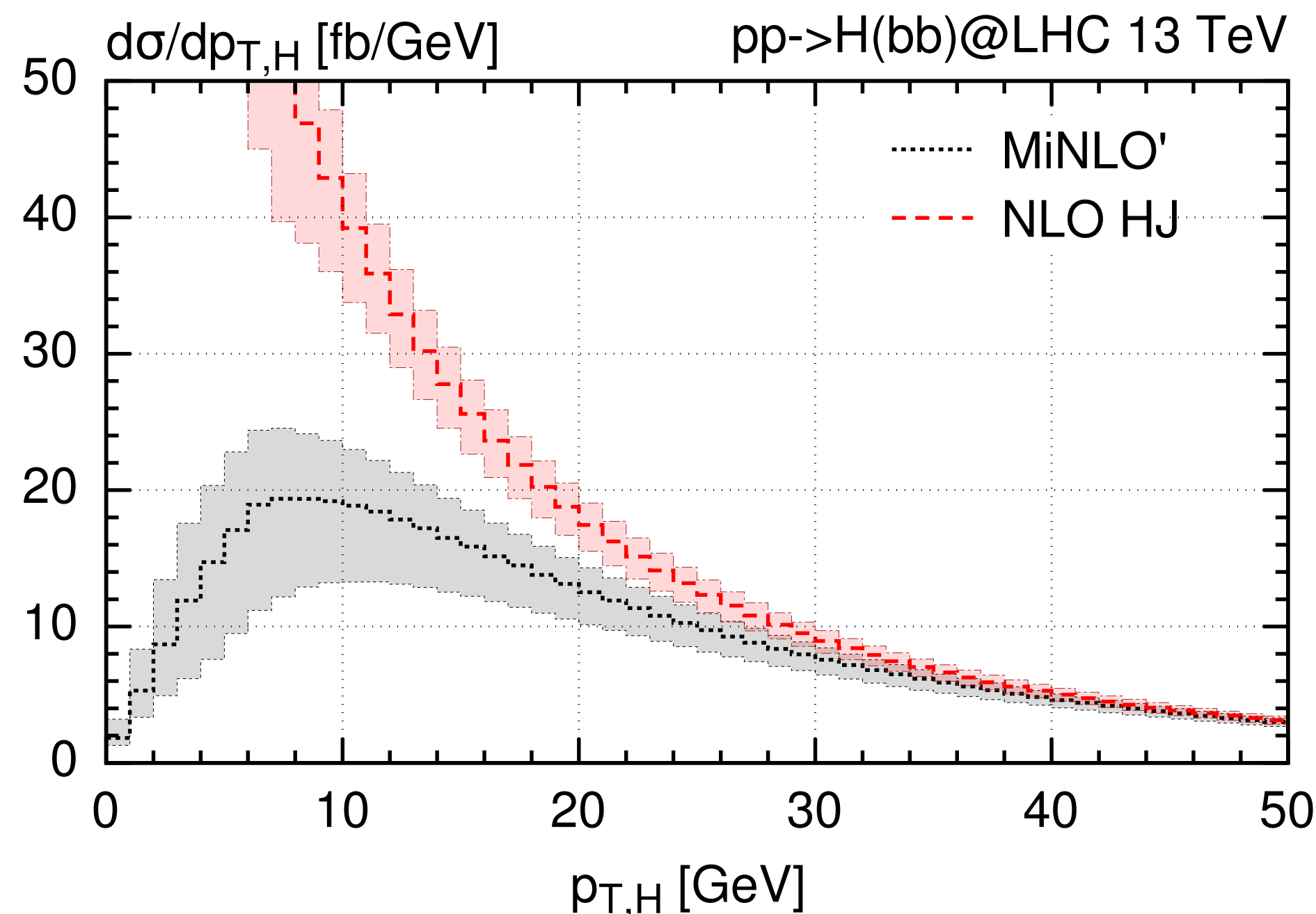
# Merging problem

$$\text{NLO } X_j \longrightarrow \text{NLO } X$$

How can we achieve NLO accuracy for inclusive X predictions from the XJ generator?

The idea of MiNLO' is to merge different multijet calculations using the **techniques of transverse momentum resummation**.

Hamilton, Nason, Zanderighi [1206.3572]  
Hamilton, Nason, Oleari, Zanderighi [1212.4504]



The merging procedure takes the advantages of two methods:

- **Flexibility of FO** and matching with PS
- **All-order control** of the resummation

with particular scale choices and without an unphysical merging scale.



# MiNNLOPs in a nutshell

$$\text{NLO } X_j \longrightarrow \text{NNLO } X$$



MiNNLOPs is an extension of MiNLO' to achieve NNLO+PS accuracy for inclusive observables.

Monni, Nason, Re, Wiesemann, Zanderighi [1206.3572]

Split the differential inclusive cross-section into the singular and regular part in the small transverse momentum limit:  $d\sigma = d\sigma^{sing} + d\sigma^{reg}$ .



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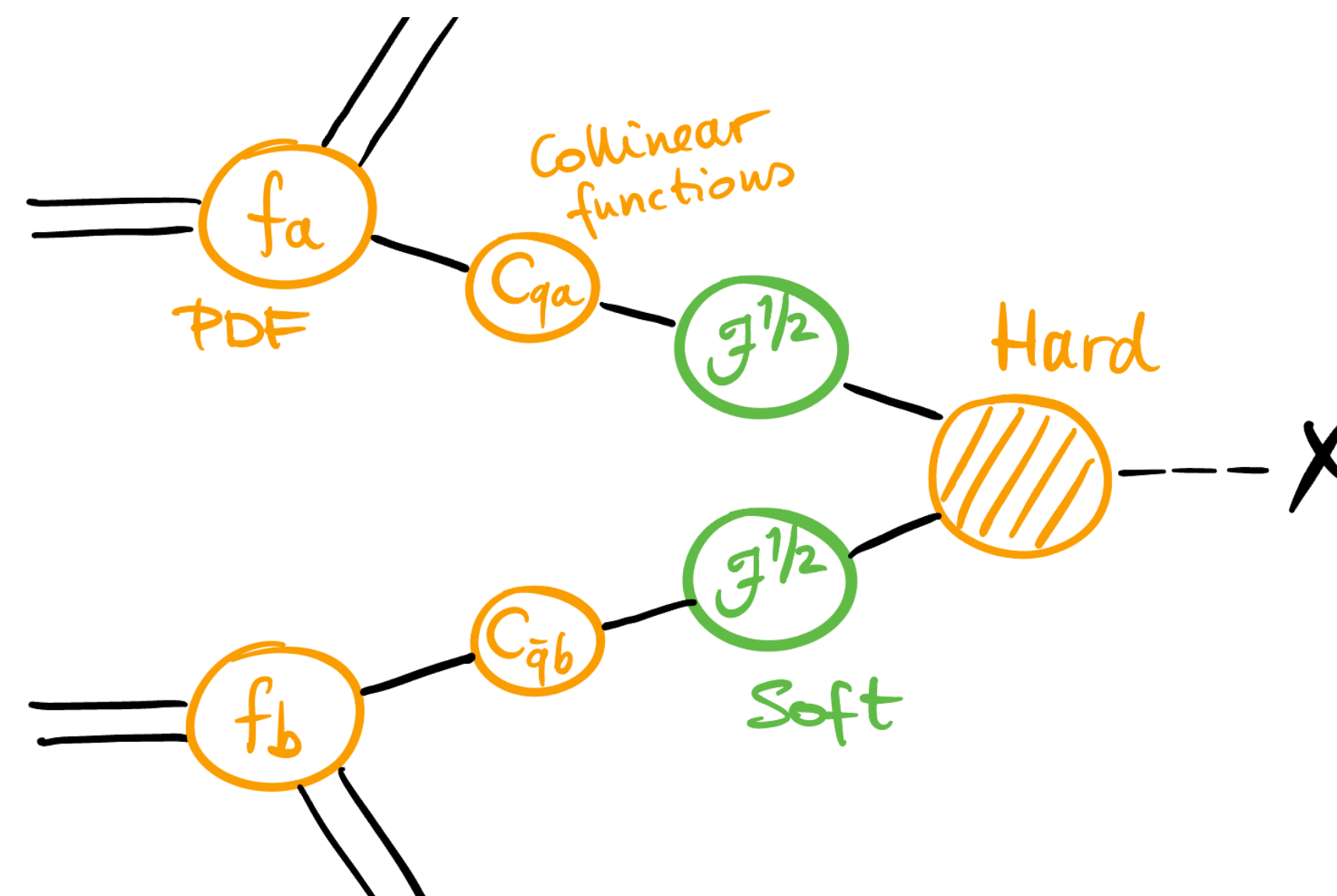
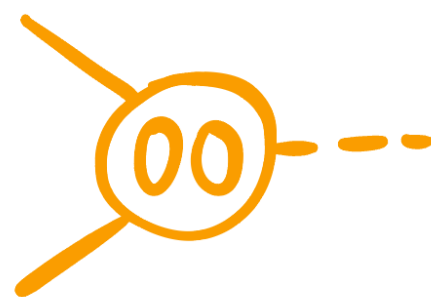
Split the differential inclusive cross-section into the singular and regular part in the small transverse momentum limit:  $d\sigma = d\sigma^{sing} + d\sigma^{reg}$ .

$$\frac{d\sigma^{sing}}{dp_T d\Phi_X} = \frac{d}{dp_T} \left\{ \mathcal{F}(p_T) \mathcal{L}(p_T) \right\} =: \exp \left[ -\tilde{S}(p_T) \right] D(p_T)$$

Sudakov form factor

$$\mathcal{F}(p_T) = \exp \left[ -\tilde{S}(p_T) \right]$$

Luminosity: it also contains







# MiNNLOps in a nutshell

$$d\sigma = d\sigma^{sing} + d\sigma^{reg}$$

The modified POWHEG function is

$$\bar{B}(\Phi_{XJ}) = e^{-\tilde{S}(p_T)} \left\{ B \left( 1 - \alpha_s(p_T) \tilde{S}^{(1)} \right) + V + \int d\phi_{rad} R + \left[ D(p_T) - D^{(1)} - D^{(2)} \right] \times F^{corr} \right\}$$

MiNLO' structure

Extra term: it ensures NNLO accuracy.

$F^{corr}$  encodes the spreading of the D-terms upon the full  $\Phi_{XJ}$ .

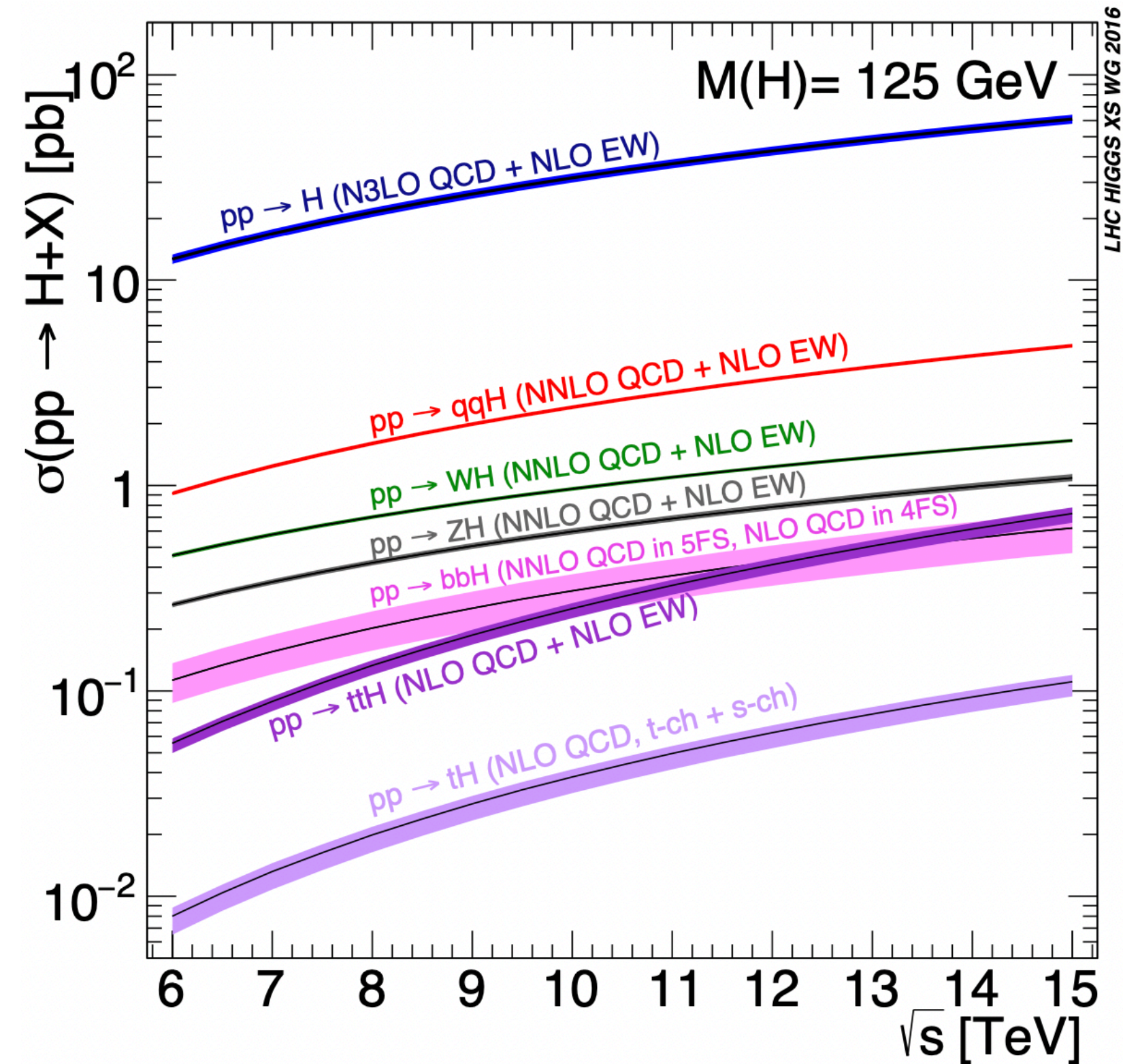
- In the singular part, the QCD scales must be  $\mu_F \sim \mu_R \sim p_T$ .
- For the regular part, different scale choices can be performed:
  - the transverse momentum  $p_T$  (original choice)
  - the **hard scale**  $Q$  (FOatQ=1)

Gavardi, Oleari, Re [2204.12602]

# The process



# Why Higgs production via bottom fusion?



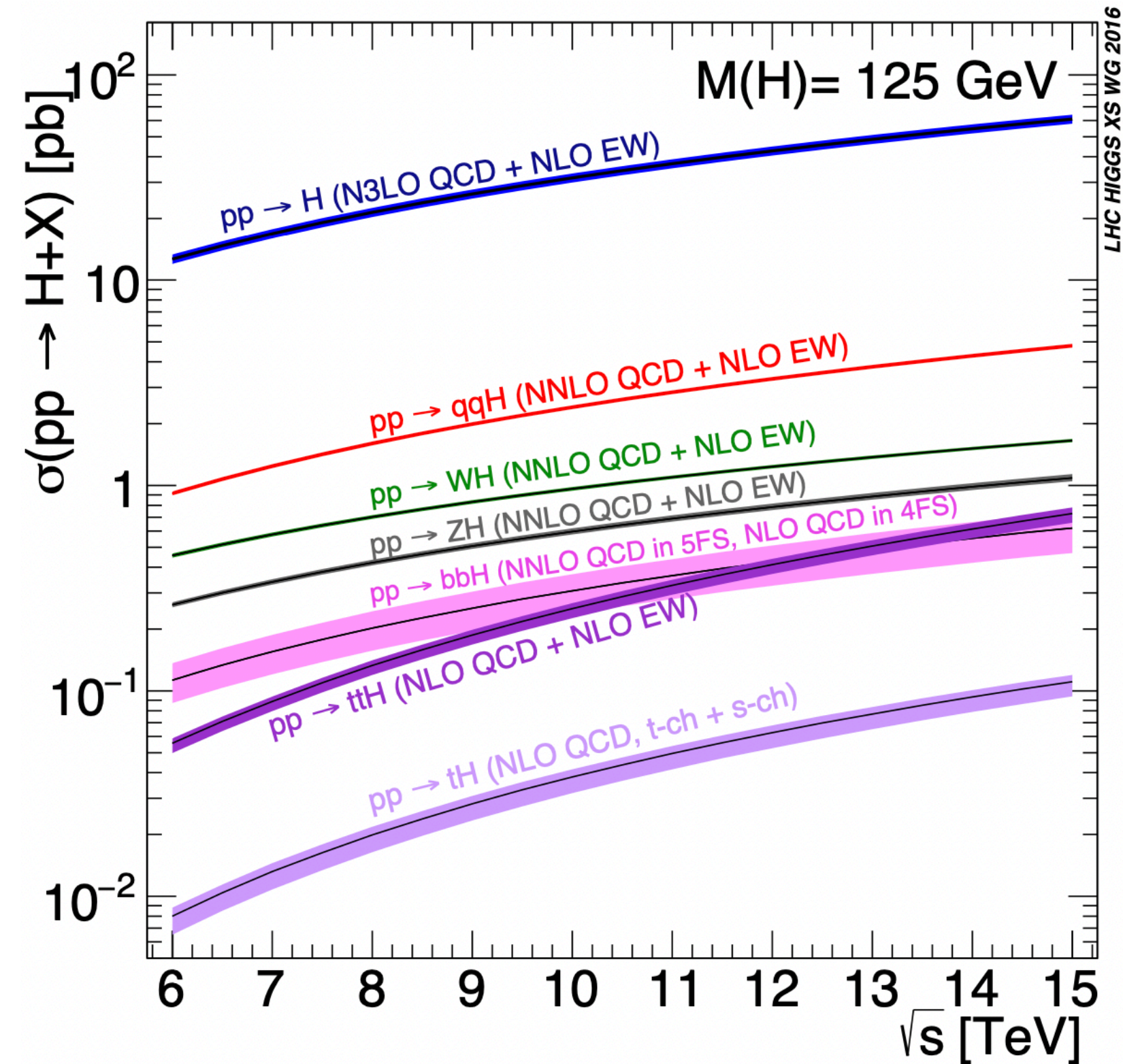
Although it is not the main production channel, the Higgs creation via bottom fusion

- allows a **direct** evaluation of the **bottom Yukawa** coupling
- is **enhanced in SUSY theories** with large  $\tan \beta$  and can become the dominant channel
- is the dominant irreducible **background** in searches for **HH production**





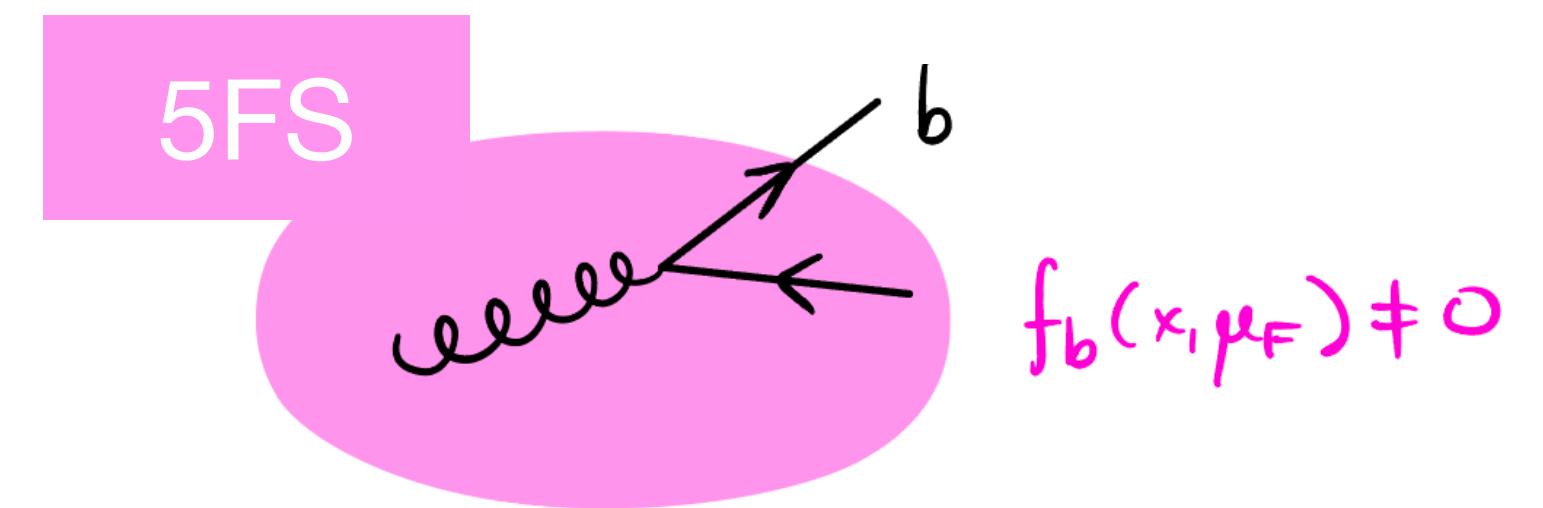
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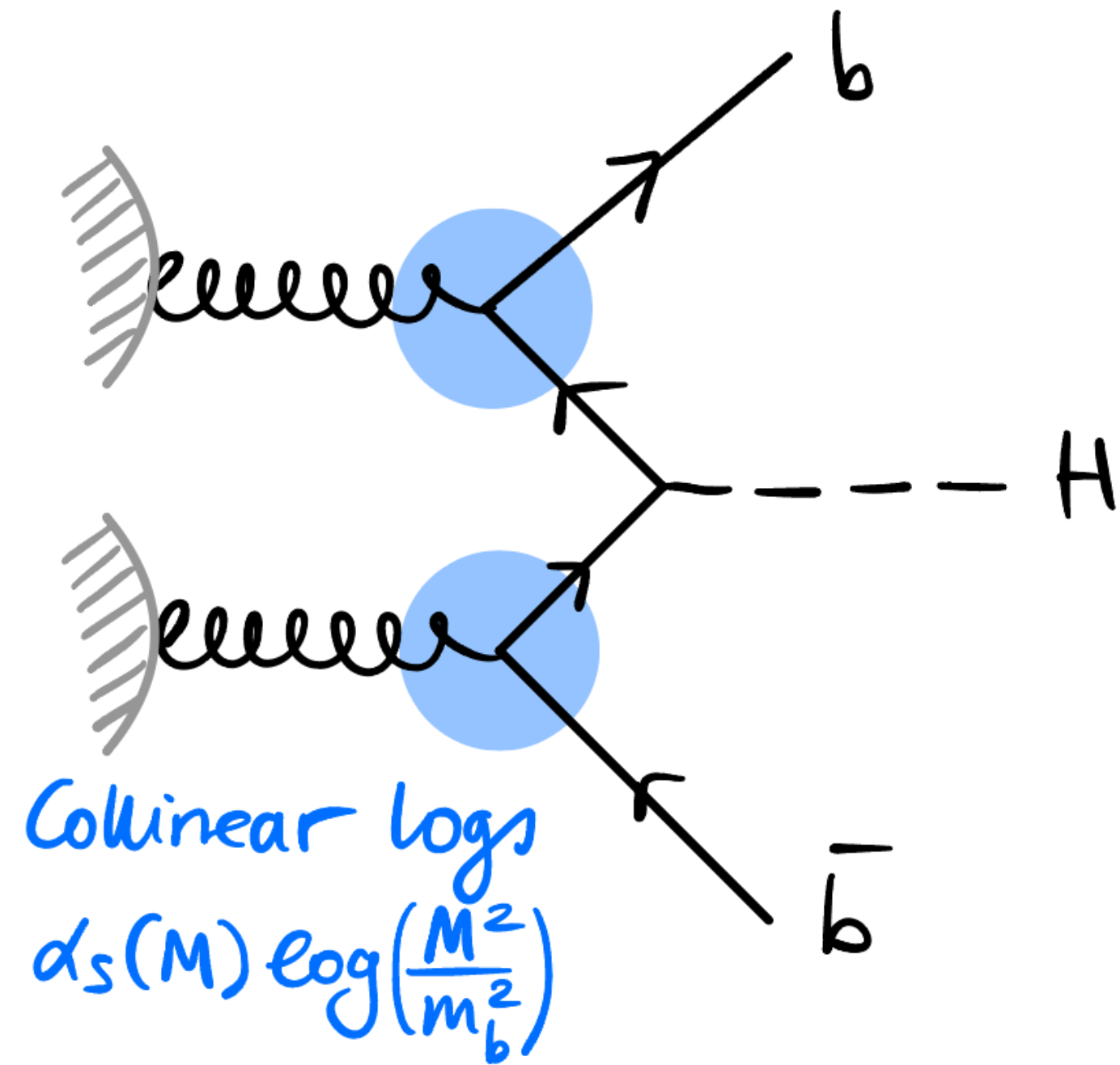
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$b\bar{b}H$  is also of theoretical interest for the **different schemes** of calculations that can be used





4FS



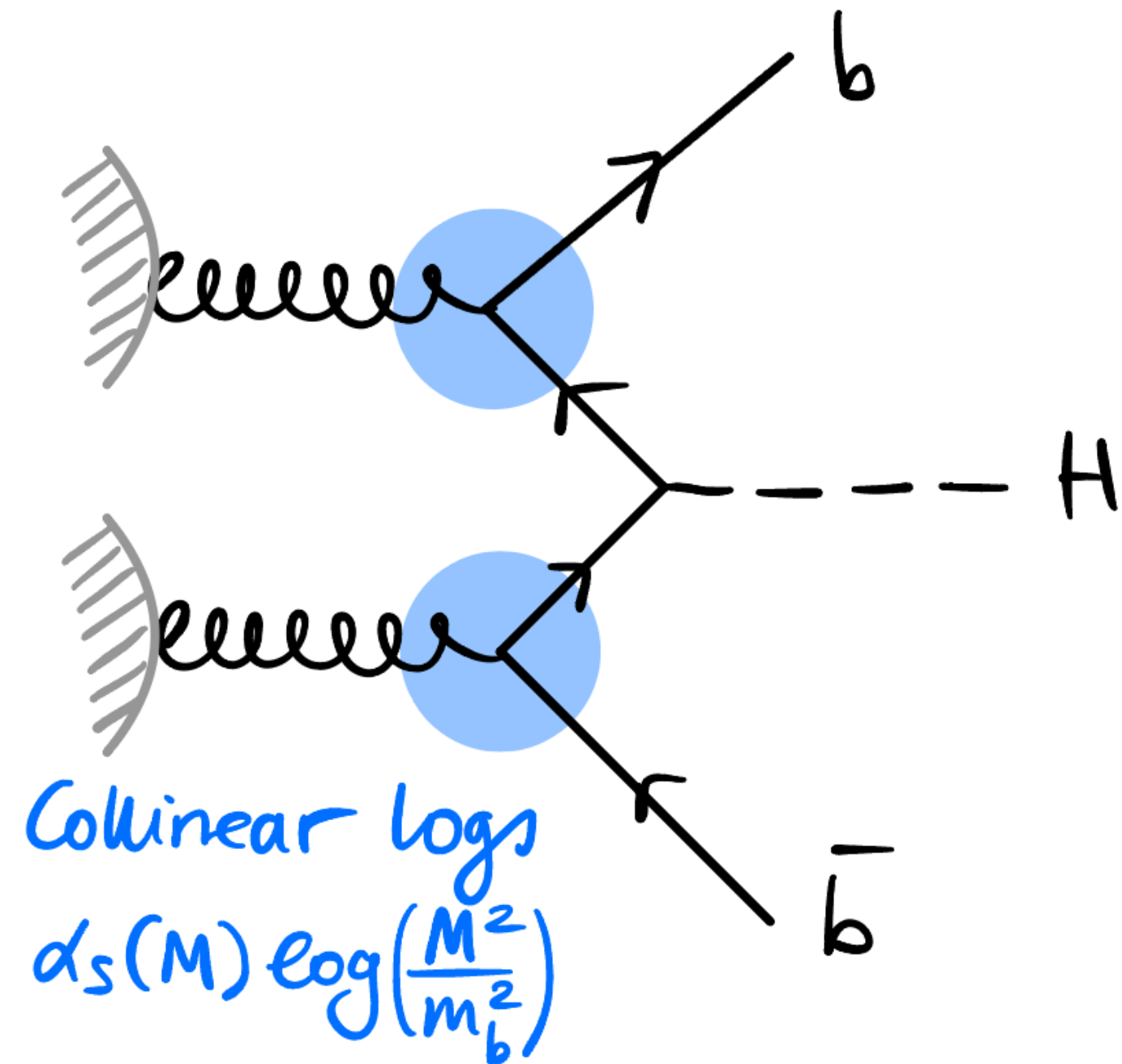
## decoupling/massive scheme

- It does not resum possibly large collinear logs
- Computing higher orders is more difficult due to higher multiplicity
- ✓ Mass effects  $O(m_b/m_H)$  are there at any order





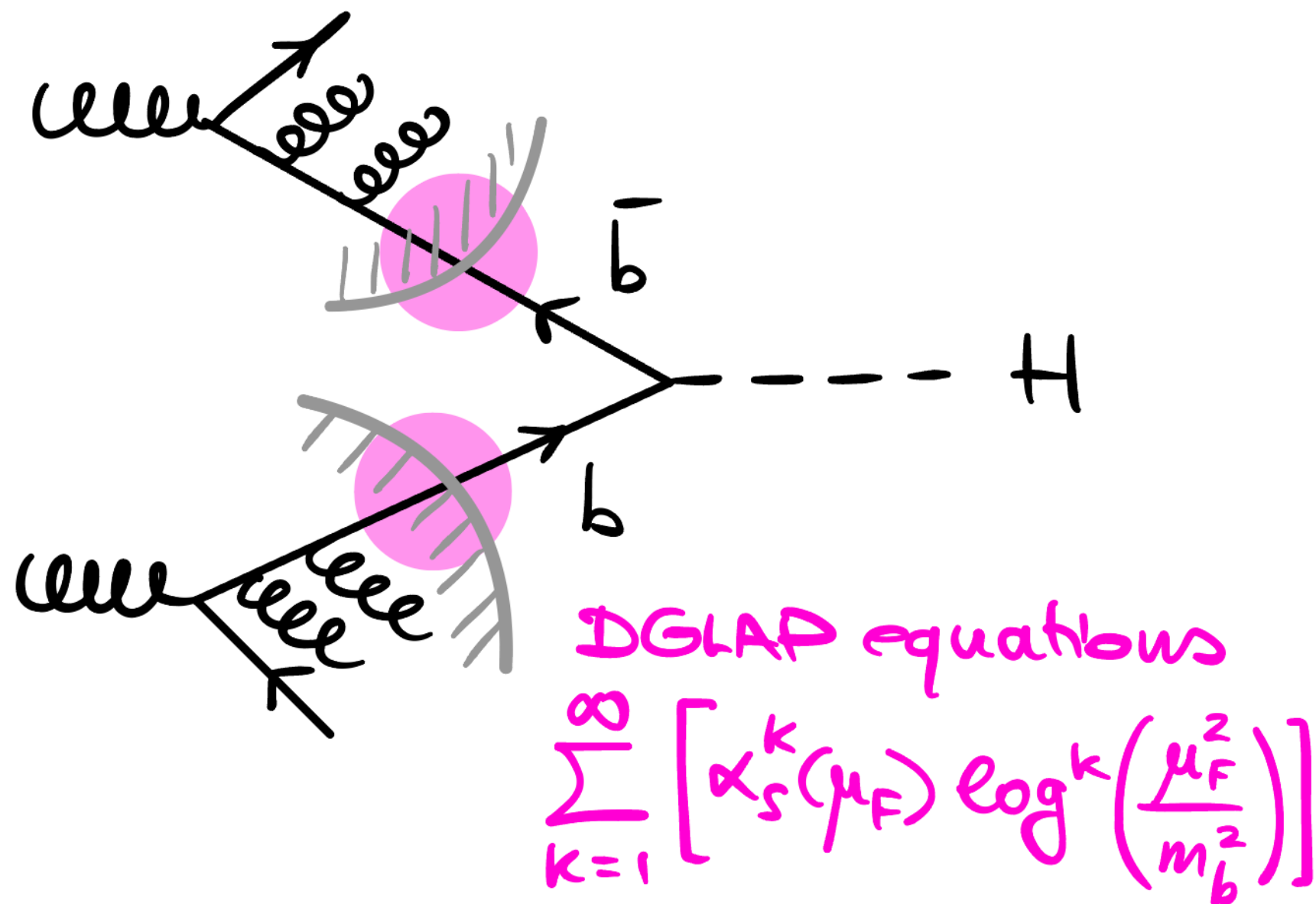
4FS



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5FS



### massless scheme

- ✓ DGLAP evolution resums initial state logs into  $f_b$
- ✓ Computing higher orders is easier
- Neglecting  $O(m_b/m_H)$ , it yields less accurate description of bottom kinematic distribution





# Current state of the art

- $N^3\text{LO}$  for the total cross section in the 5FS Duhr, Dulat, Mistlberger [1904.09990]
- $N^3\text{LO}^{5FS}$  matched to  $\text{NLO}^{4FS}$  using the FONLL matching Duhr, Dulat, Hirschi, Mistlberger [2004.04752]  
Forte, Napoletano, Ubiali [1508.01529, 1607.00389]
- $\text{NLO}^{4FS}$  matched to parton shower Wiesemann, Frederix, Frixione, Hirschi, Maltoni, Torrielli [1409.5301]  
Jäger, Reina, Wackeröth [1509.05843]
- $\text{NLO}_{QCD}^{4FS} + \text{PS}$  combined with  $\text{NLO}_{EW}^{4FS}$  Pagani, Shao, Zaro [2005.10277]



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## This talk:

We focus on the 5FS calculation of the  $b\bar{b}H$  process and we perform the **first fully-differential** calculation of **NNLO QCD** matched to **parton shower** ( $\text{NNLO}^{5FS} + \text{PS}$ )

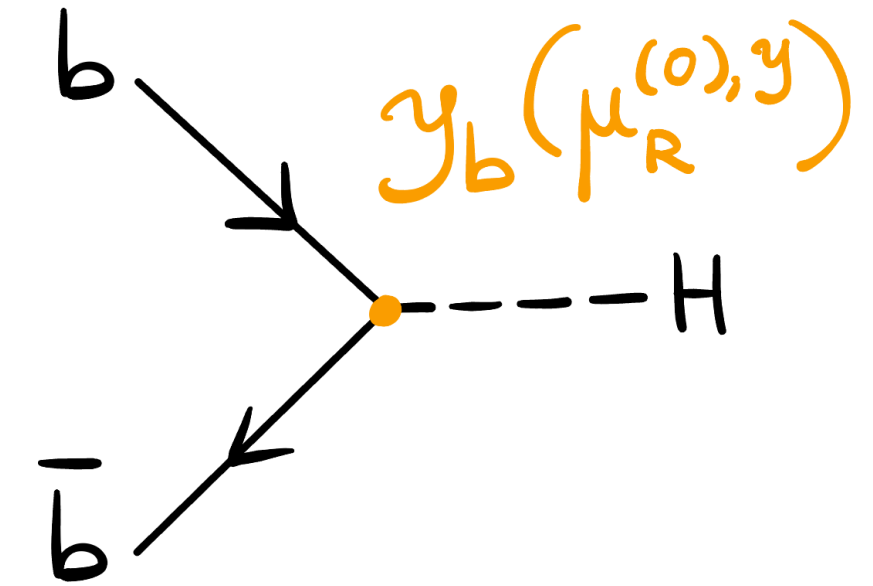


# MiNNLOPs for Yukawa induced processes

The **Yukawa coupling** is renormalised in  $\overline{\text{MS}}$  scheme.

The running of this Born coupling requires some adaptations of the MiNNLOPS method to take account the extra scale dependence.

$$H^{(1,2)} \rightarrow H^{(1,2)} \left( \log \frac{\mu_R^{(0),y}}{m_H} \right)$$

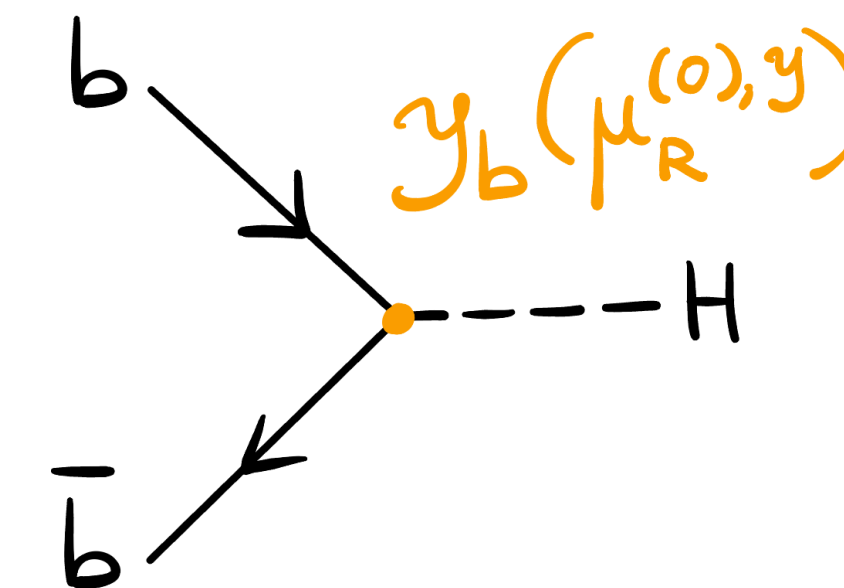




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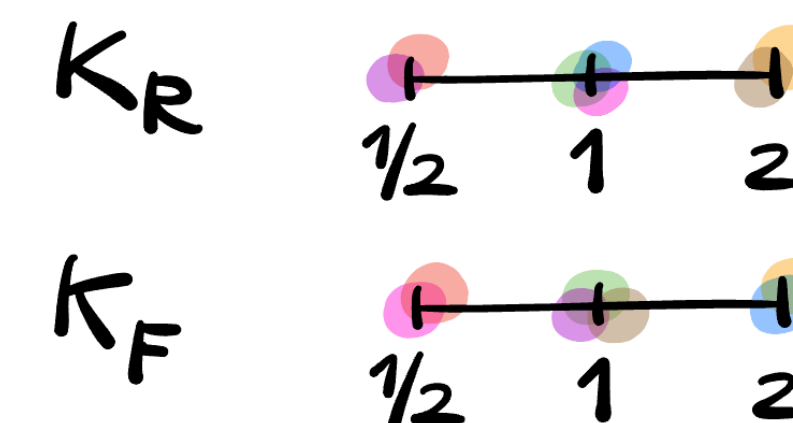
$y_b(m_b = 4.18 \text{ GeV}) \xrightarrow{\text{3-loop running}} y_b(m_H) \rightarrow y_b(K_R m_H)$

$\alpha_s(p_T) \rightarrow \alpha_s(K_R p_T)$

$f_a(p_T) \rightarrow f_a(K_F p_T)$

MiNNLO<sub>PS</sub>

7-point scale variation





# Cross-section results

Same PDFs:  
NNPDF40\_nnlo\_as\_01180  
with 5 active flavours

Comparison of the total inclusive cross section with FO results obtained with the public code **SusHi** with  $\mu_R = \mu_F = m_H$

Harlander, Lieber, Mantel [1212.3249]

Process	NLO (SUSHi)	NNLO (SUSHi)	MINLO'	MINNLO <sub>PS</sub>	MINNLO <sub>PS</sub> (FOatQ 1)
$b\bar{b} \rightarrow H$	$0.646(0)^{+10.4\%}_{-10.9\%}$ pb	$0.518(2)^{+7.2\%}_{-7.5\%}$ pb	$0.571(1)^{+17.4\%}_{-22.7\%}$ pb	$0.509(8)^{+2.9\%}_{-5.3\%}$ pb	$0.508(4)^{+3.6\%}_{-4.3\%}$ pb

- NNLO cross section is reduced by  $\sim 20\%$
- Scale uncertainties significantly reduced at NNLO
- Our MiNNLOPS predictions are in agreement with **SusHi** within the uncertainties

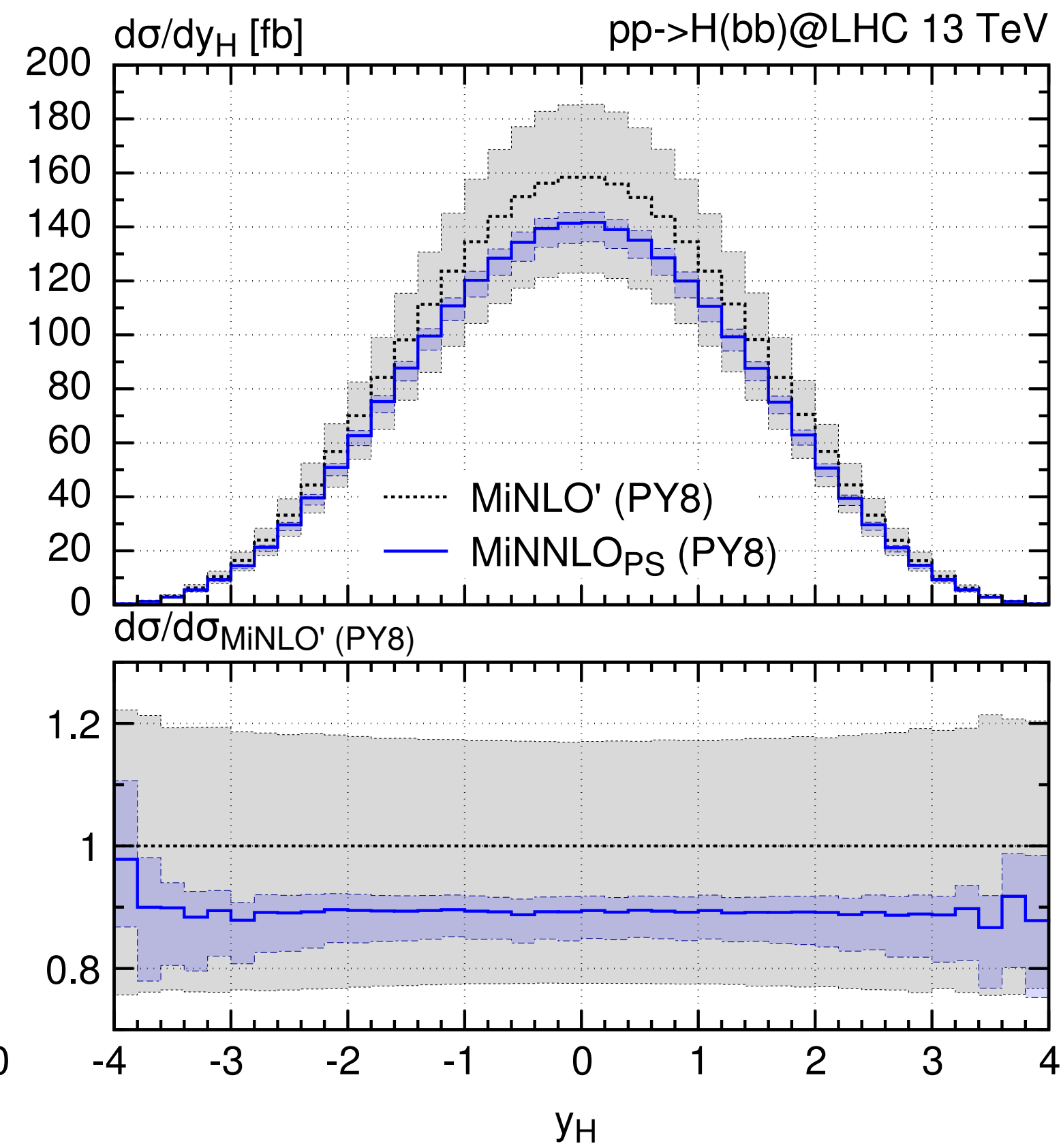
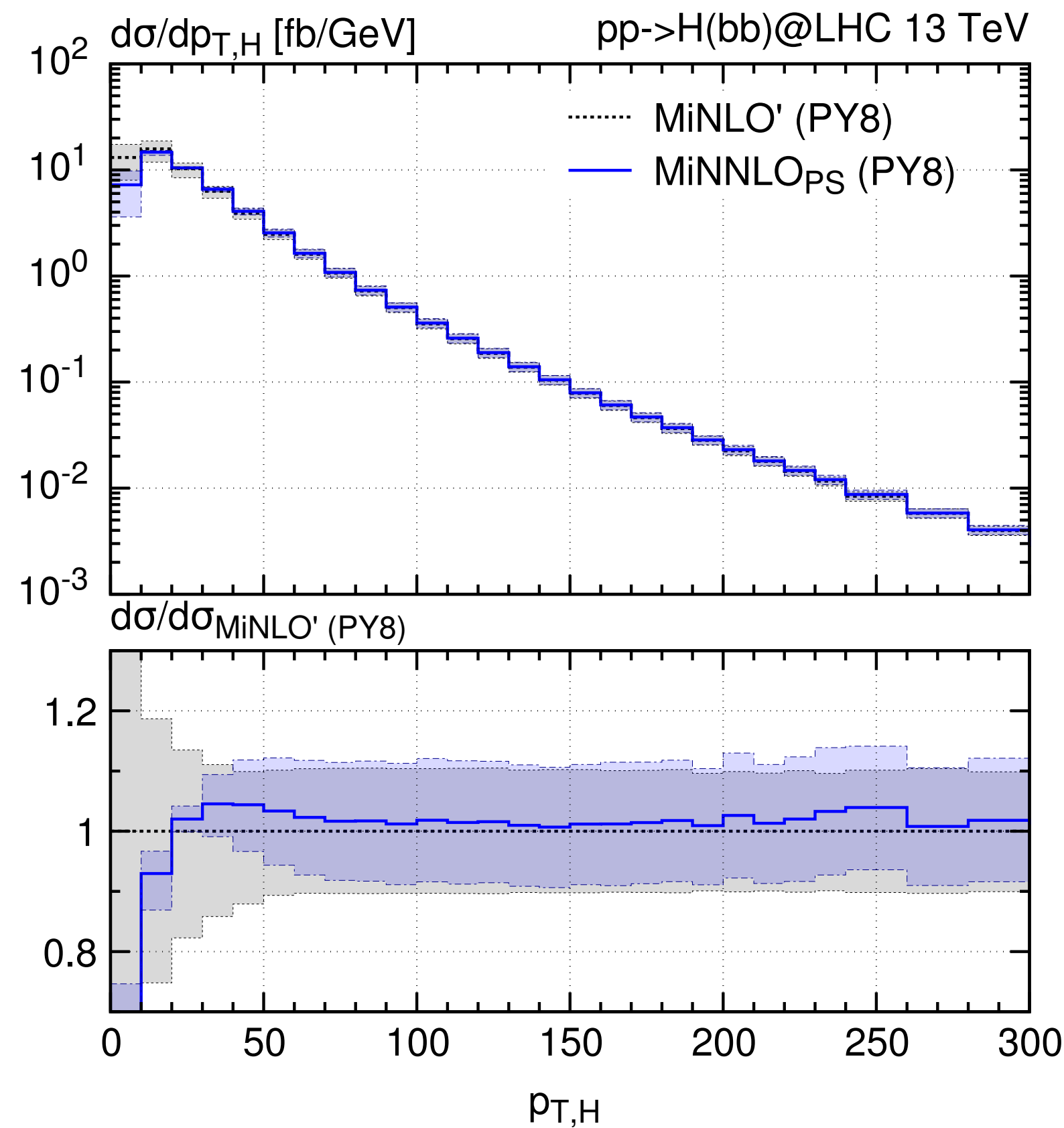




# Comparison of MiNLO' and MiNNLOPs

## Transverse momentum spectrum of the Higgs boson

## Rapidity distribution of the Higgs boson



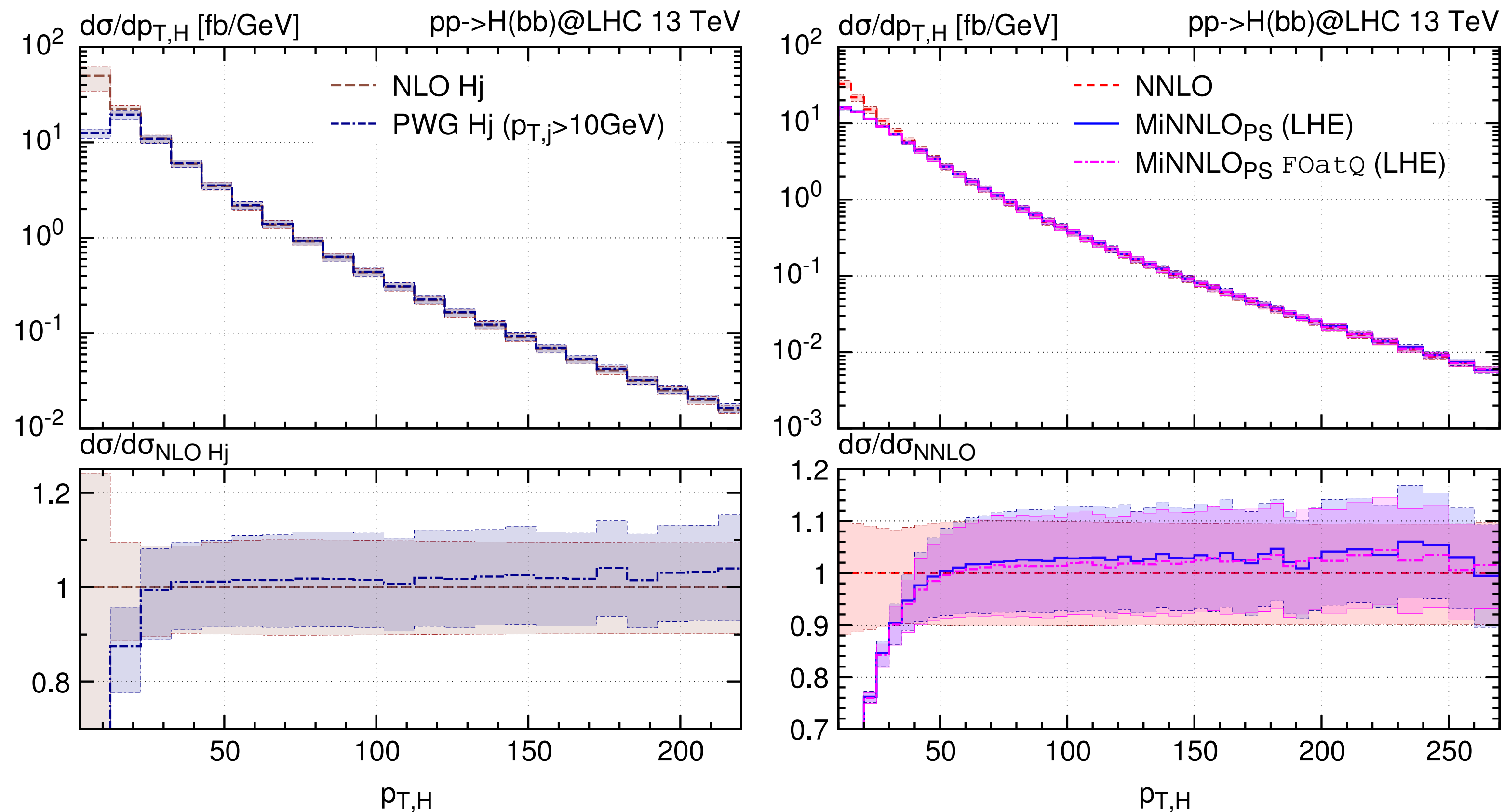
- At small  $p_{T,H}$ , **MiNNLOPs** significantly dampens the distribution.
- At high  $p_{T,H}$ , **MiNNLOPs** and **MiNLO'** coincide, both NLO accurate
- **MiNNLOPs** has a flat negative correction in the rapidity  $y_H$  distribution





# Comparison to FO results

## Transverse momentum spectrum of the Higgs



We tested our POWHEG generator before and after the MiNNLO implementation.

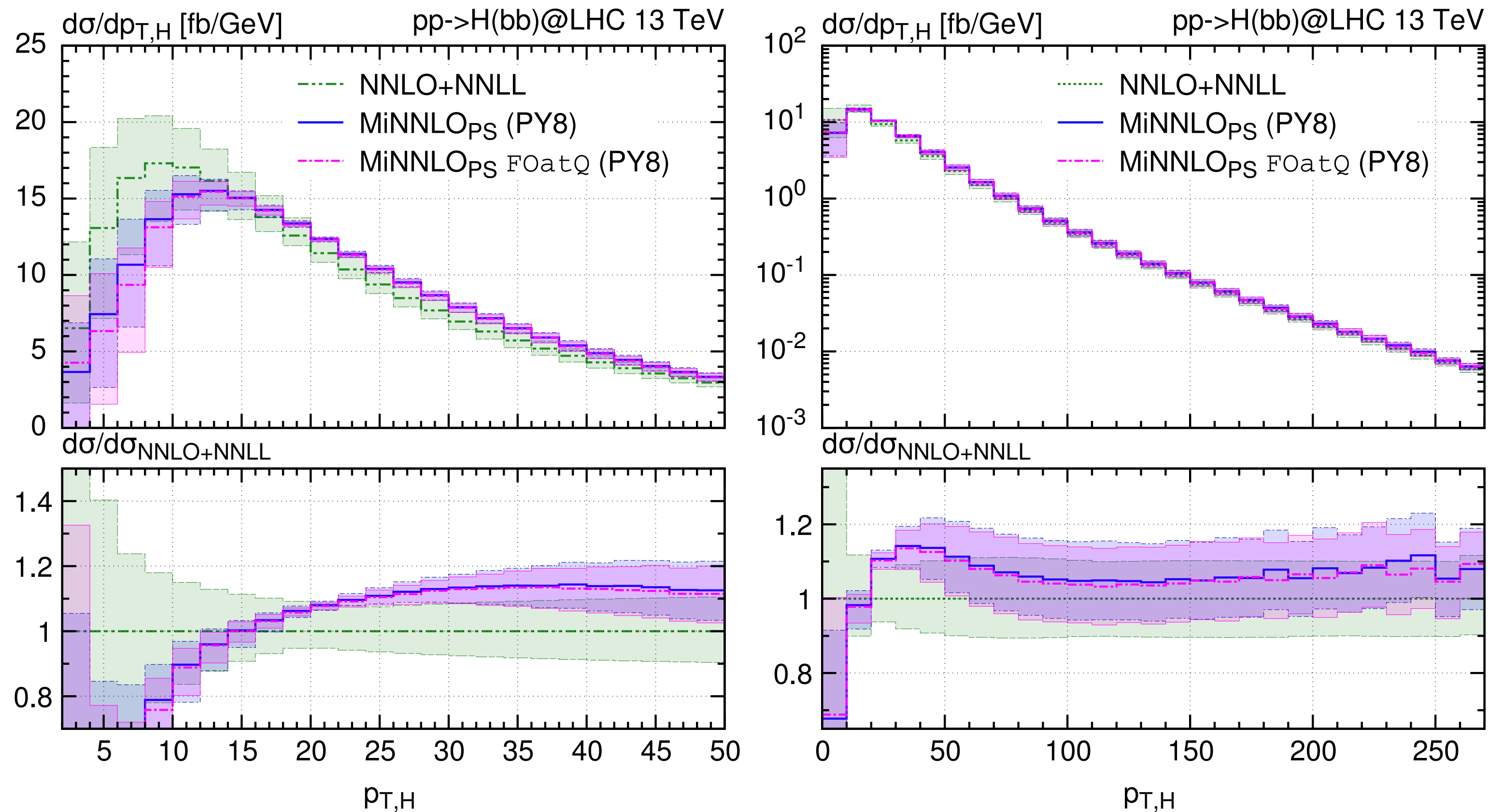
Full **agreement** at large transverse momenta  $p_{T,H}$  with analytic **Fixed-Order predictions**

**NLO H<sub>j</sub>** Harlander, Ozeren, Wiesemann [1007.5411]  
**NNLO** Harlander, Tripathi, Wiesemann [1403.7196]



# Comparison to resummed results

## Transverse momentum spectrum of the Higgs



We compare the MiNNLO implementation with the NNLO+NNLL results for low and high  $p_{T,H}$

- Acceptable agreement for small  $p_{T,H}$
- The shower has an effect on the tail

**NNLO+NNLL** Harlander, Tripathi, Wieseemann [1403.7196]



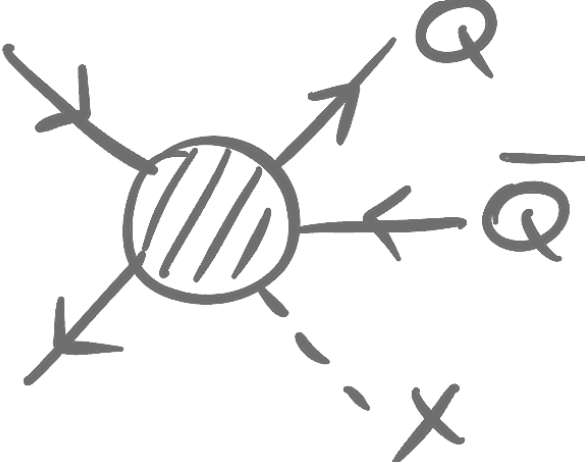
# Summary and outlook

- Presentation of **NNLO+PS predictions** for Higgs production via bottom fusion **in 5FS** which are in **agreement with fixed-order** results from literature.
- It is an **initial step** towards a complete NNLO+PS description of  $b\bar{b}H$ .



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- It is an **initial step** towards a complete NNLO+PS description of  $b\bar{b}H$ .
- We are working on the NNLO+PS implementation in **4FS**.

MiNNLO<sub>PS</sub>  + massification of  $H^{(2)}$  + two-loop finite reminder

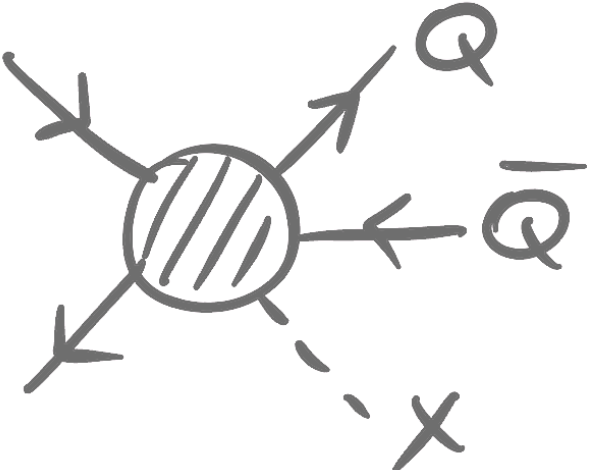
Badger, Hartanto, Kryś, Zoia [2107.14733]

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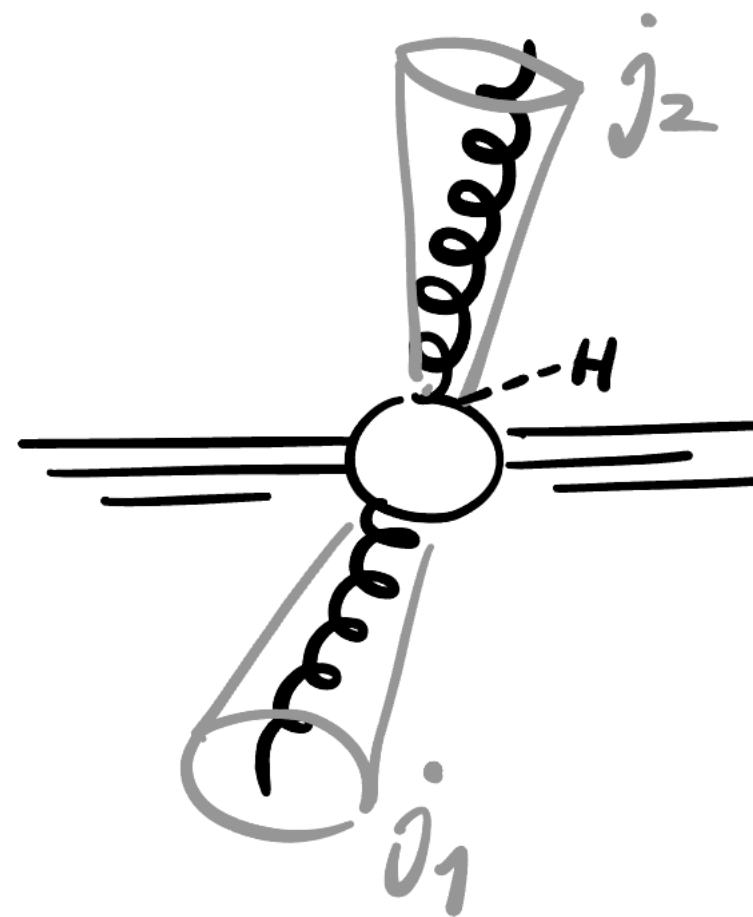
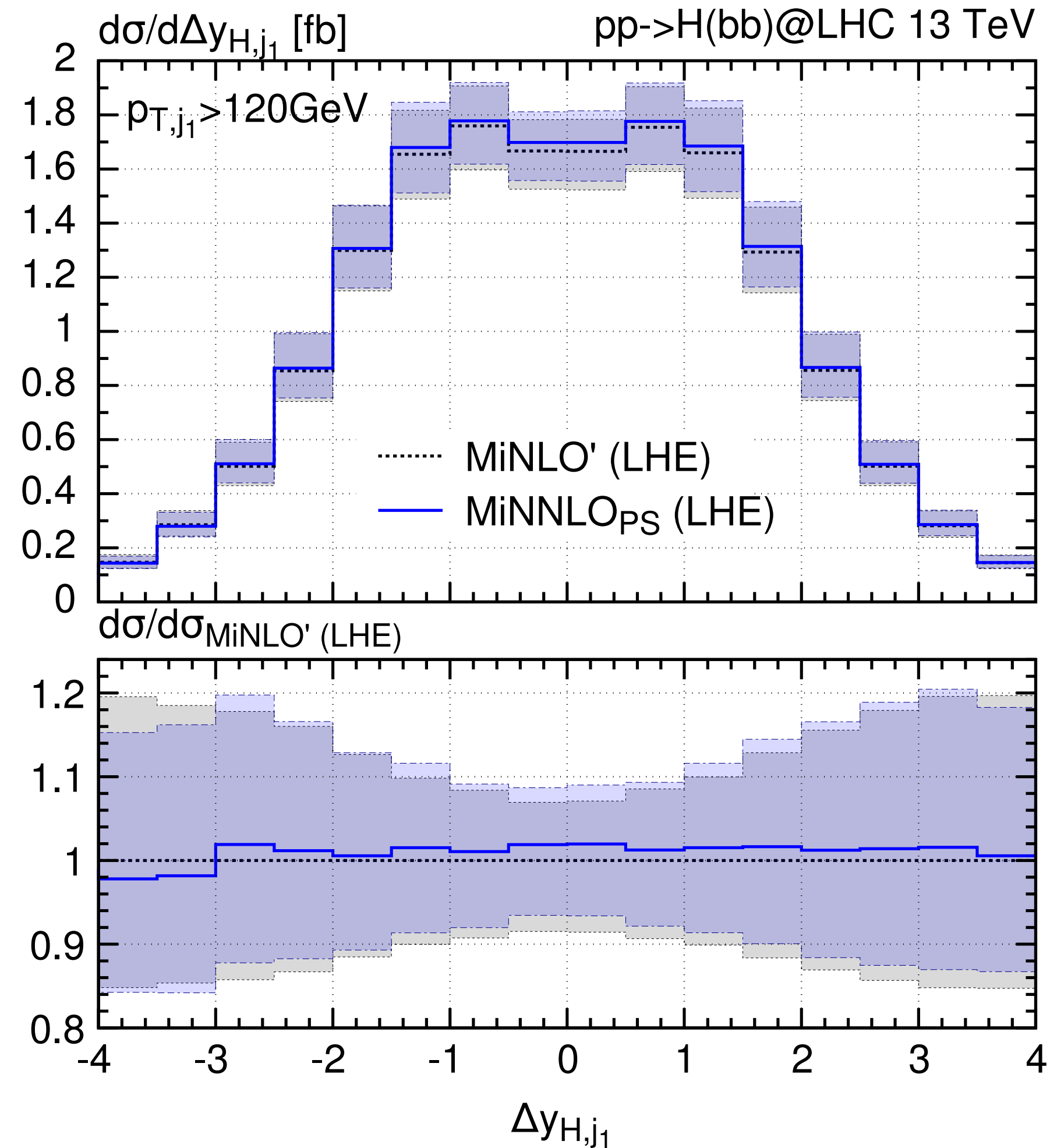
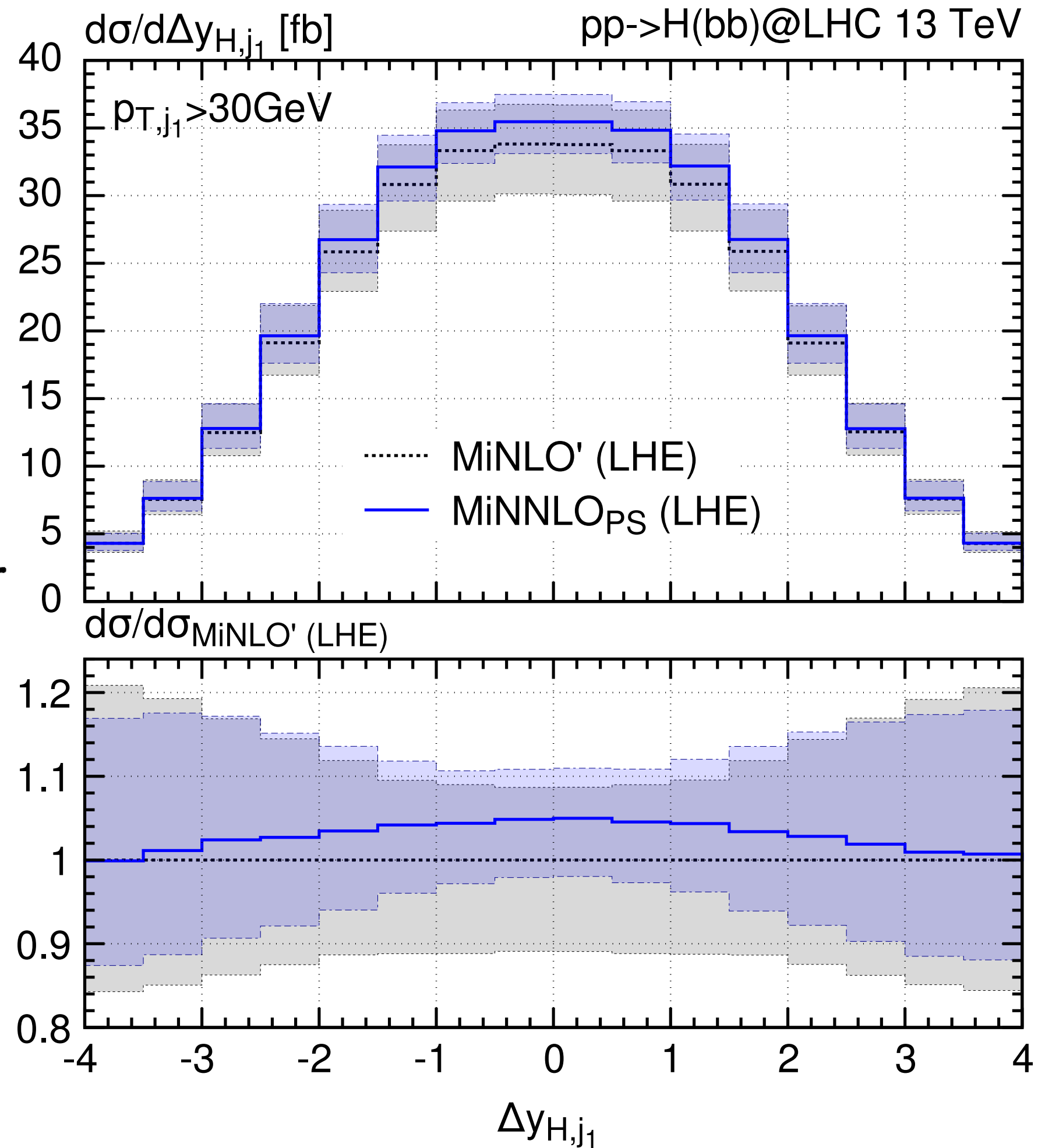
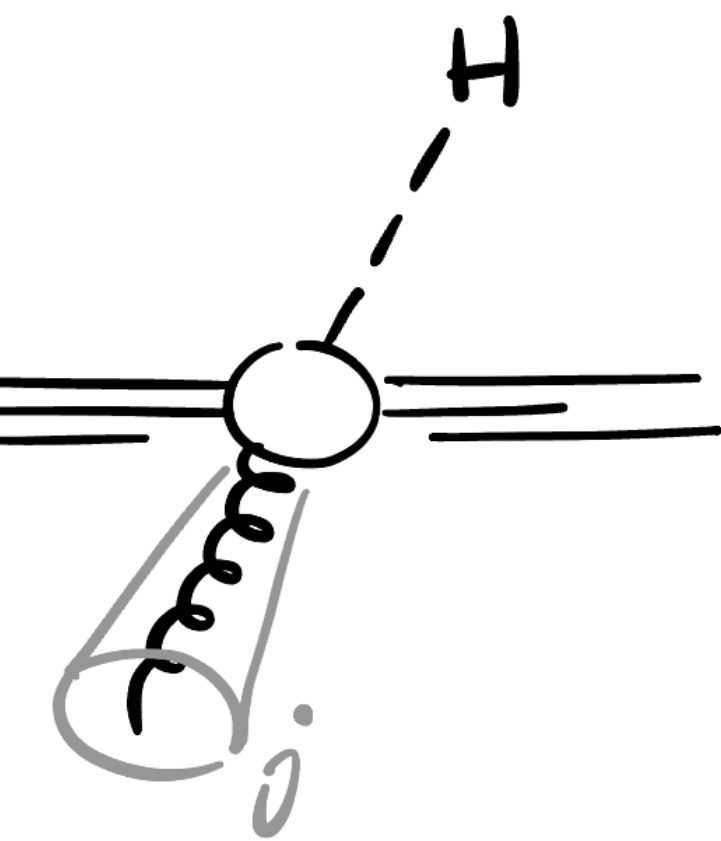
**Thank you for your attention  
and Happy Christmas!**

**Backup slides**



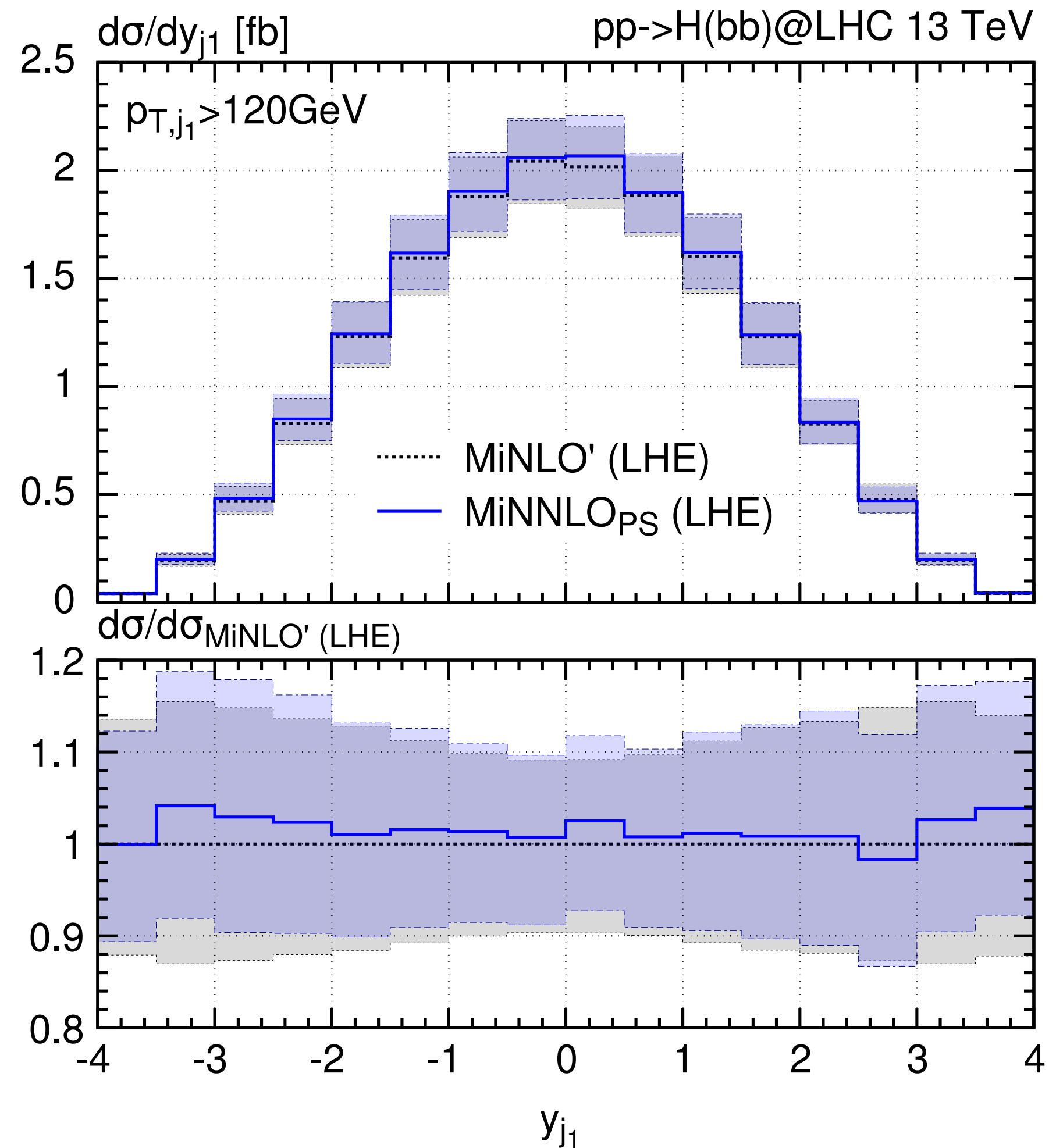
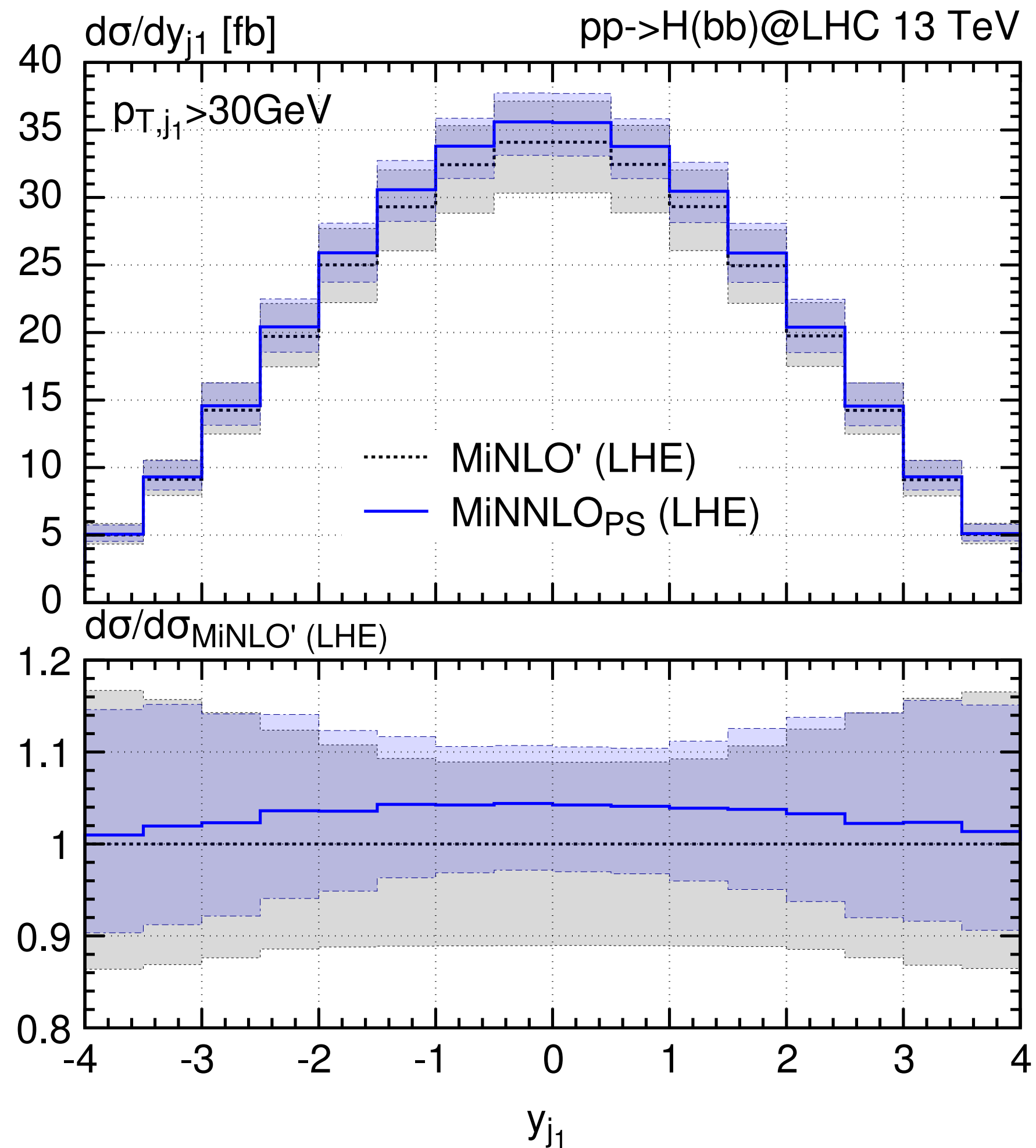


# Jet-observables: difference of rapidity





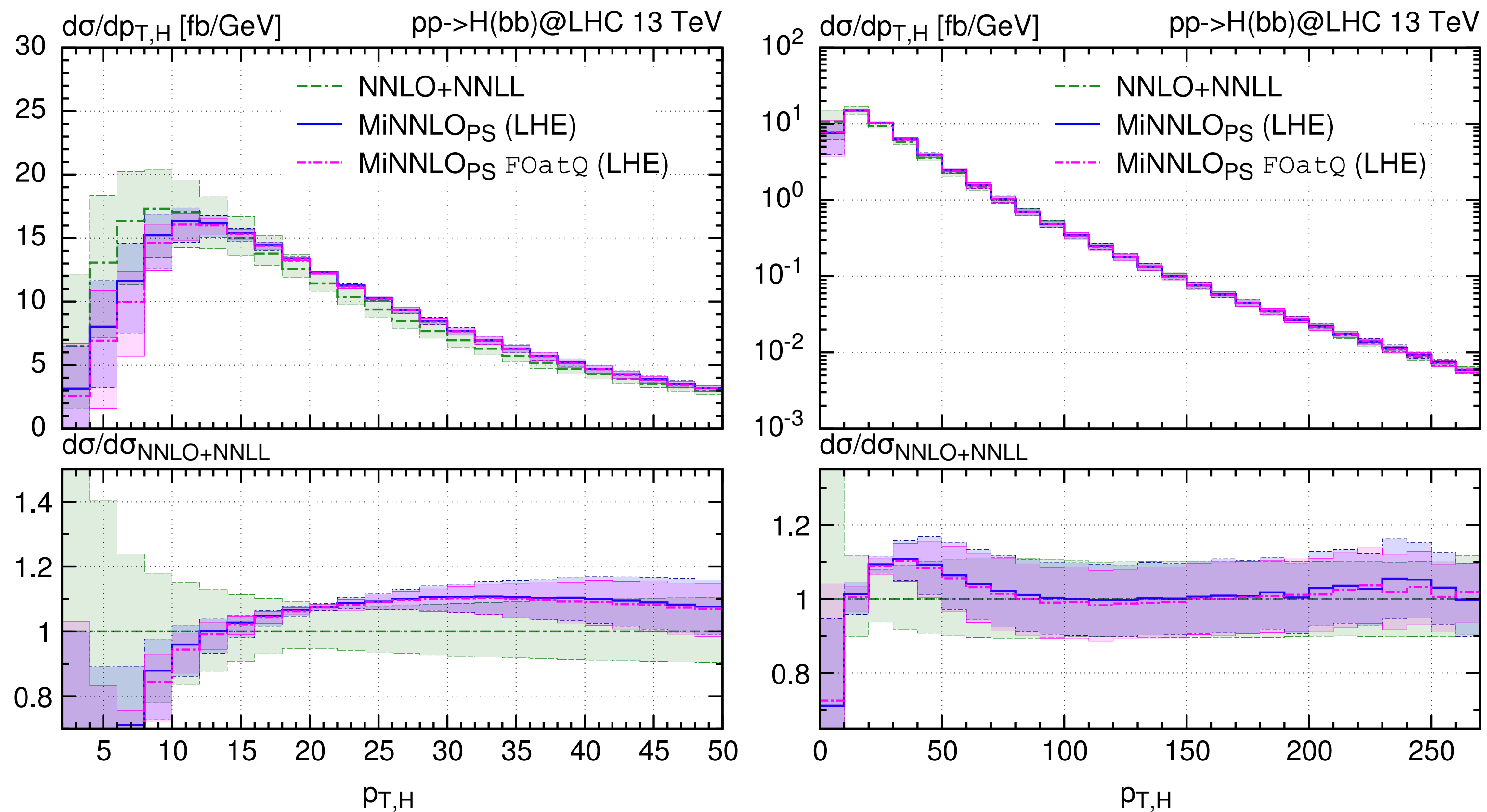
# Jet-observables: jet rapidity





# Resummed results vs LHE

## Transverse momentum spectrum of the Higgs

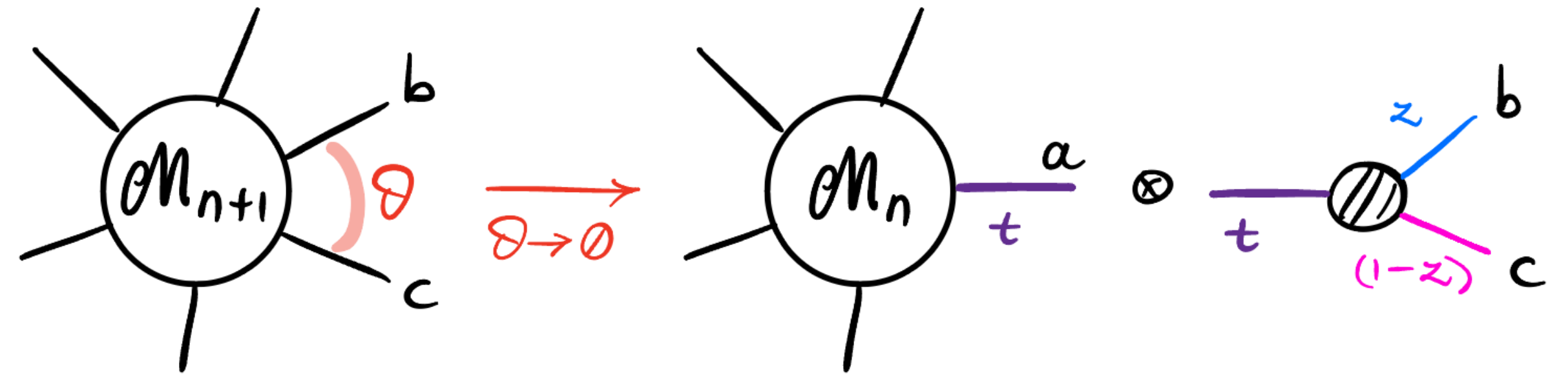


- The agreement is better before the Parton Shower
- In the case of LHE events, there is a perfect agreement at high  $p_{T,H}$  between the analytic and MiNNLOPS distributions



# Shower Monte Carlo

The Parton Shower formalism is based on **collinear factorisation** with a probabilistic description of the splitting process.



Similarly to a **radioactive decay**, the probability of evolving between two scales and emitting no gluons is

$$\Delta_t = \exp \left[ - \int_t \frac{dt'}{t'} dz' d\varphi' \frac{\alpha_s}{2\pi} P(z') \right]$$

*exp(-λt) = non-radiation probability*

Using this form factor we can deduce the SMC prediction with the first emission

$$\langle \mathcal{O} \rangle = \int d\Phi_n B(\Phi_n) \left[ \mathcal{O}(\Phi_n) \Delta_{t_0} + \int_{t_0} \frac{dt}{t} dz d\varphi \mathcal{O}(\Phi_n, \phi_r) \Delta_t \frac{\alpha}{2\pi} P(z) \right]$$

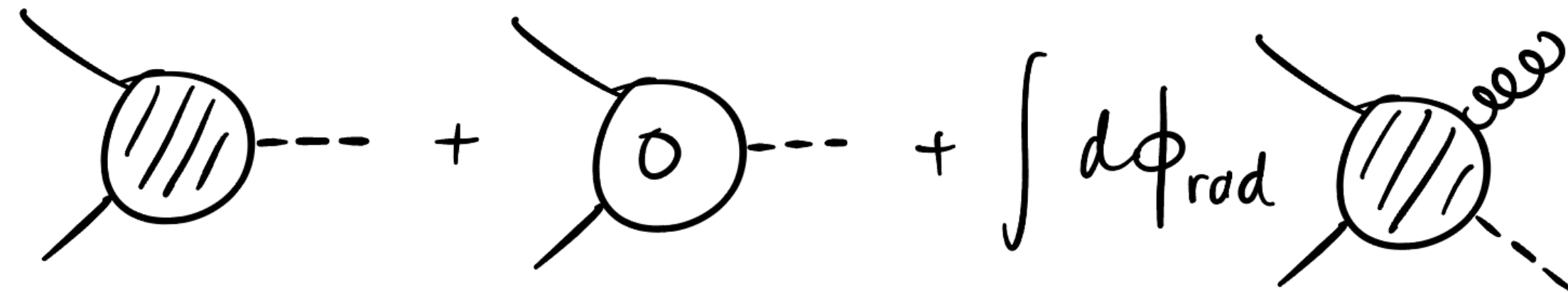
*exp(-λt) λ dt = probability of the 1<sup>st</sup> radiation*

$$\simeq \int d\Phi_n B(\Phi_n) \left[ \mathcal{O}(\Phi_n) + \int_{t_0} \frac{dt}{t} dz d\varphi \left( \mathcal{O}(\Phi_n, \phi_r) - \mathcal{O}(\Phi_n) \right) \frac{\alpha_s}{2\pi} P(z) \right]$$





# NLO



- ✓ **NLO accuracy** for inclusive observables
- ✓ Reduced theoretical uncertainty
- ✓ Correct quantum interference
- Wrong shape for **small- $p_T$**  region
- Description only at the **parton level**
- Computationally expensive

# SMC (LO<sub>PS</sub>)

- Total normalisation accurate only at **LO**
- Poor description at **high- $p_T$**
- Partial interference through shower ordering
- ✓ Sudakov suppression of small- $p_T$  emissions (LL resummation)
- ✓ Simulate high-multiplicity events at the **hadron level**
- ✓ Computationally cheap

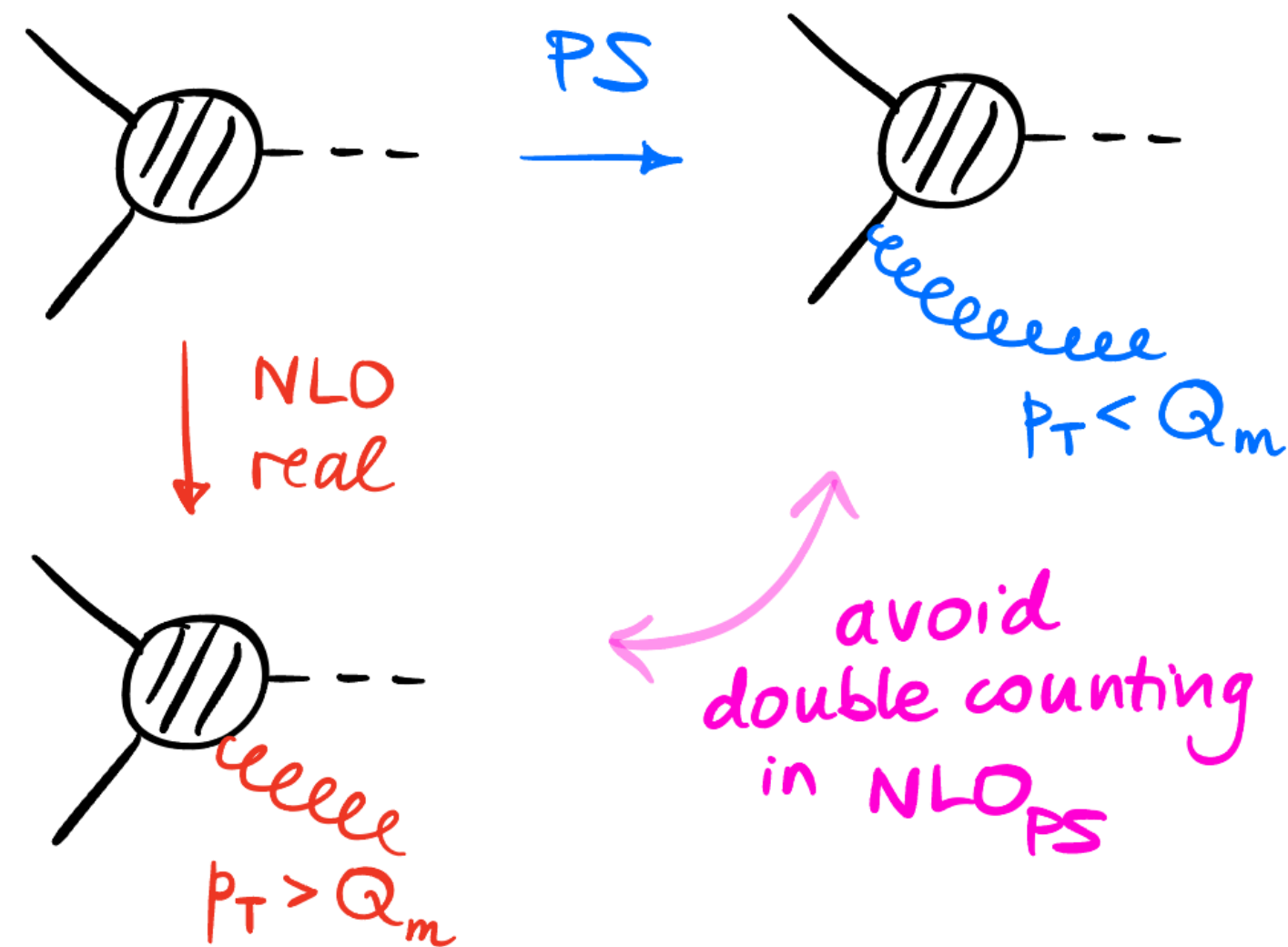
HERWIG, SHERPA, PYTHIA, ...

Approaches are complementary: combine them in a consistent way





# Matching problem



**Double counting** can be easily solved by applying a cut in phase space:

- ▶ **Reject hard jets** produced by PS with  $p_T > Q_m$

But how can we obtain smooth distributions without a critical dependence on the matching scale  $Q_m$ ?

MC@NLO [Frixione, Webber, 2002] and POWHEG [Nason, 2004] are two fully tested solutions.

## POWHEG Idea

Write a simplified Monte Carlo that generates just one emission (the hardest one) which alone gives the correct NLO result.

$$\Delta^{pwg} = \exp \left[ - \int \text{exact real-radiation probability above } p_T \right]$$



# POWHEG in a nutshell

$$\bar{B} = B + V + \int d\phi_{rad} R$$

The exact NLO prediction is

$$\langle \mathcal{O} \rangle = \int d\Phi_n \mathcal{O}(\Phi_n) \bar{B}(\Phi_n) + \int d\Phi_n d\phi_{rad} (\mathcal{O}(\Phi_n, \phi_{rad}) - \mathcal{O}(\Phi_n)) R(\Phi_n, \phi_{rad})$$

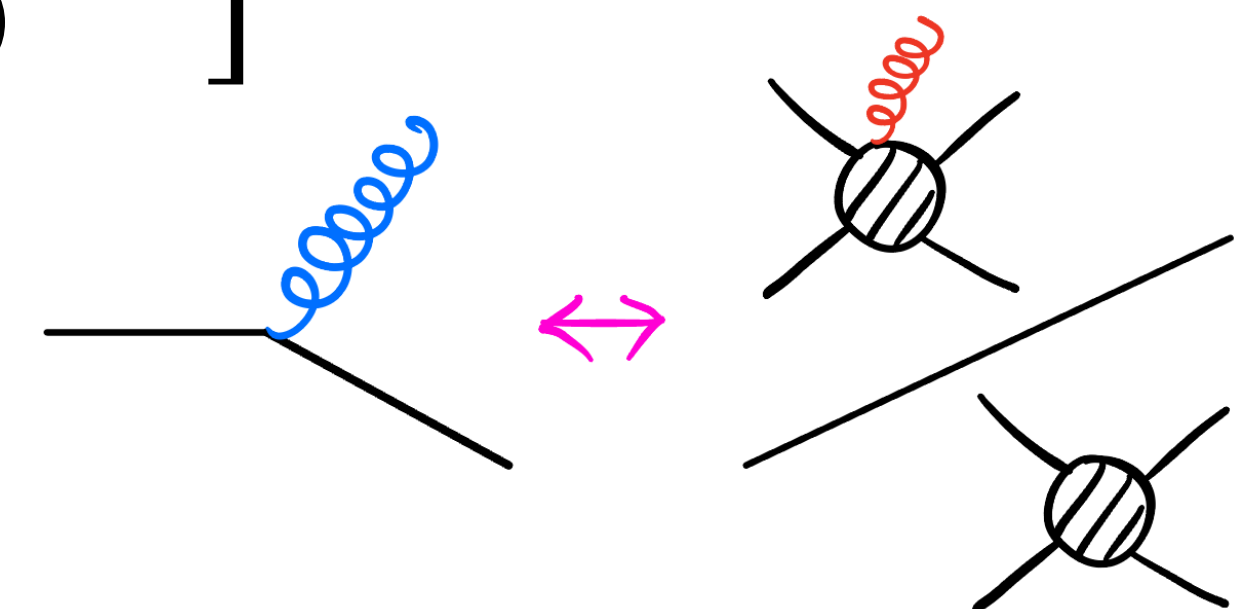
Comparing with the SMC

$$\langle \mathcal{O} \rangle_{SMC} \simeq \int d\Phi_n \left[ \mathcal{O}(\Phi_n) B(\Phi_n) + B(\Phi_n) \int_{t_0}^t \frac{dt}{t} dz d\varphi (\mathcal{O}(\Phi_n, \phi_r) - \mathcal{O}(\Phi_n)) \frac{\alpha_s}{2\pi} P(z) \right],$$

we deduce the Sudakov form factor and the shower formula in POWHEG

$$\langle \mathcal{O} \rangle = \int d\Phi_n \bar{B}(\Phi_n) \left[ \mathcal{O}(\Phi_n) \Delta_{t_0}^{pwg} + \int d\phi_{rad} \mathcal{O}(\Phi_n, \phi_{rad}) \Delta_t^{pwg} \frac{R(\Phi_n, \phi_{rad})}{B(\Phi_n)} \right]$$

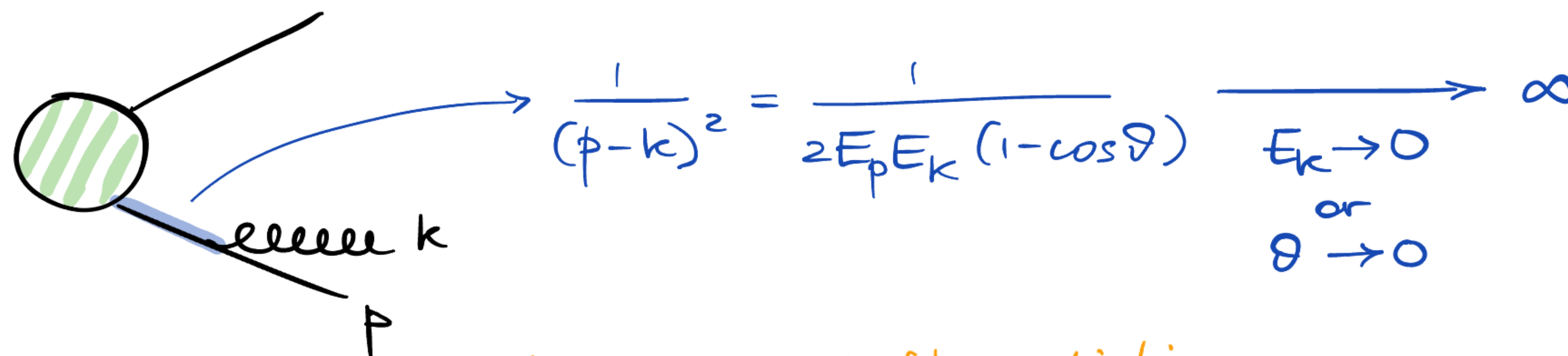
$$\text{with } \Delta_t^{pwg} = \exp \left[ - \int d\phi'_{rad} \frac{R(\Phi_n, \phi'_{rad})}{B(\Phi_n)} \Theta(t' - t) \right]$$



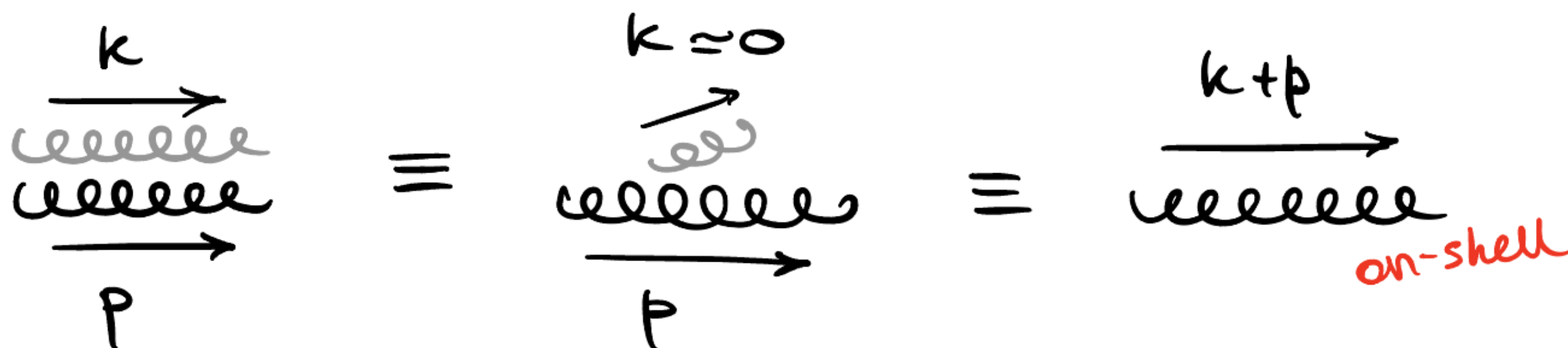
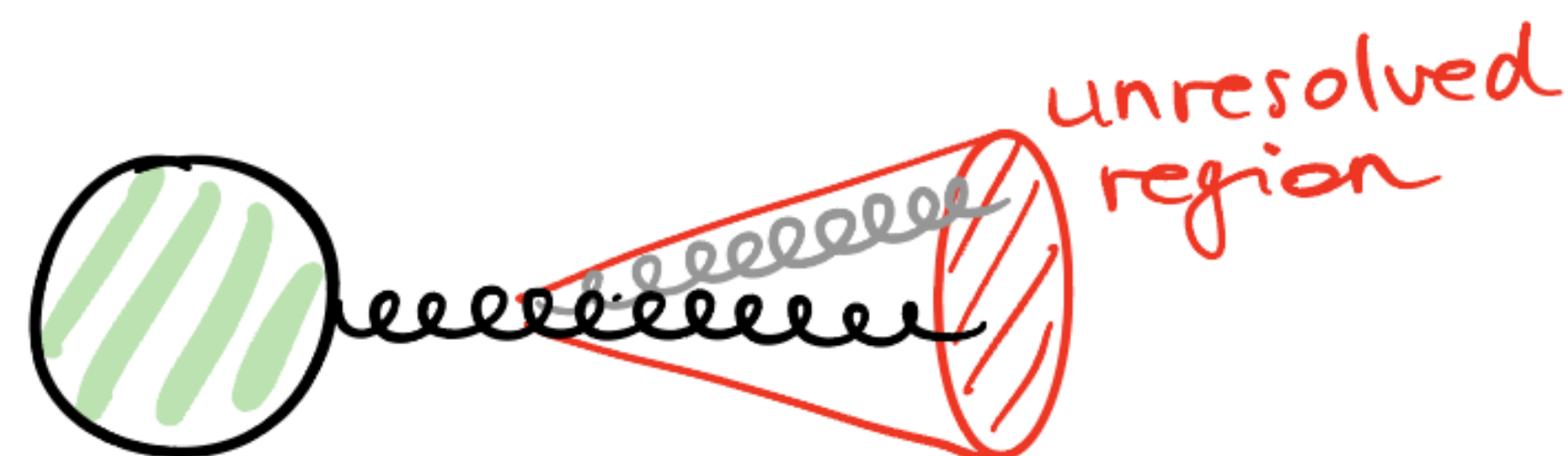


# IR divergences

The radiation of a *massless* particle produces divergences: a manifestation of the degeneration of these states



$E_k \rightarrow 0$       Soft radiation  
 $\vartheta \rightarrow 0$       Collinear radiation

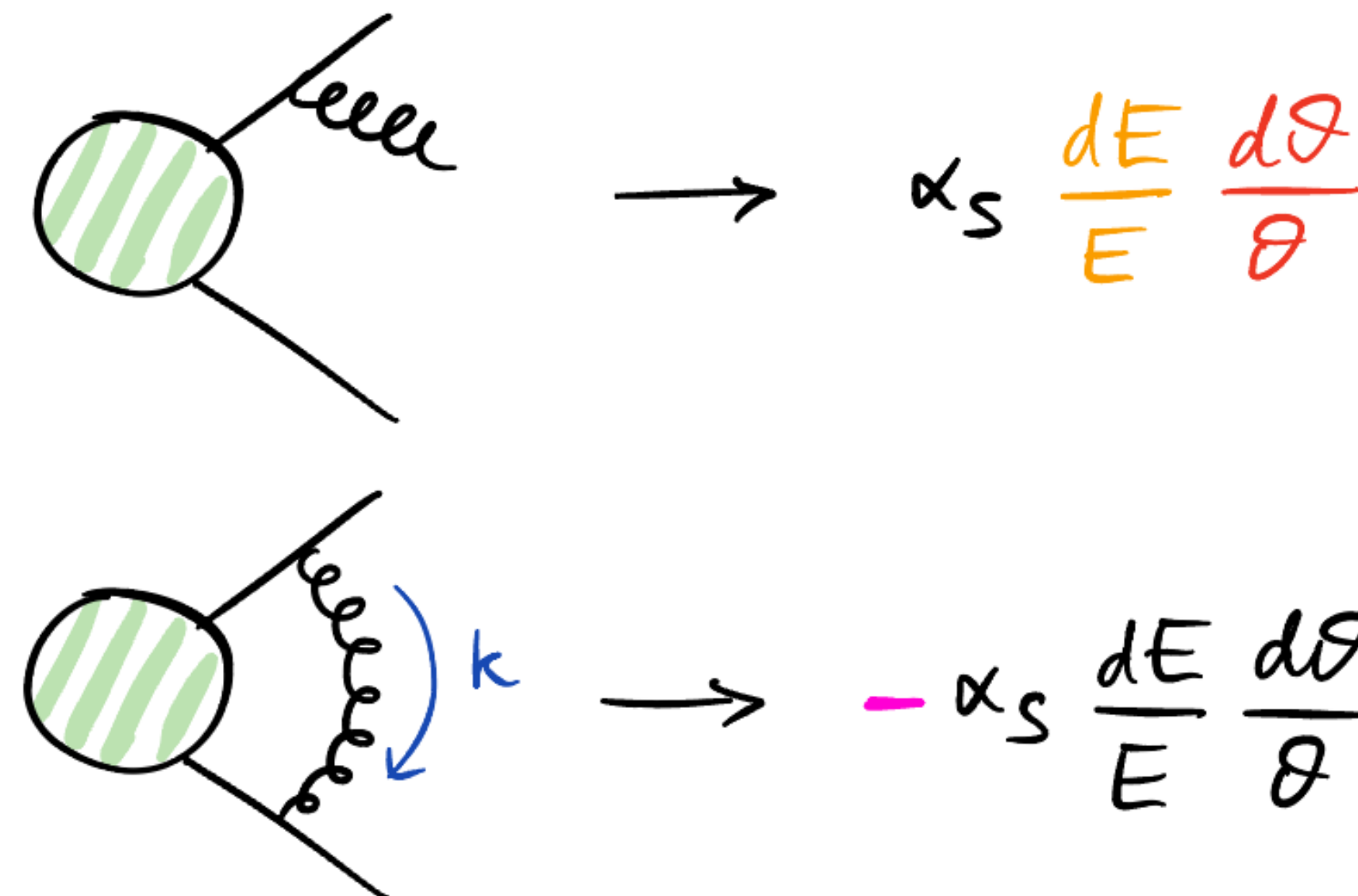




# Logs as residues of IR divergences

A divergent structure is also present in the virtual contribution.

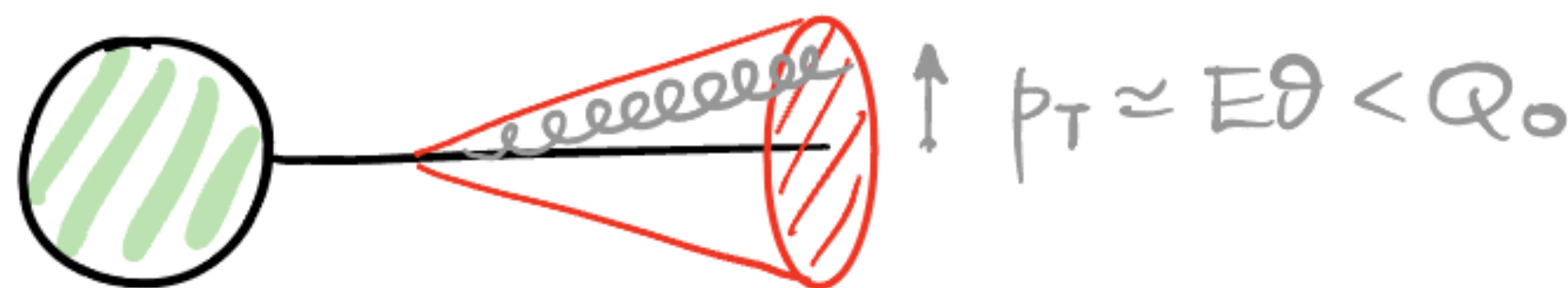
The **IR divergences cancel out** order by order in perturbation theory!



The IR divergences are cancelled, but if we are exclusive...

$$-\alpha_s \int_0^Q \frac{dE}{E} \frac{d\theta}{\theta} \Theta(E\theta < Q_0) \Big|_{\text{real}} + \alpha_s \int_0^Q \frac{dE}{E} \frac{d\theta}{\theta} \Big|_{\text{virt}} = \alpha_s \ln^2 \frac{Q}{Q_0}$$

Double Log







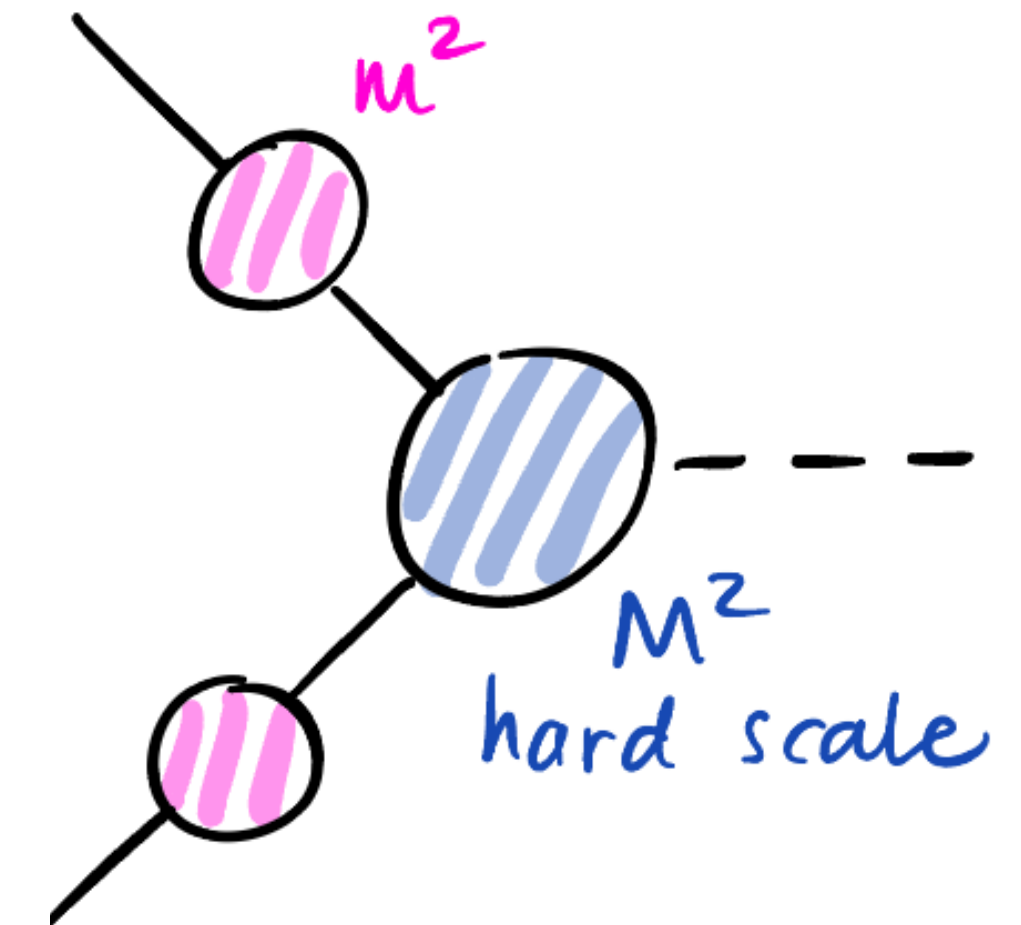
# Resummation from factorisation

Consider a physical quantity  $\mathcal{O}(M^2, m^2)$  in which  $m^2$  measures the distance from the IR region.

If  $m^2 \ll M^2$ ,

$$\mathcal{O}(M^2, m^2) = H \left( \frac{M^2}{\mu^2} \right) S \left( \frac{m^2}{\mu^2} \right)$$

Hard                      Soft



$$\mathcal{O} \text{ is } \mu \text{ - independent} \Rightarrow \frac{1}{H} \frac{d \ln H}{d \ln \mu^2} = - \frac{1}{S} \frac{d \ln S}{d \ln \mu^2} =: \gamma(\mu^2)$$

Solving the differential equation,

$$\mathcal{O}(M^2, m^2) = H(1) S(1) \exp \left[ - \int_{m^2}^{M^2} \frac{dq^2}{q^2} \gamma(q^2) \right]$$

✓ for  $m^2 \rightarrow 0$

**Sudakov form factor:**  
 it captures at *all order*  
 the log-enhanced terms





# Transverse momentum resummation

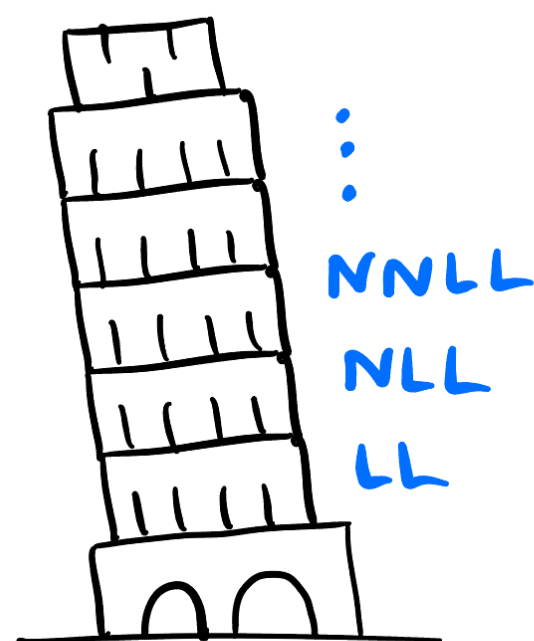
What is the probability that a boson is produced with transverse momentum  $< p_T$ ?

$$\mathcal{P} \simeq -\#\alpha_s \ln^2 \frac{Q}{p_T} + \mathcal{O}(\alpha_s^2) \rightarrow \exp \left[ -\#\alpha_s \ln^2 \frac{Q}{p_T} \right]$$

for small  $p_T$  we need to sum up the logs

In general we have a tower of logs

$$\exp \left[ -\sum_{n,m} \alpha_s^n \ln^m \frac{Q}{p_T} \right]$$



$$m = n + 1$$

→ Leading Logs (LL)

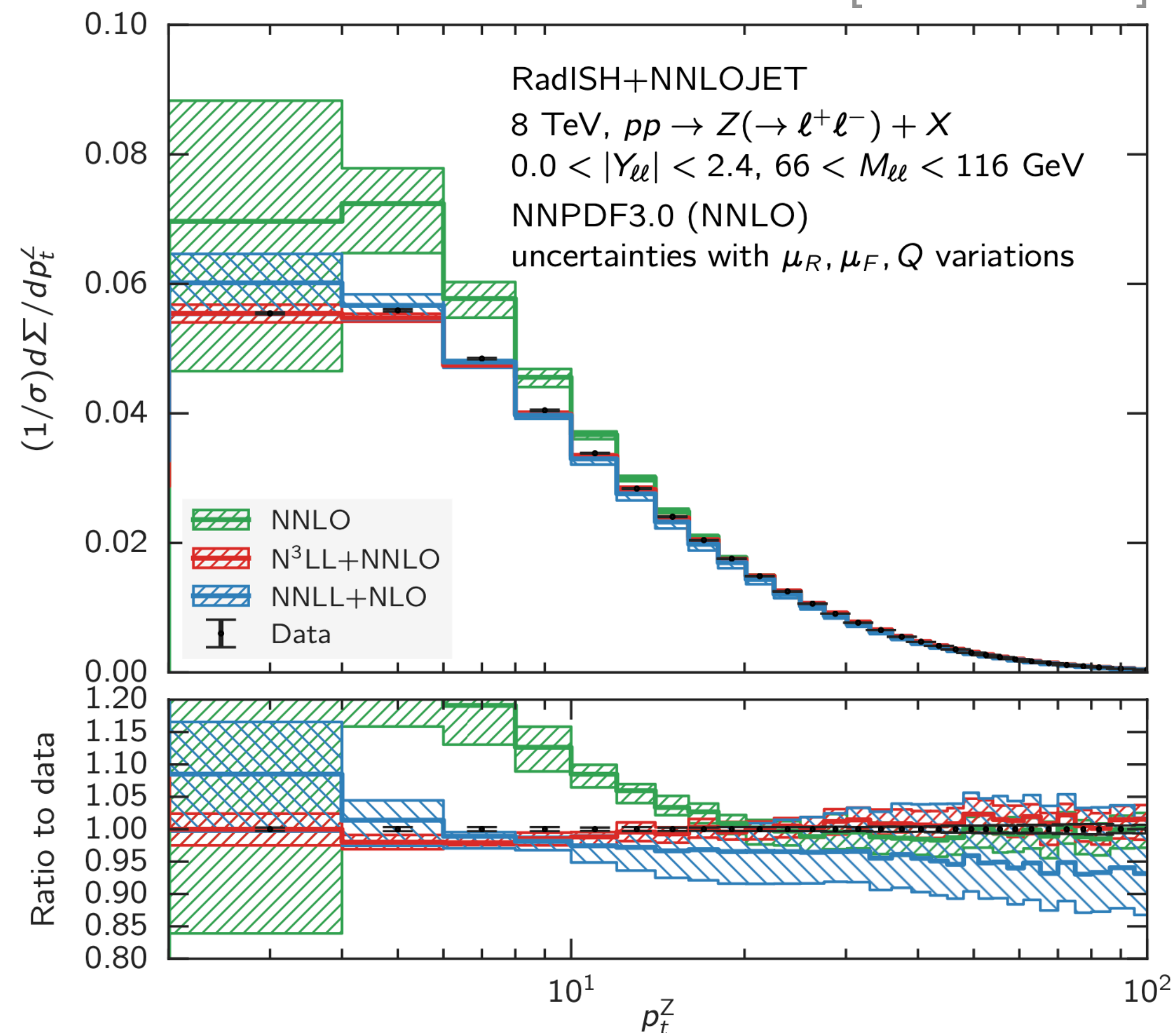
$$m = n$$

→ Next-To-LL (NLL)

$$m = n - 1$$

→ Next-To-NLL (NNLL) ...

Brizon et al. [1805.05916]



# MiLO

$$\sigma_{Xj}^{FO} = \sigma_1 + \alpha_s \sigma_2 + \dots$$

FO cross section



$$\mathcal{F} = \exp[\dots] = 1 + \alpha_s S_1 + \alpha_s^2 S_2 + \dots$$

Sudakov form factor

We introduce the MiLO cross section

$$\sigma_{MiLO} := \mathcal{F} \sigma_1 = \sigma_{Xj}^{FO} (1 + \mathcal{O}(\alpha_s))$$

We want to fix  $\mathcal{F}$  in order to obtain

$$\frac{d\sigma_{MiLO}}{dp_T^2 dy} \sim \frac{d}{dp_T^2} \left\{ \mathcal{F}(p_T, Q) f_a(x_a, p_T) f_b(x_b, p_T) \right\}$$

PDF  $f_i(x_i, \mu_F)$

such that

$$\frac{d\sigma}{dy} = \int_0^Q dp_T^2 \frac{d\sigma_{MiLO}}{dp_T^2 dy} \sim f_a(x_a, Q) f_b(x_b, Q)$$

X cross section at fixed rapidity  $y$

$$\mathcal{F}(Q, Q) = e^0 = 1$$
$$\mathcal{F}(0, Q) = e^{-\infty} = 0$$



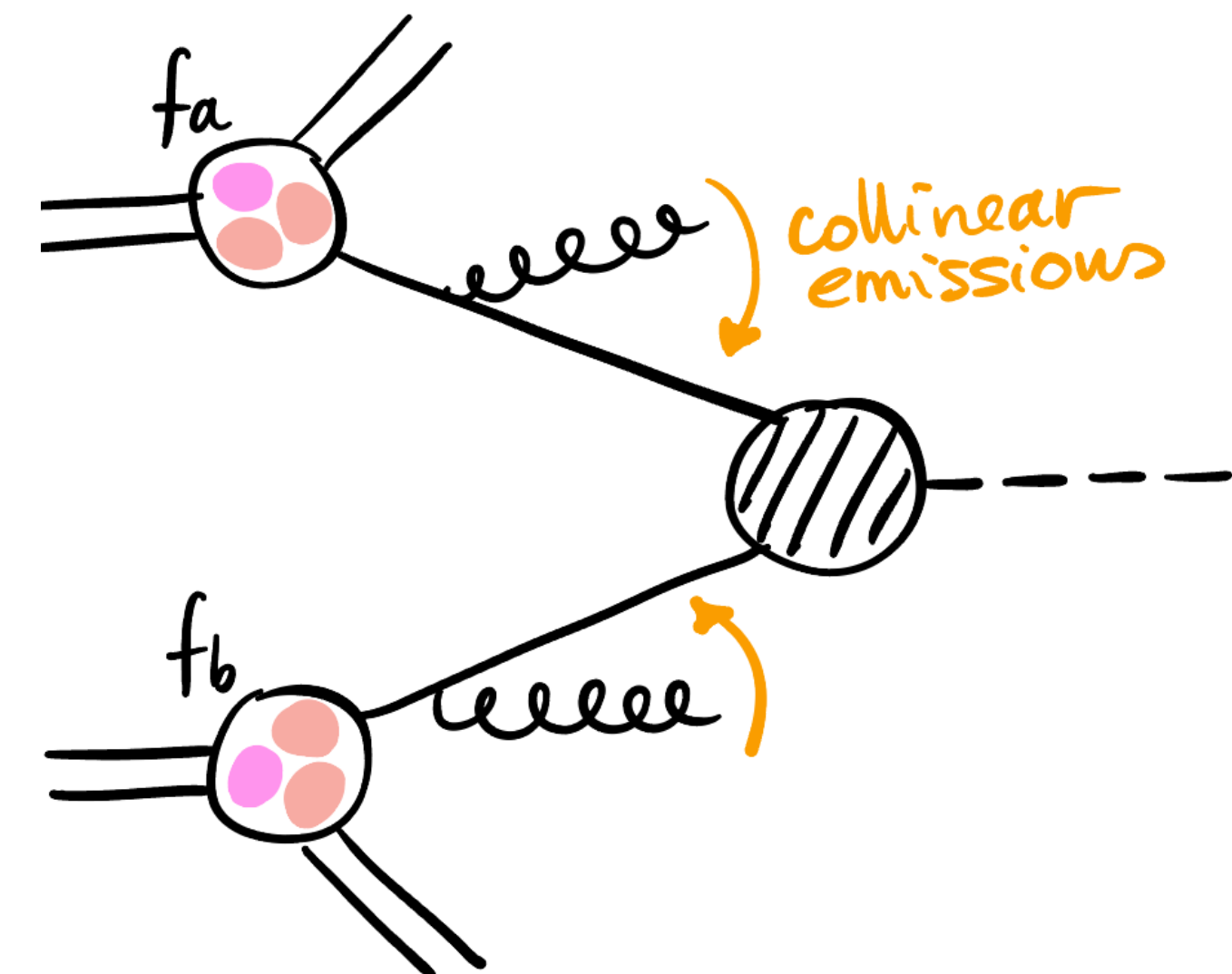
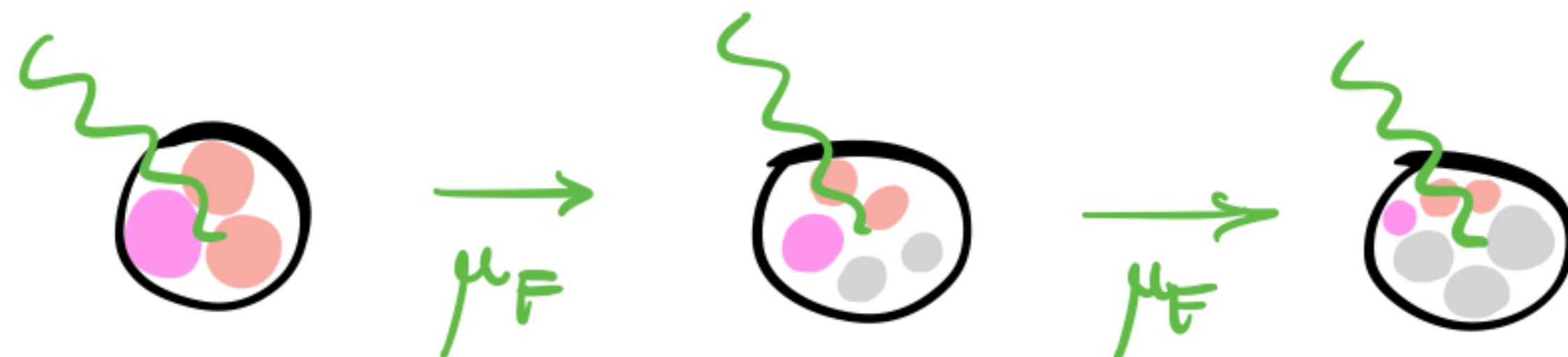
In the singular part of the  $X_j$  cross section,

$$\frac{d\sigma_{X_j}^{sing}}{dp_T^2 dy} = \frac{1}{p_T^2} \sum_{nm} \alpha_s^n(\mu_R) \mathcal{C}_{nm} \ln^m \frac{p_T^2}{Q^2} = \frac{1}{p_T^2} \alpha_s(\mu_R) \left( \mathcal{C}_{11} \ln \frac{p_T^2}{Q^2} + \mathcal{C}_{10} + \dots \right),$$

where  $\mathcal{C}_{10} \supset \frac{1}{\alpha_s(\mu_F)} \mu_F^2 \frac{d}{d\mu_F^2} (f_a f_b)$

This contribution is related to **DGLAP evolution**

The **collinear divergences** of initial states can be reabsorbed in PDFs causing their running







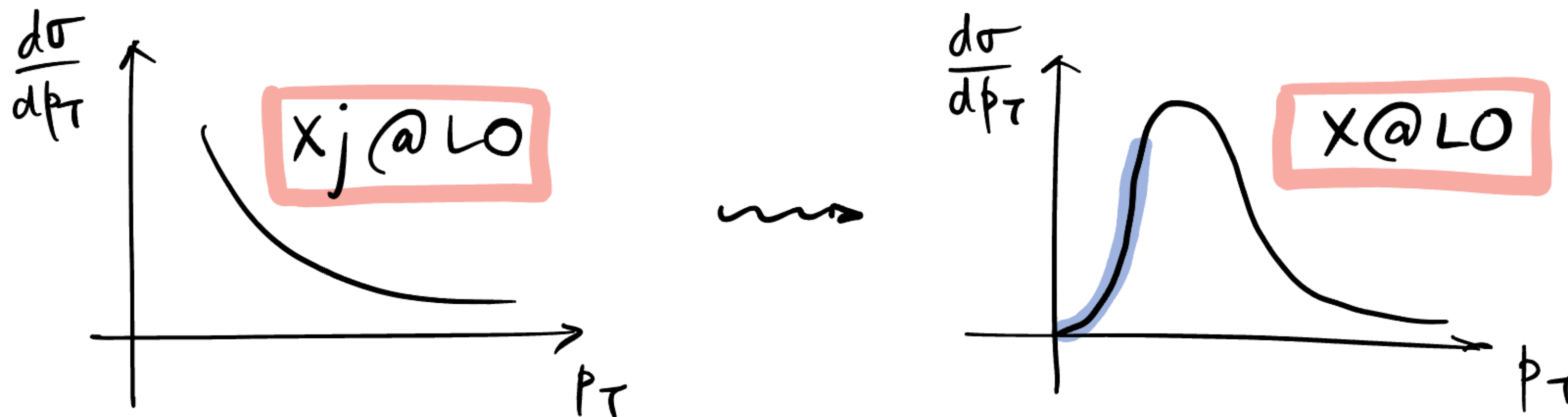
If we set  $\mu_R = p_T$  and  $\mu_F = p_T$ , the resummed cross section becomes

$$\frac{d\sigma_{\text{MiLO}}}{dp_T^2 dy} = \mathcal{F}(p_T, Q) \frac{d\sigma_{X_j}^{\text{LO}}}{dp_T^2 dy} = \frac{d}{dp_T^2} \left\{ \mathcal{F}(p_T, Q) f_a(x_a, p_T) f_b(x_b, p_T) \right\}$$

iff

$$\mathcal{F}(p_T, Q) = \exp \left[ - \int_{p_T}^Q \frac{d\mu^2}{\mu^2} \alpha_s(\mu^2) \left( A_1 \ln \frac{Q^2}{\mu^2} + B_1 \right) \right]$$

**MiLO Sudakov:**  
essential to capture  
the logs at low  $p_T$

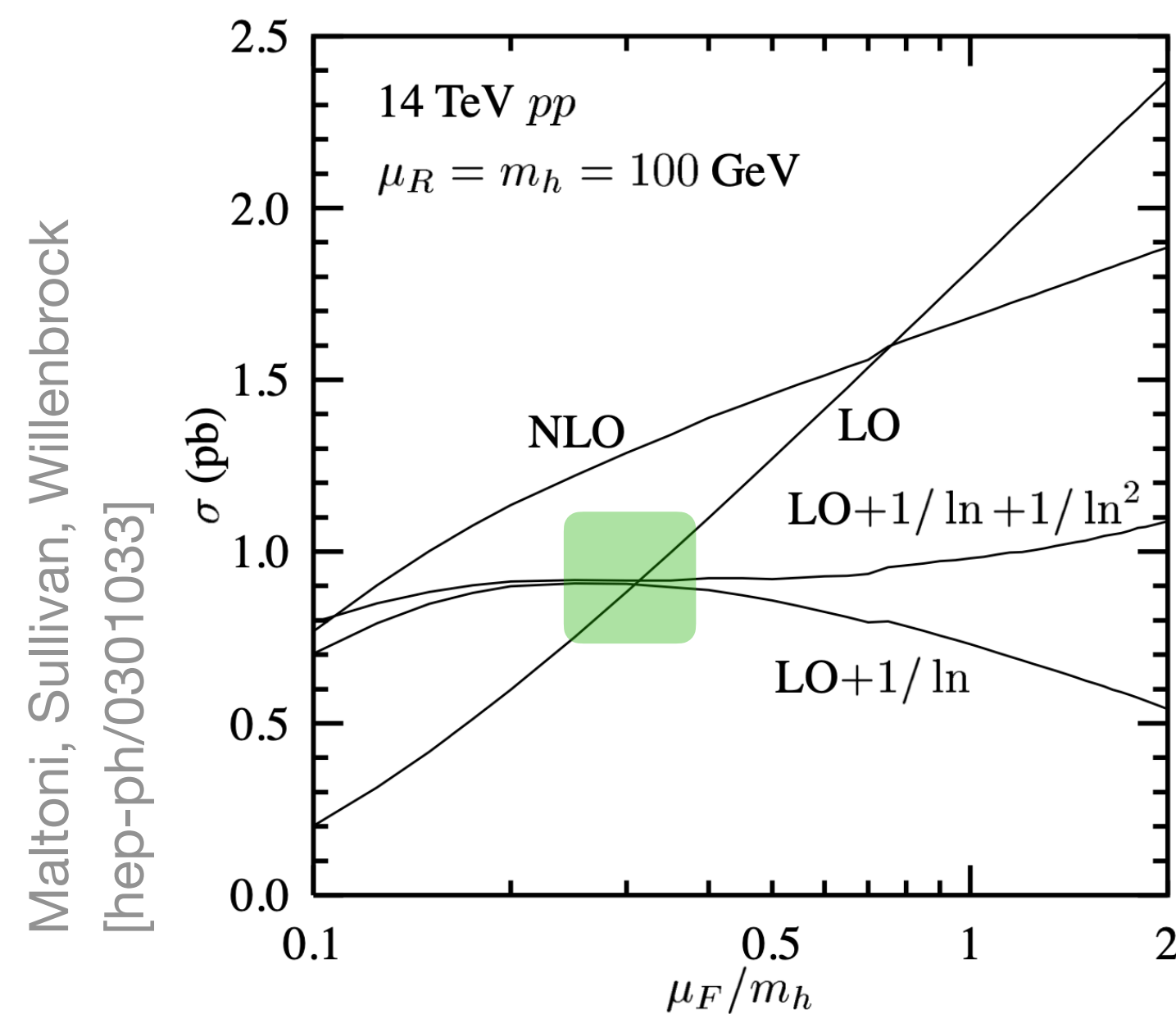
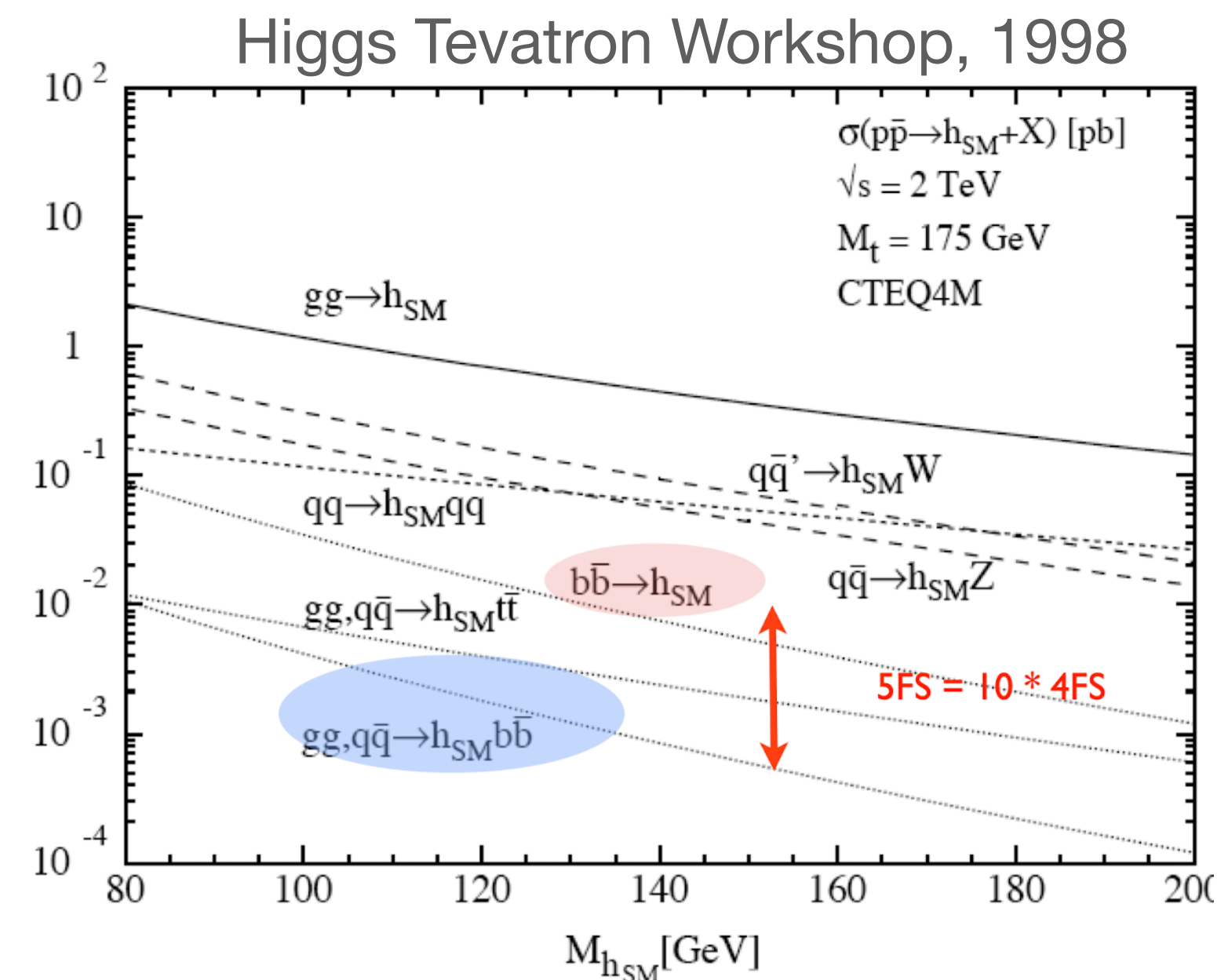


From a manipulation of FO  $X_j$  cross section, we obtained inclusive predictions.



# Historical LO comparisons

Large differences in the predictions were first observed at the leading order: the effect of collinear resummation is extremely large.



For  $\mu_F = m_H/4$ , FO computations in the different schemes become compatible, indeed the collinear logs have a small effect. This also improved the convergence of the perturbation series.

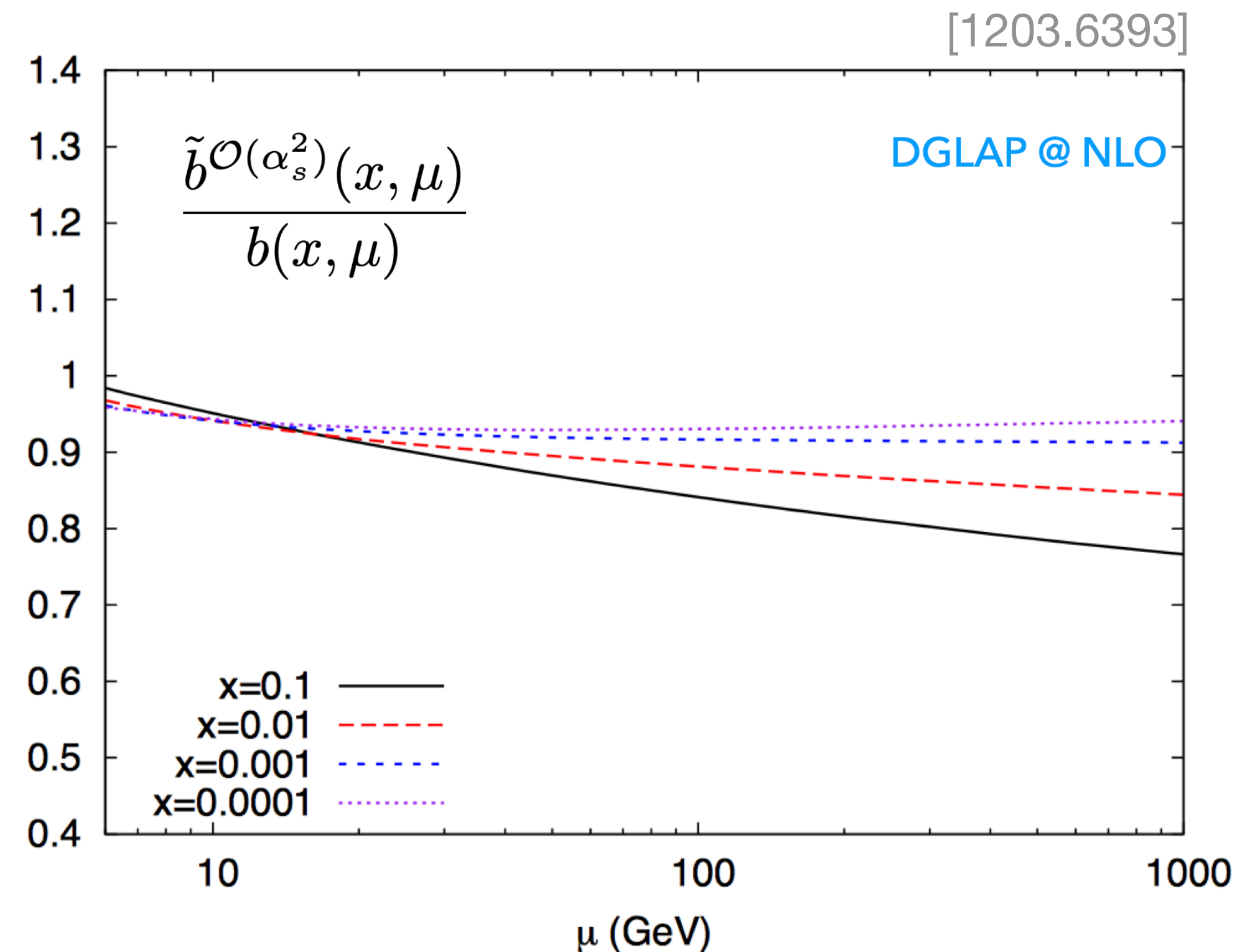
The improvement of the compatibility opens the possibility to match together the predictions at least at the inclusive level (Santander matching, FONLL...)





# Differences between schemes

Lot of progress in understanding the origin of the differences. The predictions can be merged into a consistent picture by taking into account two main results.



1. At NLO, the resummation effects of collinear logs are important only at high Bjorken- $x$
2. The possibly large ratios  $m_H^2/m_b^2$  are always accompanied by universal phase space factors  $f$

$$\ln^2 \frac{m_H^2 f}{m_b^2} = \ln^2 \frac{\tilde{\mu}^2}{m_b^2}, \quad \tilde{\mu} < m_H$$

# FONLL



- FONLL matches the flavour schemes

$$\sigma^{FONLL} = \sigma^{4FS} + \sigma^{5FS} - \text{double counting.}$$

For a consistent subtraction, we have to express the two cross-sections in terms of the same  $\alpha_s$  and PDFs.

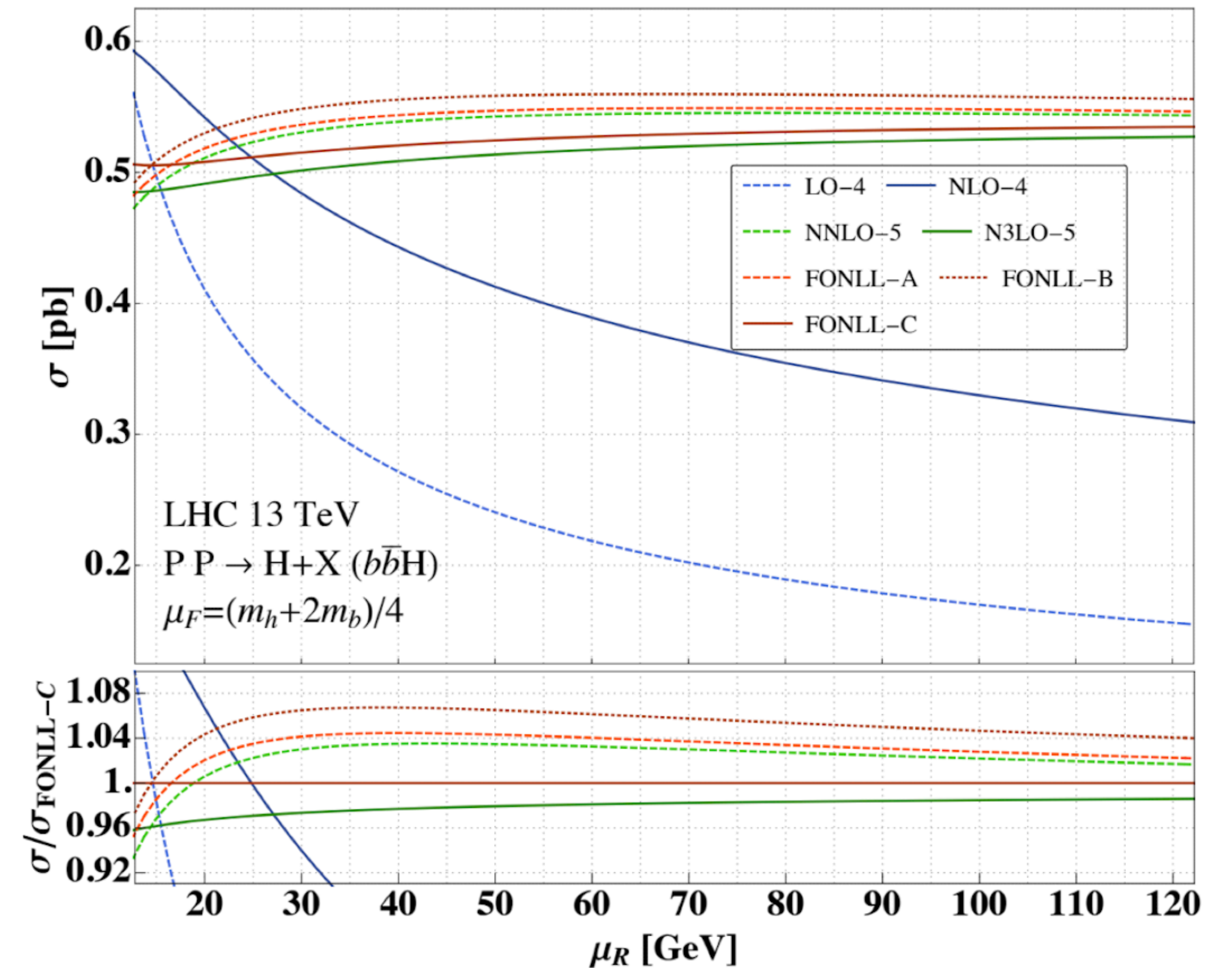
- Currently, the flavour matching for  $bbH$  is performed at

$$FONLL_C := N^3LO_{5FS} \oplus NLO_{4FS}.$$

- Differential FONLL applied for  $Z+b$ -jet

$$d\sigma^{FONLL} = d\sigma^{5FS} + \left( d\sigma_{m_b}^{4FS} - d\sigma_{m_b \rightarrow 0}^{4FS} \right)$$

Duhr, Dulat, Hirschi, Mistlberger [2004.04752]



[Gauld, Gehrmann-De Ridder,  
 Glover, Huss, Majer, 2005.03016]



# Exclusive observables

Recent developments in fully differential calculations, for example:

1. Introduce an unphysical scale  $\mu_b$  in order to switch from 4FS to 5FS in a region where mass effects and collinear logs are not crucial [Bertone, Glazov, Mitov, Papanastasiou, Ubiali, 1711.03355]
2. Massive 5FS at NLO [Krauss, Napoletano, 1712.06832]
3. Differential FONLL applied for Z+b-jet [Gauld, Gehrmann-De Ridder, Glover, Huss, Majer, 2005.03016]

$$d\sigma^{FONLL} = d\sigma^{5FS} + \left( d\sigma_{m_b}^{4FS} - d\sigma_{m_b \rightarrow 0}^{4FS} \right)$$