Heavy Quark Fragmentation to NNLO+NNLL Accuracy in Perturbative QCD in e+e- Collisions

Milan Christmas Meeting - 22/12/2023

Universität Zürich[UZH]

Swiss National Science Foundation

Leonardo Bonino
in collaboration with
Matteo Cacciari (LPTHE) and Giovanni Stagnitto (UniMiB)

Introduction

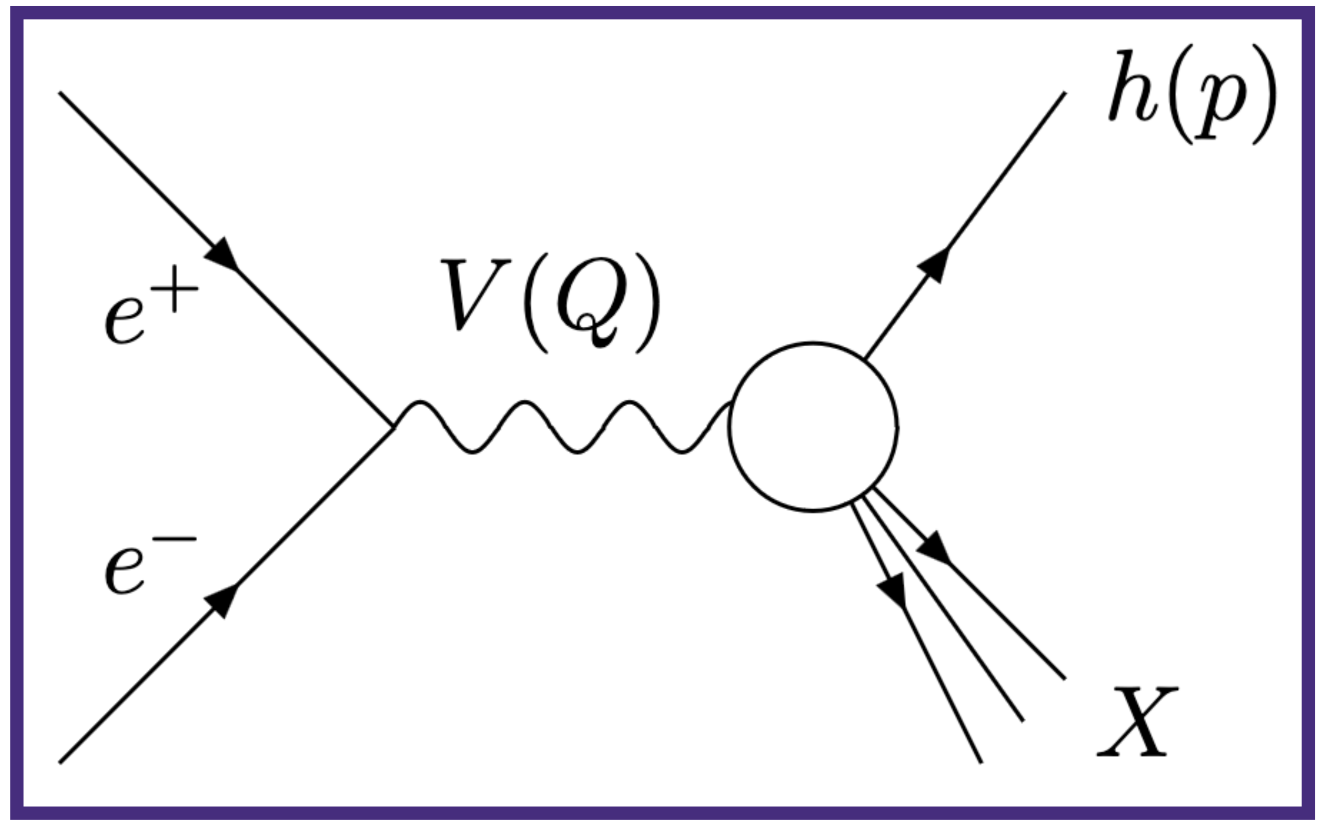
Motivation

- How well can we describe hadronisation at large-p_T ?
- "Stress-test" of perturbative QCD → improve knowledge of non-perturbative physics
- Fragmentation functions are universal (process independent)
  - e^+e^- results useful for LHC phenomenology
- Growing interest in e^+e^- fragmentation
  - NLO + NLL phenomenology available from 2005 **[Cacciari et al._0510032]**
  - Next-to-next frontier started recently
- Work based on [2312.12519]

Theory overview

Fragmentation function formalism

- Goal: describe fragmentation of heavy quarks into hadrons
- Process: $e^+e^- \to V(Q) \to h(p) + X$
- Tool: QCD perturbative fragmentation function formalism
  - Factorisation theorem → hadron cross section $\sigma_h(x, Q)$
    $$\sigma_h\left(x, Q\right) \simeq \sigma_{\mathcal{Q}}(x, Q, m) \otimes D^{np}_{\mathcal{Q}\to h}(x, \{\text{par}\})$$
  - Fully perturbative cross section for heavy quark $\mathcal{Q}$ of mass m
    $$\sigma_{\mathcal{Q}}(x, Q, m) = \hat{\sigma}_i(x, Q, \mu_F) \otimes D_{i\to\mathcal{Q}}(x, \mu_F, m) + \mathcal{O}\left((m/Q)^p\right)$$ [Mele, Nason_1991 B 361 626]
- Analytically also in Mellin space (convolution → simple product)



μ_F: factorisation scale

Theory overview

Perturbative ingredients

- $$\frac{1}{\sigma^{tot}_{\mathcal{Q}}}\sigma_{\mathcal{Q}}\left(Q,m\right) = \frac{1}{\sigma^{tot}_{\mathcal{Q}}}\sigma^{(0)}\sum_{i,j} C_i\left(Q,\mu,\mu_F\right) E_{ij}\left(\mu_F,\mu_{0F}\right) D_{j\to\mathcal{Q}}\left(\mu_{0F},m\right)$$
    - Initial conditions $D_{j\to\mathcal{Q}}$ @ NNLO [Melnikov, Mitov_0404143] [Mitov_0410205] [Maltoni et al._2207.10038]
    - DGLAP Evolution E_{ij} with MELA [Bertone et al._1501.00494] [Ridolfi et al._1911.01975]
    - Coefficient functions C_i @ NNLO [Rijken, van Neerven_B487 (1997) 233-207] [Blümlein, Ravindran_0604019] [Mitov, Moch_0604160]
- Poor behaviour in large-N $(x \to 1)$ region (Sudakov region)
    - Need resummation @ NNLL in initial conditions and coefficient functions [Cacciari, Catani_0107138] [Aglietti et. al_0610035] [Maltoni et al._2207.10038] [Czakon et al._2210.06078]

Theory overview

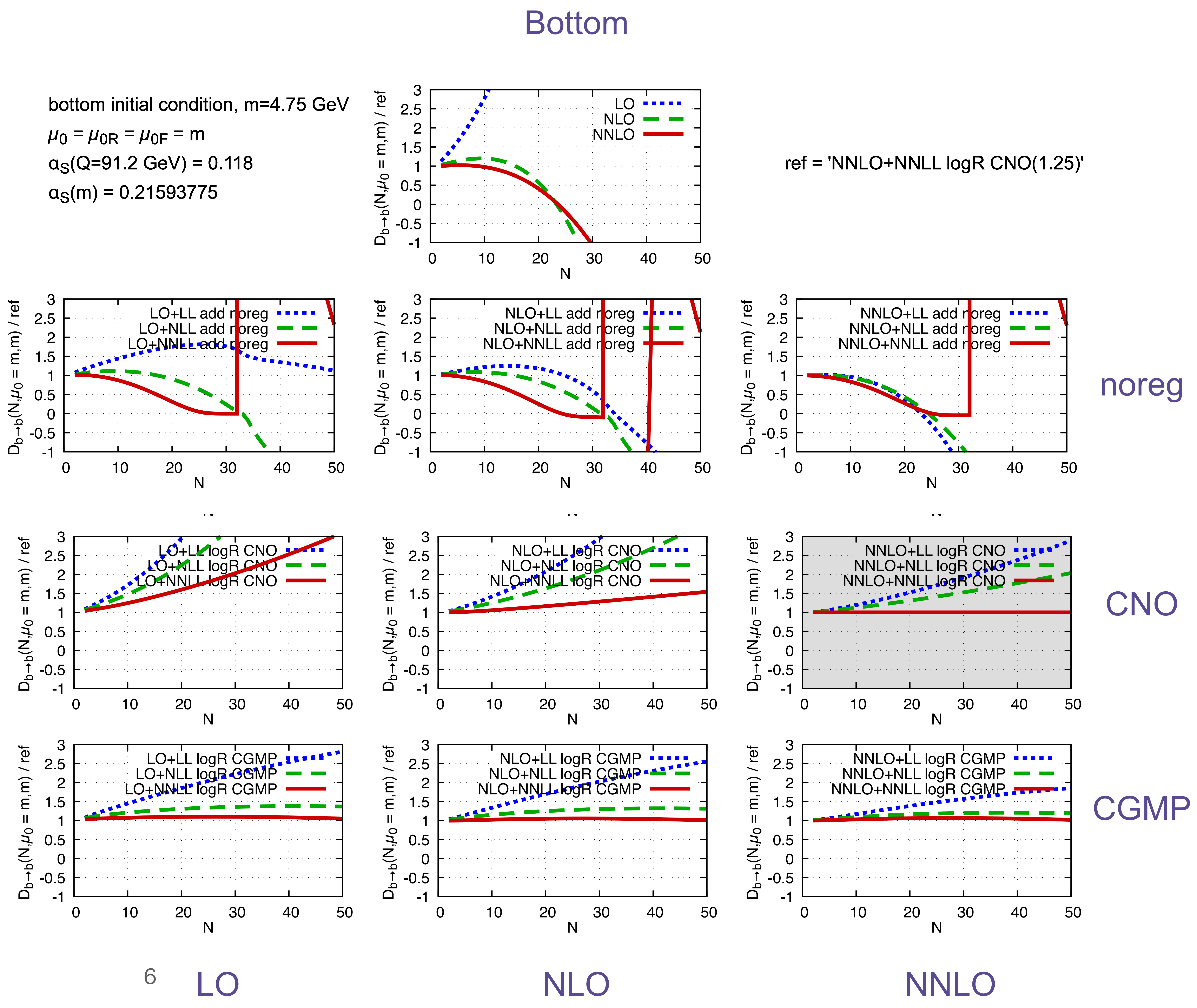
Soft-gluon resummation

- Sudakov-resummed initial conditions
  - Constant large-N limit of fixed order result & Sudakov form factor
    $$D^{res}_{Q\to Q} = \left[D_{Q\to Q}\right]_c \exp\left[\ln N g^{(1)}_{\rm ini}(\lambda_0) + g^{(2)}_{\rm ini}(\lambda_0) + \alpha_S g^{(3)}_{\rm ini}(\lambda_0)\right]$$
  - Different matchings to fixed order result (e.g. $\log \mathrm{R}$)
    $$\log D^{fo+res,\log\mathrm{R},reg}_{i\to Q} = \log D^{fo}_{i\to Q} + \log D^{res,reg}_{i\to Q} - [\log D^{res(,reg)}_{i\to Q}]_{\alpha_s^p}$$
- Landau pole in $g^{(i)}_{\rm ini}(\lambda_0) : \lambda_0 = 1/2 \to N^L_0 = \exp\left(1/(2b_0\alpha_s)\right)$
  - $N^L_0 \sim 7$ for charm @ $10.6\,\mathrm{GeV}$ & $N^L_0 \sim 32$ for bottom @ $91.2\,\mathrm{GeV}$ → (many) prescriptions
  - "CNO" (Cacciari-Nason-Oleari): shift in N **[Cacciari et al._0510032]**
  - "CGMP" (Czakon-Generet-Mitov-Poncelet): truncation of Sudakov factor **[Czakon et al._2210.06078]**

Numerical results

Heavy quark initial condition

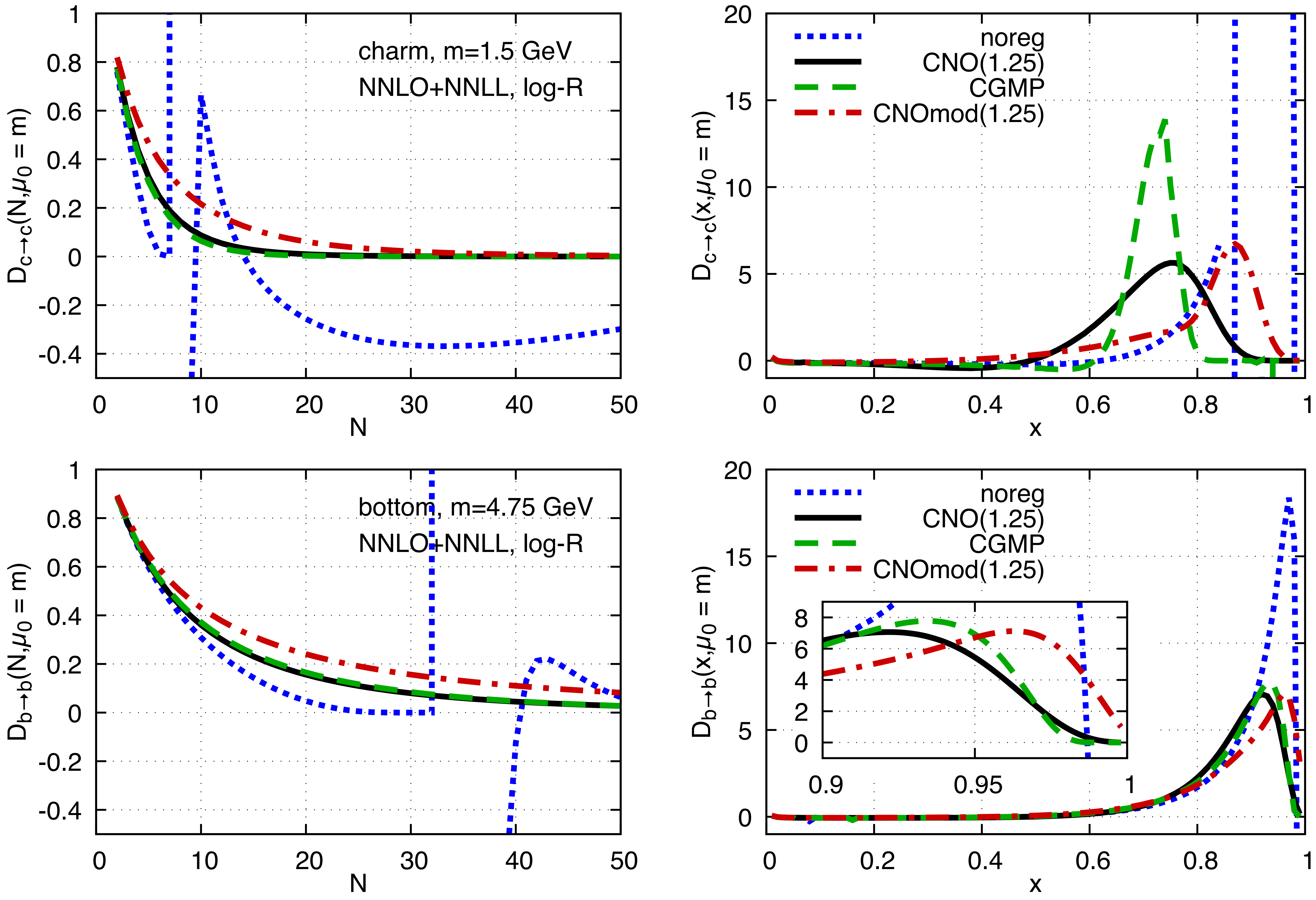
- Full range of perturbative orders
  - Ratio to NNLO+NNLL log-R CNO
- No obvious perturbative hierarchy NNLL < NLL < LL
- No systematic convergence
  - But for log-R CGMP
- NNLO+NNLL log-R CNO (default)



Numerical results

Heavy quark initial condition

- Prescription chosen for Landau pole has huge impact
- Some shapes not suited for fits
  - NNLO+NNLL log-R (default)
- Lower mass → more sensitivity to Landau
- Evolution and convolution do not help …
- What about the full FF?



Charm

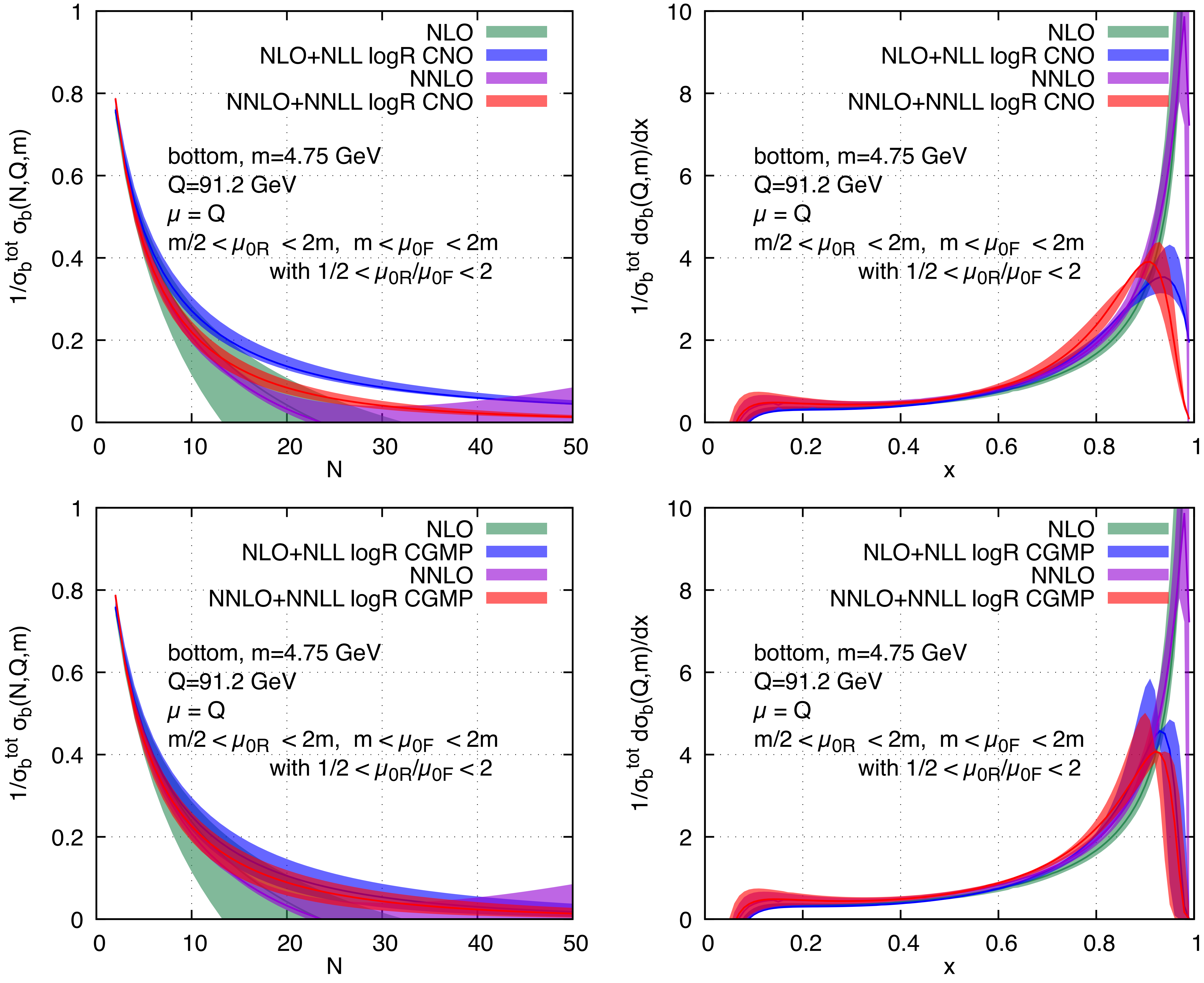
Bottom

Numerical results

Full e^+e^- fragmentation function

- Uncertainty bands for μ_{0R} and μ_{0F} 5-point scale variation in initial conditions (around m)
  - NNLO+NNLL band (red) not much narrower then NLO+NLL one (blue)
  - Bands do not overlap → poor convergence of perturbative series (CNO)
- Drastic dependence on Landau pole regularisation
- Perturbative hierarchy better respected for μ_R and μ_F variations around Q

Bottom



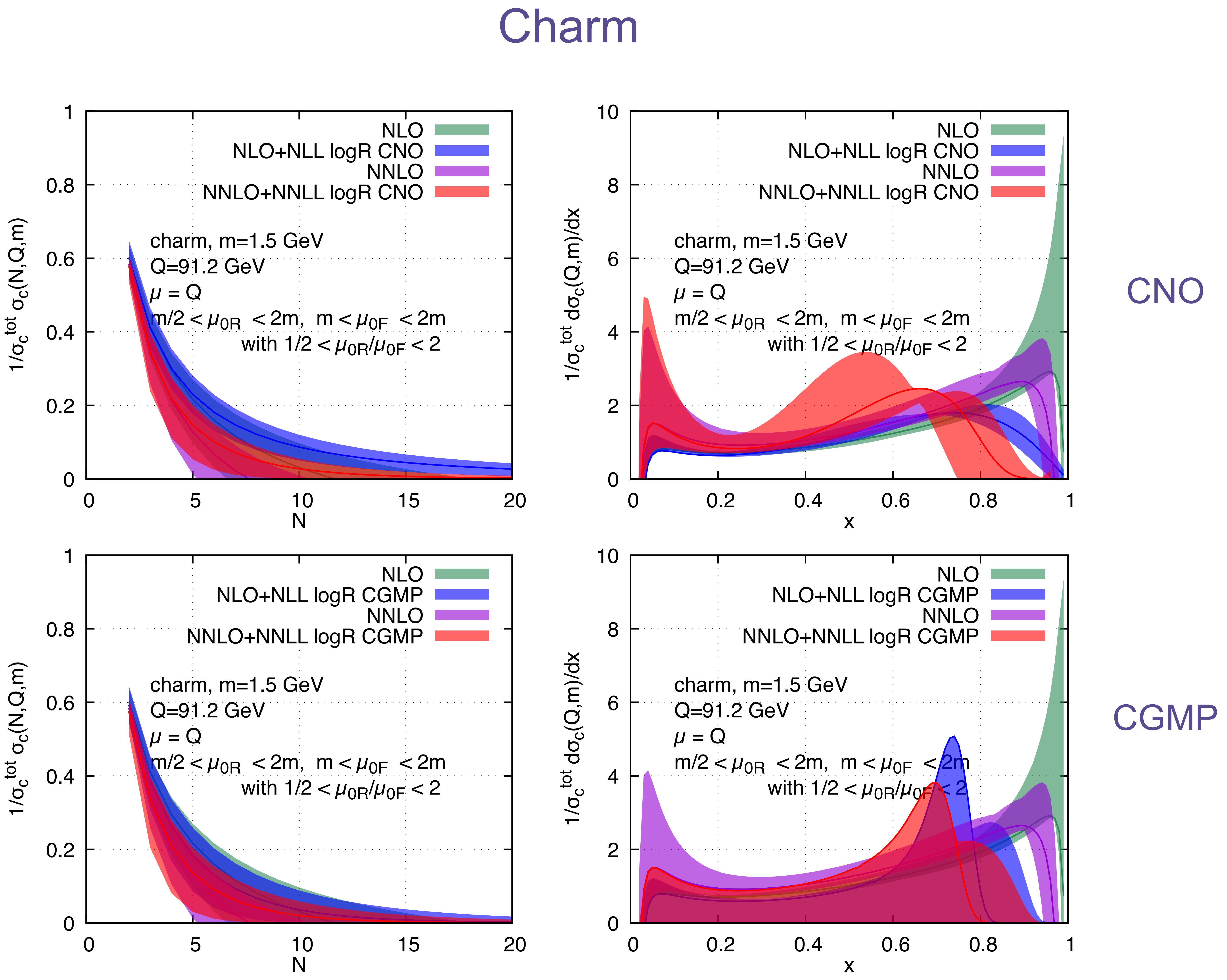
CNO

CGMP

Numerical results

Full e^+e^- fragmentation function

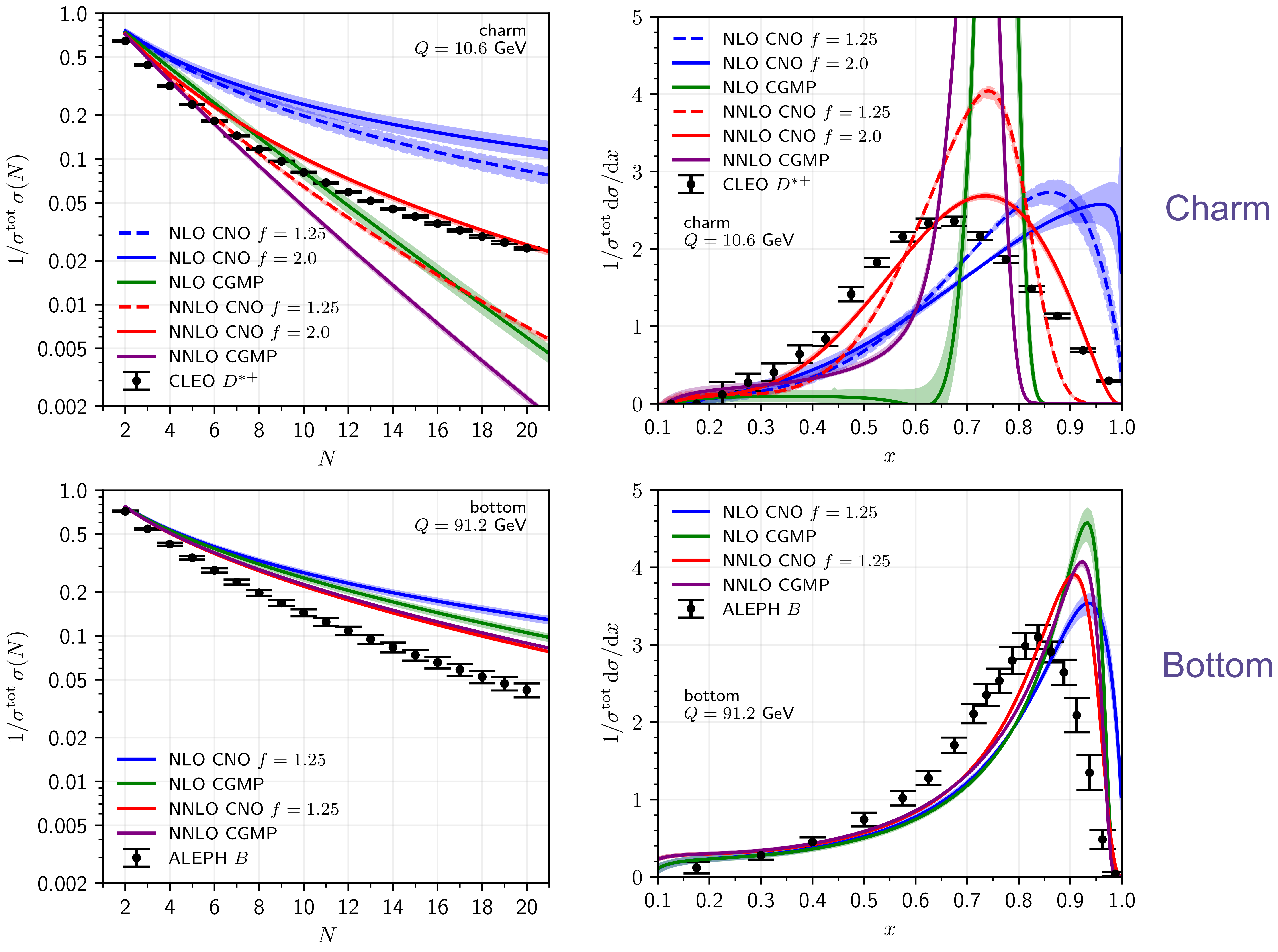
- Scale variation around m
  - Wider bands than bottom case
  - Any perturbative hierarchy essentially lost
- Larger sensitivity to Landau pole regularisation than bottom
- Can we still compare with the data?



Towards phenomenology

pQCD at comparison with data

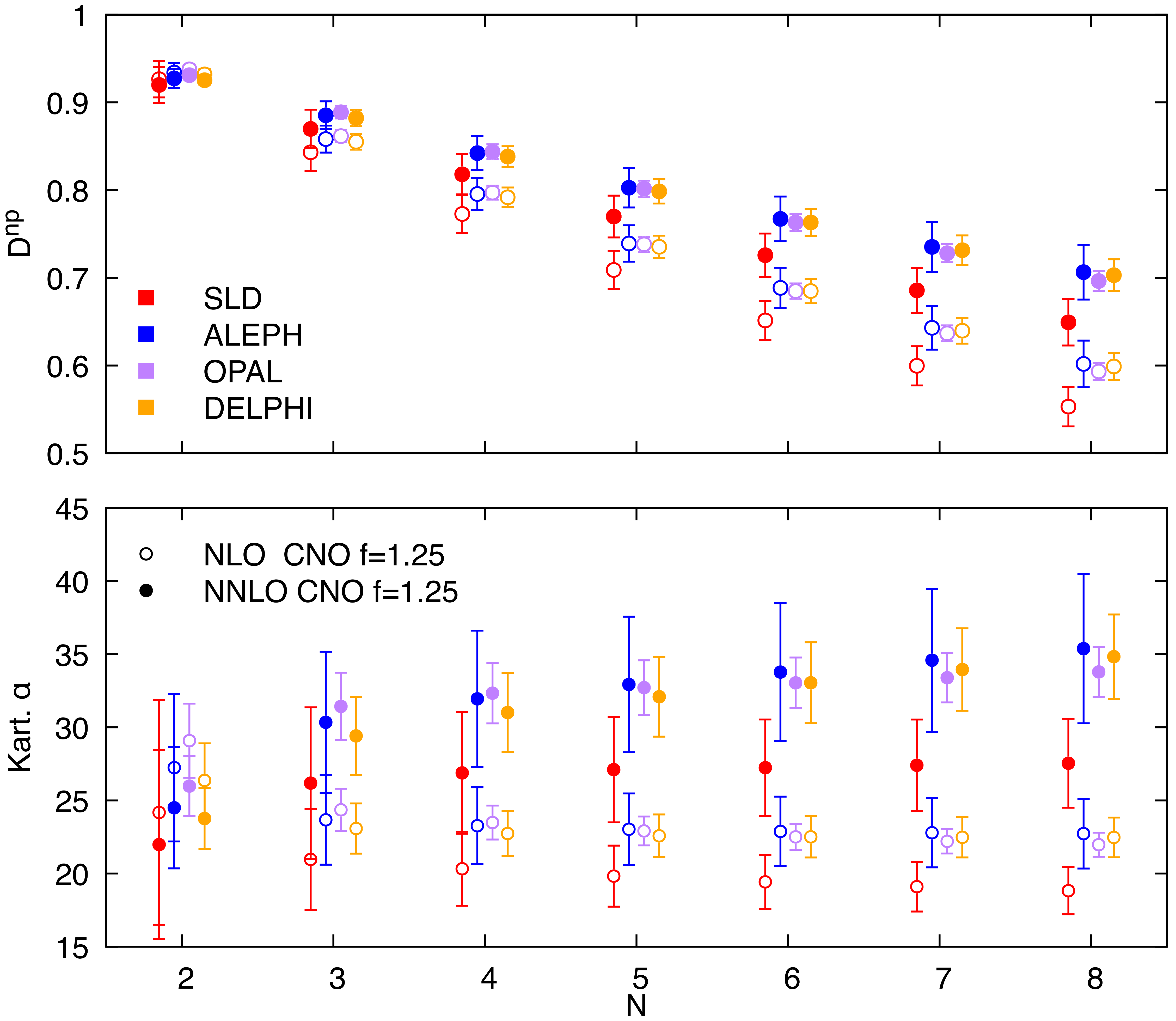
- Many data comprising bottom and charm
  - ALEPH, SLD, DELPHI, OPAL (B-mesons)
  - ALEPH, BELLE, CLEO (D-mesons)
- Bottom: all prescriptions can be used for fits
- Charm: most NNLO+NNLL curves dip below data at $N \gtrsim 6$
- D^{np} can only “lower” theoretical prediction $\rightarrow$
- Our best choice
  - Bottom: NNLO+NNLL logR CNO (f=1.25)
  - Charm: NNLO+NNLL logR CNO (f=2.0)
- Single-moment and single-parameter “fits”



Single-point fits

CNO bottom single-point fits up to NNLO + NNLL

- Recall: non-perturbative FF factorised
  $\sigma_h(Q) = \sigma_{\mathcal{Q}}(Q, m) D^{np}_{\mathcal{Q}\to h}(\{\text{par}\})$
- Our take: as simple as possible
  - (For now) single parameter non-perturbative function
    $D^{np}_K(x) = (\alpha+1)(\alpha+2)x^\alpha(1-x)$ [Kartvelishvili et al. B 78 (1978) 615.]
- "Fit" moments between 2 and 8 (relevant for hadronic collisions)
- $D^{np} = \text{data}/(\text{pert . theo})$
  - α values stable in N
  - NNLO+NNLL results closer to data → smaller non-perturbative component

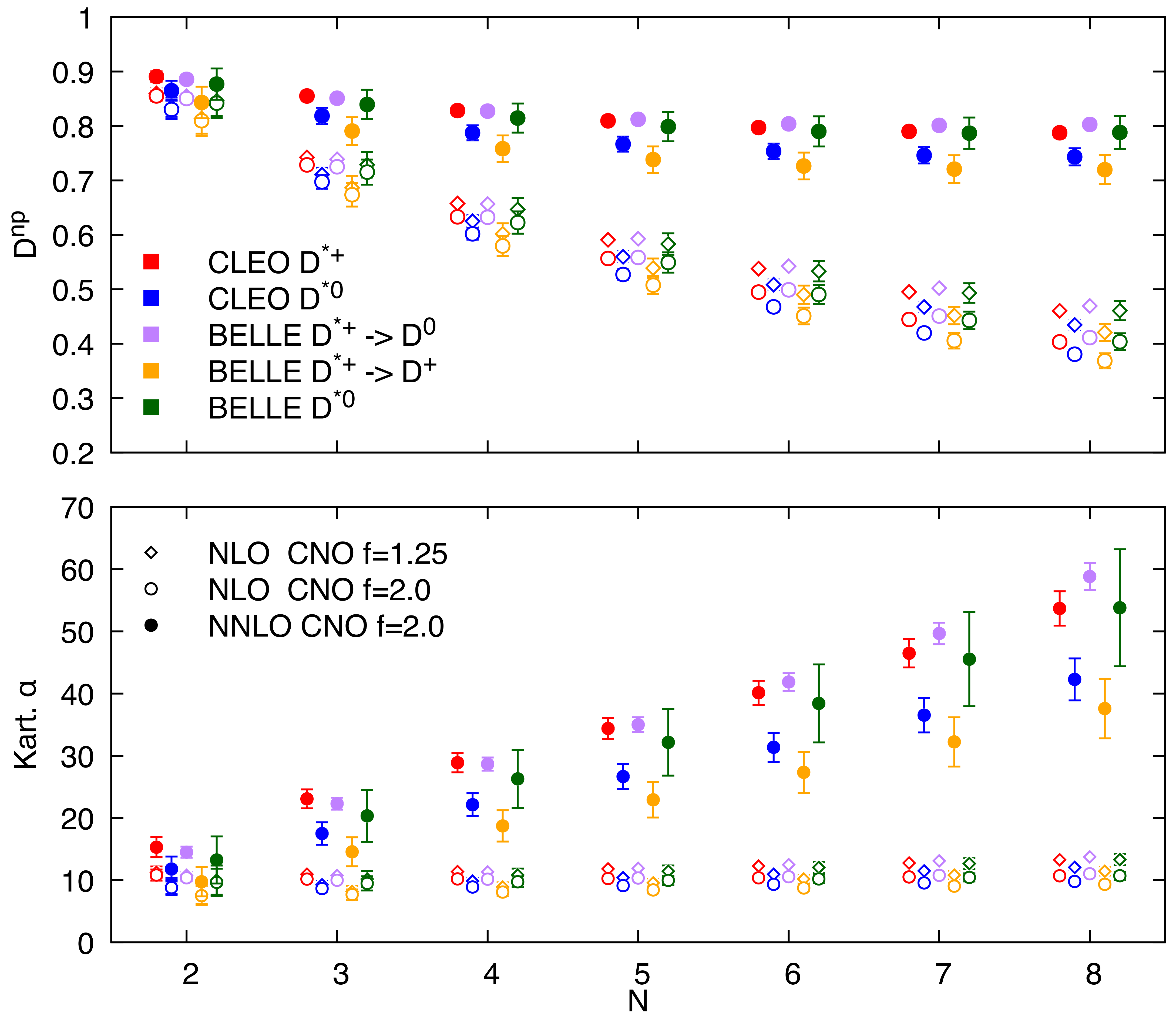


Single-point fits

CNO charm single-point fits up to NNLO + NNLL

- @ NLO+NLL
  - α values stable in N
  - Stable under variation of f
- @ NNLO+NNLL
  - large dependency of α on N
  - $f = 2.0$ mandatory choice

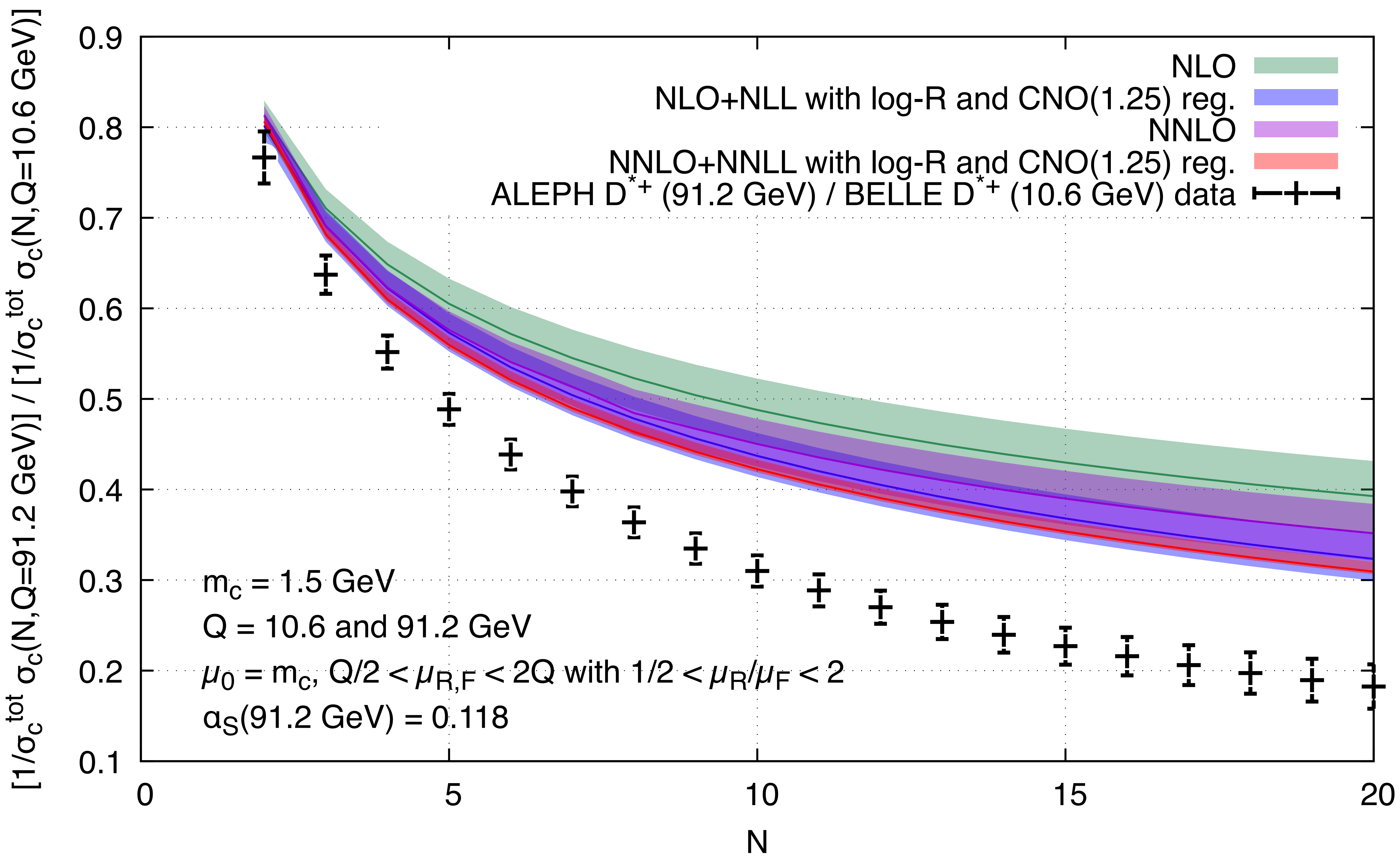
f: CNO regularisation parameter



Charm ratio

A perturbative observable

- Ratio of ALEPH ($91.2\,\mathrm{GeV}$) and BELLE ($10.6\,\mathrm{GeV}$) moments for D^{*+} (N-space)
- Non-perturbative and low-scales effects largely cancel in theory prediction → **entirely perturbative**
- Data undershoot pure QCD prediction
- Large power suppressed effects in coefficient functions?
- Observable and prediction exclusively depend upon evolution between $\Upsilon(4S)$ and Z^0 energies

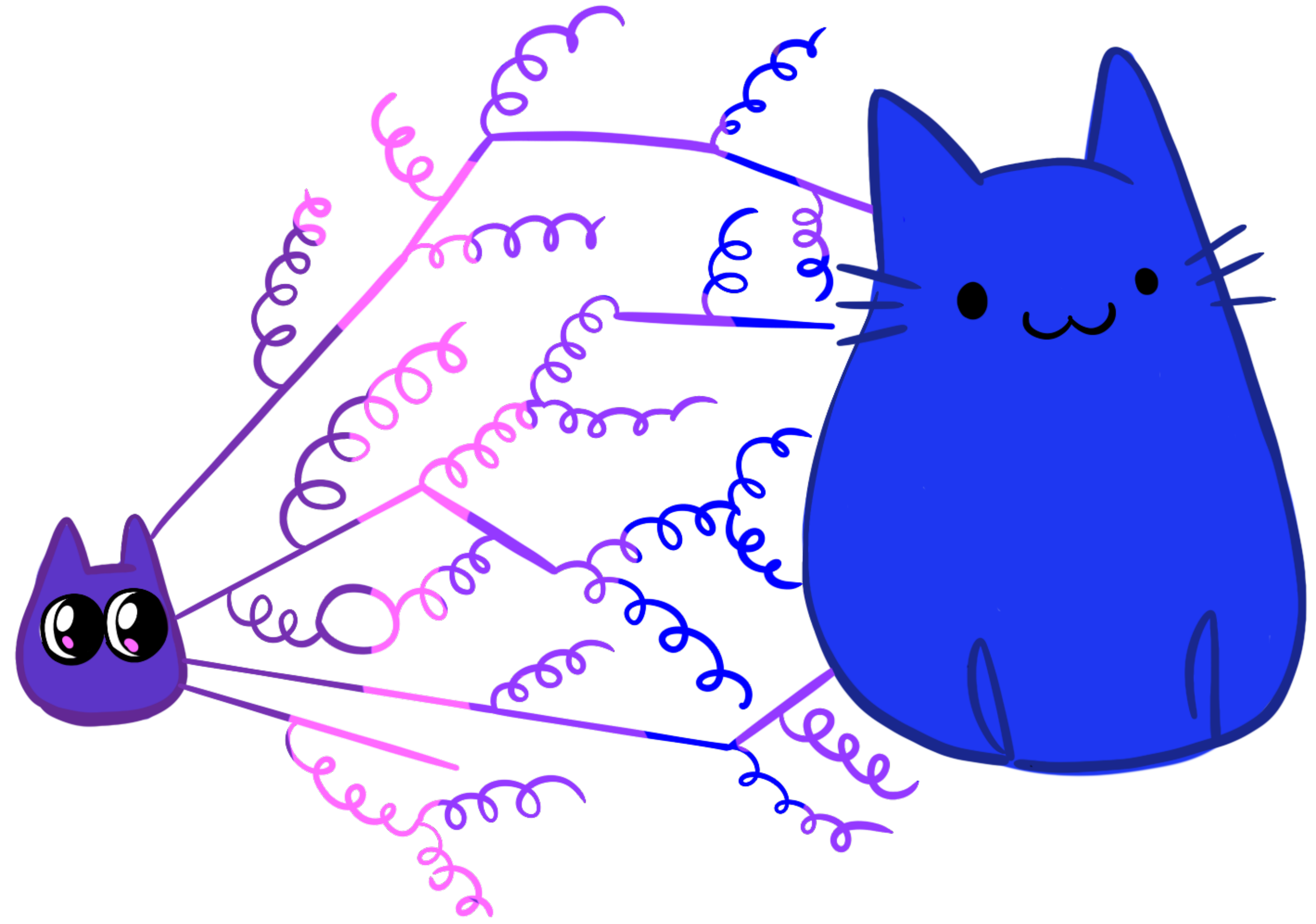


$$\frac{\sigma_{\mathcal{Q}}(M_Z, m)}{\sigma_{\mathcal{Q}}(M_\Upsilon, m)} \simeq \frac{C_q(M_Z, \mu_Z)}{\sigma_{\mathcal{Q}}^{tot}(M_Z)} E(\mu_Z, \mu_\Upsilon) \frac{\sigma_{\mathcal{Q}}^{tot}(M_\Upsilon)}{C_q(M_\Upsilon, \mu_\Upsilon)}$$

Conclusions

- NNLO + NNLL reached for bottom & charm
  - Open question on Landau pole regularisation
    - Can we do better?
    - Current limit on theoretical precision
  - Something about charm-ratio not understood
- Results important for $pp-$phenomelogy
- Public code: work in progress

Realistic representation of a heavy quark fragmenting into a hadron
by Gaia Fontana



Backup: Landau pole regularisations

CNO & CGMP prescription

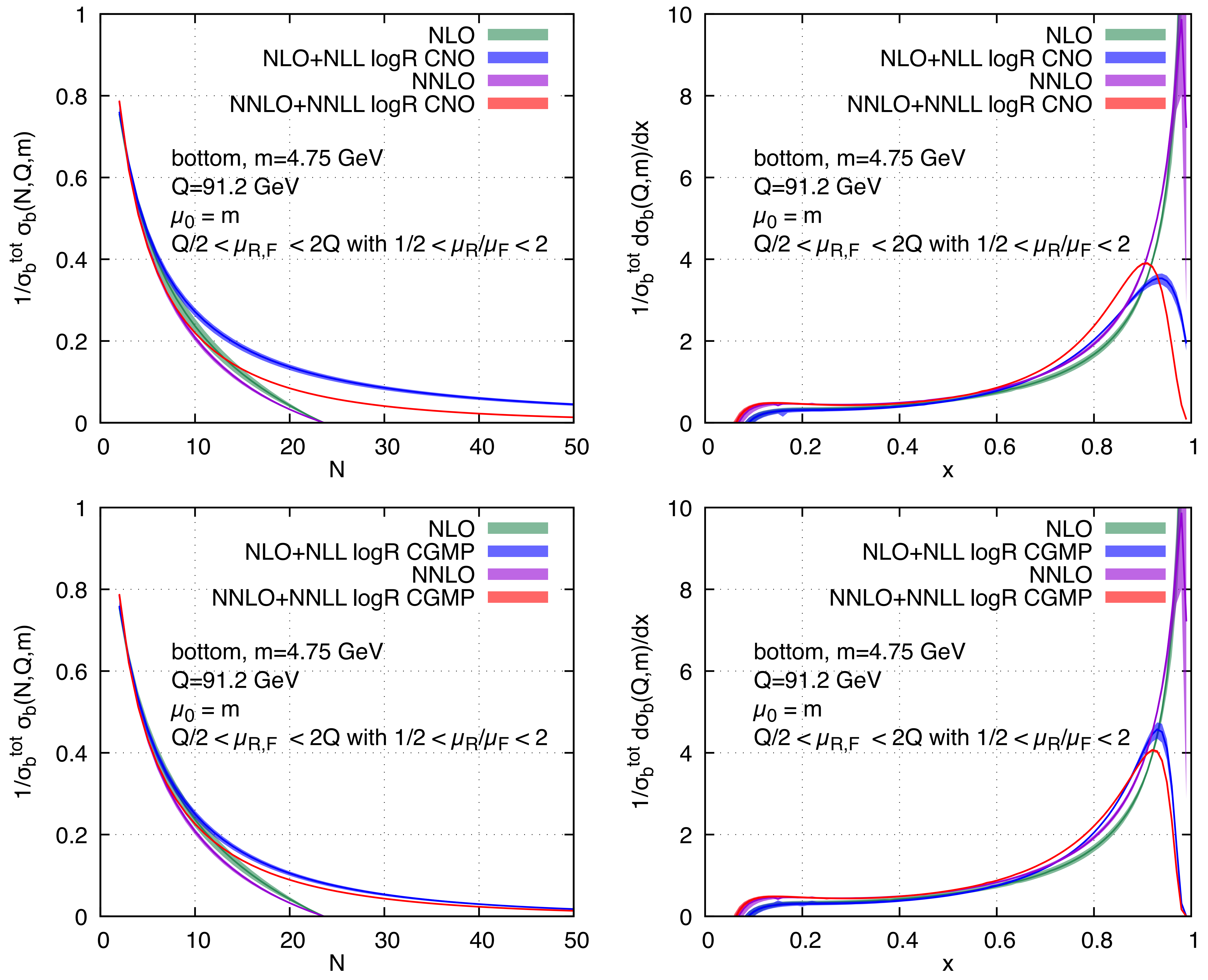
- "CNO" (Cacciari-Nason-Oleari): shift in N [Cacciari et al._0510032] $N \to N \dfrac{1 + f/N^L}{1 + fN/N^L}$
  - Consistent with all known perturbative results ✓
  - Yields physically acceptable results ✓
  - Does not introduce power corrections larger then generally expected for the process in question: $N\Lambda_{QCD}/m$ for IC and $N\Lambda_{QCD}^2/Q^2$ for CF ✓
- "CGMP" (Czakon-Generet-Mitov-Poncelet): truncation in the **exponent** of Sudakov factor [Czakon et al._2210.06078]
  - introduces power corrections larger then generally expected

Backup: numerical results

Bottom

Full e^+e^- fragmentation function

- Perturbative hierarchy better respected for μ_R and μ_F variations around Q

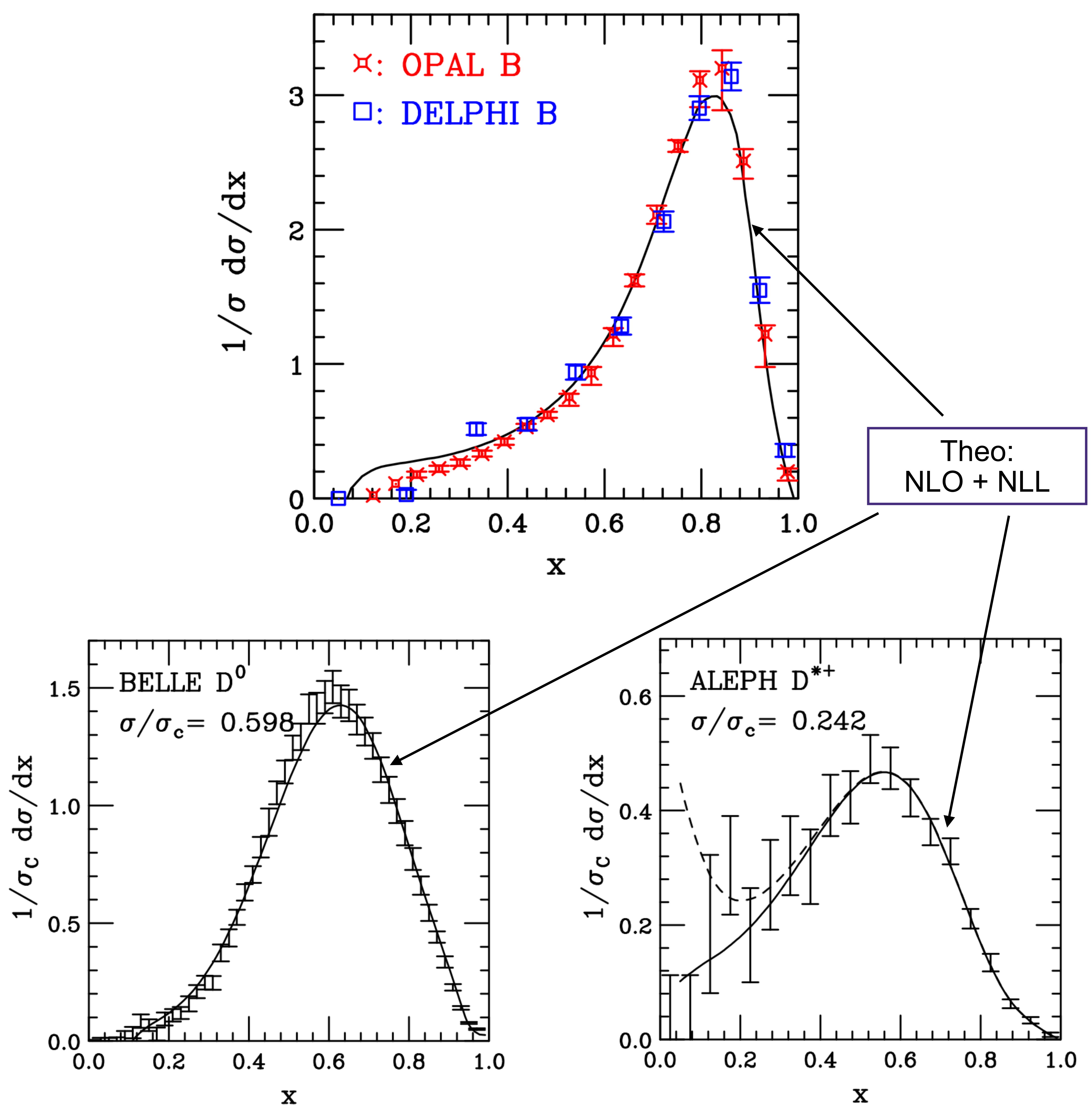


Backup: the data

At e^+e^- colliders

- Compare with the data, what do we have?
- Many data comprising bottom and charm
- B mesons @ $91.2\,\mathrm{GeV}$ (Z^0-peak):
  - ALEPH (mesons) **[ALEPH_0106051]**, SLD **[SLD_0202031]**, OPAL **[OPAL_0210031]**, DELPHI (some baryons as well) **[DELPHI_002-069 CONF 603]**
- D mesons, 2 energies:
  - ALEPH @ $91.2\,\mathrm{GeV}$ **[ALEPH_9909032]**, BELLE **[Belle_0506068]**, CLEO @ $10.6\,\mathrm{GeV}$ (ISR-corrected) **[CLEO_0402040] [CLEO_9707018]**

From: [Cacciari et al._0510032]



Backup:state of the art for fits

New results on bottom fragmentation

- Cacciari-Nason-Oleari 2005 → bottom & charm fits @ NLO + NLL
  - Good fits to D and B mesons fragmentation spectra
- Aglietti-Corcella-Ferrera 2007 → bottom & charm @ NLO + NNLL
  - Effective α_S: call for full NNLO analysis **[Aglietti et. al_0610035] [Corcella, Ferrera_0706.2357]**
- Czakon-Generet-Mitov-Poncelet 2022 → bottom @ NNLO + NNLL
  - Fits to D^{np} with 5-8 parameters
- Our contribution:
  - Charm @ NNLO + NNLL through b-threshold → fits to charm
  - Public code!

From: [Czakon et al._2210.06078]