FROM HYBRID FACTORIZATION TO NLL/NLO HIGGS + JET @LHC

Francesco Giovanni Celiberto, UAH Madrid

CHRISTMAS MEETING

MILAN

2023, DECEMBER 22ND

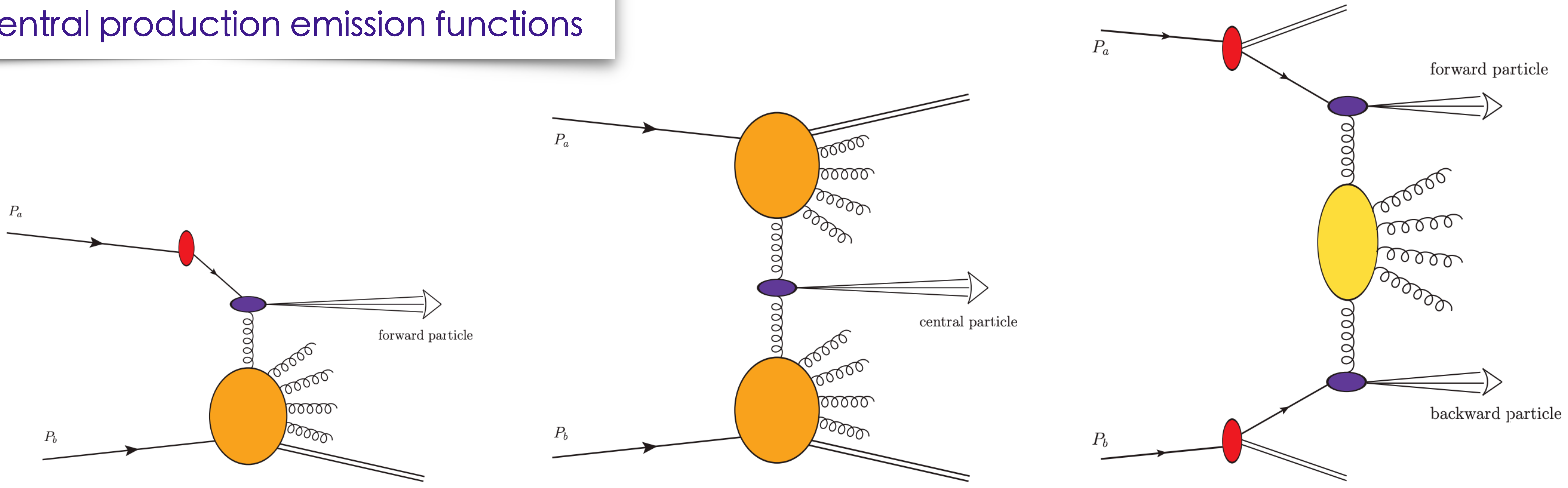
Madrid UAH

talento cm Programa de atracción de talento investigador Comunidad de Madrid

ANIVERSARIO PATRIMONIO MUNDIAL

High-energy factorization at a glance

Singly/double off-shell coefficient functions
Forward/central production emission functions



(a) Single forward (b) Single central (c) Forward-backward

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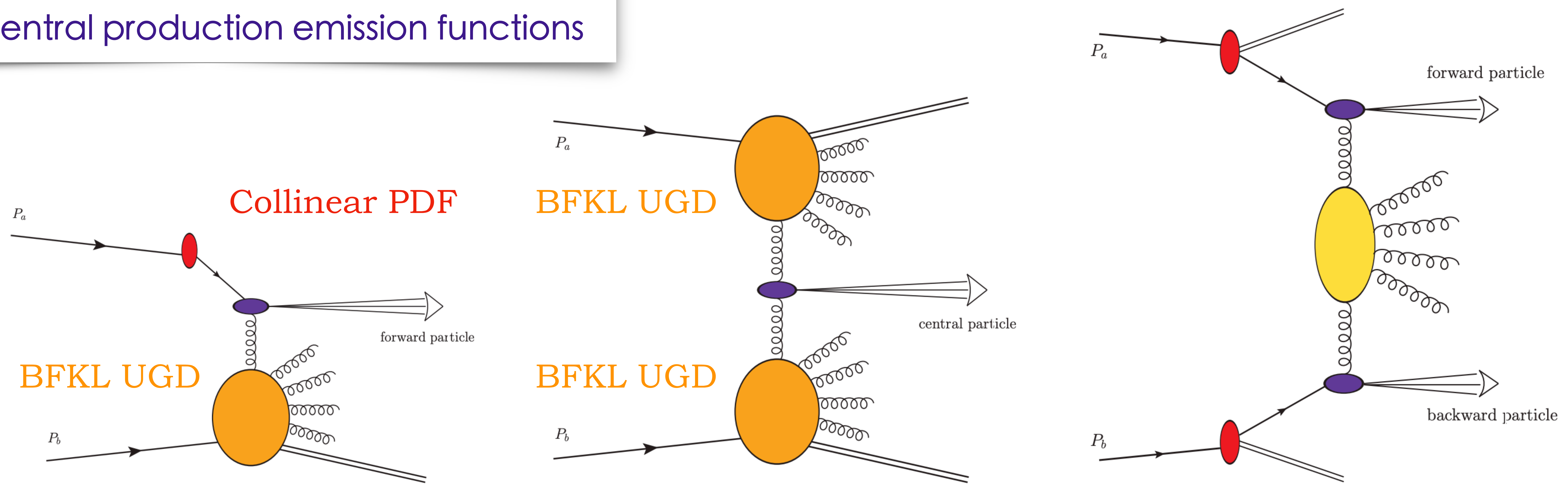

(a) Single forward (b) Single central (c) Forward-backward

Fast q/g + small-x g — gg induced

Hybrid factorization — High-energy factorization

BFKL + Threshold — BFKL or small-x improved PDFs

🔗 [M. Bonvini, S. Marzani (2018)]

High-energy factorization at a glance

Singly/double off-shell coefficient functions
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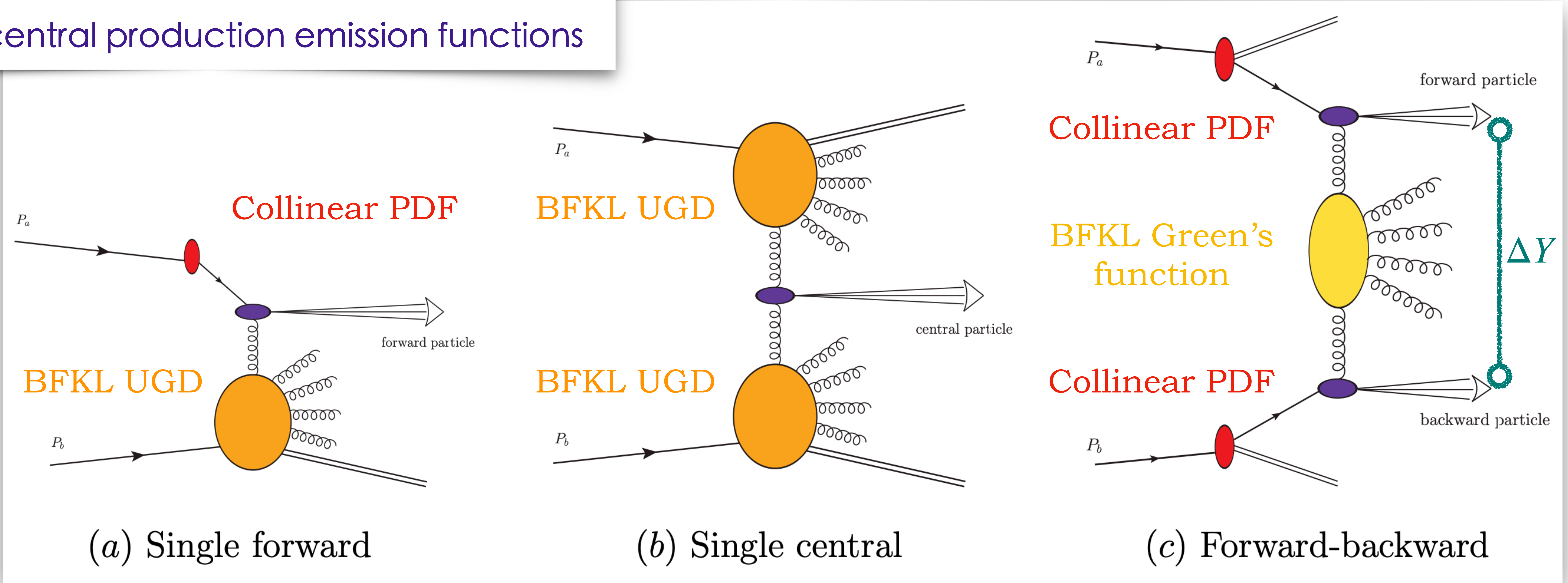


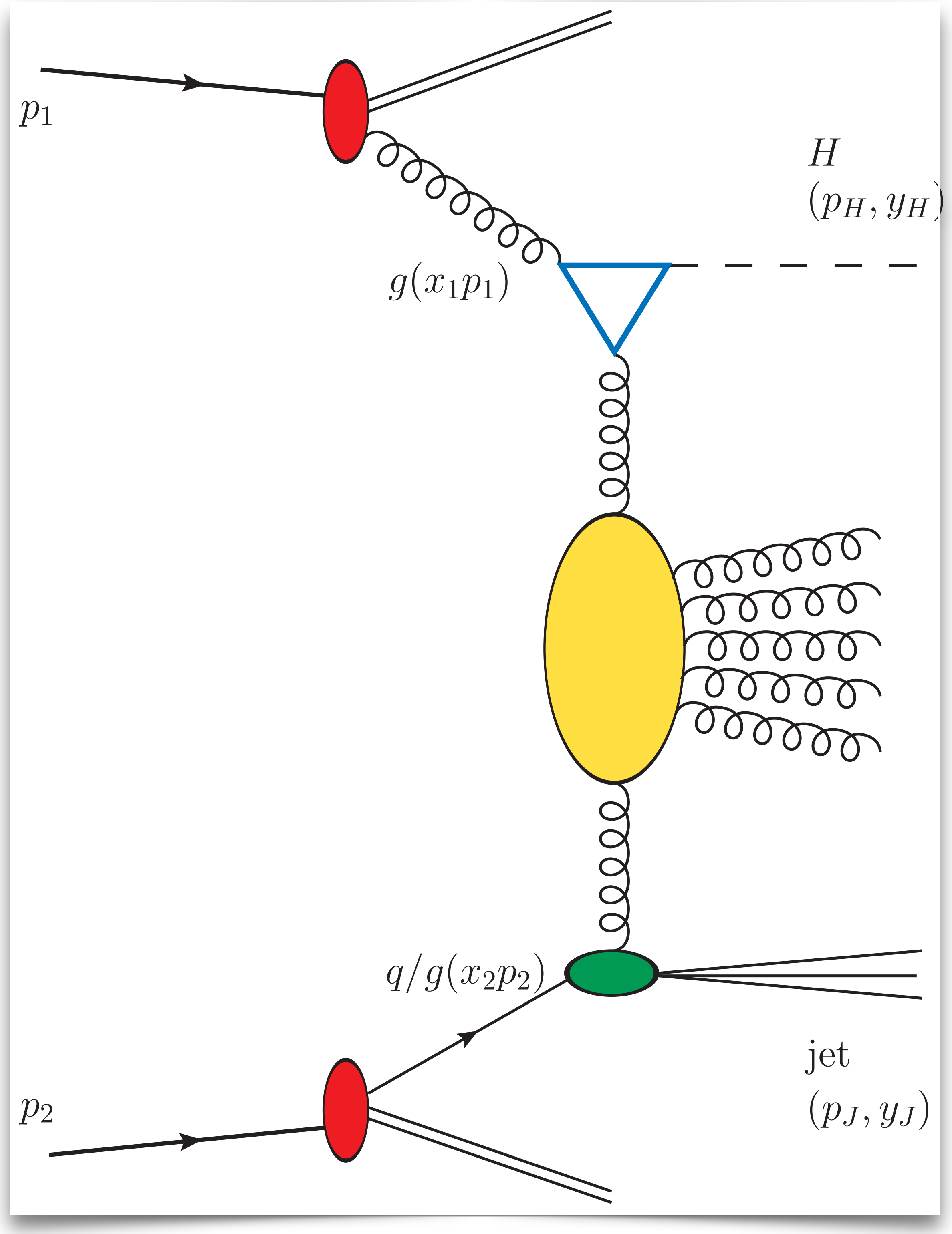
Single forward	Single central	Forward-backward
Fast q/g + small-x g	gg induced	Large rapidity distances, $\Delta Y \gg 1$
Hybrid factorization	High-energy factorization	High energies, moderate x
BFKL + Threshold	BFKL or small-x improved PDFs	PDFs + t-channel BFKL (NLL/NLO HyF)
	[M. Bonvini, S. Marzani (2018)]	Imbalance logs ← back-to-back

From Mueller-Navelet to Higgs and heavy flavor

Pheno path: hunt for channels leading to a NLL stabilization pattern at natural scales (¡!)

HIGGS BOSON

Stabilizers ⇔ large Higgs transverse masses



(Higgs + jet, NLL/NLO*) 🔗 [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

(NLO Higgs coeff. function) 🔗 [F. G. C. et al., JHEP 08 (2022) 092]

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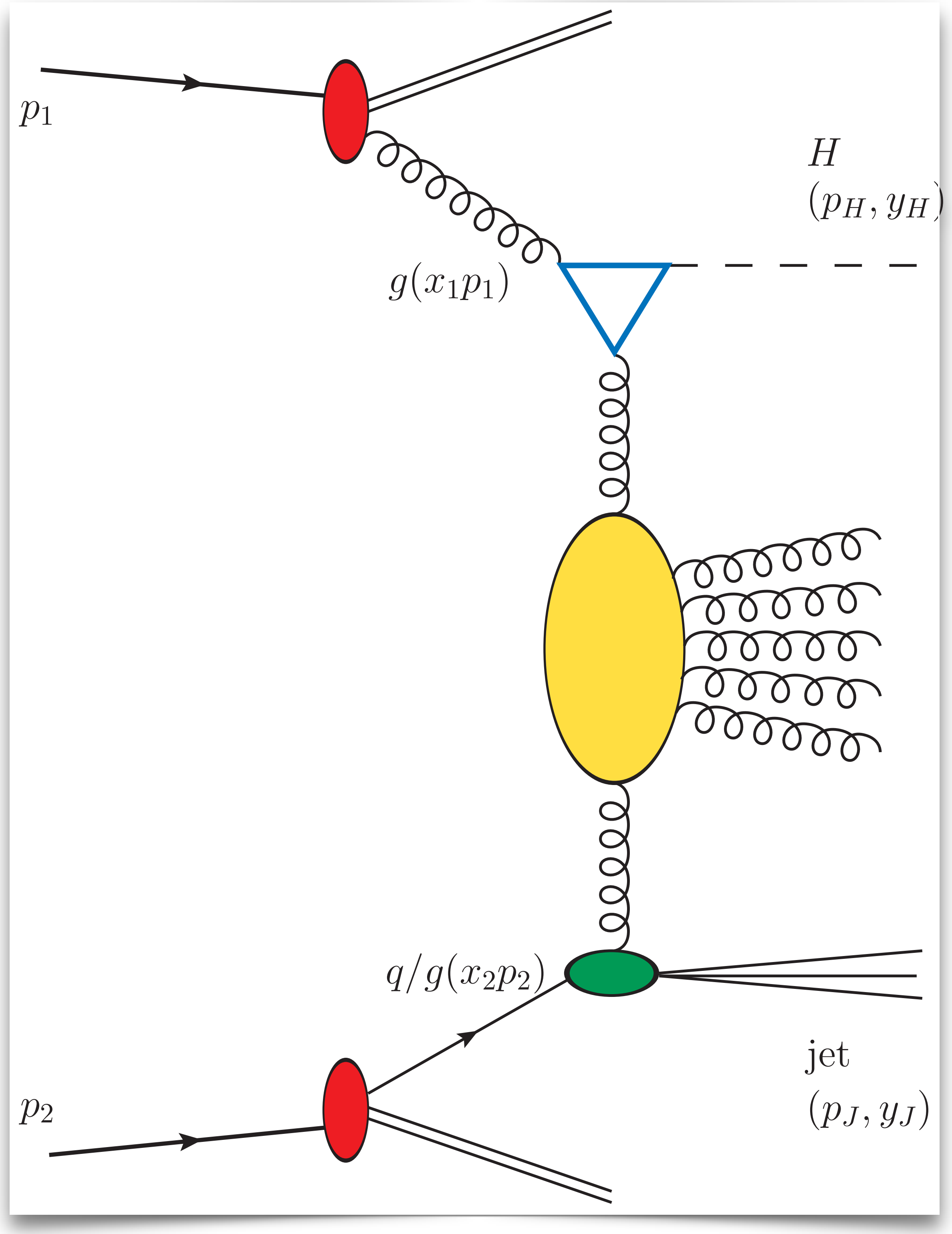

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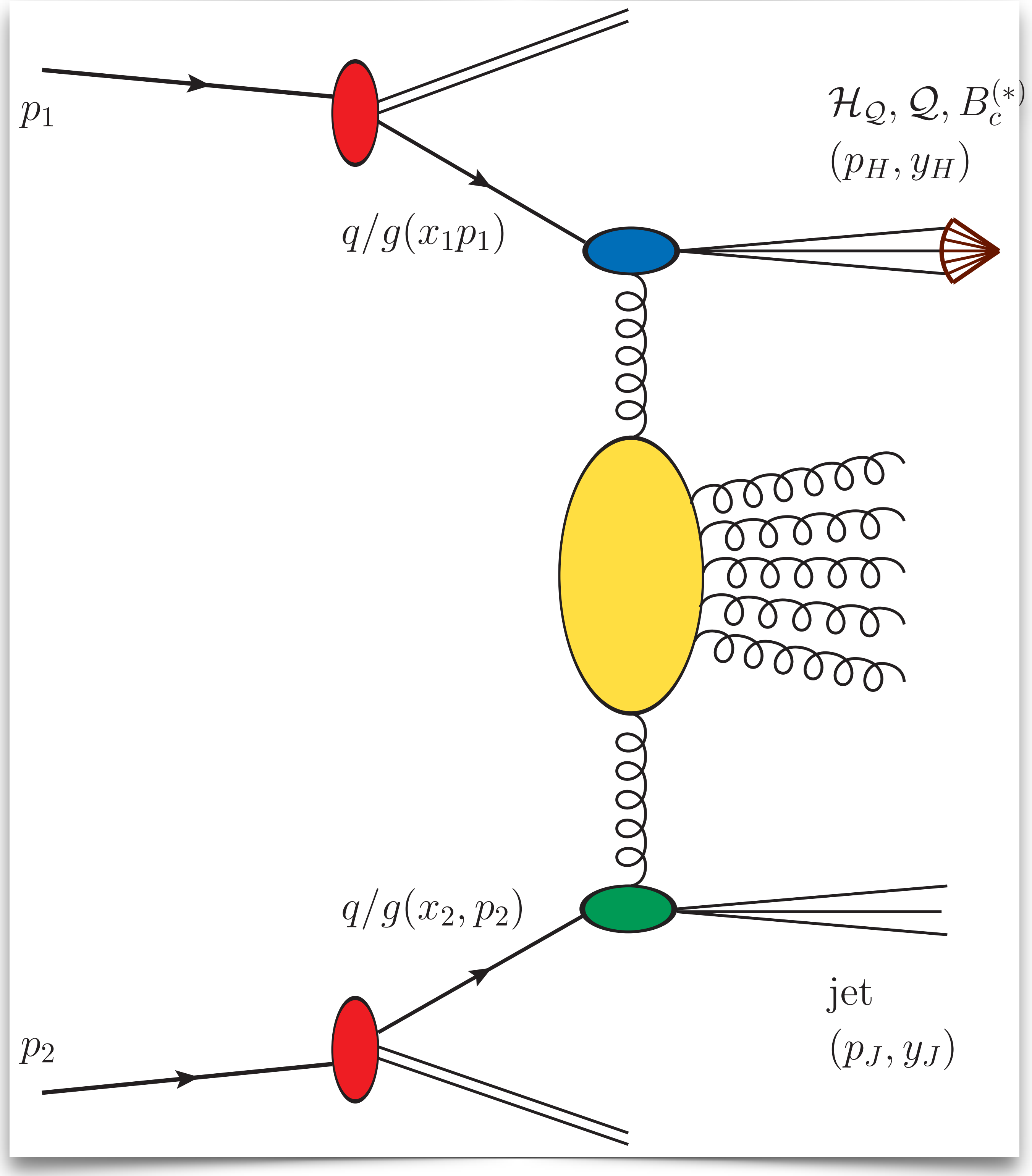
HEAVY FLAVOR AT LARGE P_T

Stabilizers ⇔ gluon fragmentation channels



($\Lambda_c^\pm$ baryons, NLL/NLO) 🔗 [F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]
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($B_c^\pm(^1S_0)$ or $B_c^{*\pm}(^3S_1)$, NLL/NLO) 🔗 [F. G. C., Phys. Lett. B 835 (2022) 137554]

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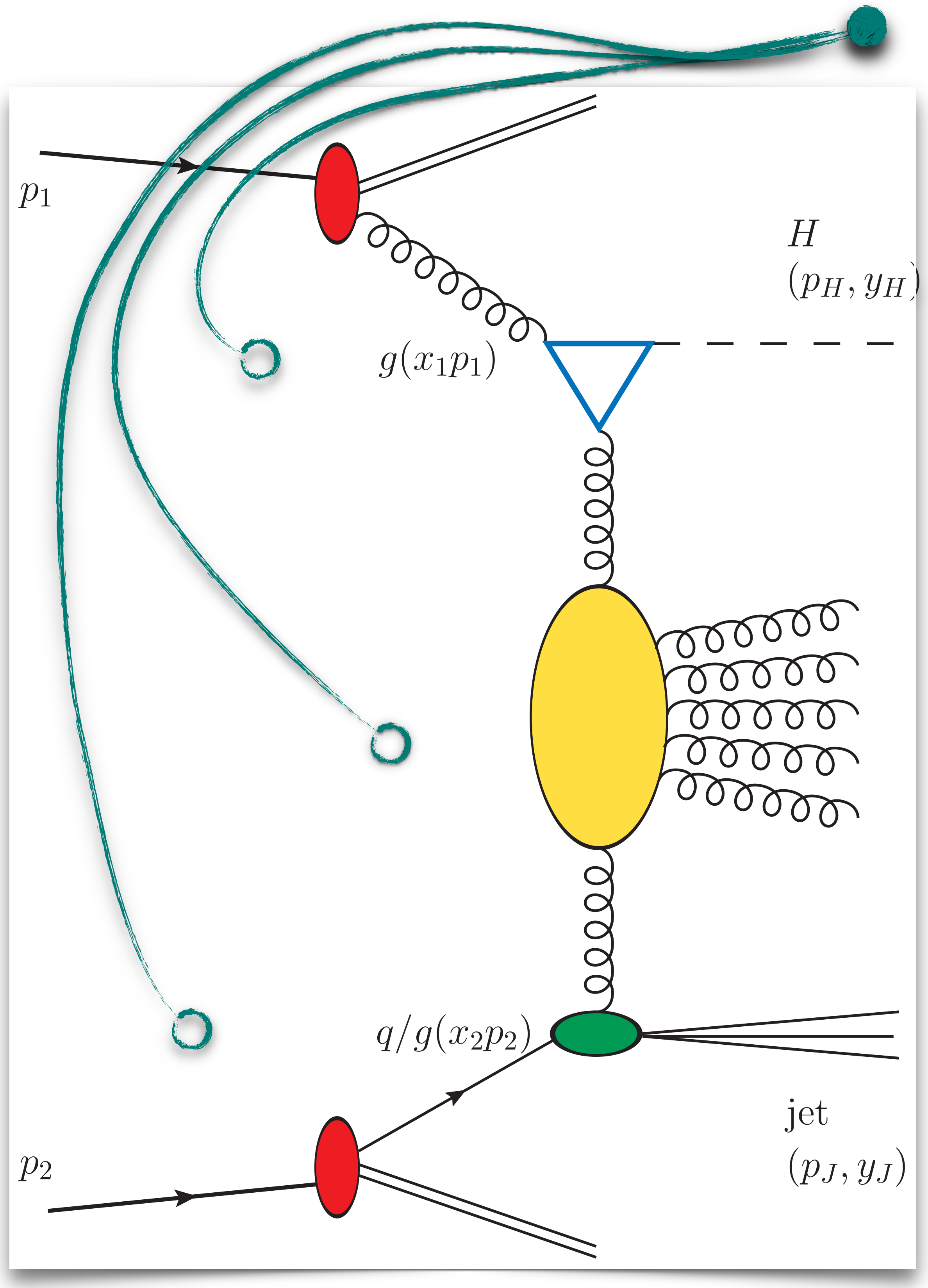
Higgs boson

Stabilizers ⇔ large Higgs transverse masses



$\mu_{F,R} \sim M_{H,\perp}$

NLO*

= LO + NLO[RGE]

NLL

NLO

(Higgs + jet, NLL/NLO*) [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

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Heavy flavor at large p_T

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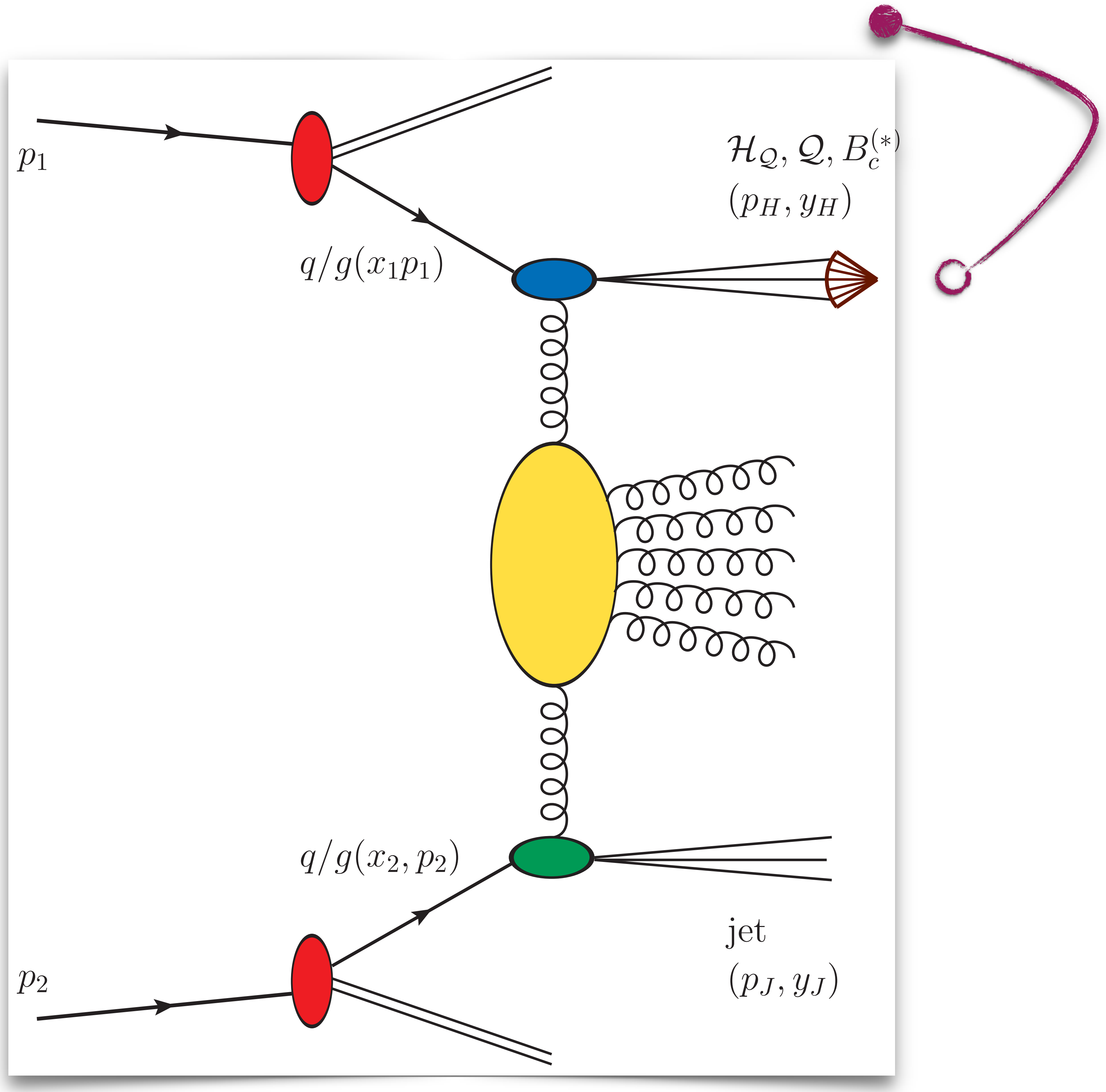

NLO(+)

NLL

NLO(+)

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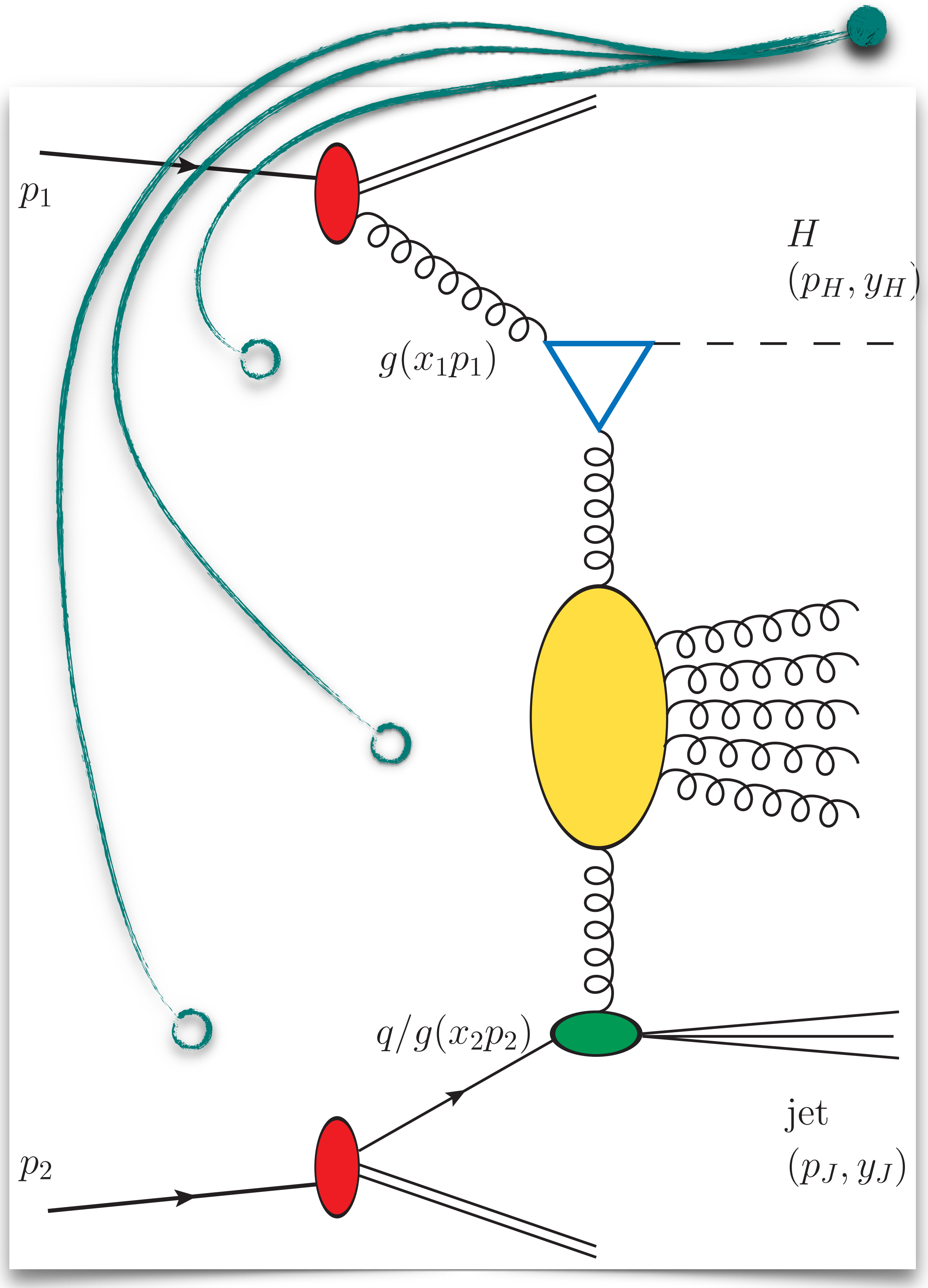

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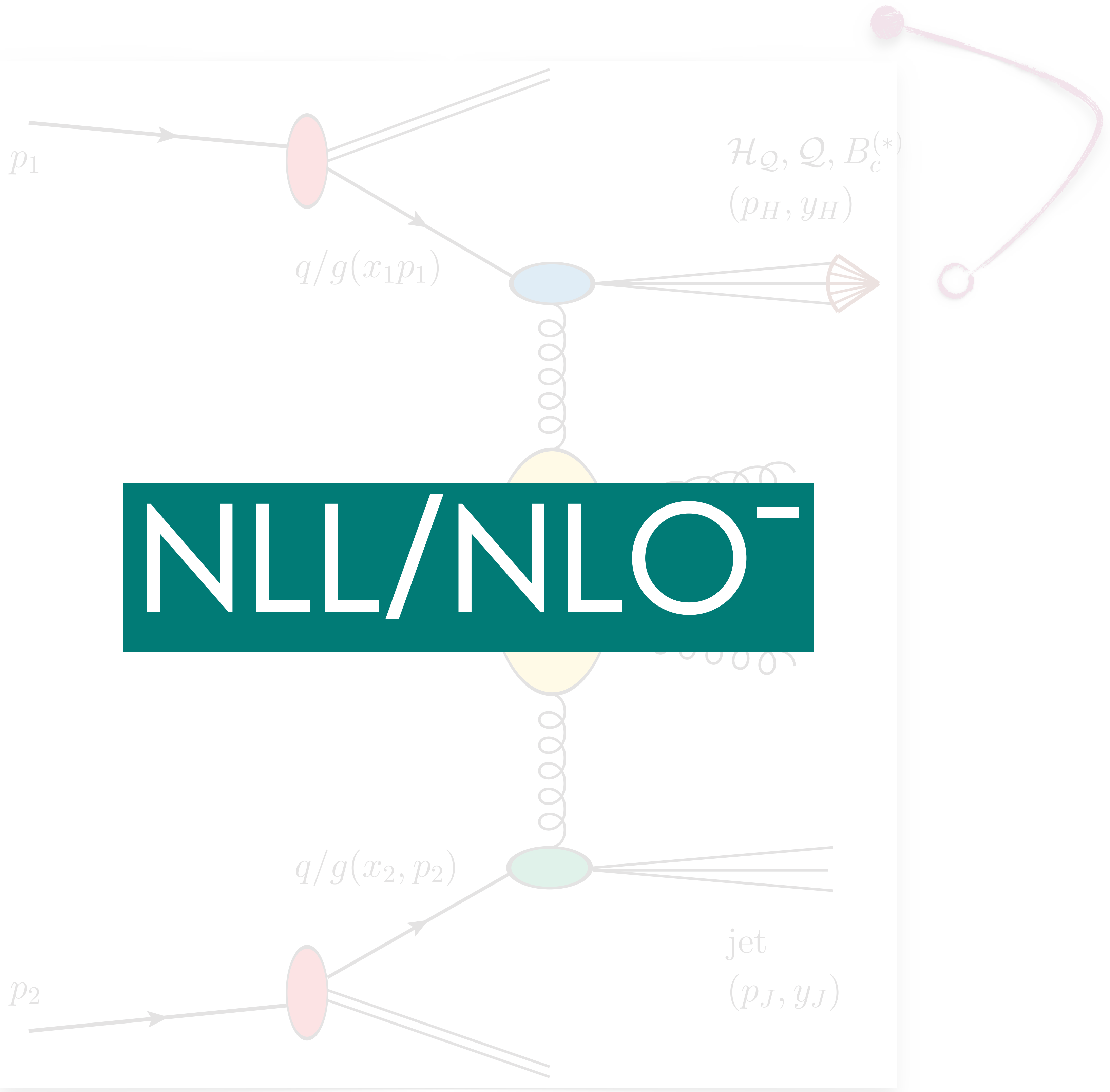
HEAVY FLAVOR AT LARGE p_T

Stabilizers ⇔ gluon fragmentation channels

NLO$^{(+)}$

NLL

NLO$^{(+)}$



NLL/NLO‾

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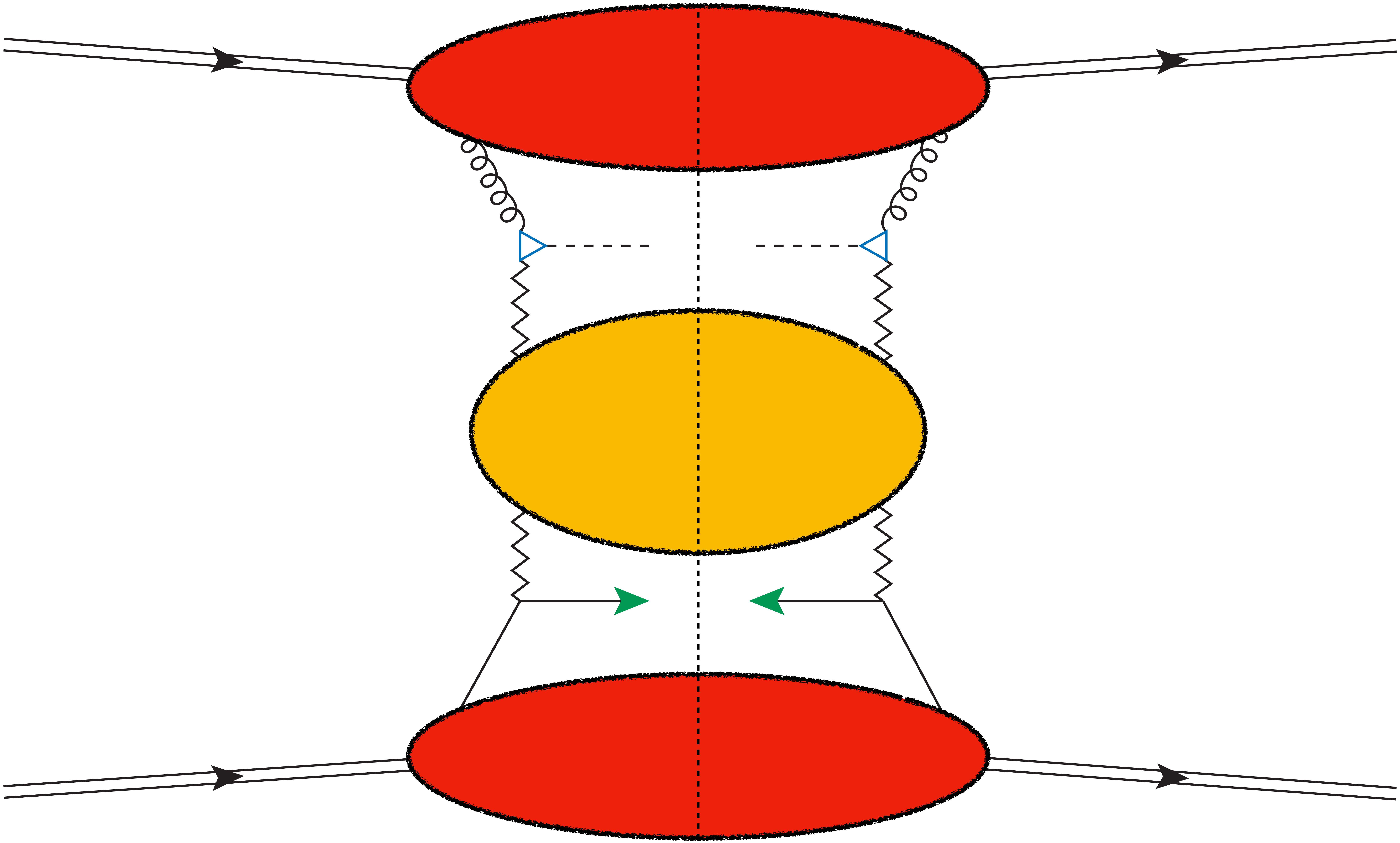
Anatomy of Higgs + jet in hybrid factorization (HyF)



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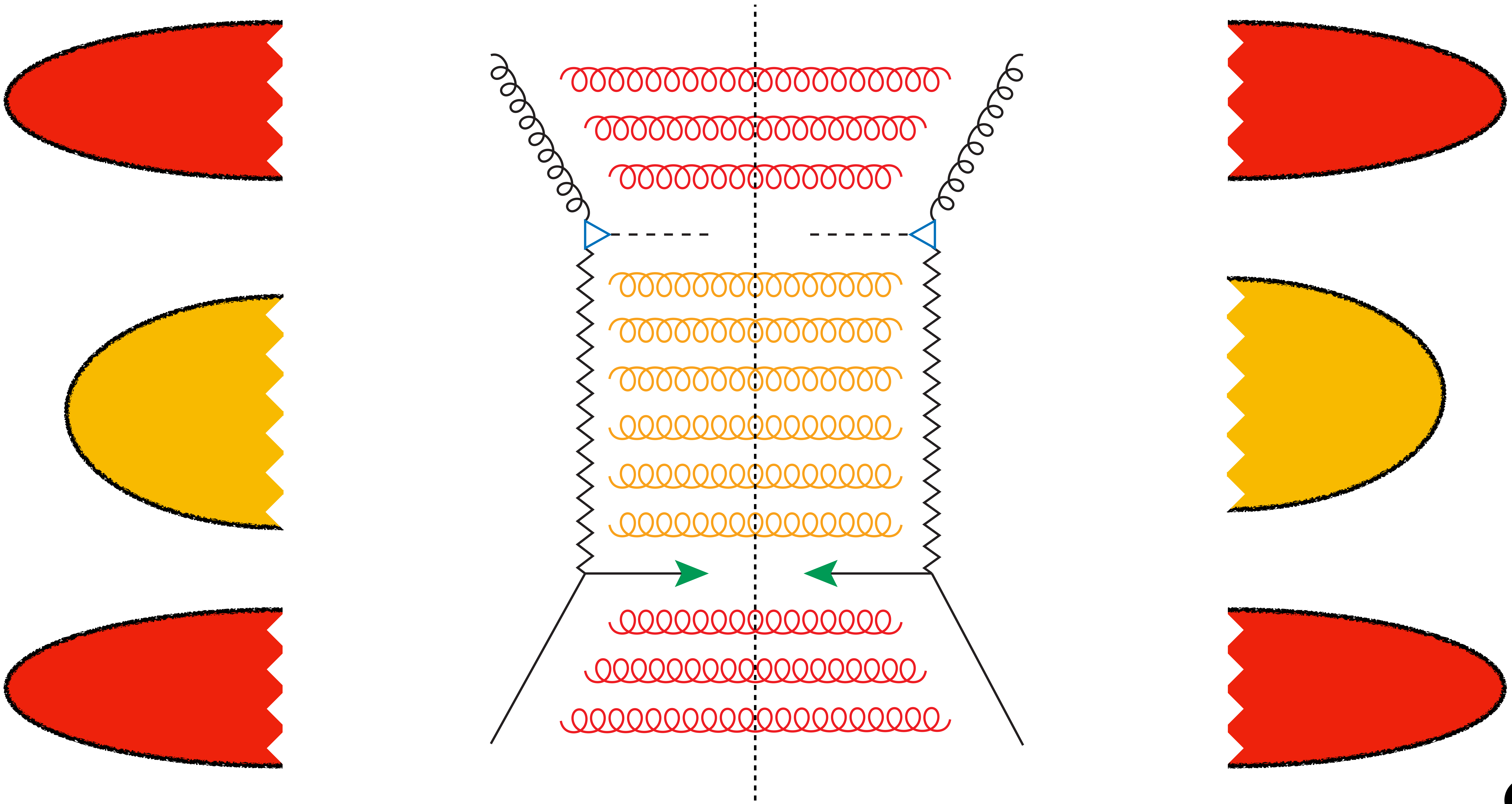

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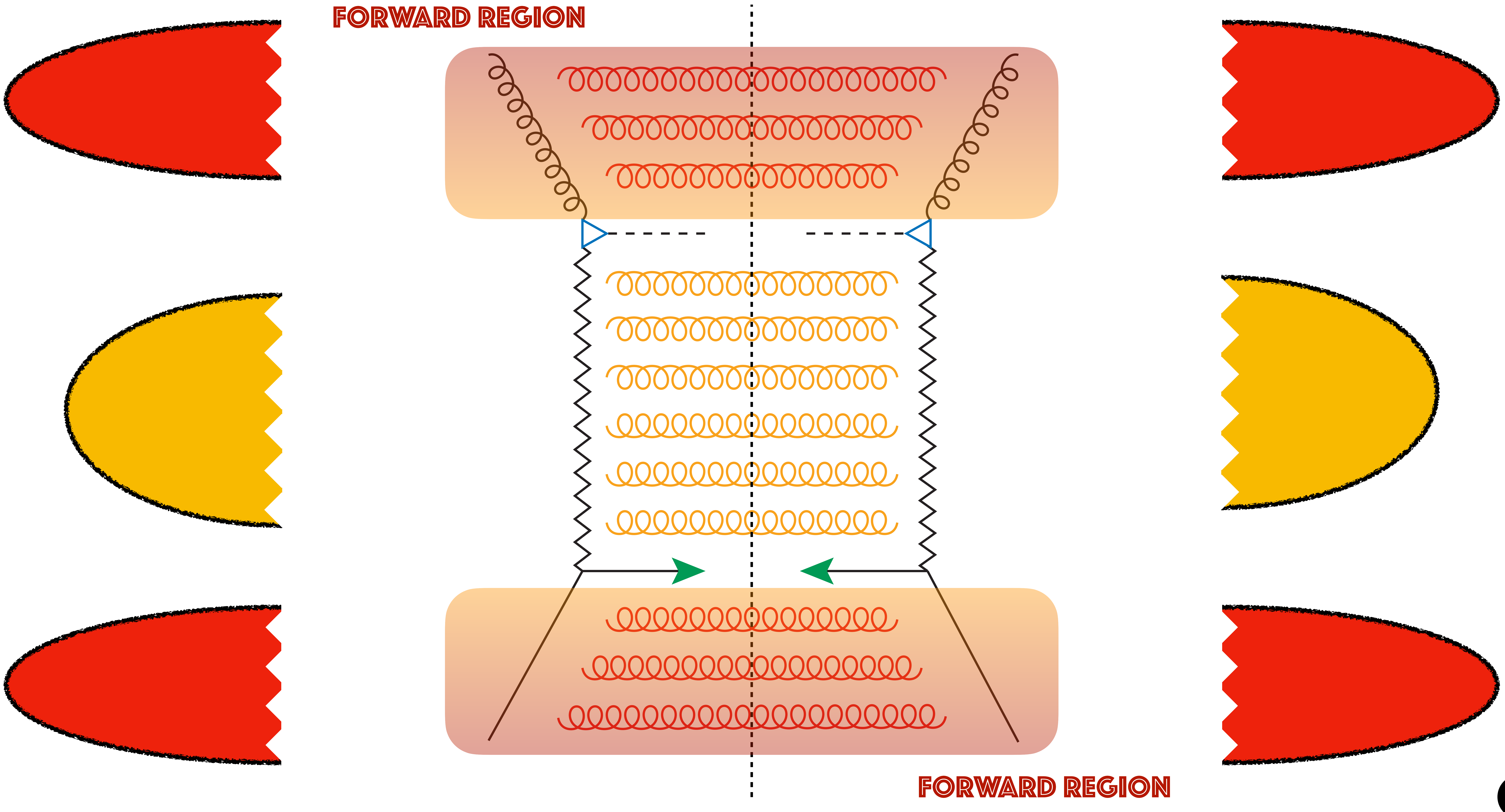

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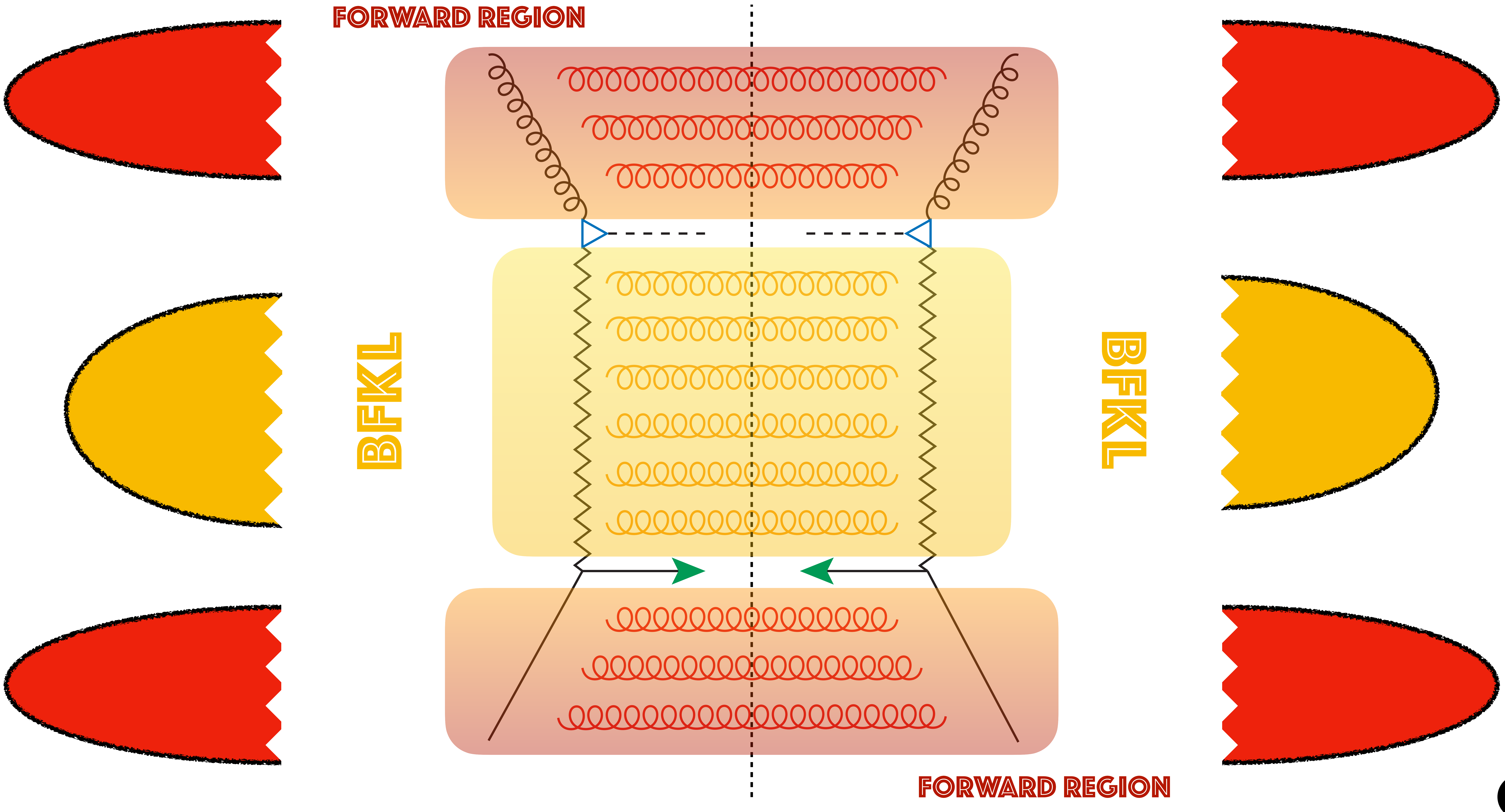


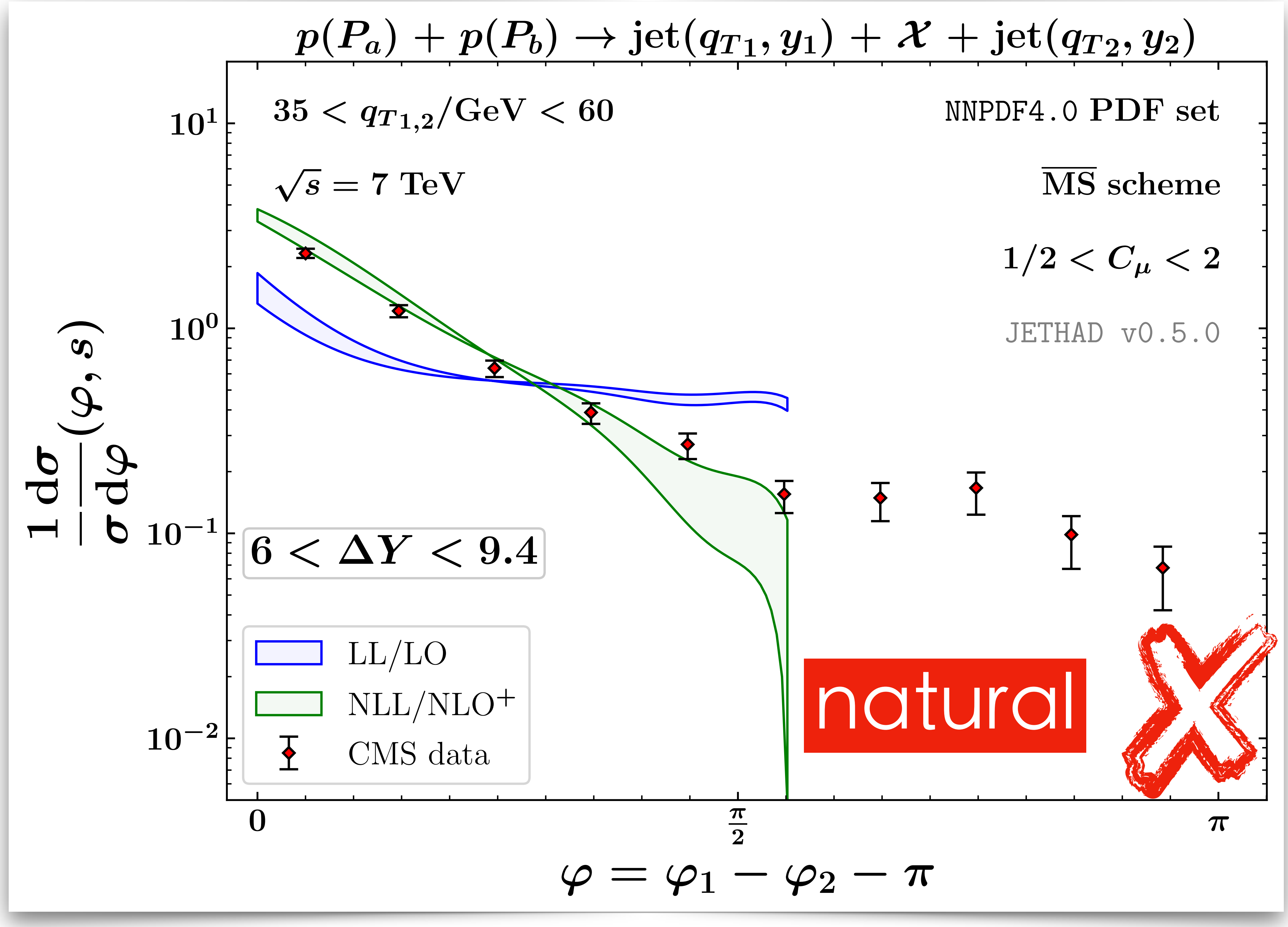
Azimuthal-angle multiplicity

$$\frac{1}{\sigma}\frac{d\sigma(\Delta Y, s)}{d\varphi} = \frac{1}{2\pi}\left\{1 + 2\sum_{n=1}^{\infty}\cos(n\varphi)\langle\cos(n\varphi)\rangle\right\}$$

MUELLER-NAVELET JETS

[B. Ducloué, L. Szymanowski, S. Wallon, Phys. Rev. Lett. 112 (2014) 082003]

(figure below) [F. G. C., A. Papa, Phys. Rev. D 106 (2022) 11, 114004]



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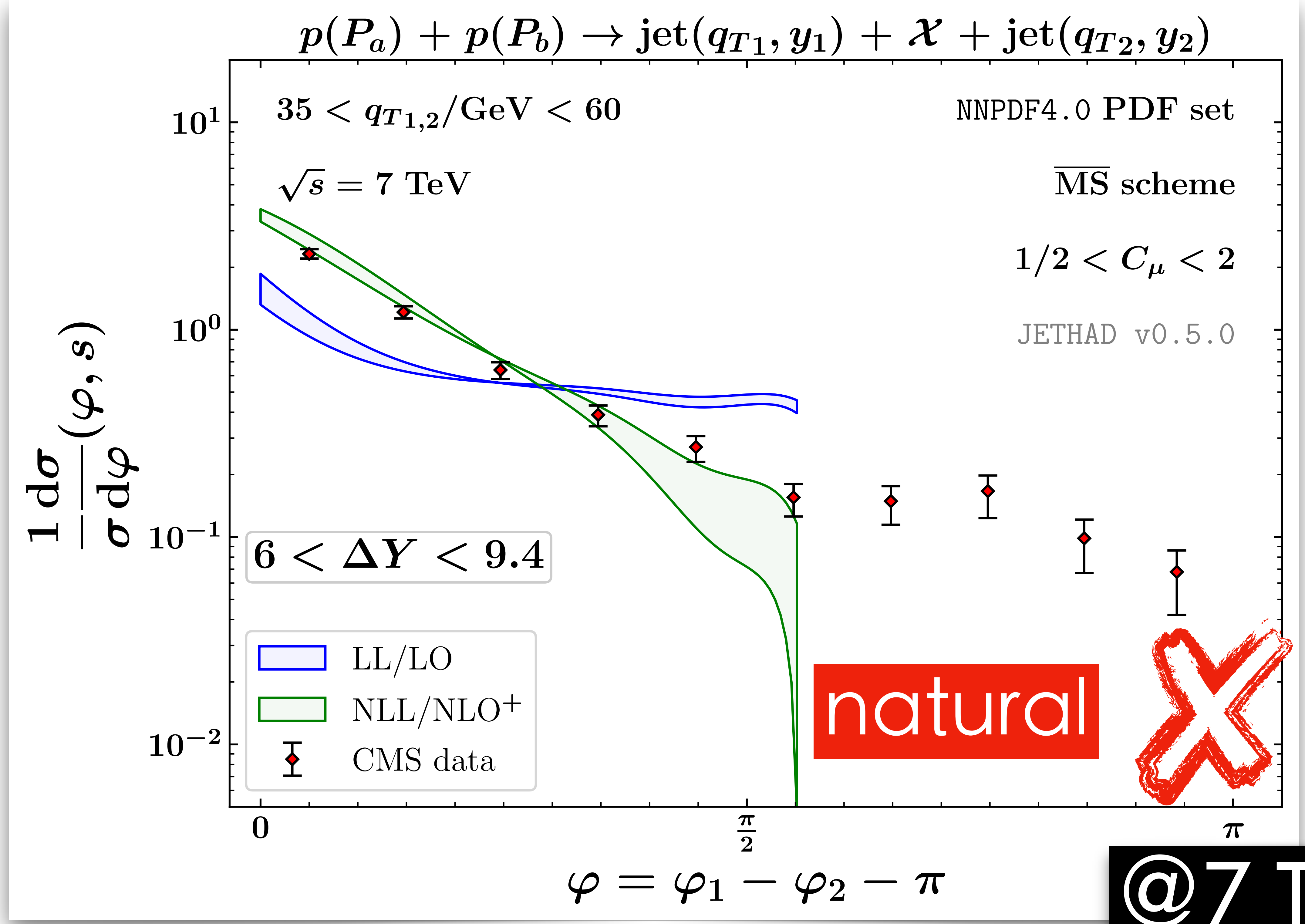
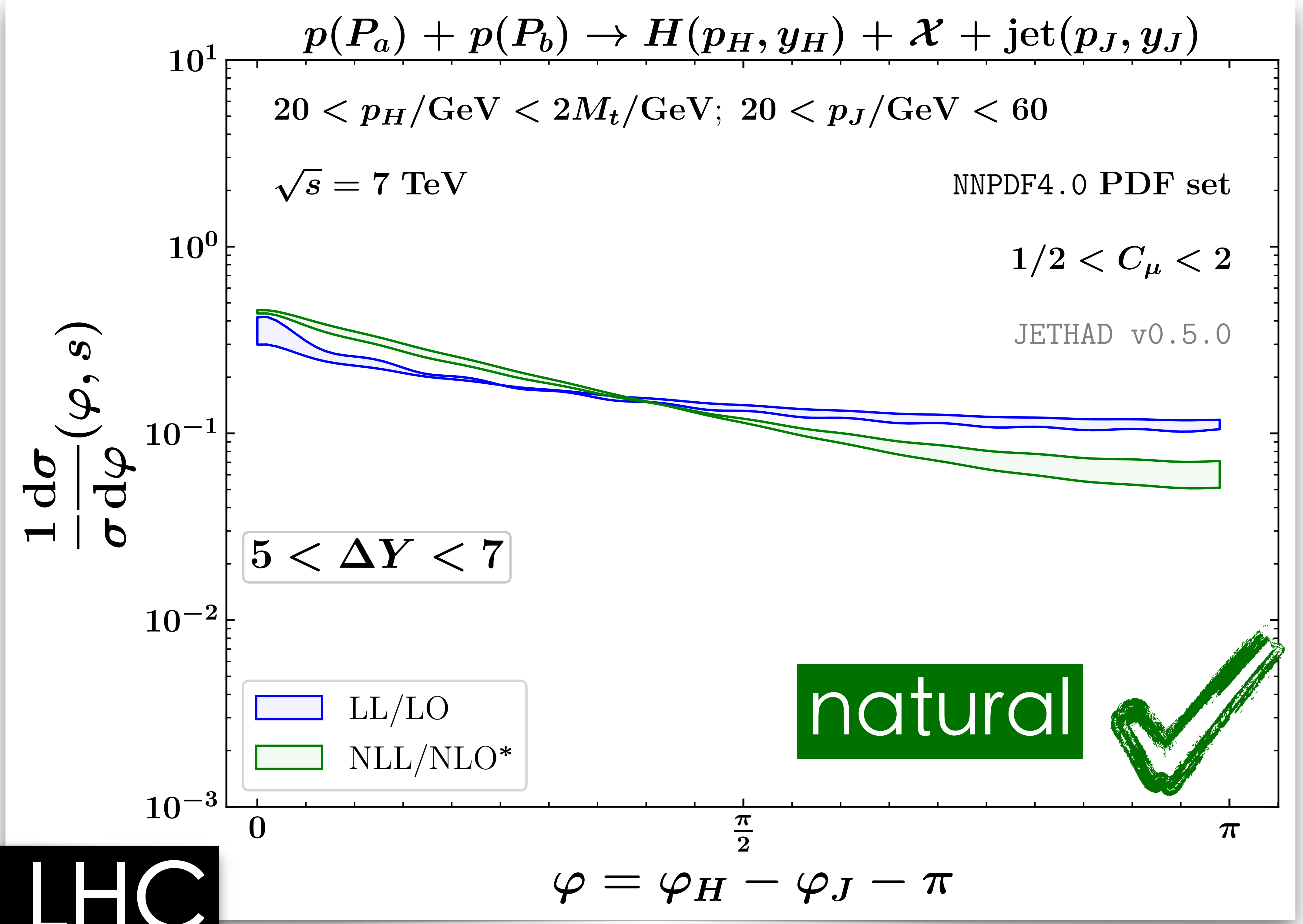
HIGGS + JET

(figure below) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 4, 293]

(NLO Higgs coefficient function) [F. G. C. et al., JHEP 08 (2022) 092]



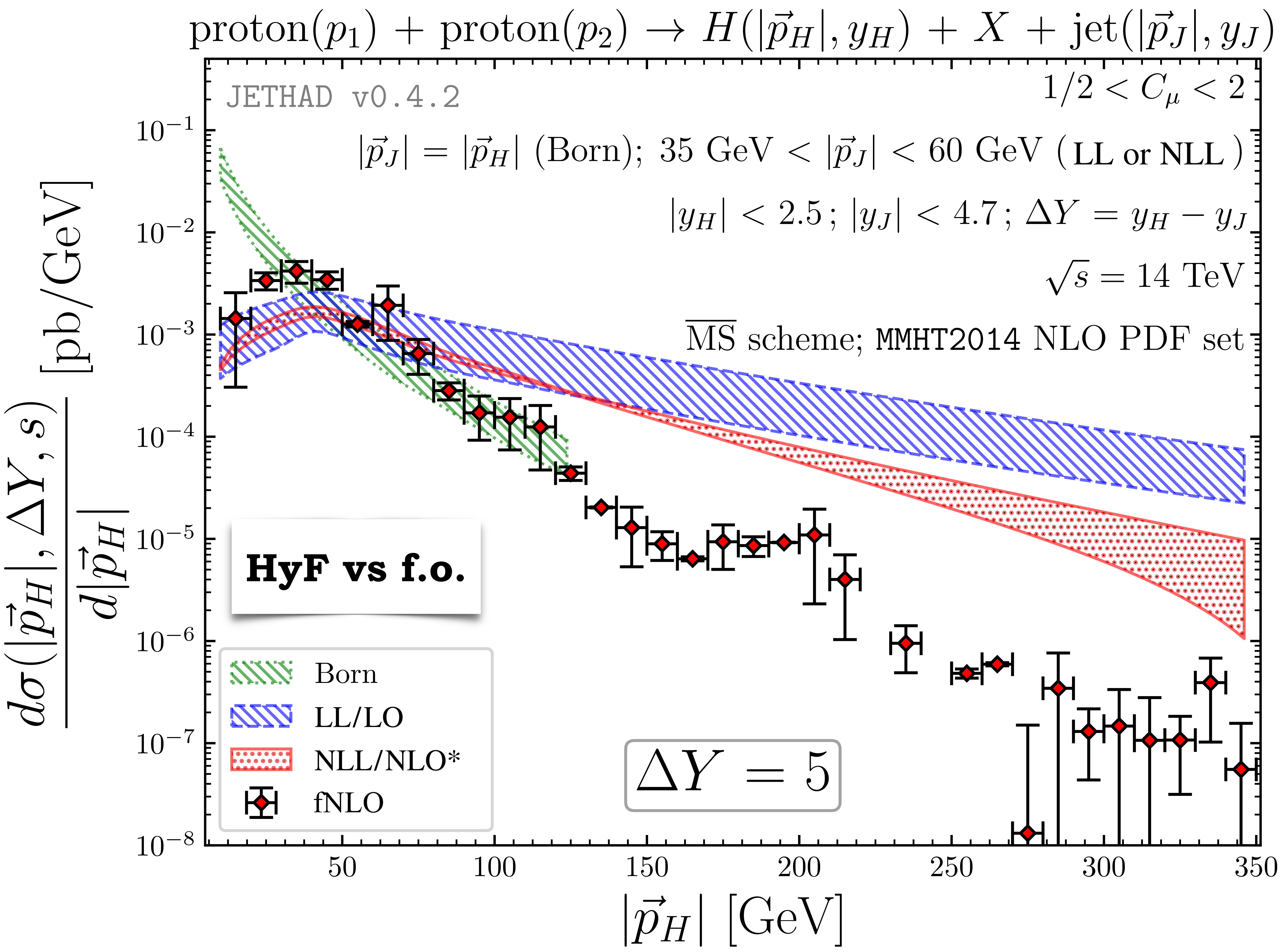


@7 TeV LHC

Hybrid factorization via the JETHAD code python + F

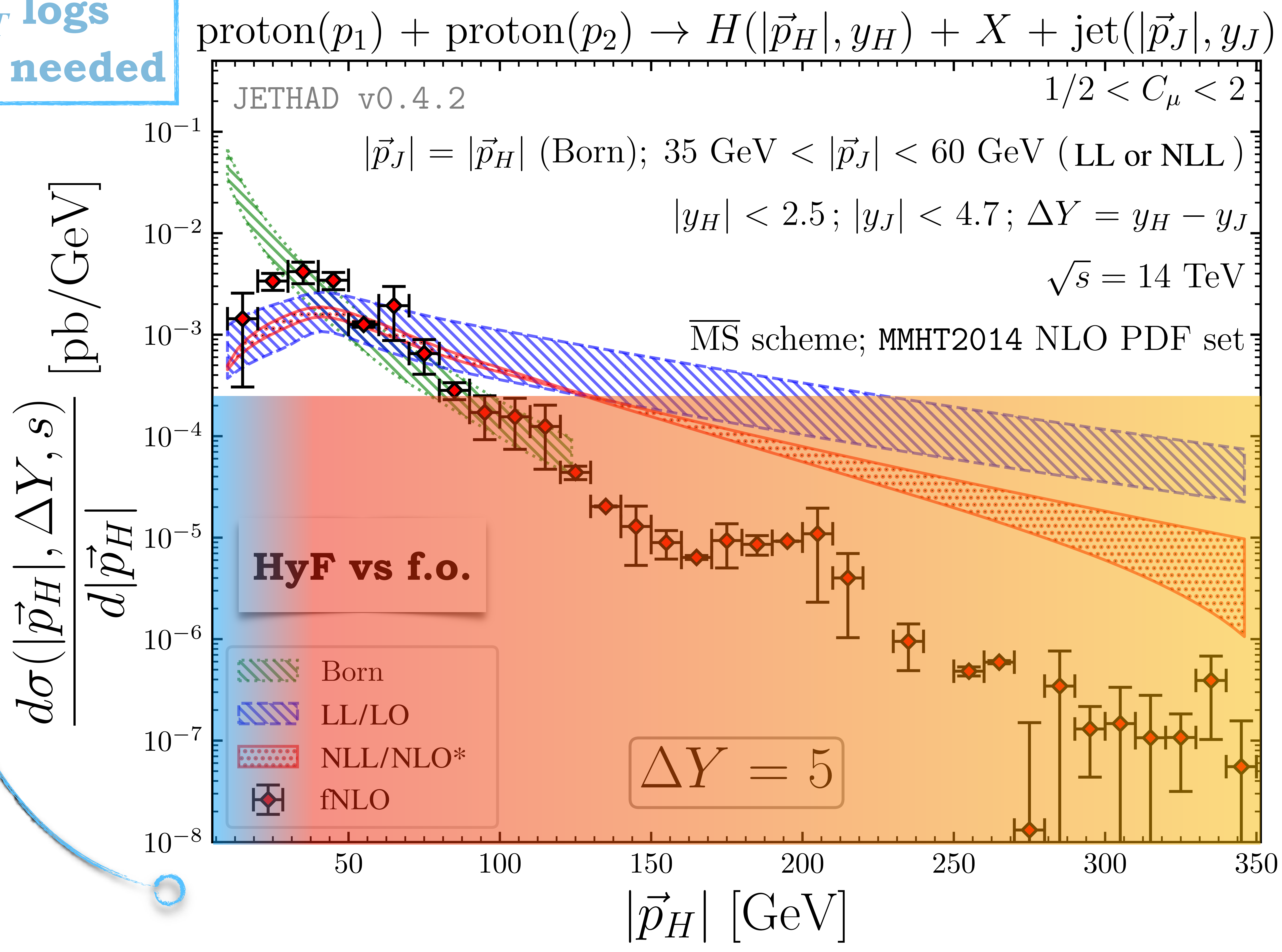
[F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]. [F. G. C. , Phys. Rev. D 105 (2022) 11, 114008]

NLL accurate predictions matched to NLO via the JETHAD Method



(Higgs + jet, NLL/NLO* HyF) 🔗 [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

large p_T logs
p_T-resum. needed



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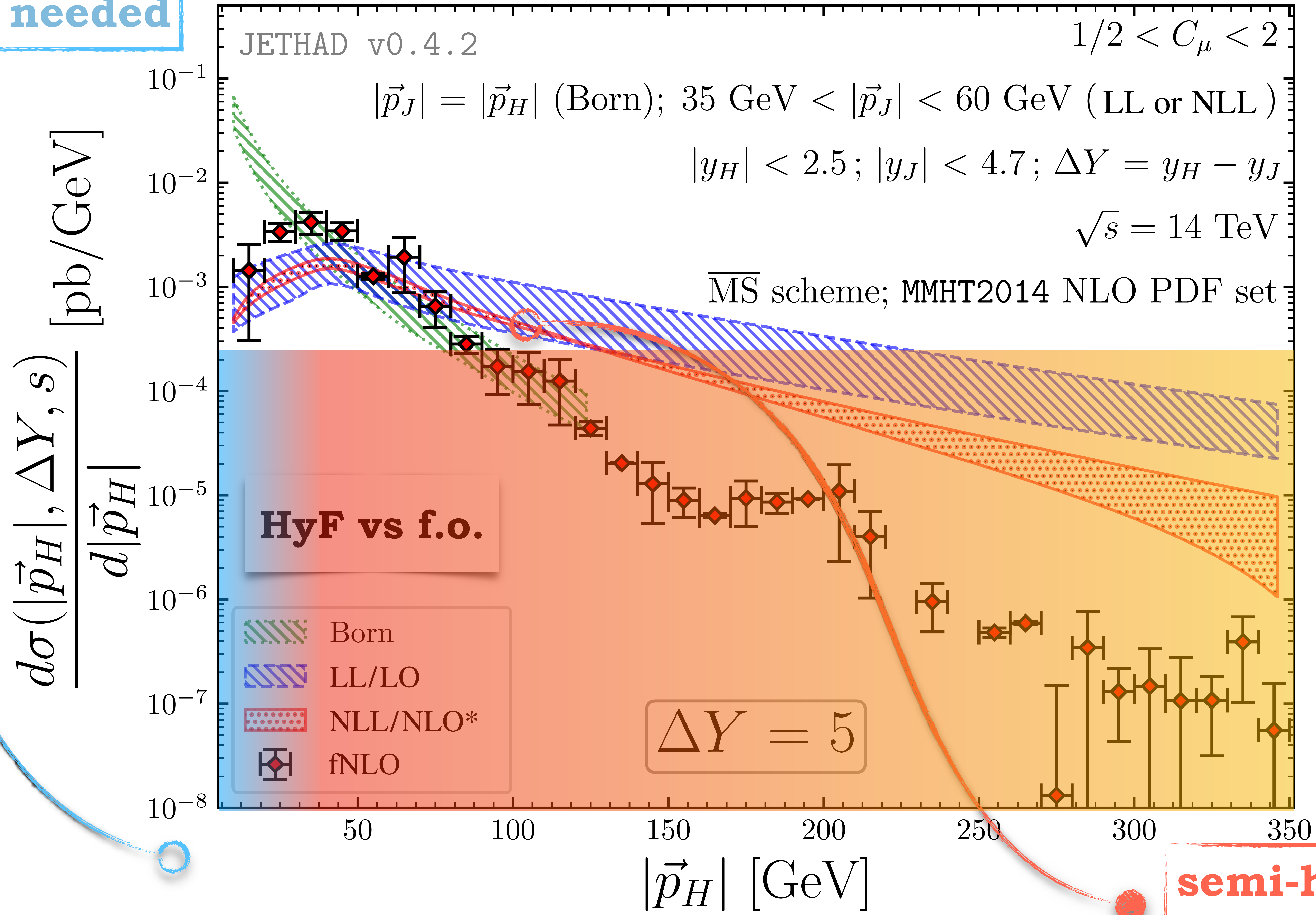
large p_T logs
p_T-resum. needed

$\text{proton}(p_1) + \text{proton}(p_2) \to H(|\vec{p}_H|, y_H) + X + \text{jet}(|\vec{p}_J|, y_J)$



semi-hard regime
BFKL expected

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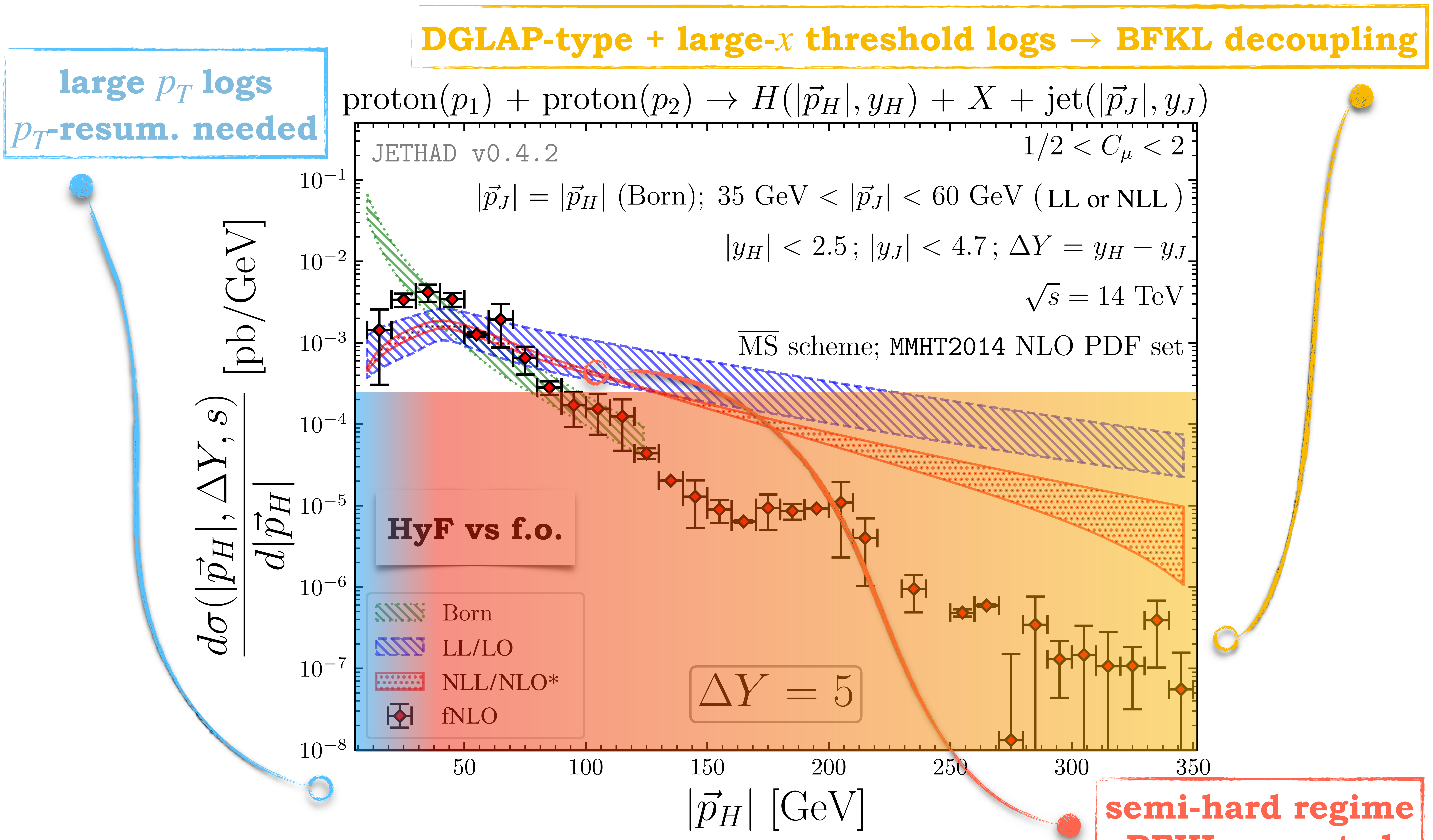
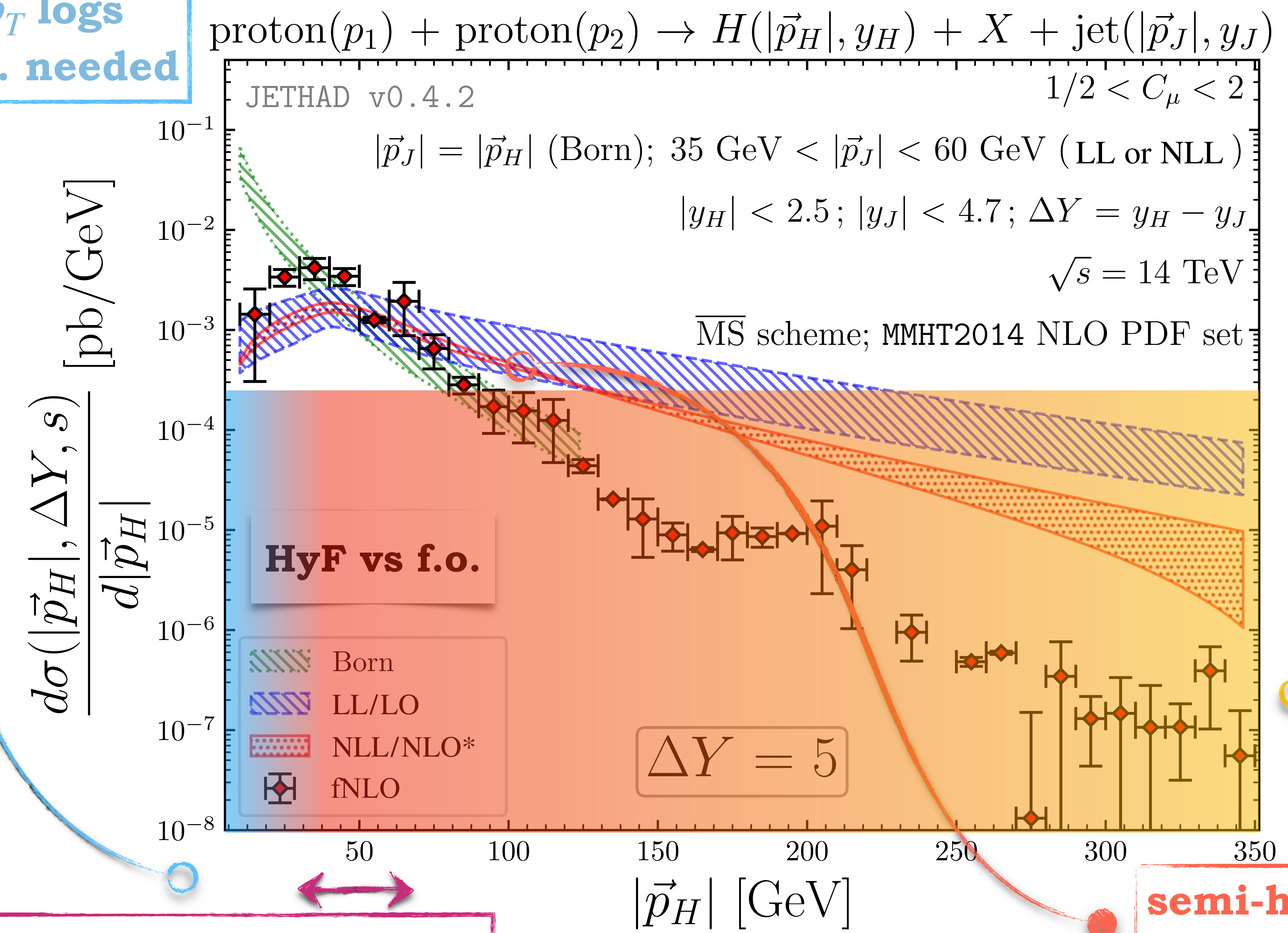


(Higgs + jet, NLL/NLO* HyF) 🔗 [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

DGLAP-type + large-x threshold logs → BFKL decoupling

large p_T logs
p_T-resum. needed

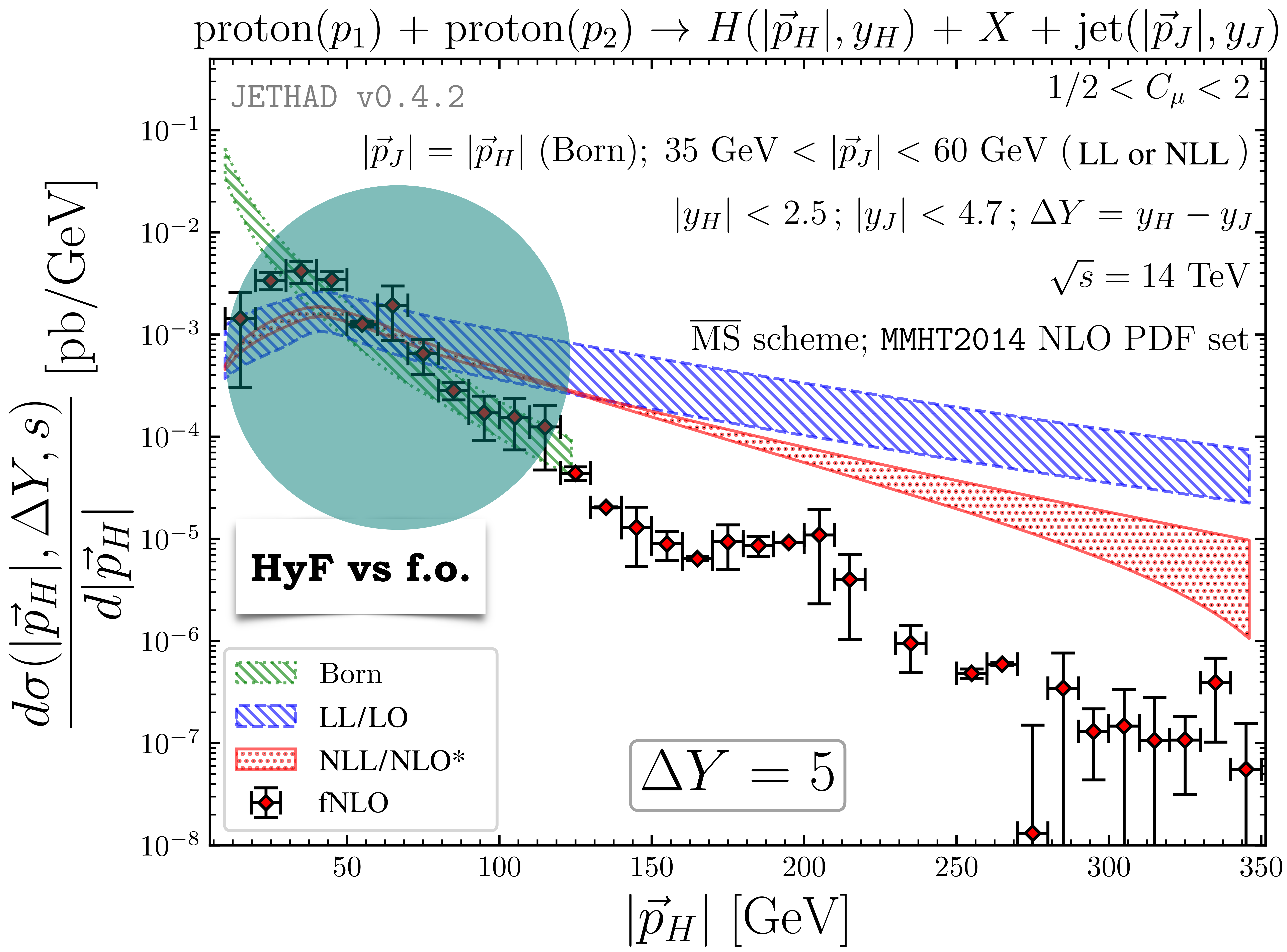


semi-hard regime
BFKL expected

almost back-to-back emissions
large imbalance double logs

(Higgs + jet, NLL/NLO* HyF) [F. G. C. et al., Eur. Phys. J. C (2021) 8, 780]

(Higgs + jet, NLL/NLO* HyF) 🔗 [F. G. C., D Yu. Ivanov, M, M. A. Mohammed, A. Papa, Eur. Phys. J. C (2021) 8, 780]



¡ **Precision corrections** *expected*

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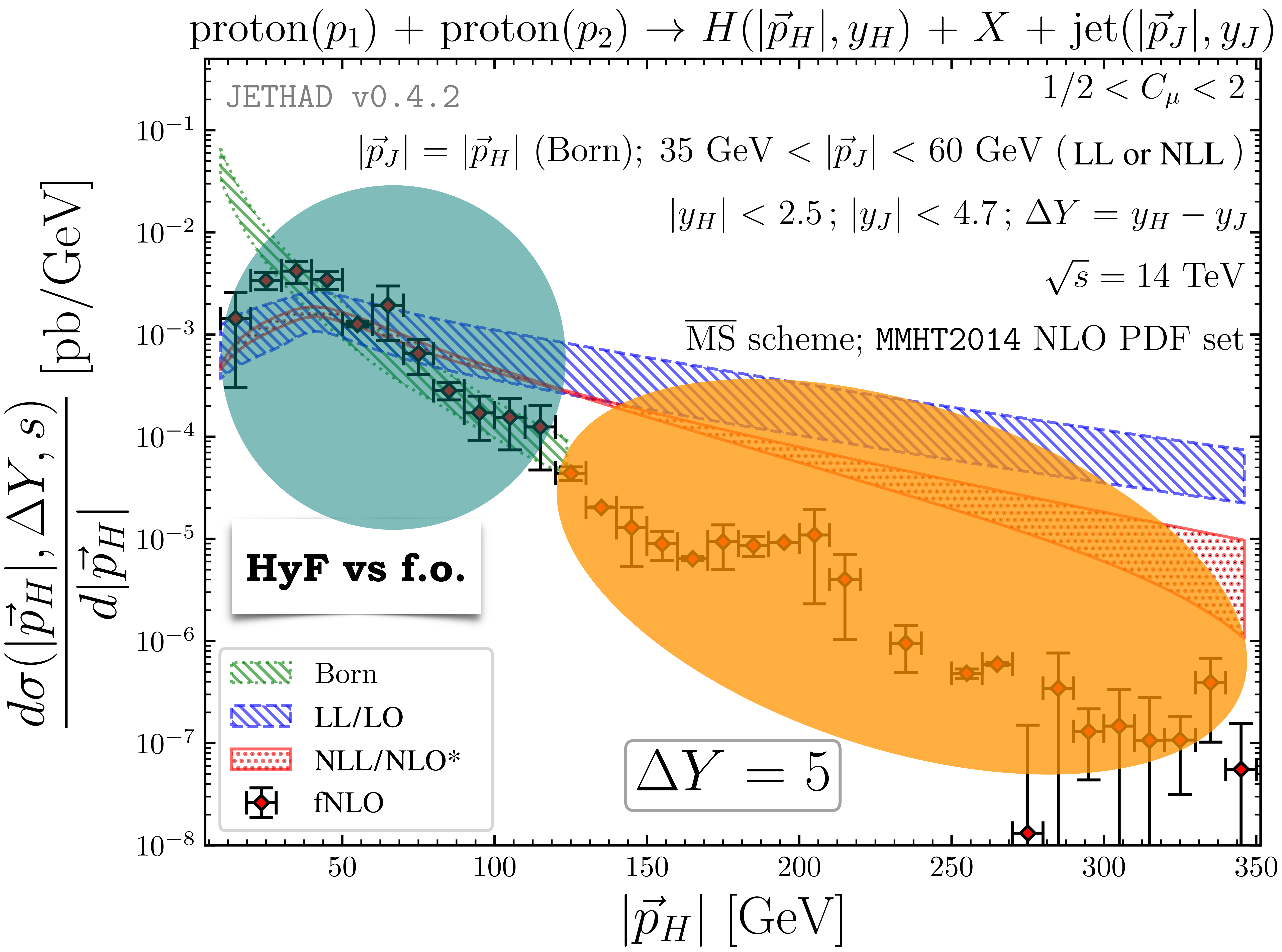

¡ **Precision corrections** *expected*, but **HyF** *predicts* **large deviations** from **f.o.** !

Matching NLL to NLO with JETHAD

¡ **Precision corrections** *expected* ⇔ *need* for an accurate NLL-to-NLO **Matching procedure** !

JETHAD Method → NLL/NLO Additive Matching (analytic: BFKL kernel + coefficient functions)

$$\underbrace{\mathrm{d}\sigma^{\mathrm{NLL/NLO^{-}}}(\Delta Y, \varphi, s)} \; = \underbrace{\underbrace{\mathrm{d}\sigma^{\mathrm{NLO}}(\Delta Y, \varphi, s)}_{\textbf{NLO}\ \text{fixed order}}}_{\textbf{NLO}\ \texttt{POWHEG}\ \text{w/o PS}} + \mathrm{d}\sigma^{\mathrm{NLL^{-}}}(\Delta Y, \varphi, s) - \Delta\mathrm{d}\sigma^{\mathrm{NLL/NLO^{-}}}(\Delta Y, \varphi, s)$$

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HELL + ggHiggs

$N^3LL_{lx}/LL_{sx}/N^3LO$

Inclusive Higgs

[M. Bonvini, S. Marzani (2018)]

Matching NLL to NLO with JETHAD

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HEJ framework

NLL_{sx}^-/NLO

Higgs + jet(s)

[J. R. Andersen et al. (2022)]

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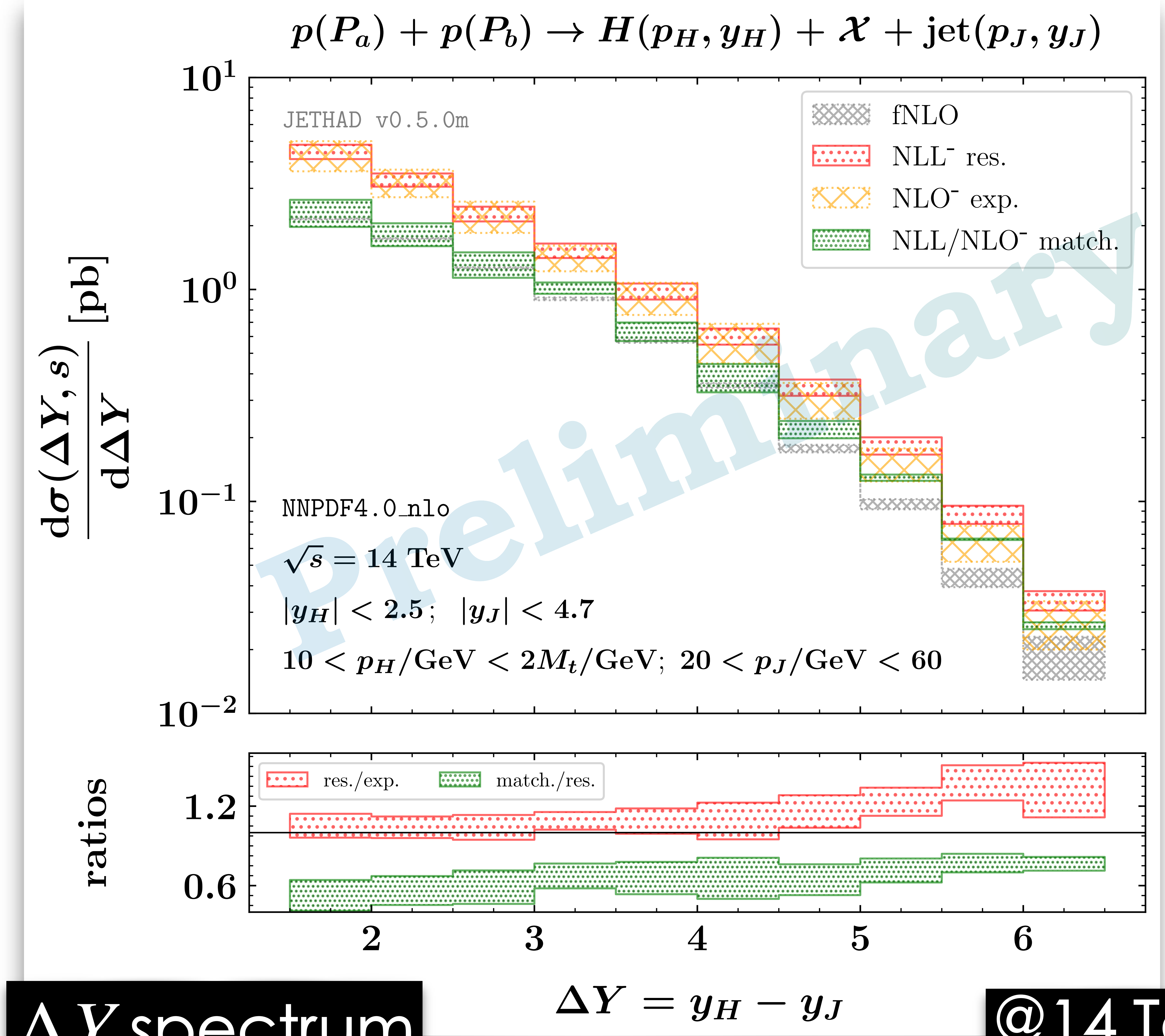
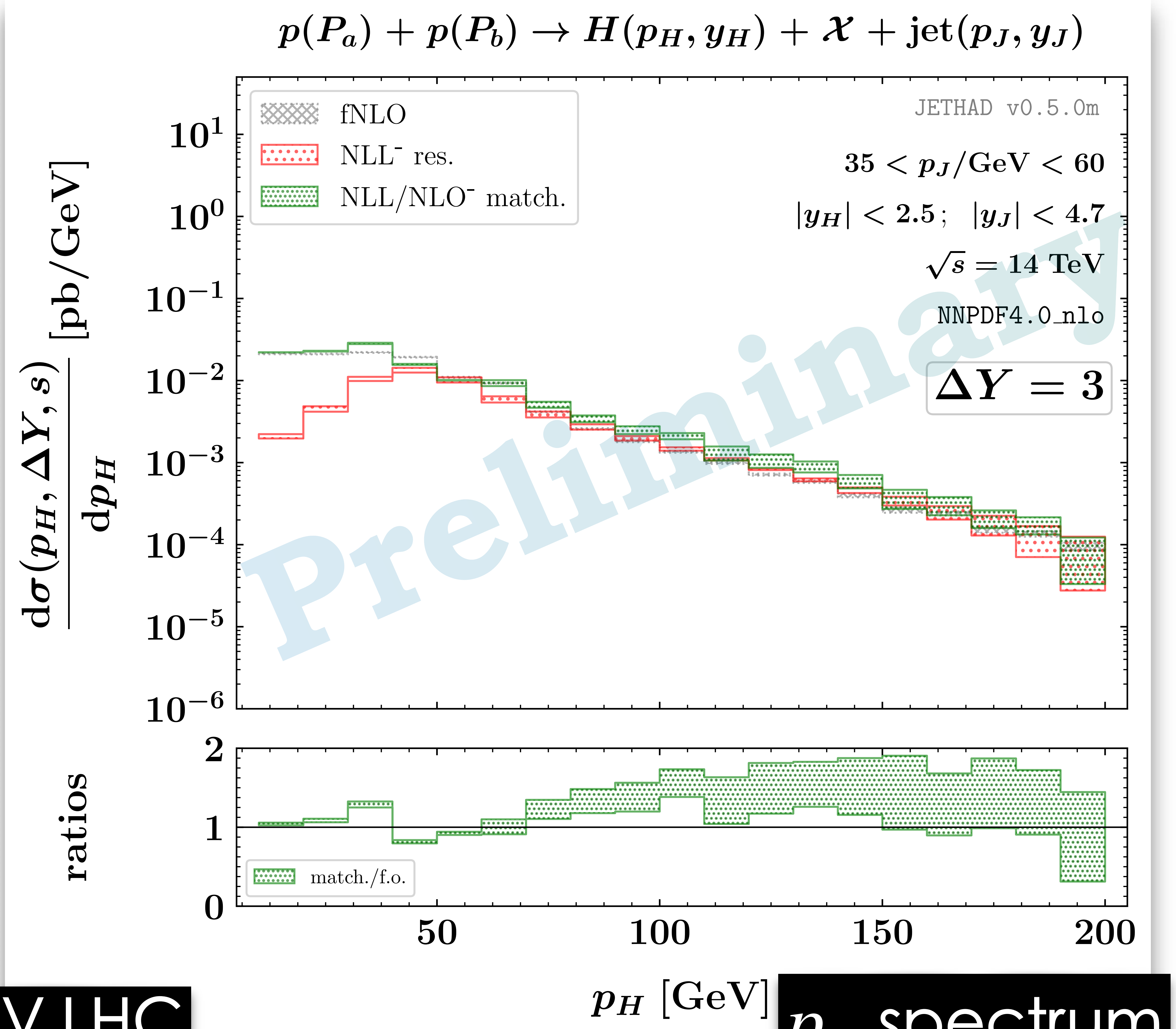
HELL + ggHiggs	HEJ framework	RadISH + MCFM-8.3
$\mathrm{N^3LL_{lx}/LL_{sx}/N^3LO}$	$\mathrm{NLL_{sx}^-/NLO}$	$\mathrm{NNLL_{TM}/NLO}$
Inclusive Higgs	Higgs + jet(s)	Higgs + jet
[M. Bonvini, S. Marzani (2018)]	[J. R. Andersen et al. (2022)]	[P.F. Monni et al. (2020)]

The Higgs + jet spectrum from POWHEG + JETHAD



ΔY spectrum

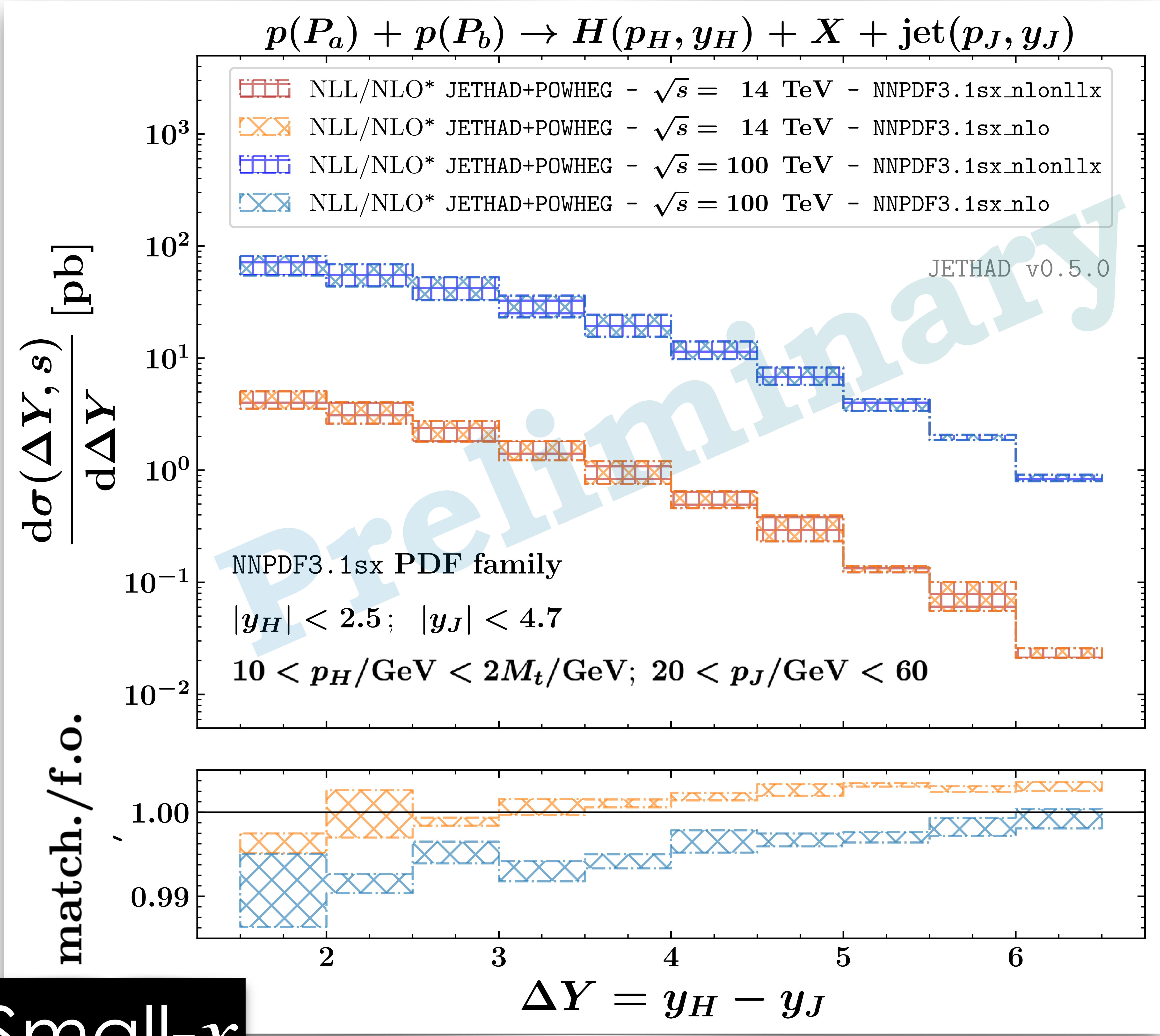


@14 TeV LHC

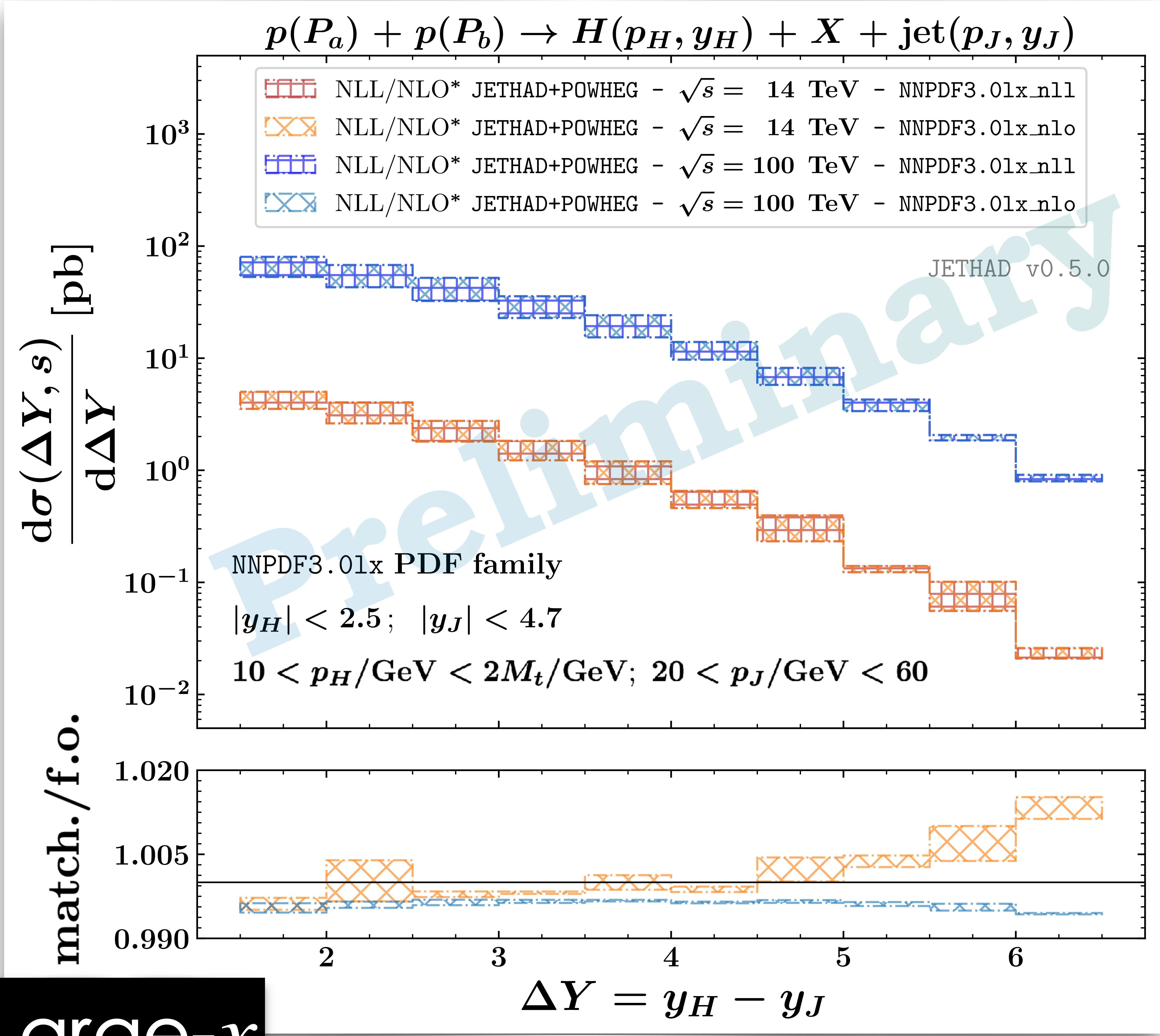
p_H spectrum

NLL matched to NLO fixed-order POWHEG + JETHAD (in progress)

Small-x and large-x enhancement from PDFs



Small-x



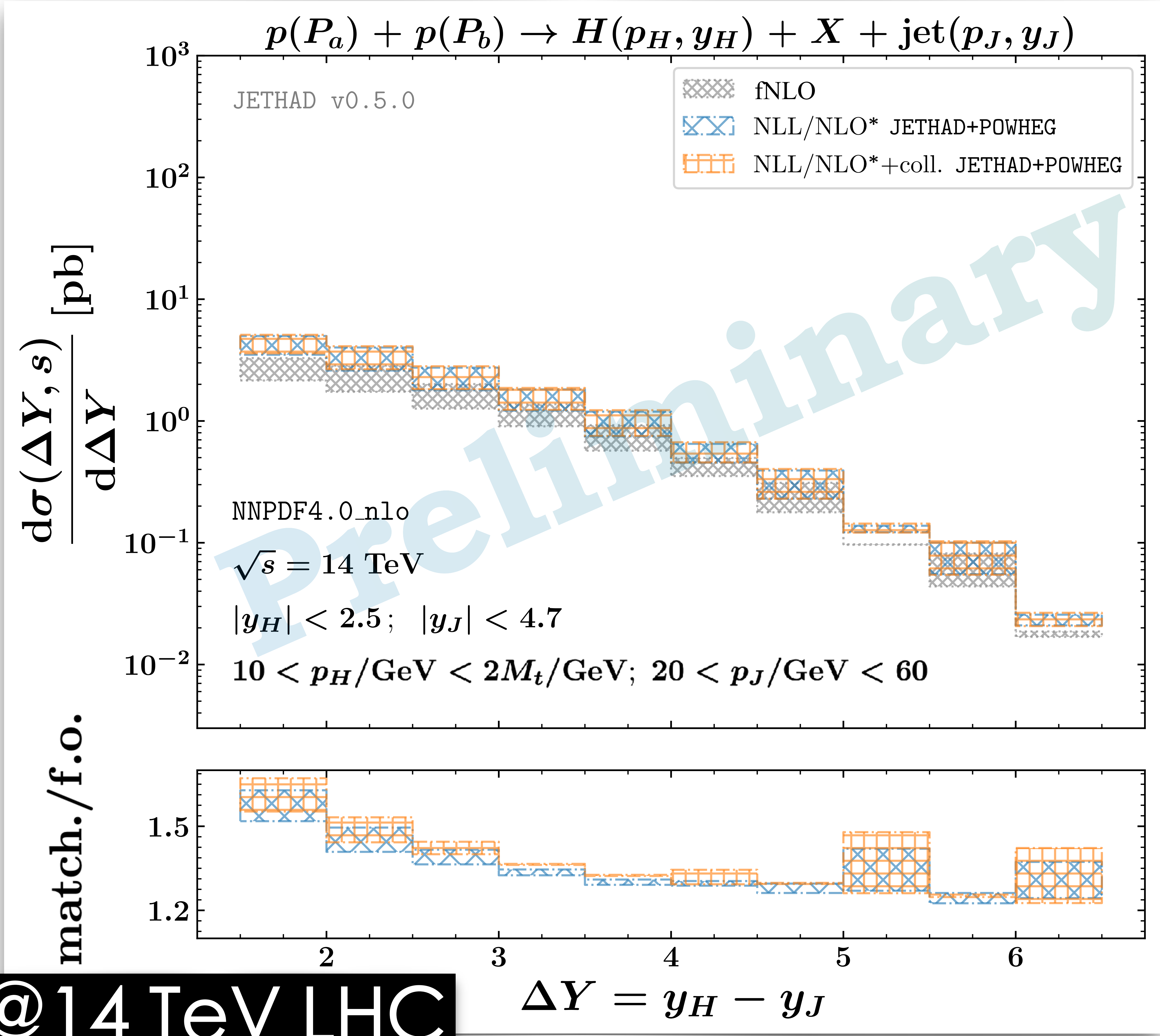
Large-x

Impact of large-x threshold logs on NLO coeff. functions to be gauged

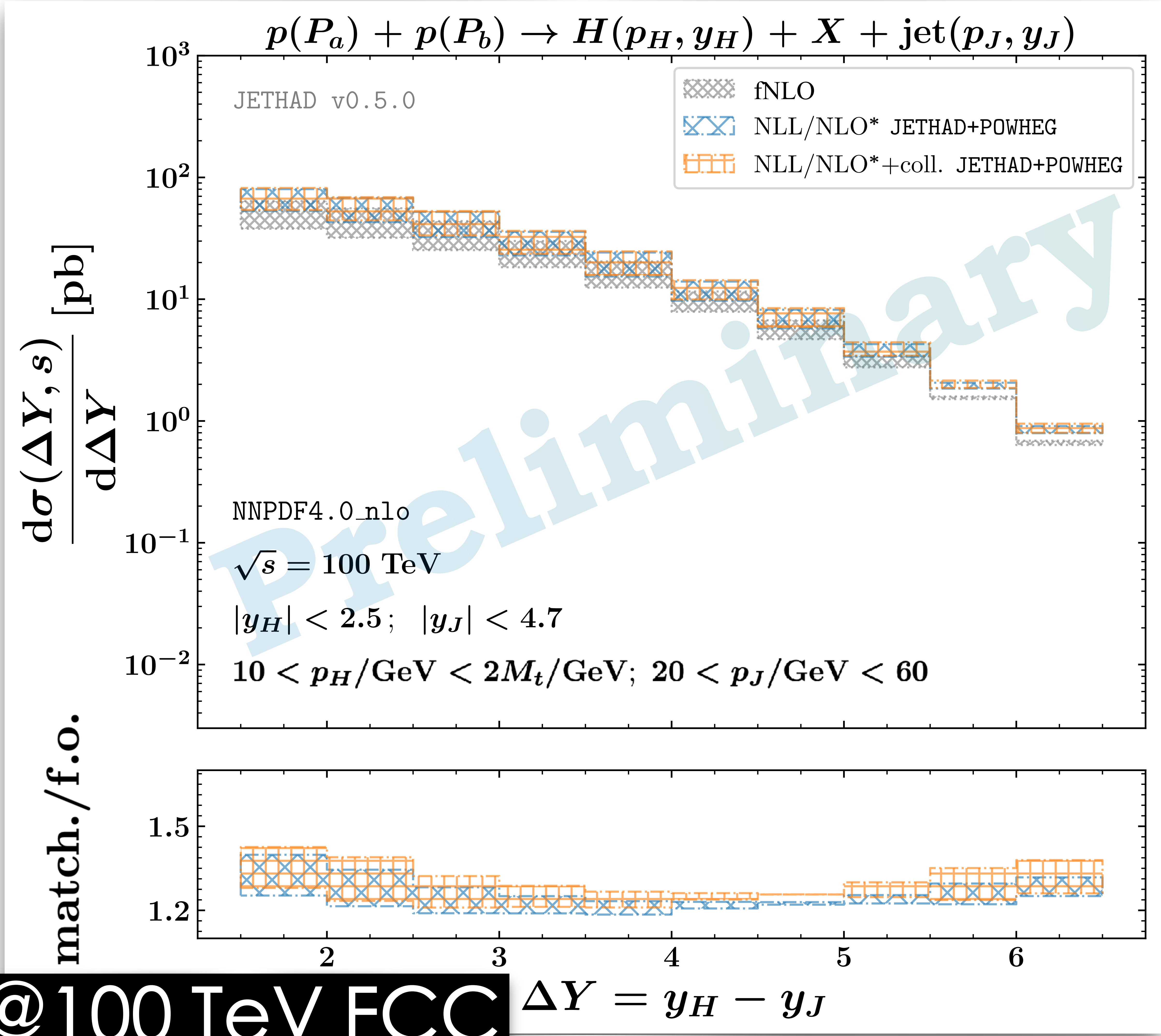
Small-x and large-x enhancement from PDFs

Impact of large-x threshold logs on NLO coeff. functions to be gauged

Effect of collinear improvement on NLL BFKL kernel



@14 TeV LHC



@100 TeV FCC

(Collinear improvement) [G.P. Salam, JHEP 07 (1998) 019]; [M. Ciafaloni et al., Phys.Lett.B 587 (2004) 87-94]; [A. Sabio Vera, Nucl.Phys.B 722 (2005) 65-80]

Effect of ABF-stabilized kernel to be gauged

Finite top- and bottom-mass corrections

Effect of finite heavy-quark masses on NLO coeff. functions to be gauged

Paving the way toward precision

- ☑ Semi-inclusive Higgs + jet as novel probe for High-Energy QCD
- ☑ Encouraging statistics for rapidity & transverse-momentum distributions
- ☑ Fair stability under NLL corrections

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- ☐ Precision studies ⇔ NLL/NLO Matching via the JETHAD Method
- ☐ Systematic uncertainties: top & bottom masses, PDF impact, Matching
- ☐ Compare TM spectrum with Parton Shower, then Match/Merge with NLL_{sx} (¿?)
- ☐ Transversal formalism as underlying staging for several resummations

EXTRAS

Higgs production from LHC to FCC

PHYSICAL REVIEW LETTERS **120,** 202003 (2018)

Double Resummation for Higgs Production

Marco Bonvini[1,*] and Simone Marzani[2,†]
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(Received 26 February 2018; published 16 May 2018)

We present the first double-resummed prediction of the inclusive cross section for the main Higgs production channel in proton-proton collisions, namely, gluon fusion. Our calculation incorporates to all orders in perturbation theory two distinct towers of logarithmic corrections which are enhanced, respectively, at threshold, i.e., large x, and in the high-energy limit, i.e., small x. Large-x logarithms are resummed to next-to-next-to-next-to-leading logarithmic accuracy, while small-x ones to leading logarithmic accuracy. The double-resummed cross section is furthermore matched to the state-of-the-art fixed-order prediction at next-to-next-to-next-to-leading accuracy. We find that double resummation corrects the Higgs production rate by 2% at the currently explored center-of-mass energy of 13 TeV and its impact reaches 10% at future circular colliders at 100 TeV.

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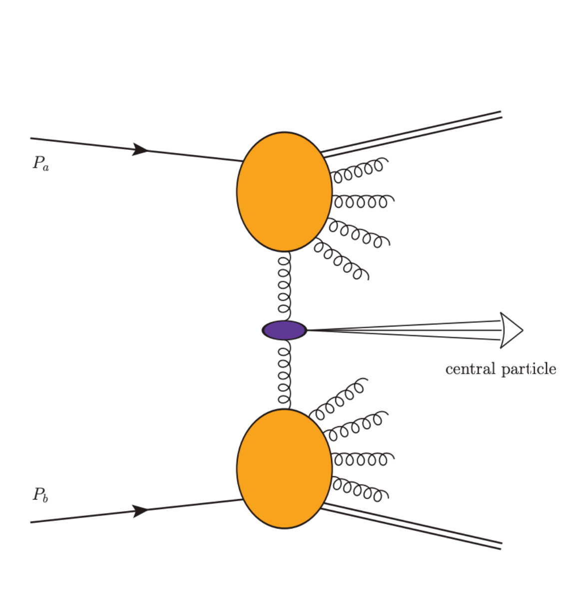

(*b*) Single central

High-energy resummation (BFKL) ⇒ PDFs at small-x

Altarelli-Ball-Forte to stabilize the NLL_{sx} BFKL kernel

N^3LL_{lx}/LL_{sx}/N^3LO rapidity-inclusive coefficient functions

$$C_{ij}(x,\alpha_s) = C^{\rm fo}_{ij}(x,\alpha_s) + \Delta C^{\rm lx}_{ij}(x,\alpha_s) + \Delta C^{\rm sx}_{ij}(x,\alpha_s)$$

Backup

Higgs production from LHC to FCC

PHYSICAL REVIEW LETTERS **120,** 202003 (2018)

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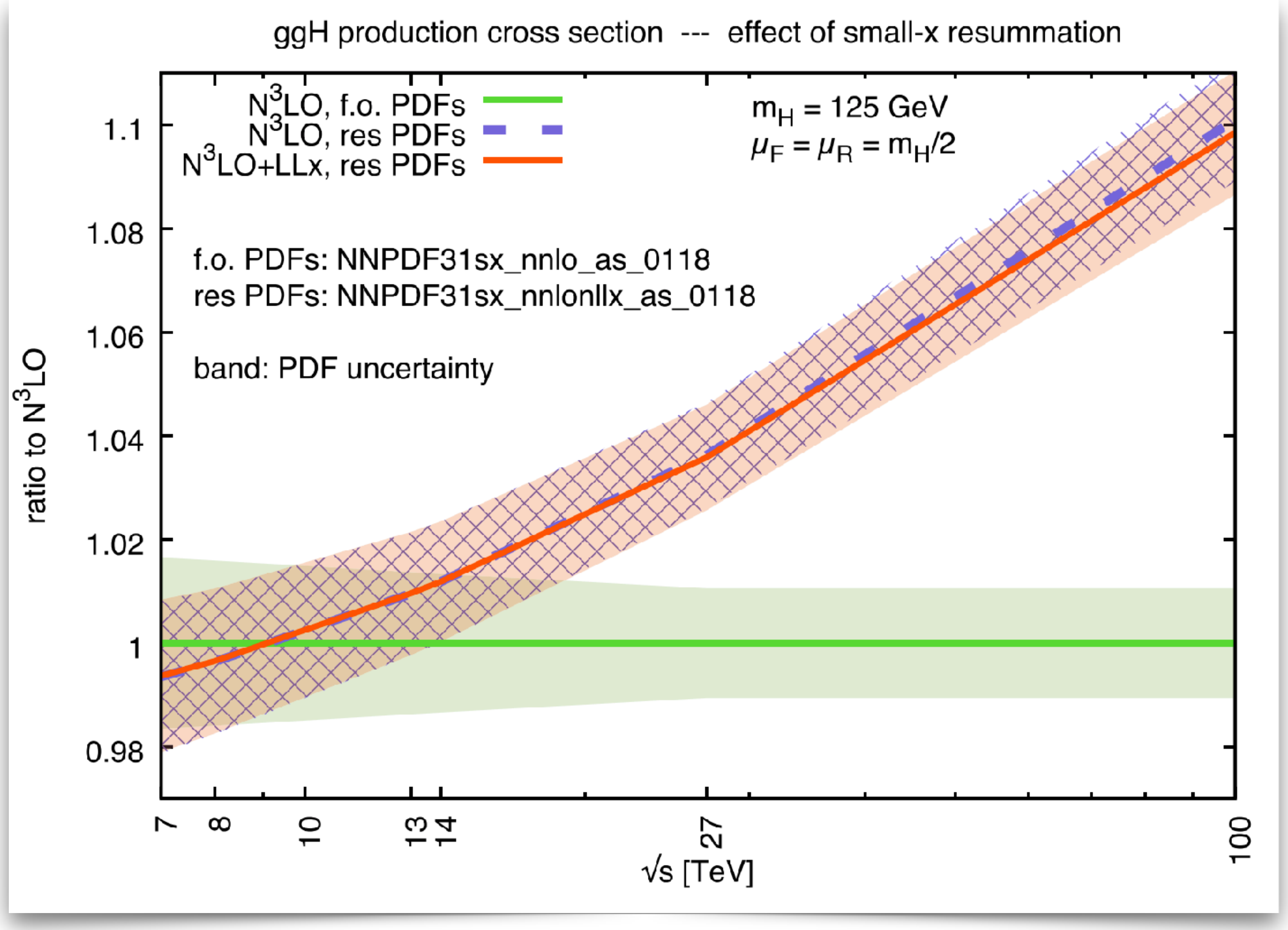

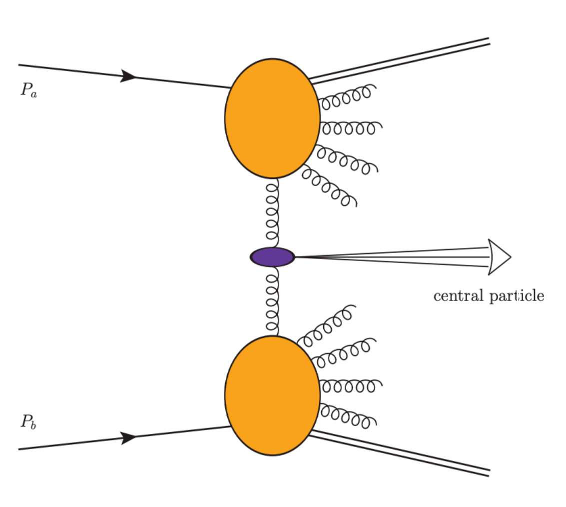

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$\mathrm{N^3LL_{lx}}$/$\mathrm{LL_{sx}}$/$\mathrm{N^3LO}$ rapidity-inclusive coefficient functions

(¡!) 100 TeV electroweak physics is small-x physics!

(¿?) Can LHC physics be BFKL physics?

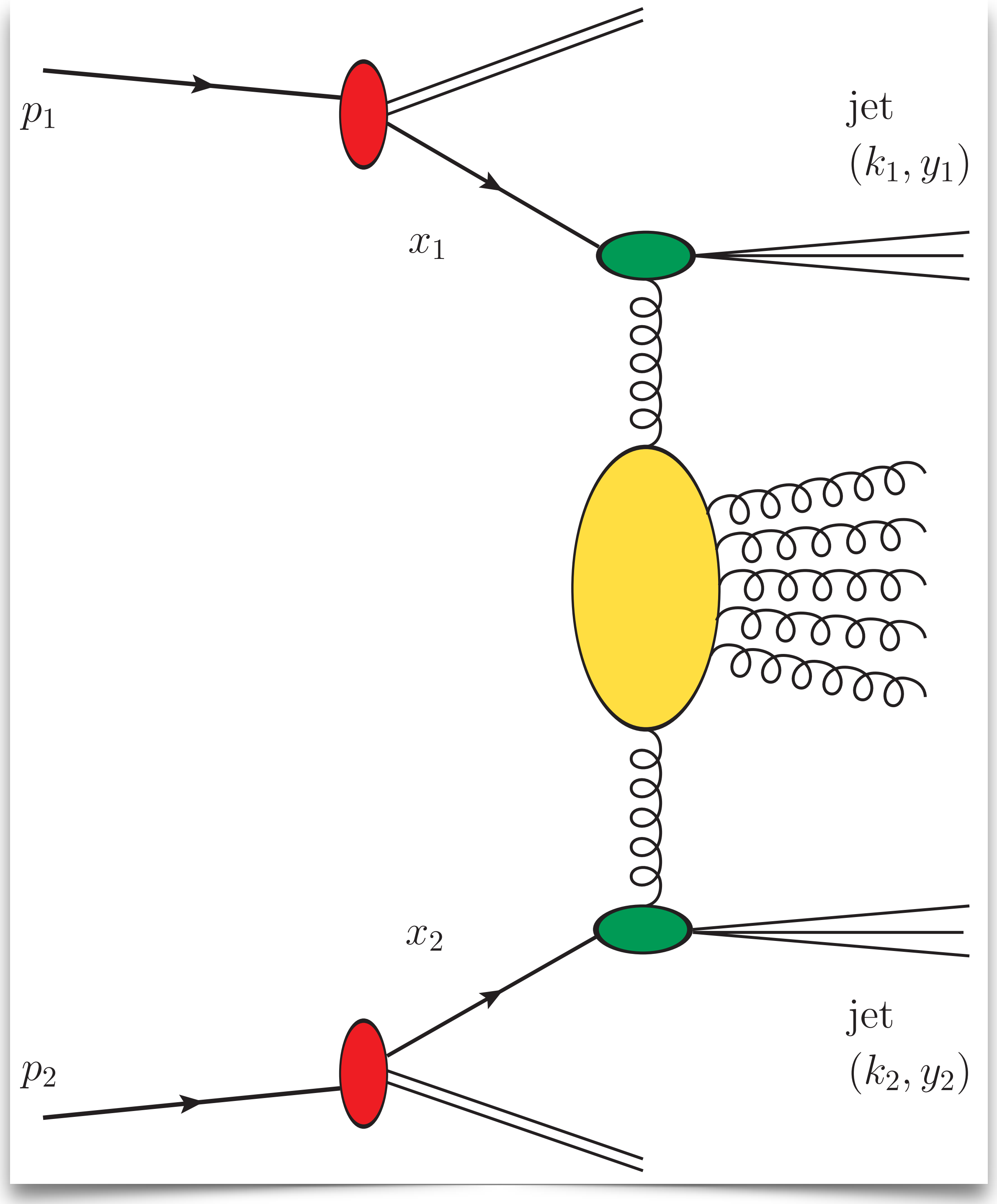
$$C_{ij}(x,\alpha_s) = C^{\mathrm{fo}}_{ij}(x,\alpha_s) + \Delta C^{\mathrm{lx}}_{ij}(x,\alpha_s) + \Delta C^{\mathrm{sx}}_{ij}(x,\alpha_s)$$

Backup

Mueller-Navelet jets @LHC & resummation instabilities

- Inclusive hadroproduction of two jets with high p_T and large rapidity separation, ΔY
- Moderate x (collinear PDFs), but t-channel p_T (BFKL resummation) $\Rightarrow$ hybrid factorization (HyF)

$$\frac{d\sigma}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2\, f_r(x_1, \mu_F)\, f_s(x_2, \mu_F)\, \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu_F)}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2}$$



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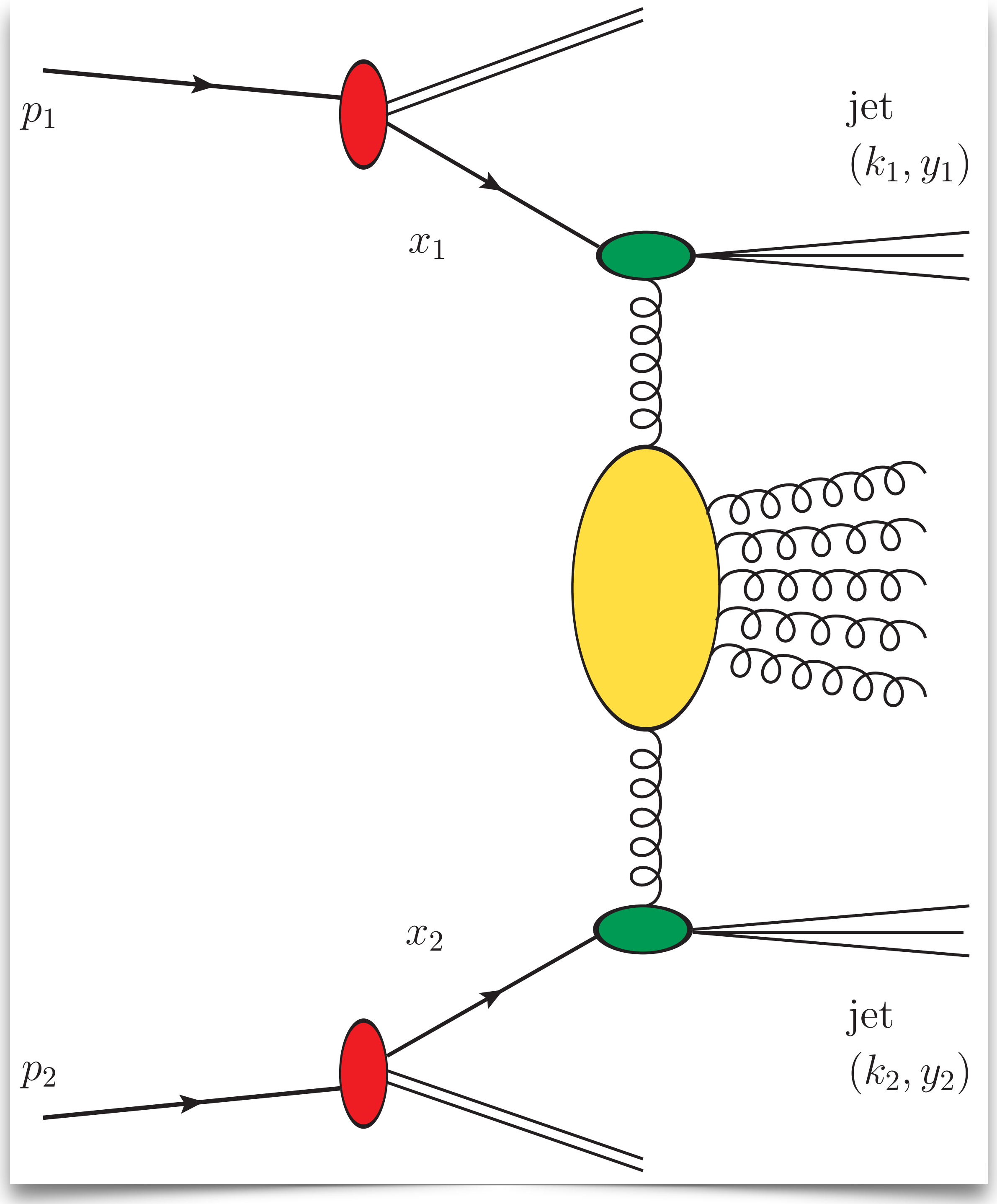
jet vertices (off-shell coefficient functions)



$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2} = \frac{1}{(2\pi)^2}$$

NLO(+) $\quad \times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\, \mathcal{V}_J^{(r)}(\vec{q}_1, s_0, x_1, \vec{k}_1)$

NLL $\quad \times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2)$

NLO(+) $\quad \times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\, \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{k}_2)$

BFKL Green's function

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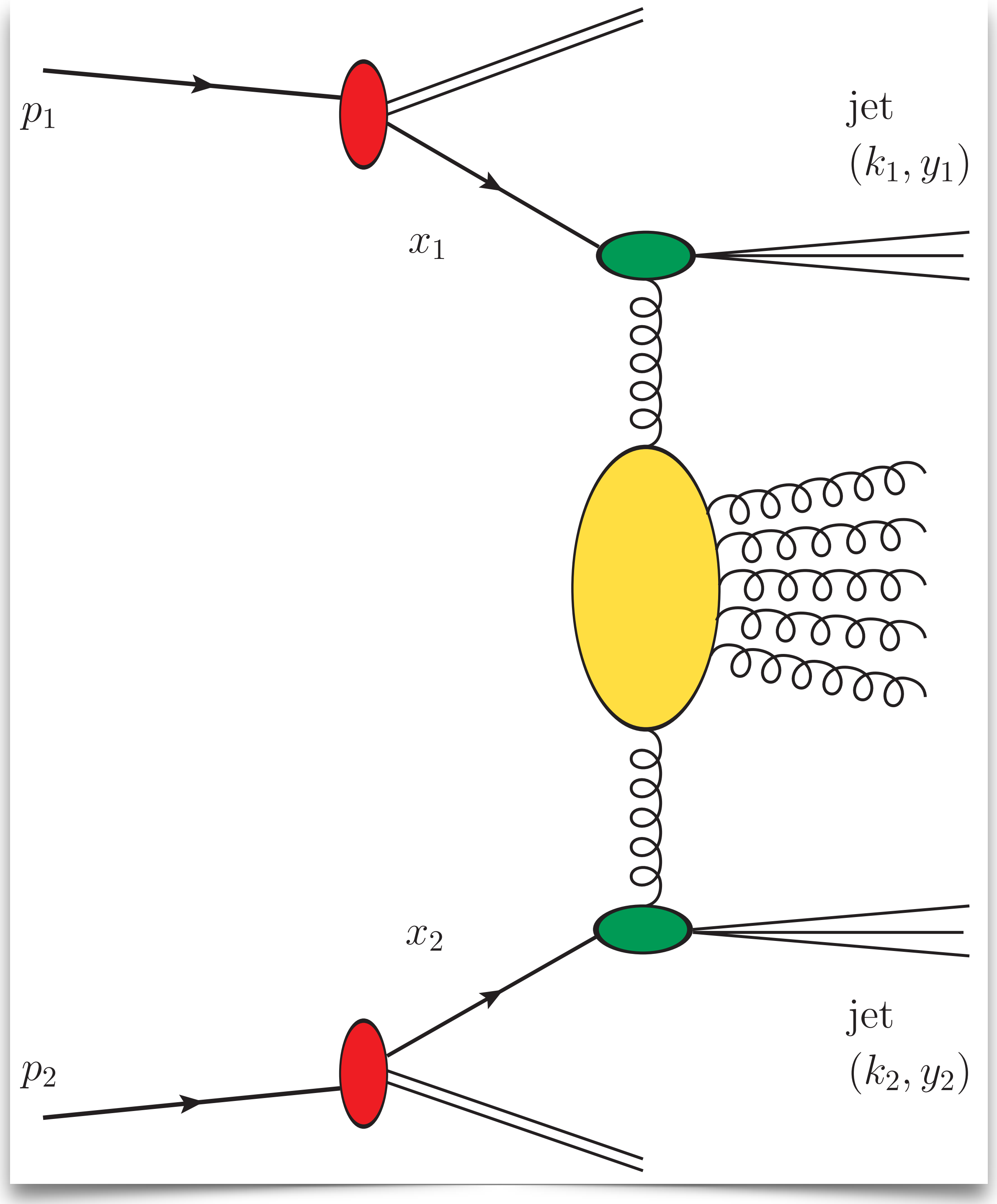

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BFKL Green's function

- NLL/LL instabilities + NLO missing threshold $\Rightarrow$ precision studies hampered

Backup

Mueller-Navelet jets @LHC & resummation instabilities

- Strong manifestation of higher-order instabilities via scale variation (¡!)
- ¡ At natural scales: NLL/LL ratio large, no agreement with data, unphysical values !
- BLM scales, theory vs experiment: CMS @7TeV with symmetric p_T-ranges, only

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[CMS Collaboration, JHEP 08 (2016) 139]
[B. Ducloué et al., Phys. Rev. Lett. 112 (2014) 082003]
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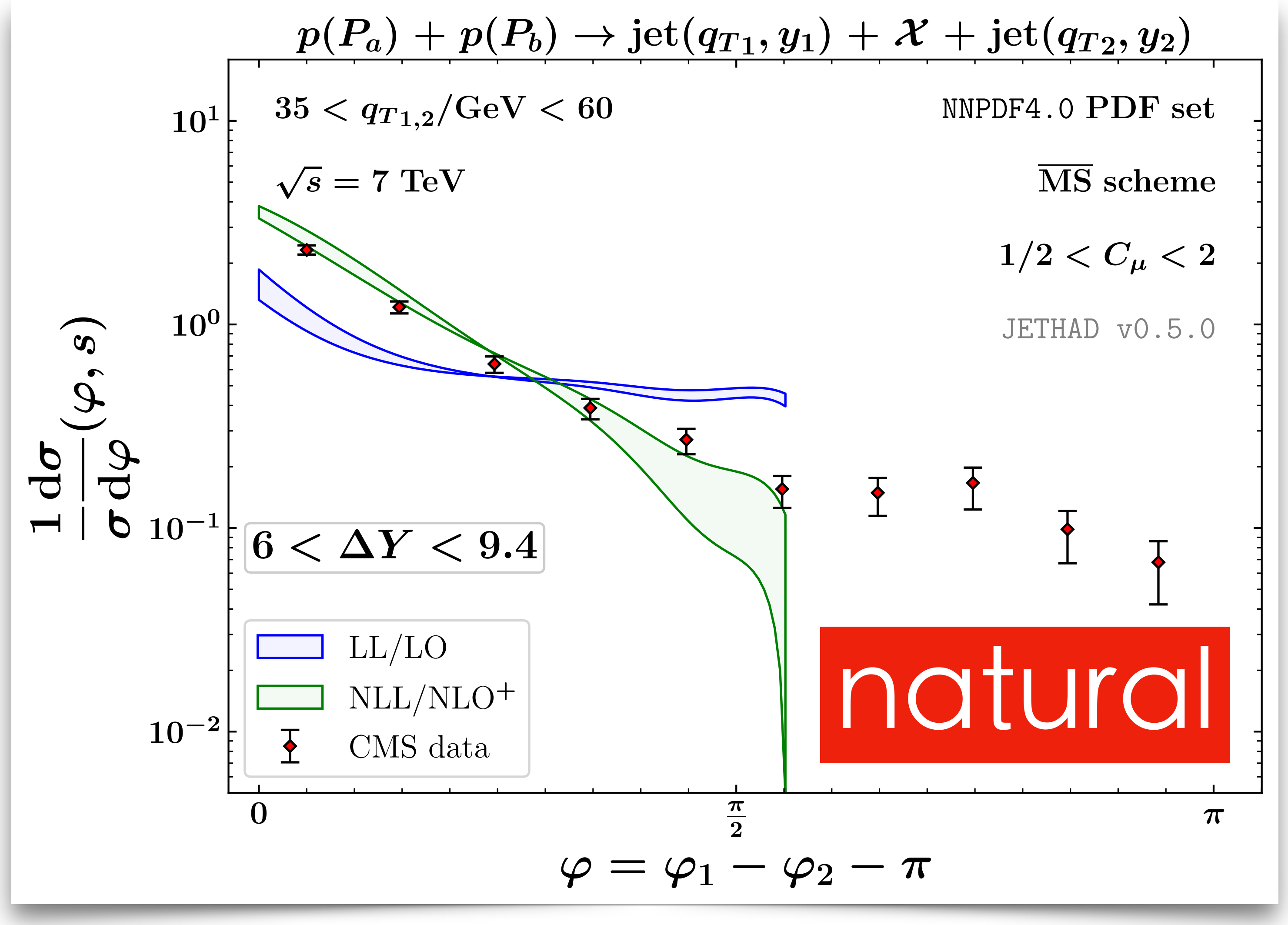

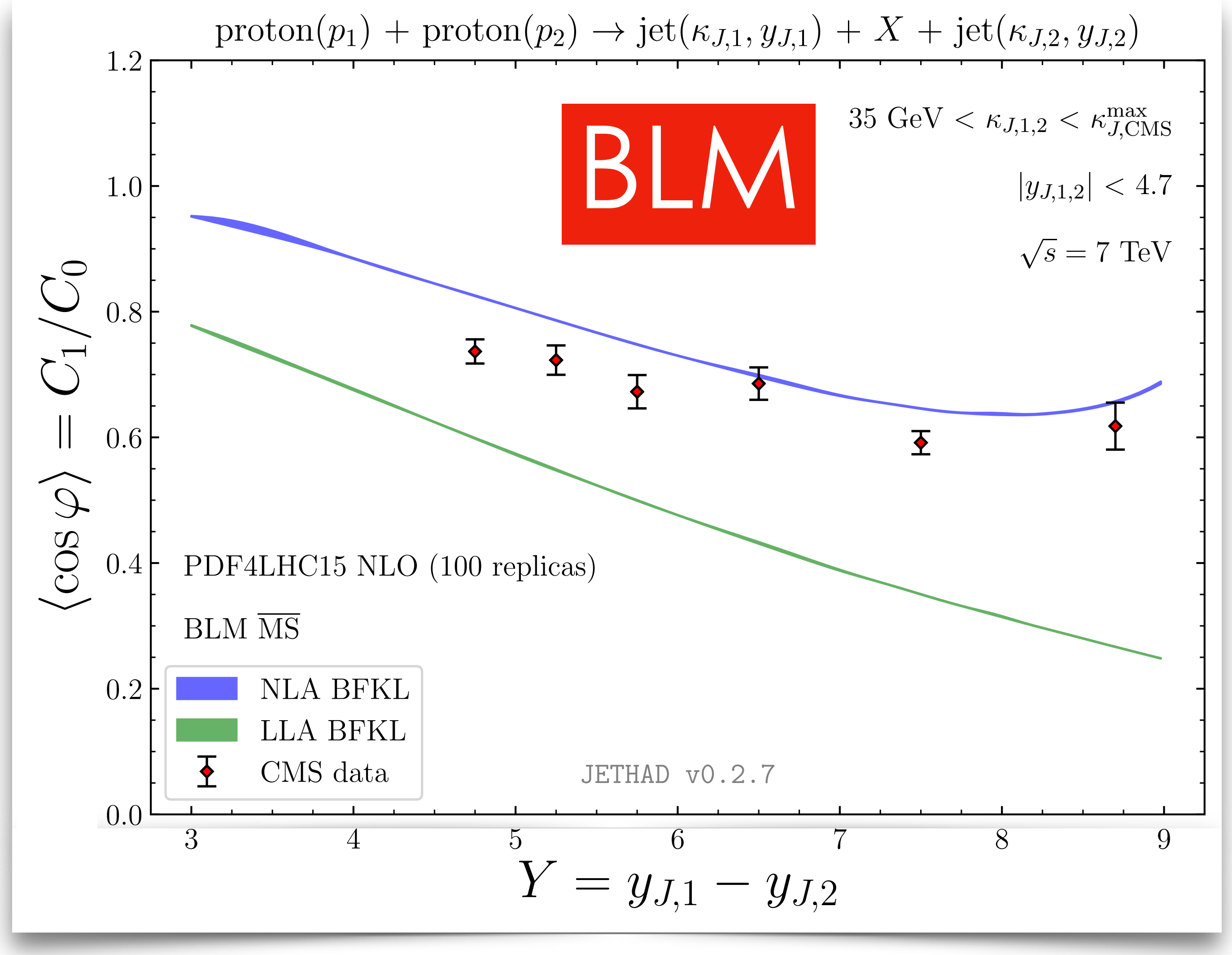

(left figure) [F. G. C., A. Papa, Phys. Rev. D 106 (2022) 11, 114004]
(right figure) [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

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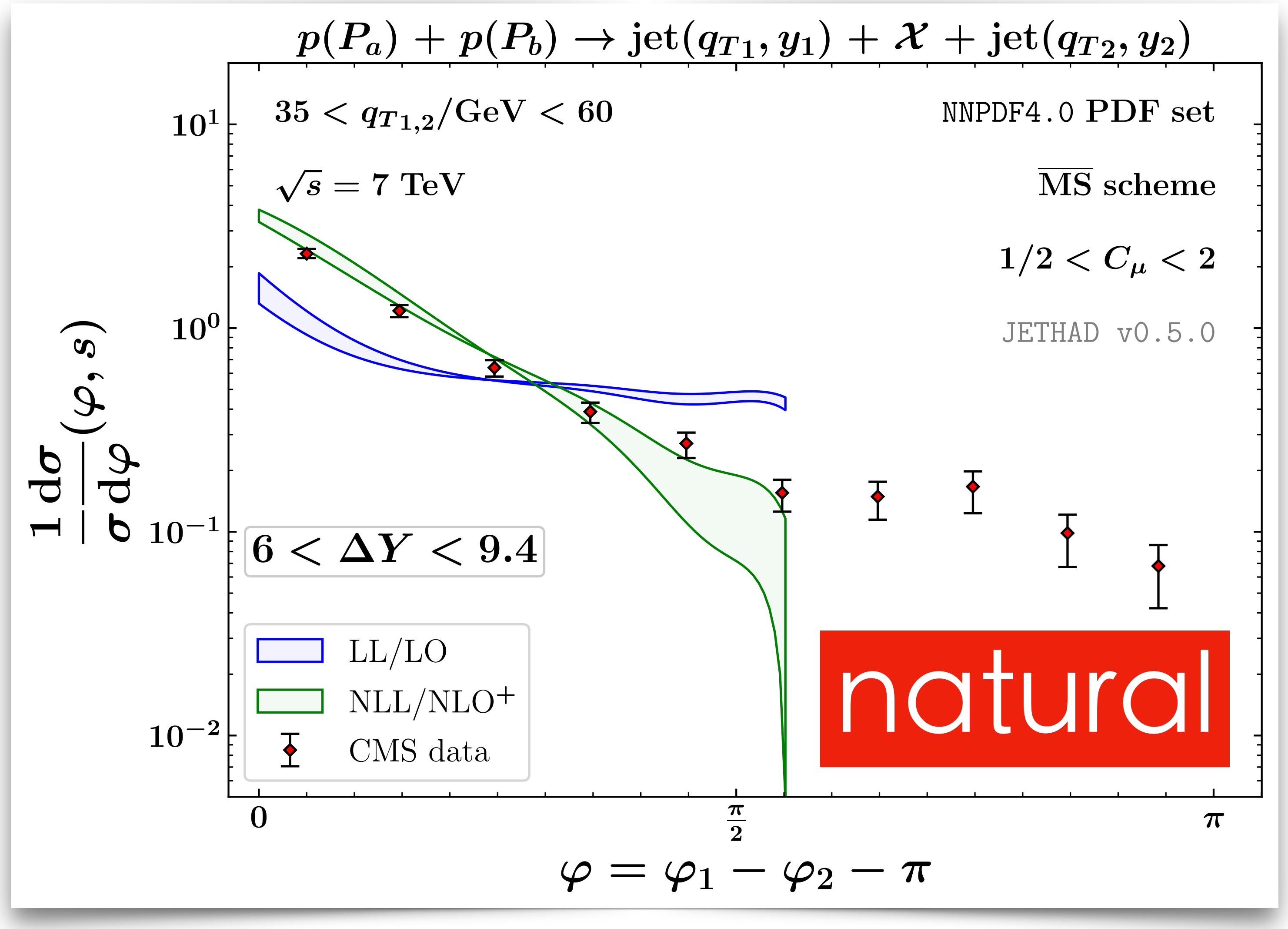

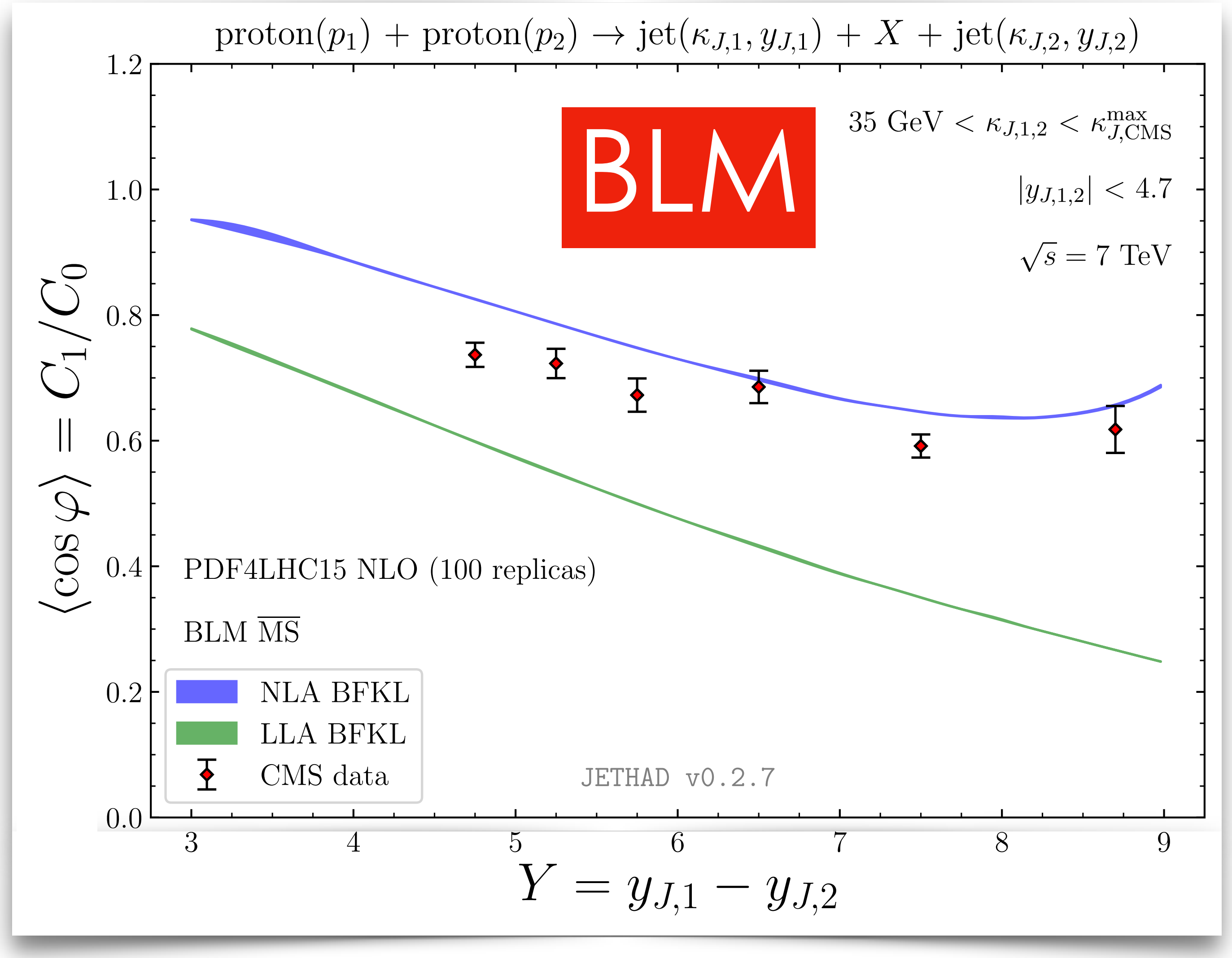

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- $\mu_R^{\text{BLM}} \gg \mu_R^{\text{nat.}} \;\Rightarrow\; d\sigma^{\text{BLM}}/d\sigma^{\text{nat.}} \sim 10^{-(1\div 2)}$ ⇒ precision studies hampered ✗
- Unsuccessful scale optimization → processes featuring natural stability (¿?)

Backup

Higgs + jet highlights from the FCC Week 2022

The high-energy QCD dynamics from Higgs+jet correlations at FCC

Francesco G. Celiberto [1,2,3] and Alessandro Papa [4,5]

FCC Week 2022, Sorbonne Université, France

Hors d'œuvre

- Higgs sector → SM benchmarks, BMS portals
- Gluon fusion → key ingredient for precision QCD
- Fixed-order ← improved by resummations
- FCC energies ↔ high-energy (HE) resummation
- Higgs+jet → golden channel to hunt for HE signals

NLL/NLO differential cross section

$$\frac{d\sigma}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2} = \sum_{r,s=q,g} \int_0^1 dx_1 \int_0^1 dx_2\, f_r(x_1,\mu_F)\, f_s(x_2,\mu_F)\, \frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu_F)}{dy_1\, dy_2\, d^2\vec{k}_1\, d^2\vec{k}_2}$$

$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_H\, dy_J\, d^2\vec{p}_H\, d^2\vec{p}_J} = \frac{1}{(2\pi)^2}$$

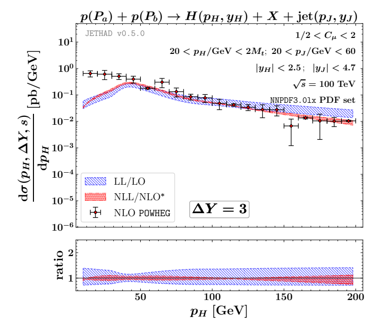
$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\, \mathcal{V}_H^{(r)}(\vec{q}_1, s_0, x_1, \vec{p}_H)$$

$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2)$$

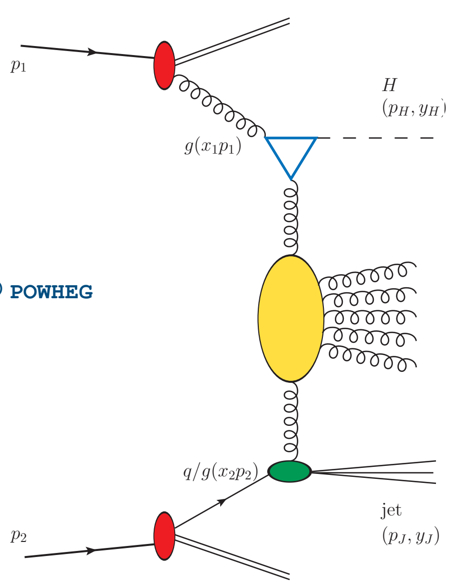
$$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\, \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{p}_J)$$

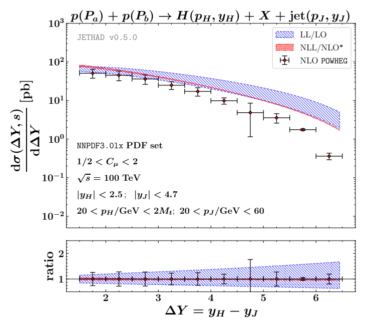
PDFs with threshold
NLO Higgs vertex
NLL BFKL kernel
NLO Jet vertex

Hybrid high-energy and collinear factorization at work

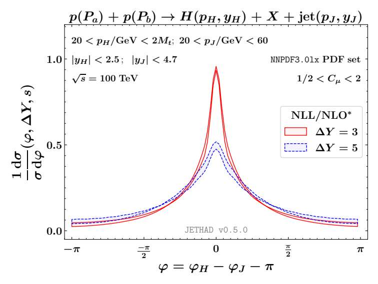


Higgs p_T distribution: NLL/NLO **JETHAD** vs NLO **POWHEG**





Rapidity distribution: NLL/NLO **JETHAD** vs NLO **POWHEG**



Azimuthal distribution at NLL/NLO

- HE resummation from **JETHAD**
- Large-x **NNPDF3.01x** PDFs with threshold
- Comparison with fixed-order from **POWHEG**
- Distributions stable under NLL corrections

A path towards precision

- ☑ NLL bands nested inside LL ones → solid stability
- ☑ HE signal clearly disengaged from NLO background
- ☑ Way toward precision studies of HE QCD (¡!)
- ☐ Multilateral formalism → encode other resummations
- ☐ A window on proton structure at small-x (¿?)

Further information

[1] ECT*, I-38123 Villazzano, Trento, Italy
[2] Fondazione Bruno Kessler (FBK), I-38123 Povo, Trento, Italy
[3] INFN-TIFPA, I-38123 Povo, Trento, Italy
[4] Università della Calabria, I-87036 Rende, Cosenza, Italy
[5] INFN-Cosenza, I-87036 Rende, Cosenza, Italy

Contact: fceliberto@ectstar.eu

Take a picture to the QR code to download the paper on Higgs+jet resummed distributions at 14TeV LHC: [FGC et al., EPJ C 81 (2021) 4, 293]

ECT* | FBK | HAS QCD | FUTURE CIRCULAR COLLIDER | UNIVERSITÀ DELLA CALABRIA DIPARTIMENTO DI FISICA | INFN | TIFPA

Backup

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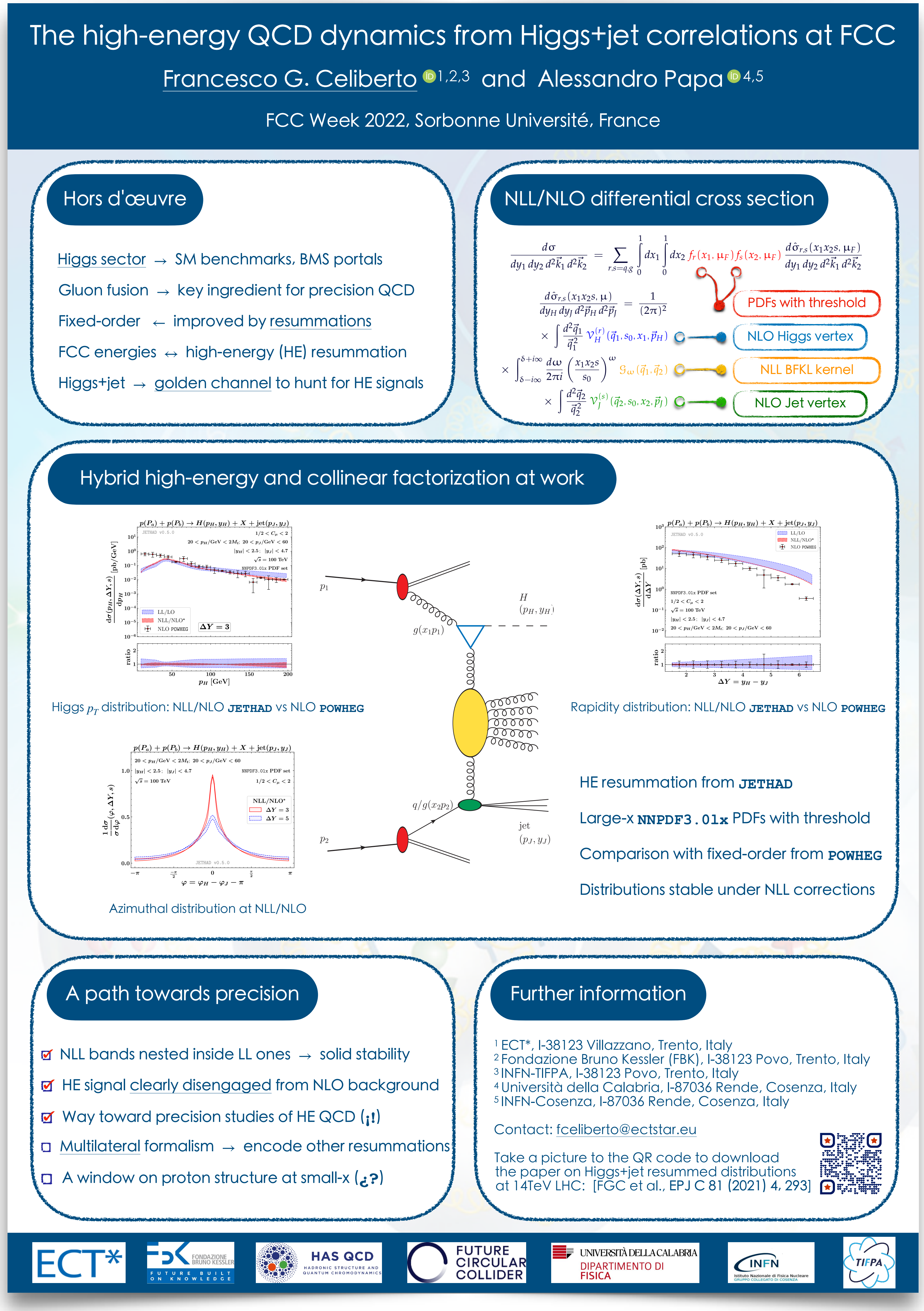
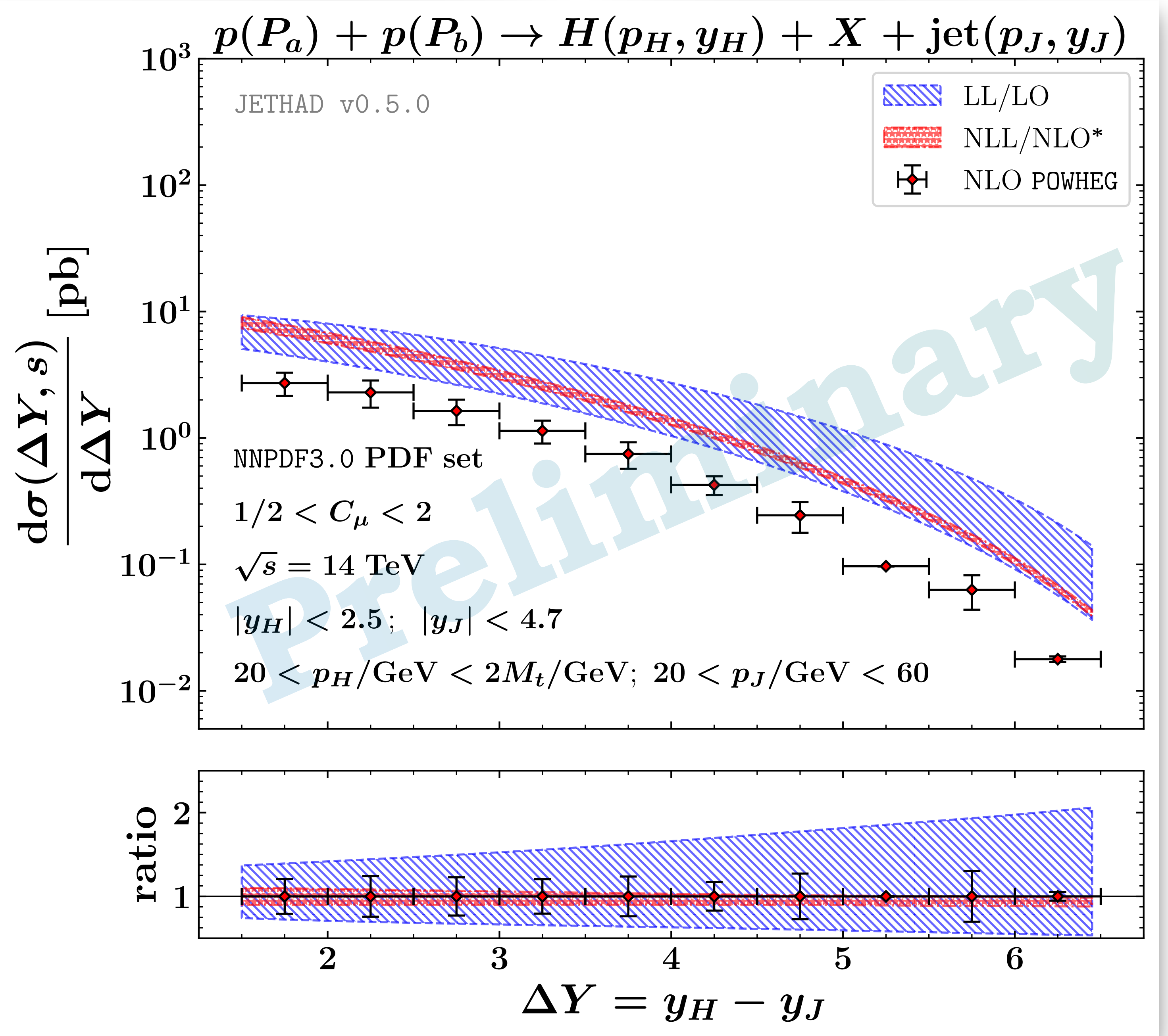
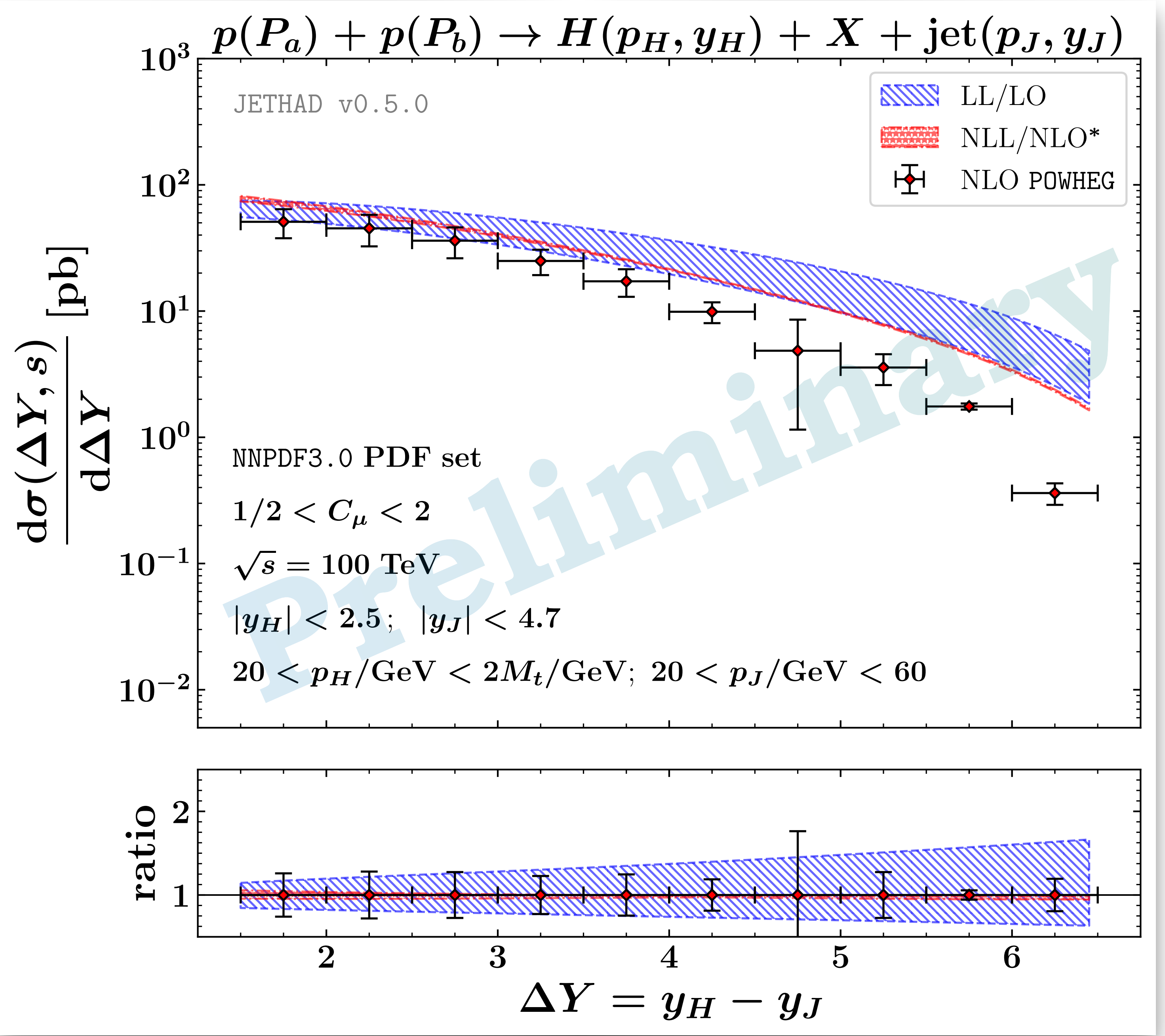


$$C_n(\Delta Y, s) = \int_{p_H^{\min}}^{p_H^{\max}} d|\vec{p}_H| \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \int_{y_H^{\min}}^{y_H^{\max}} dy_H \int_{y_J^{\min}}^{y_J^{\max}} dy_J\, \delta\left(y_H - y_J - \Delta Y\right) \mathcal{C}_n$$

Rapidity distribution: NLL/NLO* JETHAD vs NLO POWHEG



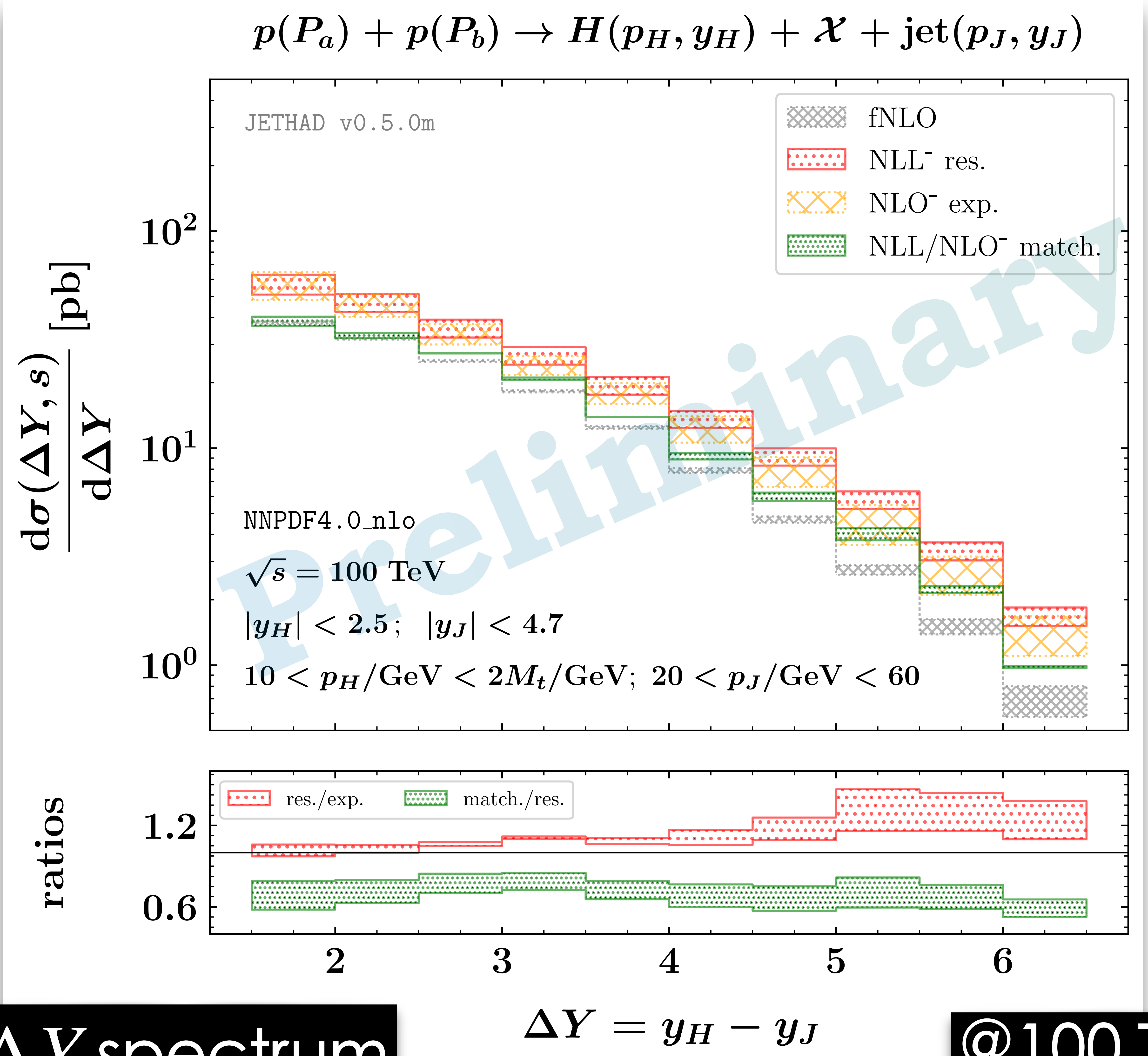
14 TeV LHC



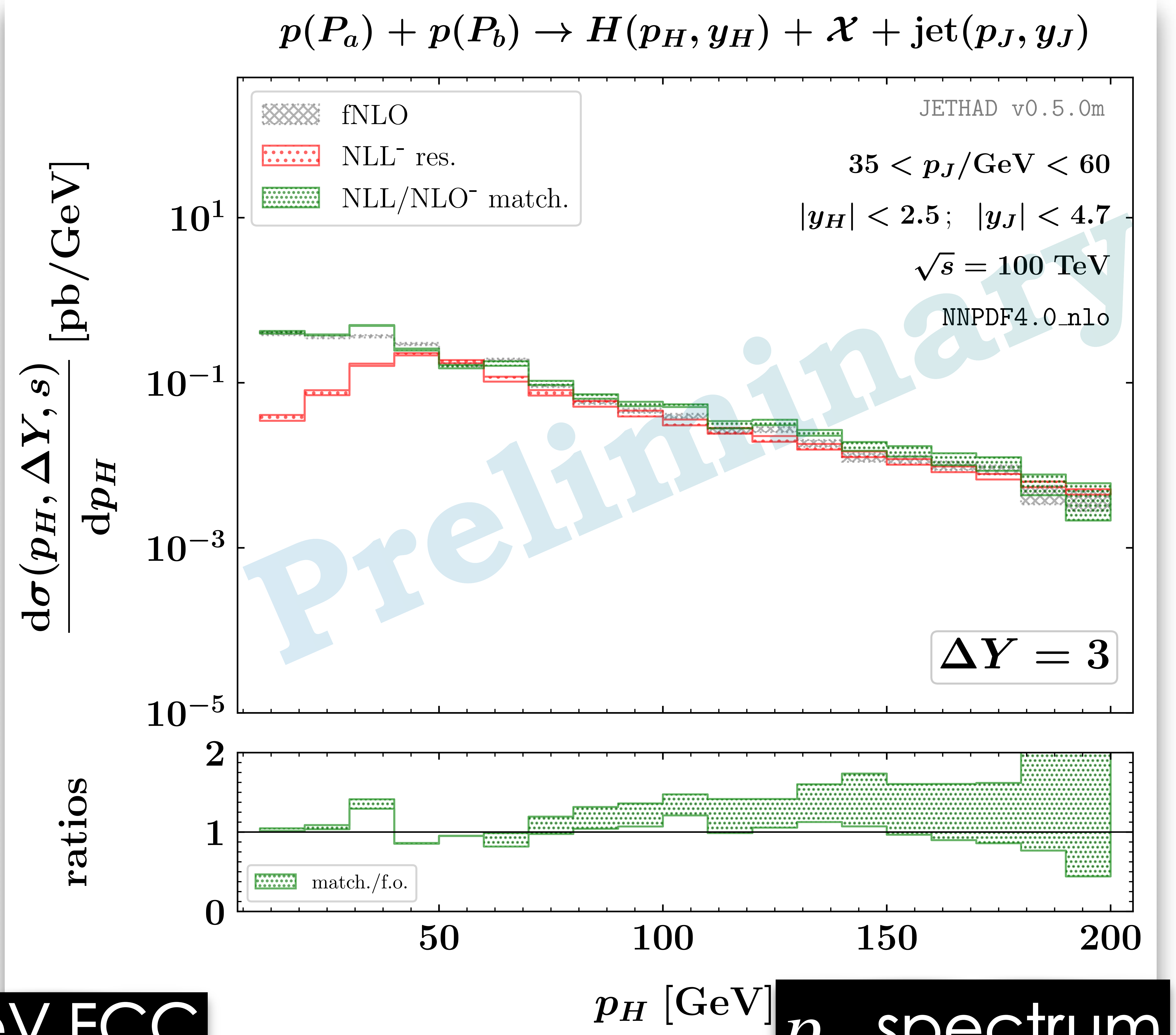
100 TeV FCC

Backup

The Higgs + jet spectrum from POWHEG + JETHAD



ΔY spectrum



@100 TeV FCC

p_H spectrum

Backup

NLL matched to NLO fixed-order JETHAD + POWHEG (in progress)

The JETHAD technology

High-energy resummation

Hunting BFKL

in semi-hard reactions

Mueller-Navelet, light hadrons

ERIS super-module

$$\alpha_s \ln(s) \lesssim 1$$

JETHAD

v0.5.0m

python + F

🔗 [Eur. Phys. J. C 81 (2021) 8, 691]

🔗 [Phys. Rev. D 105 (2022) 11, 114008]

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ERIS, 𝔇ναμις super-modules

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Forward Drell-Yan, onium, Higgs

LExA, HATHOR super-modules

Proton structure

Small-x UGD

Gluon TMD PDFs

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WORK IN PROGRESS

Quarkonium studies

from low to high p_T

NRFF1.0 onium FFs

Vectors & pseudoscalars

JETHAD + DGLAP evolution operators

Forward Drell-Yan, onium, Higgs

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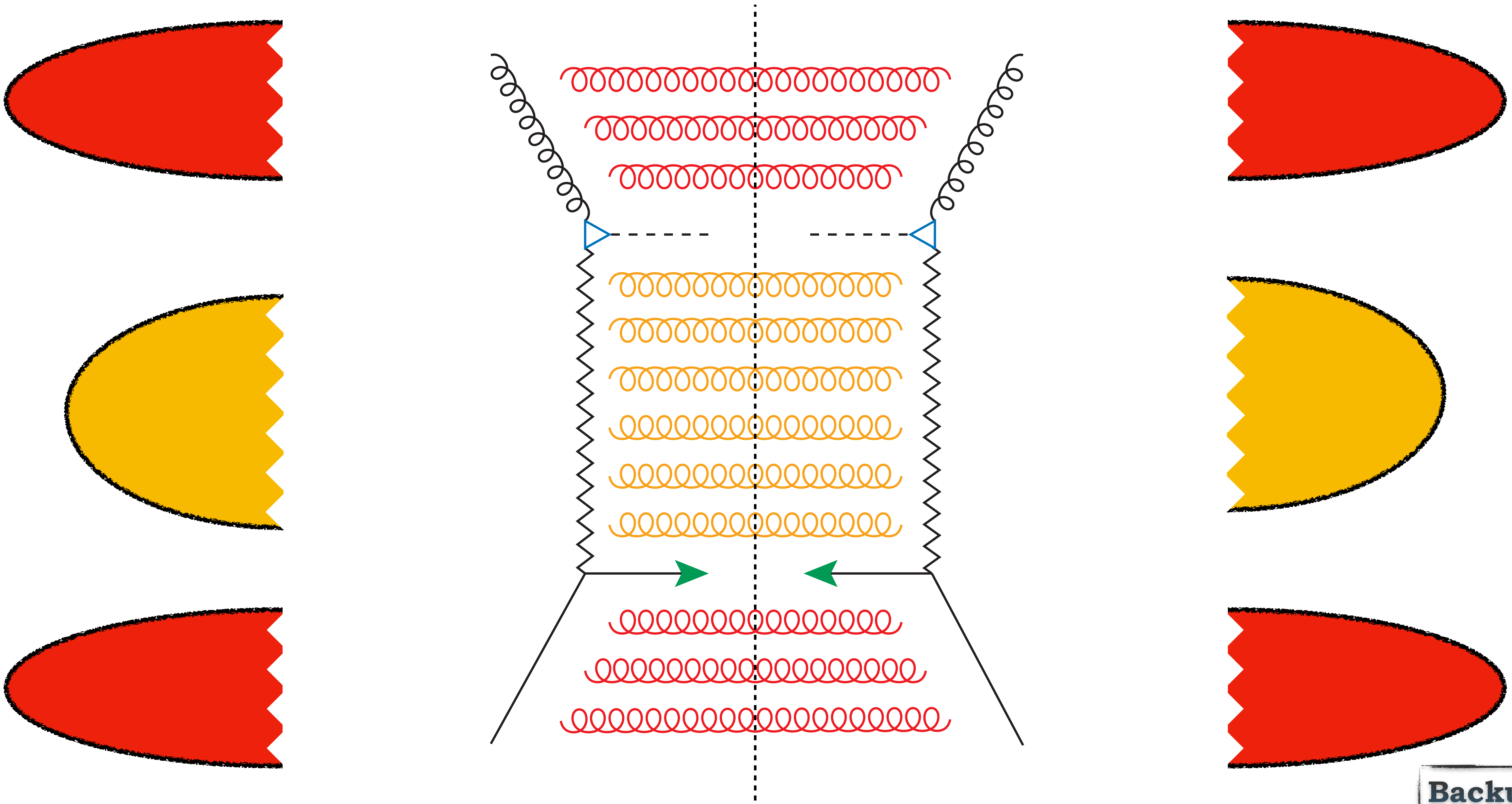
Anatomy of Higgs + jet in hybrid factorization (HyF)



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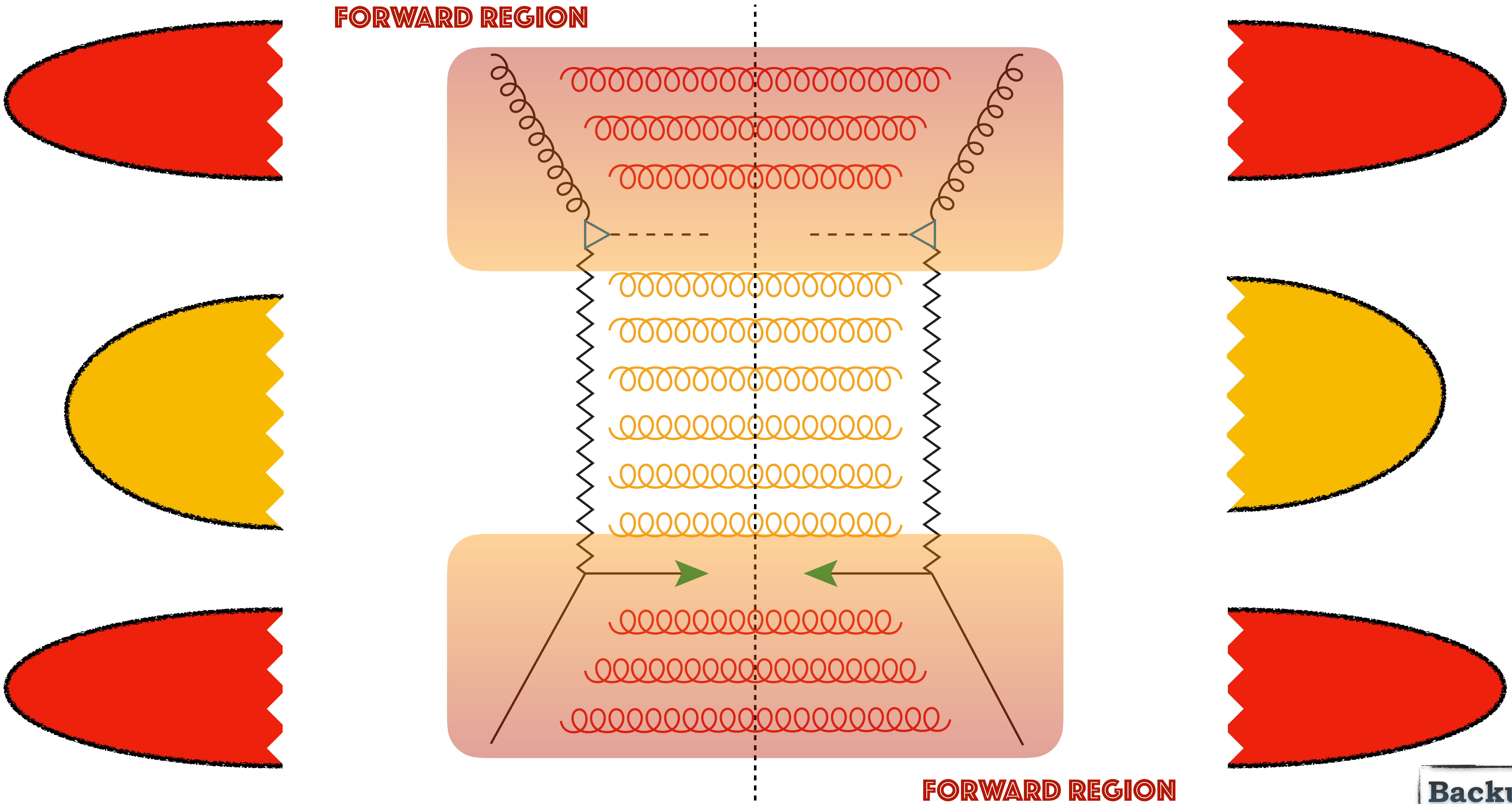

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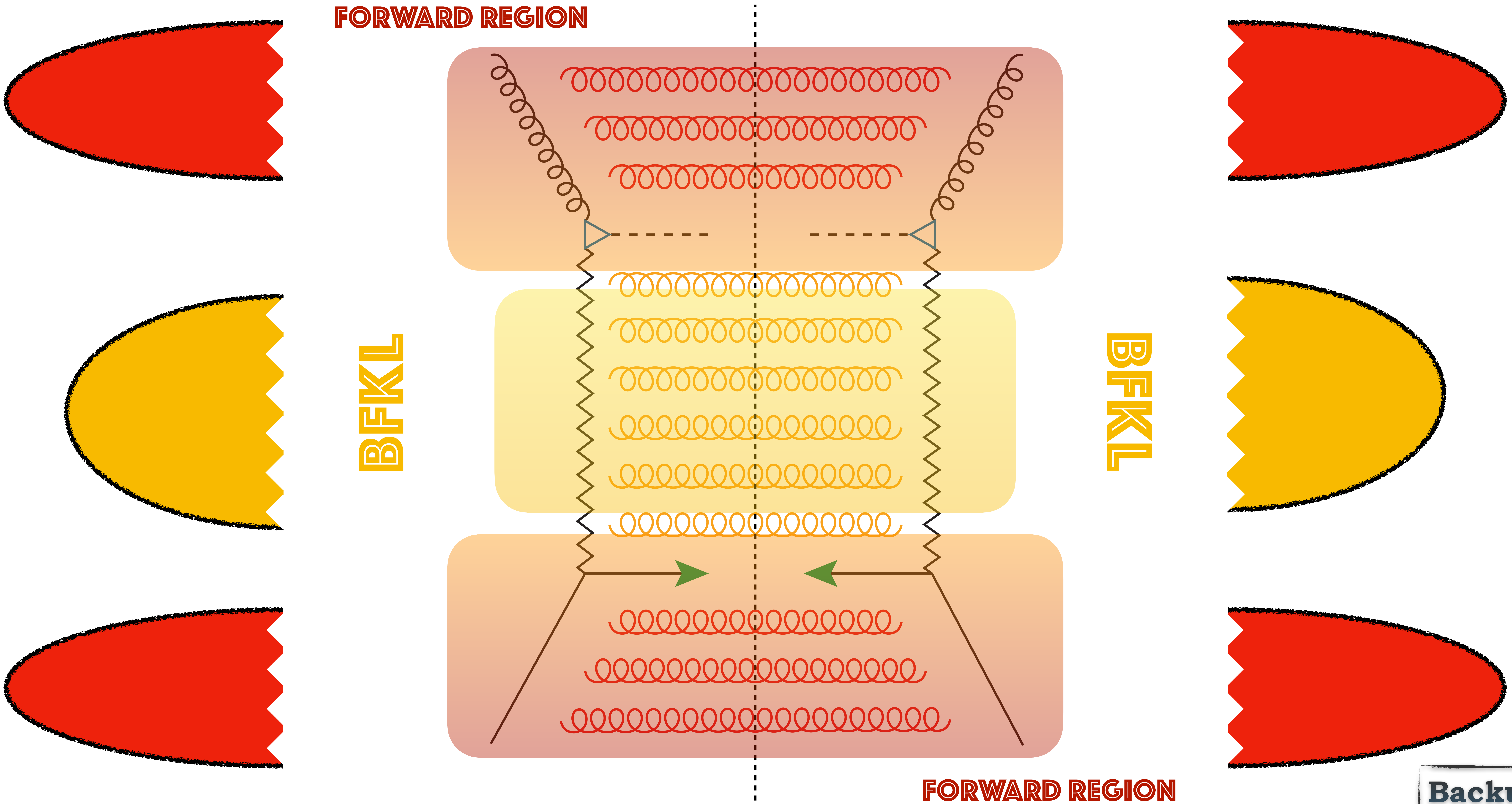

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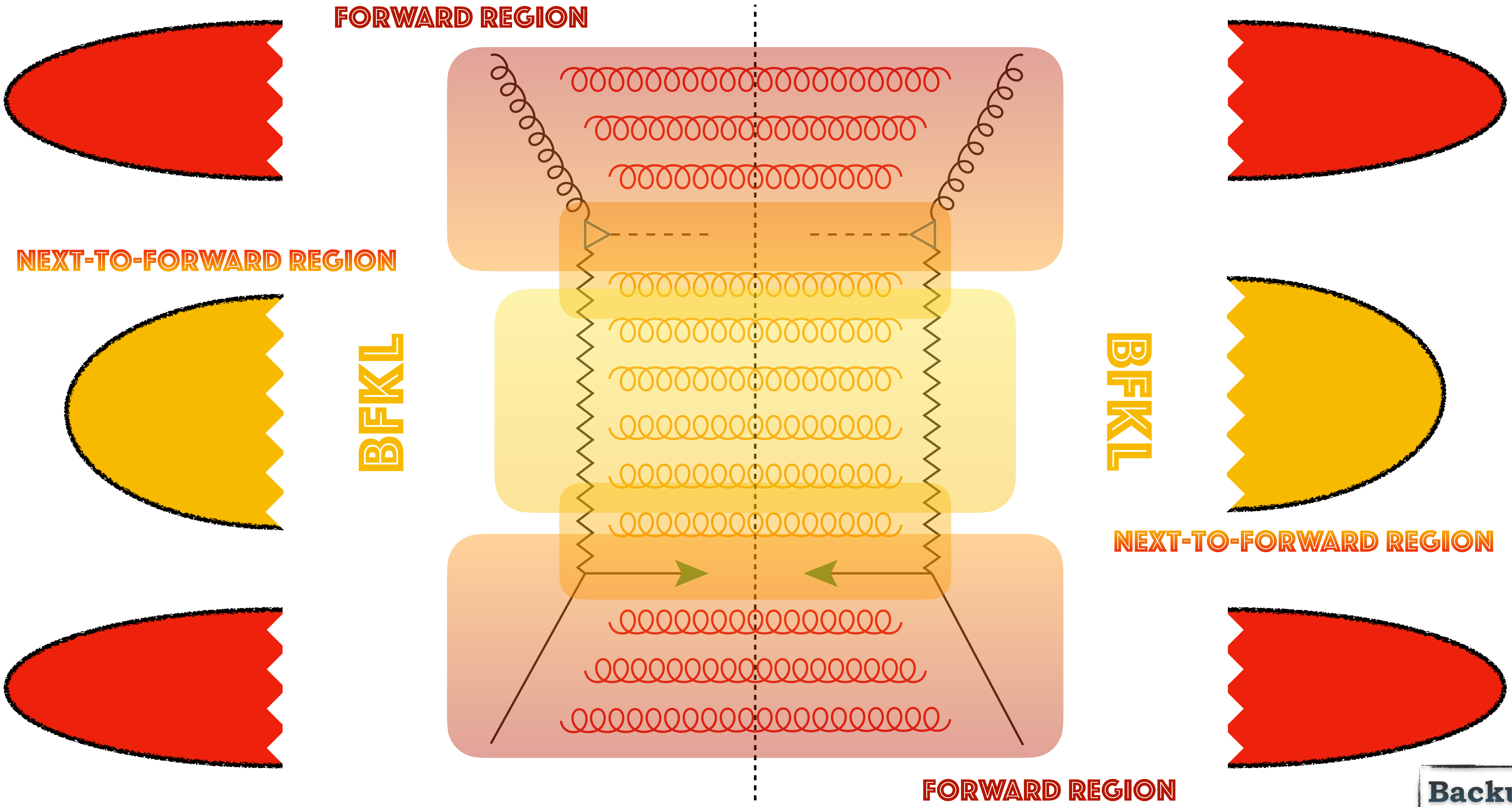
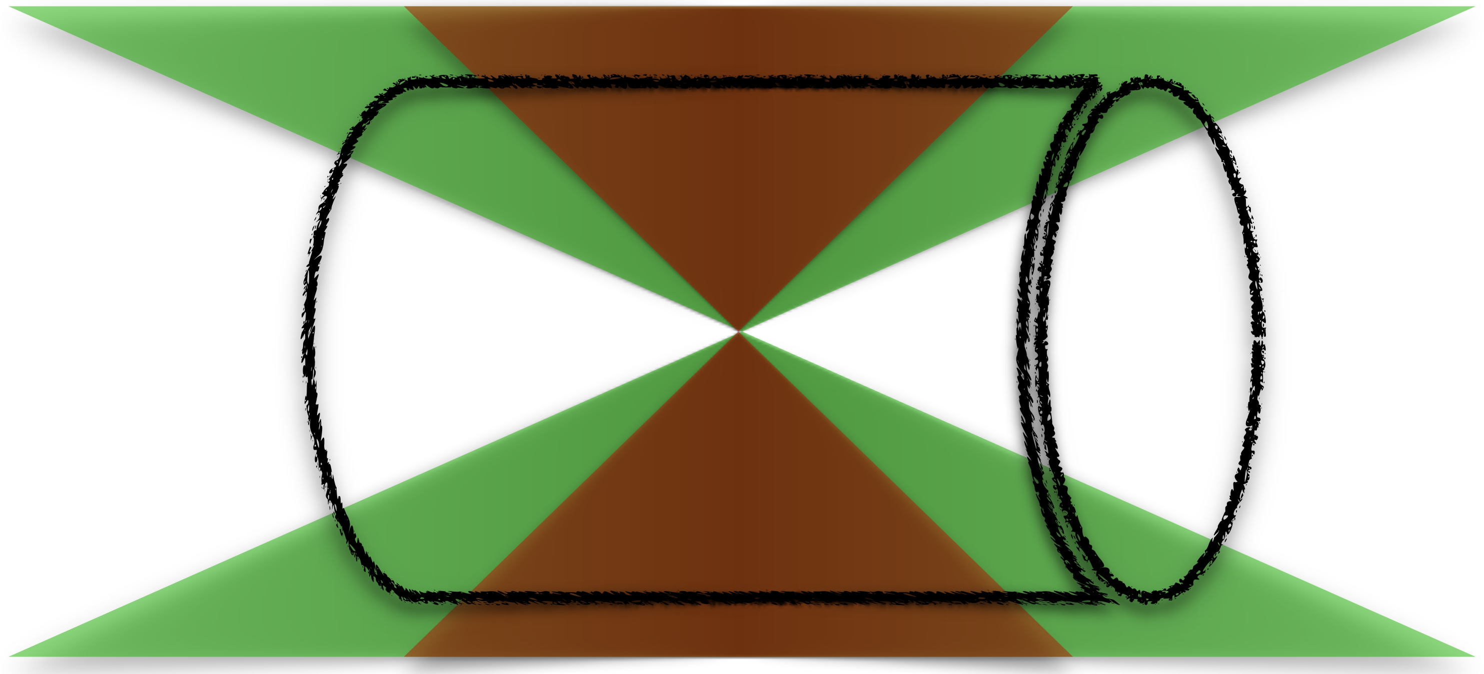


Backup

Ultraforward charm + Higgs production @14 TeV FPF+ATLAS

High-energy QCD at new-gen Forward Facilities

Forward + backward CMS detections: Mueller-Navelet, hadron-jet, di-hadron



$|y_{\rm jet}| < 4.7$

barrel + endcap

$|y_{\rm hadron}| < 2.4$

barrel

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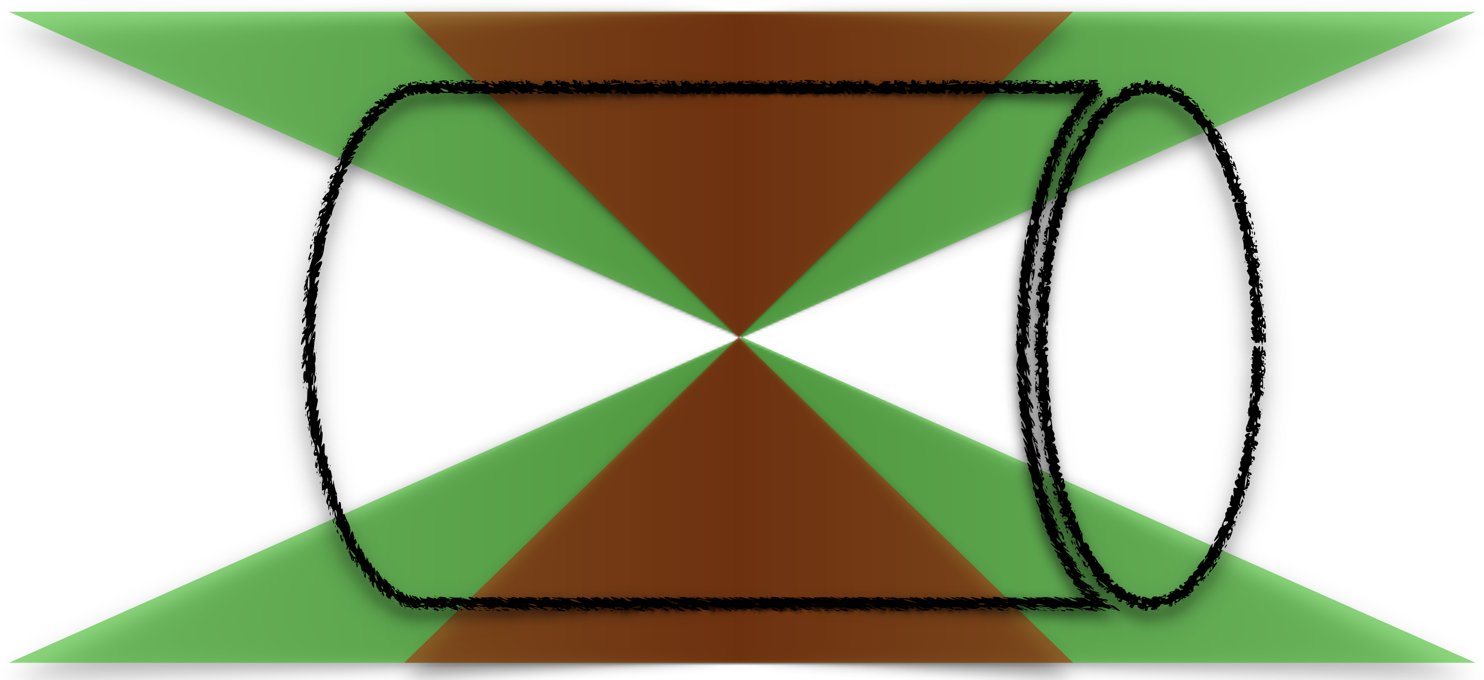
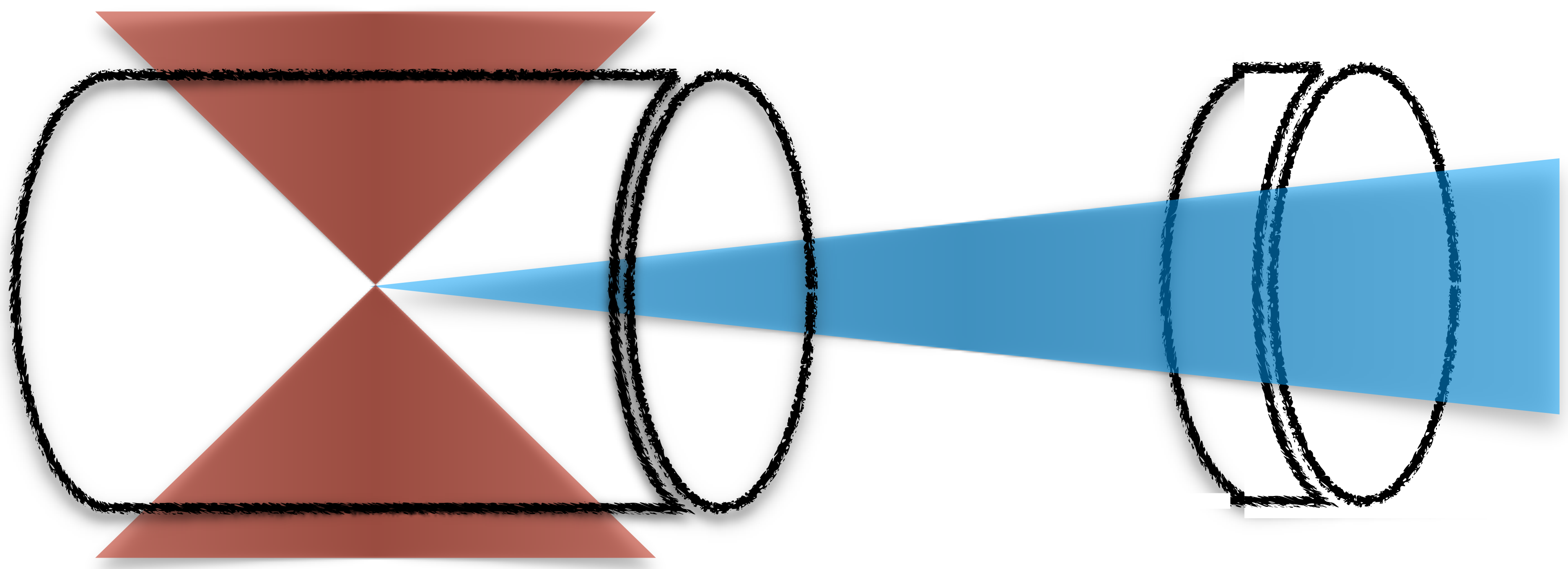


- Ultra-forward FPF + central ATLAS detections: single-charmed hadrons + Higgs



(charm + Higgs) 🔗 [F. G. C. et al., Phys. Rev. D 105 (2022) 11, 114056]

(light mesons + heavy flavor) 🔗 [F. G. C., Phys. Rev. D 105 (2022) 11, 114008]

Backup

High-energy QCD in ultraforward directions

Backup


Physics Reports 968 (2022) 1–50

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The Forward Physics Facility: Sites, experiments, and physics potential

Luis A. Anchordoqui [1], Akitaka Ariga [2,3], Tomoko Ariga [4], Weidong Bai [5], Kincso Balazs [6], Brian Batell [7], Jamie Boyd [6], Joseph Bramante [8], Mario Campanelli [9], Adrian Carmona [10], Francesco G. Celiberto [11,12,13], Grigorios Chachamis [14], Matthew Citron [15], Giovanni De Lellis [16,17], Albert De Roeck [6], Hans Dembinski [18], Peter B. Denton [19], Antonia Di Crescenzo [16,17,6], Milind V. Diwan [20], Liam Dougherty [21], Herbi K. Dreiner [22], Yong Du [23], Rikard Enberg [24], Yasaman Farzan [25], Jonathan L. Feng [26,*], Max Fieg [26], Patrick Foldenauer [27], Saeid Foroughi-Abari [28], Alexander Friedland [29], Michael Fucilla [30,31], Jonathan Gall [32], Maria Vittoria Garzelli [33,*], Francesco Giuli [34], Victor P. Goncalves [35], Marco Guzzi [36], Francis Halzen [37], Juan Carlos Helo [38,39], Christopher S. Hill [40], Ahmed Ismail [41], Ameen Ismail [42], Richard Jacobsson [6], Sudip Jana [43], Yu Seon Jeong [44], Krzysztof Jodłowski [45], Kevin J. Kelly [46], Felix Kling [29,47,**], Fnu Karan Kumar [20], Zhen Liu [48], Rafał Maciuła [49], Roshan Mammen Abraham [41], Julien Manshanden [33], Josh McFayden [50], Mohammed M.A. Mohammed [30,31], Pavel M. Nadolsky [51], Nobuchika Okada [52], John Osborne [6], Hidetoshi Otono [4], Vishvas Pandey [53,46], Alessandro Papa [30,31], Digesh Raut [54], Mary Hall Reno [55], Filippo Resnati [6], Adam Ritz [28], Juan Rojo [56], Ina Sarcevic [57], Christiane Scherb [58], Holger Schulz [59], Pedro Schwaller [60], Dipan Sengupta [61], Torbjörn Sjöstrand [62], Tyler B. Smith [26], Dennis Soldin [54], Anna Stasto [63], Antoni Szczurek [49], Zahra Tabrizi [64], Sebastian Trojanowski [65,66], Yu-Dai Tsai [26,46], Douglas Tuckler [67], Martin W. Winkler [68], Keping Xie [7], Yue Zhang [67]

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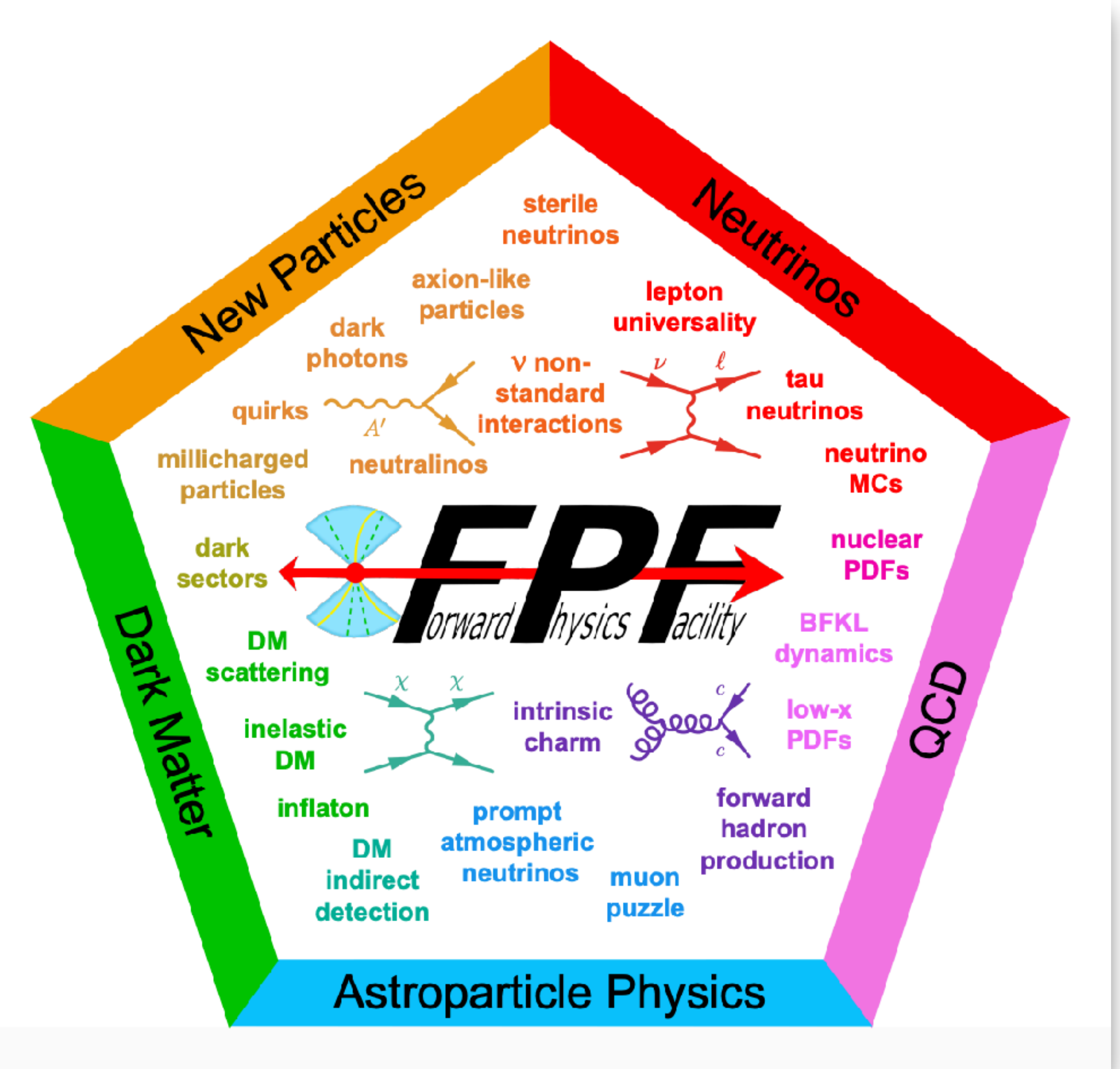
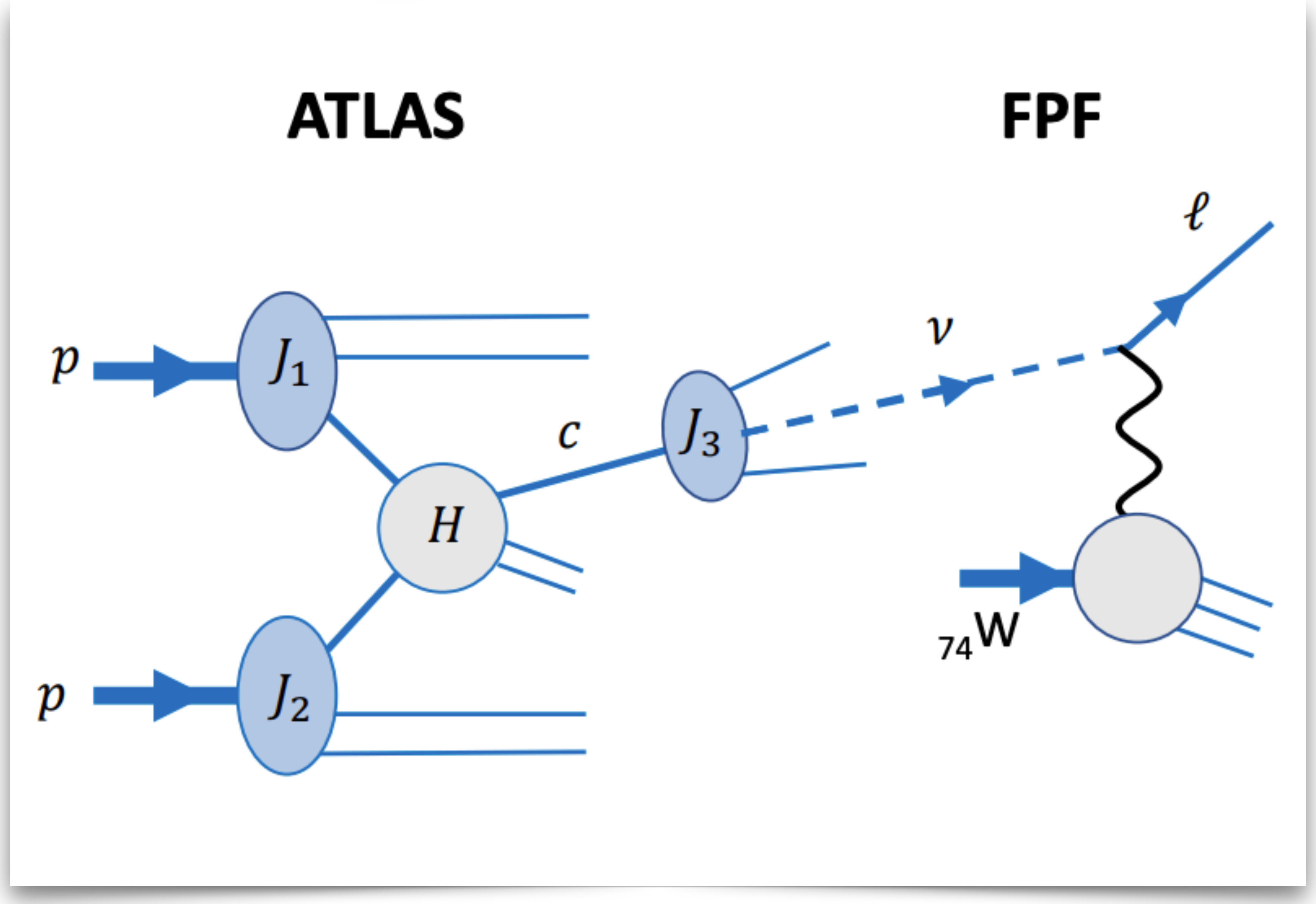


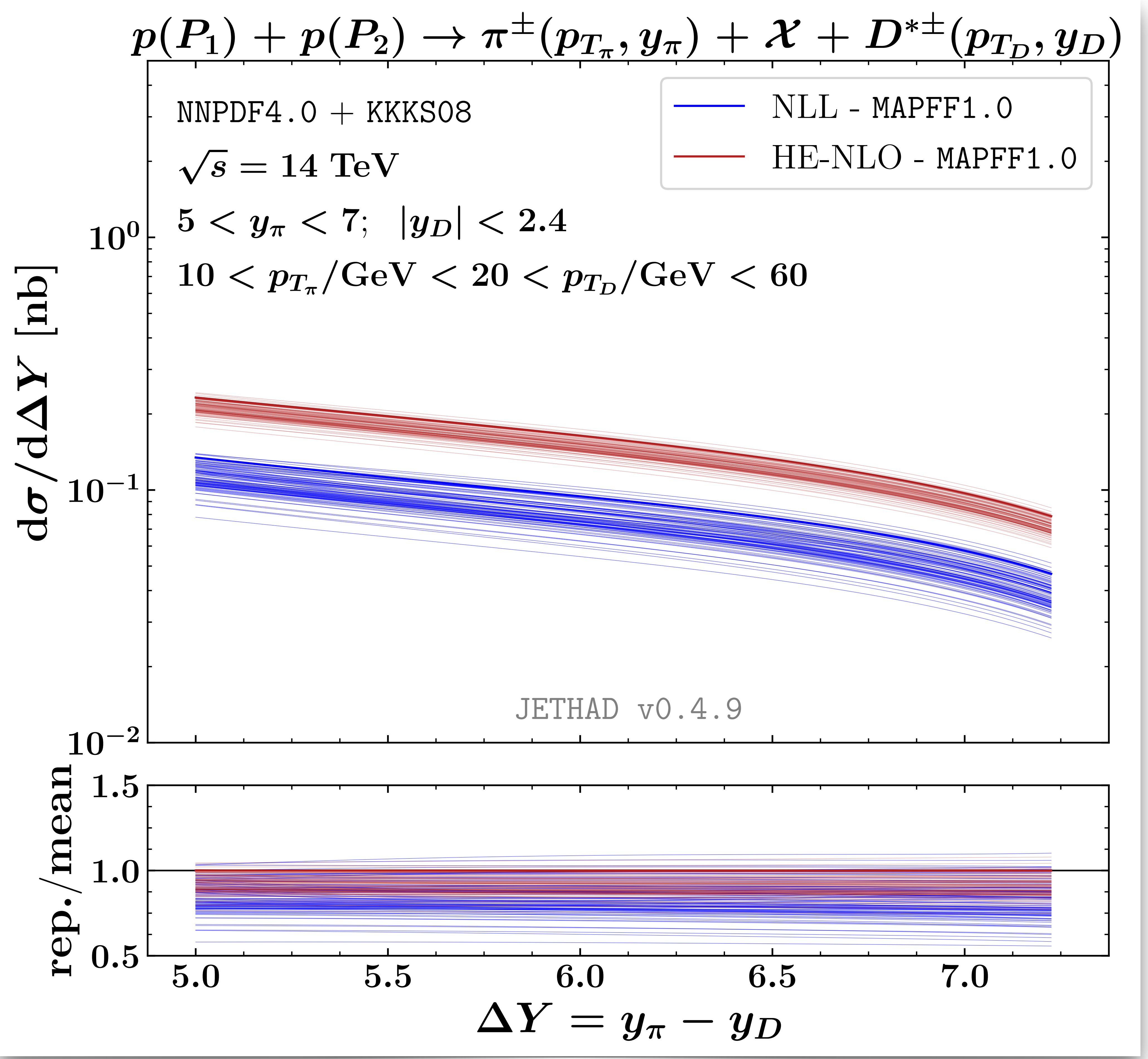
[FPF Snowmass Whitepaper]



Rapidity distributions @FPF+ATLAS

Inclusive $\pi^{\pm}$ (FPF) + $D^{*\pm}$ (ATLAS) production

[FPF Snowmass Whitepaper]

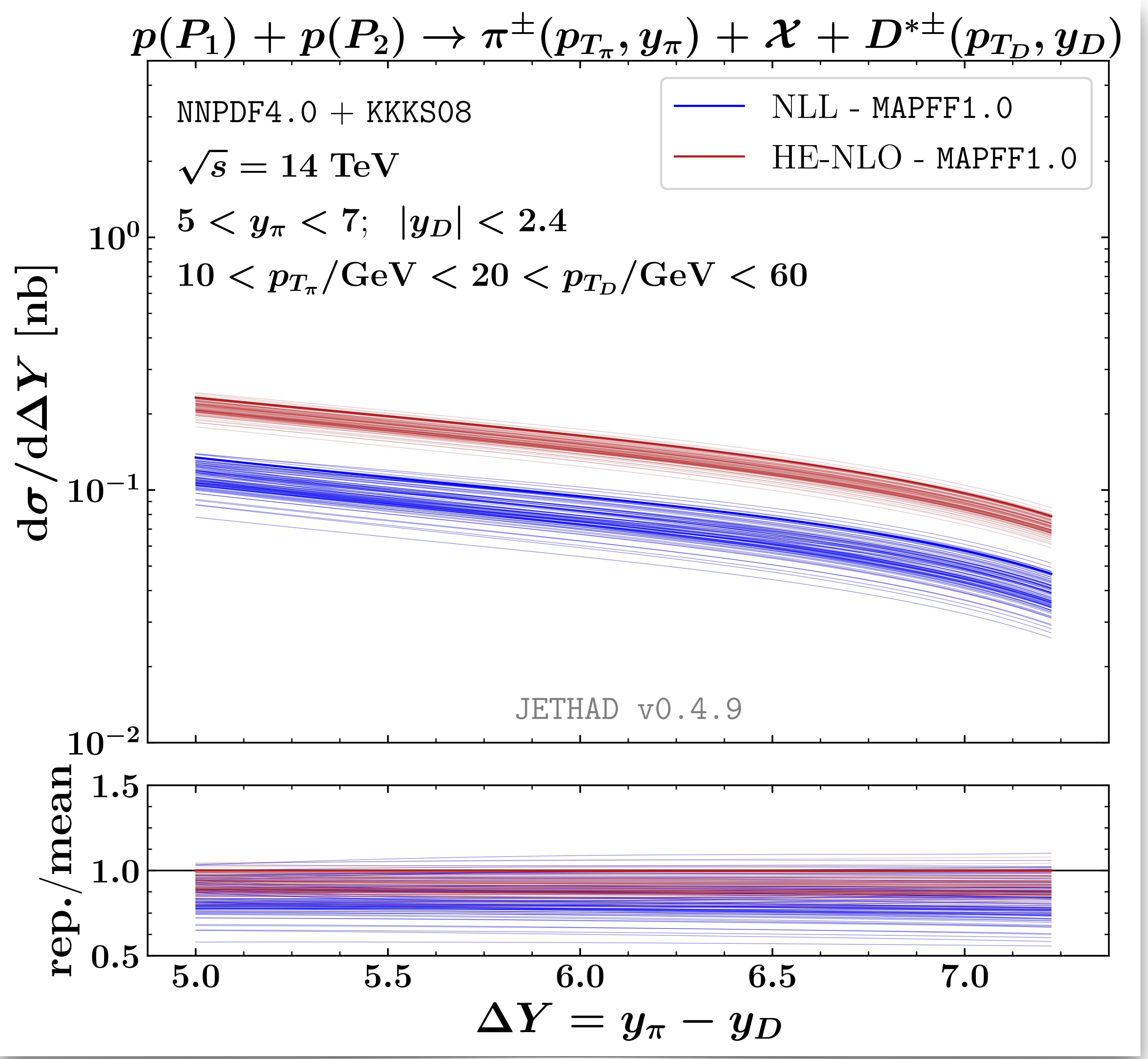


Backup

Rapidity distributions @FPF+ATLAS

Inclusive $\pi^{\pm}$ (FPF) + $D^{*\pm}$ (ATLAS) production

[FPF Snowmass Whitepaper]



- ✱ Impact of collinear FFs on ΔY-distribution
- ✱ Replica method at work

Backup

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Inclusive $\pi^{\pm}$ (FPF) + $D^{*\pm}$ (ATLAS) production

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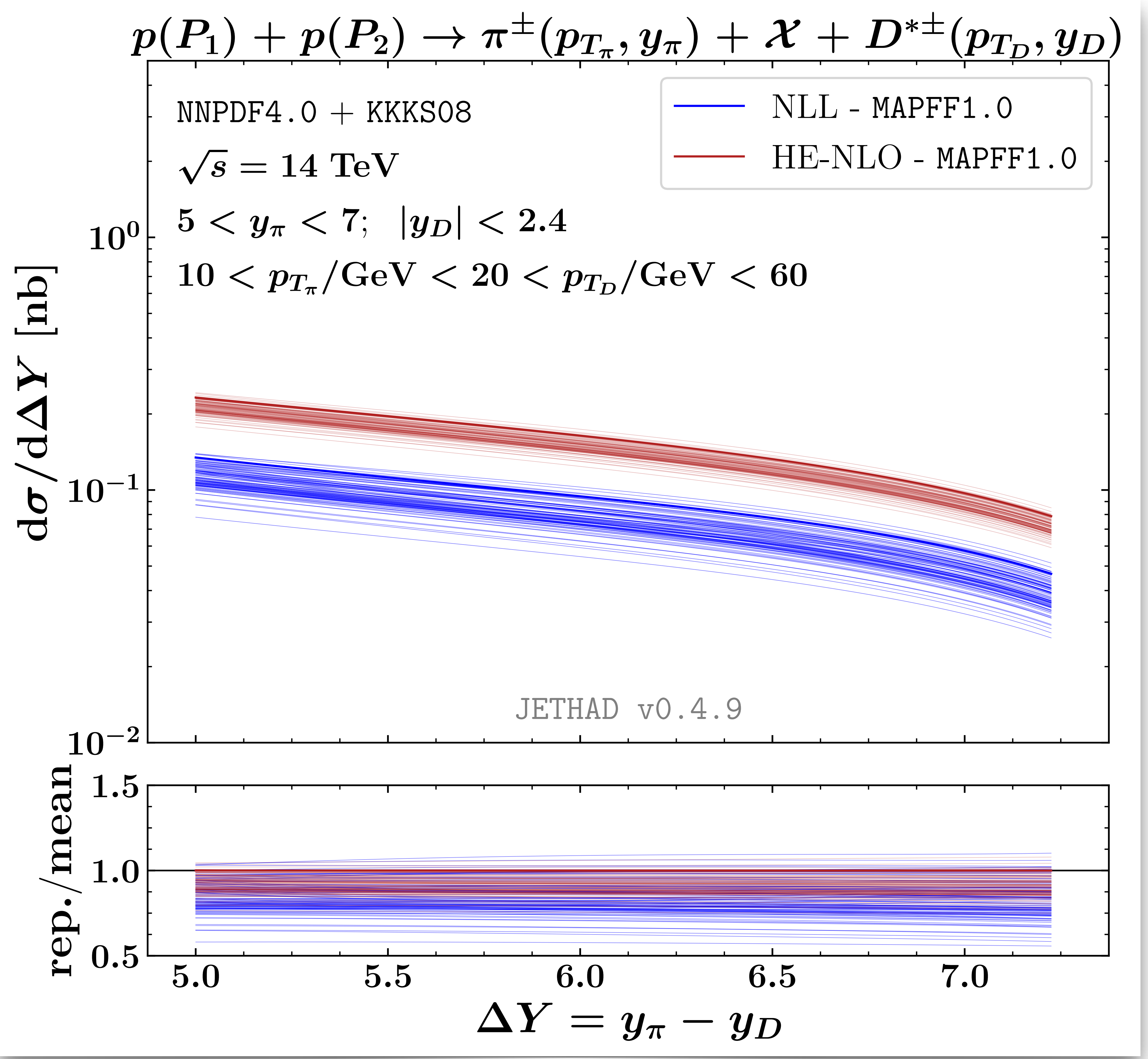


- Impact of collinear FFs on ΔY-distribution
- Replica method at work
- Larger spread of replicas at NLL
- Probe FFs in complementary ranges

  Weight of FF replicas in the same set

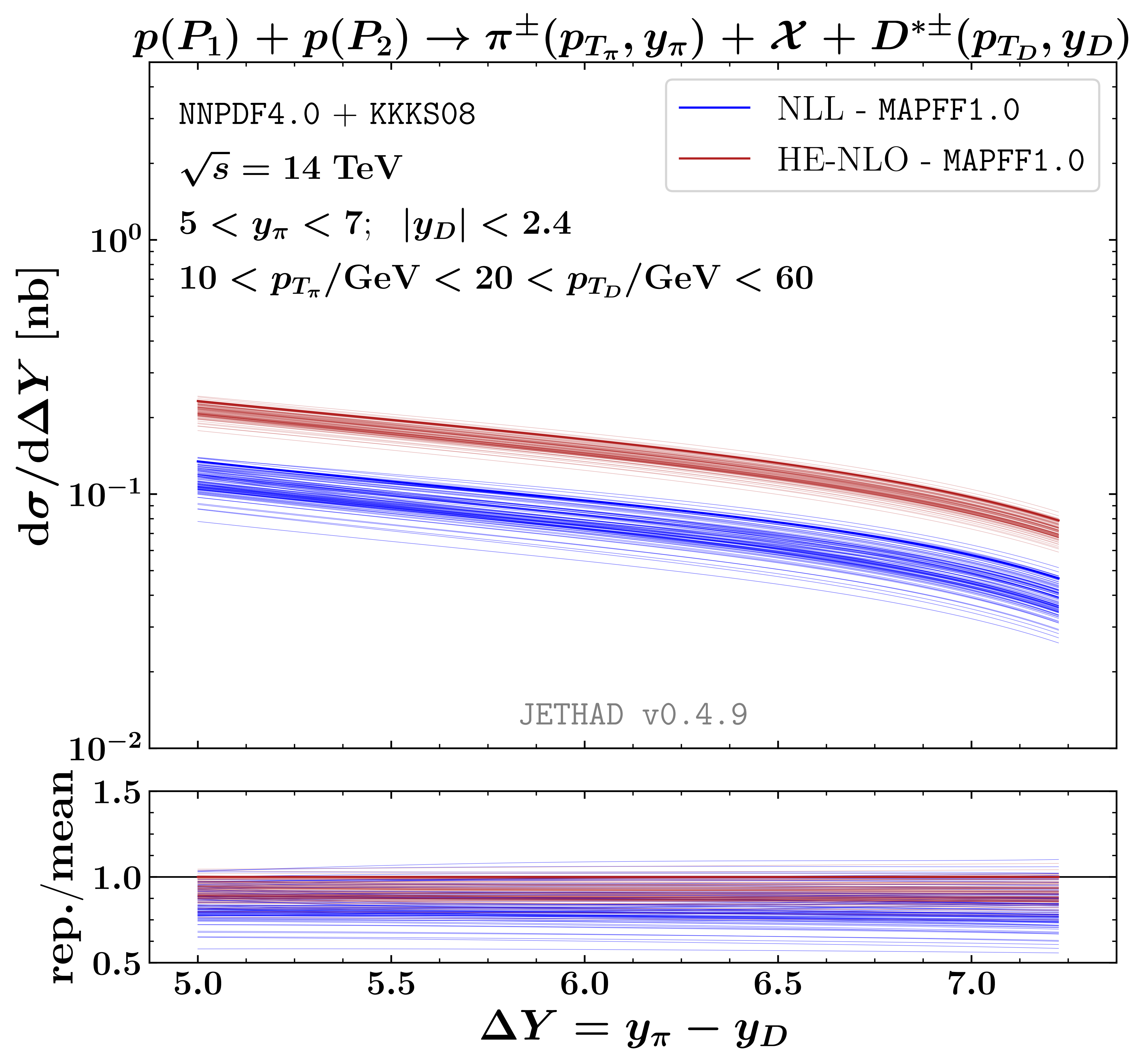
  Different sets via functional correlation?

- Complementary studies on FFs

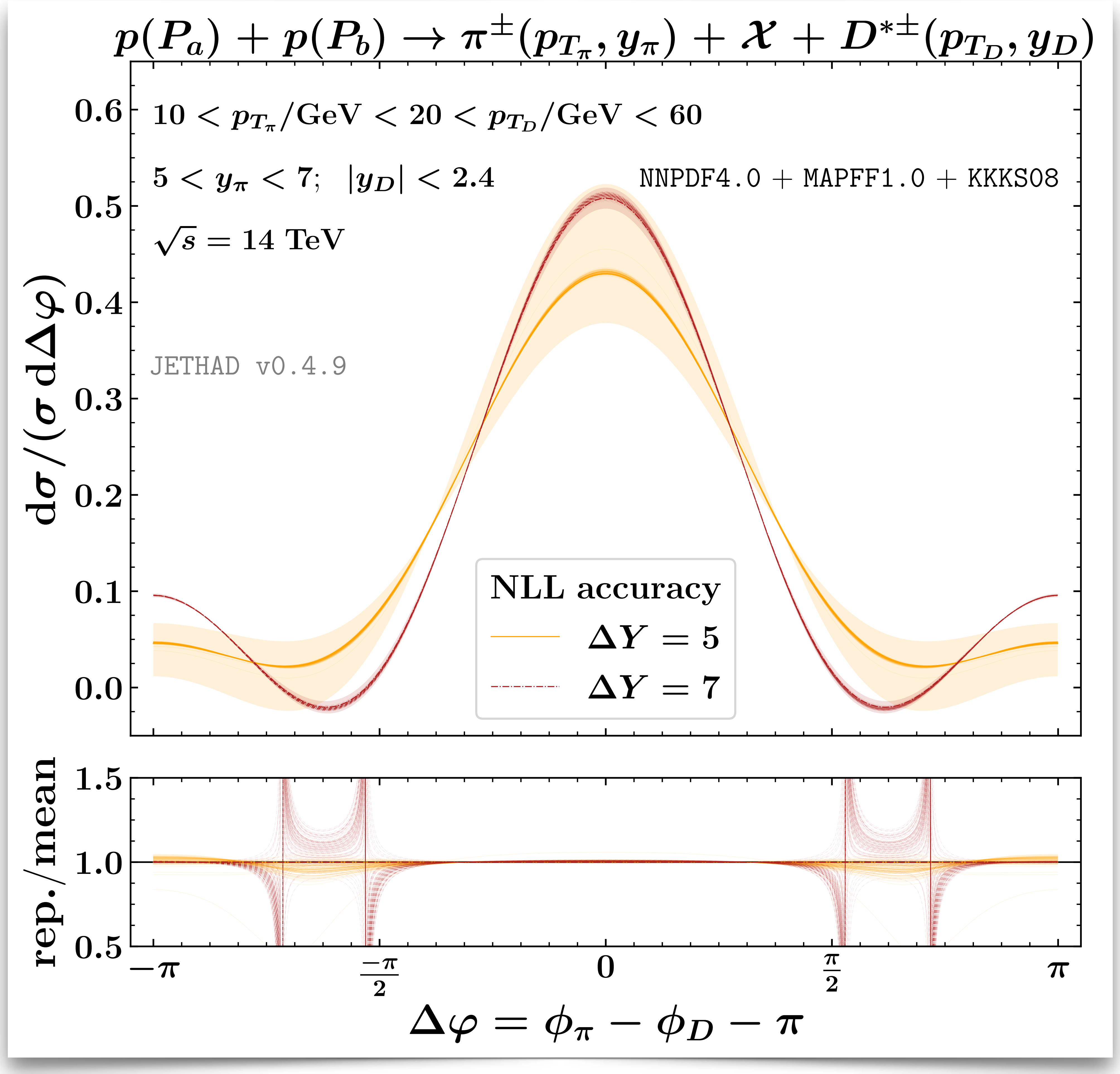
Backup

Rapidity distributions @FPF+ATLAS

Inclusive $\pi^{\pm}$ (FPF) + $D^{*\pm}$ (ATLAS) ΔY-spectrum



Inclusive $\pi^{\pm}$ (FPF) + $D^{*\pm}$ (ATLAS) $\Delta\varphi$-spectrum



[FPF Snowmass Whitepaper]

Backup

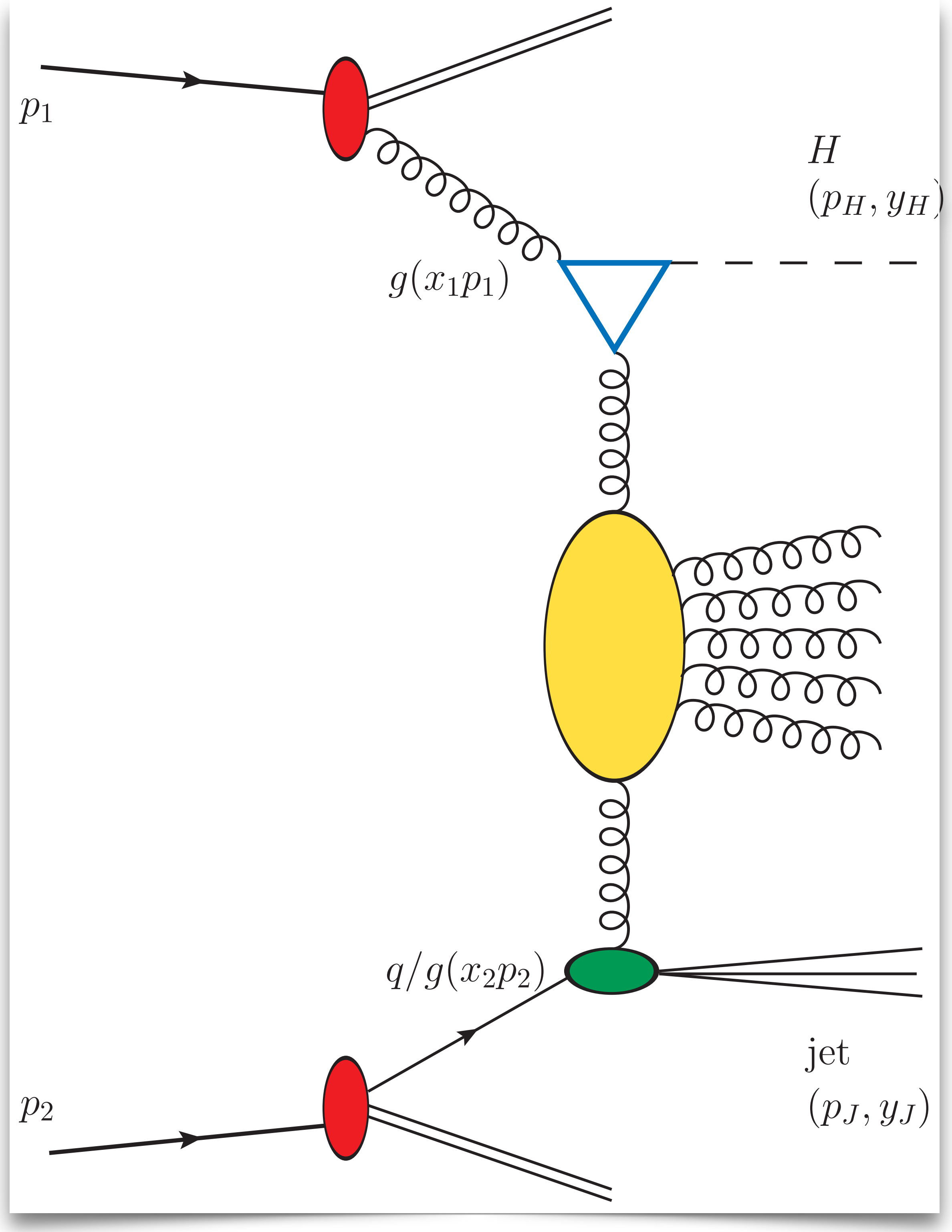
HIGGS + JET DISTRIBUTIONS

Inclusive Higgs + jet at the LHC

- Inclusive h.p. of a Higgs + jet system with high p_T and large rapidity separation, ΔY
- Large energy scales expected to stabilize the high-energy resummed series

$$\frac{d\sigma}{dx_1 dx_2\, d|\vec{p}_H|\, d|\vec{p}_J| d\varphi_H d\varphi_J} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\varphi)\,\mathcal{C}_n\right]$$

$$\varphi = \varphi_H - \varphi_J - \pi$$



Backup

Inclusive Higgs + jet at the LHC

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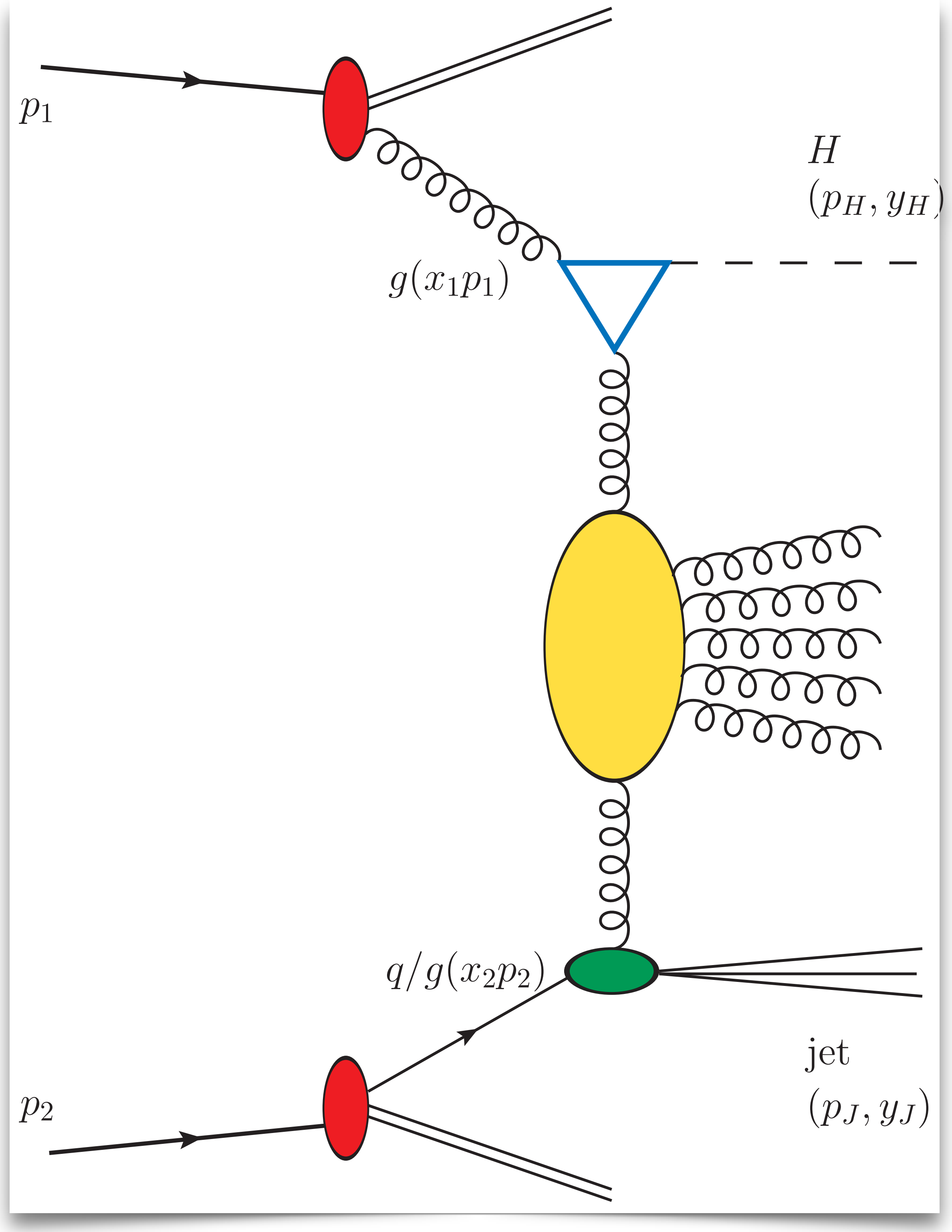
Higgs vertex (off-shell coefficient function)

jet vertex (off-shell coefficient function)

$$\varphi = \varphi_H - \varphi_J - \pi$$



$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_H\, dy_J\, d^2\vec{p}_H\, d^2\vec{p}_J} = \frac{1}{(2\pi)^2}$$

NLO* $$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\, \mathcal{V}_H^{(r)}(\vec{q}_1, s_0, x_1, \vec{p}_H)$$

NLL $$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i}\left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2)$$

NLO* $$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\, \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{p}_J)$$

BFKL Green's function

Backup

Inclusive Higgs + jet at the LHC

- Inclusive h.p. of a Higgs + jet system with high p_T and large rapidity separation, ΔY
- Large energy scales expected to stabilize the high-energy resummed series

$$\frac{d\sigma}{dx_1 dx_2\, d|\vec{p}_H|\, d|\vec{p}_J| d\varphi_H d\varphi_J} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0 + \sum_{n=1}^{\infty} 2\cos(n\varphi)\, \mathcal{C}_n\right]$$

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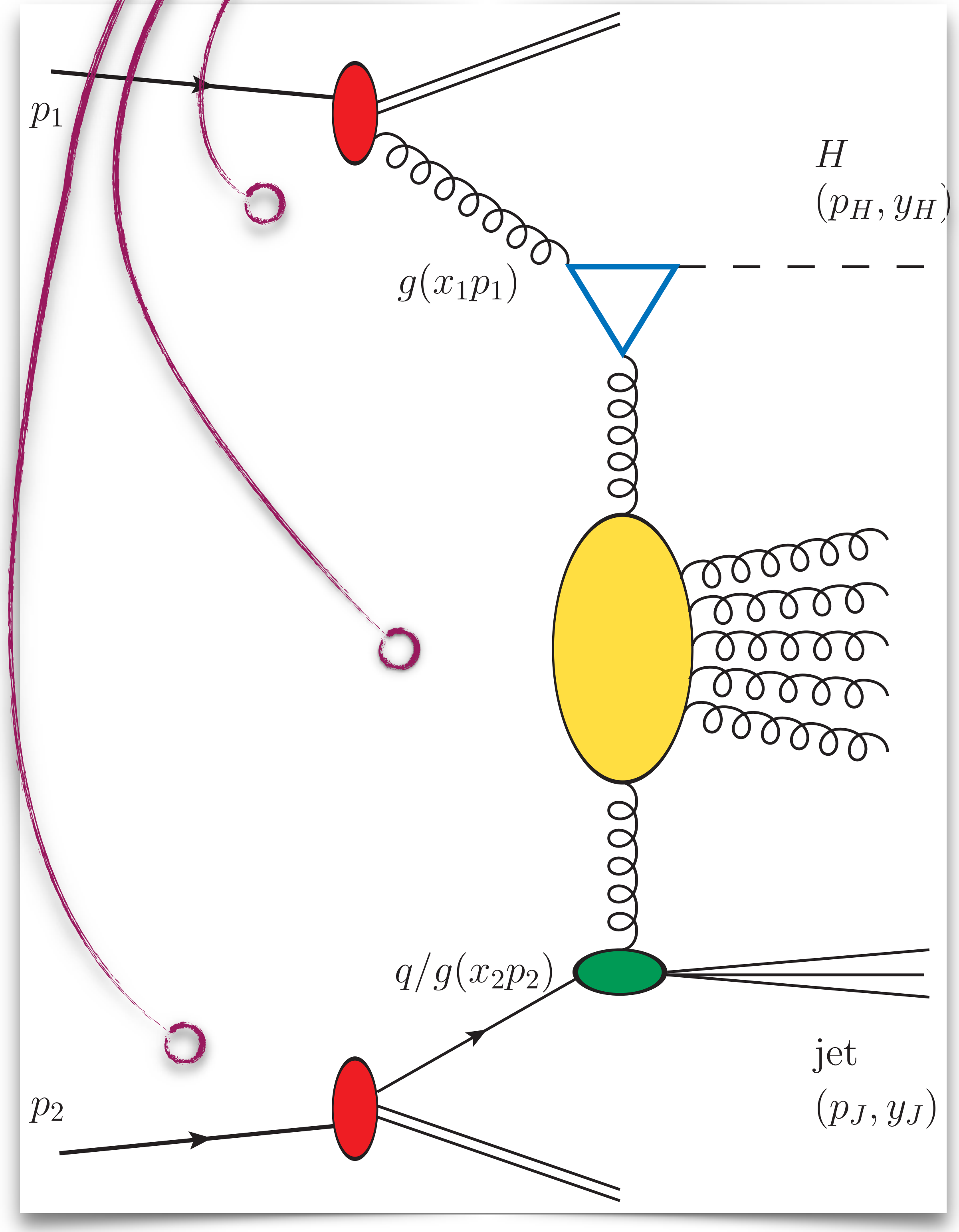


$\mu_{F,R} \sim M_{H,\perp}$

$\mu_R \sim \sqrt{M_{H,\perp}\, p_J}$

$\mu_{F,R} \sim p_J$

NLO*

NLL

NLO*

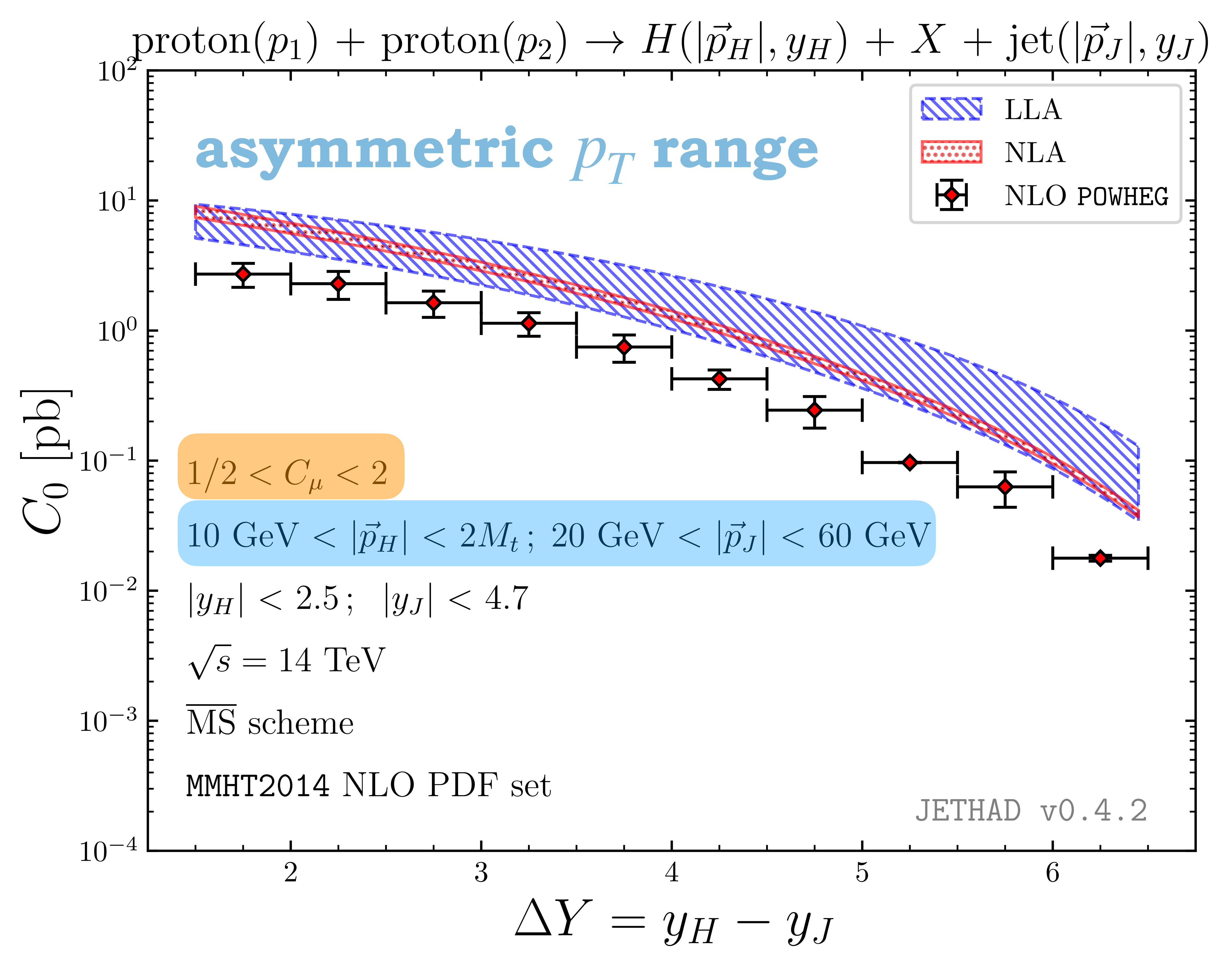
$$\frac{d\hat{\sigma}_{r,s}(x_1 x_2 s, \mu)}{dy_H\, dy_J\, d^2\vec{p}_H\, d^2\vec{p}_J} = \frac{1}{(2\pi)^2}$$
$$\times \int \frac{d^2\vec{q}_1}{\vec{q}_1^{\,2}}\, \mathcal{V}_H^{(r)}(\vec{q}_1, s_0, x_1, \vec{p}_H)$$
$$\times \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i}\left(\frac{x_1 x_2 s}{s_0}\right)^{\omega} \mathcal{G}_\omega(\vec{q}_1, \vec{q}_2)$$
$$\times \int \frac{d^2\vec{q}_2}{\vec{q}_2^{\,2}}\, \mathcal{V}_J^{(s)}(\vec{q}_2, s_0, x_2, \vec{p}_J)$$

BFKL Green's function

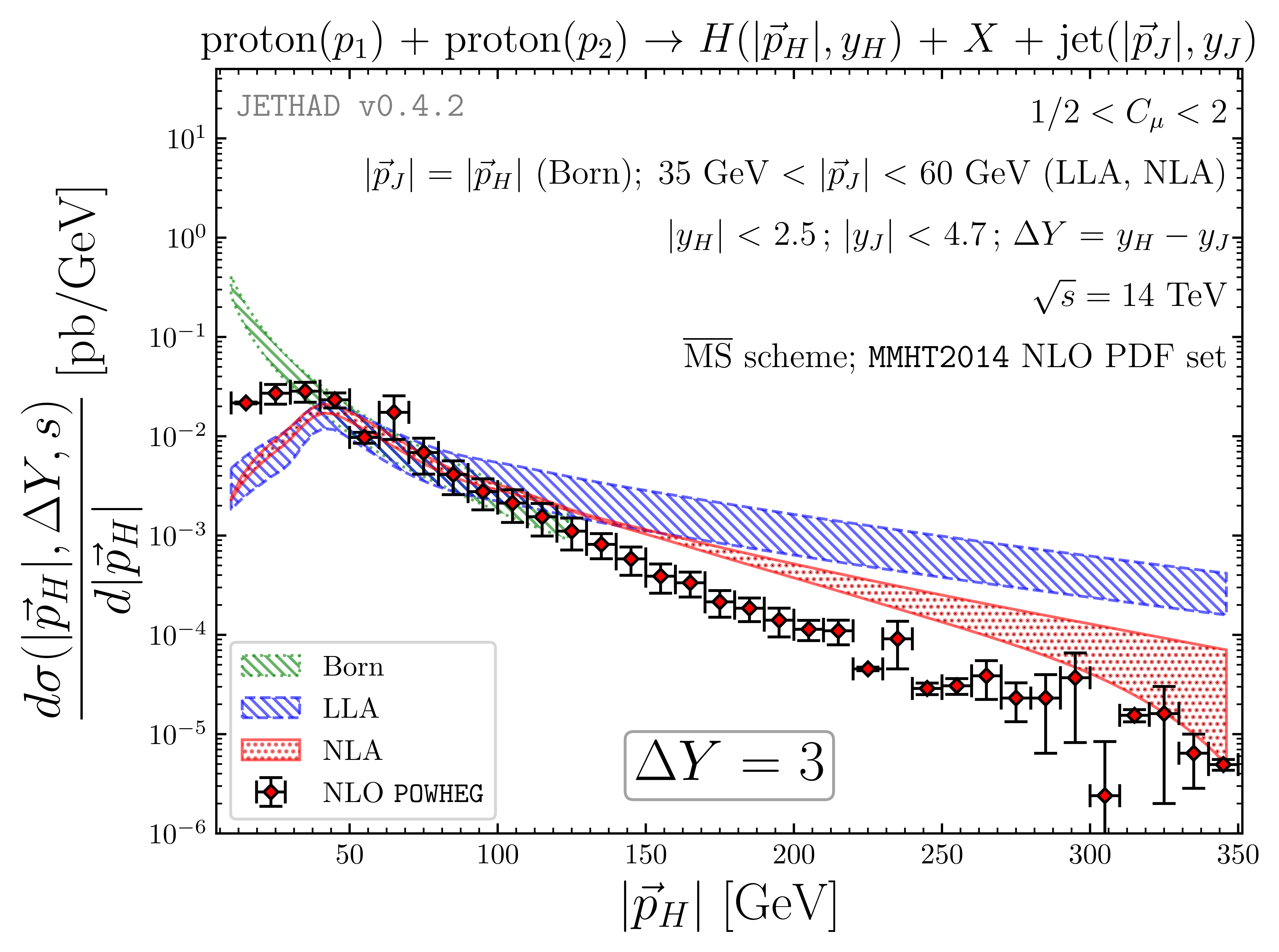
Backup

The Higgs + jet spectrum in hybrid factorization

ΔY spectrum



p_H spectrum

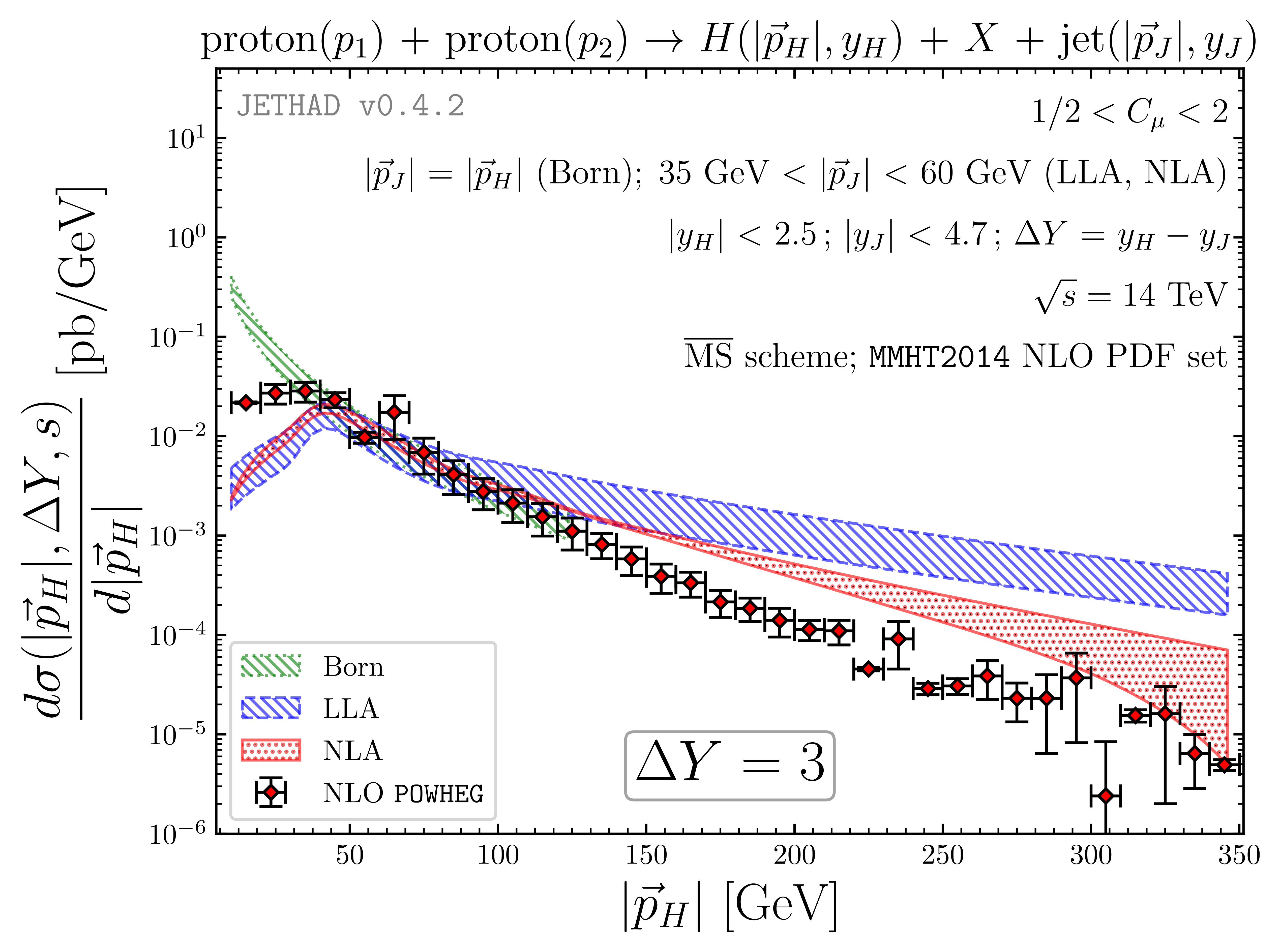


(in this slide) 🔗 [F. G. C. et al., Eur. Phys. J. C 81 (2021) 4, 293]
(JETHAD) 🔗 [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

Backup

Higgs transverse-momentum distribution

$$\frac{d\sigma(|\vec{p}_H|, \Delta Y, s)}{d|\vec{p}_H| d\Delta Y} = \int_{p_J^{\min}}^{p_J^{\max}} d|\vec{p}_J| \int_{y_H^{\min}}^{y_H^{\max}} dy_H \int_{y_J^{\min}}^{y_J^{\max}} dy_J \, \delta\left(y_H - y_J - \Delta Y\right) \mathcal{C}_0$$



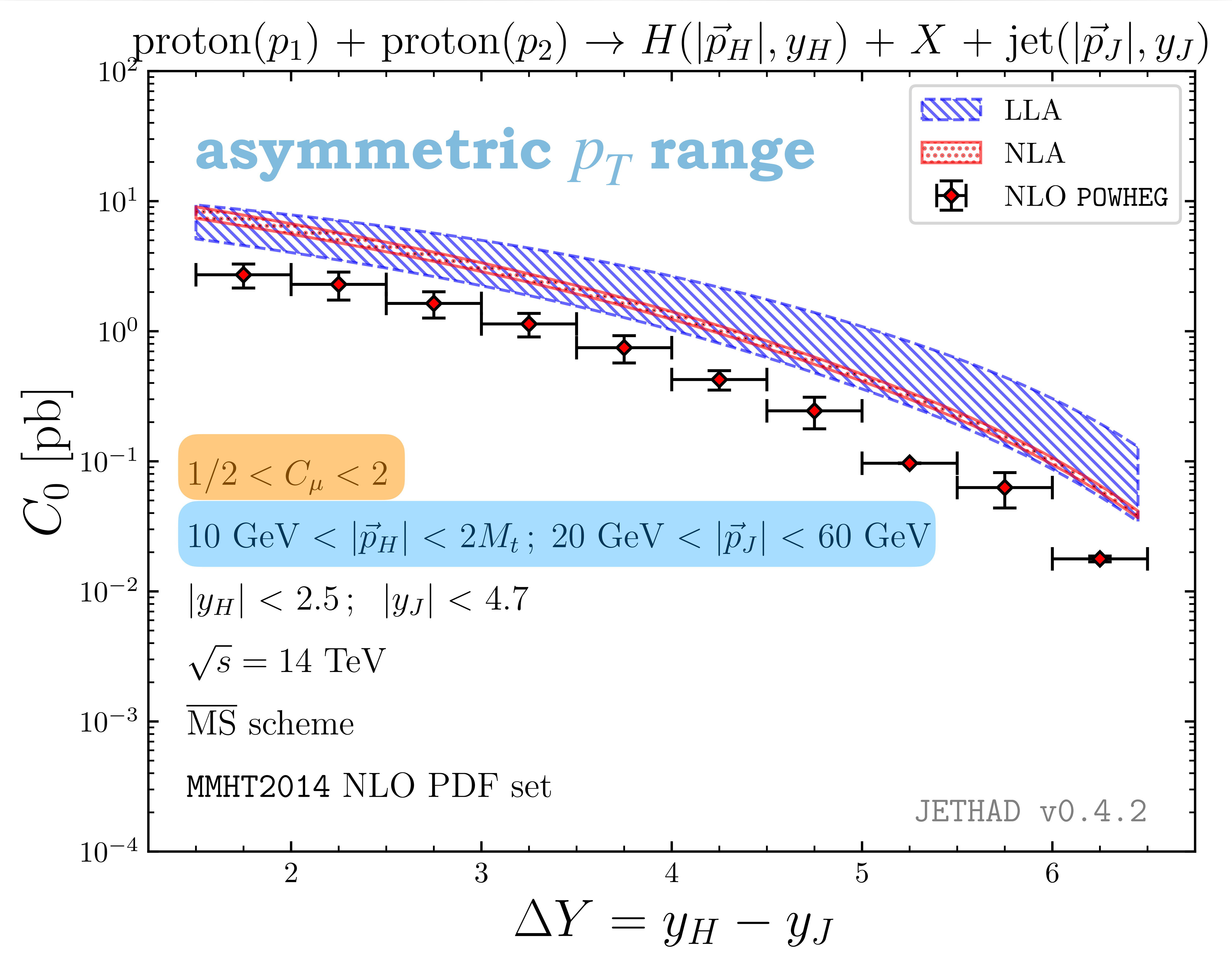
- HE resummation from JETHAD
- Comparison with fixed-order POWHEG
- Distributions stable under NLL corrections

Backup

[F. G. C., D. Yu. Ivanov, M. M. A. Mohammed, A. Papa, Eur. Phys. J. C 81 (2021) 4, 293]

ΔY-distribution

$$\mathcal{C}_n(\Delta Y, s) = \int_{p_H^{\rm min}}^{p_H^{\rm max}} d|\vec{p}_H| \int_{p_J^{\rm min}}^{p_J^{\rm max}} d|\vec{p}_J| \int_{y_H^{\rm min}}^{y_H^{\rm max}} dy_H \int_{y_J^{\rm min}}^{y_J^{\rm max}} dy_J \, \delta\left(y_H - y_J - \Delta Y\right) \mathcal{C}_n$$



Backup

[F. G. C., D. Yu. Ivanov, M. M. A. Mohammed, A. Papa, Eur. Phys. J. C 81 (2021) 4, 293]

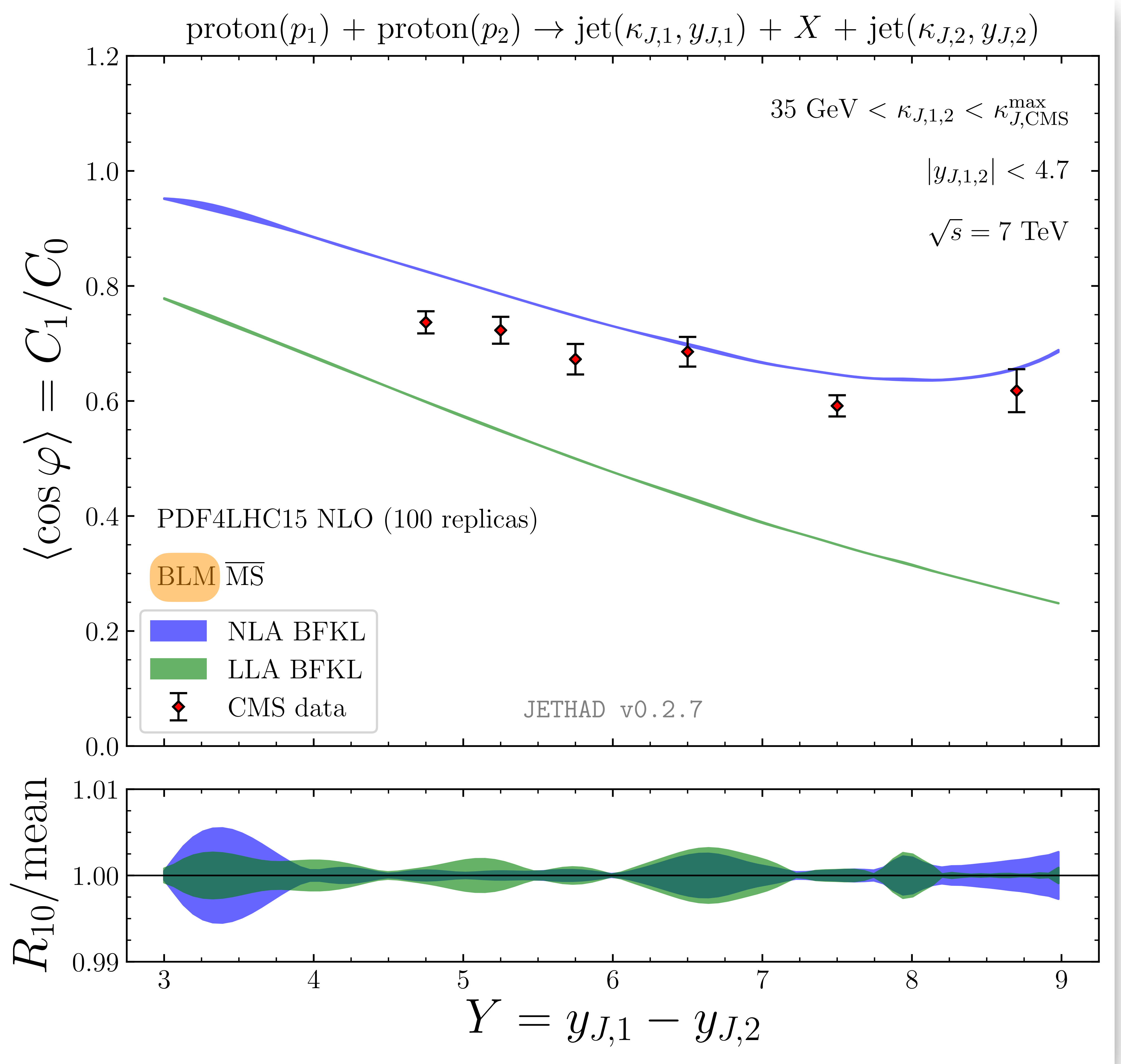
Angular correlations

$$R_{n0}(\Delta Y, s) \;=\; C_n/C_0 \;\equiv\; \langle \cos n\varphi \rangle$$

Mueller-Navelet jets

[B. Ducloué, L. Szymanowski, S. Wallon, Phys.Rev.Lett. 112 (2014) 082003]

(figure below) [F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]



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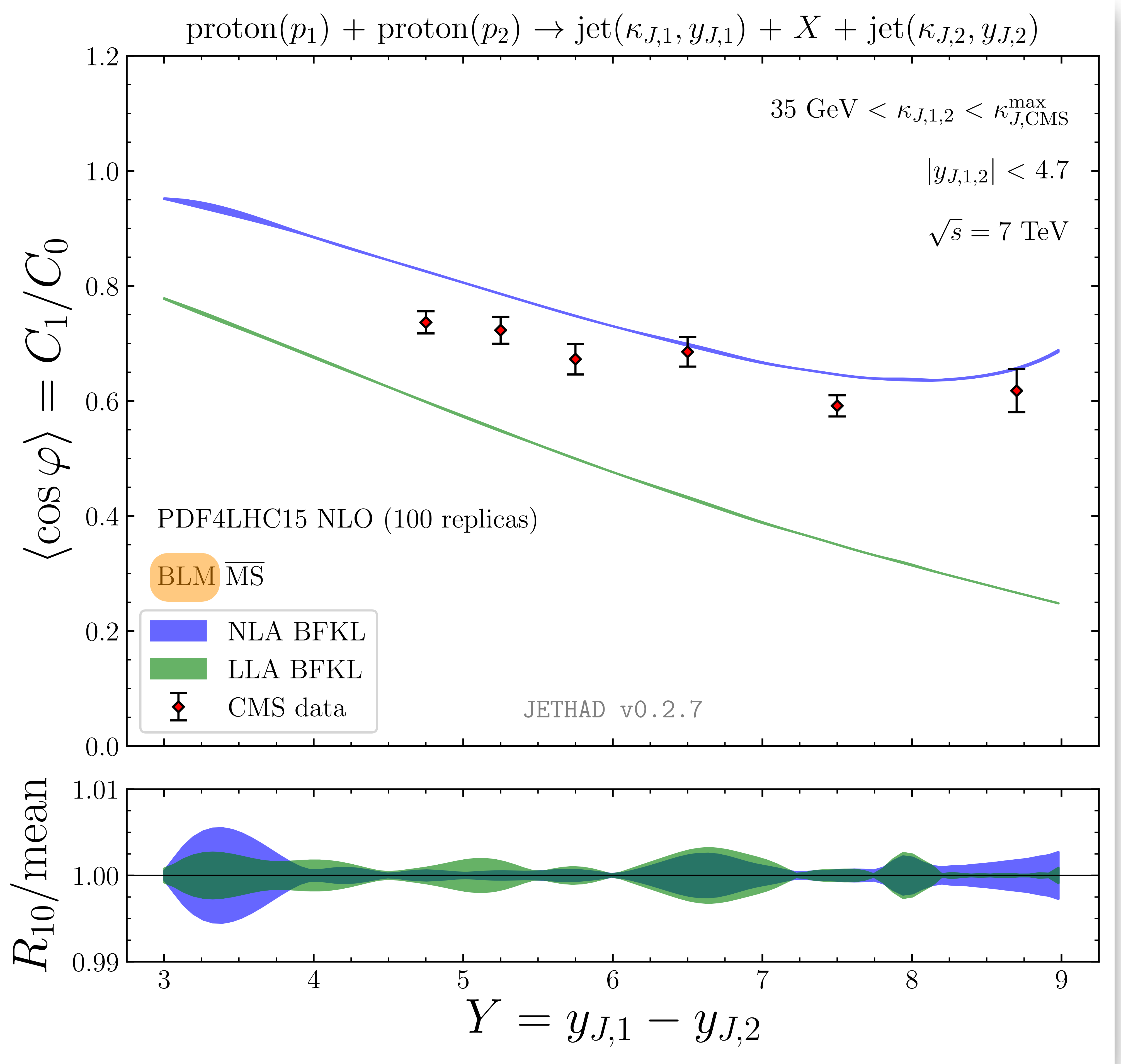
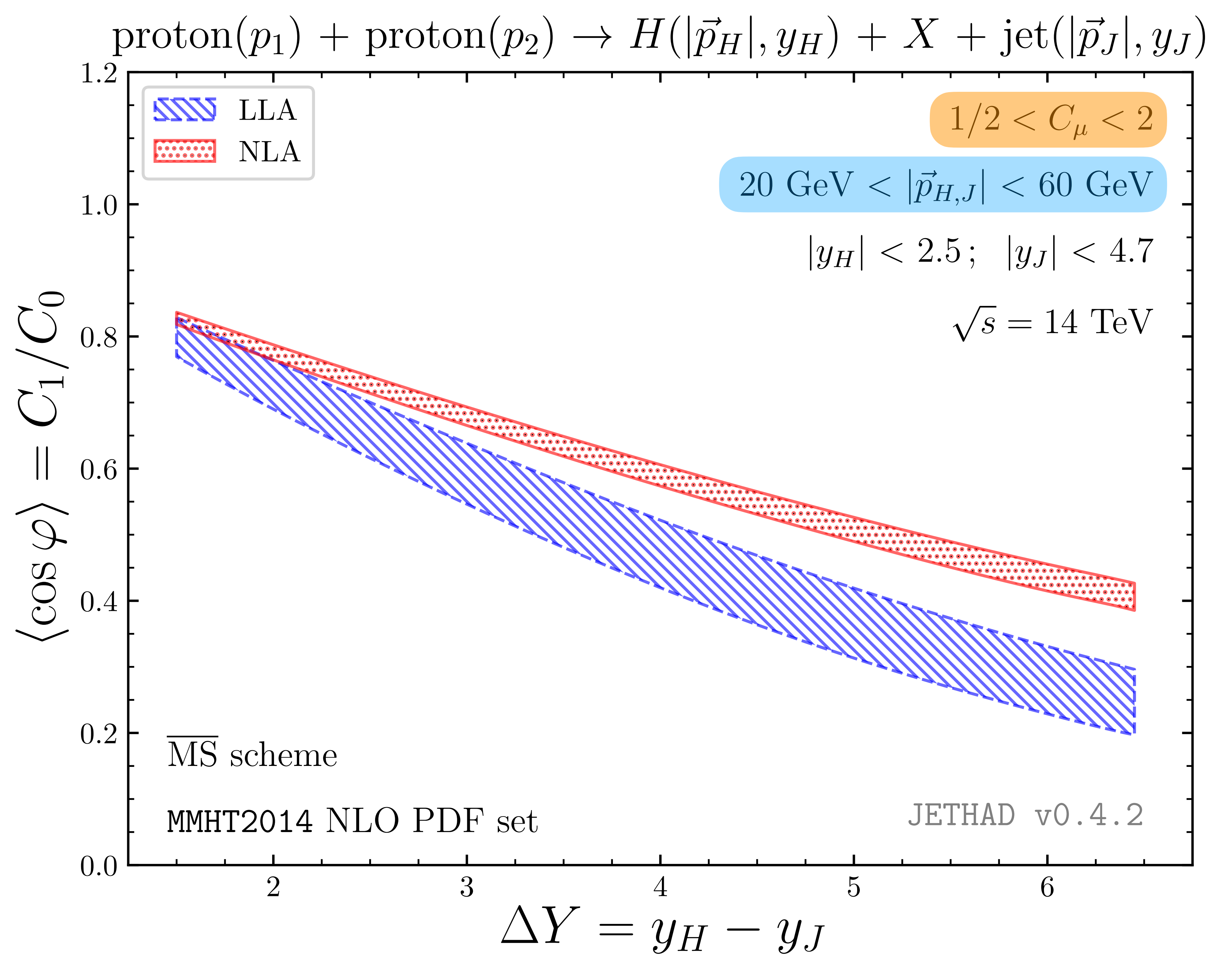


Higgs + jet

(figure below) 🔗 [F. G. C. et al., Eur. Phys. J. C 81 (2021) 4, 293]

(NLO Higgs impact factor) 🔗 [F. G. C. et al., under review (2022)]



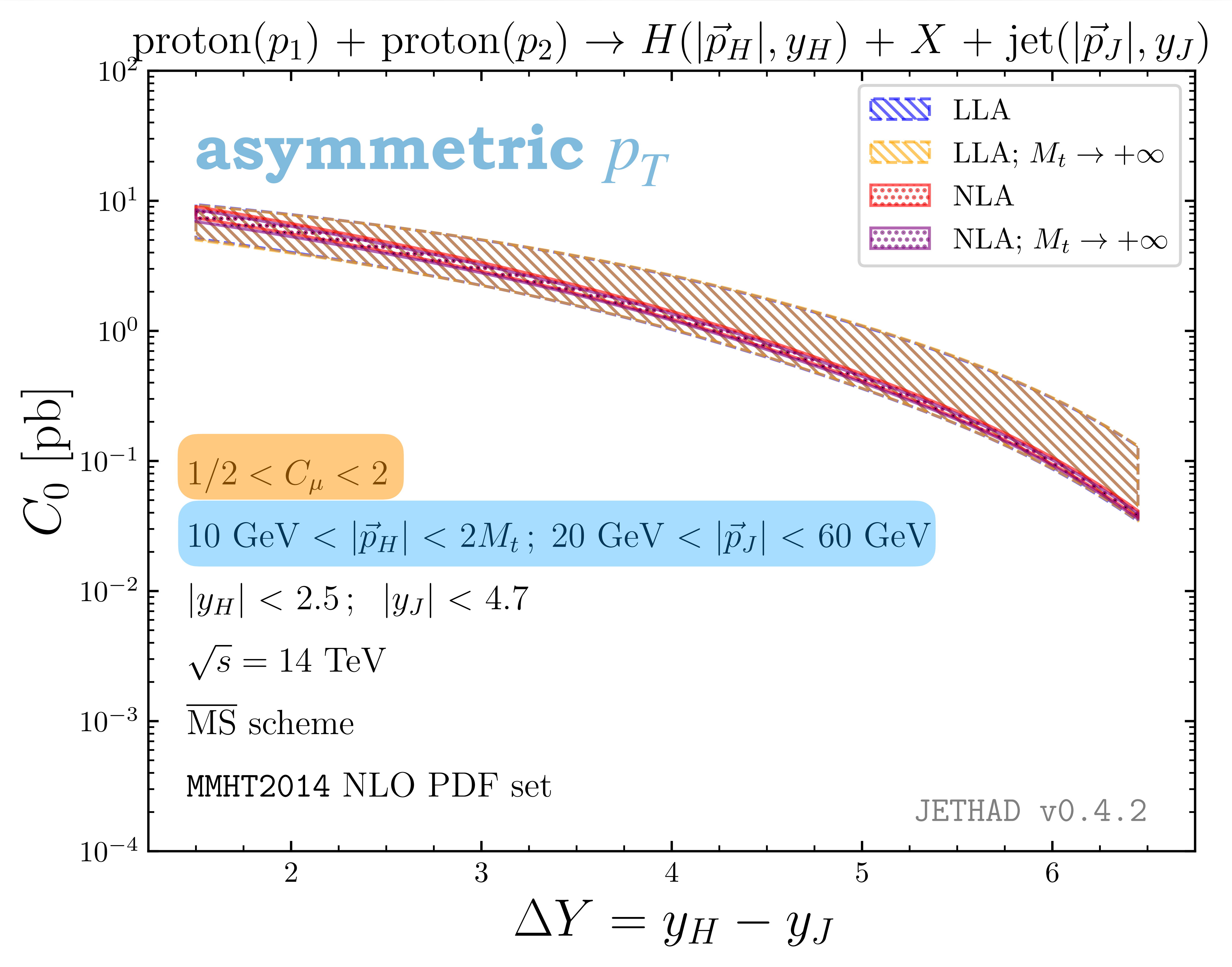
natural scales

symmetric p_T range

Backup

ΔY-distribution in the infinite top-mass limit

$$C_n(\Delta Y, s) = \int_{p_H^{\rm min}}^{p_H^{\rm max}} d|\vec{p}_H| \int_{p_J^{\rm min}}^{p_J^{\rm max}} d|\vec{p}_J| \int_{y_H^{\rm min}}^{y_H^{\rm max}} dy_H \int_{y_J^{\rm min}}^{y_J^{\rm max}} dy_J \, \delta\left(y_H - y_J - \Delta Y\right) \mathcal{C}_n$$



Backup

[F. G. C., D. Yu. Ivanov, M. M. A. Mohammed, A. Papa, Eur. Phys. J. C 81 (2021) 4, 293]

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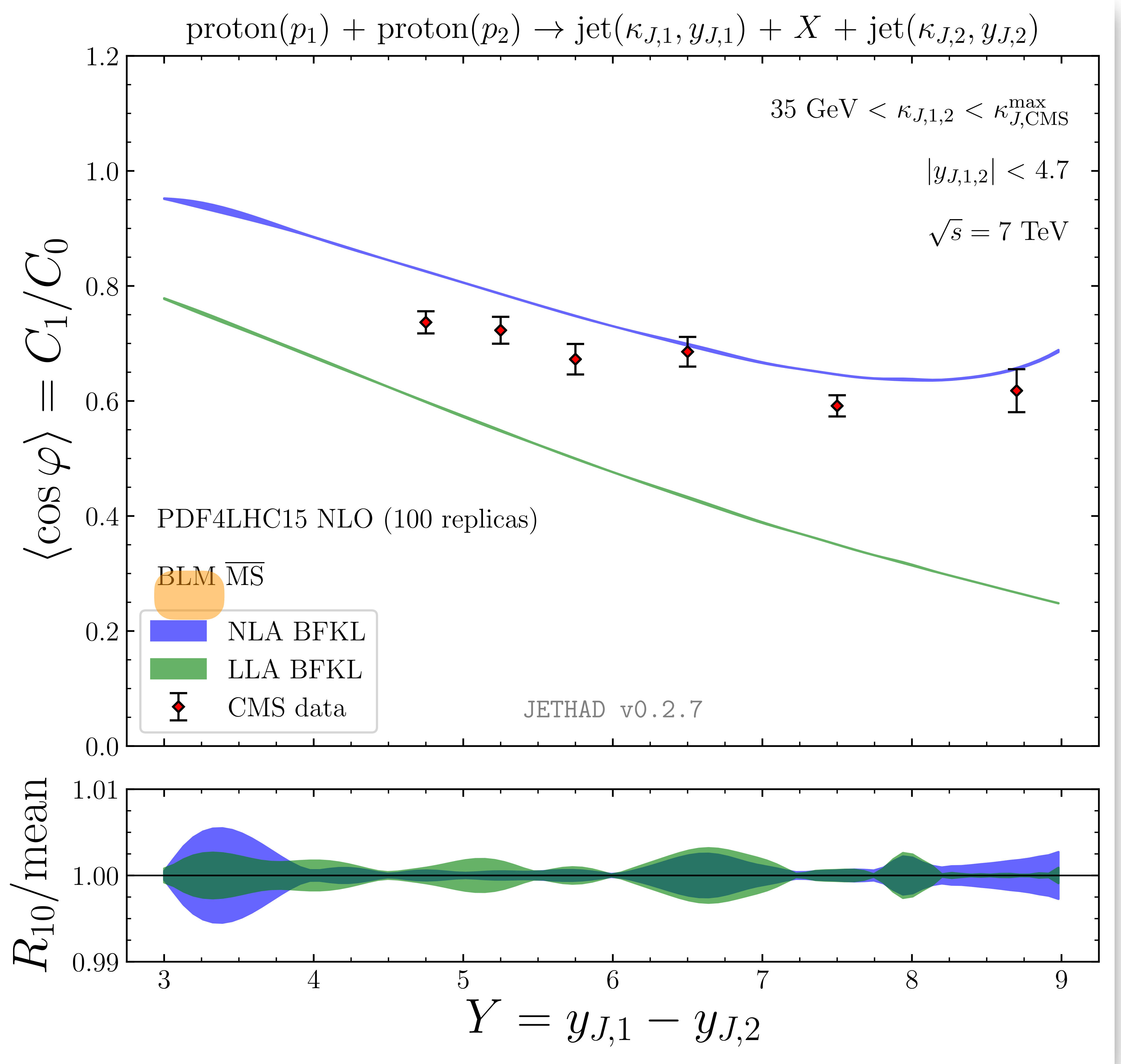

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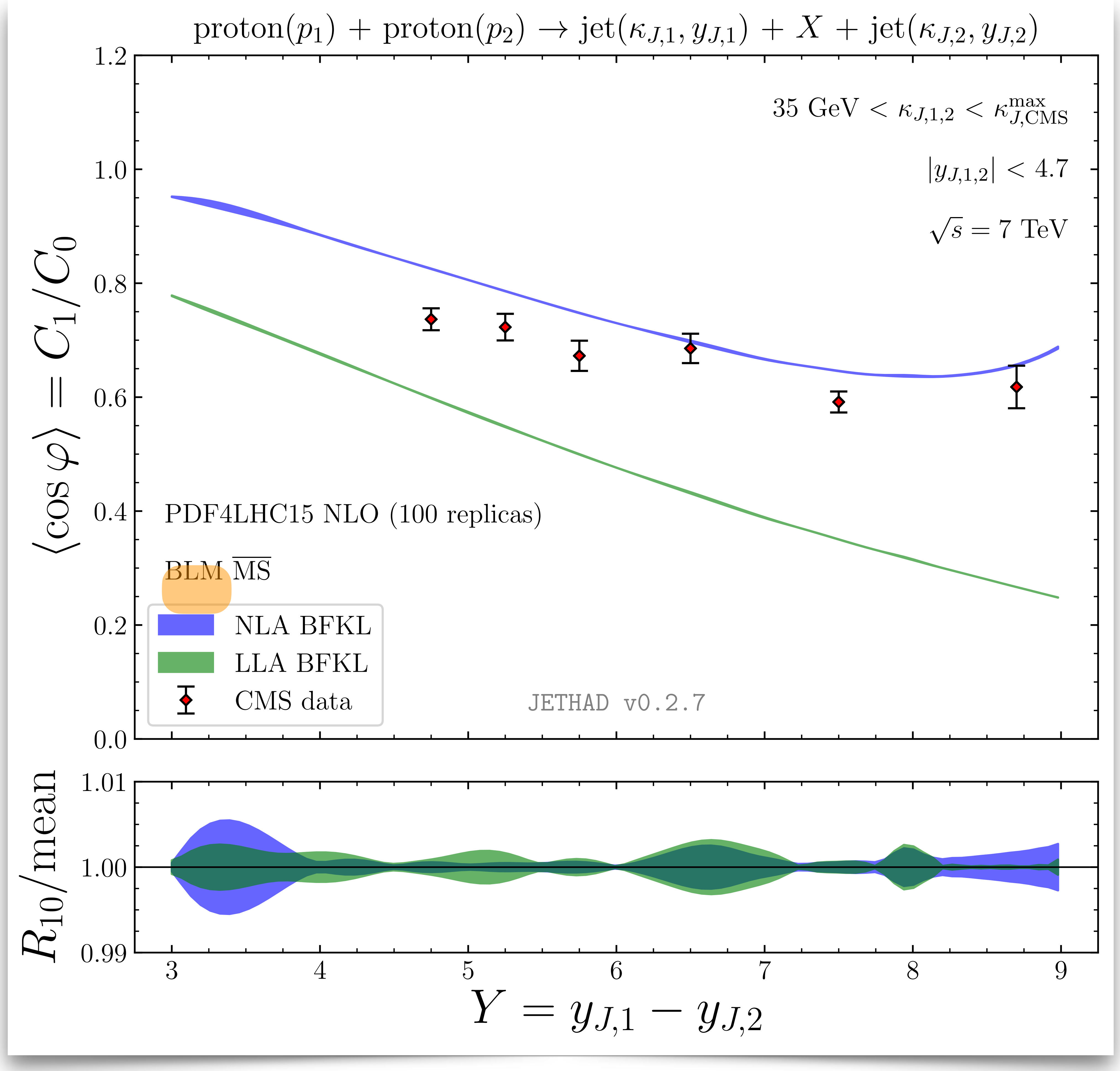

Higgs + jet

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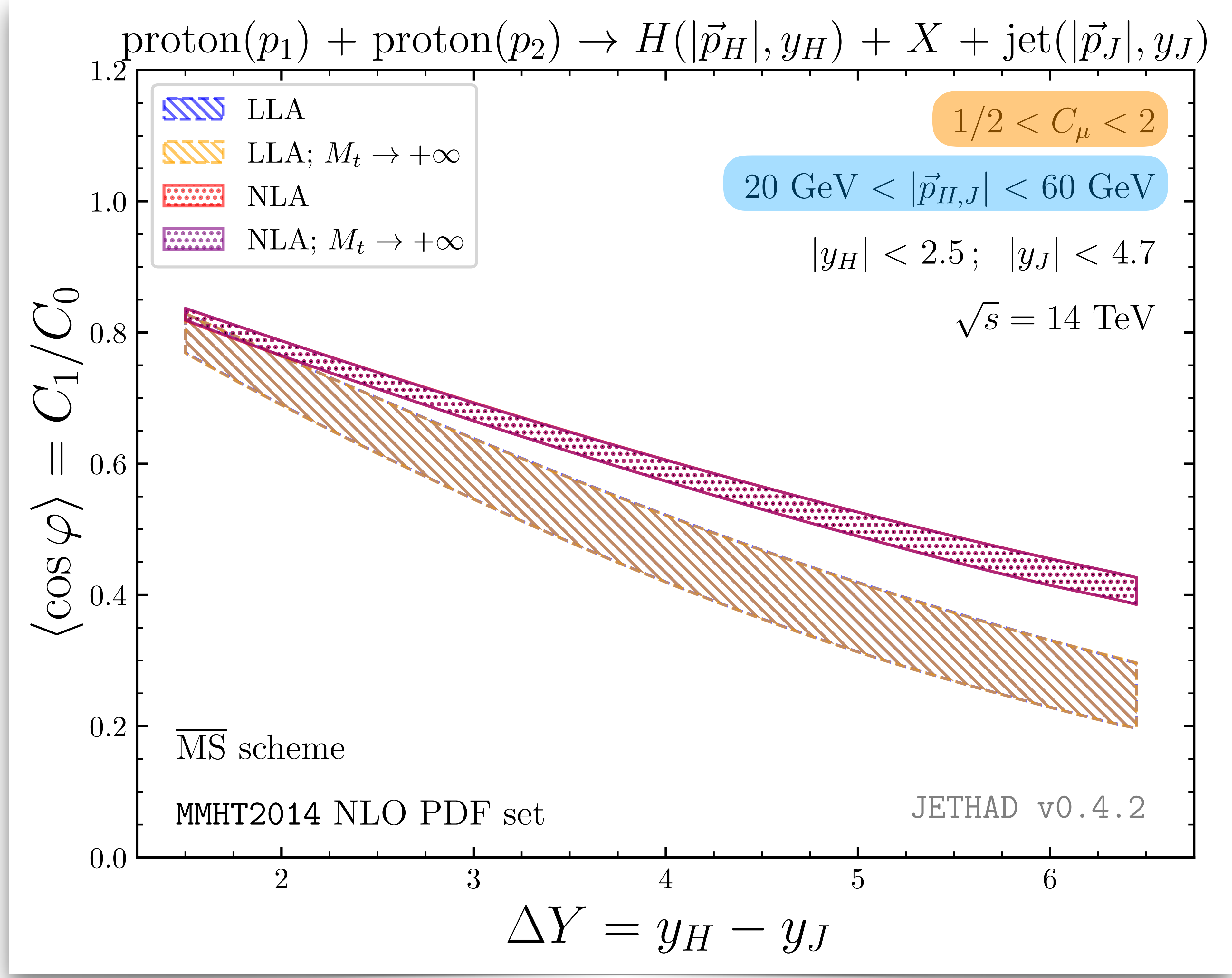
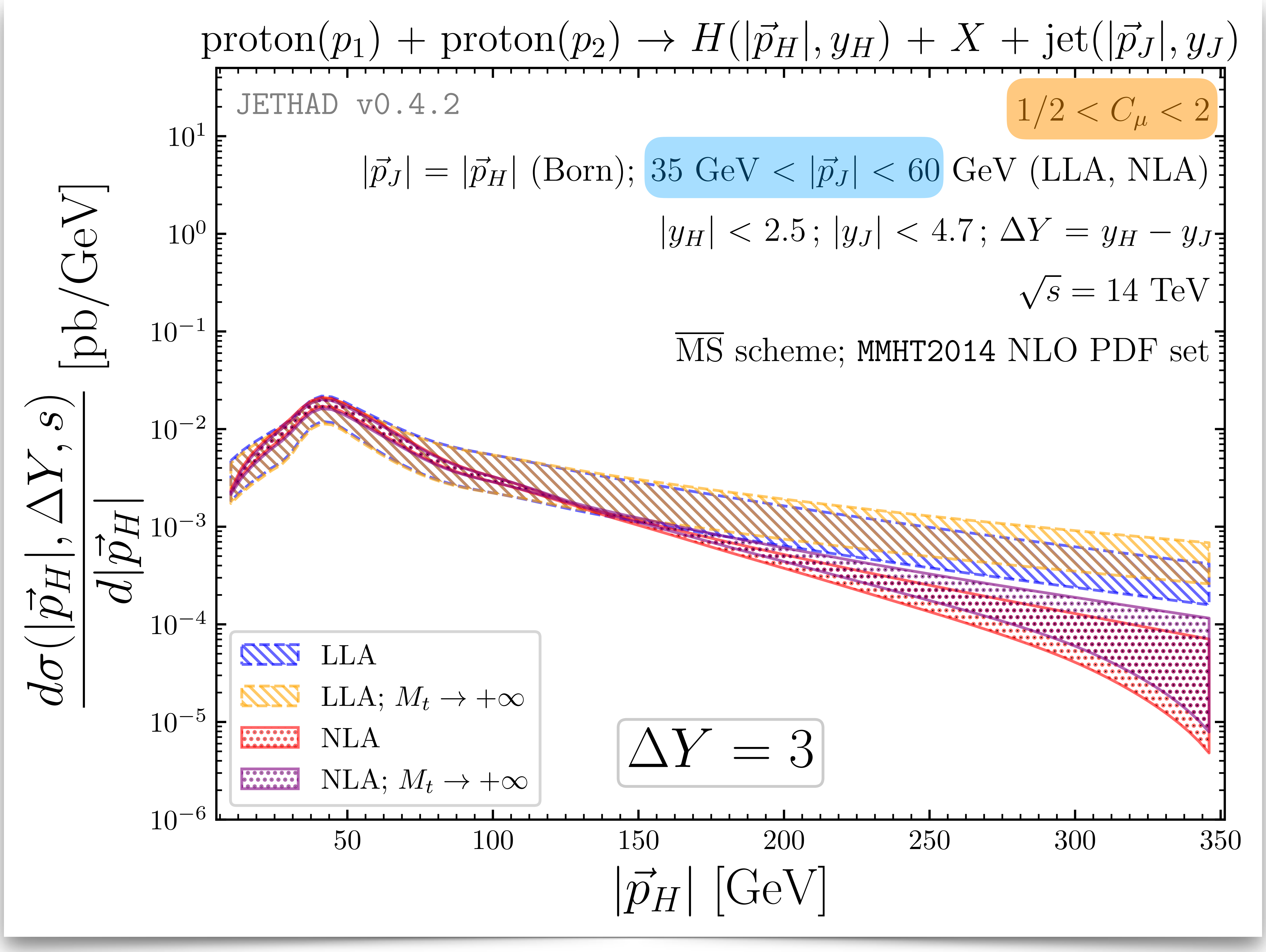


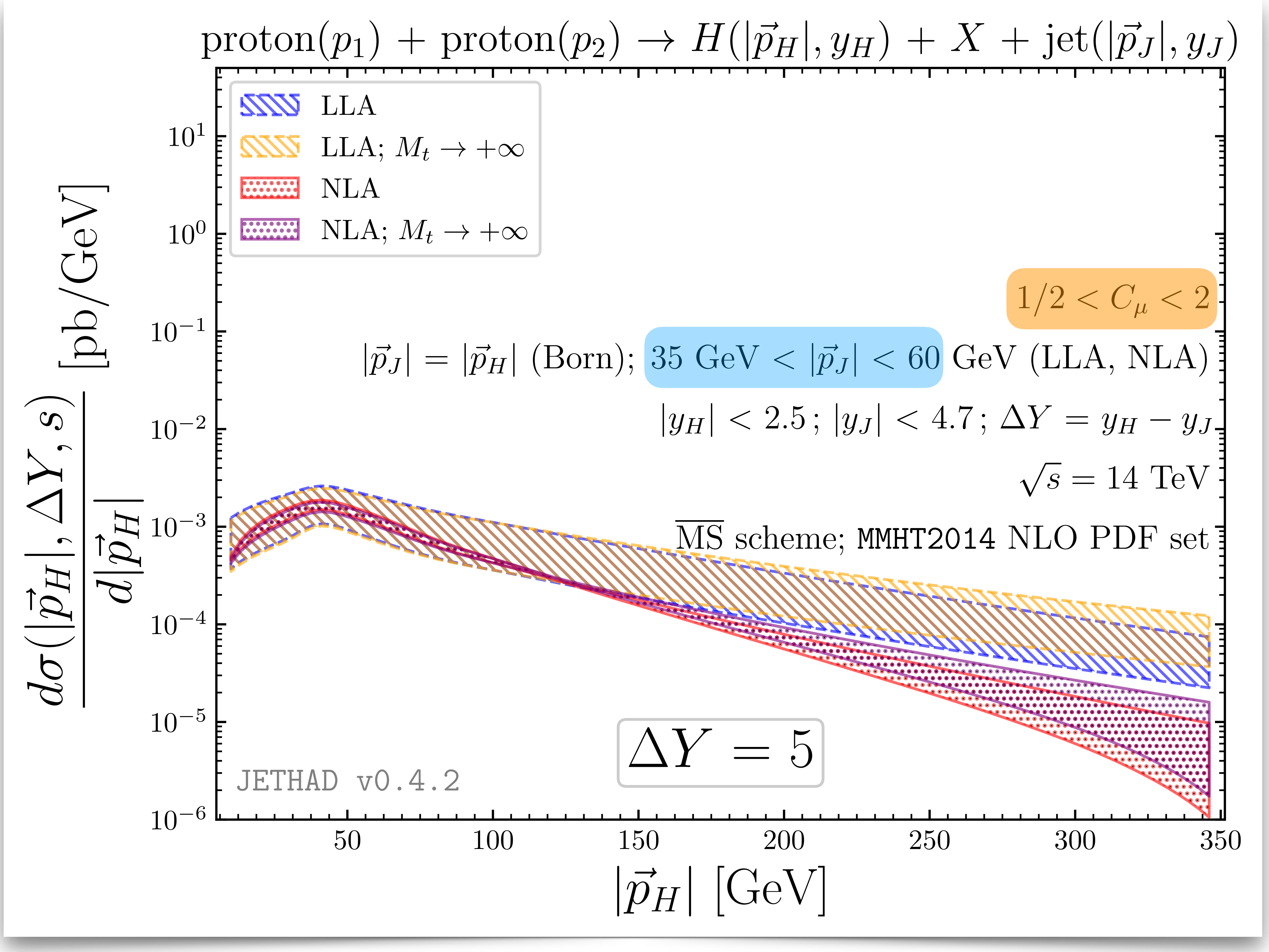
natural scales

symmetric p_T range

Backup

Higgs transverse-momentum distribution for $(M_t \to +\infty)$



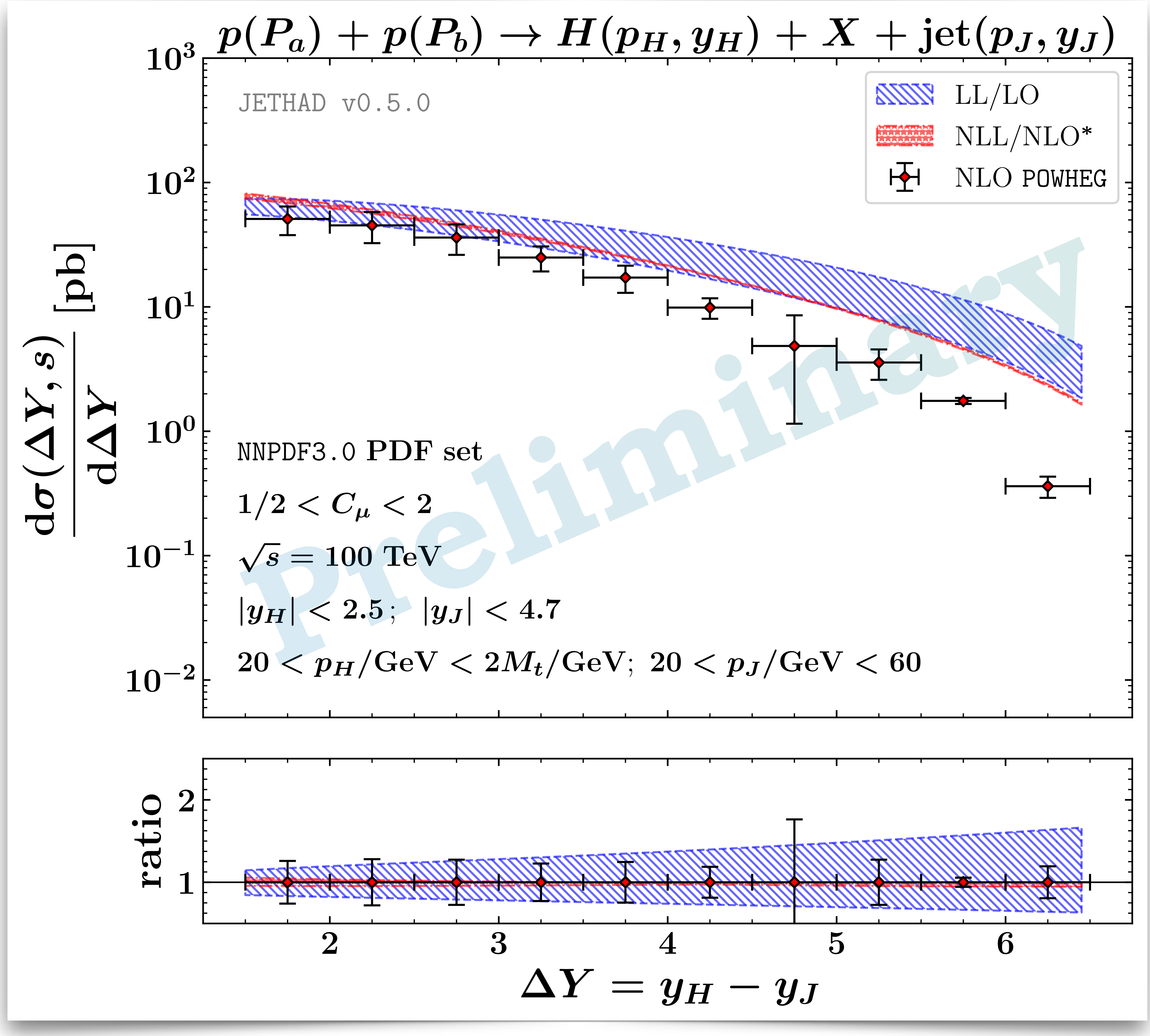


🔗 [F. G. C., D. Yu. Ivanov, M. M. A. Mohammed, A. Papa, Eur. Phys. J. C 81 (2021) 4, 293]

Backup

Higgs + jet at @FCC: small-x enhancement from PDFs

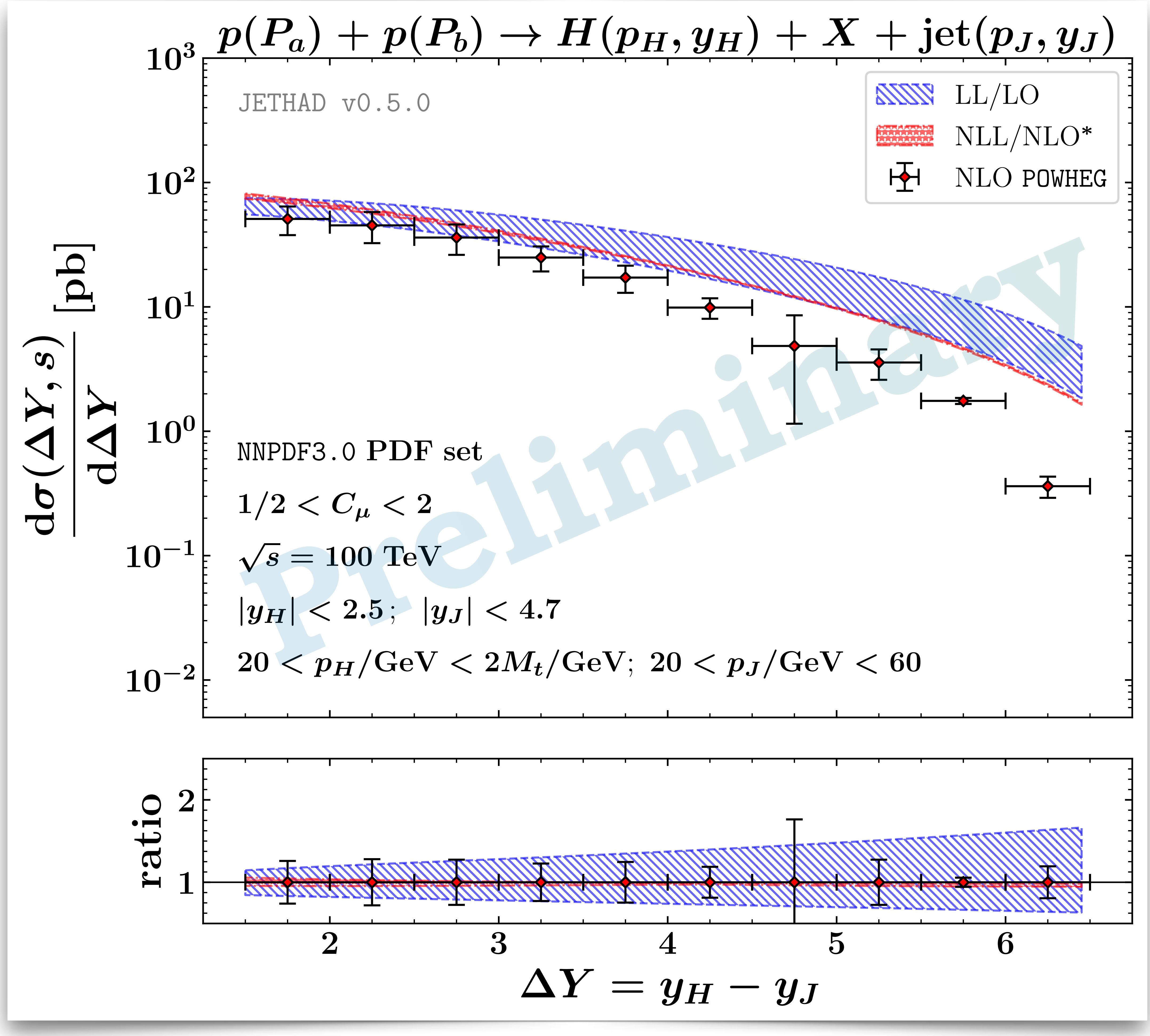
High-energy resummation + **NNPDF3.0** 🔗



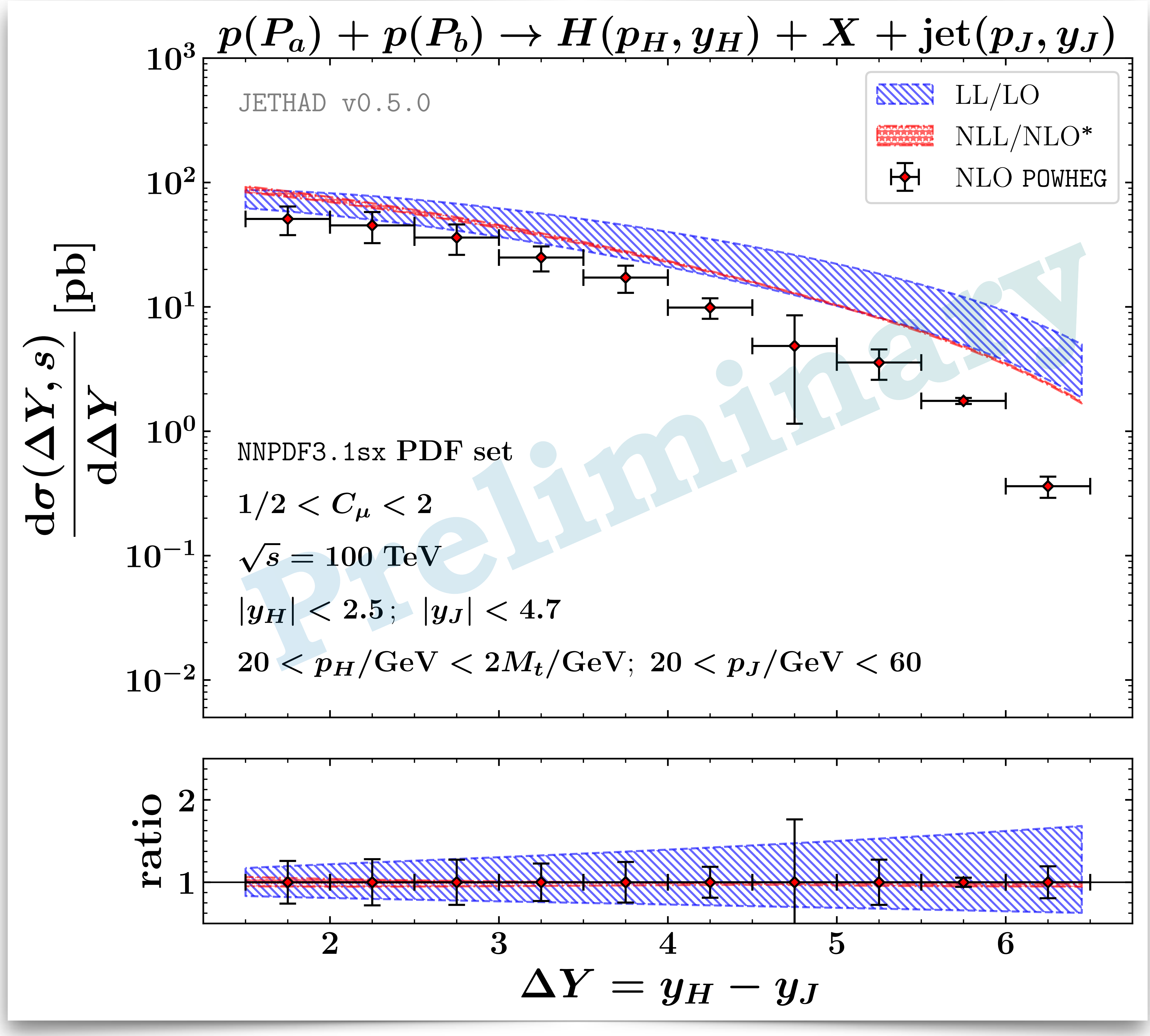
Backup

Higgs + jet at @FCC: small-x enhancement from PDFs

High-energy resummation + **NNPDF3.0**



High-energy resummation + **NNPDF3.1sx**

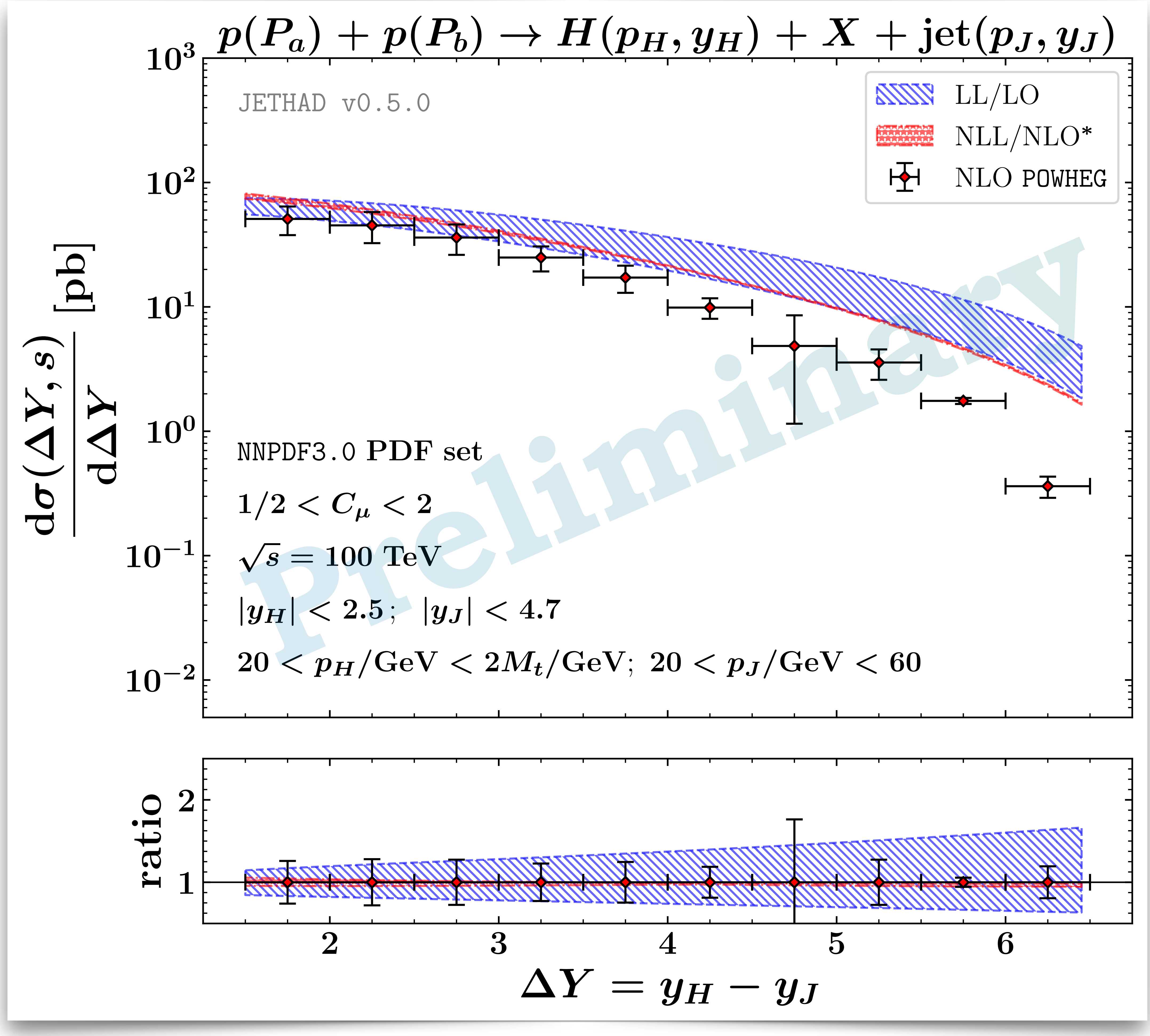


Backup

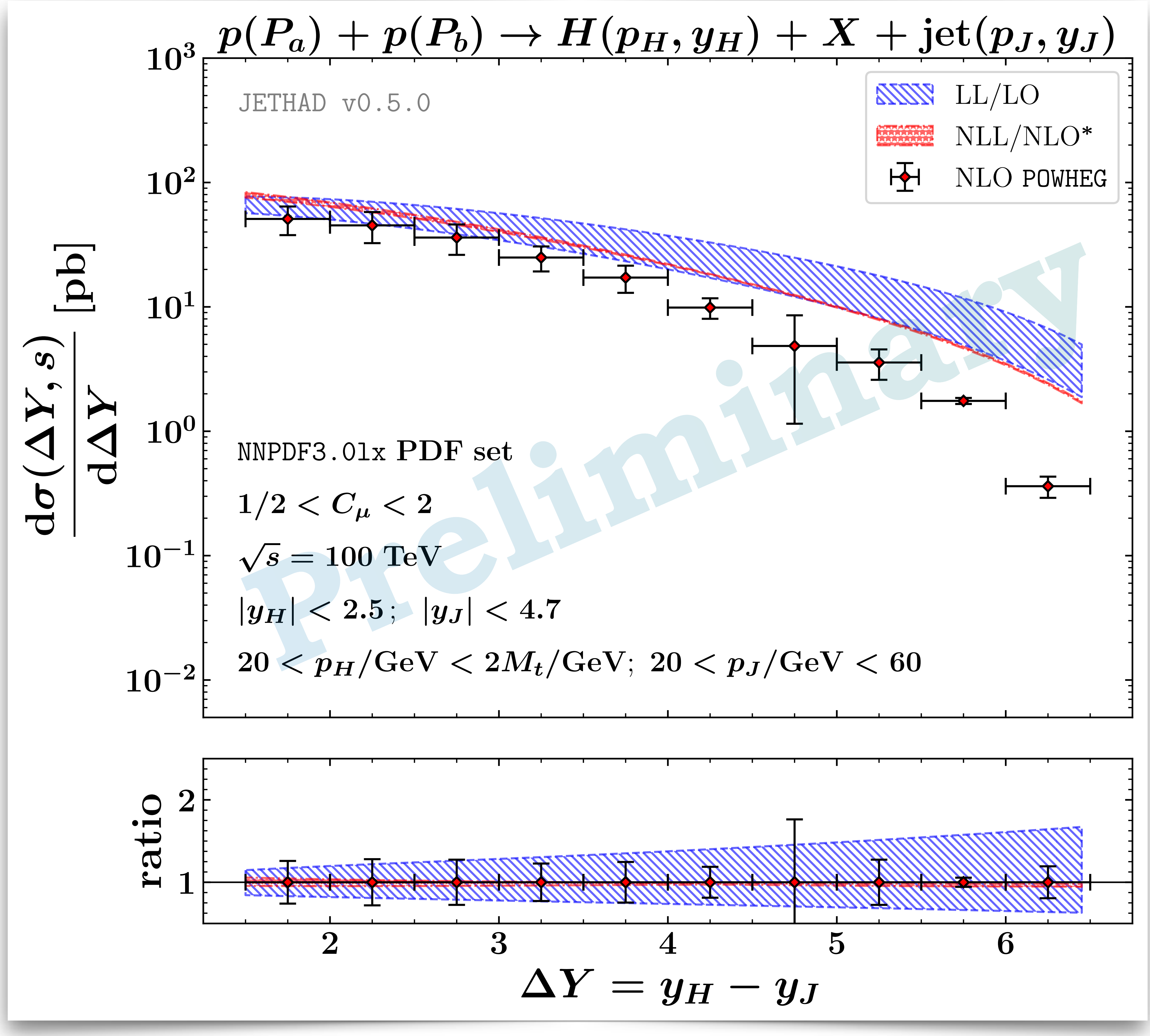
Small-x resummation on PDFs $\Rightarrow$ $+(13.5 \div 2.10)\,\%$ @NLL/NLO*

Higgs + jet at @FCC: large-x enhancement from PDFs

High-energy resummation + **NNPDF3.0**



High-energy resummation + **NNPDF3.0lx**



Backup

Threshold resummation on PDFs $\Rightarrow$ $+(10.7 \div 2.15)\,\%$ @NLL/NLO*

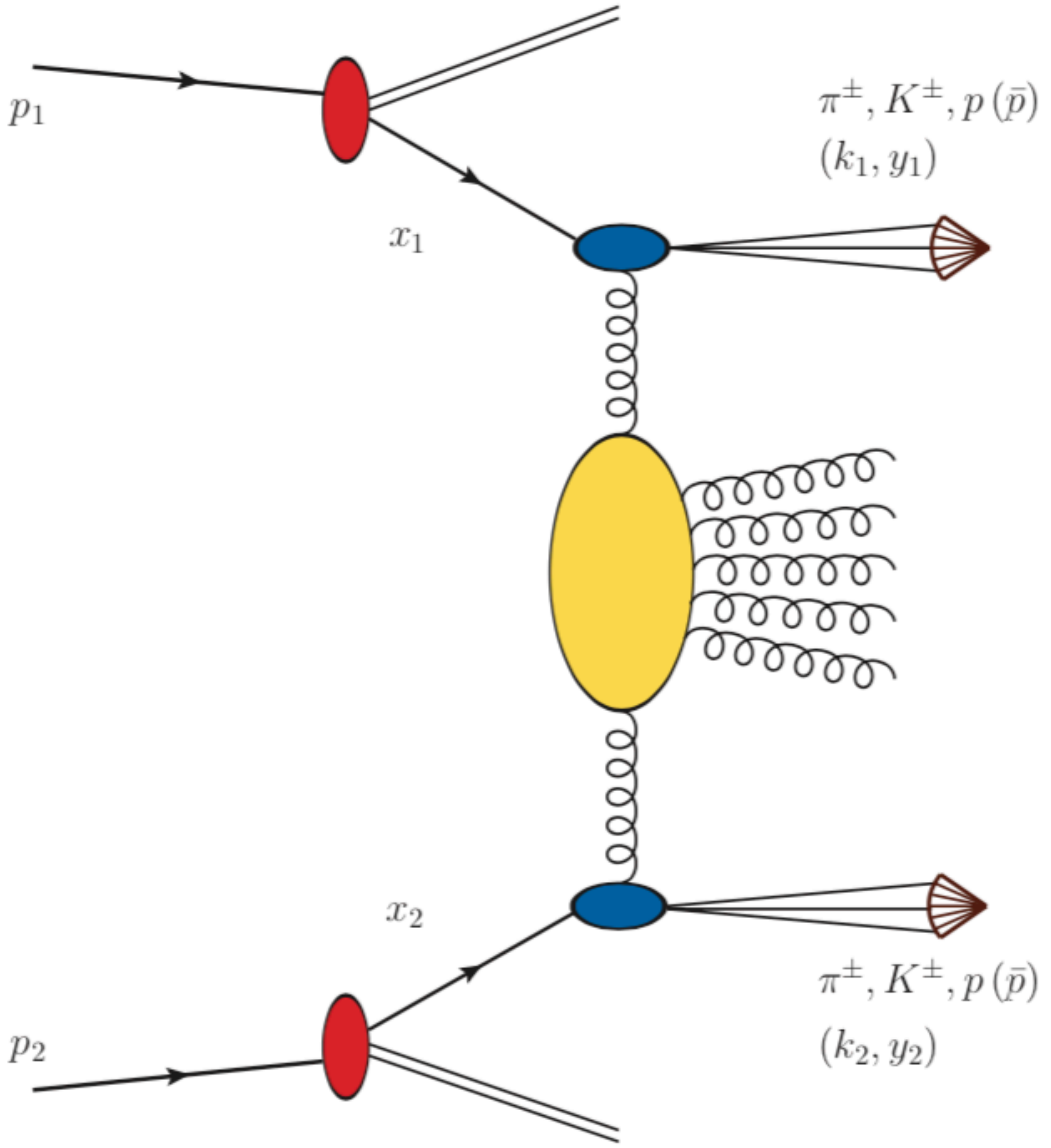
HEAVY-LIGHT HADRONS

From Higgs + jet to bound states

Di-hadron and hadron-jet correlations

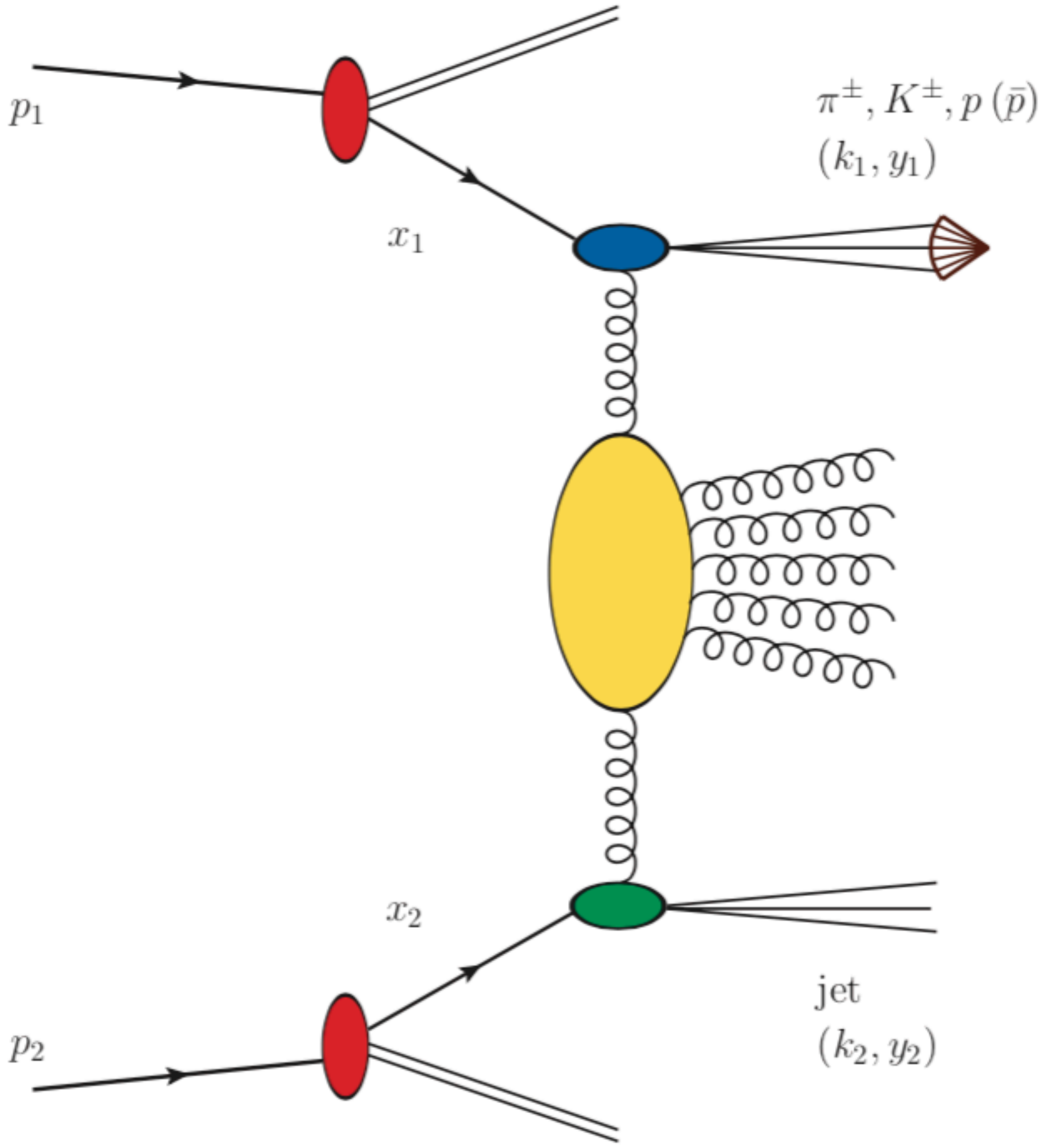
Inclusive di-hadron production

[D.Yu. Ivanov, A. Papa (2012)] (NLO forward-hadron impact factor)
[F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]



Inclusive hadron-jet production

[A.D. Bolognino, F.G.C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]
[F.G.C. (in preparation)]



- ◇ NLO impact factors known $\Rightarrow$ full NLA BFKL analysis feasible
- ◇ PDFs + FFs at work (both), hadrons at smaller rapidities than jets (di-hadron)
- ◇ genuine *asymmetric* cuts in transverse momenta (hadron-jet)

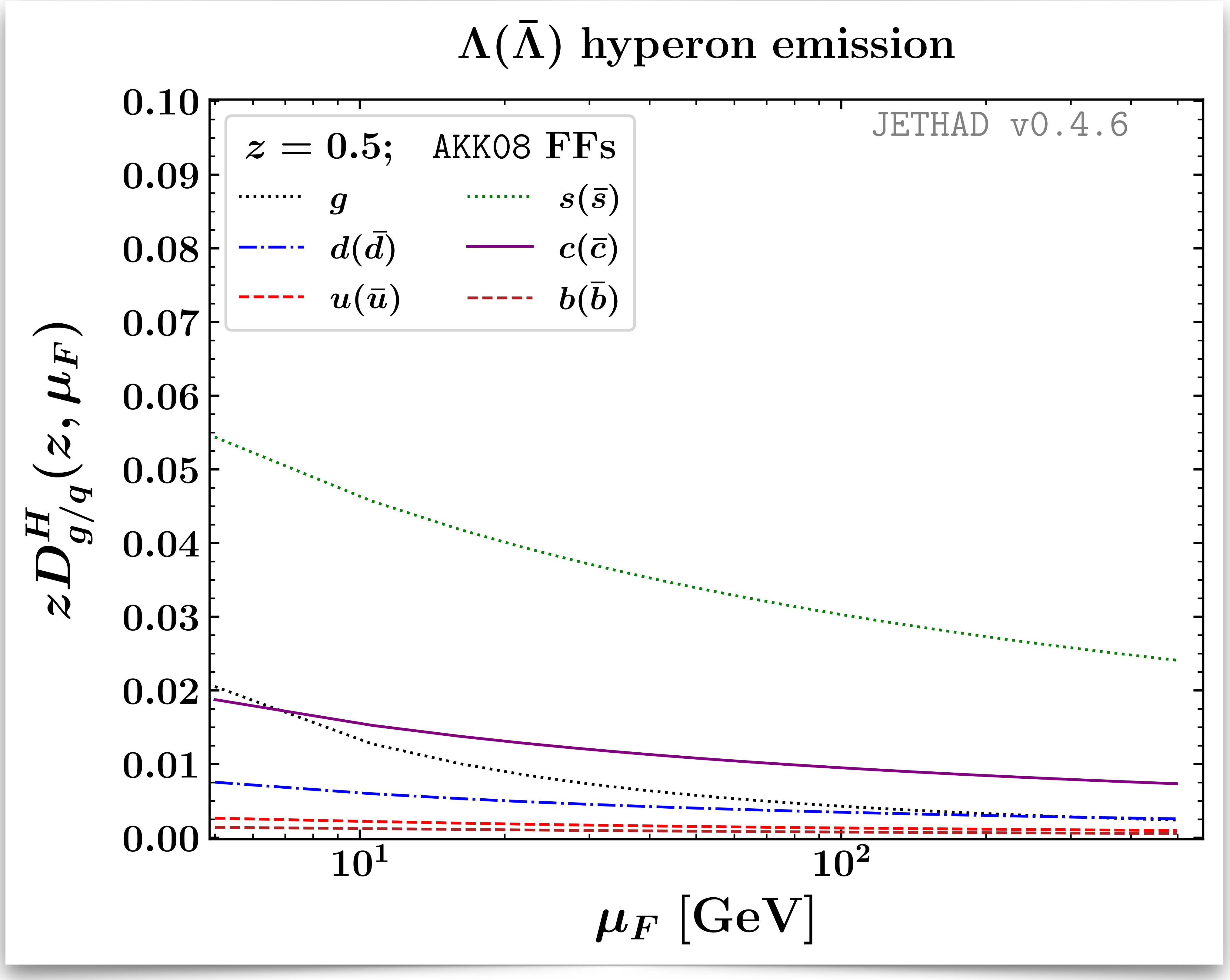
[F. G. C., Eur. Phys. J. C 81 (2021) 8, 691]

Backup

Stabilizing effects of heavy-flavor fragmentation

AKK08 VFNS collinear FFs for Λ hyperon: $|uds\rangle$

[S. Albino et al., Nucl. Phys. B 803 (2008) 42-104]



Backup

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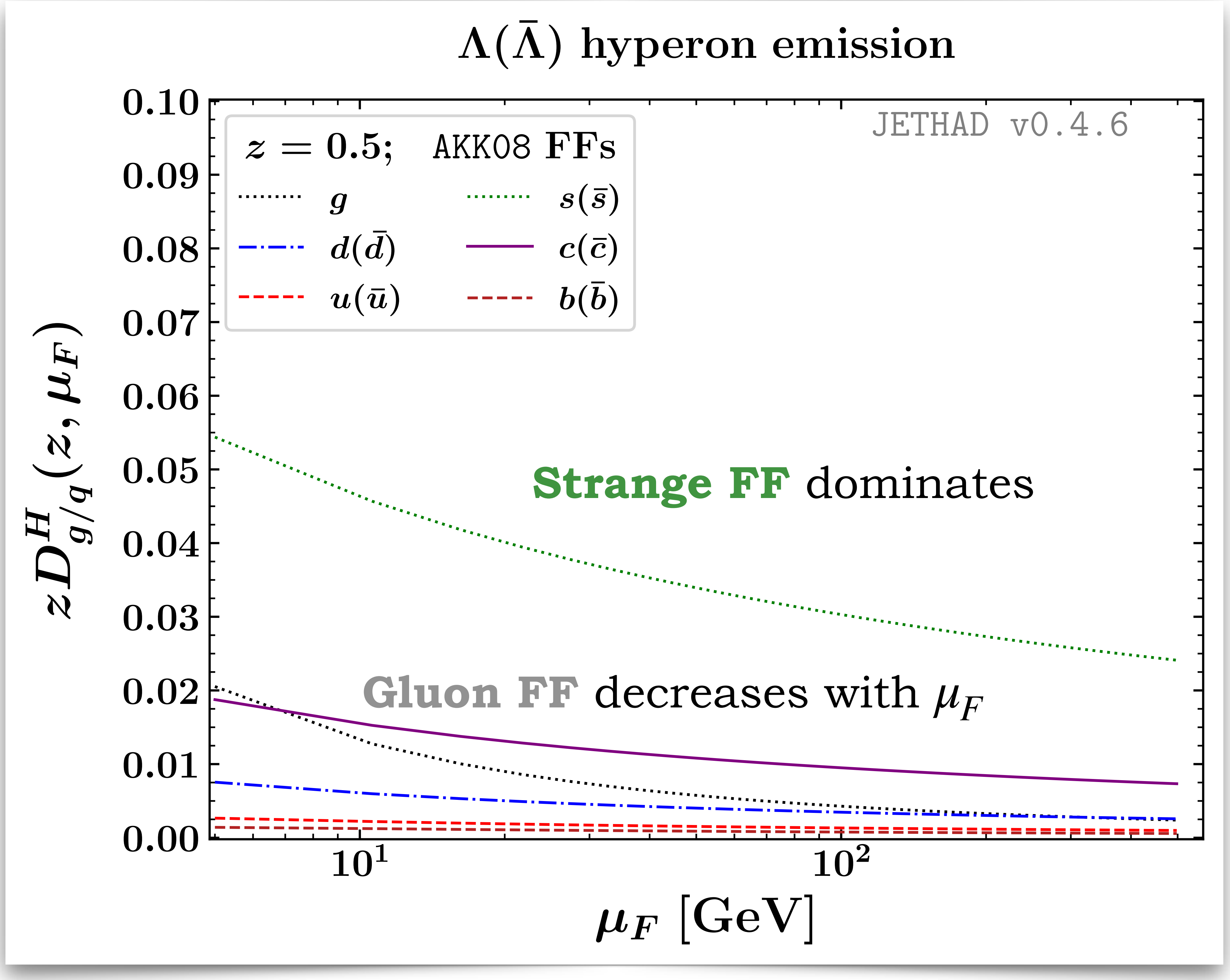

Backup

Stabilizing effects of heavy-flavor fragmentation

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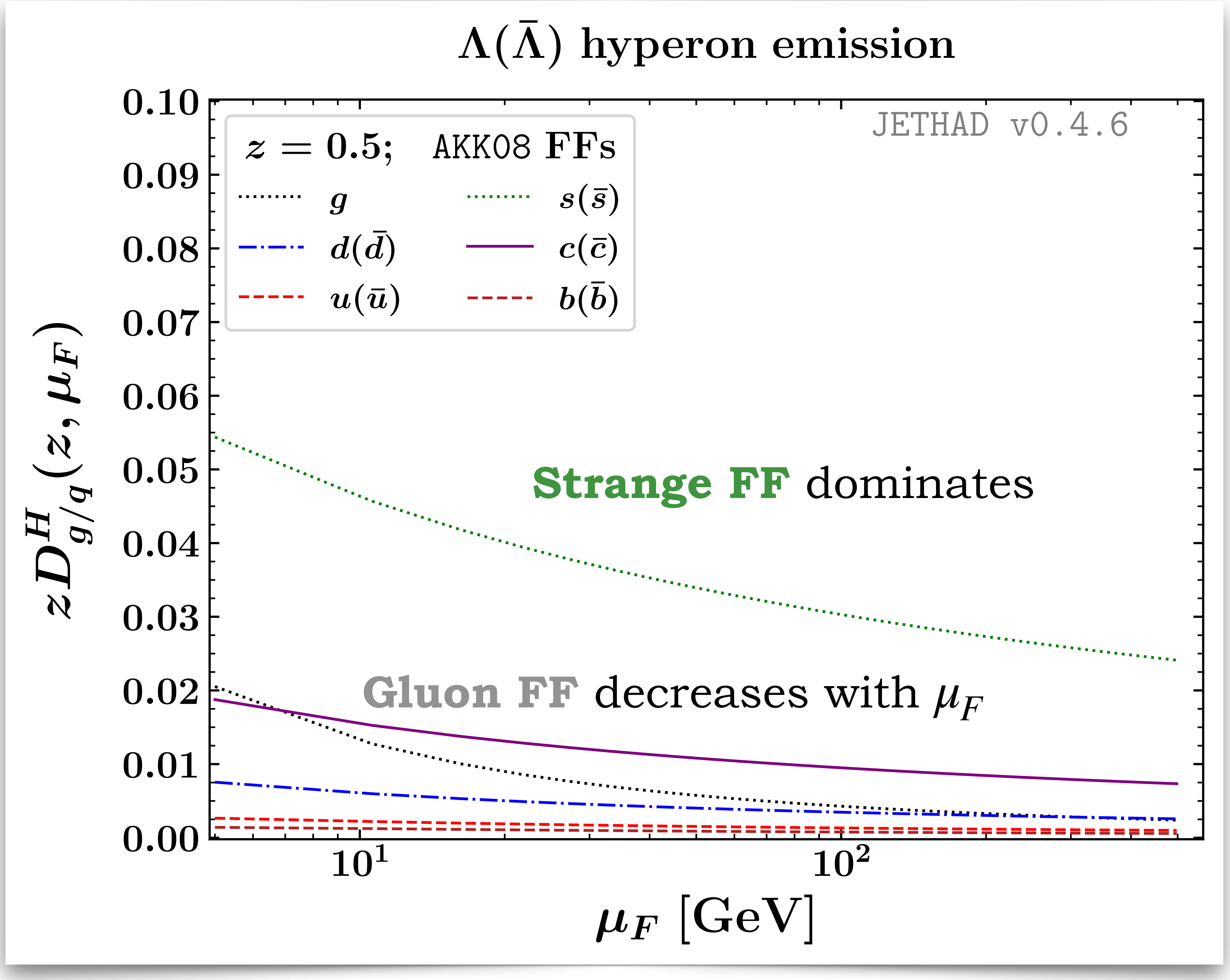
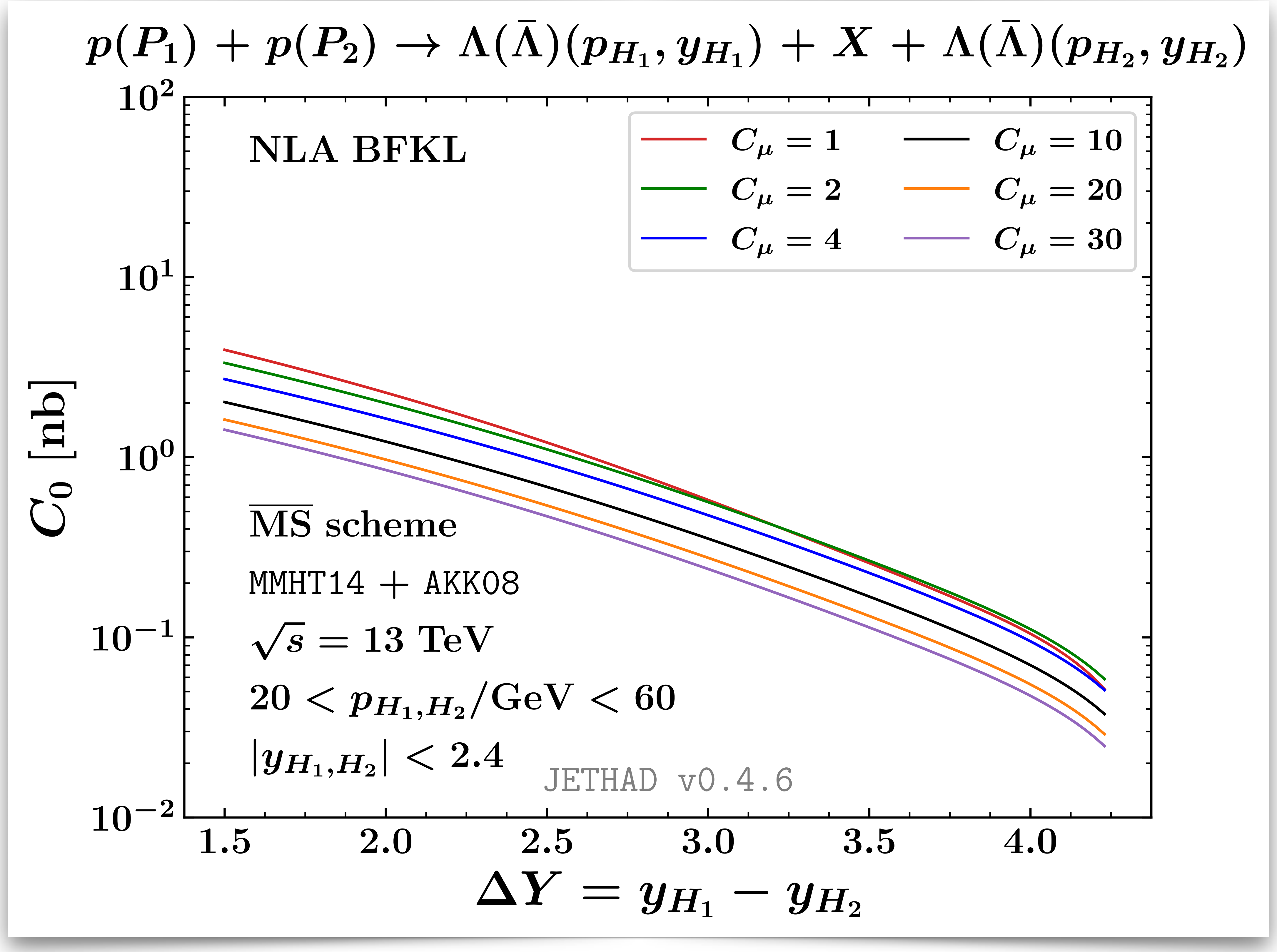




Rapidity distribution sensitive to scale variations

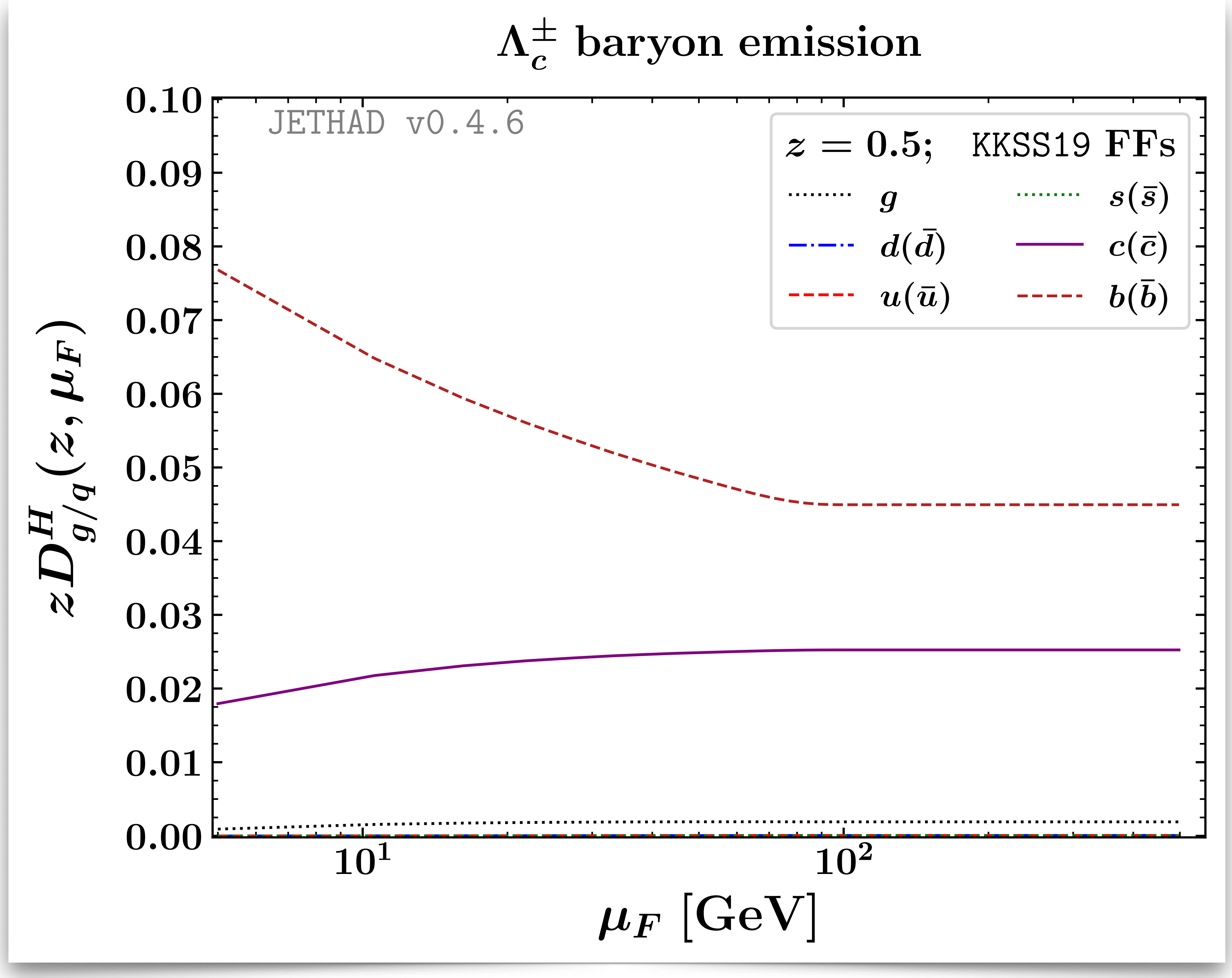
(Λ hyperons) [F. G. C. et al., Phys. Rev. D 102 (2020) 9, 094019]

(cascade Ξ baryons) [F. G. C., Eur. Phys. J. C (in press)]

Backup

Stabilizing effects of heavy-flavor fragmentation

KKSS19 VFNS collinear FFs for Λ_c baryons: $|udc\rangle$ [B. A. Kniehl et al., Phys. Rev. D 101 (2020) 11, 114021]



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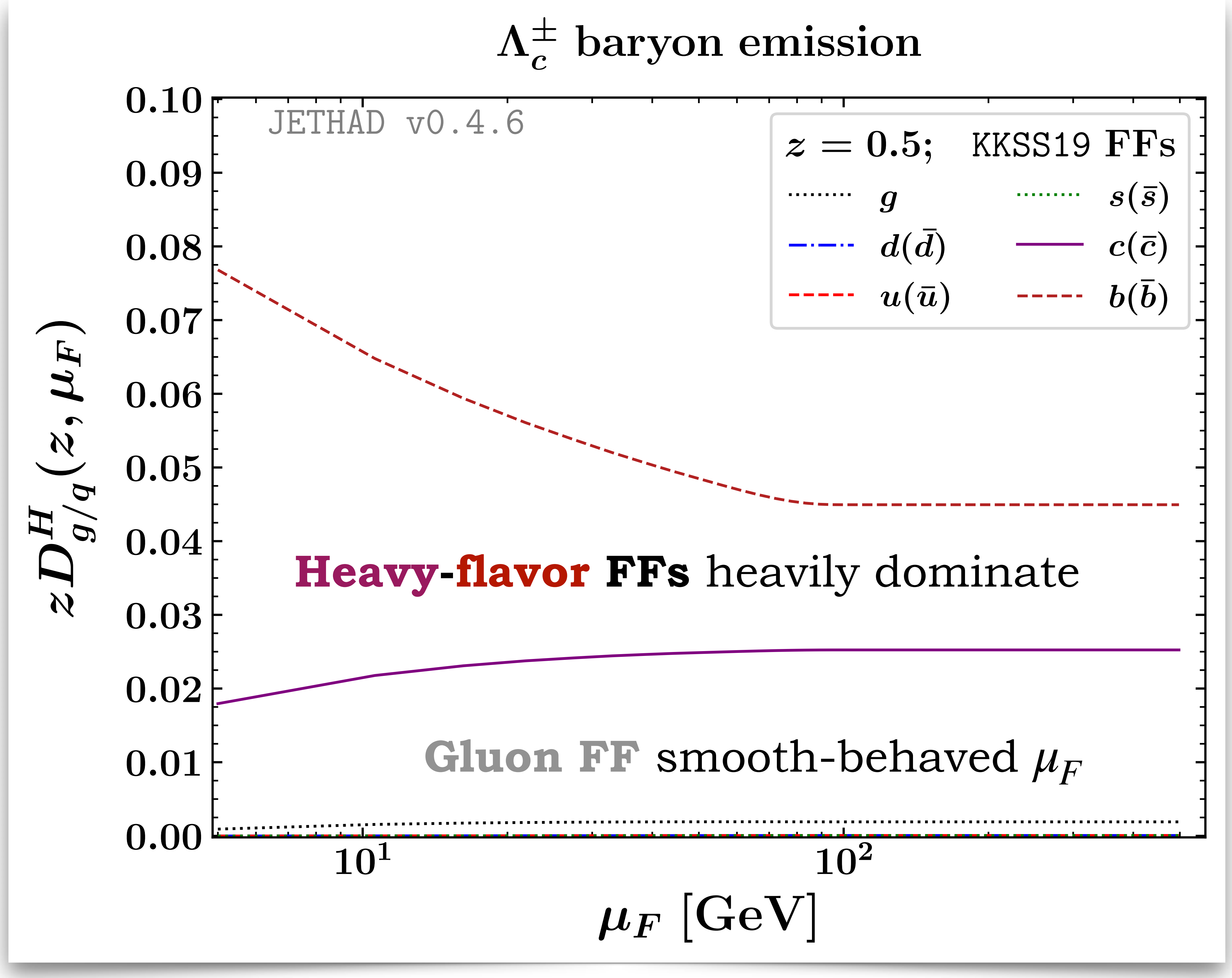

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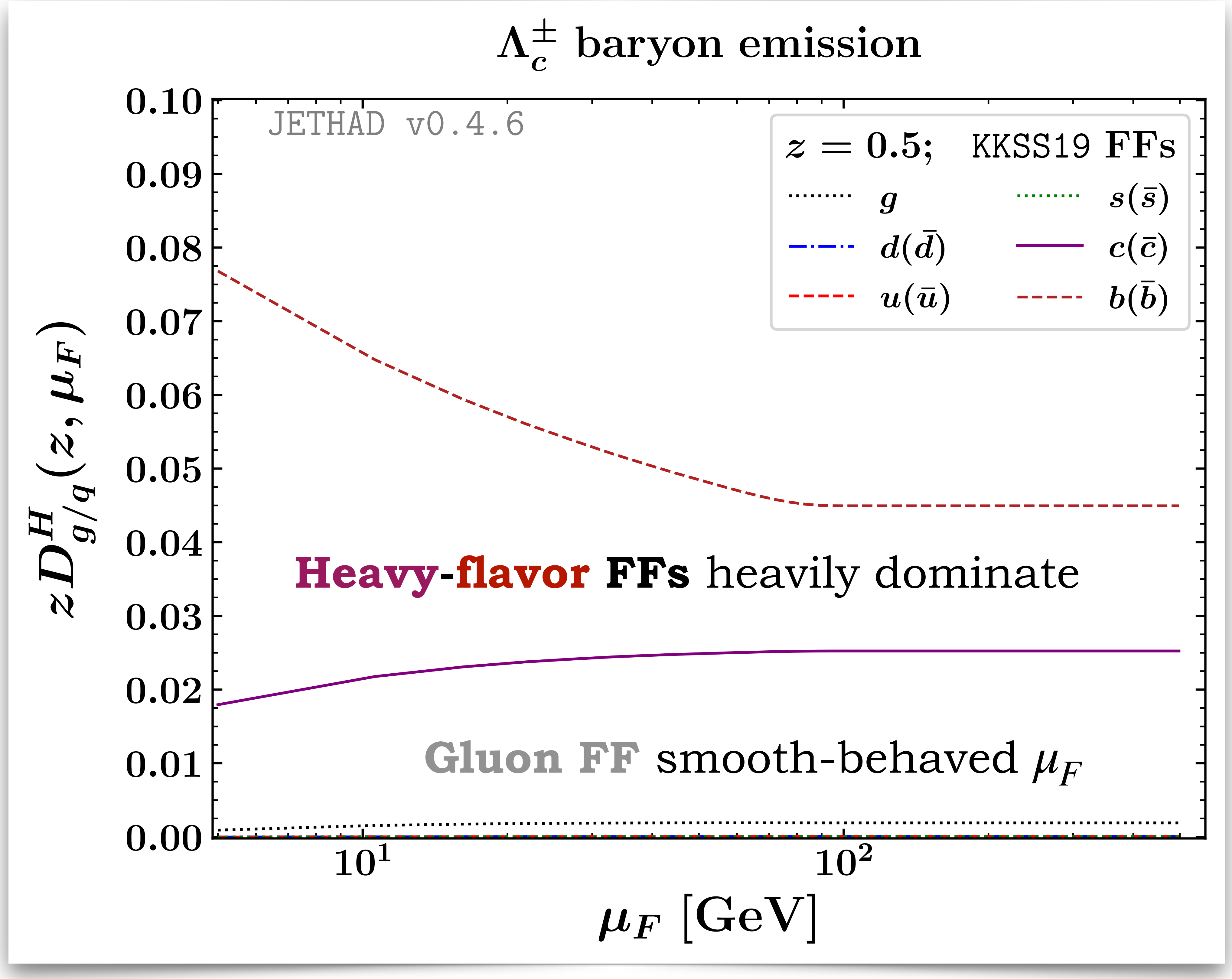
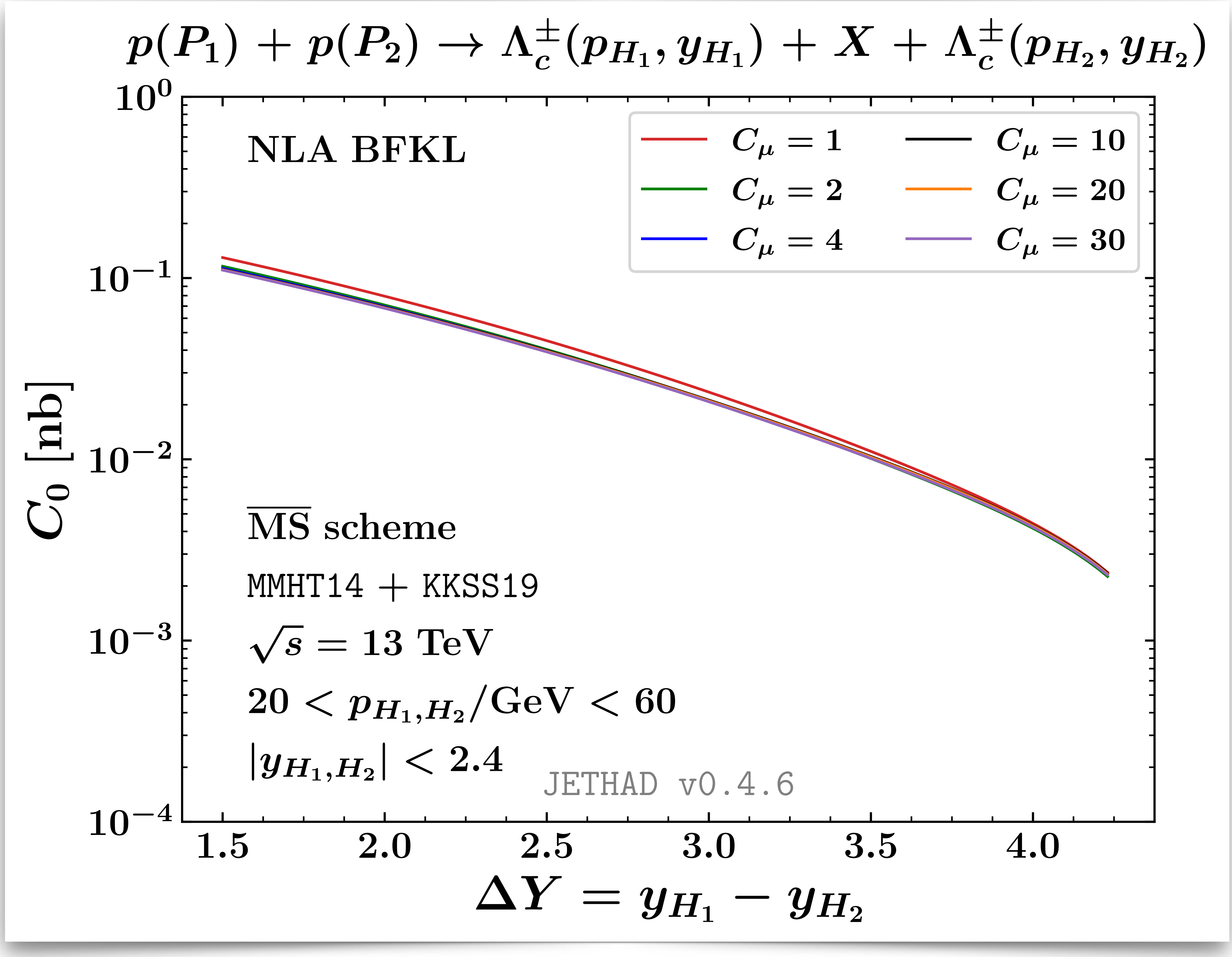




Rapidity distribution stable under scale variations

($B_c^{(*)}$ hadrons) [F. G. C., Phys. Lett. B 835 (2022) 137554]

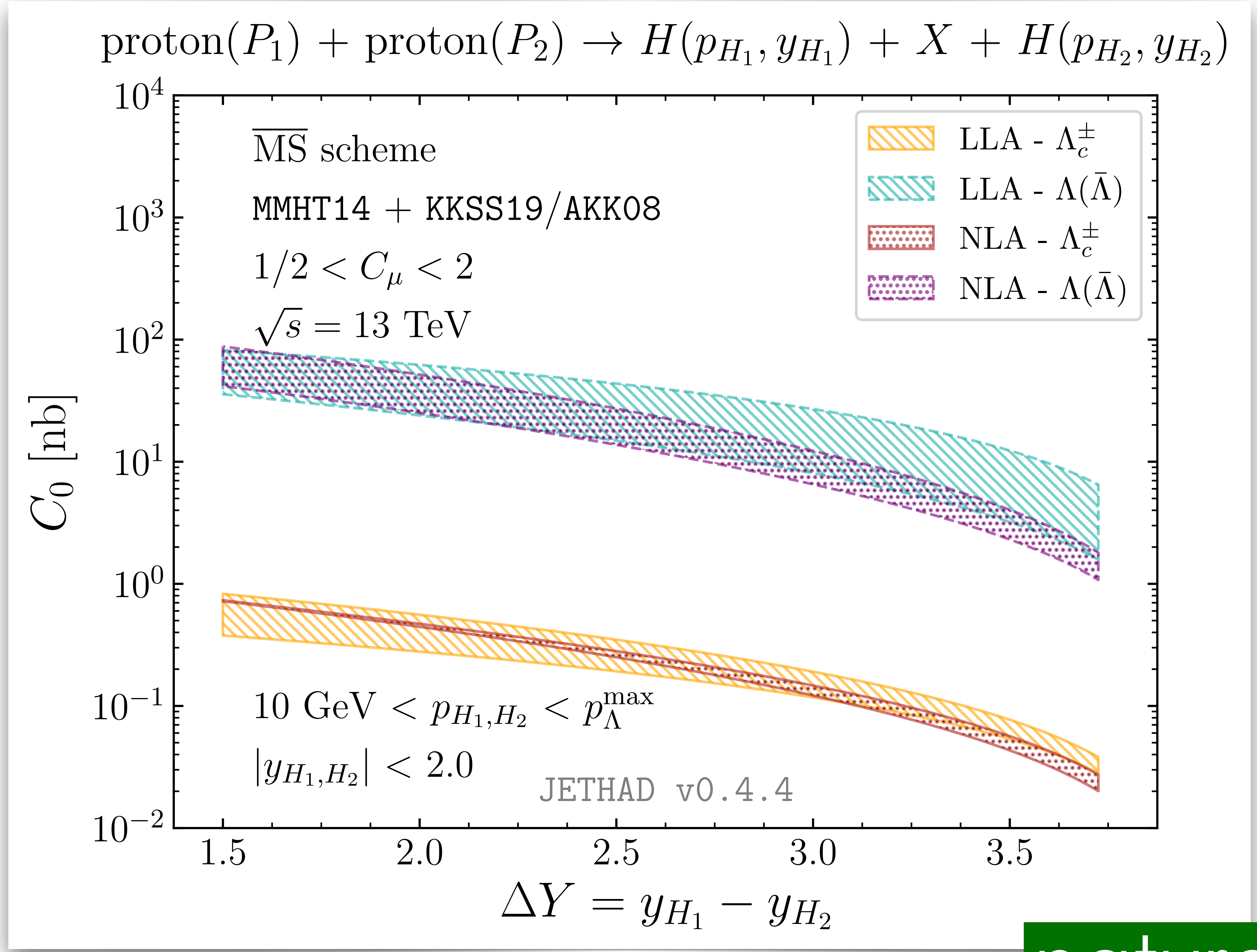
(Λ_c baryons, in this slide) [F. G. C. et al., Eur. Phys. J. C 81 (2021) 8, 780]

(H_b hadrons) [F. G. C. et al., Phys. Rev. D 104 (2021) 11, 114007]

Backup

Stability under scale variations & NLL corrections

Hybrid factorization @work: Λ_c baryons $|udc\rangle$ versus Λ hyperons $|uds\rangle$



natural

Backup

Stability under scale variations & NLL corrections

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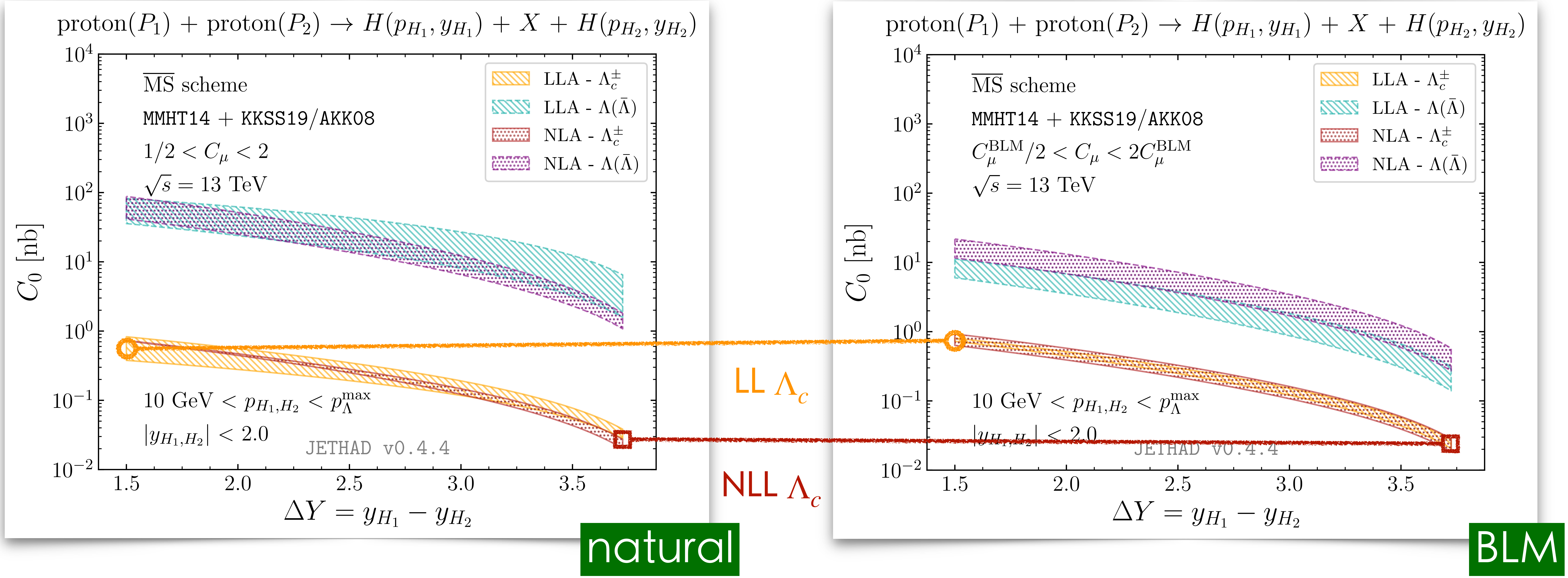

- NLL corrections: rapidity distribution stable for Λ_c

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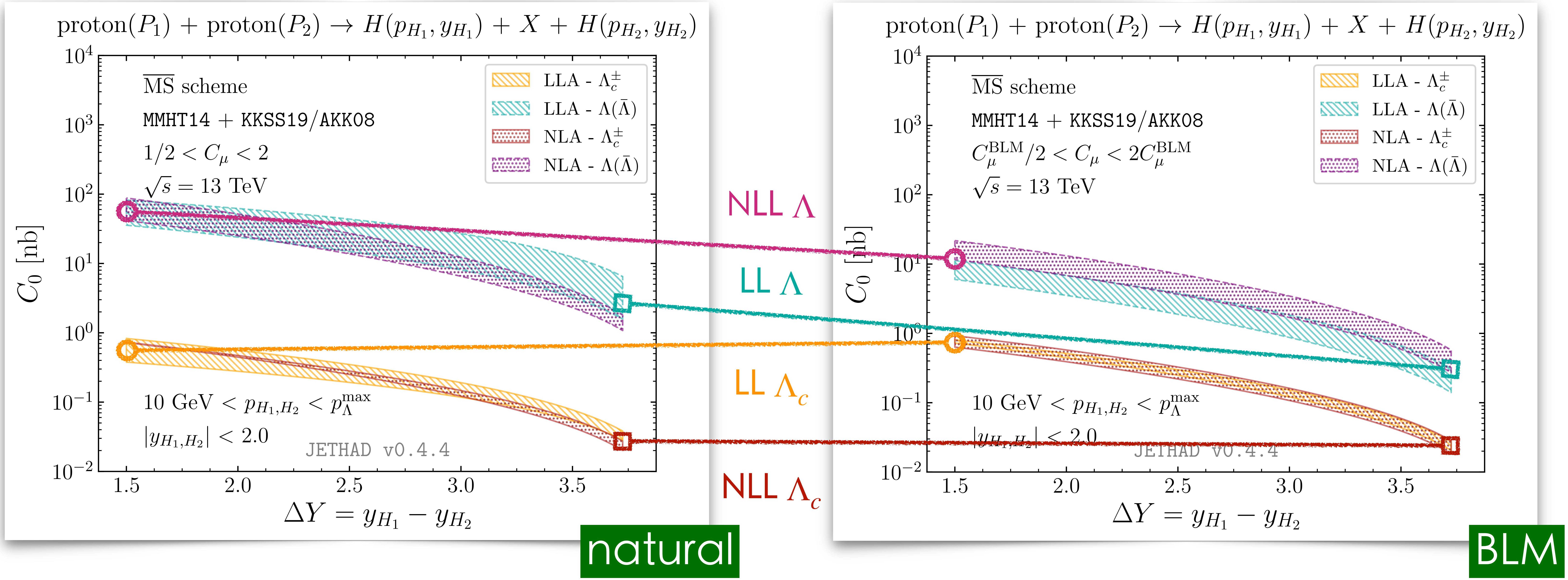

- NLL corrections: rapidity distribution stable for Λ_c, loses $\sim 10^1$ magnitude for Λ

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Backup

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KKSS07 VFNS collinear FFs for:

$$H_b = B^{\pm}, B^0, B_s^0, \Lambda_b$$

🔗 [B. A. Kniehl, H. Spiesberger, Phys. Rev. D 98 (2018) 11, 114010]

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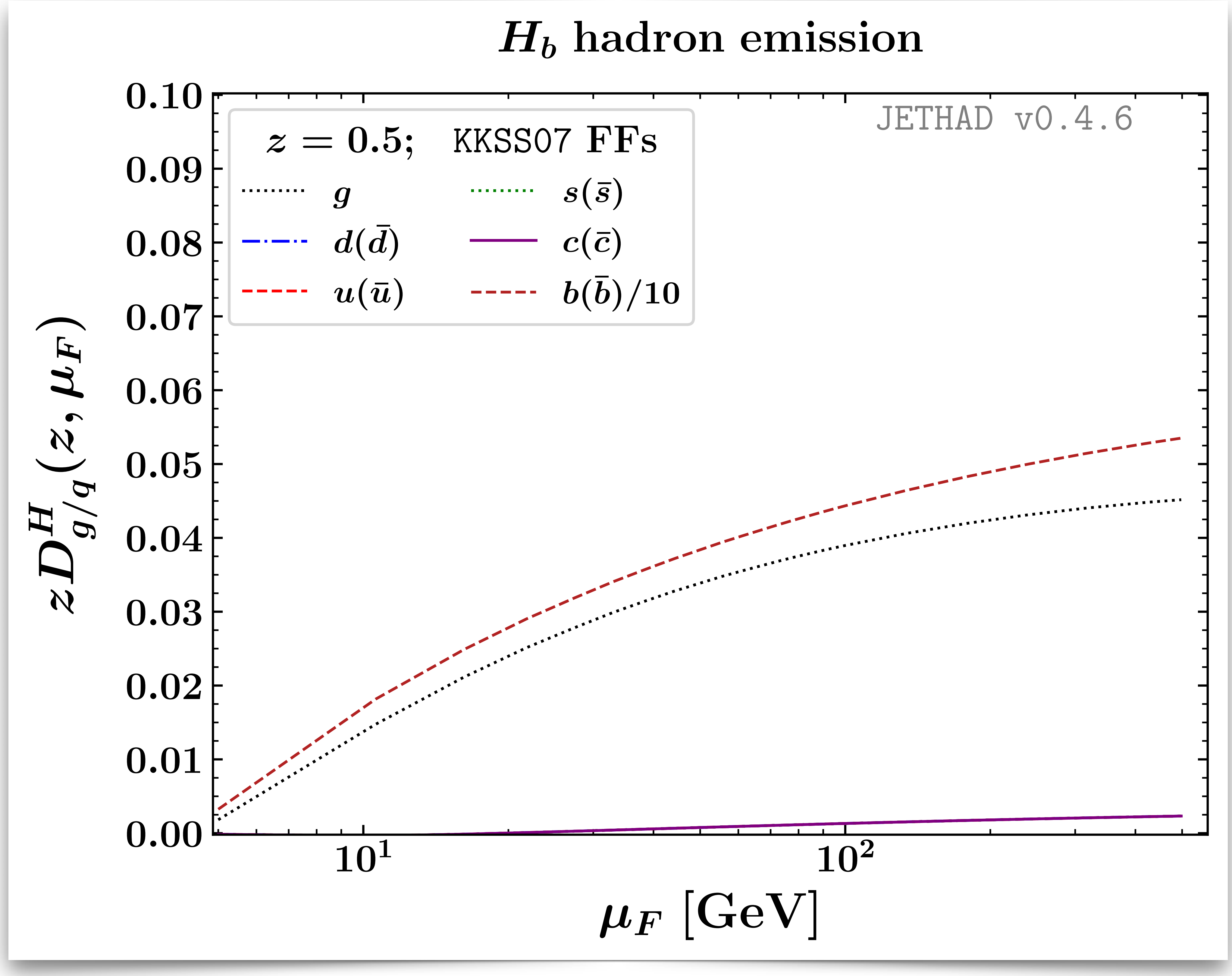

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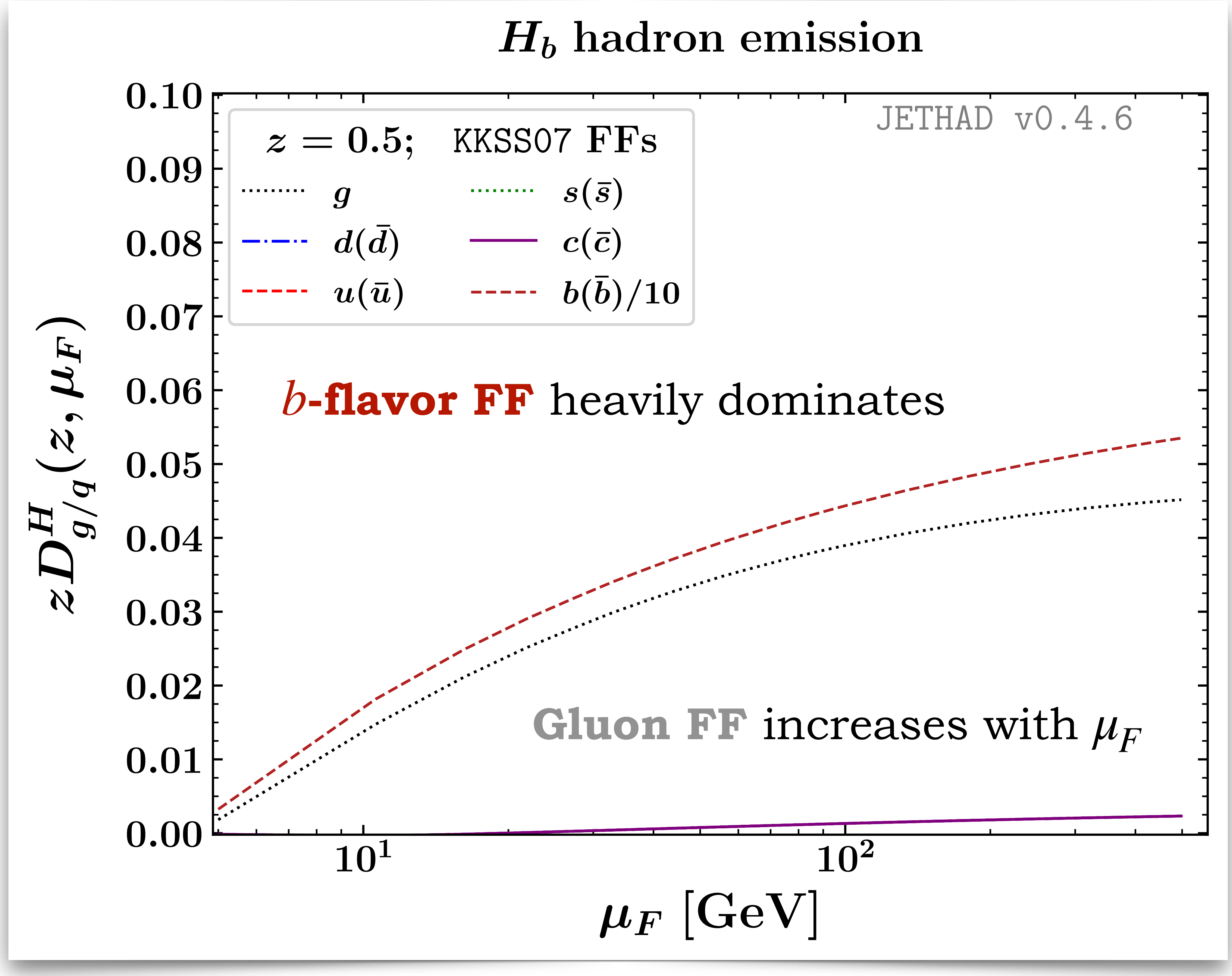

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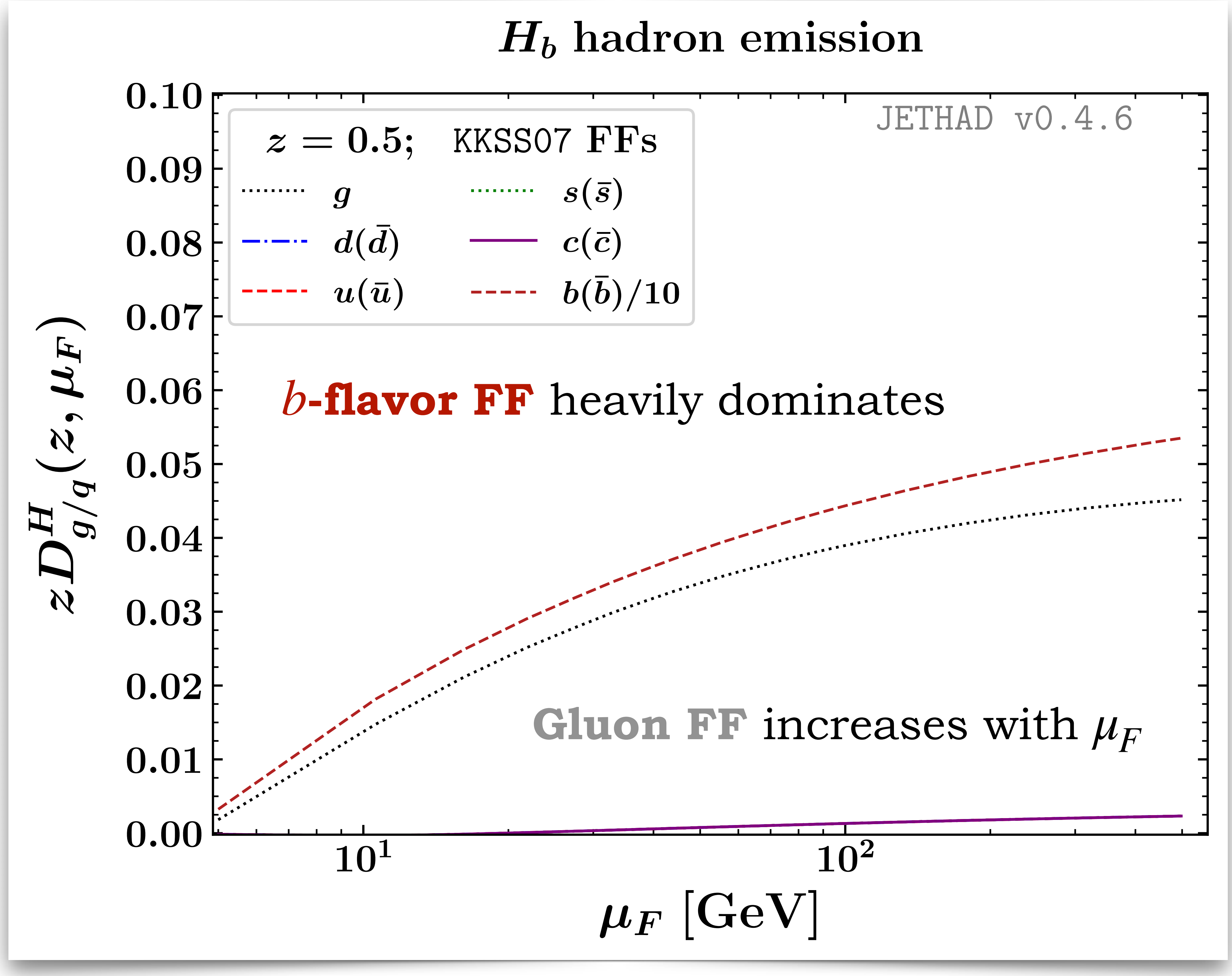

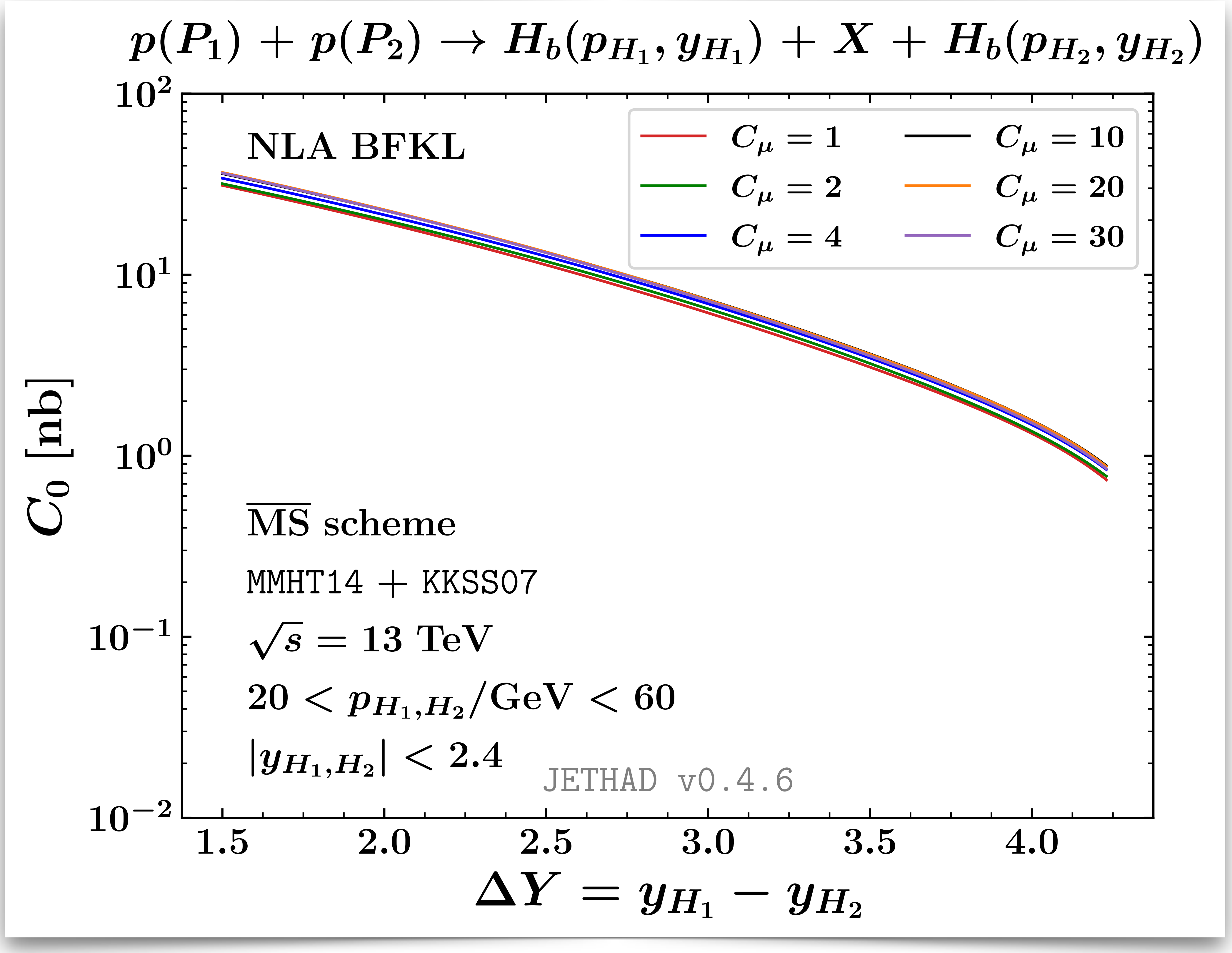

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Backup

Stabilizing effects of heavy-flavor fragmentation

- Stabilization mechanism encoded in the heavy-flavor gluon FF

- Forward-hadron LO impact factor $\Rightarrow$ gluon FF enhanced by gluon PDF in collinear convolution

$$c_\Lambda(n,\nu,|\vec{p}|,x) = 2\sqrt{\frac{C_F}{C_A}}(|\vec{p}|^2)^{i\nu-1/2}\int_x^1 \frac{\mathrm{d}z}{z}\left(\frac{z}{x}\right)^{2i\nu-1}\left[\frac{C_A}{C_F} f_g(z) D_g^\Lambda\left(\frac{x}{z}\right) + \sum_{a=q,\bar{q}} f_a(z) D_a^\Lambda\left(\frac{x}{z}\right)\right]$$

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- Forward-hadron NLO impact factor $\Rightarrow$ a non-diagonal heavy-flavor channel open...

$$c_1^{(1)}(n,\nu,|\vec{k}_1|,\alpha_1) = 2\sqrt{\frac{C_F}{C_A}}\left(\vec{k}_1^2\right)^{i\nu-\frac{1}{2}}\frac{1}{2\pi}\int_{\alpha_1}^1\frac{dx}{x}\int_{\frac{\alpha_1}{x}}^1\frac{d\zeta}{\zeta}\left(\frac{x\zeta}{\alpha_1}\right)^{2i\nu-1}$$
$$\times\left[\frac{C_A}{C_F}f_g(x)D_g^h\left(\frac{\alpha_1}{x\zeta}\right)C_{gg}(x,\zeta) + \sum_{a=q,\bar{q}} f_a(x) D_a^h\left(\frac{\alpha_1}{x\zeta}\right)C_{qq}(x,\zeta)\right.$$
$$\left. + D_g^h\left(\frac{\alpha_1}{x\zeta}\right)\sum_{a=q,\bar{q}} f_a(x) C_{qg}(x,\zeta) + \frac{C_A}{C_F} f_g(x) \sum_{a=q,\bar{q}} D_a^h\left(\frac{\alpha_1}{x\zeta}\right) C_{gq}(x,\zeta)\right]$$

...but $|C_{gg}| \sim 50 \div 10^4 |C_{gq}|$

Backup

(NLO impact factor) 🔗 [D. Yu. Ivanov, A. Papa JHEP 07 (2012) 045]

Stabilizing effects of heavy-flavor fragmentation

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- Gluon FF rises with energy ⇒ this compensates PDF and BFKL kernel decreasing behavior

Backup

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VECTOR QUARKONIA

Is the natural stability robust?

(1) **KKSS07** and **KKSS19** VFNS collinear FFs share the same extraction technology

¿ *Might natural stability be related to the given FF determination(s)* ?

Backup

Is the natural stability robust?

(1) **KKSS07** and **KKSS19** VFNS collinear FFs share the same extraction technology

¿ *Might natural stability be related to the given FF determination(s)* ?

(2) **KKSS07** and **KKSS19** VFNS collinear FFs assume no initial-scale gluon, but evolution-driven

¿ *Might natural stability be artificially generated by this Ansatz* ?

Backup

Vector quarkonium from single-parton fragmentation

(1) ¡ *Let us consider J/ψ and Υ at large p_T $\rightarrow$ single-parton **fragmentation** from **NRQCD** !*

Backup

Vector quarkonium from single-parton fragmentation

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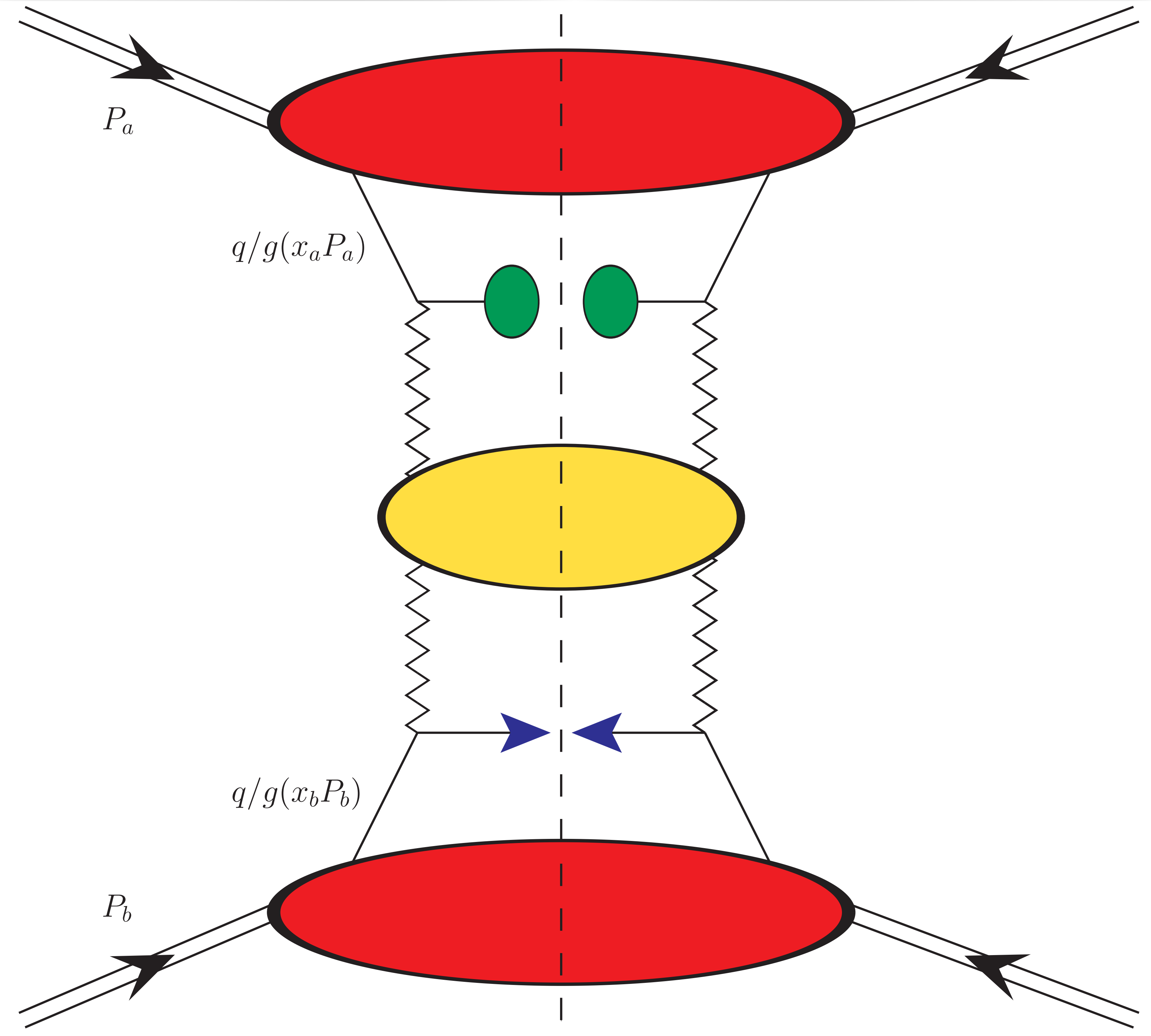

2.1 High-energy resummed cross section

The process under investigation is

$$p(P_a) + p(P_b) \to \mathcal{Q}(p_{\mathcal{Q}}, y_{\mathcal{Q}}) + X + \text{jet}(p_J, y_J) \,, \tag{1}$$

where $p(P_{a,b})$ stands for an initial proton with momentum $P_{a,b}$, $\mathcal{Q}(p_{\mathcal{Q}}, y_{\mathcal{Q}})$ is a J/ψ or a Υ emitted with momentum $p_{\mathcal{Q}}$ and rapidity $y_{\mathcal{Q}}$, the light jet is tagged with momentum p_J and rapidity y_J, and X denotes all the undetected products of the reaction. High observed transverse momenta, $|\vec{p}_{\mathcal{Q},J}|$, together with a large rapidity separation, $\Delta Y = y_{\mathcal{Q}} - y_J$, are required conditions to get a diffractive semi-hard configuration in the final state. Furthermore the transverse-momentum ranges need to be enough large to ensure the validity of description of the quarkonium production mechanism in terms of single-parton VFNS collinear fragmentation.

🔗 [F. G. C. et al., Eur. Phys. J. C 82 (2022) 10, 929]

Backup

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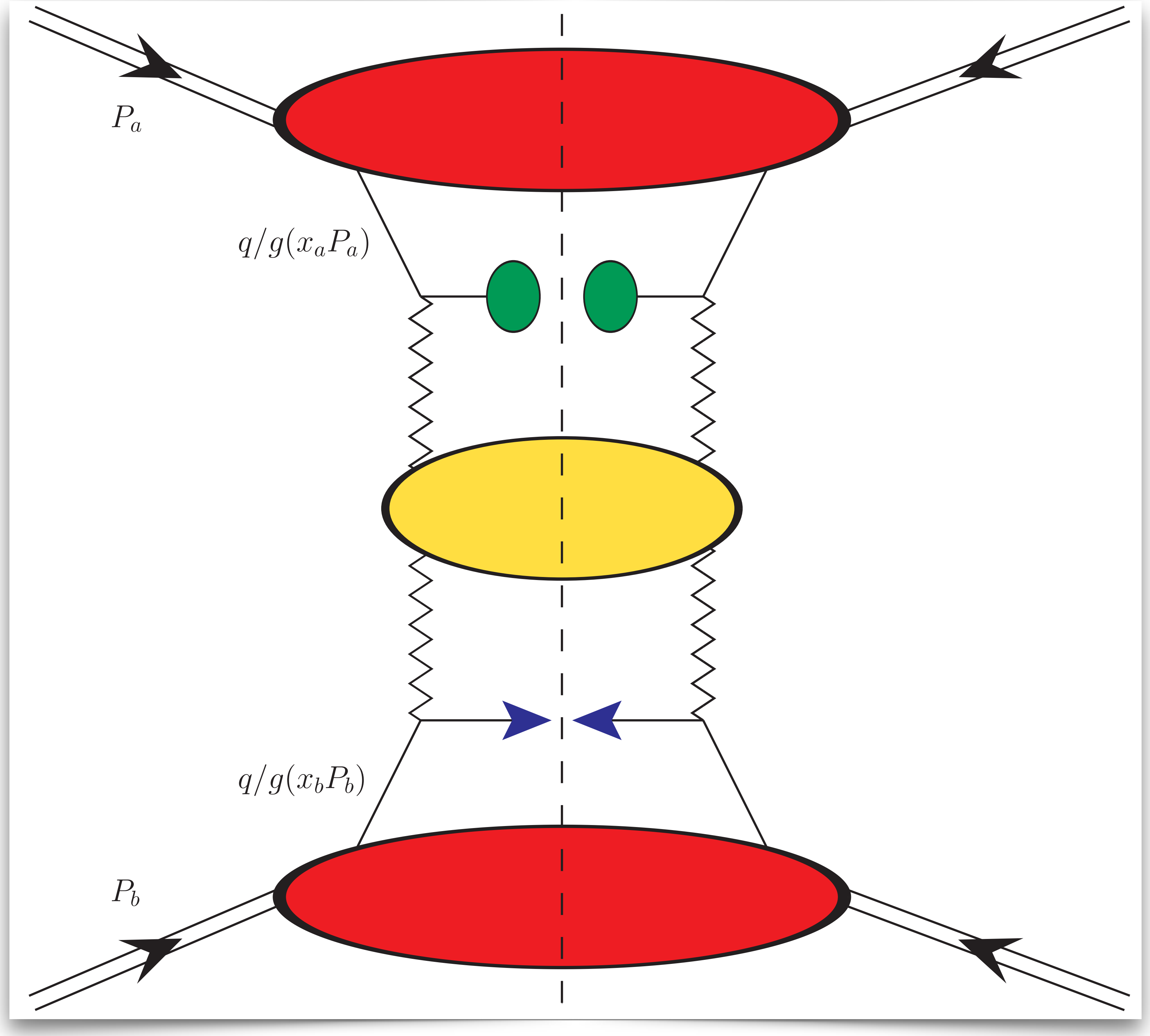

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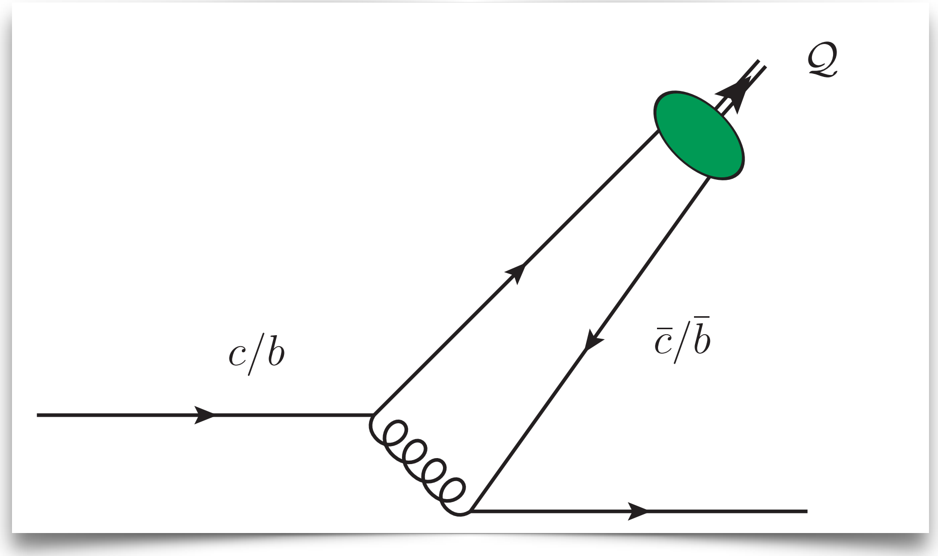

(LO) 🔗 [E. Braaten et al., Phys. Rev. D 48 (1993) 4230-4235]
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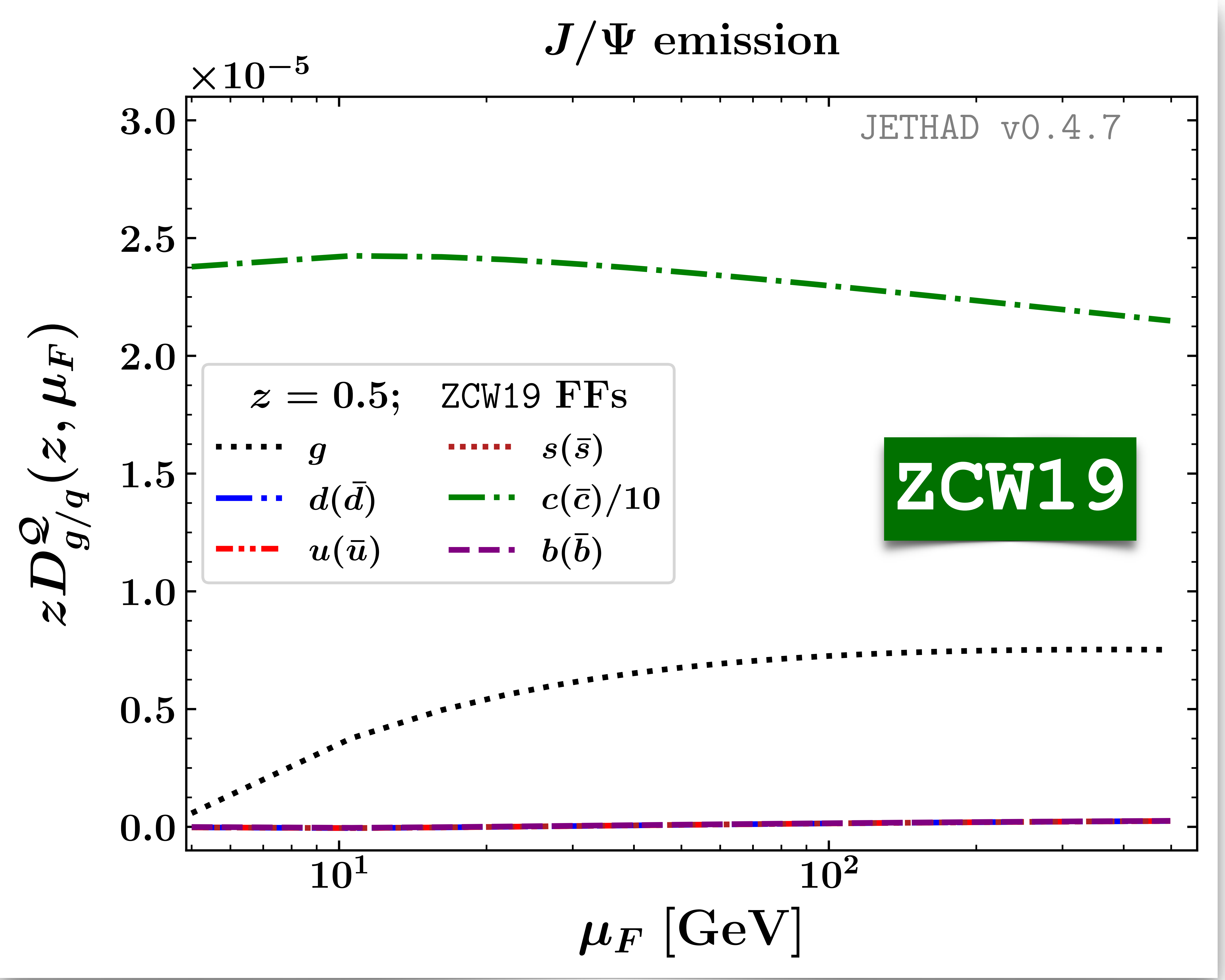
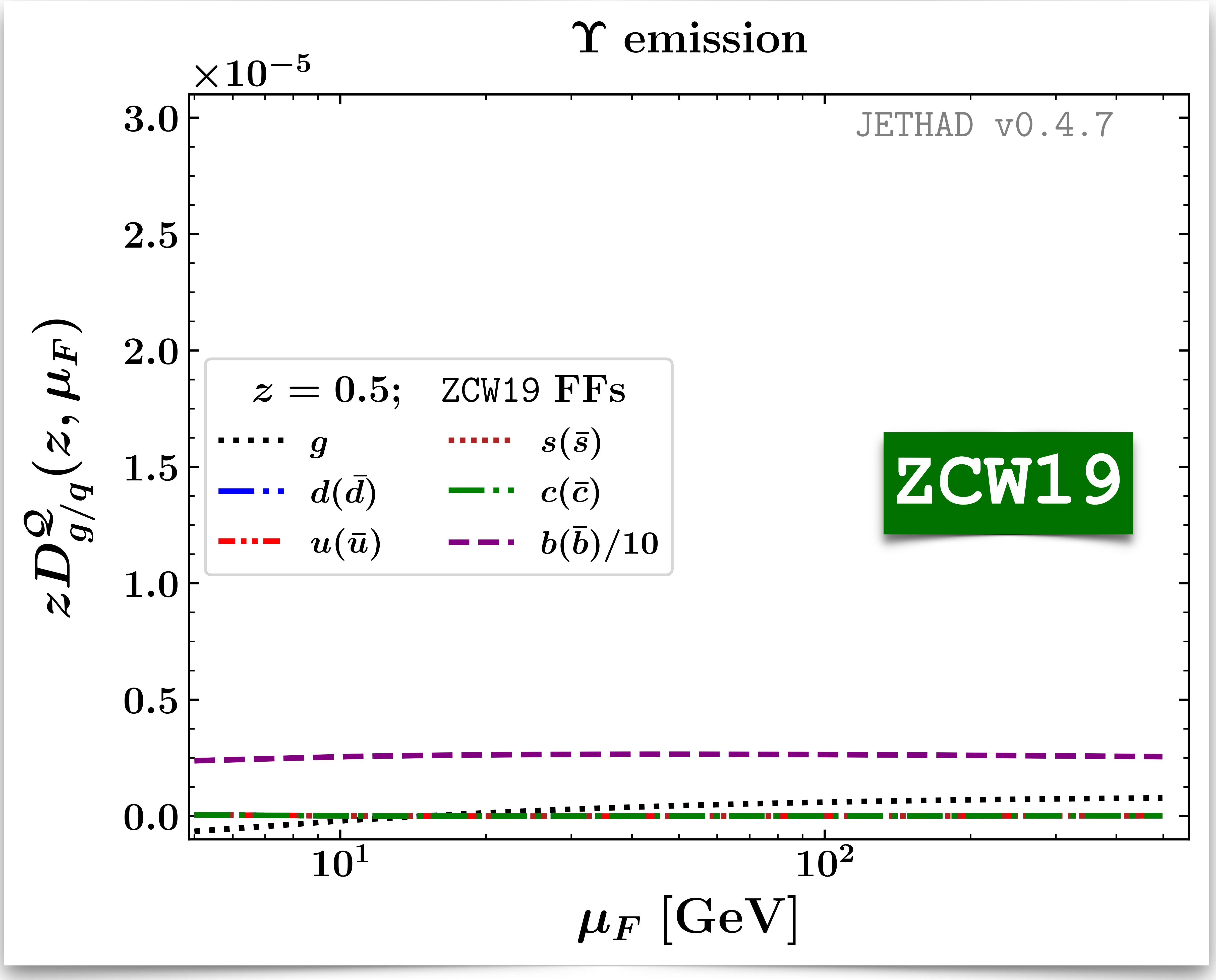
Backup

Vector quarkonium + jet at the LHC

J/ψ collinear FFs



Υ collinear FFs



Backup

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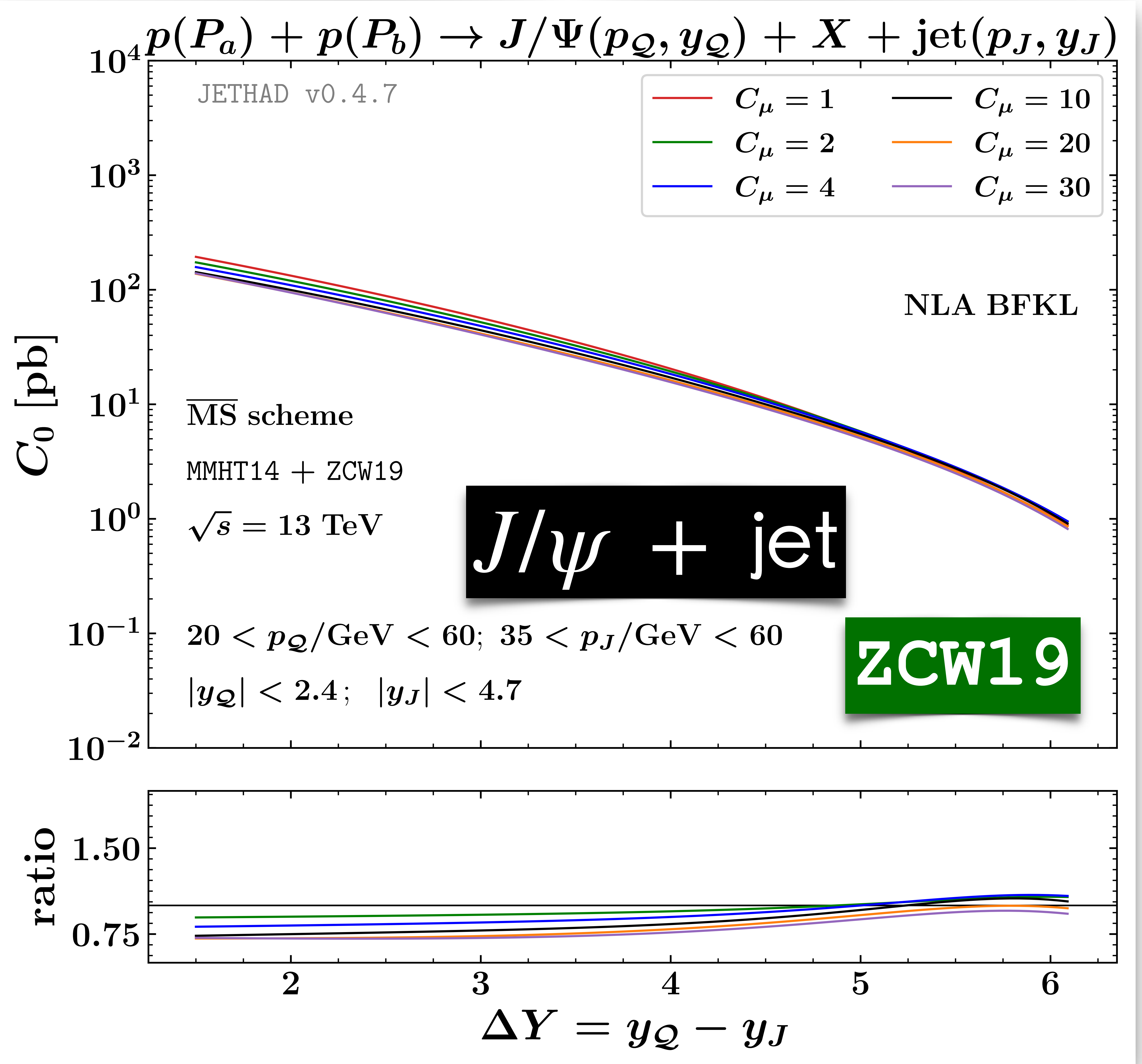

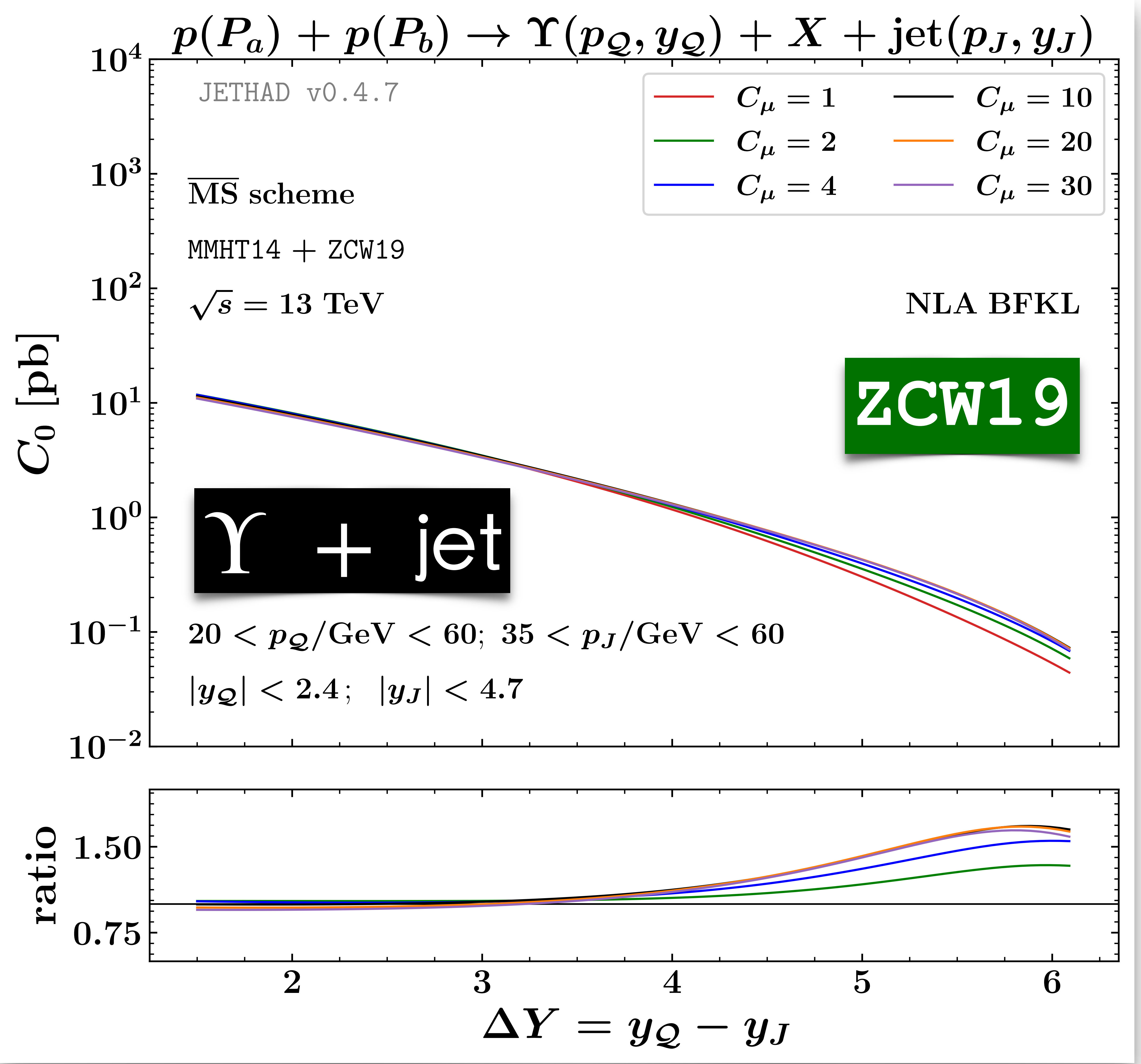

Backup

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Vector quarkonium from single-parton fragmentation

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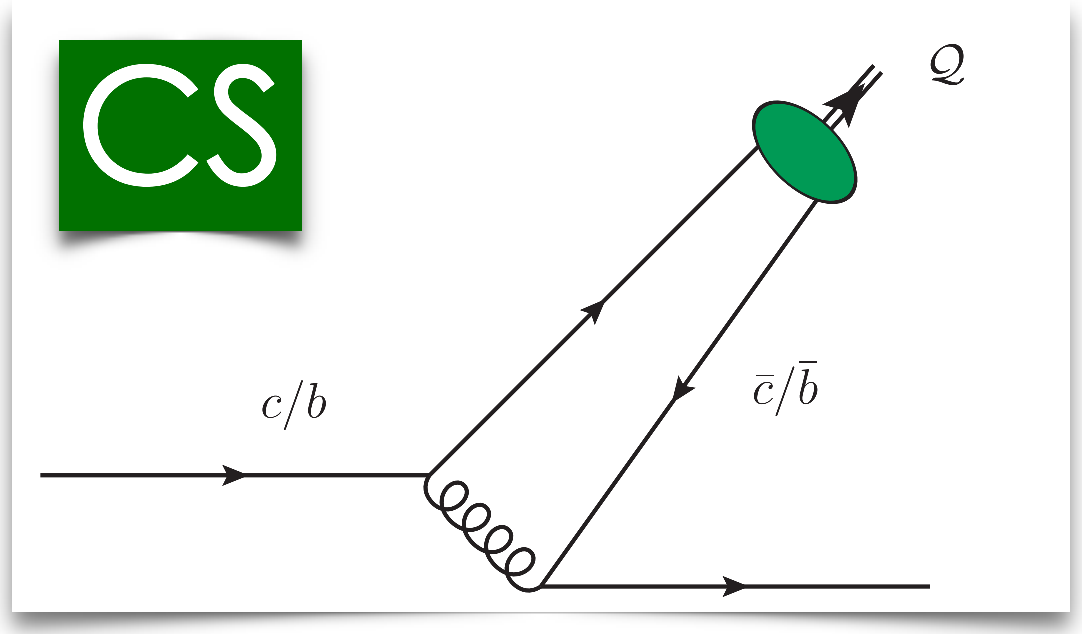

$$D_Q^{\mathcal{Q}}(z, \mu_F \equiv \mu_0) = D_Q^{\mathcal{Q},\mathrm{LO}}(z) + \frac{\alpha_s^3(\mu_R)}{m_Q^3} |\mathcal{R}_{\mathcal{Q}}(0)|^2 \, \Gamma^{\mathcal{Q},\mathrm{NLO}}(z)$$

$(Q \to \mathcal{Q}\, Q)$ at $\mu_0 = 3m_Q$

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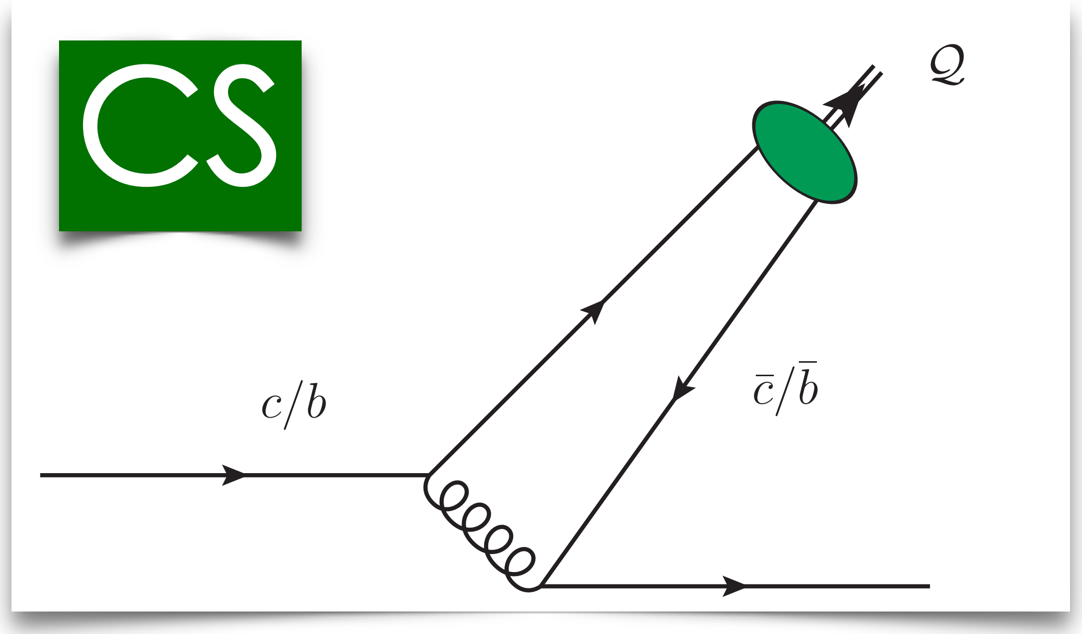

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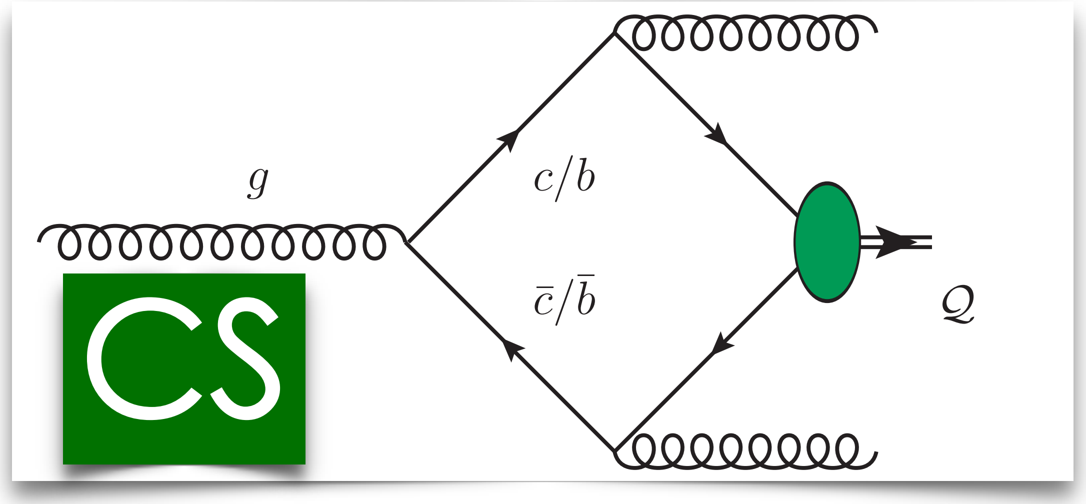

$$D_g^{\mathcal{Q}}(z, 2m_Q) = \frac{5}{36(2\pi)^2} \alpha_s^3(2m_Q) \frac{|\mathcal{R}_{\mathcal{Q}}(0)|^2}{m_Q^3} \int_0^z \mathrm{d}\xi \int_{(\xi+z^2)/2z}^{(1+\xi)/2} \mathrm{d}\tau \frac{1}{(1-\tau)^2(\tau-\xi)^2(\tau^2-\xi)^2}$$
$$\sum_{i=1}^{2} z^i \left[f_i^{(g)}(\xi,\tau) + g_i^{(g)}(\xi,\tau) \frac{1+\xi-2\tau}{2(\tau-\xi)\sqrt{\tau^2-\xi}} \ln\left(\frac{\tau-\xi+\sqrt{\tau^2-\xi}}{\tau-\xi-\sqrt{\tau^2-\xi}} \right) \right],$$

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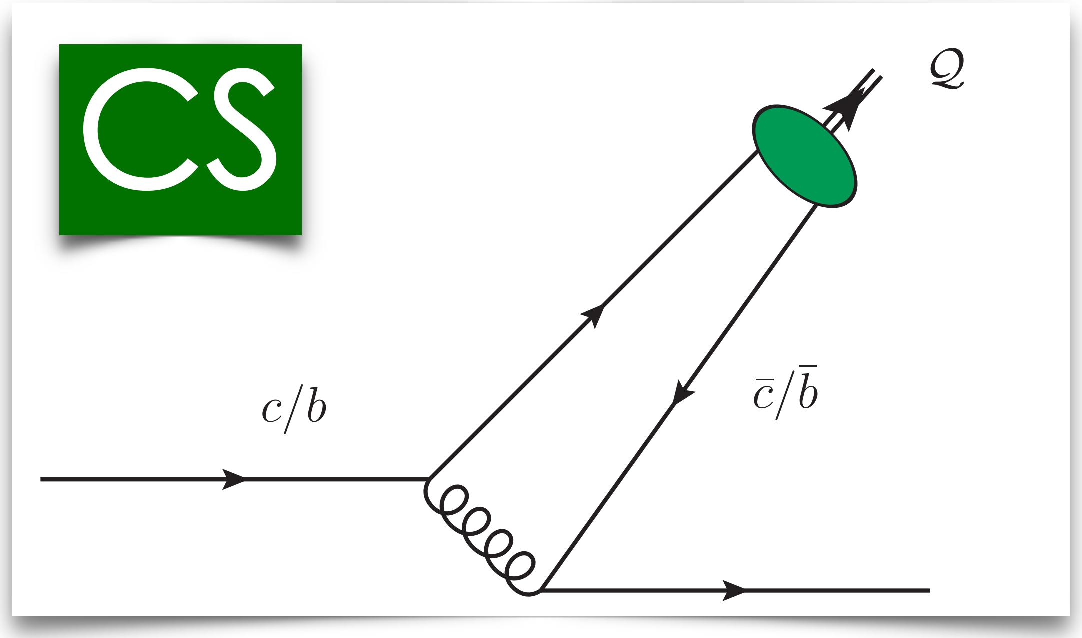

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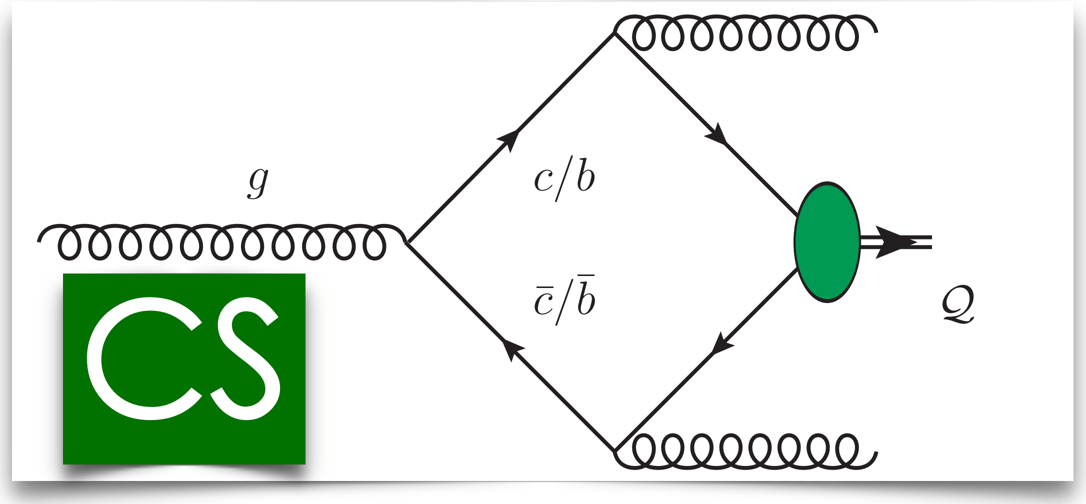

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⊗

🔗 [F. G. C. et al., Eur. Phys. J. C 82 (2022) 10, 929]

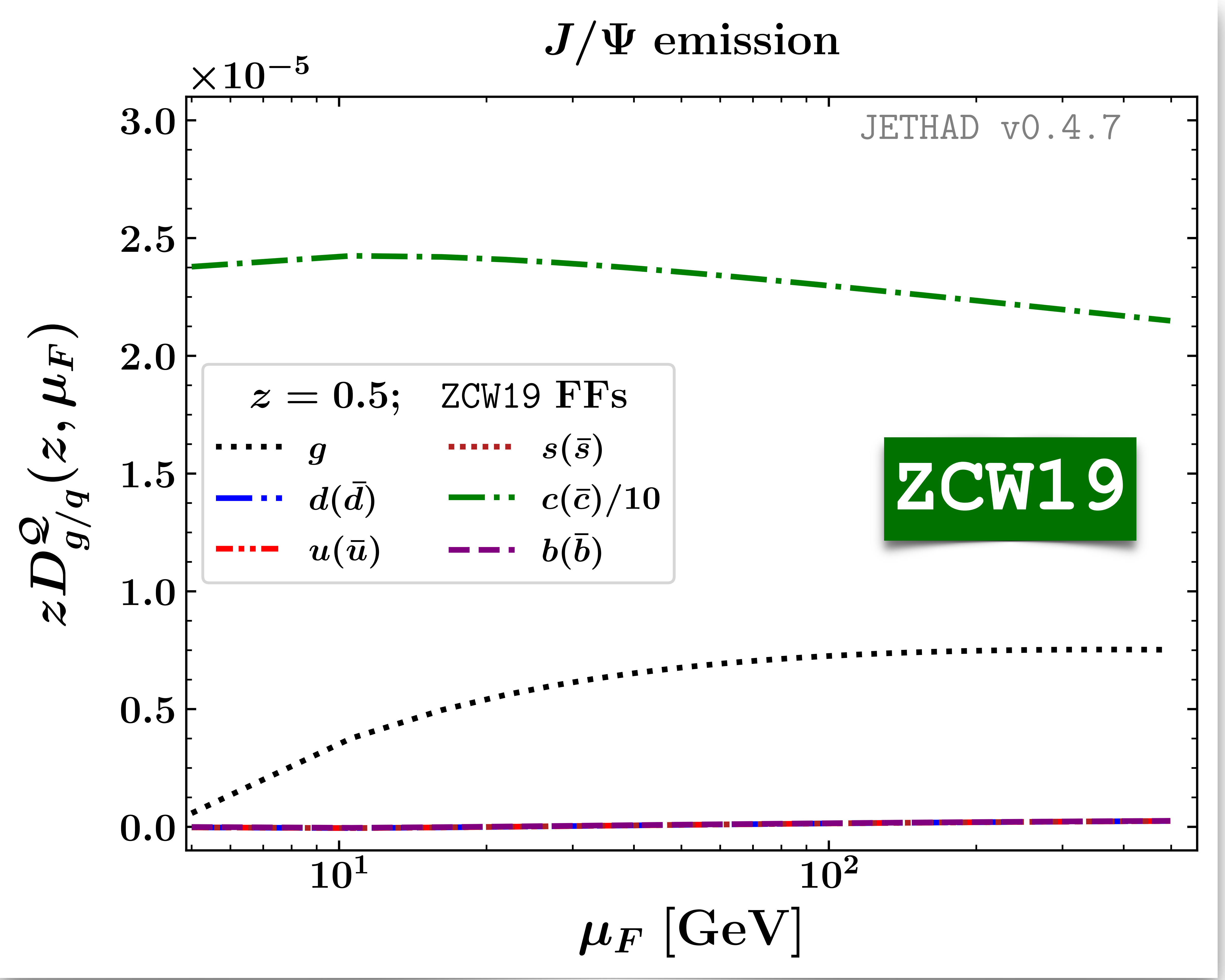
ZCW19+

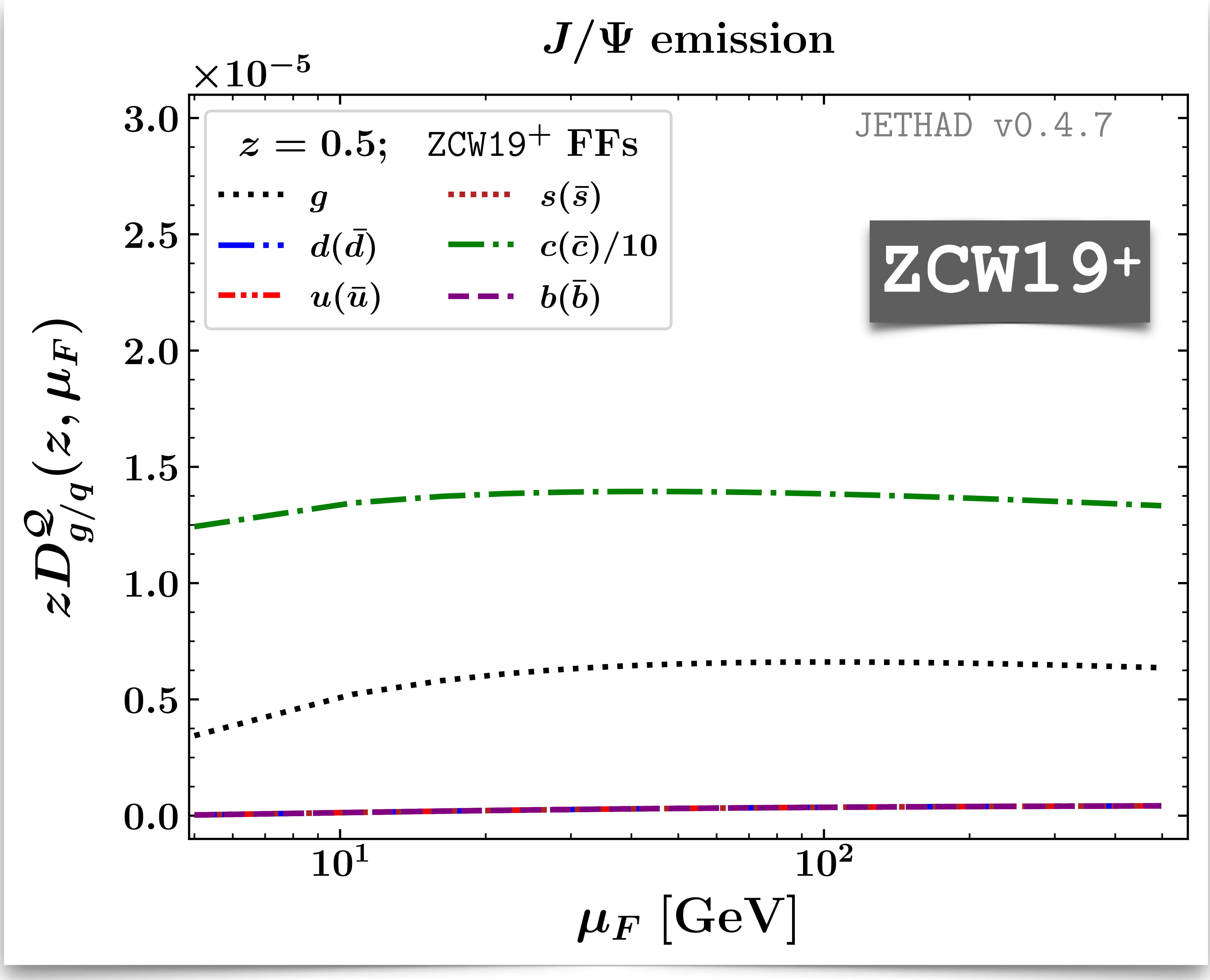
onium FFs = **APFEL++**

Backup

Vector quarkonium + jet at the LHC

J/ψ collinear FFs



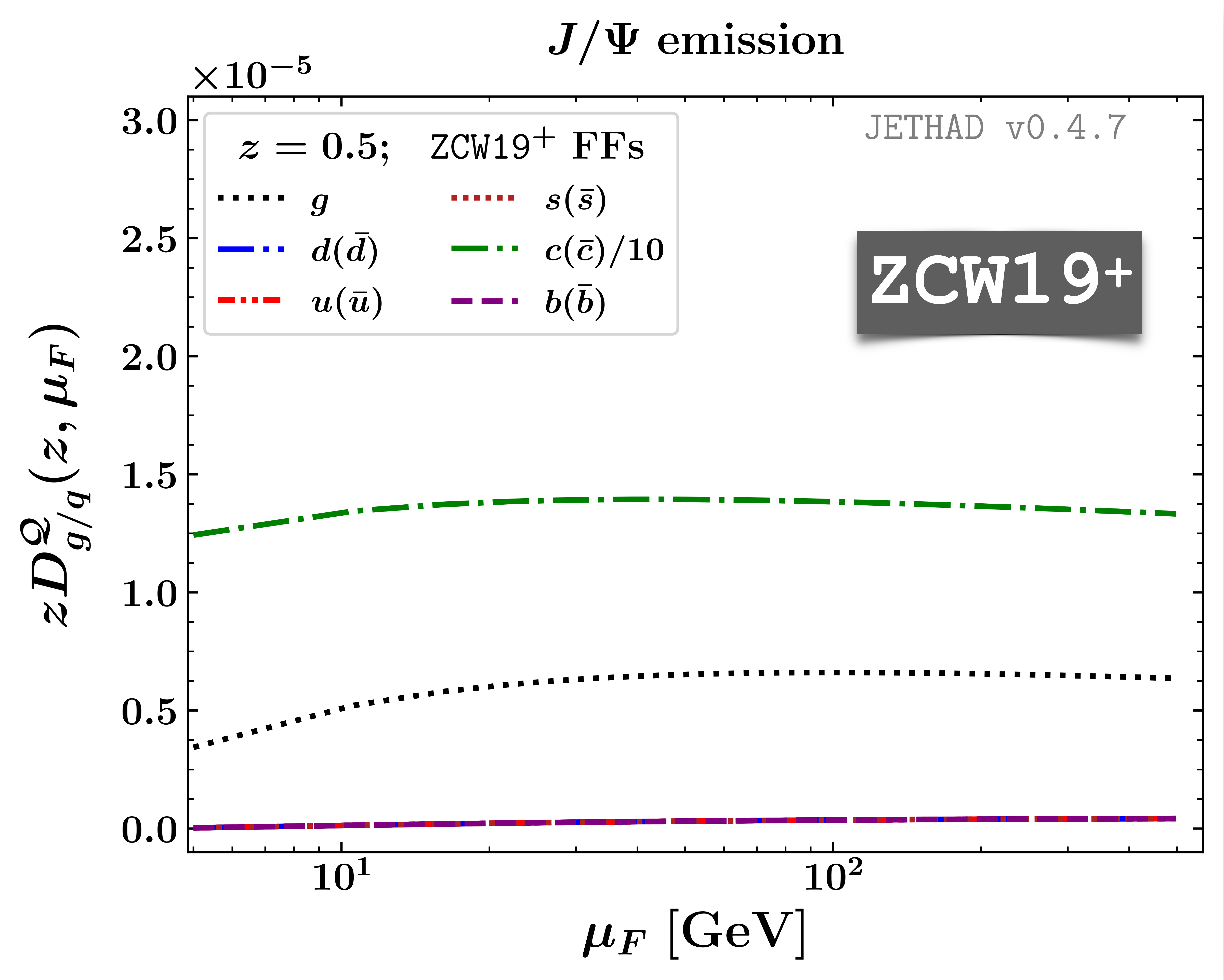


[F. G. C. et al., Eur. Phys. J. C 82 (2022) 10, 929]

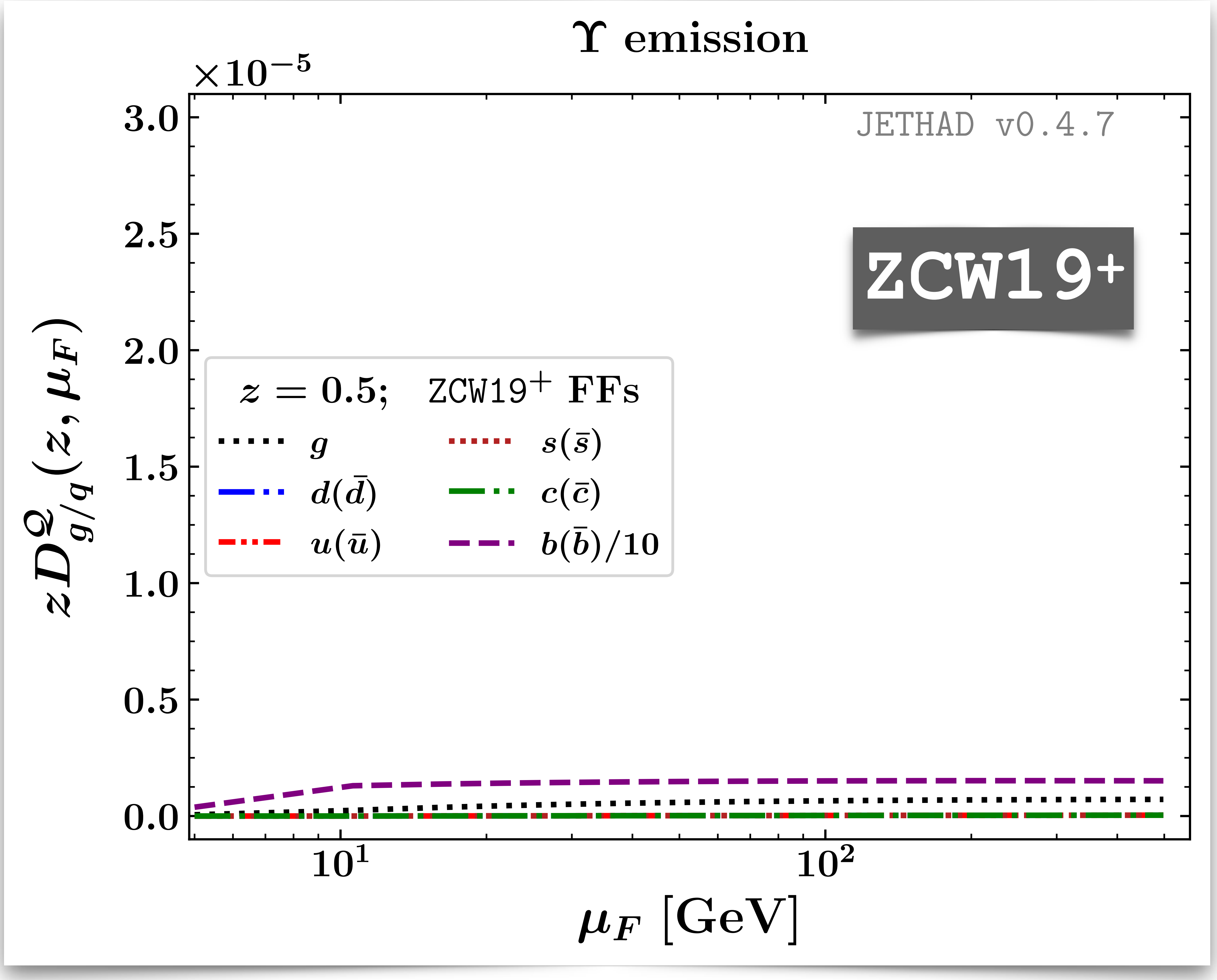
Backup

Vector quarkonium + jet at the LHC

J/ψ collinear FFs



Υ collinear FFs

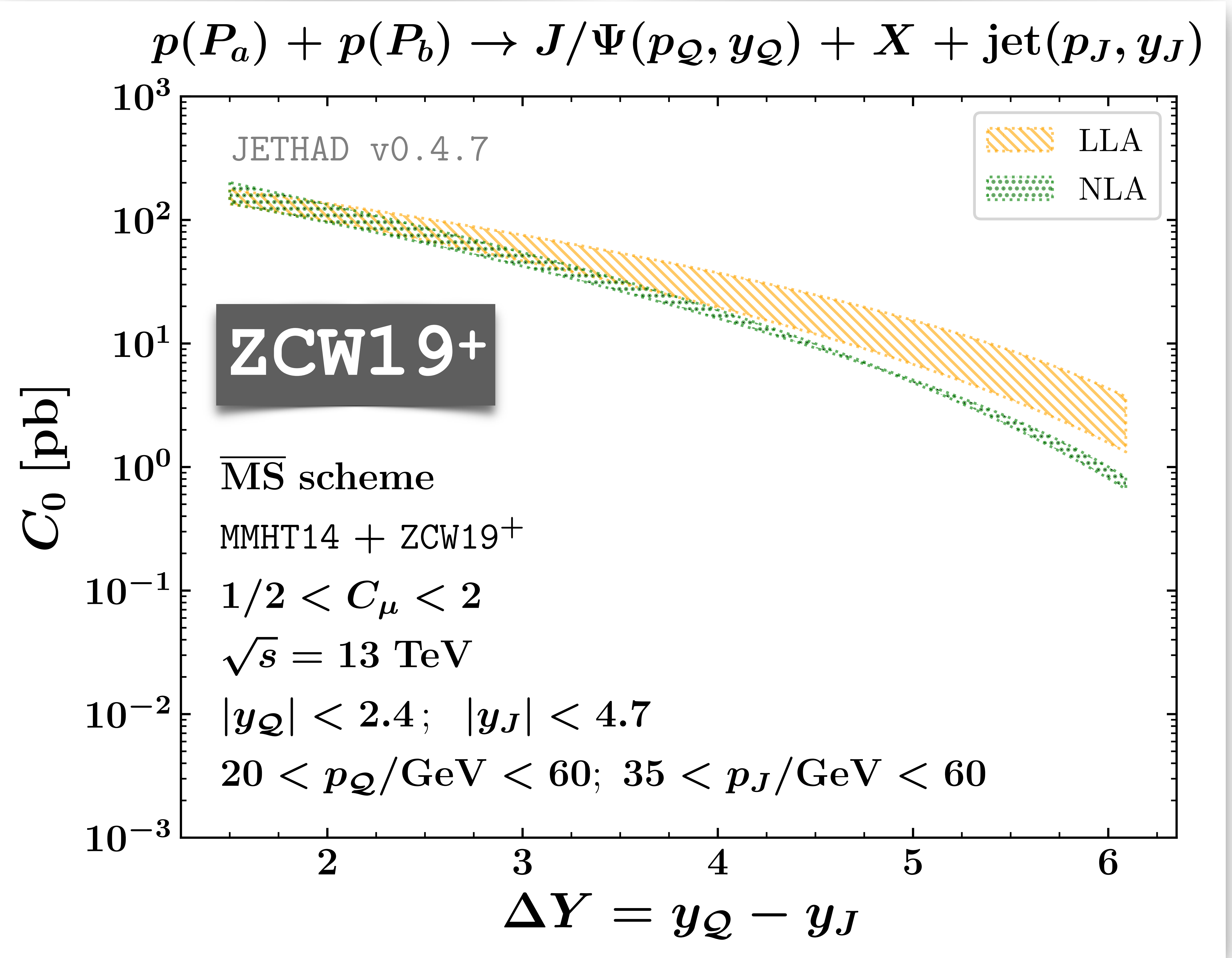


[F. G. C. et al., Eur. Phys. J. C 82 (2022) 10, 929]

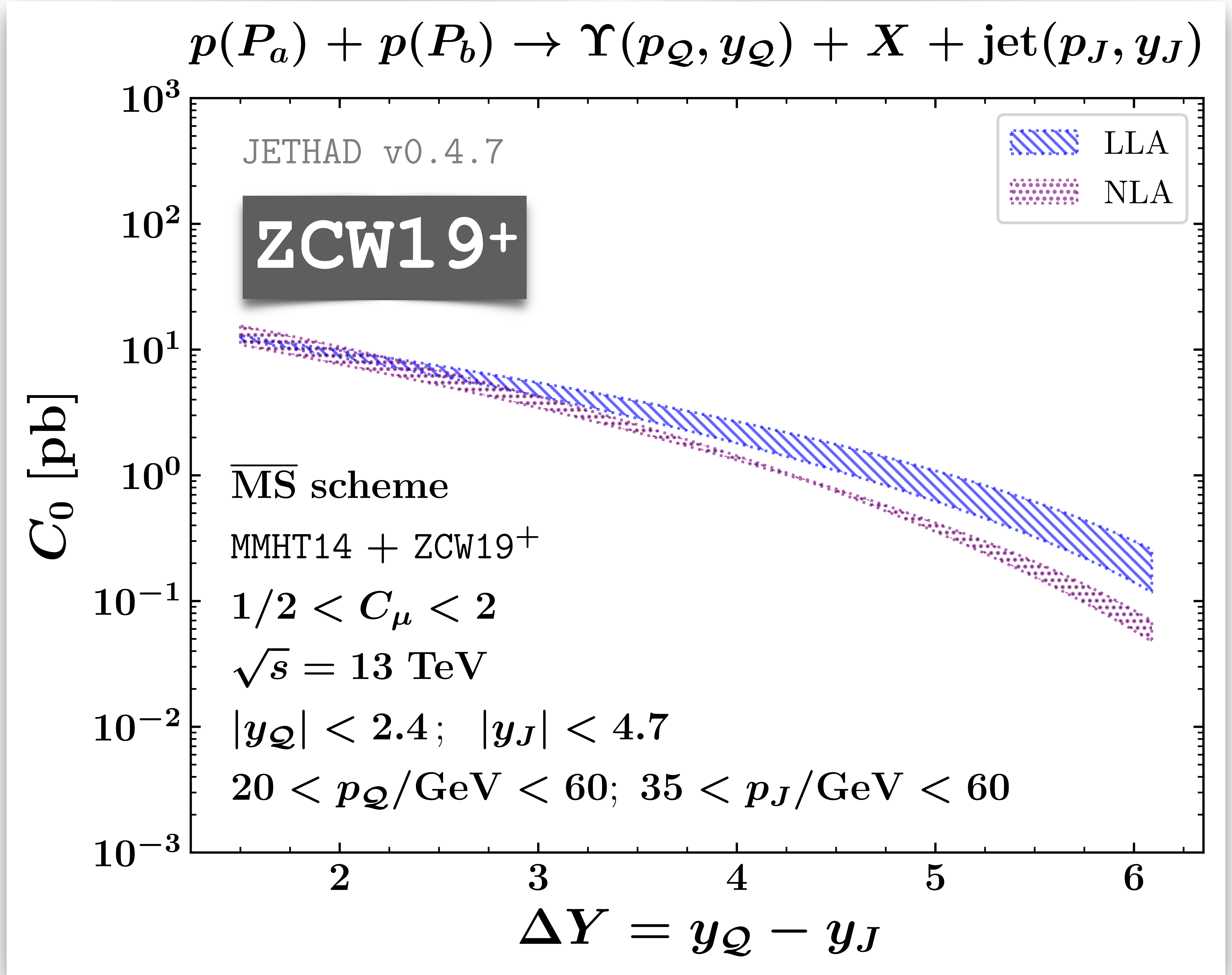
Backup

Vector quarkonium + jet at the LHC

J/ψ + jet



Υ + jet

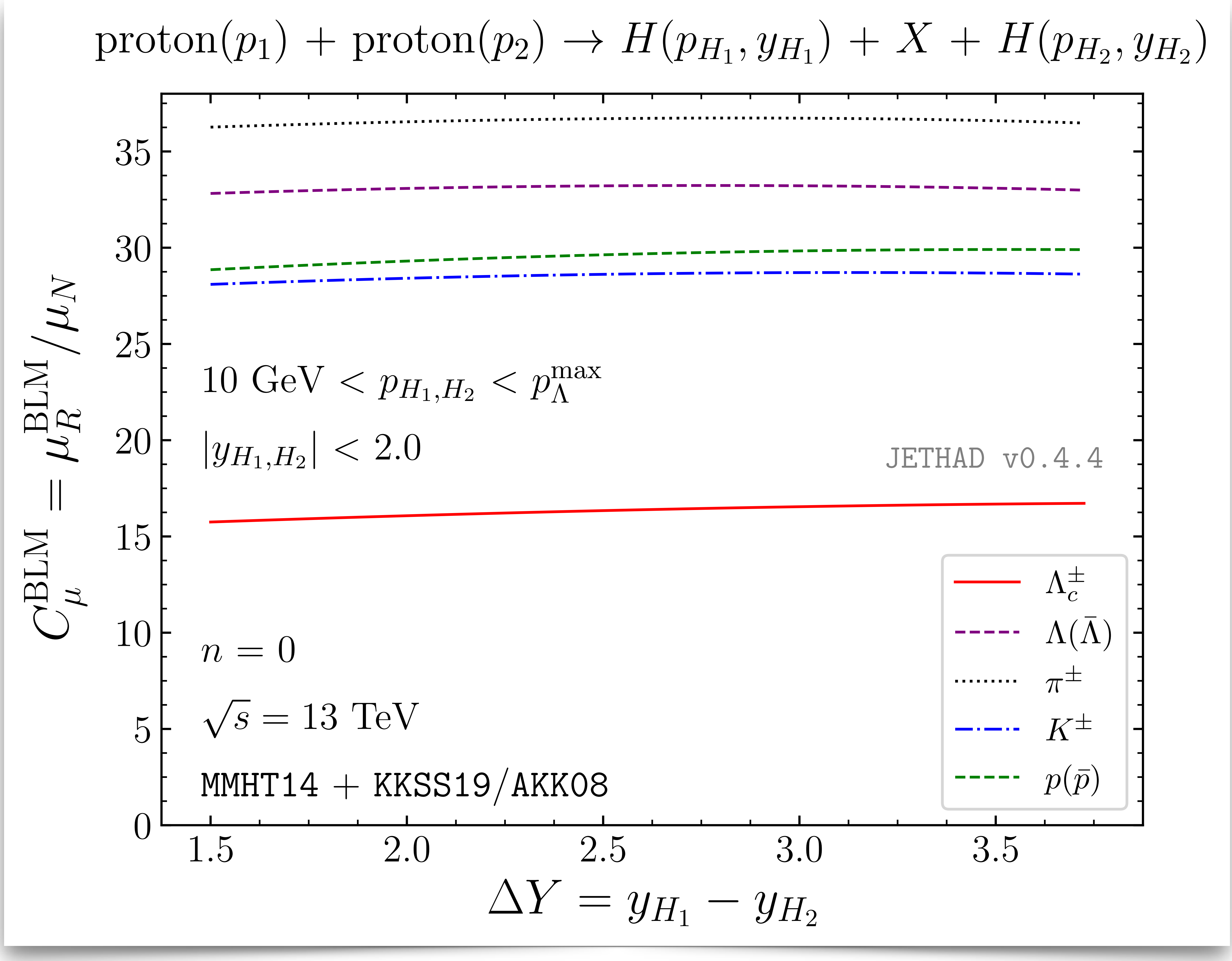


[F. G. C. et al., Eur. Phys. J. C 82 (2022) 10, 929]

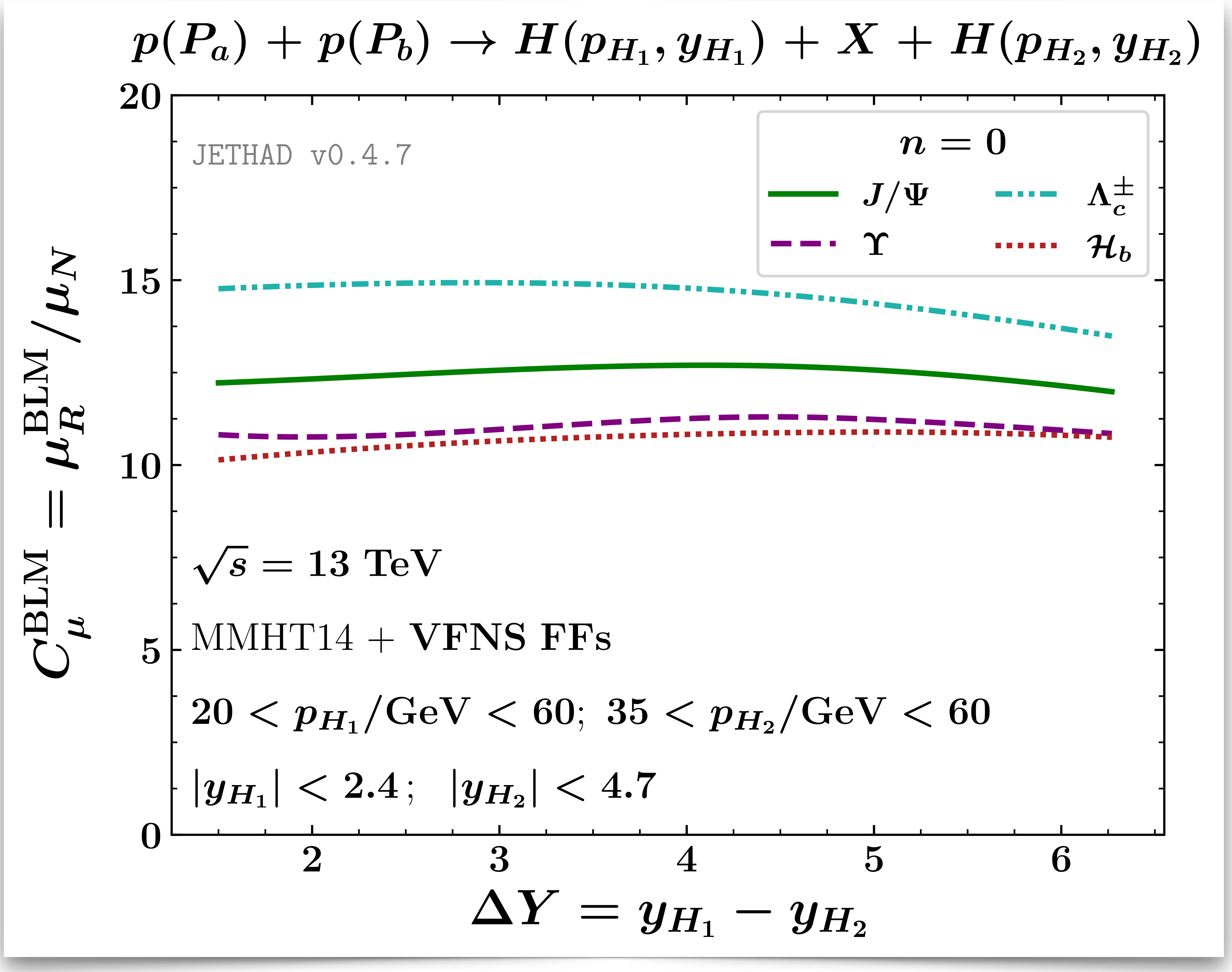
Backup

Heavy flavor at the LHC: BLM scales

Heavy-light hadrons



Vector quarkonia

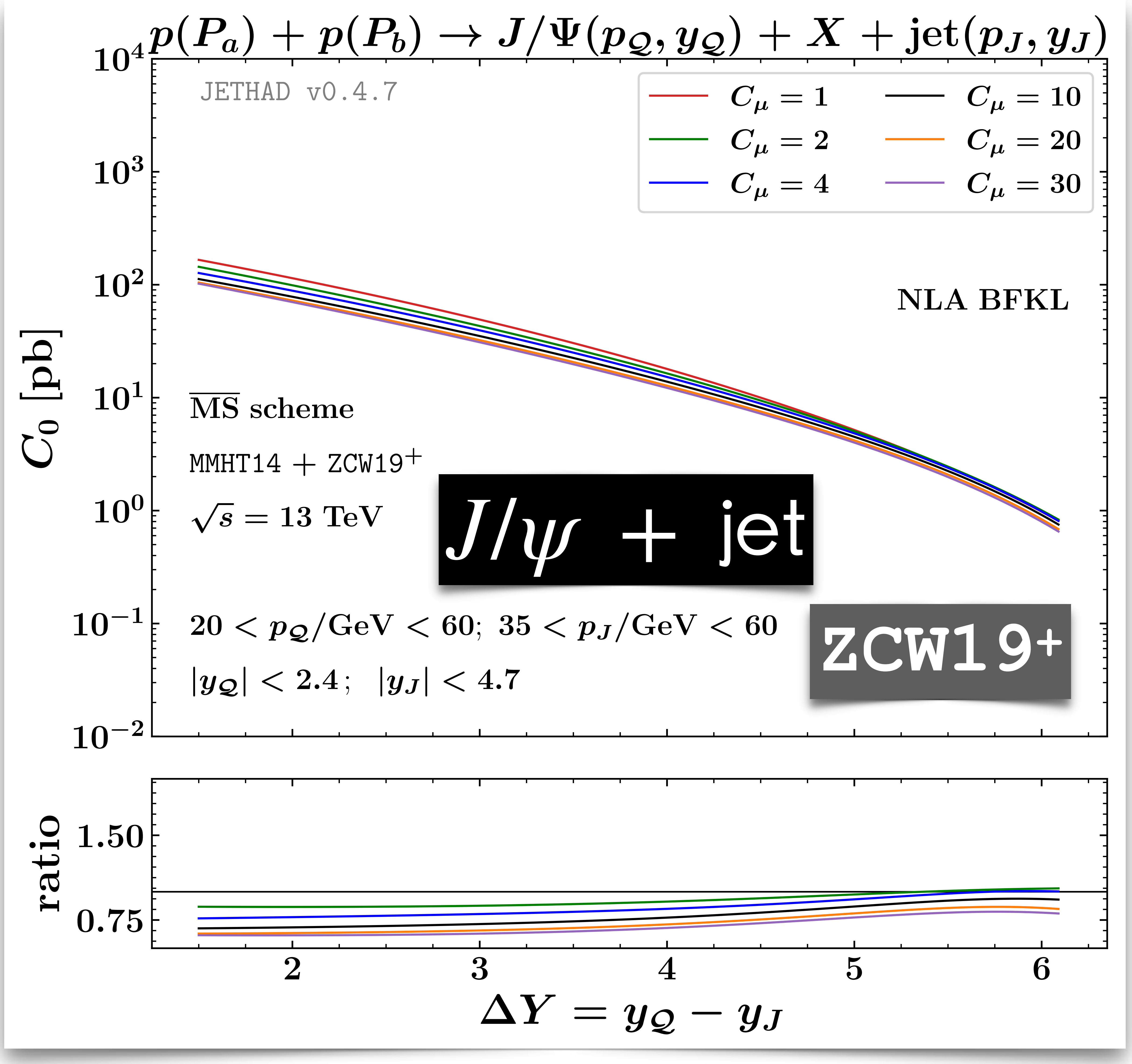


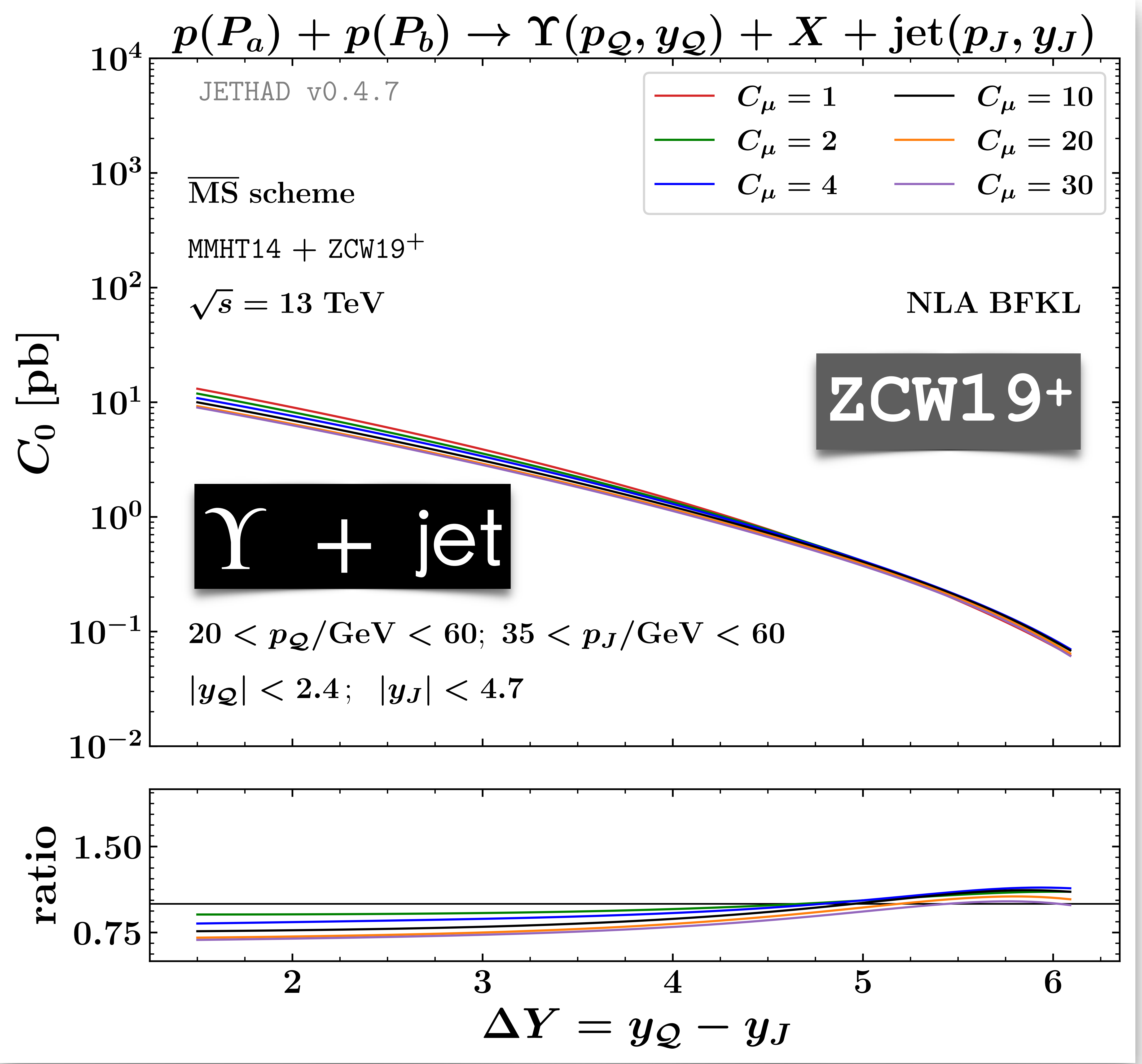
[F. G. C. et al., Eur. Phys. J. C 81 (2021) 8, 780]

Backup

[F. G. C. et al., Eur. Phys. J. C 82 (2022) 10, 929]

Vector quarkonium + jet at the LHC





[F. G. C. et al., Eur. Phys. J. C 82 (2022) 10, 929]

Backup

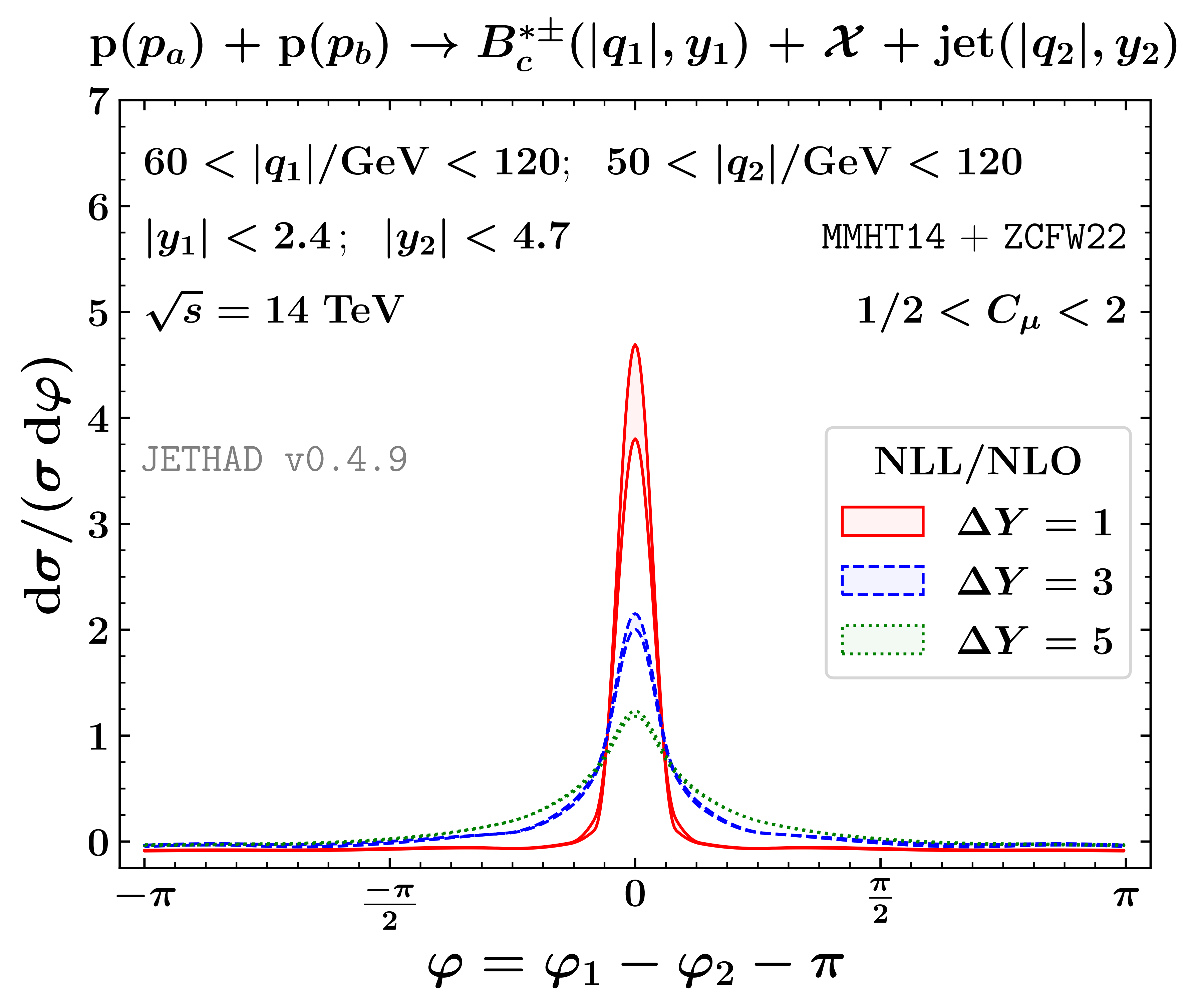
CHARMED B MESONS

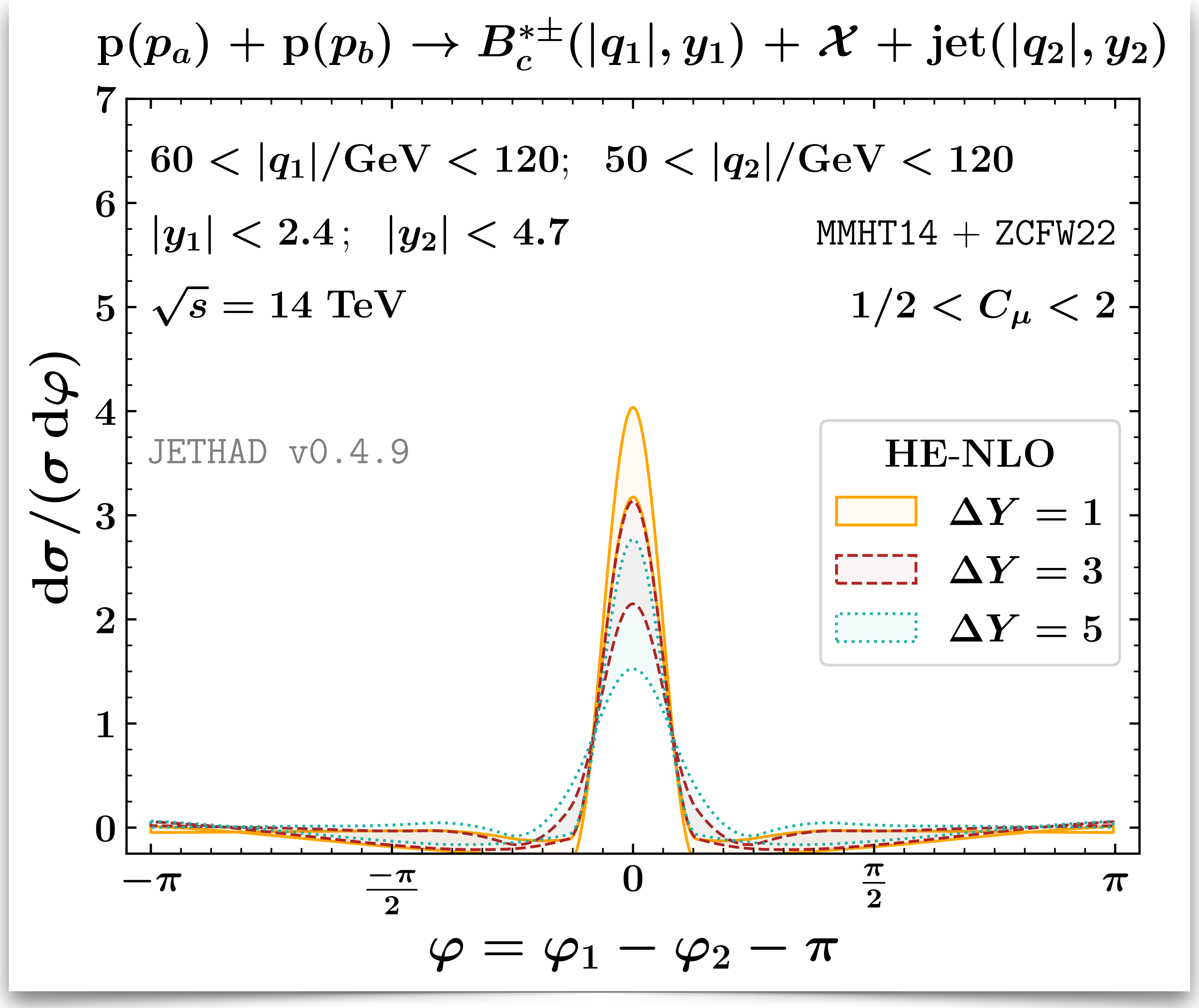
Charmed B-mesons from single-parton fragmentation

(2) ¡ *Let us consider $B_c(^1S_0)$ and $B_c(^3S_1)$ at large p_T* $\rightarrow$ *single-parton* ***fragmentation*** *from* ***NRQCD*** !

(NLO heavy quark) 🔗 [X. Zheng et al., Phys. Rev. D 100 (2019) 3, 034004]

(NLO gluon) 🔗 [X. Zheng et al., JHEP 05 (2022) 036]



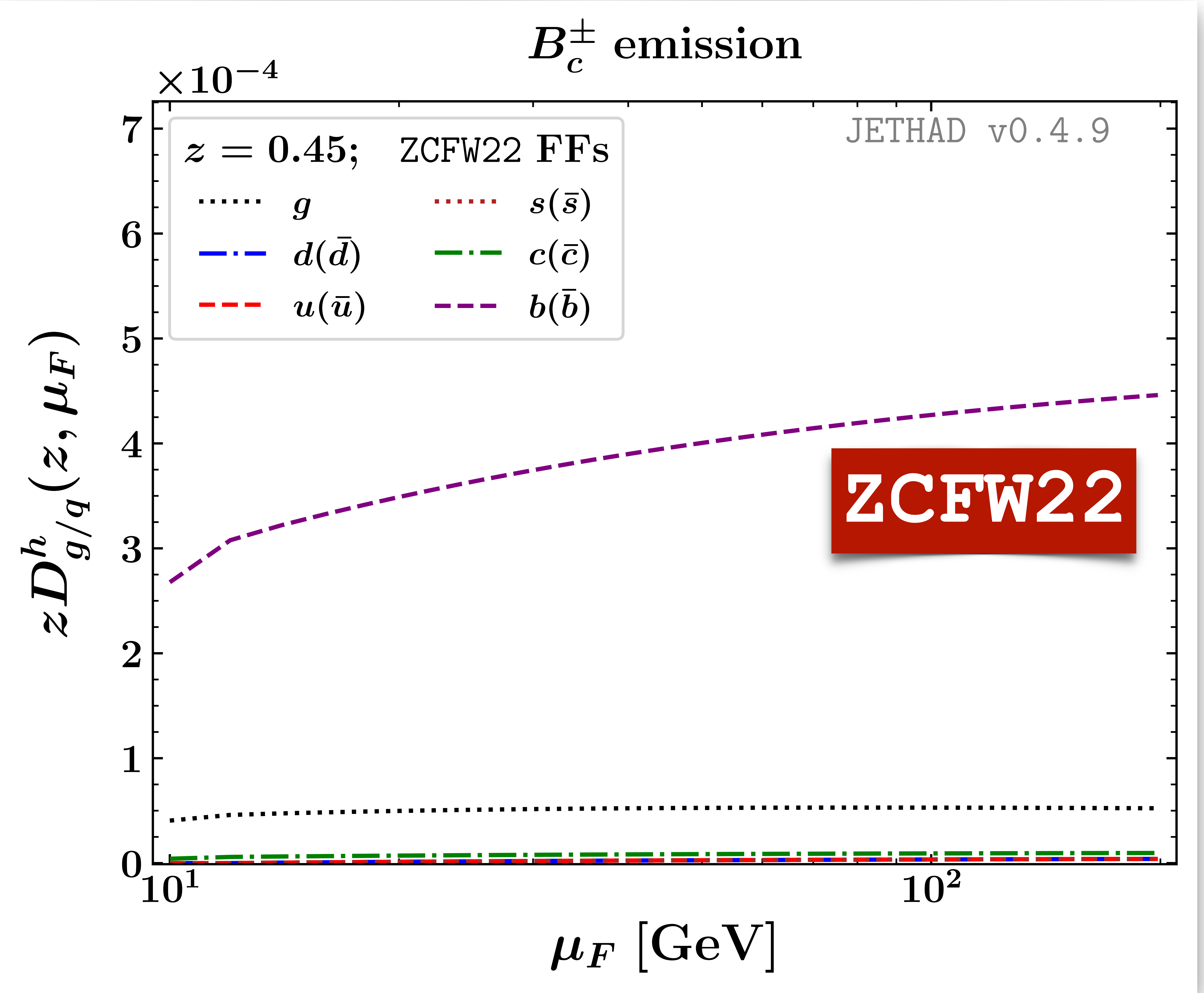


🔗 [F. G. C., Phys. Lett. B 835 (2022) 137554]

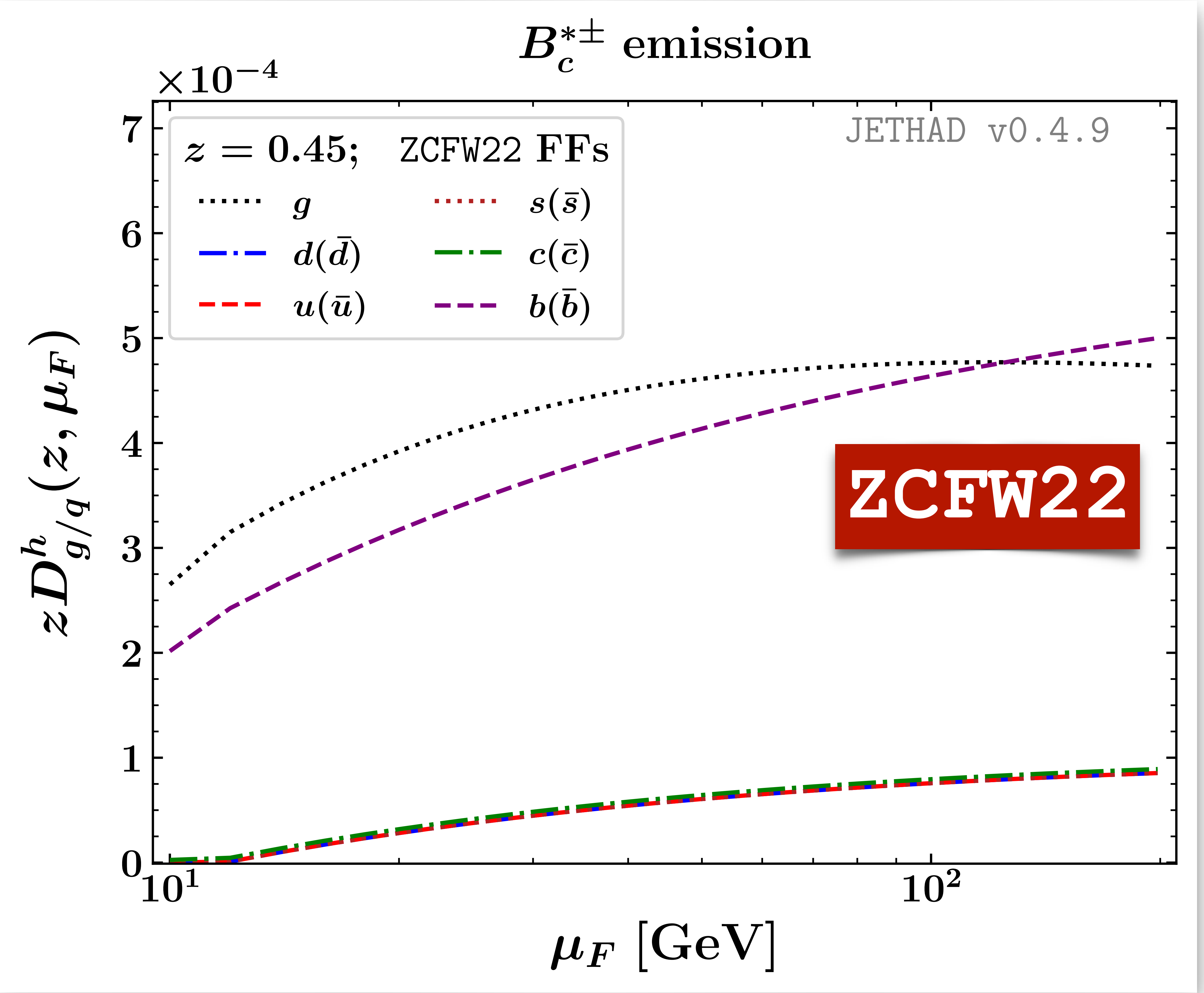
Backup

Charmed B-mesons + jet at the HL-LHC

$B_c^{\pm}(^1S_0)$ collinear FFs



$B_c^{\pm}(^3S_1)$ collinear FFs

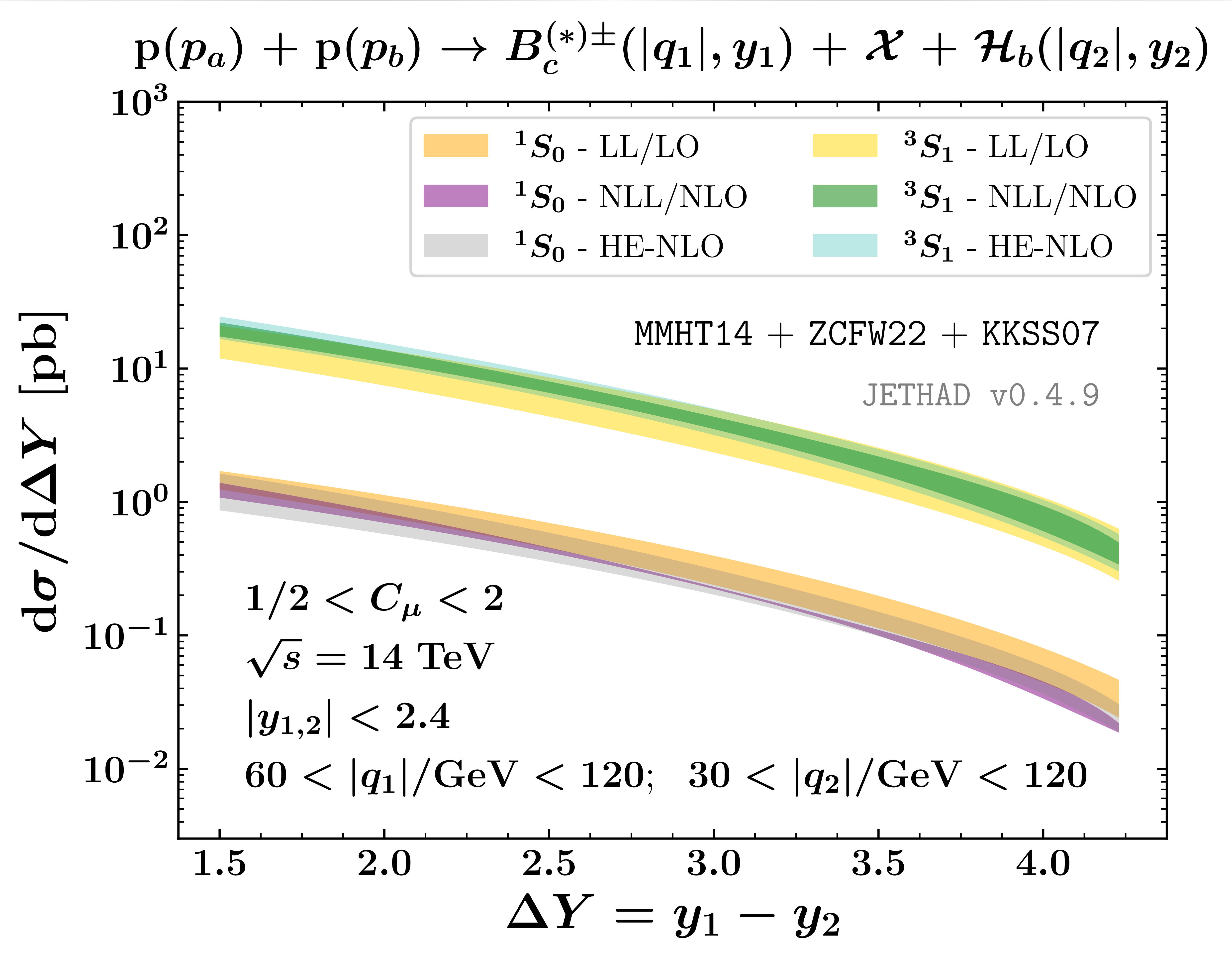


[F. G. C., Phys. Lett. B 835 (2022) 137554]

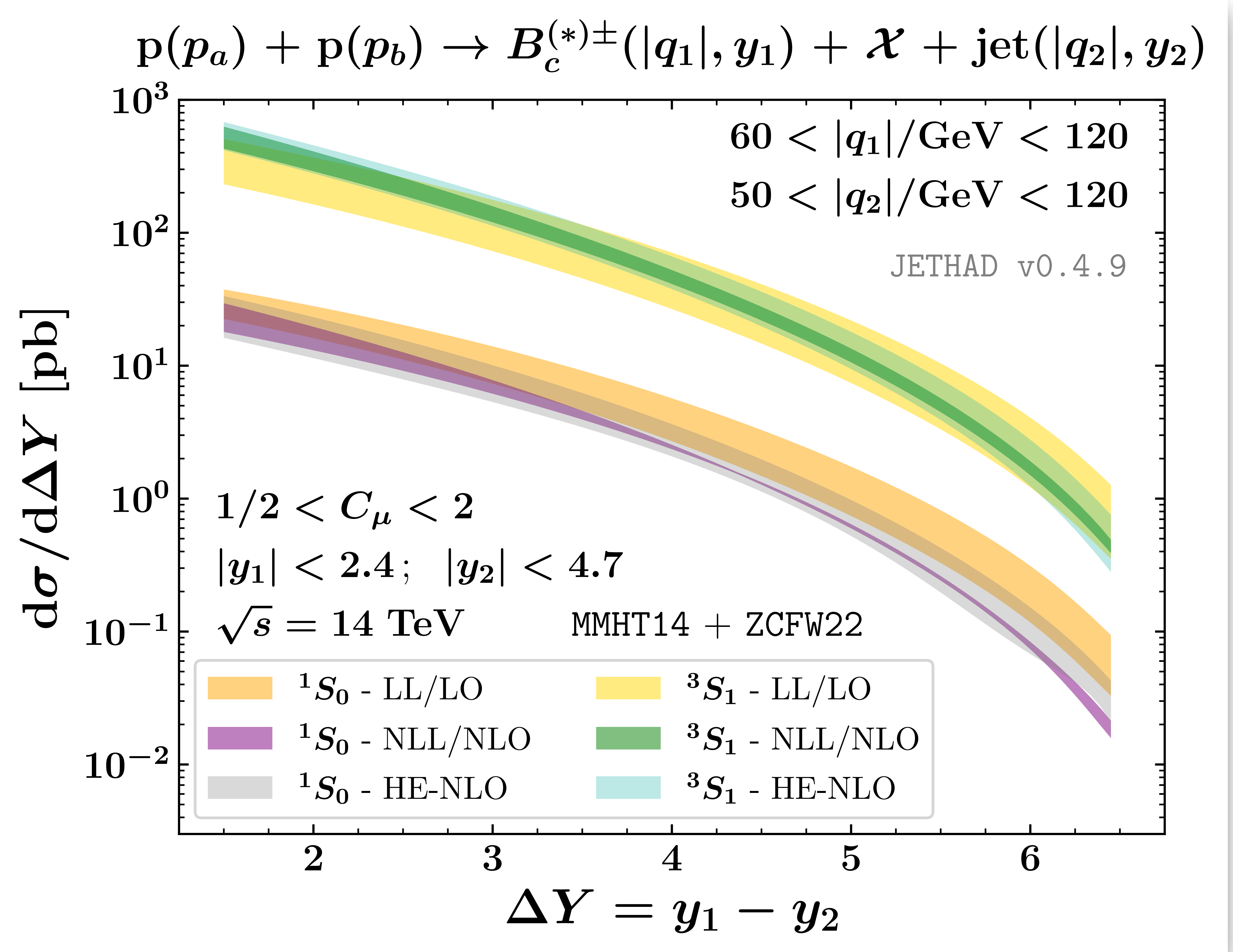
Backup

Charmed B-mesons + jet at the HL-LHC

$B_c^{\pm}(^1S_0)$ + b-hadron



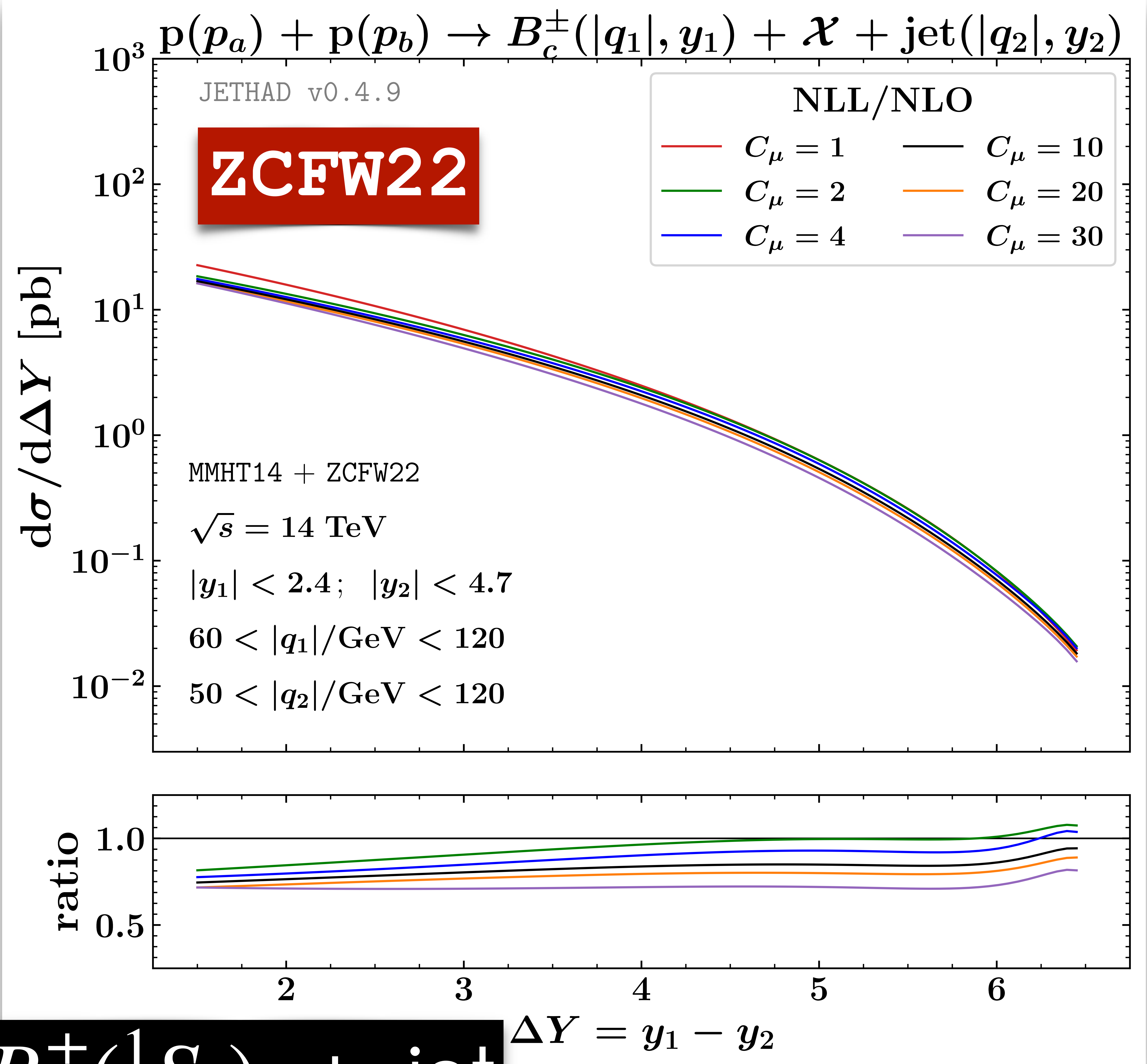
$B_c^{\pm}(^3S_1)$ + jet



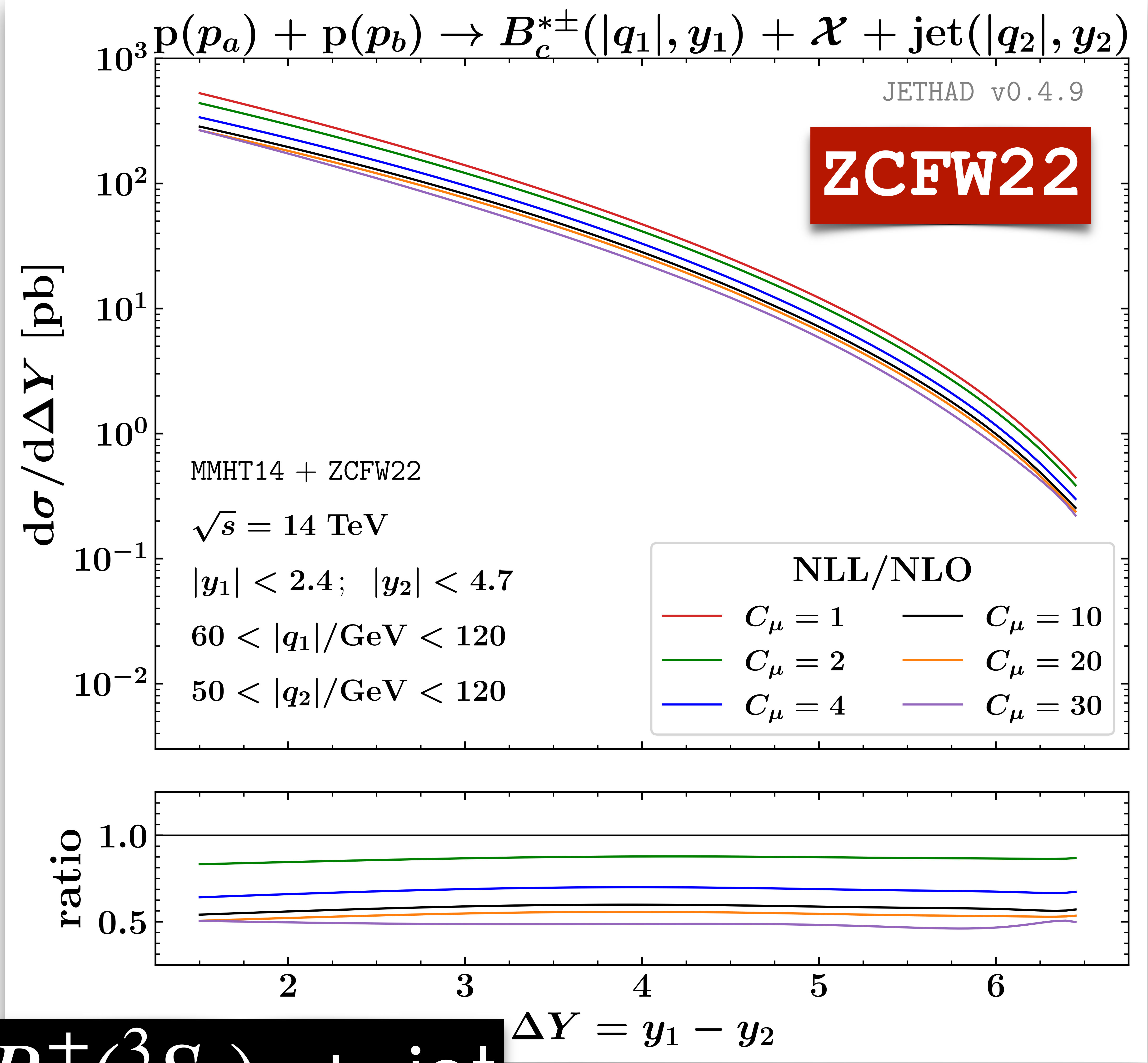
[F. G. C., Phys. Lett. B 835 (2022) 137554]

Backup

Charmed B-mesons + jet at the HL-LHC



$B_c^{\pm}(^1S_0)$ + jet



$B_c^{\pm}(^3S_1)$ + jet

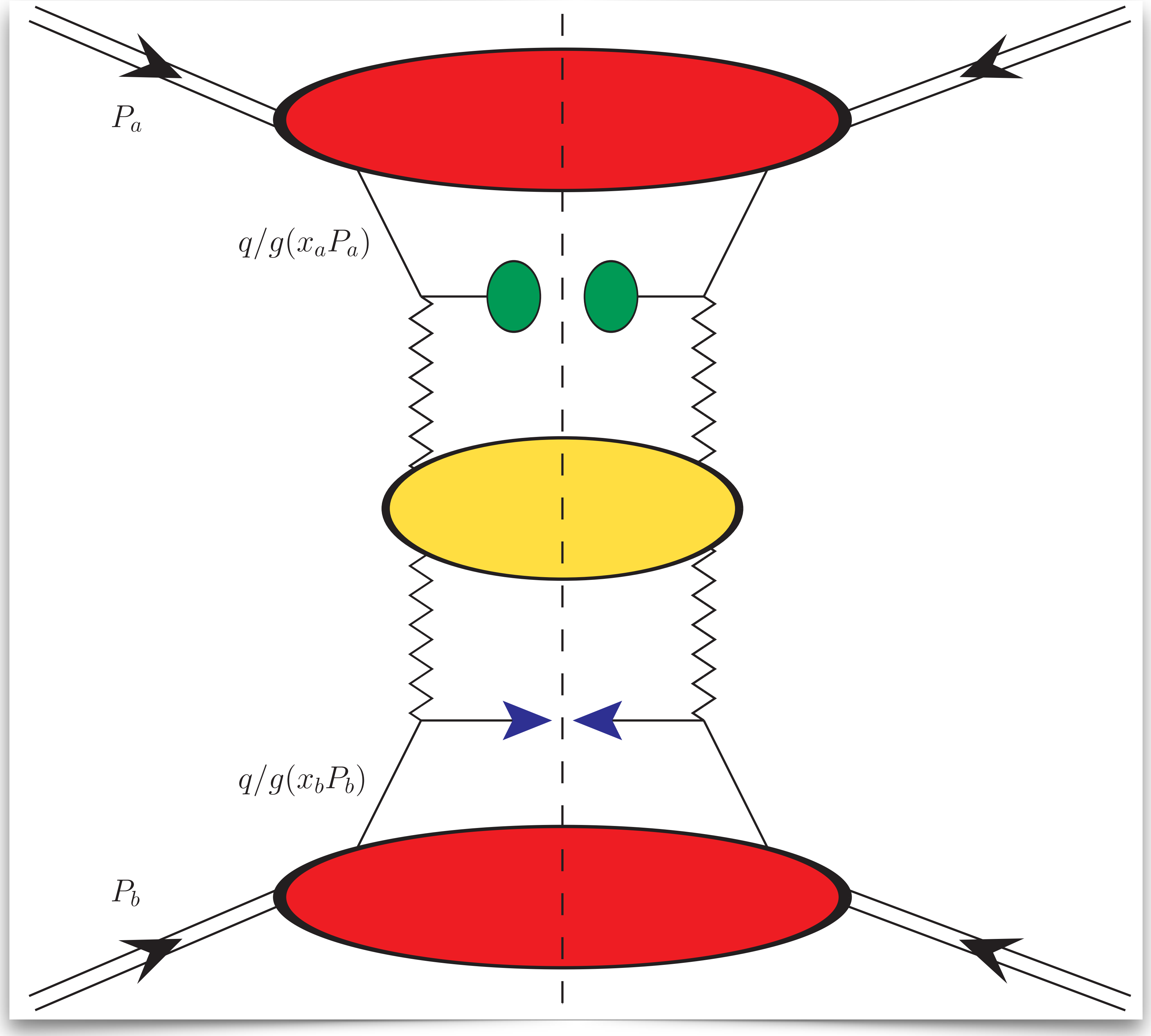
[F. G. C., Phys. Lett. B 835 (2022) 137554]

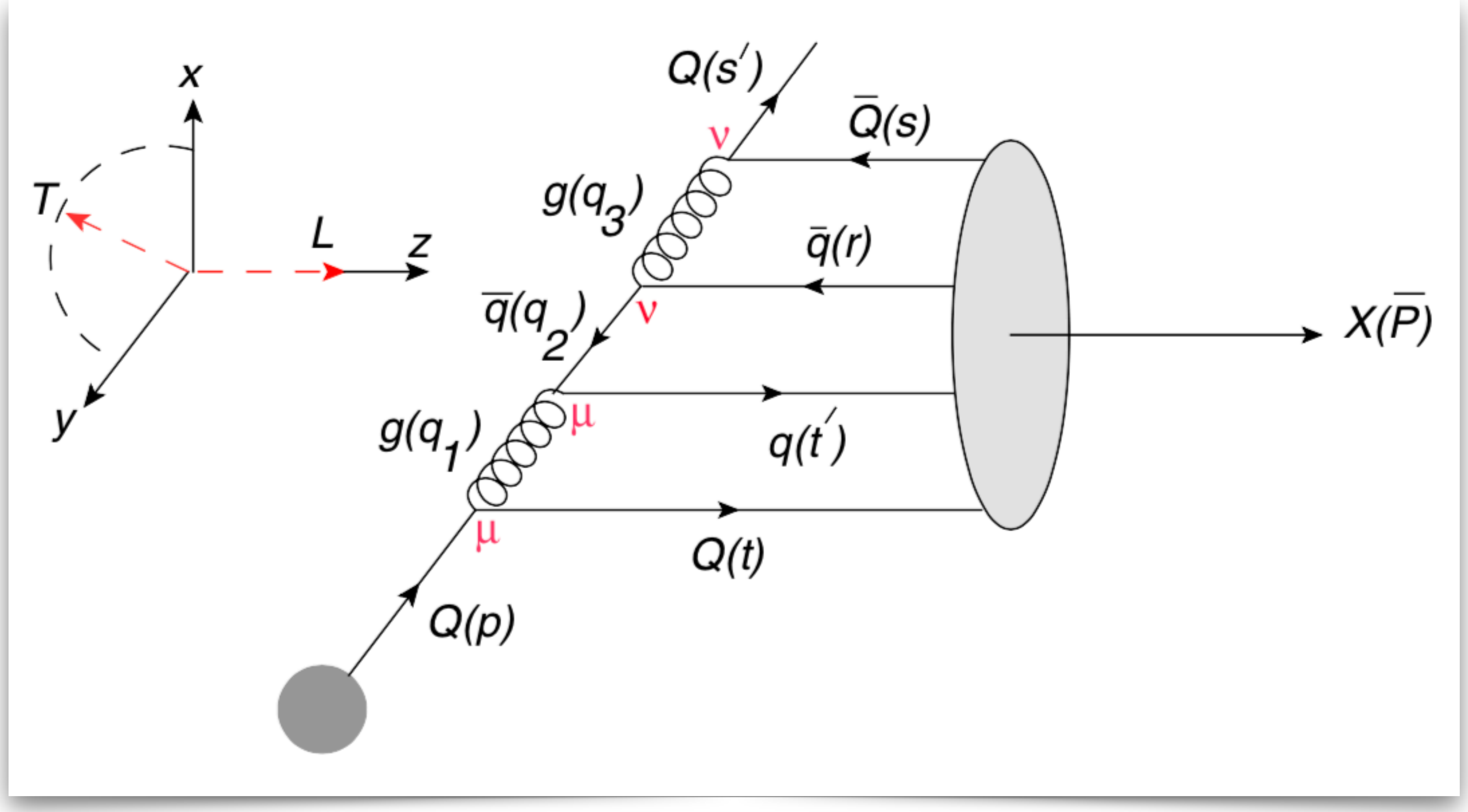
Backup

A HIGH-ENERGY QCD PORTAL TO EXOTIC MATTER

Heavy-light tetraquark from single-parton fragmentation

¡ *Let us consider heavy-light* $X_{Qq\bar{Q}\bar{q}}$ *tetraquarks at large* p_T → *single-parton* ***fragmentation*** !





🔗 [F. G. C., A. Papa, to appear in PLB]

Backup

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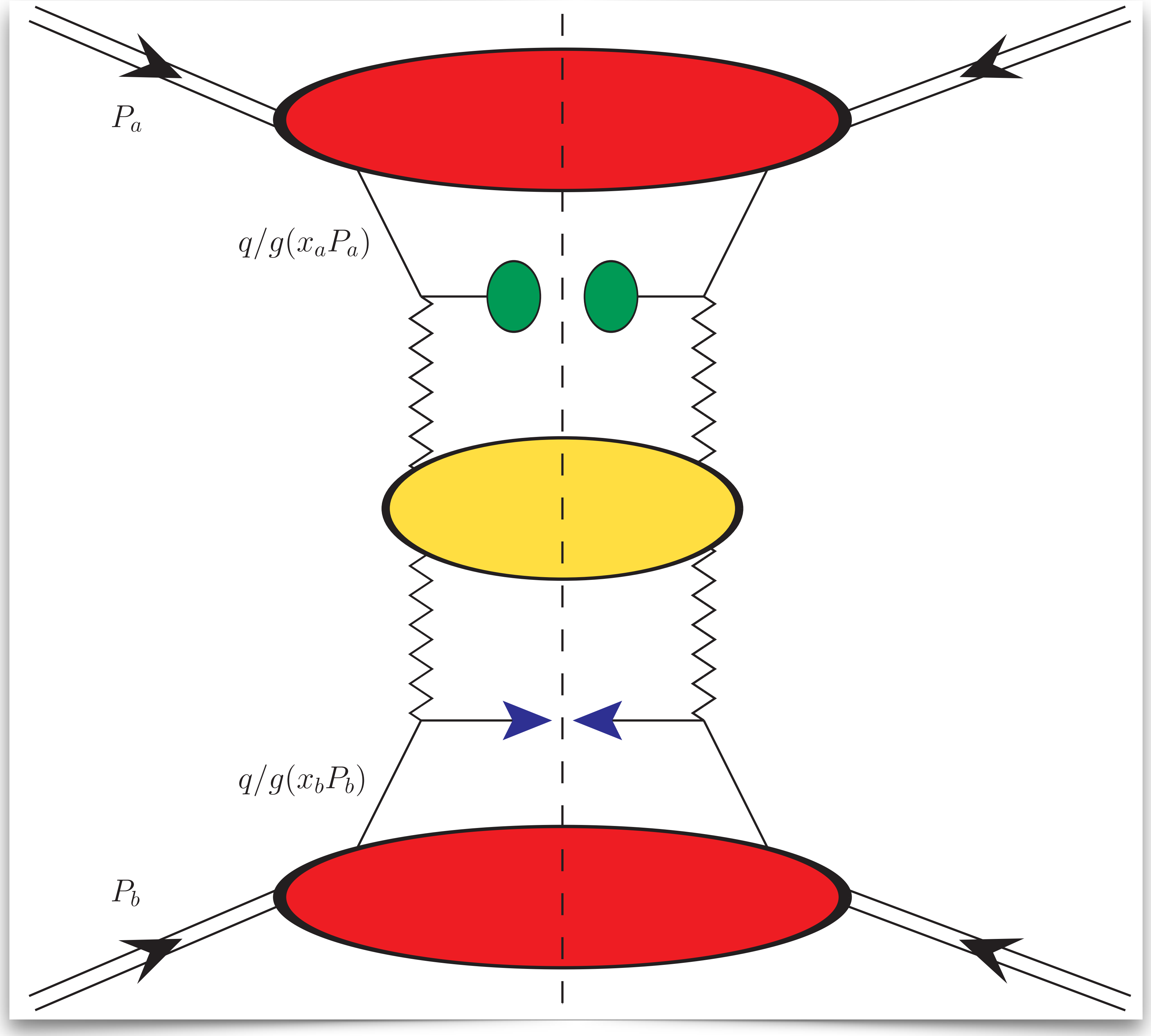
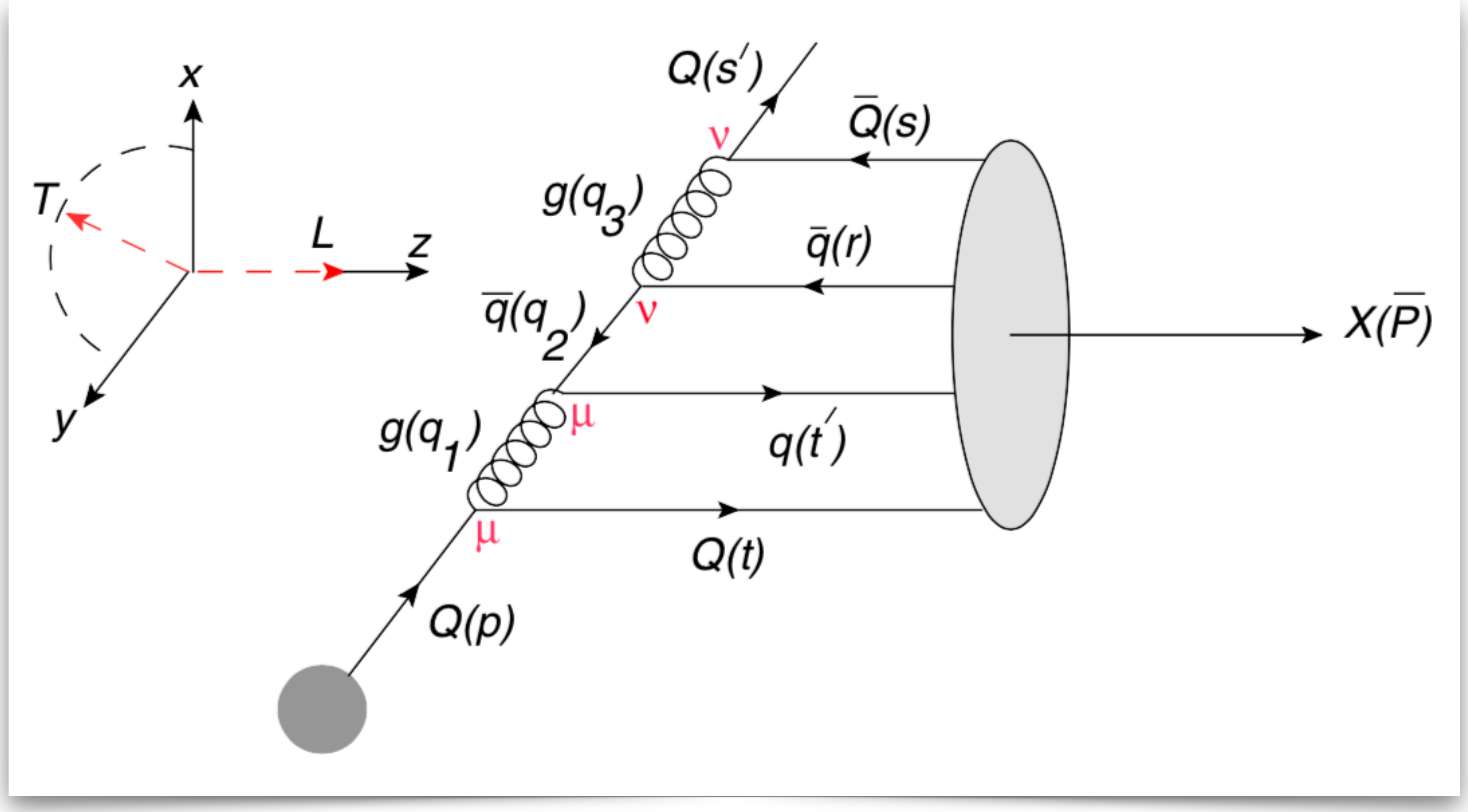




🔗 [F. G. C., A. Papa, to appear in PLB]

TQHL1.0 FFs: $(Q \rightarrow X_{Qq\bar{Q}\bar{q}}) \otimes$ **APFEL++**

$[\mu_0 = m_X + m_Q]$

S-wave

$$D_Q^X(z,\mu_0) = N \frac{z \times \Sigma_{\rm spin} \Gamma \bar{\Gamma}}{(m_X^2 - 2m_Q^2 + 2p \cdot s')^2}$$

$$= N \frac{z \times \Sigma_{\rm spin} \Gamma \bar{\Gamma}}{[m_X^2 - (m_Q^2 + \langle p_T^2 \rangle)(1 + z - \frac{1}{1-z})]^2}$$

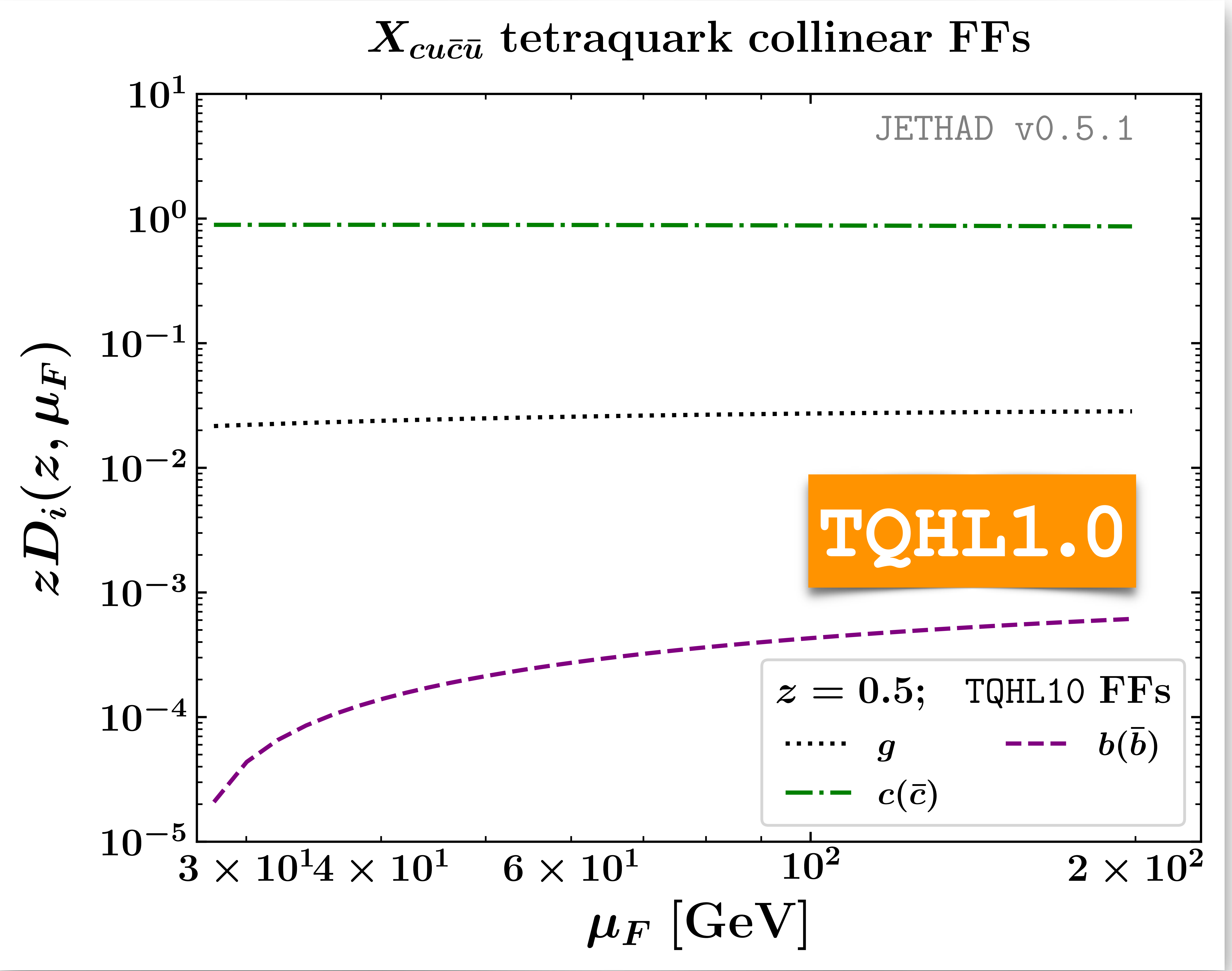
(LO) 🔗 [S. M. Moosavi Nejad, Phys. Rev. D 05 (2022) 3, 034001]

(framework) 🔗 [M. Suzuki, Phys. Rev. D 33 (1986) 676]

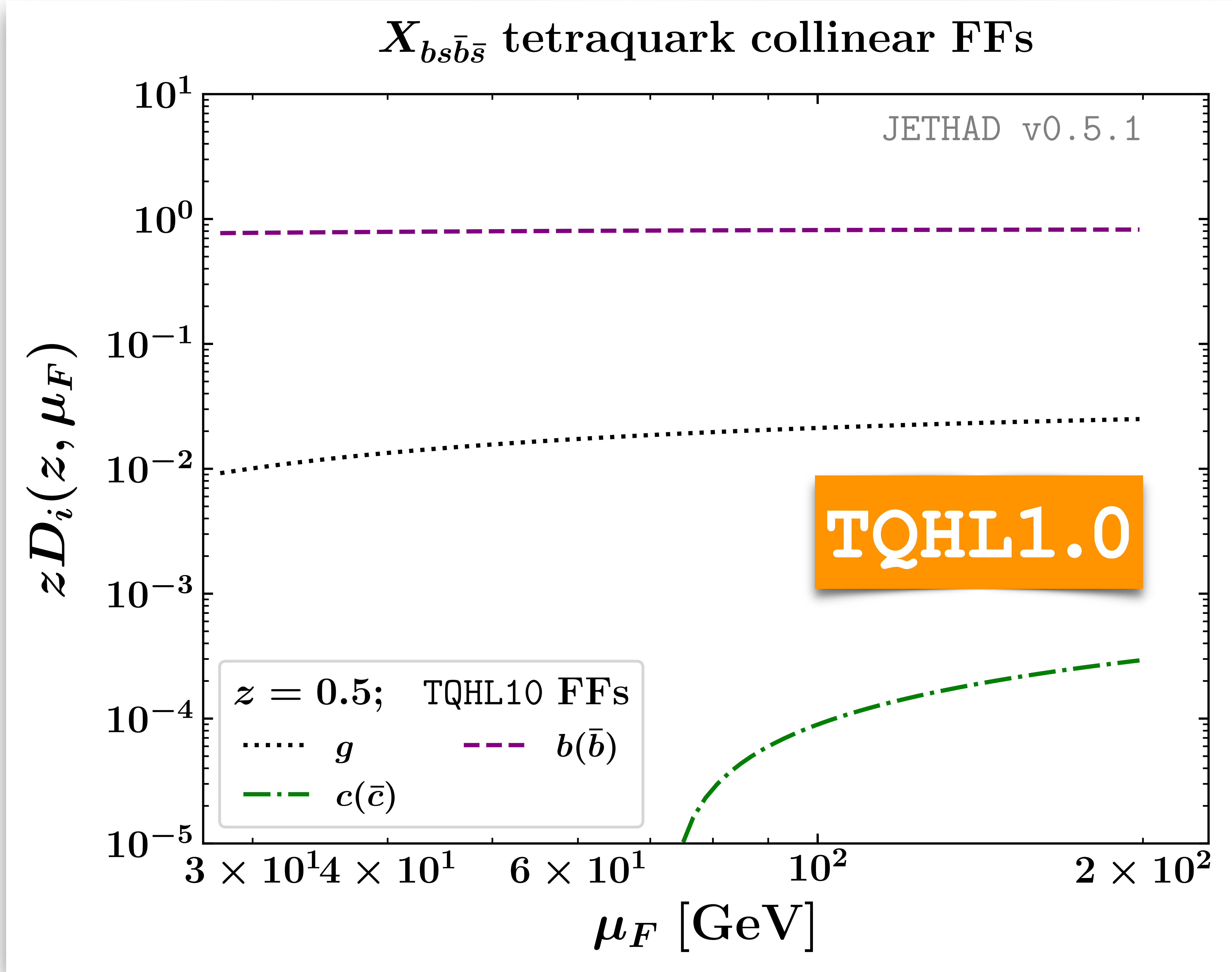
Backup

Heavy-light tetraquarks at the HL-LHC

$X_{cu\bar{c}\bar{u}}$ collinear FFs



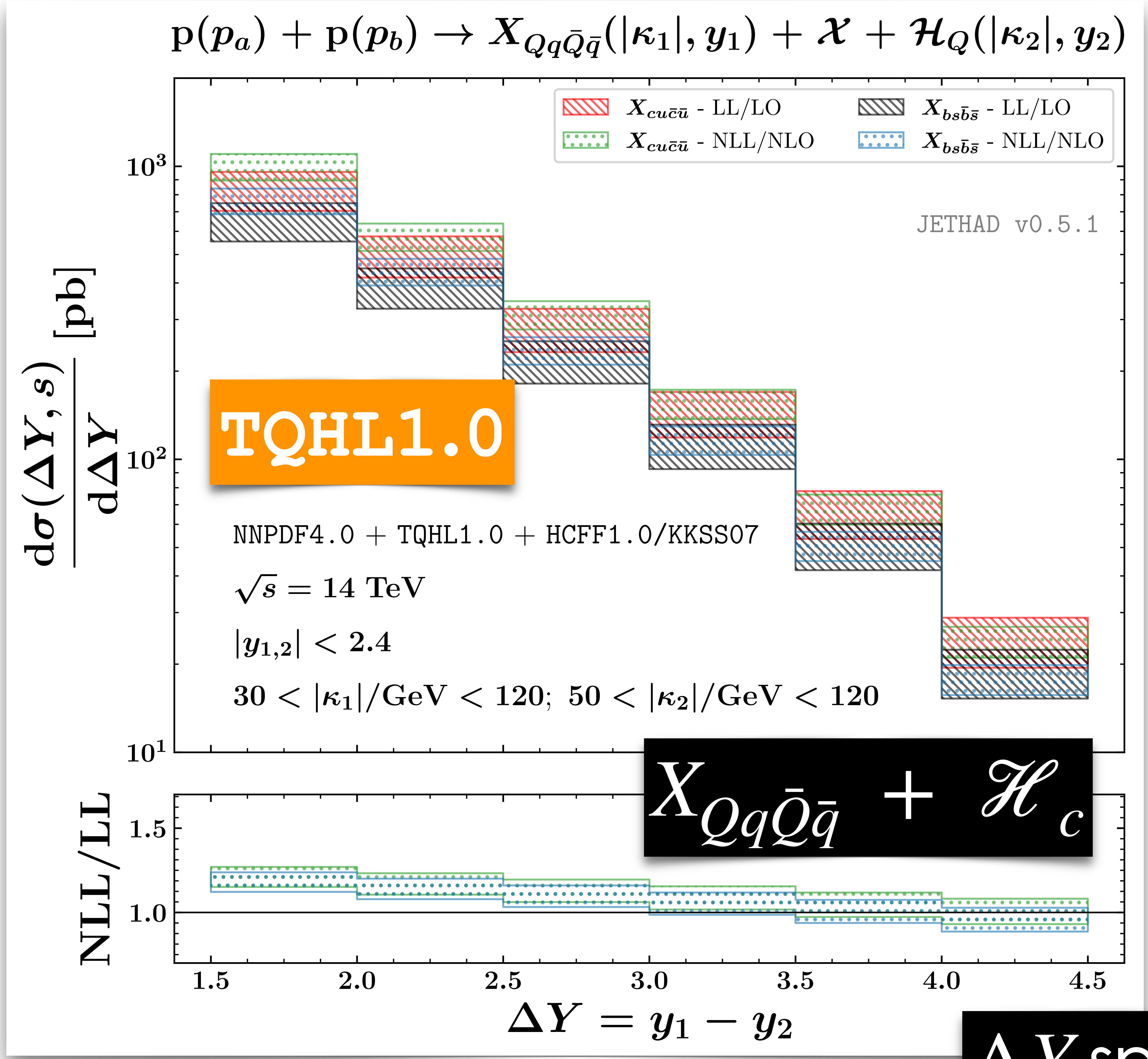
$X_{bs\bar{b}\bar{s}}$ collinear FFs



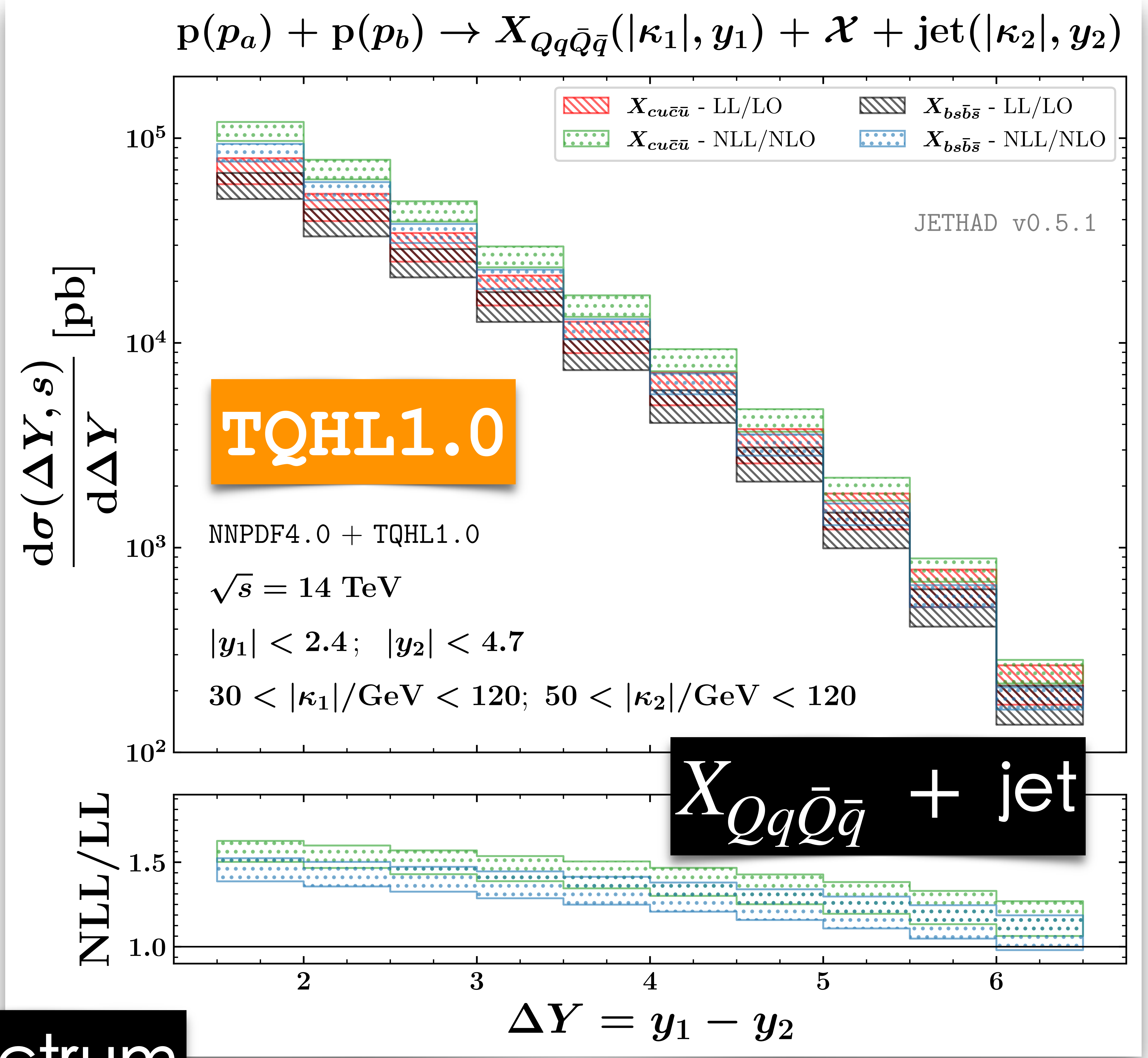
[F. G. C., A. Papa, to appear in PLB]

Backup

Heavy-light tetraquarks at the HL-LHC



$X_{Qq\bar{Q}\bar{q}} + \mathcal{H}_c$



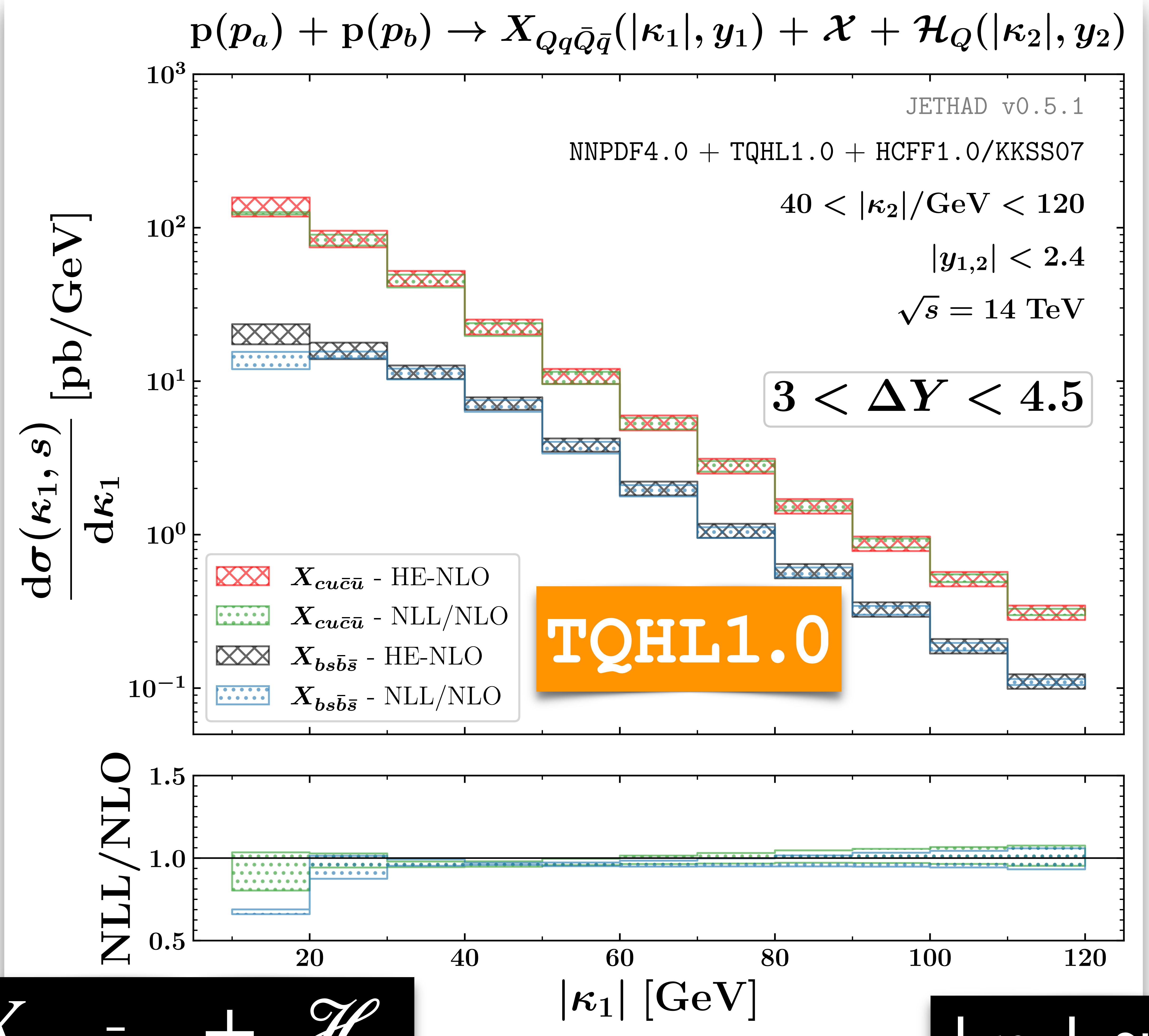
$X_{Qq\bar{Q}\bar{q}}$ + jet

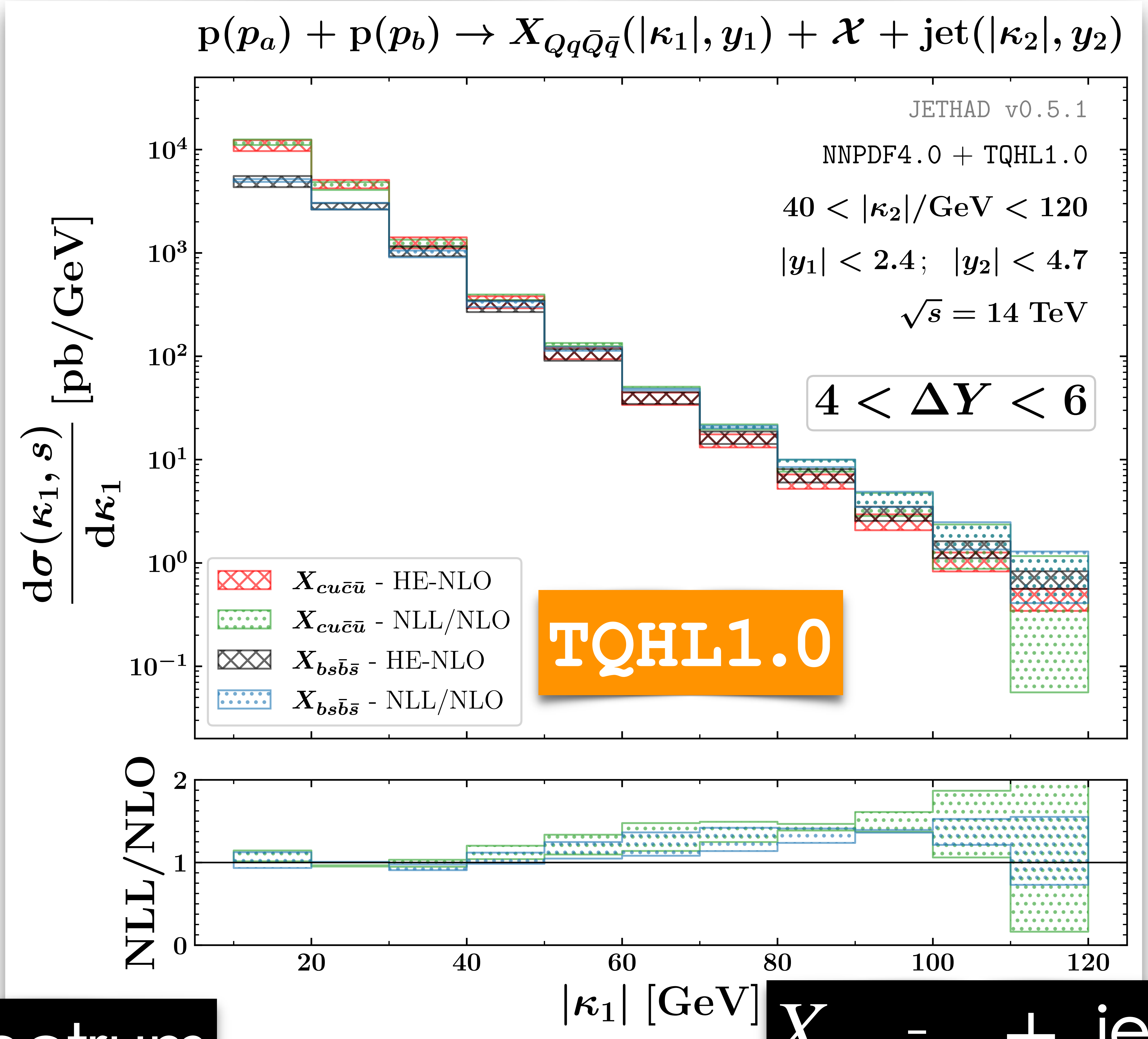
ΔY spectrum

[F. G. C., A. Papa, to appear in PLB]

Backup

Heavy-light tetraquarks at the HL-LHC





$X_{Qq\bar{Q}\bar{q}} + \mathscr{H}_c$

$|\kappa_1|$ spectrum

$X_{Qq\bar{Q}\bar{q}}$ + jet

[F. G. C., A. Papa, to appear in PLB]

Backup

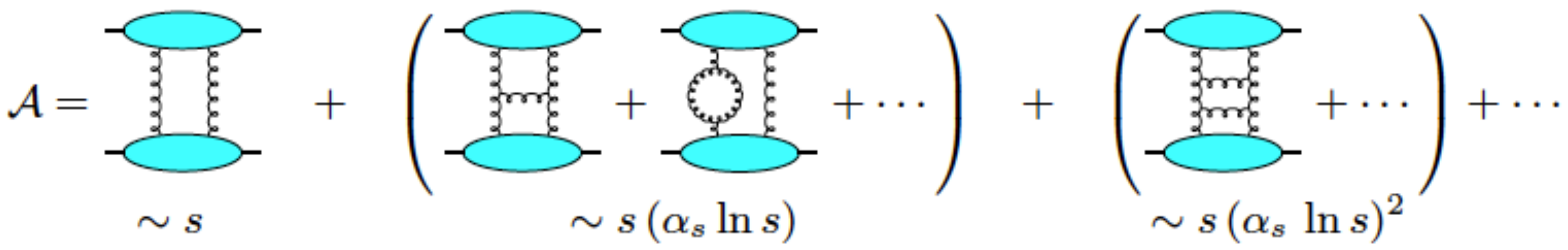
BASICS OF BFKL

The high-energy resummation

- **BFKL resummation**: [V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975, 1976, 1977); Y.Y. Balitskii, L.N. Lipatov (1978)]

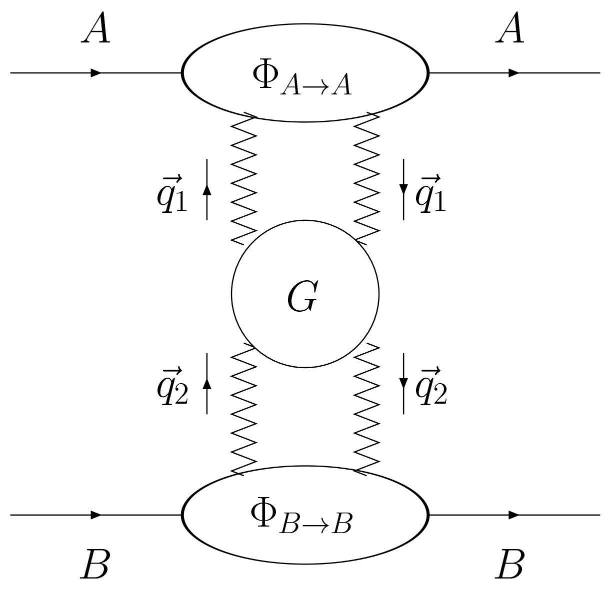
  $\xrightarrow{\text{based on}}$ **gluon Reggeization**

  leading logarithmic approximation (LL): $\alpha_s^n (\ln s)^n$



  next-to-leading logarithmic approximation (NLL): $\alpha_s^{n+1} (\ln s)^n$

Total cross section for $A + B \to X$: $\sigma_{AB}(s) = \frac{\mathcal{I}m_s\{\mathcal{A}_{AB}^{AB}\}}{s}$ $\Leftarrow$ ***optical theorem***



▶ $\mathcal{I}m_s\{\mathcal{A}_{AB}^{AB}\}$ factorization:

convolution of the **Green's function** of two interacting Reggeized gluons with the **impact factors** of the colliding particles

Green's function is **process-independent**, describes energy dependence and obeys BFKL equation; impact factors are known in the **NLL just for few processes**

Backup

The high-energy resummation

$$\mathrm{Im}_s\left(\mathcal{A}\right) = \frac{s}{(2\pi)^{D-2}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}} \Phi_A\left(\vec{q}_1, s_0\right) \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}} \Phi_B\left(-\vec{q}_2, s_0\right) \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega\left(\vec{q}_1, \vec{q}_2\right)$$

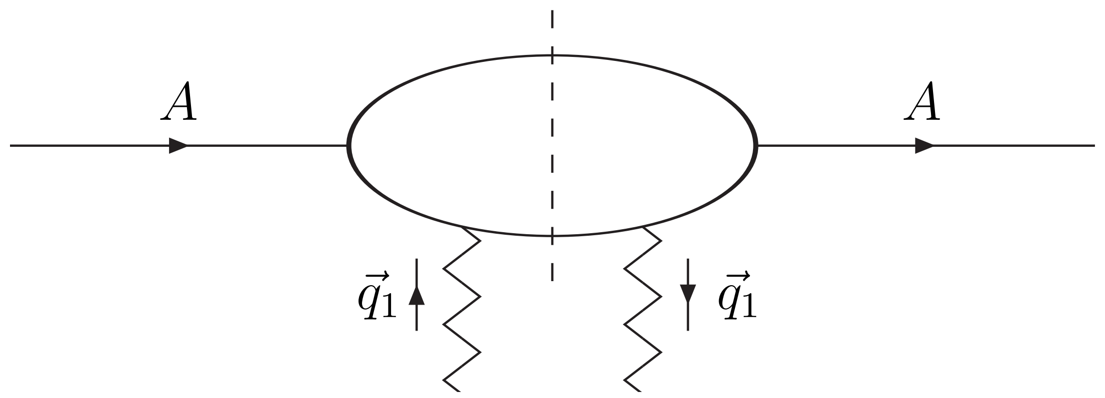
- **Green's function** is **process-independent** and takes care of the **energy dependence**

  $\longrightarrow$ determined through the **BFKL equation**

  [Ya.Ya. Balitskii, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

- **Impact factors** are **process-dependent** and depend on the hard scale, but not on the energy

  $\longrightarrow$ known in the NLA just for few processes



- <u>Successful tests</u> of NLA BFKL in the **Mueller–Navelet** channel with the advent of the LHC; nevertheless, *new BFKL-sensitive observables* as well as *more exclusive final-state reactions* are needed (**di-hadron**, **hadron-jet**, **heavy-quark pair**, **multi-jet**, production processes,...)

(**MN jets**) [B. Ducloué, L. Szymanowski, S. Wallon (2014); F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2015, 2016)]
(**di-hadron**) [F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2016, 2017)]
(**four-jet**) [F. Caporale, F.G.C., G. Chachamis, A. Sabio Vera (2016)]
(**multi-jet**) F. Caporale, F.G.C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016, 2017, 2017)]
(**heavy-quark pair**) [F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018); A.D. Bolognino, F.G.C., D.Yu. Ivanov, M. Fucilla, A. Papa (2018)]
(**hadron-jet**) [M.M.A. Mohammed, MD thesis (2018); A.D. Bolognino, F.G.C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

Backup

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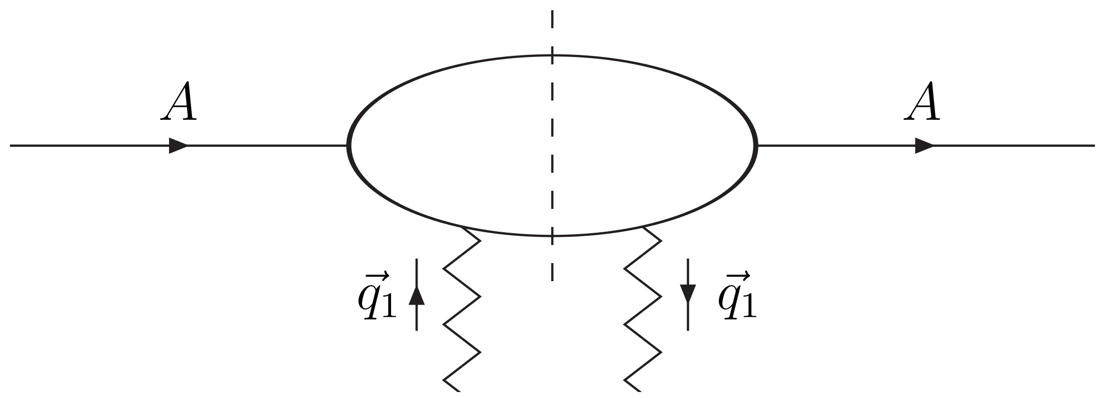

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(**multi-jet**) F. Caporale, F.G.C., G. Chachamis, D. Gordo Gómez, A. Sabio Vera (2016, 2017, 2017)]
(**heavy-quark pair**) [F.G.C., D.Yu. Ivanov, B. Murdaca, A. Papa (2018); A.D. Bolognino, F.G.C., D.Yu. Ivanov, M. Fucilla, A. Papa (2018)]
(**hadron-jet**) [M.M.A. Mohammed, MD thesis (2018); A.D. Bolognino, F.G.C., D.Yu. Ivanov, M.M.A. Mohammed, A. Papa (2018)]

¡ NEW ! NLO HIGGS

(κ_T space) [M. Hentschinski et al. (2021)]
(κ_T & Mellin) [F.G.C. et al. (2022)]

Backup

The high-energy resummation

Gluon Reggeization in perturbative QCD

◇ Gluon quantum numbers in the t-channel: 8^- representation

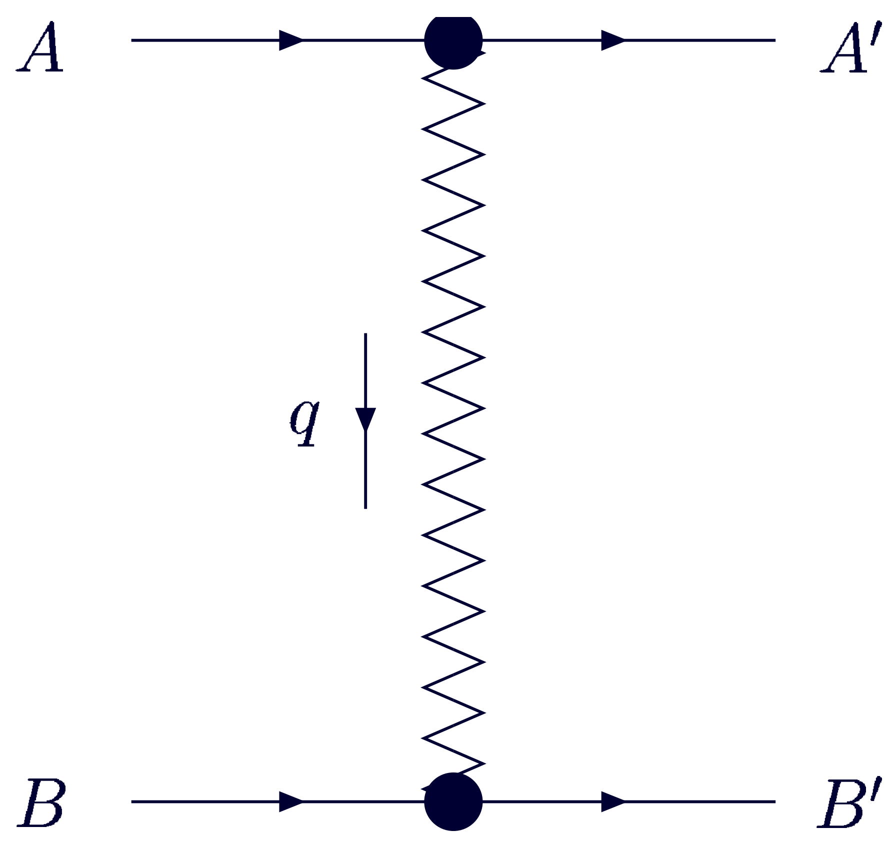
◇ Regge limit: $s \simeq -u \to \infty$, t not growing with s

$\to$ amplitudes governed by **gluon Reggeization** $\to$ $D_{\mu\nu} = -i\frac{g_{\mu\nu}}{q^2}\left(\frac{s}{s_0}\right)^{\alpha_g(q^2)-1}$

$\xrightarrow{\text{feature}}$ all-order resummation: **LLA** $[\alpha_s^n(\ln s)^n]$ + **NLA** $[\alpha_s^{n+1}(\ln s)^n]$

$\xrightarrow{\text{consequence}}$ factorization of elastic and real part of inelastic amplitudes

$\xrightarrow{\text{example}}$ Elastic scattering process: $A + B \longrightarrow A' + B'$



$$(\mathcal{A}_8^-)_{AB}^{A'B'} = \Gamma^c_{A'A}\left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{s}{-t}\right)^{j(t)}\right]\Gamma^c_{B'B}$$

$$j(t) = 1 + \omega(t)\,, \qquad j(0) = 1$$

$\omega(t) \ \to$ Reggeized gluon trajectory

$\Gamma^c_{A'A} = g\langle A'|T^c|A\rangle\Gamma_{A'A} \ \to$ PPR vertex

$T^c \ \to$ fundamental (q) or adjoint (g)

- QCD is the unique SM theory where all elementary particles reggeize
- Possible extensions: N=4 SYM, AdS/CFT,...

Backup

The high-energy resummation

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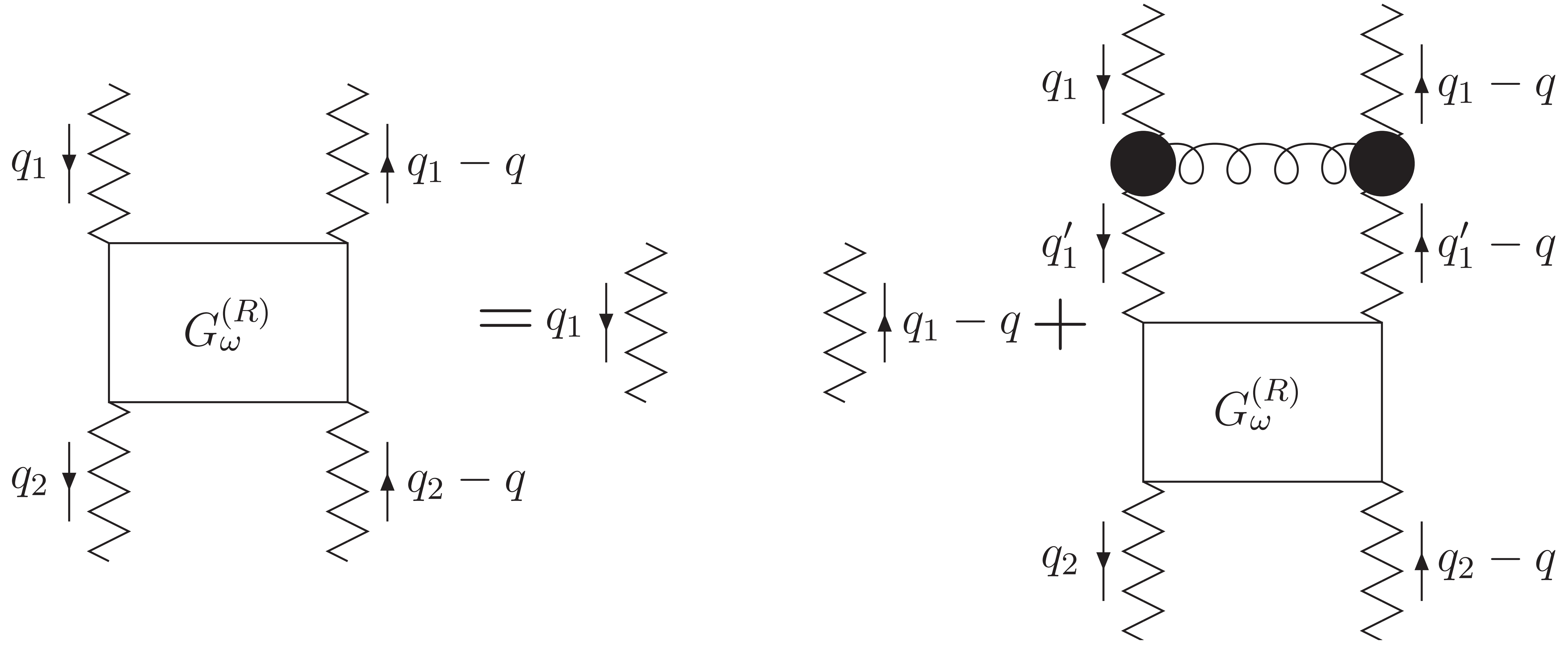
$\longrightarrow$ determined through the **BFKL equation**

[Ya.Ya. Balitskii, V.S. Fadin, E.A. Kuraev, L.N. Lipatov (1975)]

$$\omega\, G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}q\, K(\vec{q}_1, \vec{q})\, G_\omega(\vec{q}, \vec{q}_1)\,.$$



Backup

The high-energy resummation

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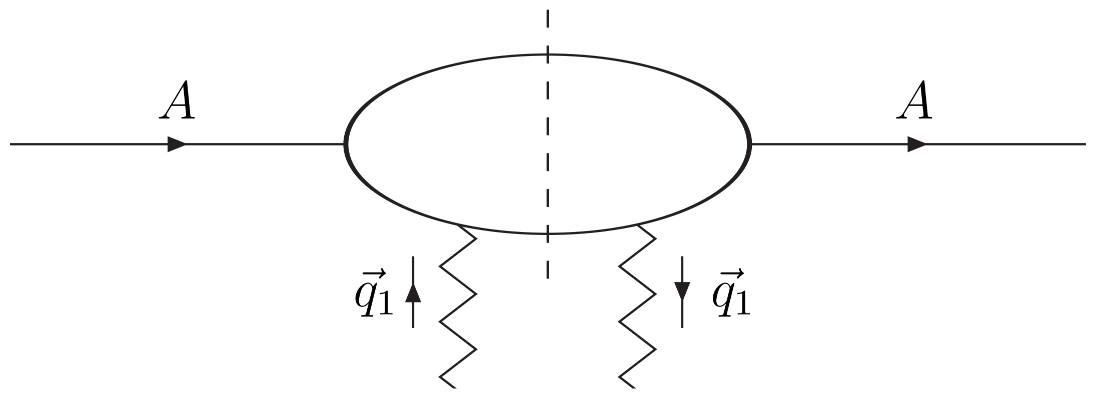


◇ **colliding partons**

[V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
[M. Ciafaloni, G. Rodrigo (2000)]

◇ $\gamma^* \longrightarrow V$, with $V = \rho^0,\ \omega,\ \phi$, forward case

[D.Yu. Ivanov, M.I. Kotsky, A. Papa (2004)]

◇ forward jet production

[J. Bartels, D. Colferai, G.P. Vacca (2003)]
(exact IF) [F. Caporale, D.Yu. Ivanov, B. Murdaca, A. Papa, A. Perri (2012)]
(small-cone IF) [D.Yu. Ivanov, A. Papa (2012)]
(several jet algorithms discussed) [D. Colferai, A. Niccoli (2015)]

◇ forward identified hadron production

[D.Yu. Ivanov, A. Papa (2012)]

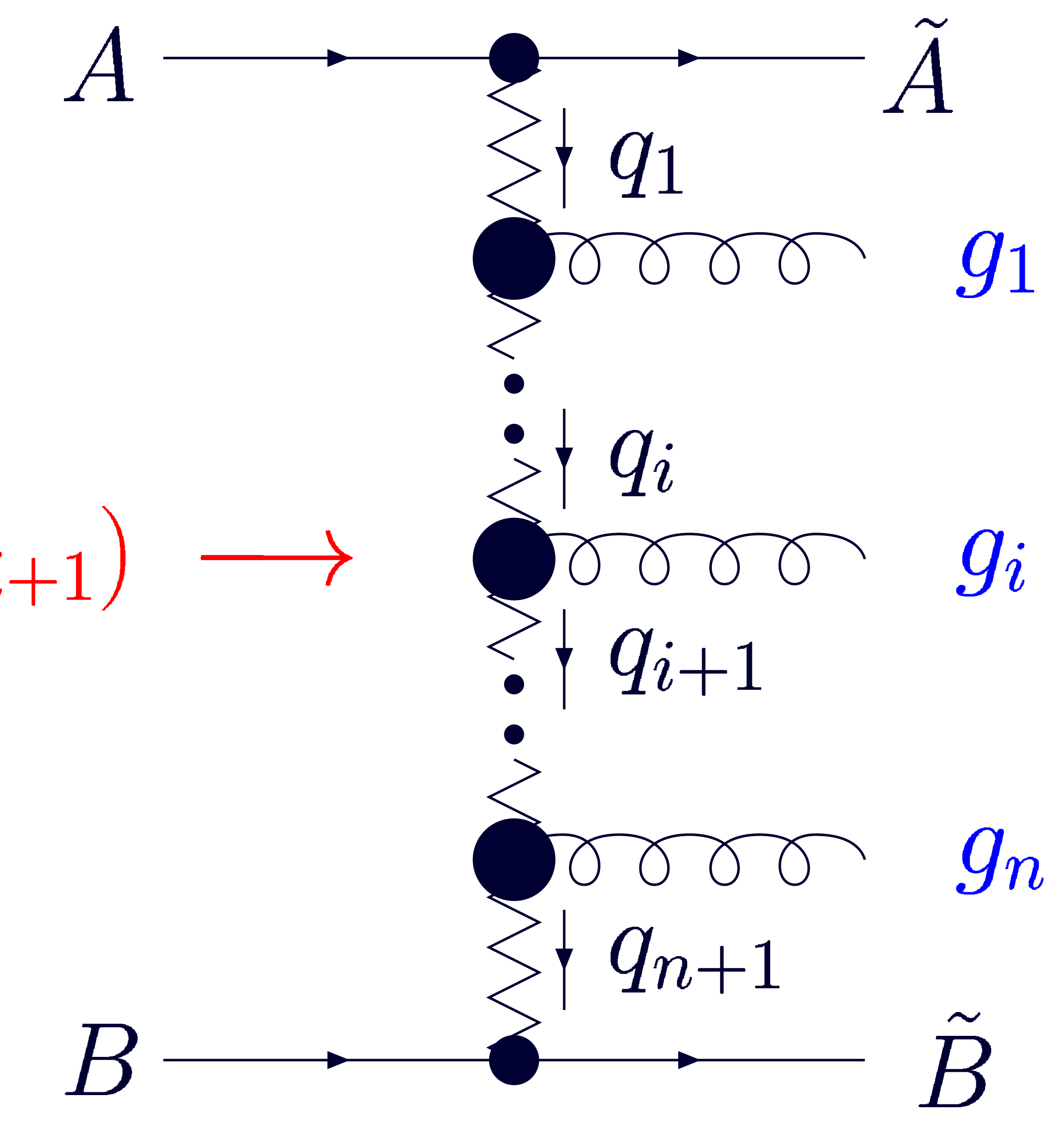
◇ $\gamma^* \longrightarrow \gamma^*$

[J. Bartels *et al.* (2001), I. Balitsky, G.A. Chirilli (2011, 2013)]

Backup

BFKL in the LLA (I)

Inelastic scattering process $A + B \;\; \rightarrow \;\; \tilde{A} + \tilde{B} + n$ in the LLA



multi-Regge kinematics

$$\mathrm{Re}\mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\,\Gamma^{c_1}_{\tilde{A}A}\left(\prod_{i=1}^{n}\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1})\left(\frac{s_i}{s_R}\right)^{\omega(t_i)}\frac{1}{t_i}\right)\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_R}\right)^{\omega(t_{n+1})}\Gamma^{c_{n+1}}_{\tilde{B}B}$$

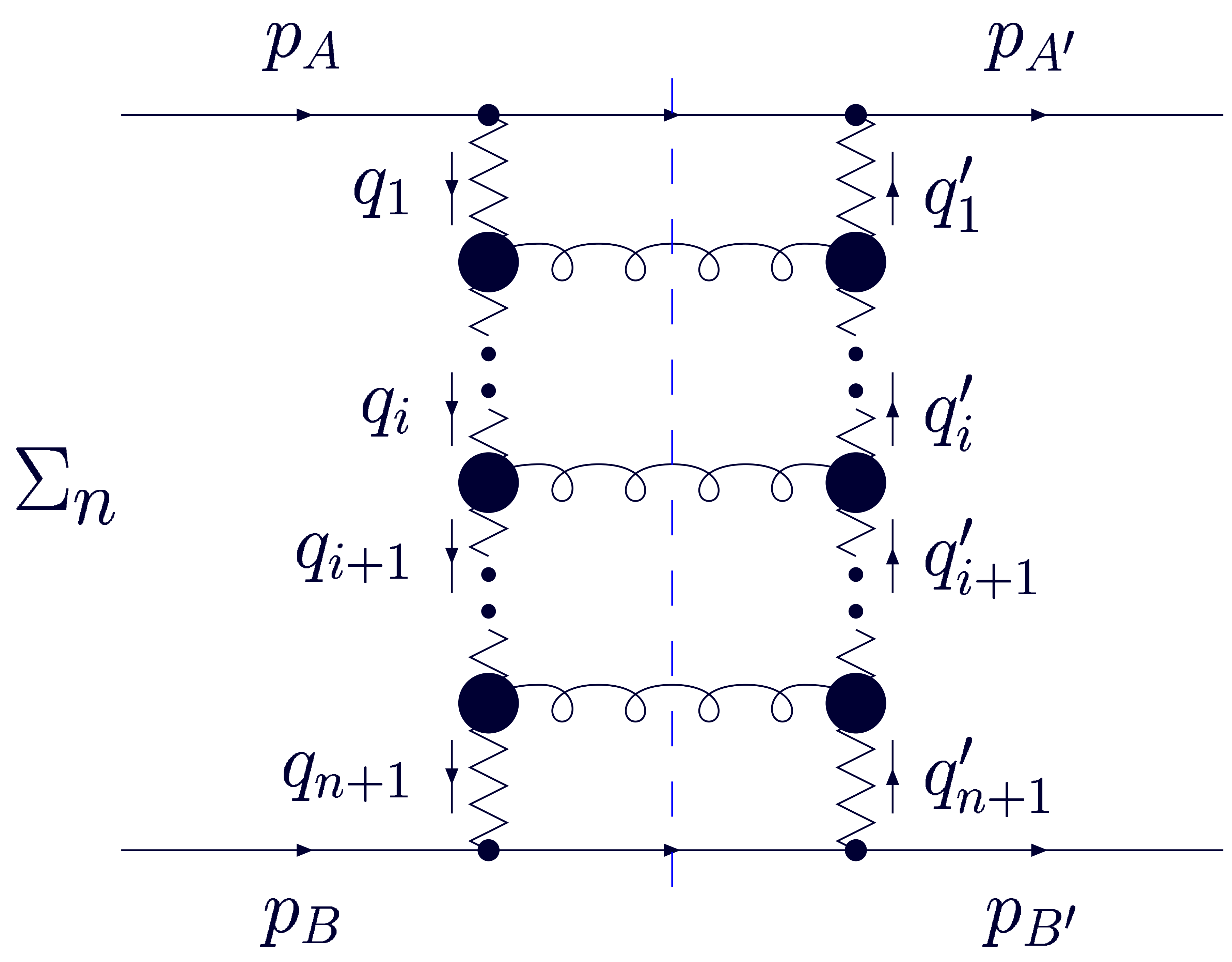
$\gamma^{P_i}_{c_i c_{i+1}}(q_i, q_{i+1}) \;\; \rightarrow$ RRG vertex

$s_R \;\; \rightarrow$ energy scale, irrelevant in the LLA

Backup

BFKL in the LLA (II)

Elastic amplitude $A + B \longrightarrow A' + B'$ in the LLA via s-channel unitarity



$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} (\mathcal{A}_{\mathcal{R}})_{AB}^{A'B'} \,, \qquad \mathcal{R} = 1 \text{ (singlet)}, \, 8^- \text{ (octet)}, \ldots$$

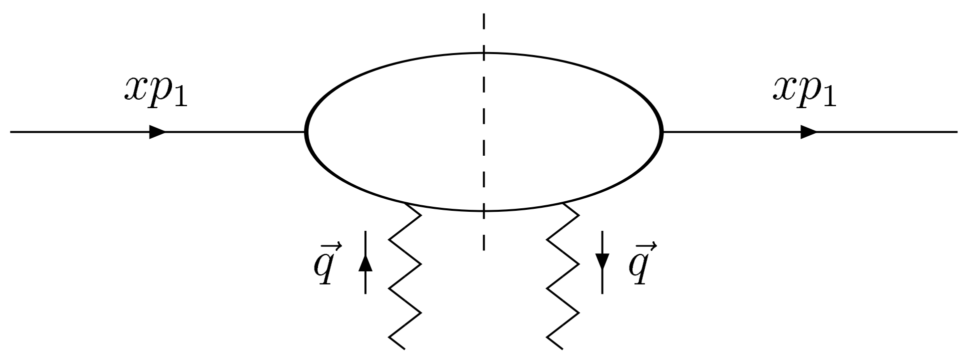
The 8^- color representation is important for the bootstrap, i.e. the consistency between the above amplitude and that with one Reggeized gluon exchange

Backup

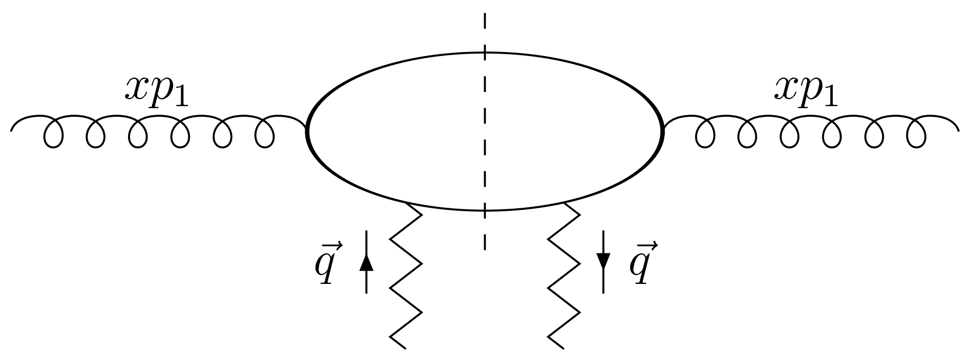
Forward-jet impact factor

- take the impact factors for **colliding partons**

  [V.S. Fadin, R. Fiore, M.I. Kotsky, A. Papa (2000)]
  [M. Ciafaloni and G. Rodrigo (2000)]

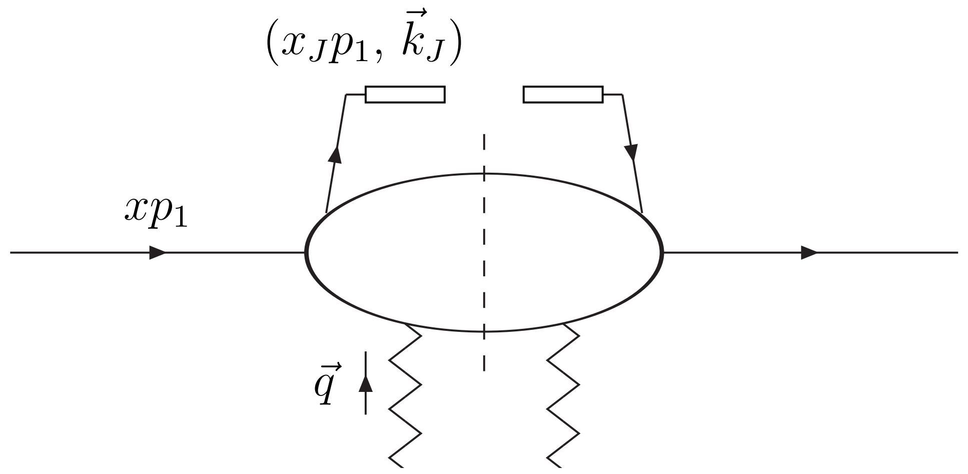


quark vertex

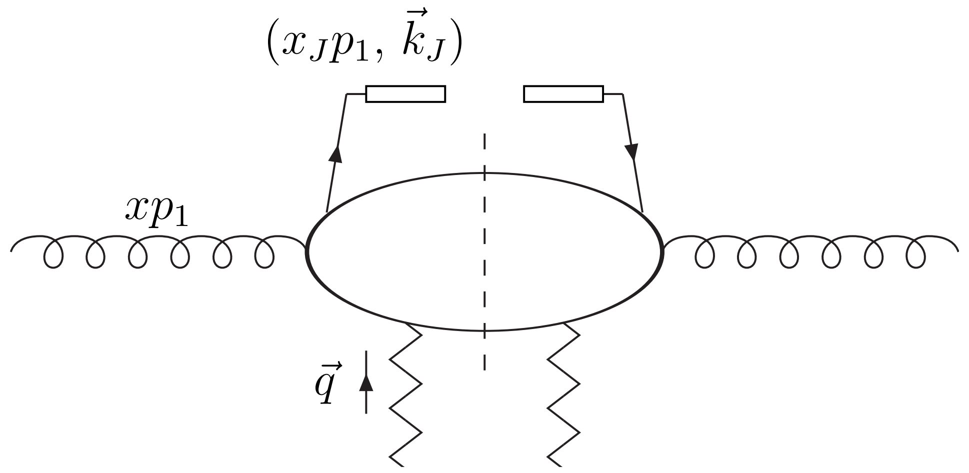


gluon vertex

- "open" one of the integrations over the phase space of the intermediate state to allow one parton to generate the jet



quark jet vertex

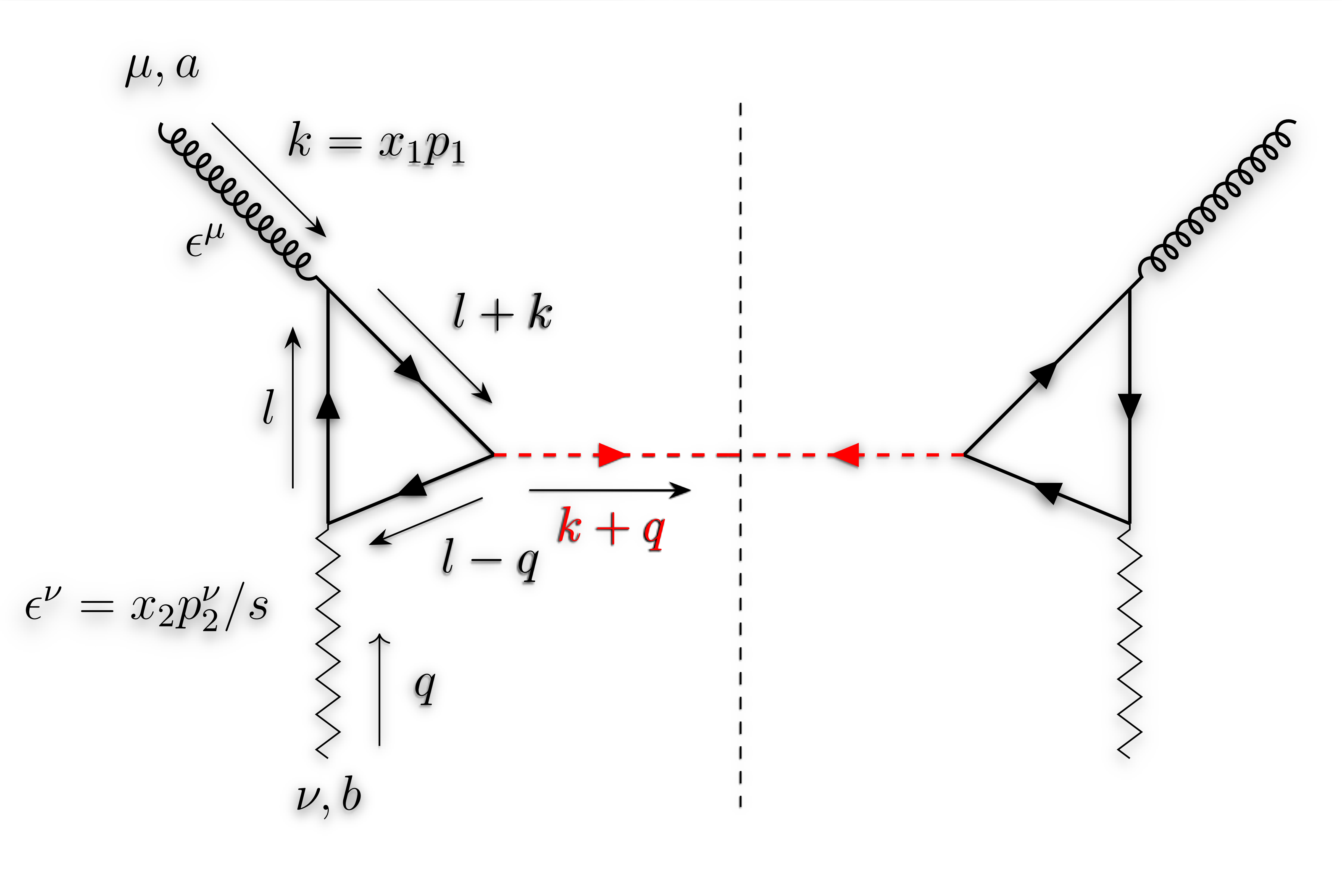


gluon jet vertex

- use QCD collinear factoriz.: $\sum_{s=q,\bar{q}} f_s \otimes$ [quark vertex] $+ f_g \otimes$ [gluon vertex]

Backup

Forward-Higgs LO impact factor



$$\frac{d\Phi_J^{(0)}(\nu, n)}{dx_J d^2\vec{p}_J} = 2\alpha_s \sqrt{\frac{C_F}{C_A}} (\vec{p}_J^{\,2})^{i\nu - 3/2} \left(\frac{C_A}{C_F} f_g(x_J) + \sum_{a=q\bar{q}} f_a(x_J) \right) e^{in\phi_J}$$

Backup

Forward-Higgs NLO-RG impact factor

$$\tilde{c}_H^{(1)}(n,\nu,|\vec{p}_H|,x_H) = c_H(n,\nu,|\vec{p}_H|,x_H)\left\{\frac{\beta_0}{4N_c}\left(2\ln\frac{\mu_{R_1}}{|\vec{p}_H|}+\frac{5}{3}\right)+\frac{\chi\left(n,\nu\right)}{2}\ln\left(\frac{s_0}{M_{H,\perp}^2}\right)\right.$$

$$+\frac{\beta_0}{4N_c}\left(2\ln\frac{\mu_{R_1}}{M_{H,\perp}}\right)$$

$$\left.-\frac{1}{2N_c f_g(x_H,\mu_{F_1})}\ln\frac{\mu_{F_1}^2}{M_{H,\perp}^2}\int_{x_H}^1\frac{dz}{z}\left[P_{gg}(z)f_g\left(\frac{x_H}{z},\mu_{F_1}\right)+\sum_{a=q,\bar{q}}P_{ga}(z)f_a\left(\frac{x_H}{z},\mu_{F_1}\right)\right]\right\}$$

Backup

Forward-jet NLO-RG impact factor

$$\tilde{c}_J^{(1)}(n,\nu,|\vec{p}_J|,x_J) = c_J(n,\nu,|\vec{p}_J|,x_J)\left\{\frac{\beta_0}{4N_c}\left(2\ln\frac{\mu_{R_2}}{|\vec{p}_J|}+\frac{5}{3}\right)+\frac{\chi\left(n,\nu\right)}{2}\ln\left(\frac{s_0}{|\vec{p}_J|^2}\right)\right.$$

$$-\frac{1}{2N_c\left(\frac{C_A}{C_F}f_g(x_J,\mu_{F_2})+\sum_{a=q,\bar{q}}f_a(x_J,\mu_{F_2})\right)}\ln\frac{\mu_{F_2}^2}{|\vec{p}_J|^2}$$

$$\times\left(\frac{C_A}{C_F}\int_{x_J}^1\frac{dz}{z}\left[P_{gg}(z)f_g\left(\frac{x_J}{z},\mu_{F_2}\right)+\sum_{a=q,\bar{q}}P_{ga}(z)f_a\left(\frac{x_J}{z},\mu_{F_2}\right)\right]\right.$$

$$\left.\left.+\sum_{a=q,\bar{q}}\int_{x_J}^1\frac{dz}{z}\left[P_{ag}(z)f_g\left(\frac{x_J}{z},\mu_{F_2}\right)+P_{aa}(z)f_a\left(\frac{x_J}{z},\mu_{F_2}\right)\right]\right)\right\}\,.$$

Backup

Inclusive Higgs + jet: NLL/NLO* azimuthal coefficients

$$\mathcal{C}_n = \frac{e^{\Delta Y}}{s} \frac{M_{H,\perp}}{|\vec{p}_H|}$$

$$\times \int_{-\infty}^{+\infty} d\nu \left(\frac{x_J x_H s}{s_0} \right)^{\bar{\alpha}_s(\mu_{R_c}) \left\{ \chi(n,\nu) + \bar{\alpha}_s(\mu_{R_c}) \left[\bar{\chi}(n,\nu) + \frac{\beta_0}{8N_c} \chi(n,\nu) \left[-\chi(n,\nu) + \frac{10}{3} + 4 \ln \left(\frac{\mu_{R_c}}{\sqrt{\vec{p}_H \vec{p}_J}} \right) \right] \right] \right\}}$$

$$\times \left\{ \alpha_s^2(\mu_{R_1}) c_H(n, \nu, |\vec{p}_H|, x_H) \right\} \left\{ \alpha_s(\mu_{R_2}) [c_J(n, \nu, |\vec{p}_J|, x_J)]^* \right\}$$

$$\times \left\{ 1 + \bar{\alpha}_s(\mu_{R_1}) \frac{\tilde{c}_H^{(1)}(n, \nu, |\vec{p}_H|, x_H)}{c_H(n, \nu, |\vec{p}_H|, x_H)} + \bar{\alpha}_s(\mu_{R_2}) \left[\frac{\tilde{c}_J^{(1)}(n, \nu, |\vec{p}_J|, x_J)}{c_J(n, \nu, |\vec{p}_J|, x_J)} \right]^* \right\} .$$

Backup