Threshold resummation of rapidity distributions

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based on
MB, Giulia Marinelli
On the approaches to threshold resummation of rapidity distributions for the Drell-Yan process
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Threshold resummation - the inclusive case

Focus on Drell-Yan. Threshold logarithms appear only in the $q\bar{q}$ channel

$$\frac{d^2\sigma_{q\bar{q}}}{dQ^2} = \tau\sigma_0 \int_\tau^1 \frac{dz}{z}\, L_{q\bar{q}}\left(\frac{\tau}{z}\right) C(z) \qquad \tau = \frac{Q^2}{S}, \quad z = \frac{Q^2}{s}$$

with

$$L_{q\bar{q}}(x) = \sum_q c_{q\bar{q}} \int_x^1 \frac{dx'}{x'} f_q(x', \mu_{\rm F}^2)\, f_{\bar{q}}\left(\frac{x}{x'}, \mu_{\rm F}^2\right)$$

and

$$C(z) = \delta(1-z) + \frac{\alpha_s}{\pi} C_1(z) + \left(\frac{\alpha_s}{\pi}\right)^2 C_2(z) + \mathcal{O}(\alpha_s^3)$$

Threshold enhanced logarithms appear as double logs in the perturbative coefficients

$$C_n(z) = \sum_{k=0}^{2n-1} c_{n,k} \left(\frac{\log^k(1-z)}{1-z}\right)_+ + d_n \delta(1-z) + \text{power suppressed}$$

They are large in the threshold limit $z \to 1$, i.e. $s \to Q^2$, and can be resummed to all orders with well established techniques [Sterman 1987] [Catani,Trentadue 1989]

Rapidity distributions are usually written in two equivalent ways

$$\frac{d^2\sigma_{q\bar{q}}}{dQ^2\,dY} = \tau\sigma_0 \int_{x_a}^1 \frac{dz_a}{z_a} \int_{x_b}^1 \frac{dz_b}{z_b}\, \tilde{\mathscr{L}}_{q\bar{q}}\left(\frac{x_a}{z_a}, \frac{x_b}{z_b}\right)\, \tilde{C}(z_a, z_b)$$
$$= \tau\sigma_0 \int_{\tau}^1 \frac{dz}{z} \int_0^1 du\, \mathscr{L}_{q\bar{q}}(z,u)\, C(z,u)$$

where

$$\mathscr{L}_{q\bar{q}}(z,u) = \tilde{\mathscr{L}}_{q\bar{q}}(x_1, x_2) = \sum_q c_{q\bar{q}} f_q\left(x_1, \mu_{\mathrm{F}}^2\right) f_{\bar{q}}\left(x_2, \mu_{\mathrm{F}}^2\right)$$

and

$$x_1 = \frac{x_a}{z_a} = \sqrt{\frac{\tau}{z}}e^{Y-y} = \sqrt{\frac{\tau}{z}}e^{Y}\sqrt{\frac{z+(1-z)u}{1-(1-z)u}} \qquad x_a = \sqrt{\tau}e^{Y}$$

$$x_2 = \frac{x_b}{z_b} = \sqrt{\frac{\tau}{z}}e^{y-Y} = \sqrt{\frac{\tau}{z}}e^{-Y}\sqrt{\frac{1-(1-z)u}{z+(1-z)u}} \qquad x_b = \sqrt{\tau}e^{-Y}$$

The variable u is related to the parton-level rapidity y, but it is more convenient as it ranges between 0 and 1.

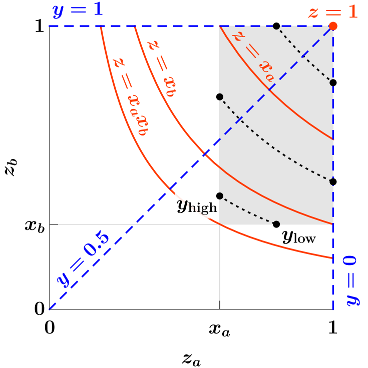
Using the first formulation, $\tilde{C}(z_a, z_b)$ contains logarithms of the form

$$\alpha_s^n \left(\frac{\log^k(1-z_a)}{1-z_a} \right)_+ \quad \text{and} \quad \alpha_s^n \left(\frac{\log^k(1-z_b)}{1-z_b} \right)_+, \qquad 0 \le k < 2n.$$

Using the second formulation, $C(z,u)$ contains logarithms of the form

$$\alpha_s^n \left(\frac{\log^k(1-z)}{1-z} \right)_+, \qquad 0 \le k < 2n,$$

and also plus distributions in u.



In the plot, taken from [Lustermans,Michel,Tackmann 1908.00985], the variable y is our u

Note that $z = z_a z_b$, so the threshold limit $z \to 1$ corresponds to both $z_a, z_b \to 1$

Explicit NLO result

$$
\begin{aligned}
\frac{\tilde{C}_1(z_a, z_b)}{C_F} &= (3\zeta_2 - 4)\delta(1-z_a)\delta(1-z_b) \\
&+ \left(\frac{\log(1-z_a)}{1-z_a}\right)_+ \delta(1-z_b) + \delta(1-z_a)\left(\frac{\log(1-z_b)}{1-z_b}\right)_+ + \left(\frac{1}{1-z_a}\right)_+ \left(\frac{1}{1-z_b}\right)_+ \\
&- \left(\frac{1}{1-z_a}\right)_+ \frac{1+z_b}{2} - \frac{1+z_a}{2}\left(\frac{1}{1-z_b}\right)_+ \\
&+ \delta(1-z_a)\left[\frac{1-z_b}{2} - \frac{1+z_b}{2}\log(1-z_b) + \frac{1}{2}\frac{1+z_b^2}{1-z_b}\log\frac{2}{1+z_b}\right] \\
&+ \delta(1-z_b)\left[\frac{1-z_a}{2} - \frac{1+z_a}{2}\log(1-z_a) + \frac{1}{2}\frac{1+z_a^2}{1-z_a}\log\frac{2}{1+z_a}\right] \\
&+ \frac{(z_a^2+z_b^2)[(1+z_a)^2+(1+z_b)^2+2z_a z_b(3+z_a+z_b+z_a z_b)]}{2(1+z_a)(1+z_b)(z_a+z_b)^2}
\end{aligned}
$$

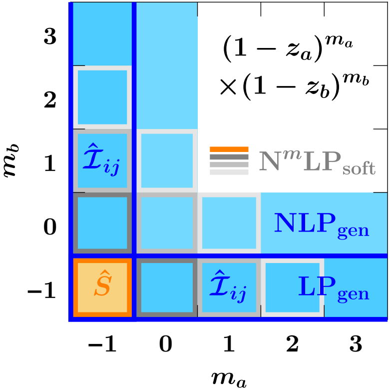
$$
\begin{aligned}
\frac{C_1(z,u)}{C_F} &= \frac{\delta(1-u)+\delta(u)}{2}\left[(2\zeta_2-4)\delta(1-z) + 2(1+z^2)\left(\frac{\log(1-z)}{1-z}\right)_+ - \frac{1+z^2}{1-z}\log(z) + 1 - z\right] \\
&+ \frac{1}{2}\frac{1+z^2}{1-z}\left[\left(\frac{1}{u}\right)_+ + \left(\frac{1}{1-u}\right)_+\right] - (1-z)
\end{aligned}
$$

$$
\frac{C_1(z)}{C_F} = (2\zeta_2-4)\delta(1-z) + 2(1+z^2)\left(\frac{\log(1-z)}{1-z}\right)_+ - \frac{1+z^2}{1-z}\log(z)
$$

Four approaches to threshold resummation:

- BNX [Becher,Neubert,Xu 0710.0680]
  resums logs of z, based on the observation that the u dependence in the PDF luminosity is next-to-leading power at large z
- BFR [Bonvini,Forte,Ridolfi 1009.5691]
  resums logs of z, based on an argument in Mellin-Fourier space
- BDDR [Banerjee,Das,Dhani,Ravindran 1805.01186]
  resums logs of z_a, z_b, based on a two-scale extension of the original approaches to rapidity-integrated resummation, recently extended to next-to-leading power at large z: AMRST [Ajjath,Mukherjee,Ravindran,Sankar,Tiwari, 2112.14094]
- LMT [Lustermans,Michel,Tackmann 1908.00985]
  resums logs of z_a, z_b, based on a soft-collinear effective theory framework to factorise the cross section in terms of beam functions and a soft function

Formally, BDDR and LMT are more accurate as they resum more.
LMT also contains subleading power contributions wrt BDDR, some of which are also predicted by AMRST.



- BDDR: $(-1,-1)$
- AMRST: $(-1,-1)+(-1,0)+(0,-1)$
- LMT: $(-1,\text{any})+(\text{any},-1)$ minus double counting
- BNX/BFR: a part of the $(-1,-1)$ box

LMT criticisms

The LMT paper has criticised the BNX and BFR approaches, claiming that they miss leading power (LP) contributions at threshold.

In other words, they say that BNX and BFR are wrong.

This motivated our work...

Our BNX/BFR proof

We constructed a new, explicit proof of BNX/BFR

$$
\begin{aligned}
\frac{1}{\tau\sigma_0}\frac{d^2\sigma_{q\bar q}}{dQ^2\,dY} &= \int_\tau^1 \frac{dz}{z}\int_0^1 du\; \mathscr{L}_{q\bar q}(z,u)\; C(z,u) \\
&= \int_\tau^1 \frac{dz}{z}\int_0^1 du\; [\mathscr{L}_{q\bar q}(1,\cdot) + \mathcal{O}(1-z)]\, C(z,u) \\
&= \int_\tau^1 \frac{dz}{z}\int_0^1 du\; \mathscr{L}_{q\bar q}(1,\cdot)\, [1 + \mathcal{O}(1-z)]\, C(z,u) \\
&= \mathscr{L}_{q\bar q}(1,\cdot) \int_\tau^1 \frac{dz}{z}\int_0^1 du\; C(z,u)\, [1 + \mathcal{O}(1-z)] \\
&= \mathscr{L}_{q\bar q}(1,\cdot) \int_\tau^1 \frac{dz}{z}\; C(z)\, [1 + \mathcal{O}(1-z)] \\
&= \int_\tau^1 \frac{dz}{z}\; \mathscr{L}_{q\bar q}(z,\bar u)\, C(z)\, [1 + \mathcal{O}(1-z)] \\
&= \int_\tau^1 \frac{dz}{z}\; \mathscr{L}_{q\bar q}(z,\bar u)\, C_{\rm thr}(z)\, [1 + \mathcal{O}(1-z)]
\end{aligned}
$$

based on the observation that $\mathscr{L}_{q\bar q}(z,u)$ does not depend on u in $z=1$:

$$
\mathscr{L}_{q\bar q}(1,u) = \sum_q c_{q\bar q} f_q\big(\sqrt{\tau}e^{Y},\mu_{\rm F}^2\big) f_{\bar q}\big(\sqrt{\tau}e^{-Y},\mu_{\rm F}^2\big)
$$

BNX and BFR

- BNX corresponds to the average of $\bar{u} = 0$ and $\bar{u} = 1$:

$$\frac{1}{\tau\sigma_0}\frac{d^2\sigma^{\rm BNX}_{q\bar{q}}}{dQ^2\,dY} \equiv \int_\tau^1 \frac{dz}{z}\,\frac{\mathscr{L}_{q\bar{q}}(z,0)+\mathscr{L}_{q\bar{q}}(z,1)}{2}C(z)$$

that corresponds to the "approximation" of the coefficient function

$$C^{\rm BNX}(z,u) \equiv \frac{\delta(1-u)+\delta(u)}{2}C(z)$$

which is exact at NLO.

- BFR corresponds to $\bar{u} = 1/2$

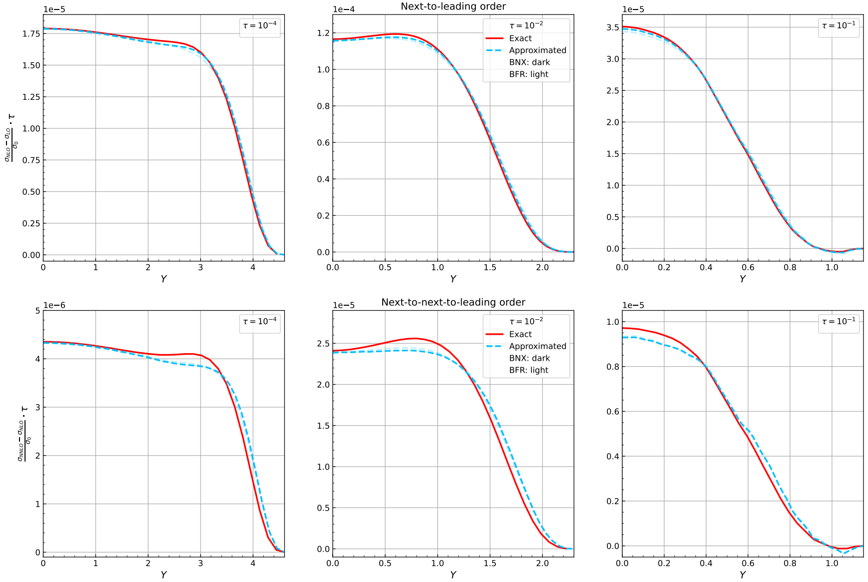
$$\frac{1}{\tau\sigma_0}\frac{d^2\sigma^{\rm BFR}_{q\bar{q}}}{dQ^2\,dY} \equiv \int_\tau^1 \frac{dz}{z}\,\mathscr{L}_{q\bar{q}}\left(z,\frac{1}{2}\right)C(z)$$

corresponding to

$$C^{\rm BFR}(z,u) \equiv \delta\left(u-\frac{1}{2}\right)C(z)$$

It is easy to prove that BNX and BFR are equivalent up to NNLP, namely relative corrections suppressed by $(1-z)^2$

Validation of BNX/BFR luminosity approximation



[NNLO computed with `Vrap`]

Do BNX/BFR miss leading power contributions?

$$\frac{C_1(z,u)}{C_F} = \frac{\delta(1-u)+\delta(u)}{2}\left[(2\zeta_2-4)\delta(1-z) + 2(1+z^2)\left(\frac{\log(1-z)}{1-z}\right)_+ - \frac{1+z^2}{1-z}\log(z) + 1 - z\right]$$
$$+ \frac{1}{2}\frac{1+z^2}{1-z}\left[\left(\frac{1}{u}\right)_+ + \left(\frac{1}{1-u}\right)_+\right] - (1-z)$$

The first term in the last line cannot be reproduced by BNX/BFR, as it vanishes after integration over u... but it looks LP

Moreover it corresponds to LP terms in z_a, z_b

$$\frac{1}{1-z}\left[\left(\frac{1}{u}\right)_+ + \left(\frac{1}{1-u}\right)_+\right] \to \zeta_2\delta(1-z_a)\delta(1-z_b) + \left(\frac{1}{1-z_a}\right)_+\left(\frac{1}{1-z_b}\right)_+$$
$$- \left(\frac{\log(1-z_a)}{1-z_a}\right)_+\delta(1-z_b) - \delta(1-z_a)\left(\frac{\log(1-z_b)}{1-z_b}\right)_+$$
$$+ \delta(1-z_a)\frac{1}{1-z_b}\log\frac{2z_b}{1+z_b} + \delta(1-z_b)\frac{1}{1-z_a}\log\frac{2z_a}{1+z_a} + \ldots$$

Are LMT right?

Do BNX/BFR miss LP terms?

[How could this be compatible with the good agreement of BNX/BFR with the exact result at NLO and NNLO?]

Not really...

They are LP contributions in the coefficient function that contribute at (N)NLP to the cross section:

$$\int_0^1 du\, \frac{1}{1-z}\left[\left(\frac{1}{u}\right)_+ + \left(\frac{1}{1-u}\right)_+\right]\mathscr{L}_{q\bar q}(z,u)$$
$$= \frac{1}{1-z}\left(\int_0^1 \frac{du}{u}\left[\mathscr{L}_{q\bar q}(z,u) - \mathscr{L}_{q\bar q}(z,0)\right] + \int_0^1 \frac{du}{1-u}\left[\mathscr{L}_{q\bar q}(z,u) - \mathscr{L}_{q\bar q}(z,1)\right]\right)$$

and the luminosity differences can be expanded at large z

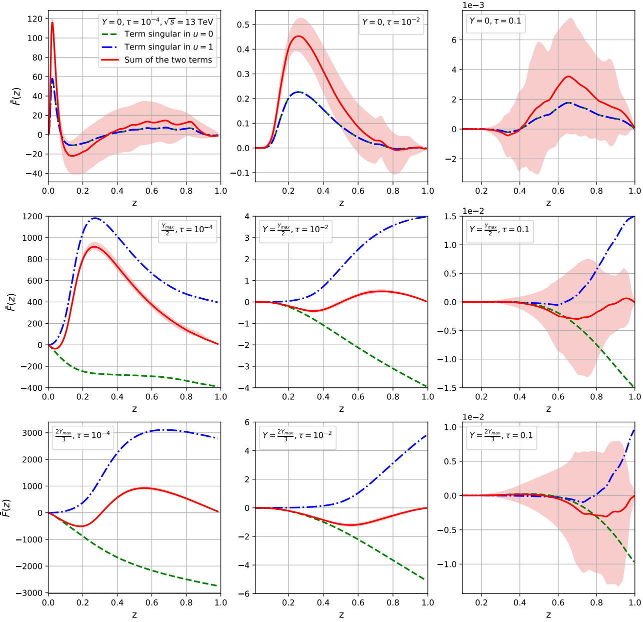
$$\mathscr{L}_{q\bar q}(z,u) - \mathscr{L}_{q\bar q}(z,0) = \cancel{\mathscr{L}_{q\bar q}(1,u) - \mathscr{L}_{q\bar q}(1,0)}$$
$$- \left(\mathscr{L}'_{q\bar q}(1,u) - \mathscr{L}'_{q\bar q}(1,0)\right)(1-z) + \mathcal{O}[(1-z)^2]$$

$$\mathscr{L}_{q\bar q}(z,u) - \mathscr{L}_{q\bar q}(z,1) = \cancel{\mathscr{L}_{q\bar q}(1,u) - \mathscr{L}_{q\bar q}(1,1)}$$
$$- \left(\mathscr{L}'_{q\bar q}(1,u) - \mathscr{L}'_{q\bar q}(1,1)\right)(1-z) + \mathcal{O}[(1-z)^2],$$

In fact, any function that integrates to zero over u contributes at NLP

This is why in that term the $\frac{1}{1-z}$ factor does not need a plus distribution

Numerical check of the integral from the previous slide

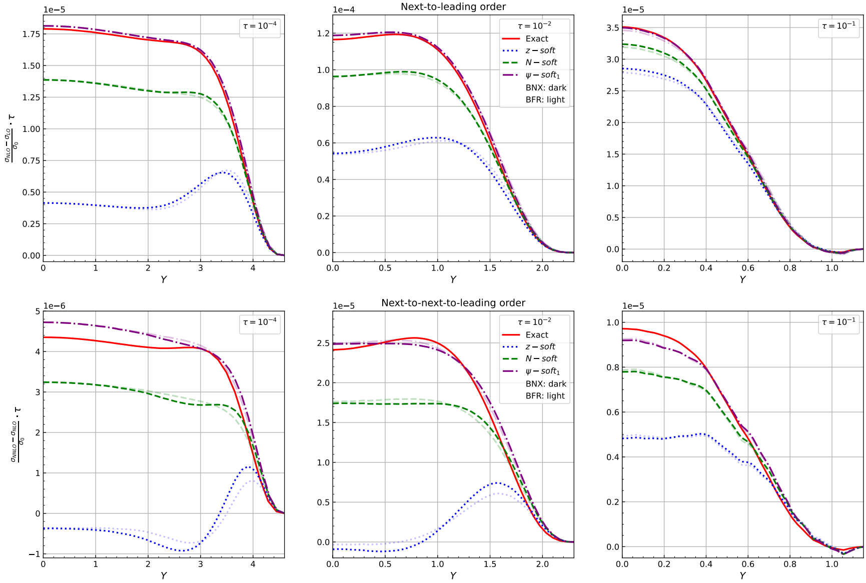


Definition of threshold logarithms in BNX/BFR

In order to use BNX/BFR for resummation, $C(z)$ must be resummed to all orders.
A determination of how good BNX/BFR can be is obtained comparing fixed-order predictions with the expansion of resummation.
The quality of resummation depends on the definition of threshold logs.
Options:

- z-soft: use z-space distributions $\left(\frac{\log^k(1-z)}{1-z}\right)_+$
- N-soft: use N-space logs $\log^k N$, the large-N limit of the Mellin of the distributions above, corresponding in z space to distributions $\left(\frac{\log^k \log\frac{1}{z}}{\log\frac{1}{z}}\right)_+$
- ψ-soft: use a N-space variant in which $\log^k N \to \psi_0^k(N+1)$, corresponding in z space to distributions $z\left(\frac{\log^k \frac{1-z}{\sqrt{z}}}{1-z}\right)_+$ [MB,Marzani 1405.3654]

Definition of threshold logarithms in BNX/BFR



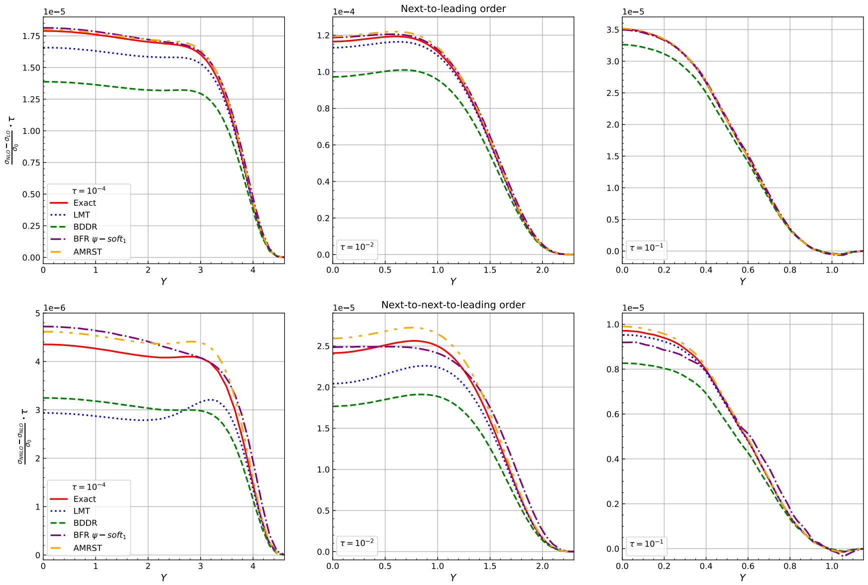
Not bad!

When using ψ-soft, the quality of BNX/BFR is very good, similar to what we got without approximating $C(z)$

Ultimately due to the high accuracy of ψ-soft for approximating the rapidity-integrated cross section even far from threshold, thanks to the inclusion of important subleading power contributions [MB,Marzani 1405.3654]

How do BNX/BFR compare with the other approaches?

Comparison with other approaches

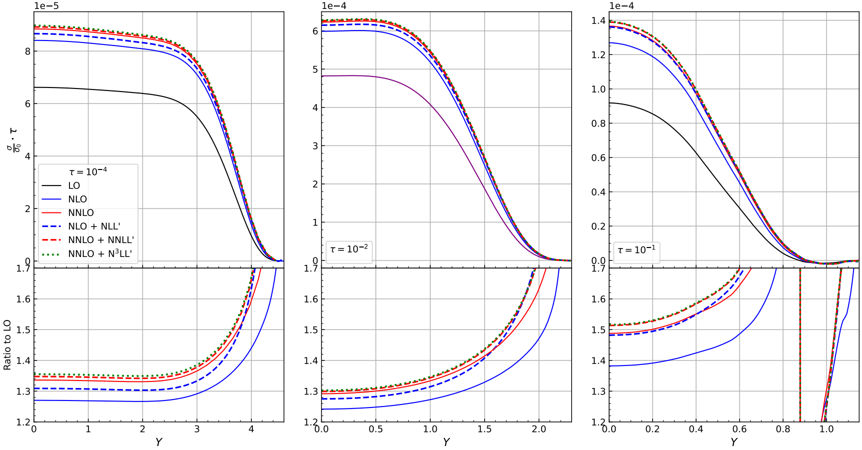


Explanation of the comparison

- BDDR and AMRST are essentially N-soft, which is not as good as ψ-soft but decent
- since AMRST includes NLP contributions, it is closer to ψ-soft, and technically better
- LMT includes more terms than all the others, but it is based on z-soft, which is a bad approximation far from threshold
- the would-be LP contributions missed by BNX/BFR seem to affect the shape in rapidity (bump at NNLO not reproduced)

Resummed BFR results

FO and BFR-resummation, $\sqrt{s} = 13$ TeV, $q\bar{q}$ channel, photon exchange only



All-order resummation à la BFR with ψ-soft available through TROLL

Summary and outlook

- **BNX/BFR approaches**:
  - new formal proof
  - no LP terms to the cross section are missing
  - rather accurate when using ψ-soft
  - competitive with other formally better approaches
- **Public code**: `TROLL` `www.roma1.infn.it/~bonvini/troll`
- **Outlook**:
  - test against $\mathrm{N^3LO}$
  - possible improvement of LMT approach (à la ψ-soft)
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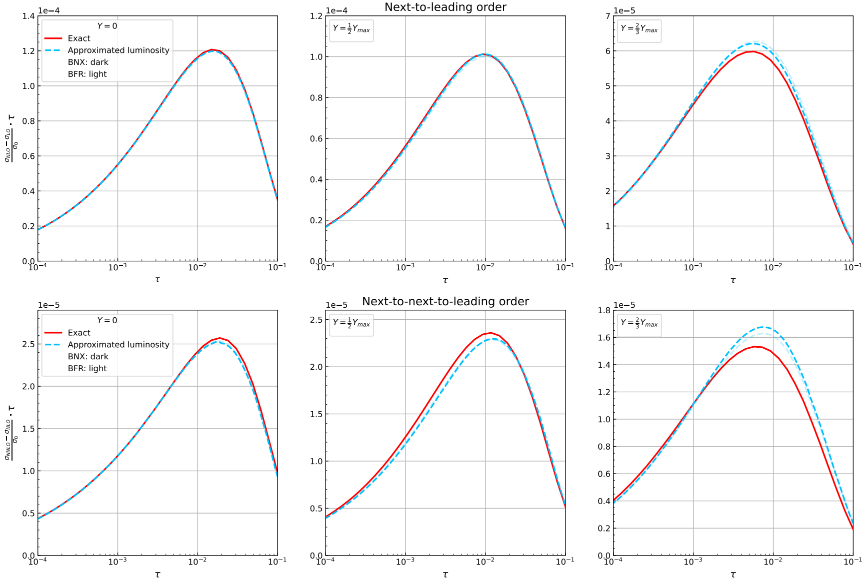

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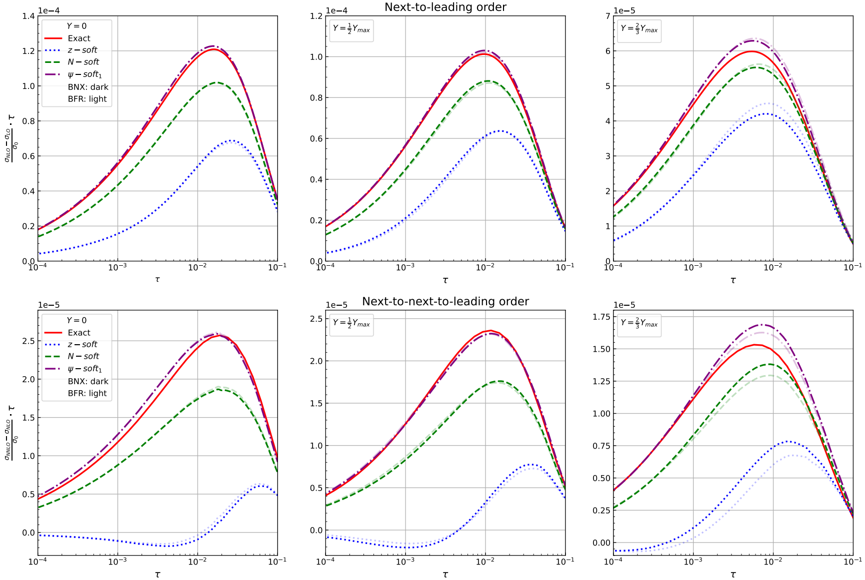

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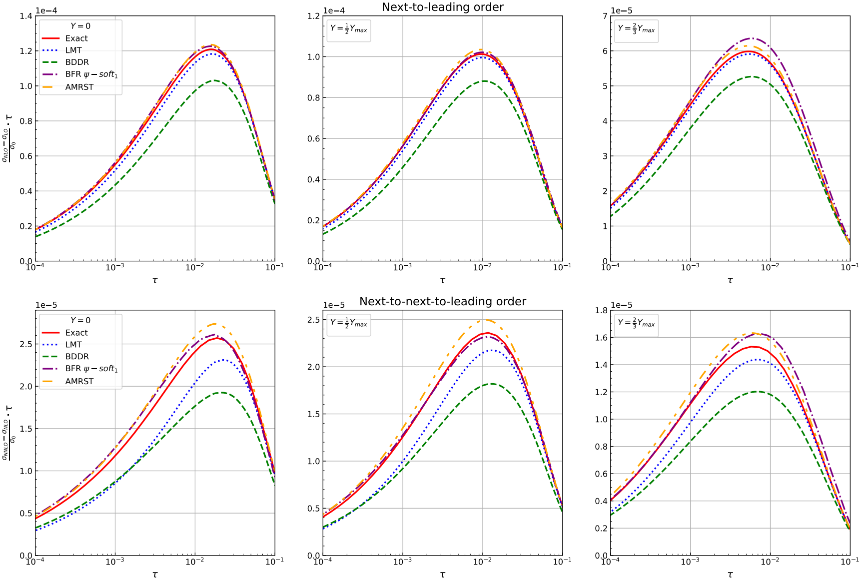

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