Dark Matter annihilation

&

quasi-bound states

Milan

Christmas Meeting 2023

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Outline

- Dark Matter annihilation
- Sommerfeld enhancement
- S-wave
- P-wave

Model

Cold Dark Matter

Heavy Fermion χ

Dark Interaction

$g\varphi\bar{\chi}\chi,\ g\bar{\chi}\gamma^{\mu}\chi\varphi_{\mu}, \ldots$

Light Mediator

Scalar, Vector, … $\varphi^{(\mu)}$

Annihilation process

$\bar{\chi}\chi \rightarrow \varphi^{(\mu)}\varphi^{(\mu)} \rightarrow \mathrm{SM}$

Mixed with SM

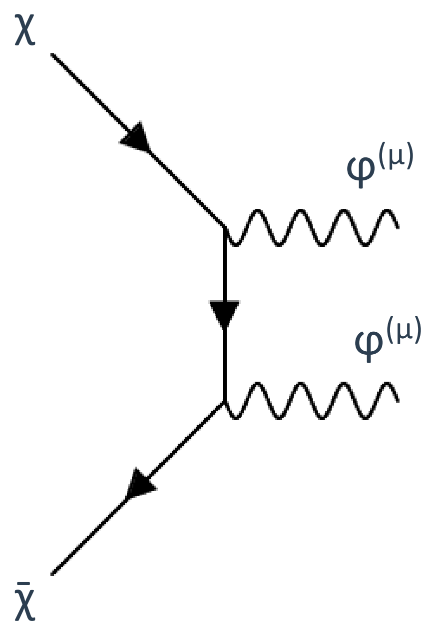
Mixed with Higgs,

Dark U(1) mixed with $U(1)_Y$

DM annihilation

Annihilation process

$$\bar{\chi}\chi \to \varphi^{(\mu)}\varphi^{(\mu)} \to \text{SM}$$



Heavy Dark Matter

↓

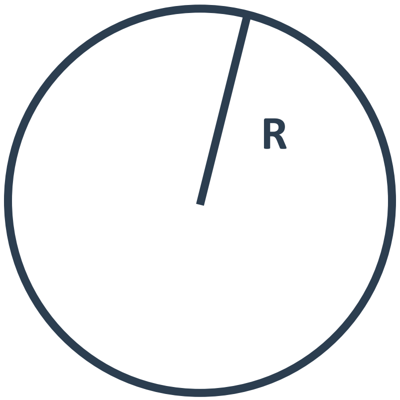
Non-relativistic limit of annihilation cross section

Classical analogy

Classical analogy

[N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer, N. Weiner, 1502.01281]

Slow object



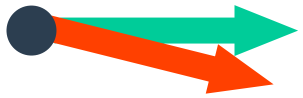
$\sigma_0 = \pi R^2$

Classical analogy

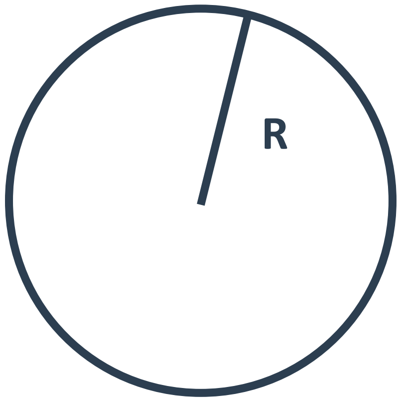
Classical analogy

[N. Arkani-Hamed, D. P. Finkbeiner, T. R. Slatyer, N. Weiner, 1502.01281]

Slow object



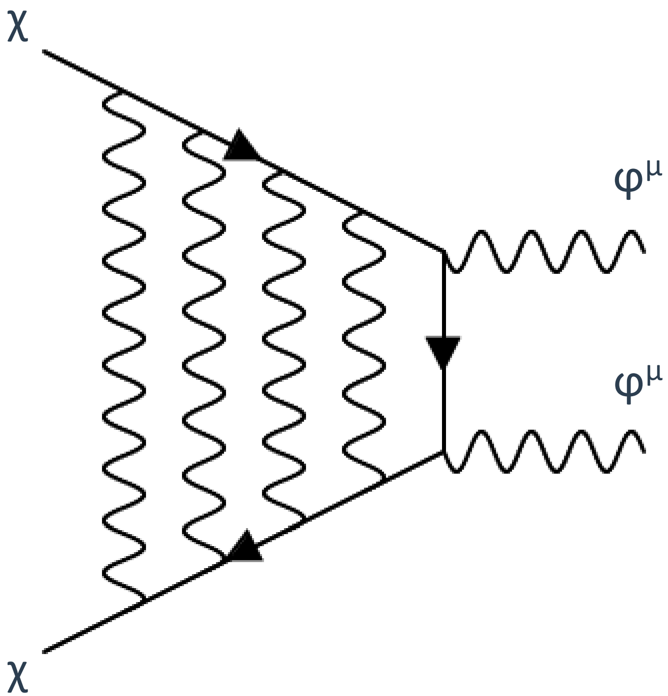
With gravitational attraction

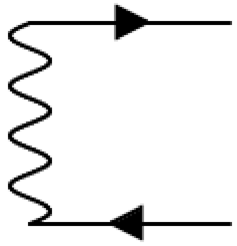


$$\sigma = \sigma_0(1+v_{esc}^2/v^2)$$

The cross section is enhanced at low velocity

Non-relativistic annihilation





Every ladder insertion

$\propto \alpha / v_{rel}$

for $v_{rel} \lesssim \alpha$

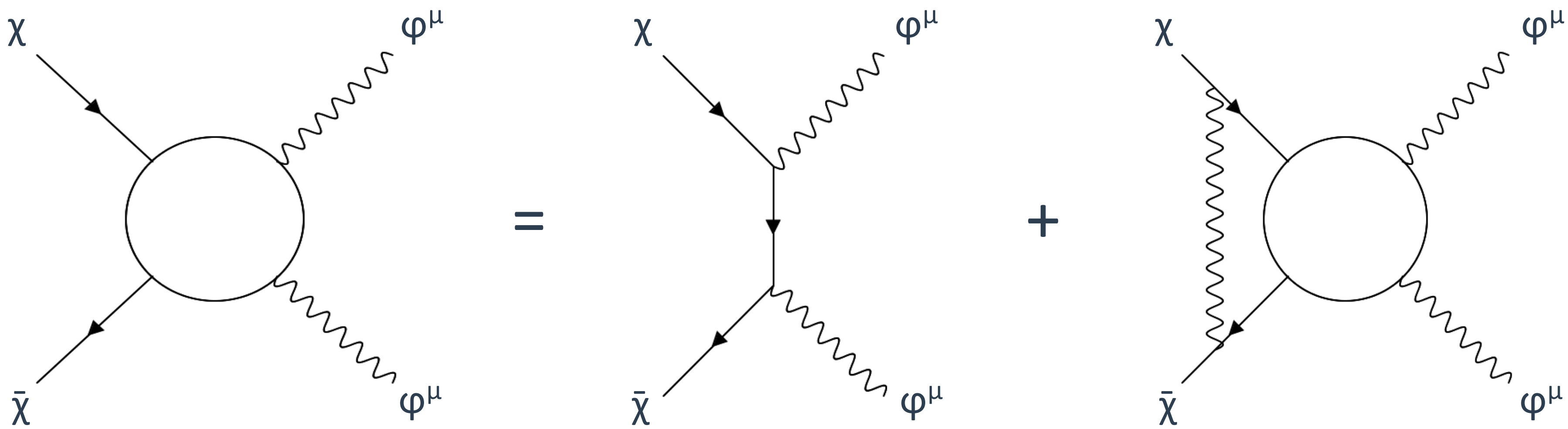
ladder resummation

is needed

Diagrammatic approach

Bethe-Salpeter equation

[R. Iengo, 0902.0688][S. Cassel, 0903.5307]



$$|A^\ell|^2 = |A_0{}^\ell|^2\, S^\ell(v_{rel})$$

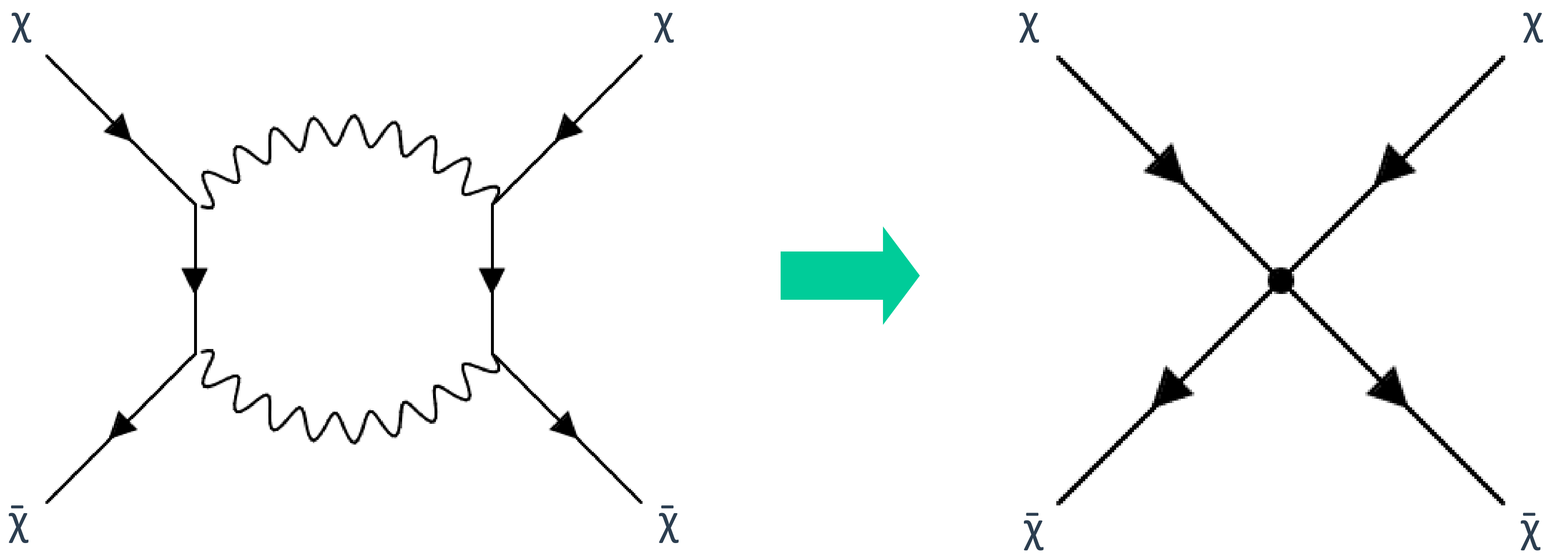
Sommerfeld factor

$$S^{(\ell)}(v_{rel}) = |u^{(\ell)}(0)|^2$$

Schrödinger equation

$$-u''(r)+V^\ell(r)u(r)= E\,u(r)$$

NREFT approach

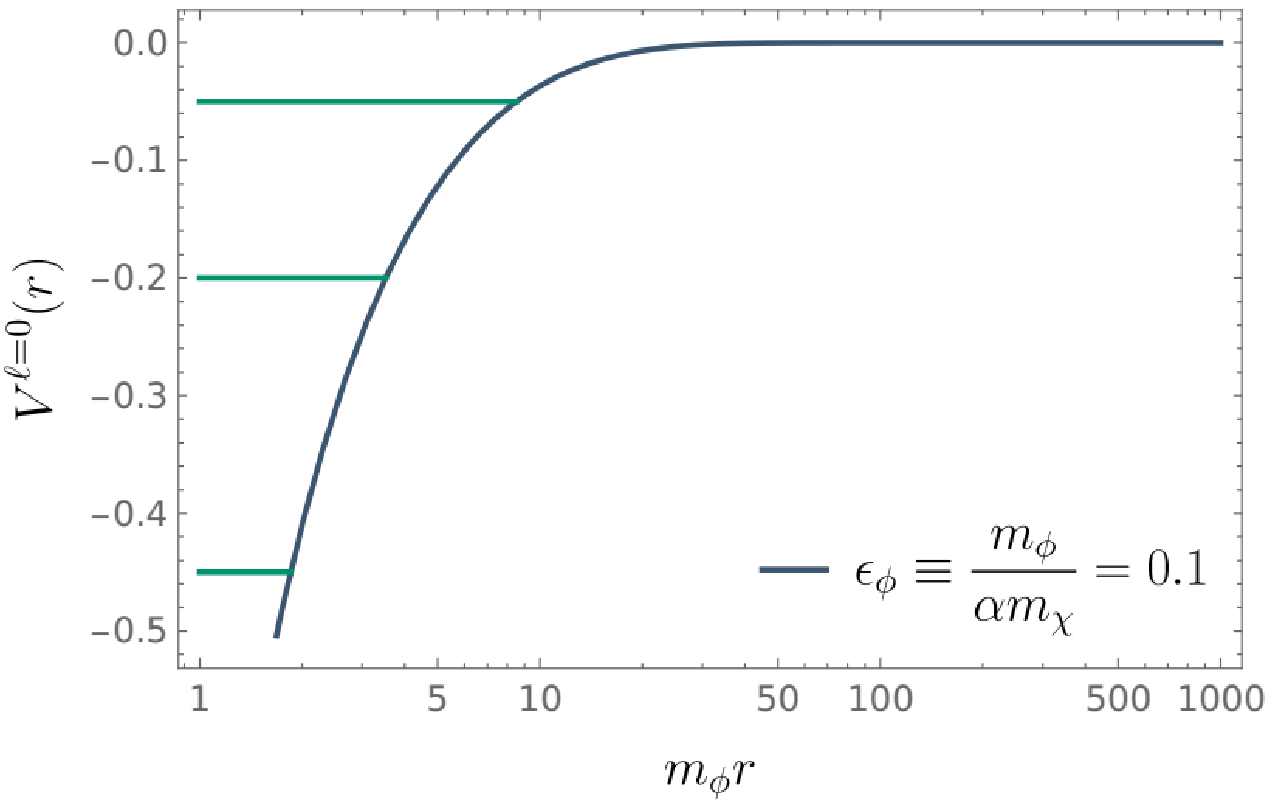


$$O^{\ell} = \bar{\chi}(\sigma)\partial^{(\ell)}\chi\,\bar{\chi}(\sigma)\partial^{(\ell)}\chi$$

$$S^{\ell} \sim \langle\bar{\chi}\chi|O^{\ell}|\bar{\chi}\chi\rangle \sim |u^{(\ell)}(0)|^2$$

S-wave Sommerfeld factor

$$-u''(r)+V^{\ell=0}(r)u(r) = E\,u(r)$$



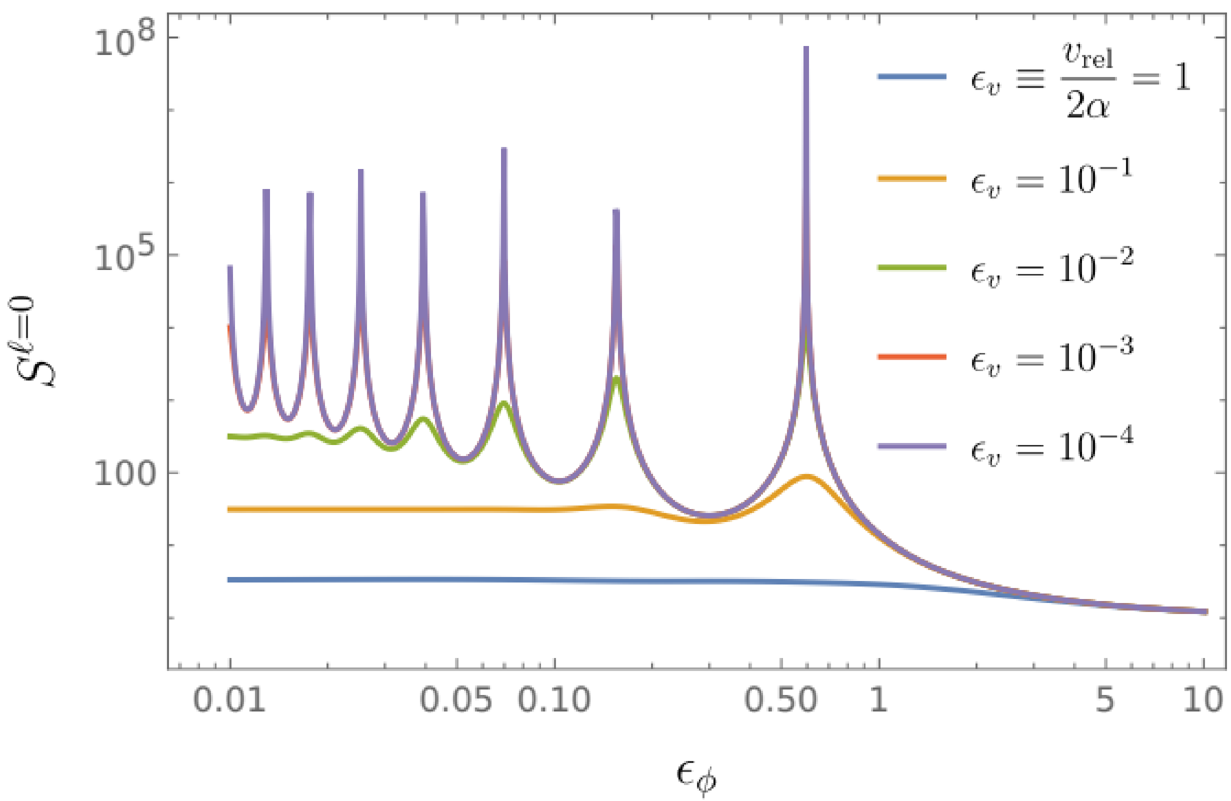
Yukawa potential

$$V^{\ell=0}(r) = -\alpha e^{-m_\varphi r}/r$$

Finite number of bound-states

S-wave Sommerfeld factor

$-u''(r)+V^{\ell=0}(r)u(r) = E\,u(r)$



Yukawa potential

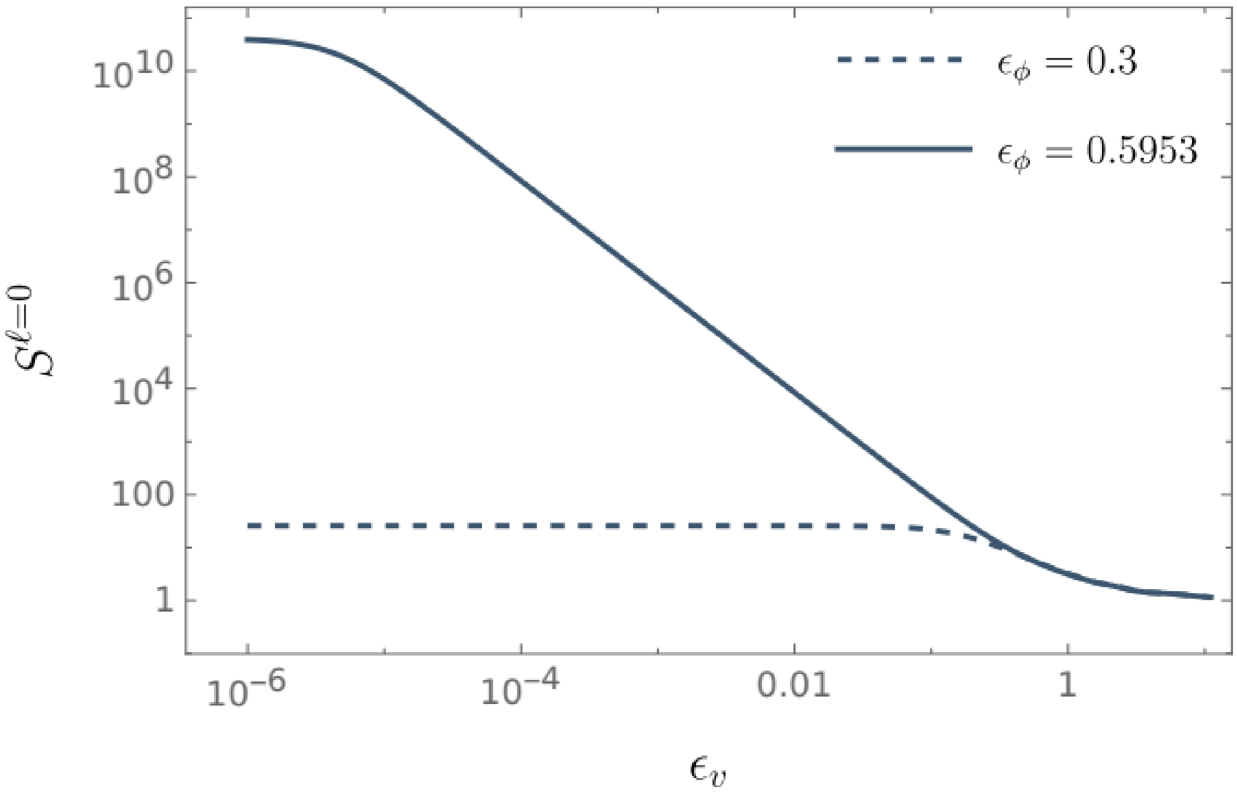
$V^{\ell=0}(r) = -\,\alpha e^{-m_\varphi r}/r$

Finite number of bound-states

Critical values of ϵ_φ threshold for the appearance of a new bound-state

S-wave Sommerfeld factor

$$-u''(r)+V^{\ell=0}(r)u(r) = E\,u(r)$$



Yukawa potential

$$V^{\ell=0}(r) = -\alpha e^{-m_\varphi r}/r$$

Finite number of bound-states

Critical values of ϵ_φ threshold for the appearance of a new bound-state

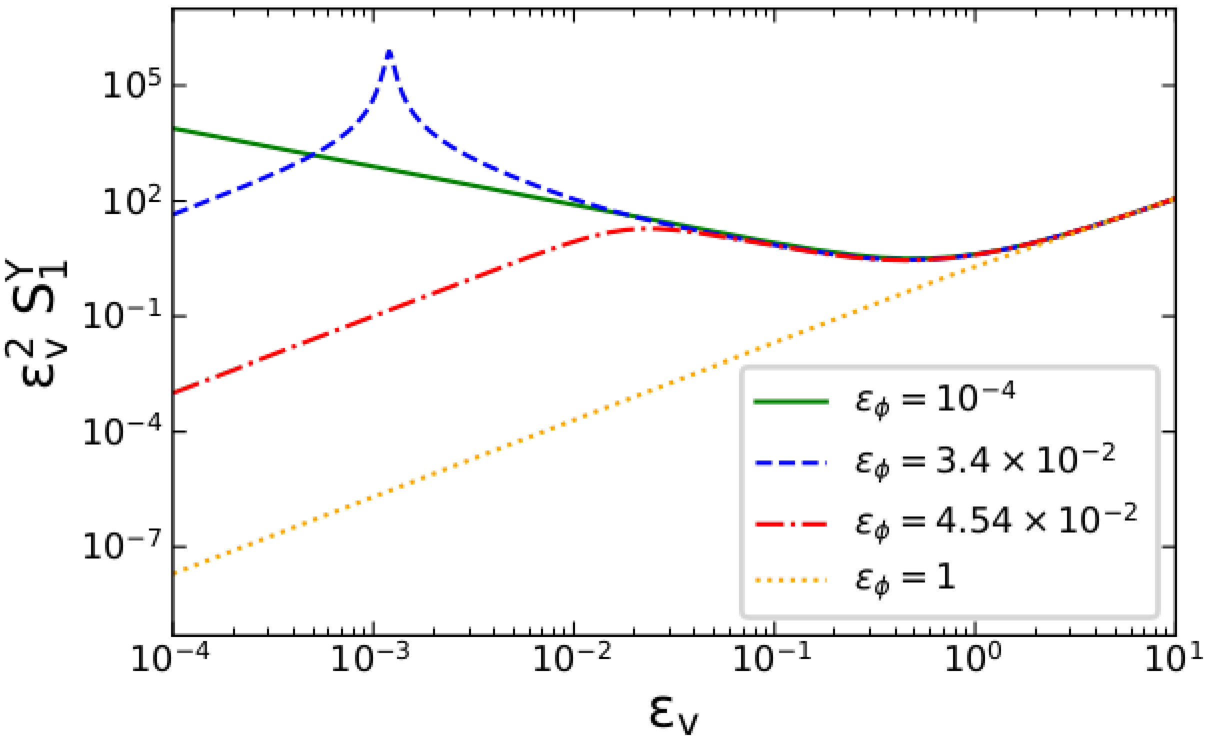
$1/v_{rel}^2$ **scaling close to critical ϵ_φ**

P-wave Sommerfeld factor

$$-u''(r)+V^{\ell}(r)u(r) = E\,u(r)$$

Yukawa potential

$$V^{\ell}(r) = -\,\alpha e^{-m_{\varphi} r}/r+\ell(\ell+1)/r^2$$



[Y. Ding, Y. Ku, C. Wei, Y. Zhou, 2104.14881]

[A. Kamada , T. Kuwaharab, A. Patel, 2303.17961]

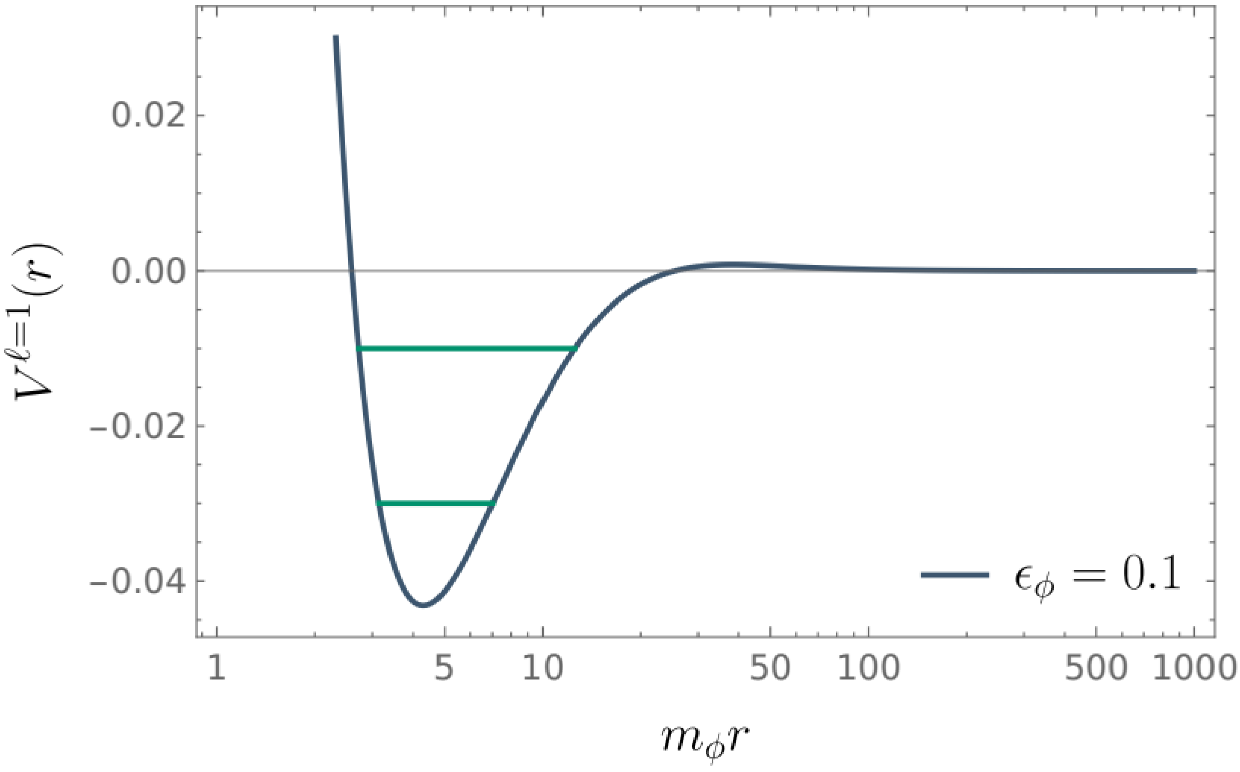
P-wave Sommerfeld factor

$$-u''(r)+V^{\ell}(r)u(r) = E\, u(r)$$

Yukawa potential

$$V^{\ell}(r) = -\alpha e^{-m_\varphi r}/r+\ell(\ell+1)/r^2$$

Finite number of bound-states

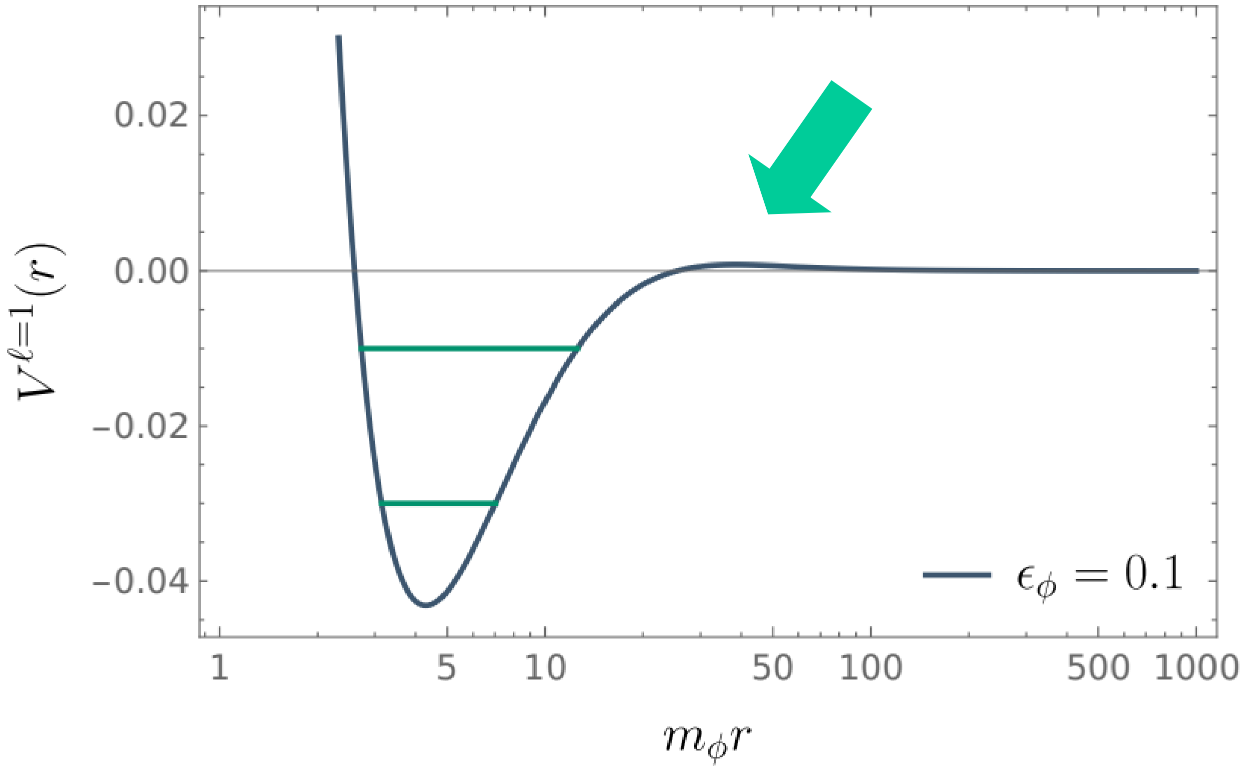


P-wave Sommerfeld factor

$$-u''(r)+V^{\ell}(r)u(r) = E\,u(r)$$

Yukawa potential

$$V^{\ell}(r) = -\alpha e^{-m_{\varphi} r}/r+\ell(\ell+1)/r^2$$



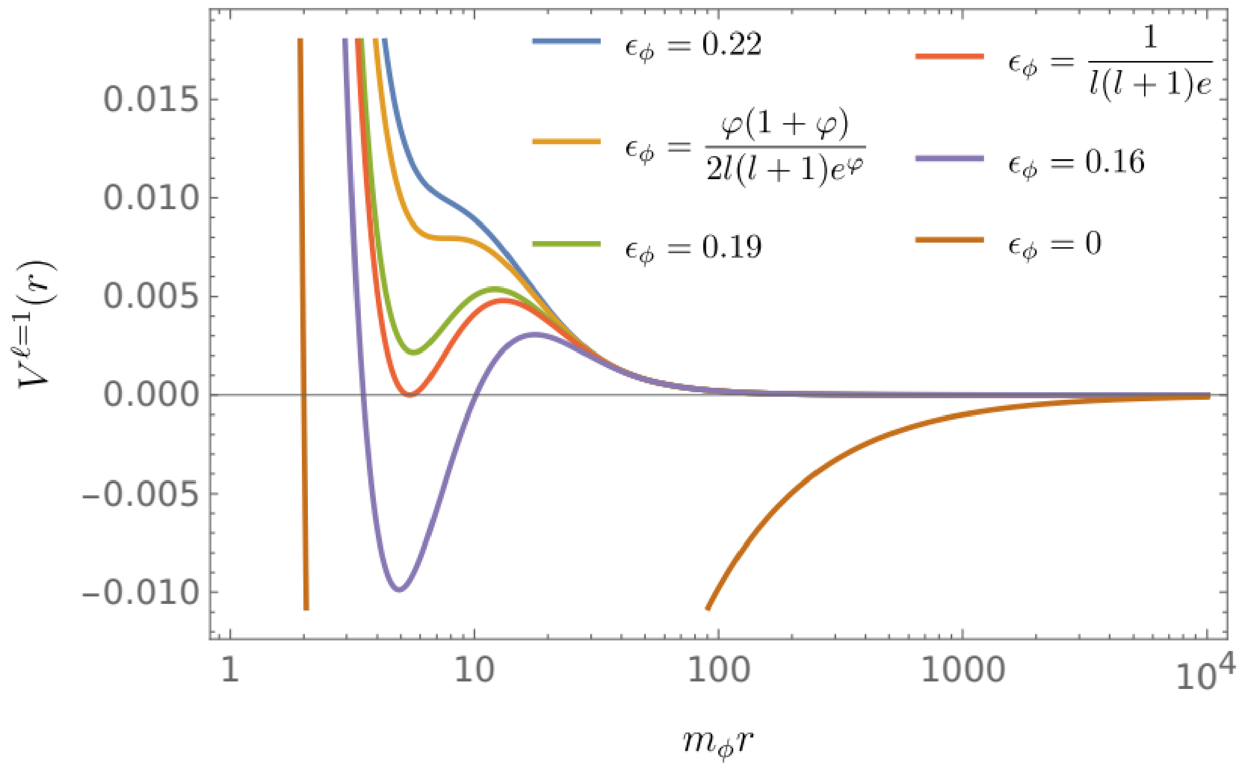
Finite number of bound-states

P-wave Sommerfeld factor

$-u''(r)+V^{\ell}(r)u(r) = E\,u(r)$

Yukawa potential

$V^{\ell}(r) = -\alpha e^{-m_\varphi r}/r+\ell(\ell+1)/r^2$



Finite number of bound-states

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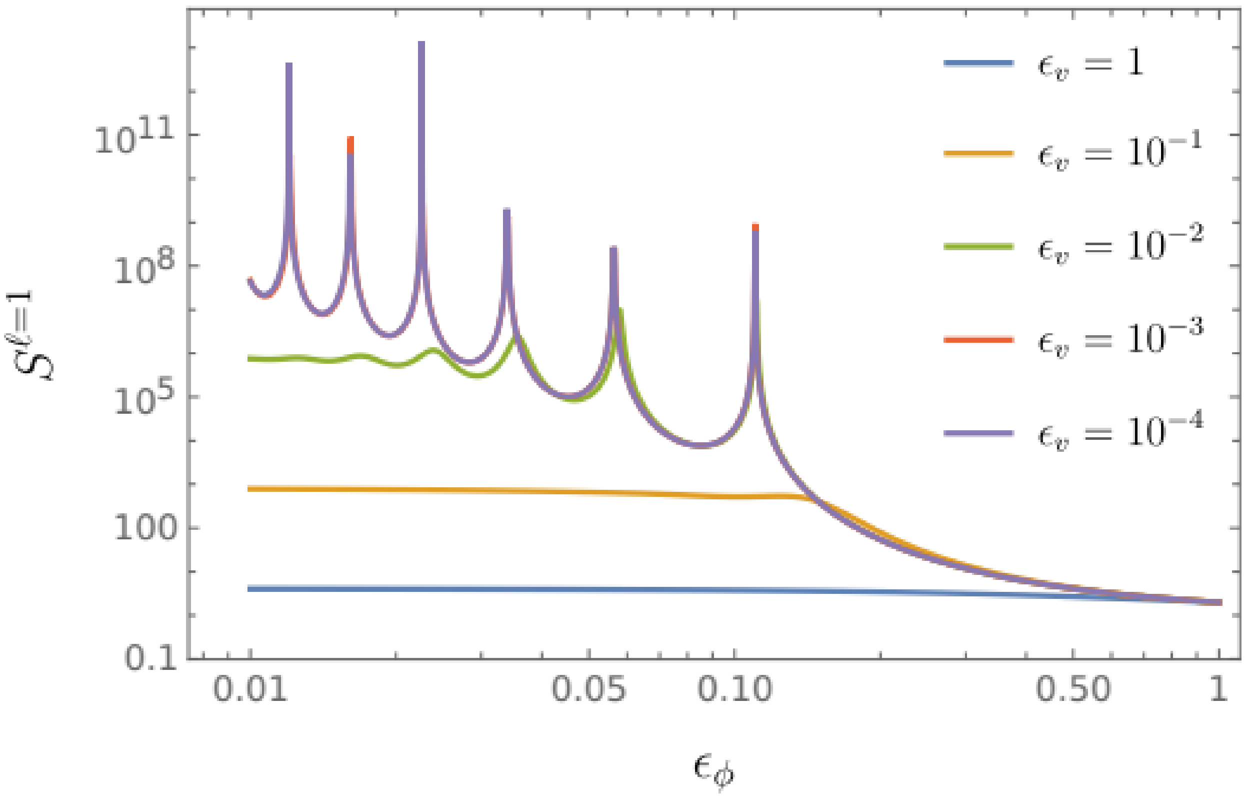
Centrifugal barrier

P-wave Sommerfeld factor

$$-u''(r)+V^{\ell}(r)u(r) = E\, u(r)$$



Yukawa potential

$$V^{\ell}(r) = -\alpha e^{-m_\varphi r}/r+\ell(\ell+1)/r^2$$

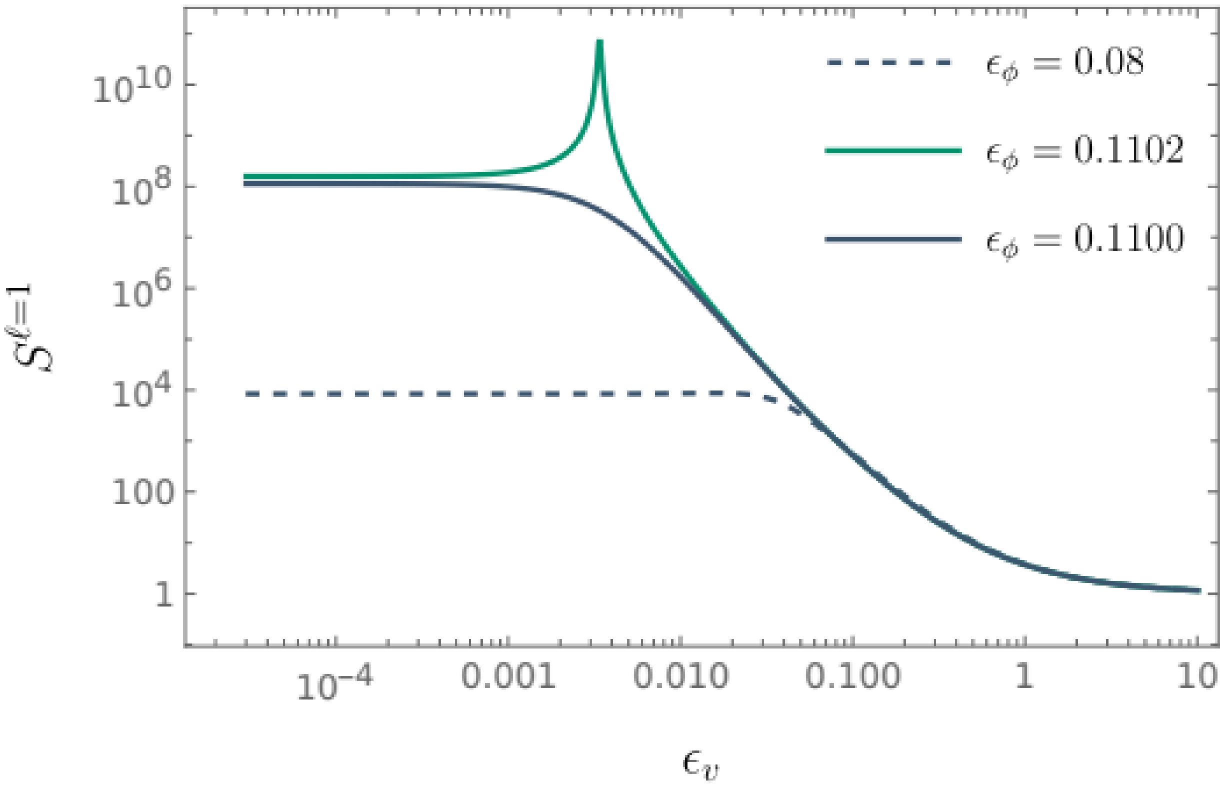
Finite number of bound-states

&

Centrifugal barrier

P-wave Sommerfeld factor

$-u''(r)+V^{\ell}(r)u(r) = E\,u(r)$



Yukawa potential

$V^{\ell}(r) = -\alpha e^{-m_\varphi r}/r+\ell(\ell+1)/r^2$

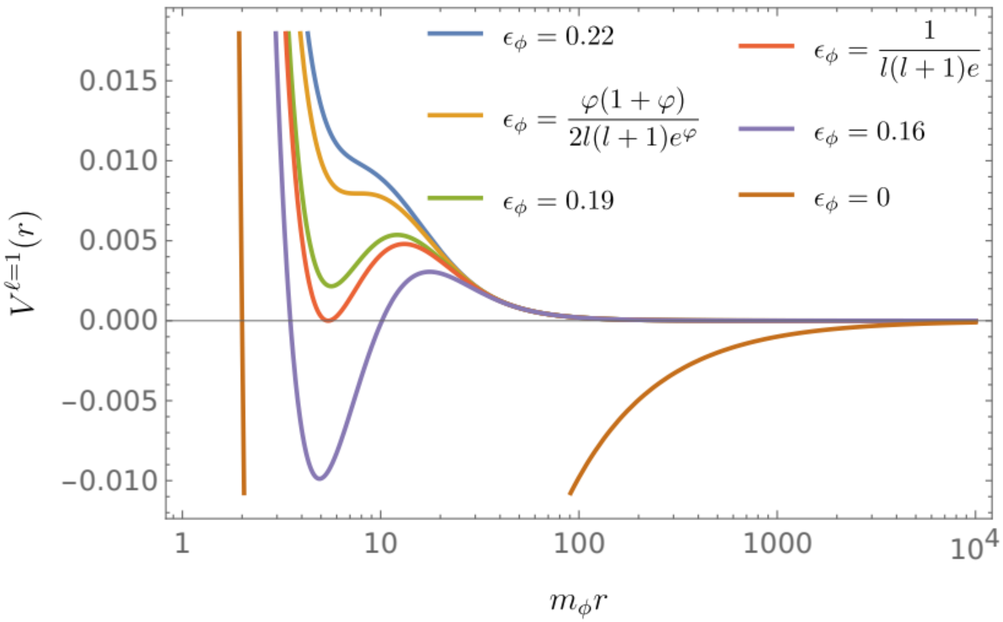
Finite number of bound-states

&

Centrifugal barrier

Resonances corresponding to *quasi*-bound states

WKB estimates



$\Gamma \sim v_{rel}^{2\ell+1}$

$I_w = (n+1/2)\pi$

E_{QBS}

$$S_l \approx \frac{[(2l+1)!!]^2 e^{2I_b}}{(l+1/2)\epsilon_v^{2l+1}} \frac{(\Gamma_{nl}/2)^2}{(\epsilon_v^2-\gamma_{nl})^2+(\Gamma_{nl}/2)^2} \times \exp\left\{2\int_0^{x_1} dx' \, \ln(x') \frac{d}{dx'}[x'p(x')]\right\}$$

Wino limit of MSSM

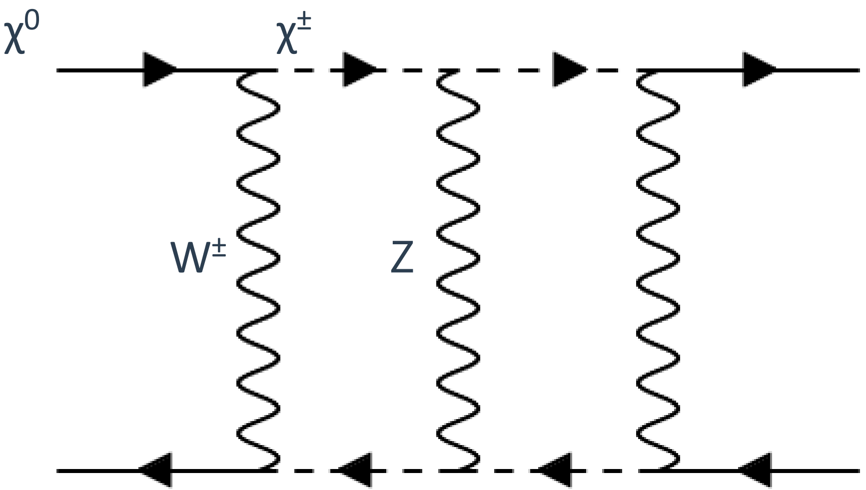
Lightest neutralino Dark Matter χ^0

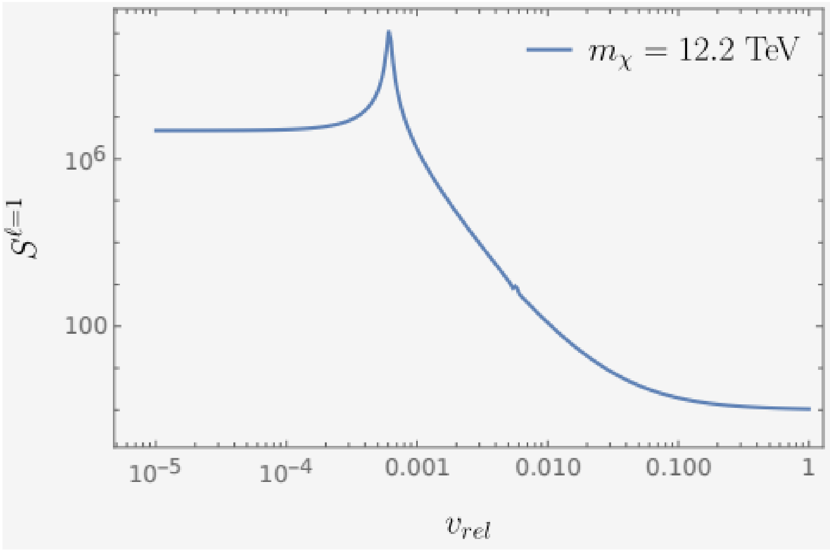
Charginos $\chi^{\pm}$
with mass splitting
$\delta m = m^{\pm} - m^0$

2-channel interaction potential

$$\begin{pmatrix} 0 & -\sqrt{2}\,\alpha_2 \frac{e^{-M_W r}}{r} \\ -\sqrt{2}\,\alpha_2 \frac{e^{-M_W r}}{r} & -\frac{\alpha}{r} - \alpha_2 c_W^2 \frac{e^{-M_Z r}}{r} \end{pmatrix}$$

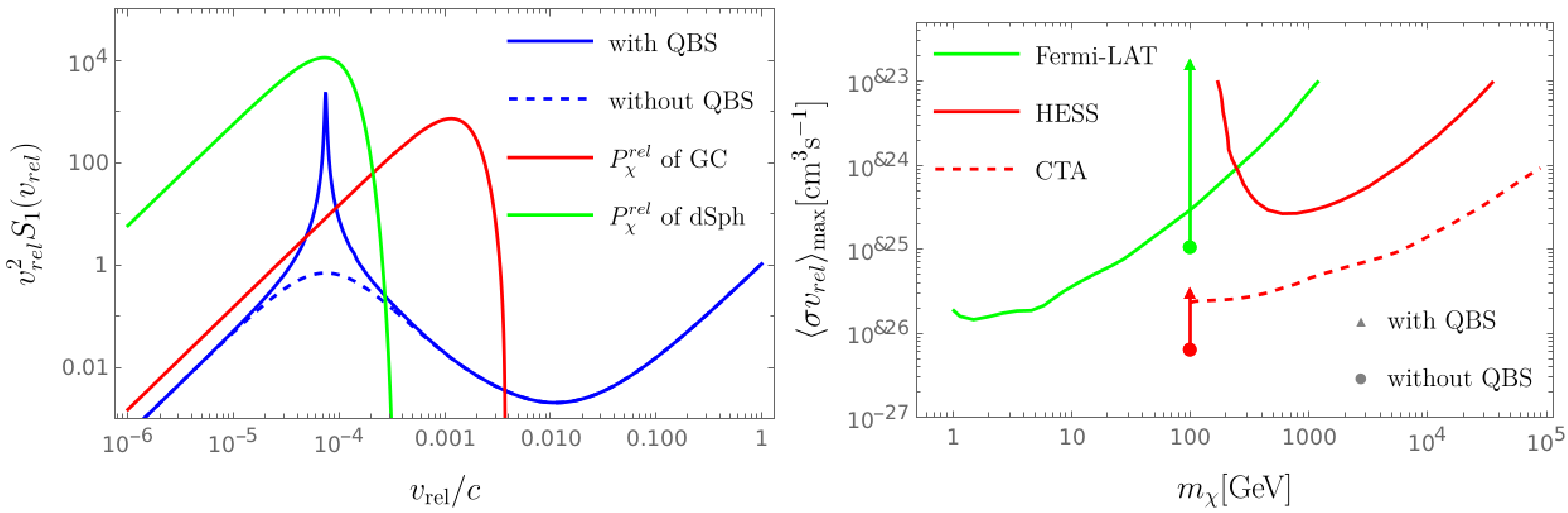
[M. Beneke, C. Hellmann, P. Ruiz-Femenía, 1411.6924]





Indirect Detection

Preliminary

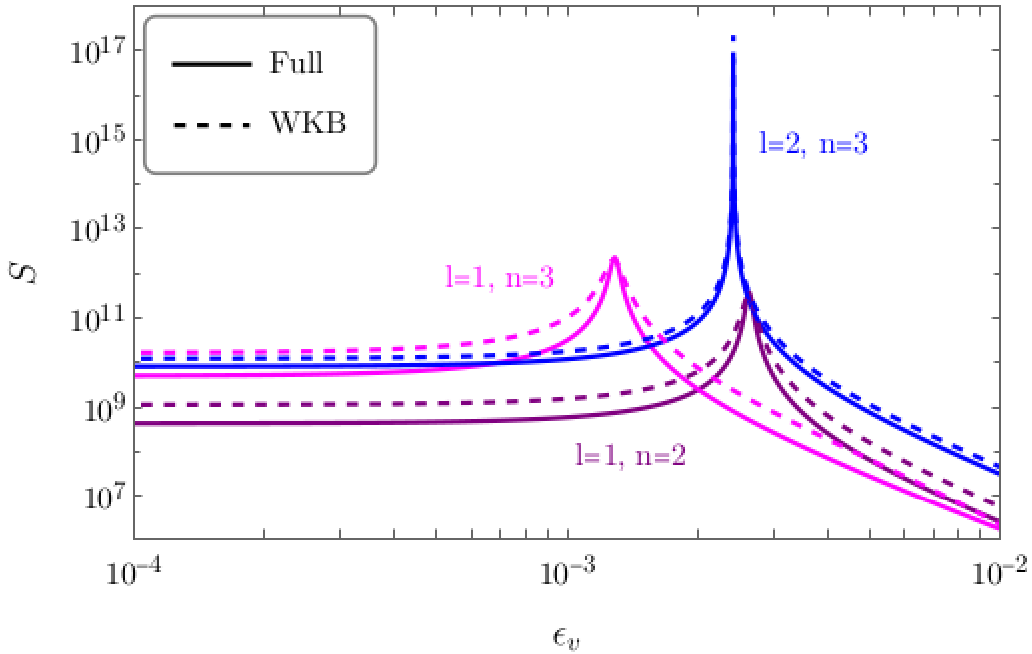


Conclusion

- Explanation for spiky P-wave Sommerfeld factor
- Wino quasi-bound states
- Impact on indirect detection

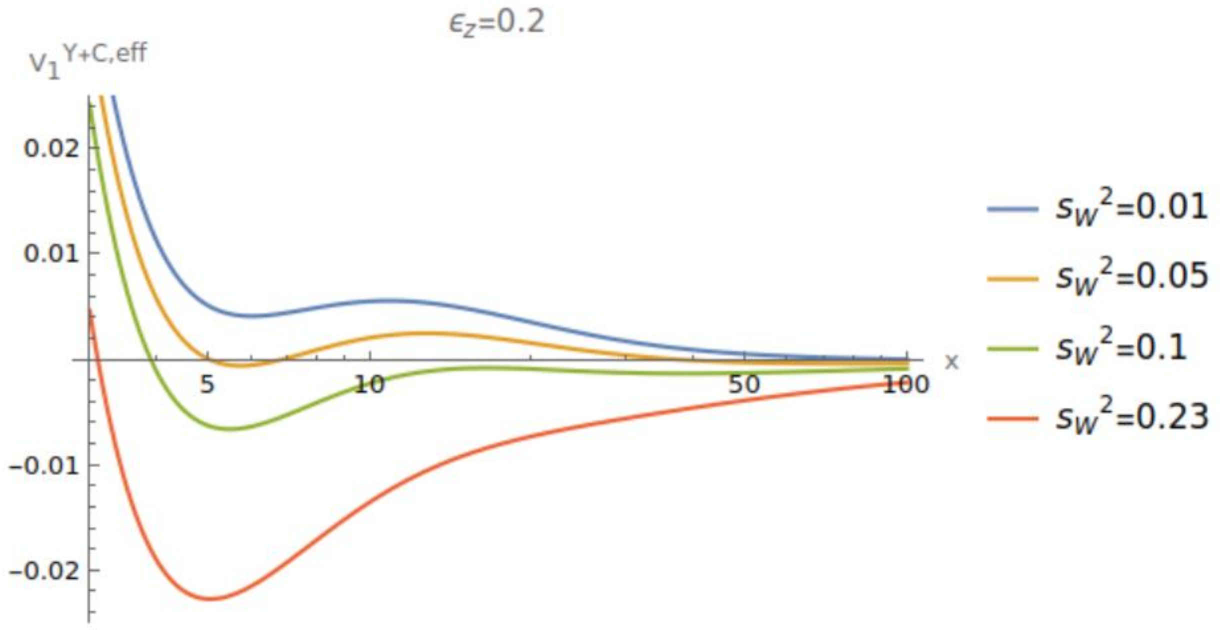
Thank you and Merry Christmas!

WKB



Full
WKB
l=2, n=3
l=1, n=3
l=1, n=2
S
10^{17}
10^{15}
10^{13}
10^{11}
10^{9}
10^{7}
10^{-4}
10^{-3}
10^{-2}
ϵ_v

Chargino potential



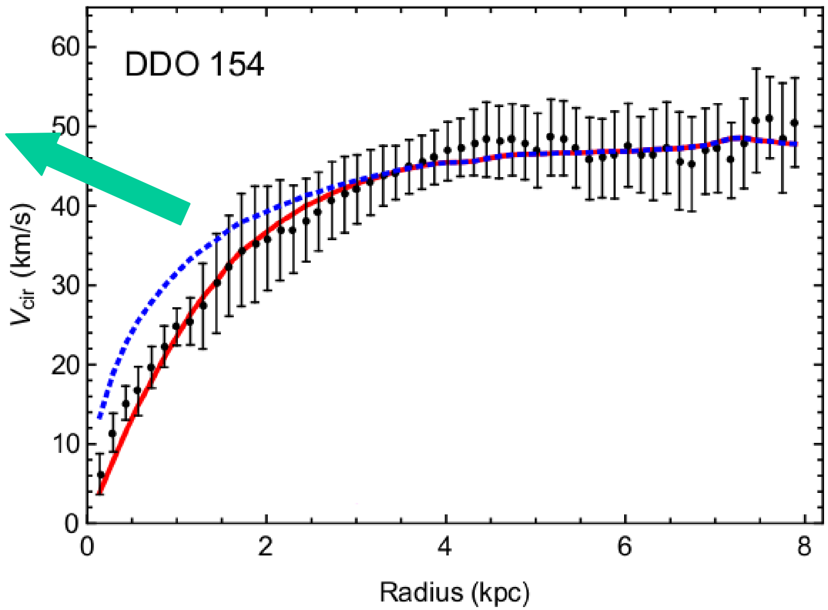
Core-cusp problem

From observed circular velocity $v_{cir}(r)$ **of baryonic matter**

↓

Dark Matter density profile $\rho_{DM}(r)$

Collisionless DM



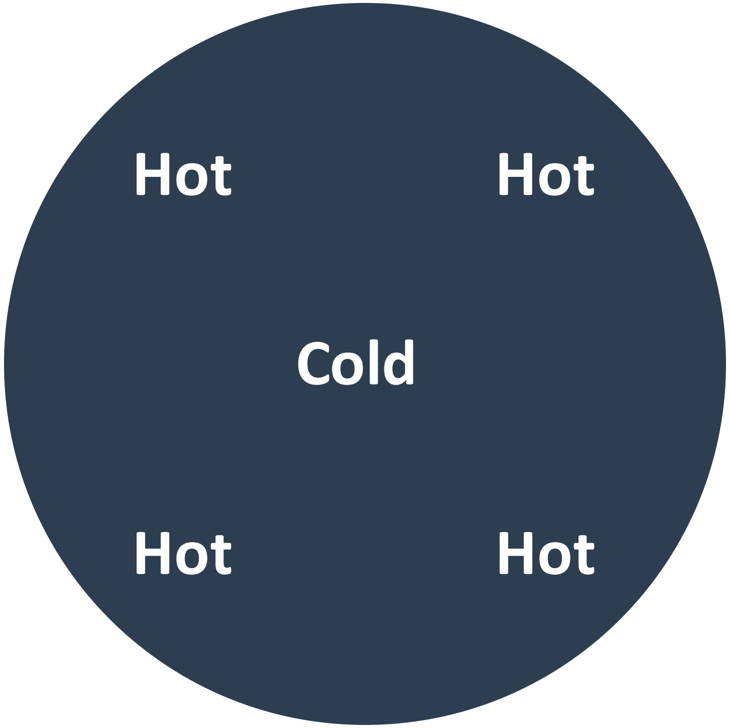
[S. Oh et al, 1502.01281]

Collisionless DM simulations → **Cusp:** $\rho_{DM}(r) \sim 1/r$ **for** $r \to 0$

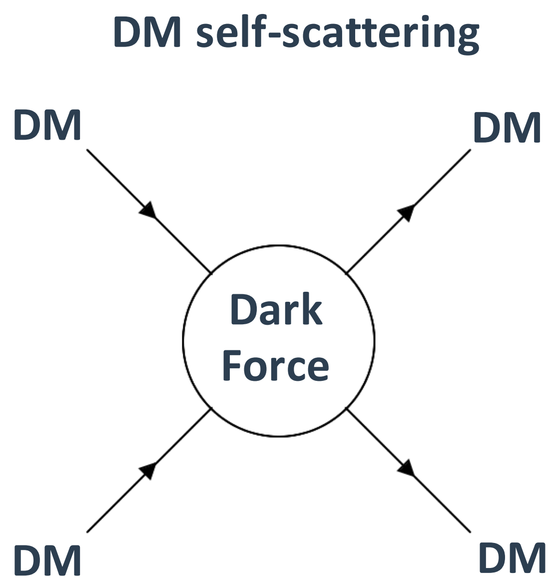
Observations → **Core:** $\rho_{DM}(r) \sim \text{const}$ **for** $r \to 0$

SIDM solution

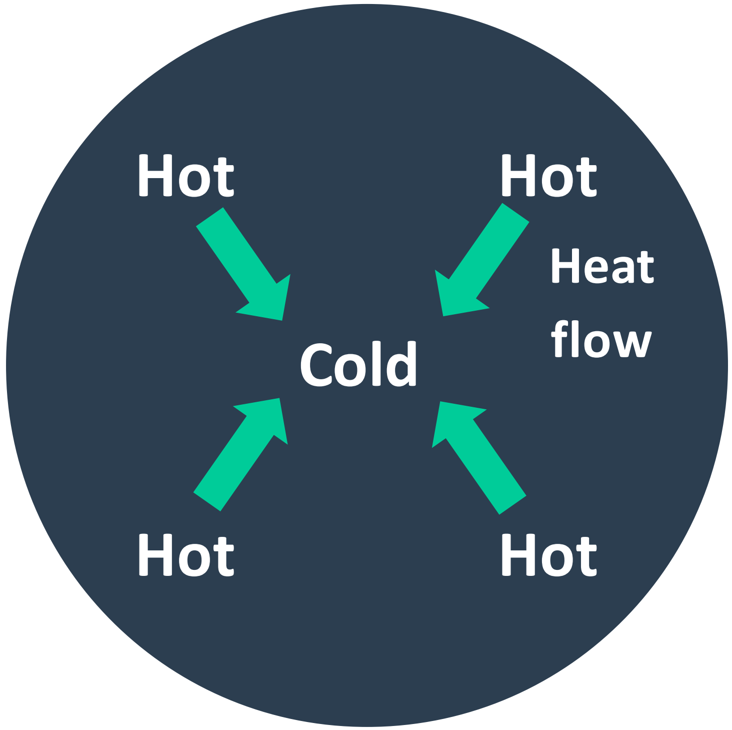
Collisionless
DM Halo



SIDM solution



SIDM Halo

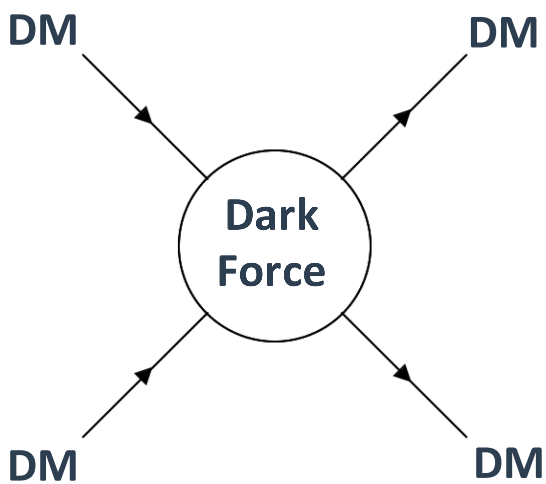


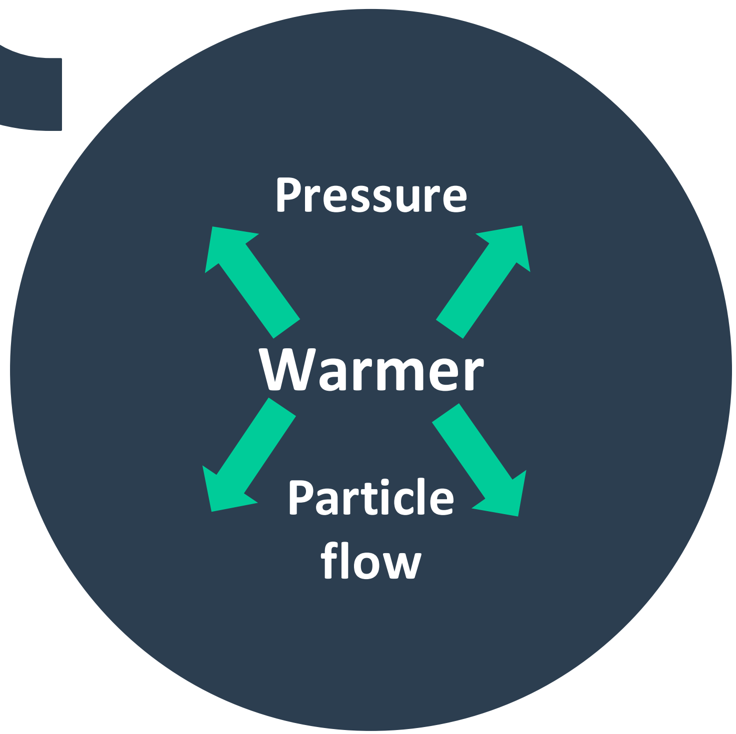
SIDM solution

Less dense halo center

SIDM Halo

DM self-scattering





SIDM solution

Similar issues are observed also for galaxy clusters

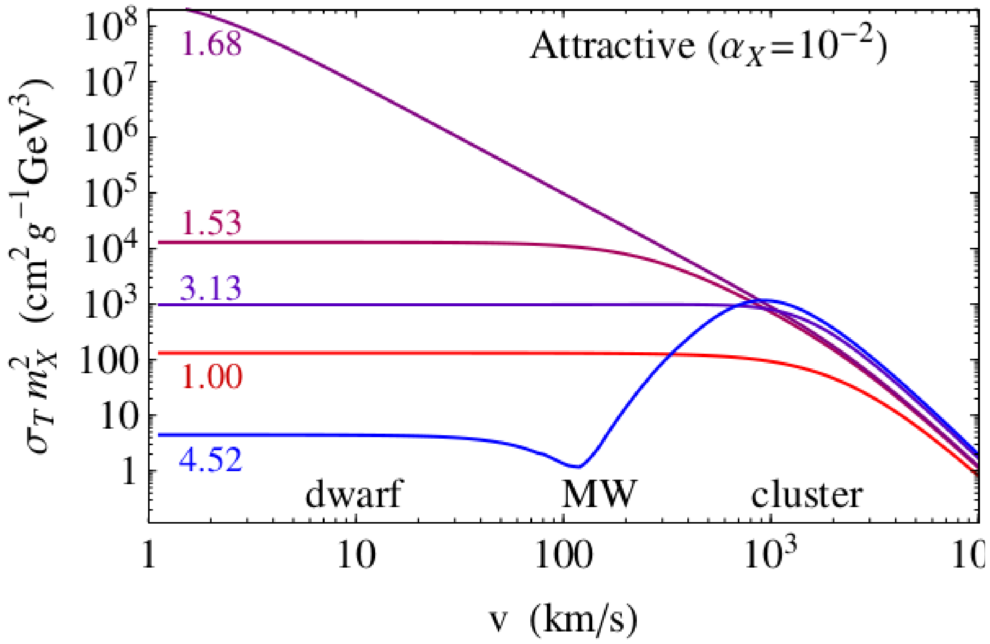
Dwarf galaxy:	$v_{rel} \approx 10^{-4}$ km/s,	$\sigma_{sc}/m_{DM} \approx 1$ cm^2/g
Galaxy Cluster:	$v_{rel} \approx 10^{-2}$ km/s,	$\sigma_{sc}/m_{DM} \approx 0.1$ cm^2/g

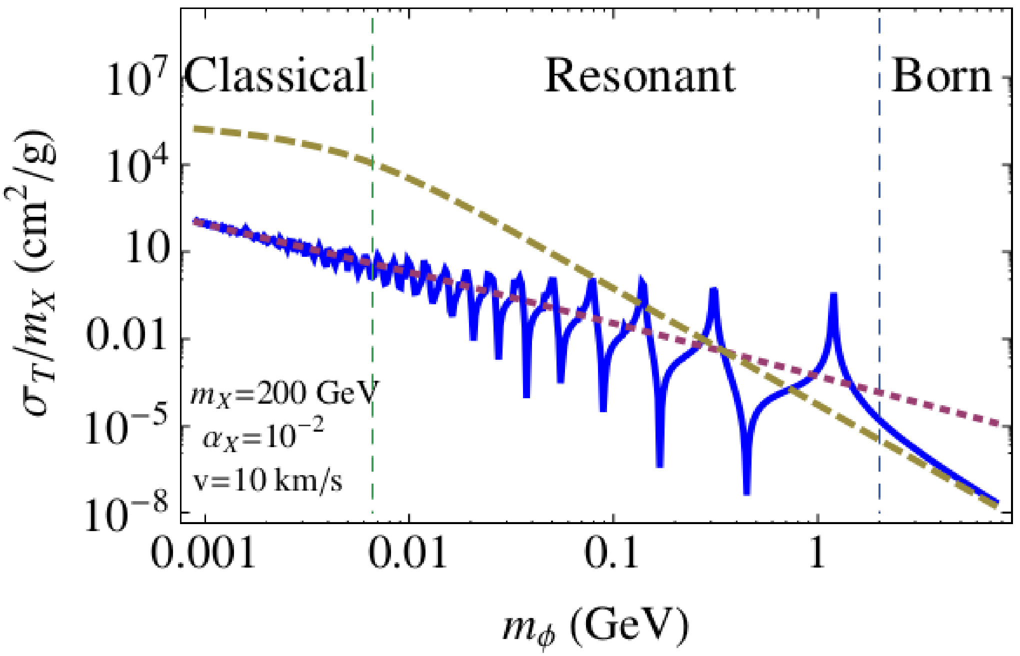
But the required cross section is different

$\sigma_{sc}(v_{rel})$

velocity-dependent cross section

DM-DM scattering





[S. Tulin, H. Yu, K. M. Zuerk, 1302.3898]