# A consistent resummation of mass and soft logarithms in processes with heavy quarks

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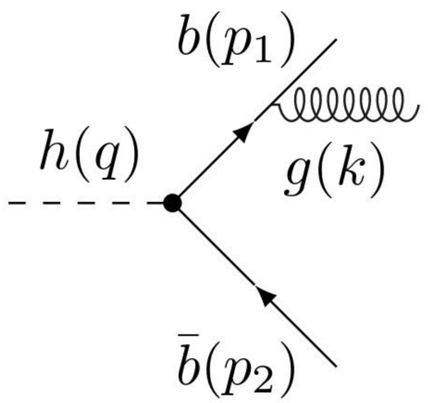
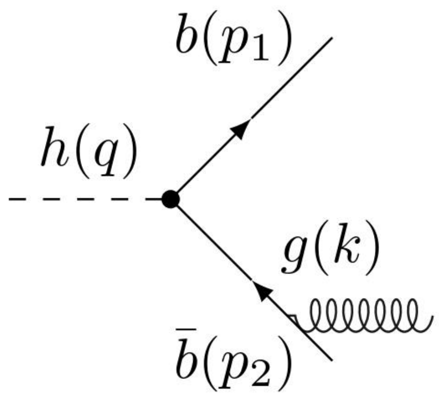
Based on arXiv:2309.06139, in collaboration with S.Marzani and G.Ridolfi

# Introduction

We consider heavy flavour production from the decay of a Higgs boson (colour singlet)




- We want to compute the differential decay rate over $x = \frac{2\, p_1 \cdot q}{q^2}$ in the large $x$ limit.
- $x$ tends to 1 for $k \to 0$ or in the collinear limit.

# Massless vs Massive Scheme Approach

## **Massless Scheme**:

- Quark mass used as a regulator
- Cross section computed as a convolution of a coefficient function times a fragmentation function
- Logs of $\xi = \frac{m^2}{q^2}$ resummed through DGLAP

## **Massive Scheme**:

- All mass dependence taken into account
- Kinematics treated correctly at every order
- Large logs spoil the convergence of the series.

# FONLL

Matching resummed scheme with fixed order calculations gives better prediction

in the study of differential decay rate in various regions of $\xi$:

$$\widetilde{\Gamma}(N,\xi) = \underbrace{\widetilde{\Gamma}_k^{(4)}(N,\xi)}_{\xi = \mathcal{O}(1)} + \underbrace{\widetilde{\Gamma}_\ell^{(5)}(N,\xi)}_{\xi \ll 1} - \text{double counting}$$

- $\tilde{\Gamma}$ denotes the Mellin tranform of the differential decay rate: $\tilde{\Gamma}(N) = \int_0^1 dx\, x^{N-1}\, \frac{1}{\Gamma_0}\frac{d\Gamma}{dx}$
- In the following we will restrict to the case $\ell = 1$ (M.Cacciari, M. Greco, P. Nason)

# Problems with Merging

We want to merge the two different calculations of the differential decay rate resumming logs of $N$ in the large $N$ limit ($x \to 1$).

**Massless Scheme**

- Double logs of $N$ with mass independent coefficients (M.Cacciari, S. Catani; F. Maltoni et al, 2022)

**Massive Scheme**

- Single logs of $N$ with mass dependent coefficient
- If we perform the limit $\xi \to 0$ after the large $N$ limit, we do not recover the massless case

Different logarithmic structure of the bremsstrahlung radiation in the two cases, in the threshold limit

# Example at Fixed order

If we compute the process with one emission at fixed order in the small mass and soft limit we find (G.Corcella, A. Mitov; D.Gaggero et al)

**Massless Scheme**

Performing the massless limit first :

$$\lim_{x\to 1}\lim_{\xi\to 0}\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = -\frac{\alpha_s C_F}{\pi}\left[\frac{\log\xi}{1-x} + \frac{\log(1-x)}{1-x} + \frac{7}{4}\frac{1}{1-x} + \dots\right]$$

**Massive Scheme**

Performing the soft limit first

$$\lim_{\xi\to 0}\lim_{x\to 1}\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = -\frac{2\alpha_s C_F}{\pi}\left[\frac{1+\log\xi}{1-x} + \dots\right]$$

**The two limits do not commute, comparing the accuracy of the soft logs can be confusing**

# Problem with merging

The soft resummation formula in the massive scheme is the product of a coefficient function times a soft function (E. Laenen, G. Oderda, G. Sterman)

$$\widetilde{\Gamma}(N,\xi) = C(\xi,\alpha_s)e^{-2\int_0^1 \mathrm{d}x \frac{x^{N-1}-1}{1-x}\gamma_{\text{soft}}\left(\beta,\alpha_s\left((1-x)^2 q^2\right)\right)}$$

where $\gamma_{soft}$ is the soft anomalous dimension.

If we perform the massless limit of the first order coefficient we find:

$$C^{(1)}(\xi) = C_F\left(\frac{1}{2}\log^2\xi + \log\xi + \frac{\pi^2}{2} + \mathcal{O}(\xi)\right)$$

This term is not predicted by DGLAP!

# Our strategy

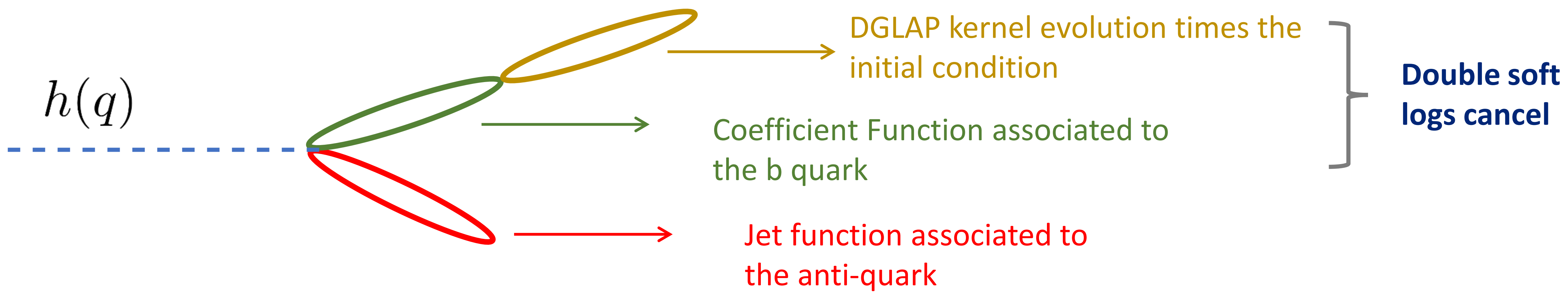
In the same spirit of FONLL, we would like to define a matching scheme merging togheter:

$$\widetilde{\Gamma}^{(4)}(N,\xi) = \widetilde{\Gamma}_k^{(4)}(N,\xi) + \widetilde{\Gamma}_{\ell_1}^{(4,\mathrm{res})}(N,\xi) - \text{double counting},$$
$$\widetilde{\Gamma}^{(5)}(N,\xi) = \widetilde{\Gamma}_\ell^{(5)}(N,\xi) + \widetilde{\Gamma}_{\ell\ell_2}^{(5,\mathrm{res})}(N,\xi) - \text{double counting}.$$

- We cannot identify an all-order subtraction term.
- We start from the resummed massless scheme expression taking into account mass effects in the regime in which $1 - x \ll \xi$.

# The origin of the problem

At NLL one can see the threshold resumed expression in the massless framework as a product of two independent jet functions:




In the measured leg the double logarithmic structure cancel between the quark jet function and the fragmentation function.

# Towards a Solution

- The tagged jet function is computed considering the $b$ massless above the 5/4 flavor threshold, and massive below ⟹ double logs cancel.

- The recoiling jet function exhibit double logs. In the massless approach, the $\bar{b}$ is always retained to be massless.

⬇

We have to consider also the recoiling jet function in the quasi-collinear limit so that:

1. When $1 - x \gg \xi$ we recover Cacciari-Catani formula.
2. When $1 - x \ll \xi$ we recover the resummed calculation made by Laenen-Oderda-Sterman

# Resummation of the cumulative distribution in momentum space

- The resummation of the cumulative distribution related to the observable $x$ coincide with the computation of the jet functions provided that we identify $1-x=\frac{1}{N\,e^{\gamma_E}}$.
- We employ the quasi-collinear limit keeping $\xi \simeq \theta^2$.

Emission quasi collinear to the $b$

$$j(1-x,\xi) = -\int_0^1 \mathrm{d}z_1 \int_{z_1^2 m^2}^{q^2} \frac{\mathrm{d}k_t^2}{k_t^2} \frac{\alpha_s^{\mathrm{CMW}}(k_t^2)}{2\pi} P_{gb}(z_1, k_t^2 - z_1^2 m^2)\Theta(\eta_1)\,\Theta(z_1-(1-x))\,,$$

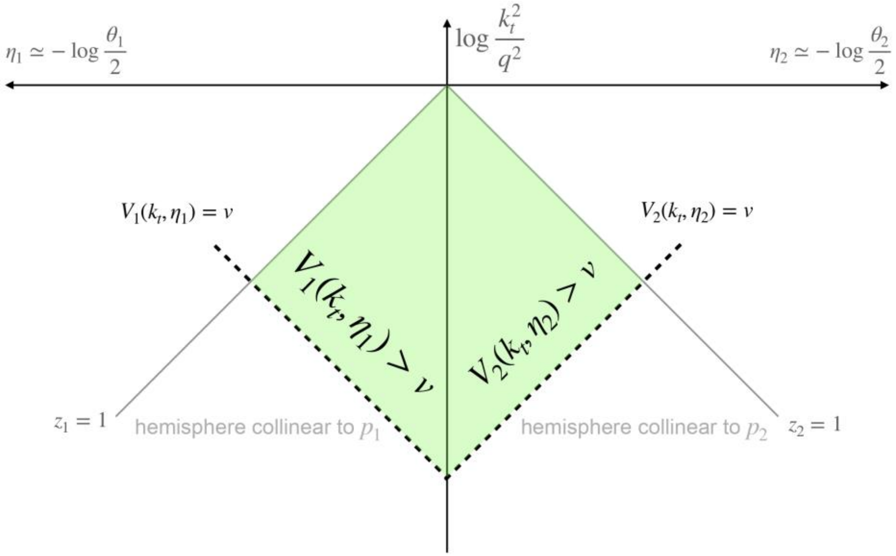
$$\bar{j}(1-x,\xi) = -\int_0^1 \mathrm{d}z_2 \int_{z_2^2 m^2}^{q^2} \frac{\mathrm{d}k_t^2}{k_t^2} \frac{\alpha_s^{\mathrm{CMW}}(k_t^2)}{2\pi} P_{gb}(z_2, k_t^2 - z_2^2 m^2)\Theta(\eta_2)\,\Theta\left(\frac{k_t^2}{q^2 z_2}-(1-x)\right).$$

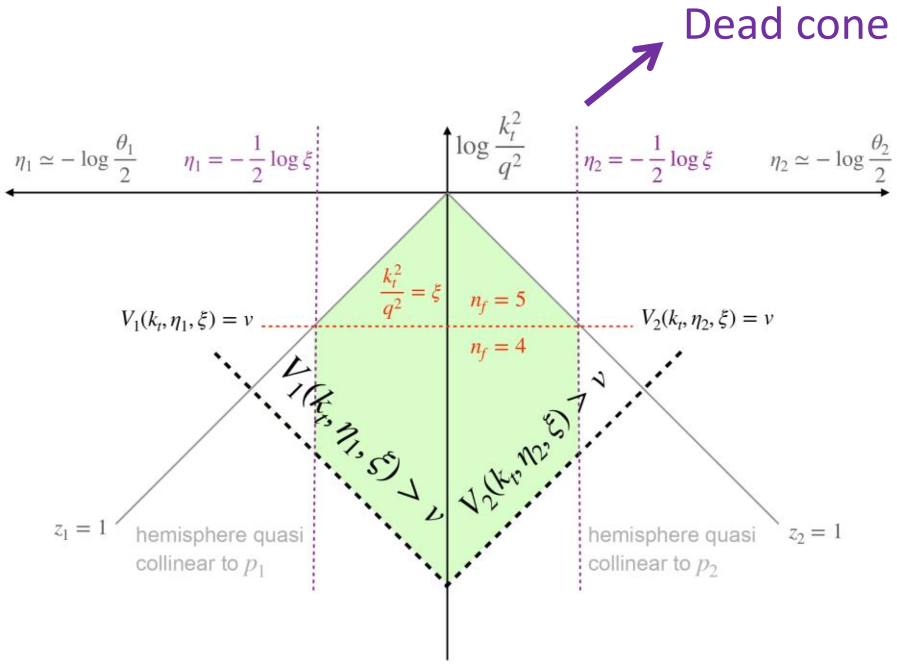
Emission quasi collinear to the $\bar{b}$

- Decoupling scheme is employed for the running coupling.

# Lund Plane

The observable $1 - x$ has two different parametrization in the two collinear regions: $V = z_1$ (gluon energy fraction), $\bar{V} = z_2\, \bar{\theta}^2$ (jet invariant mass).

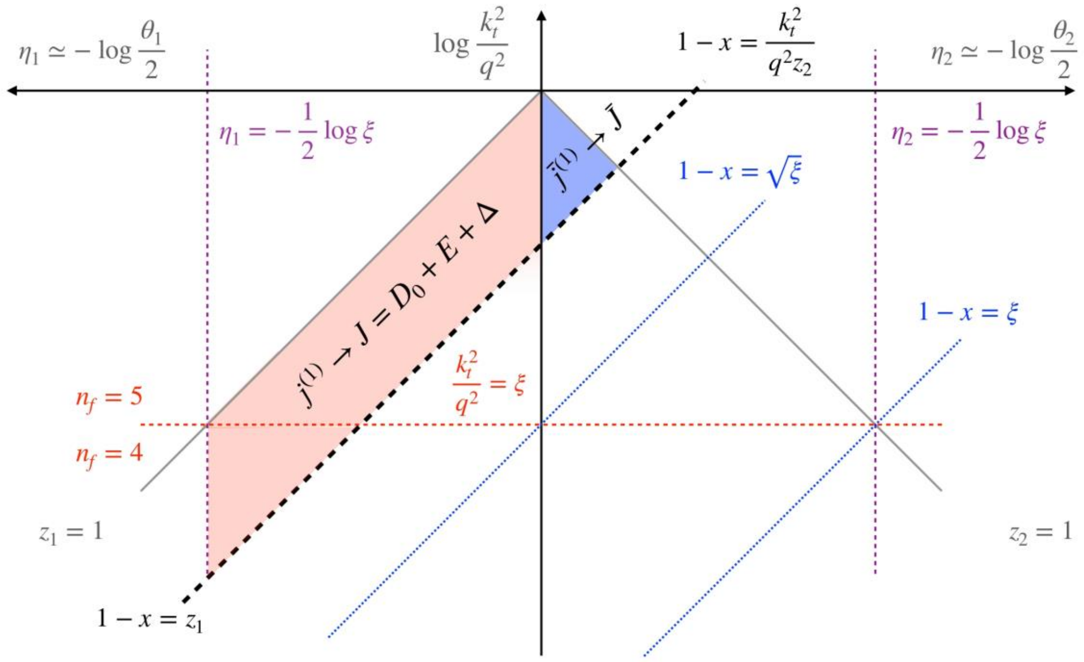


# Region 1: $1 - x > \sqrt{\xi}$

- We find 3 different regions, depending on the hierarchy between $1 - x, \sqrt{\xi}, \xi$.

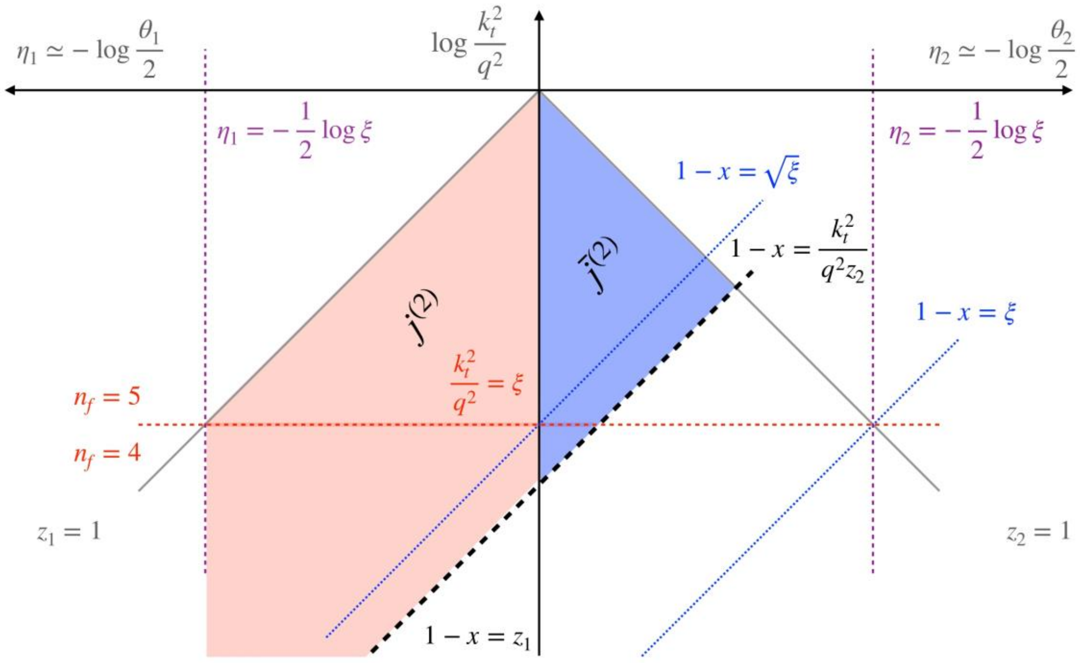
- When $1 - x > \sqrt{\xi}$, we recover the massless calculation



# Region 2: $\sqrt{\xi} > 1 - x > \xi$

- When $\sqrt{\xi} > 1 - x > \xi$ we enter a transition region that interpolates the 4 flavor calculation with the 5 flavor one.

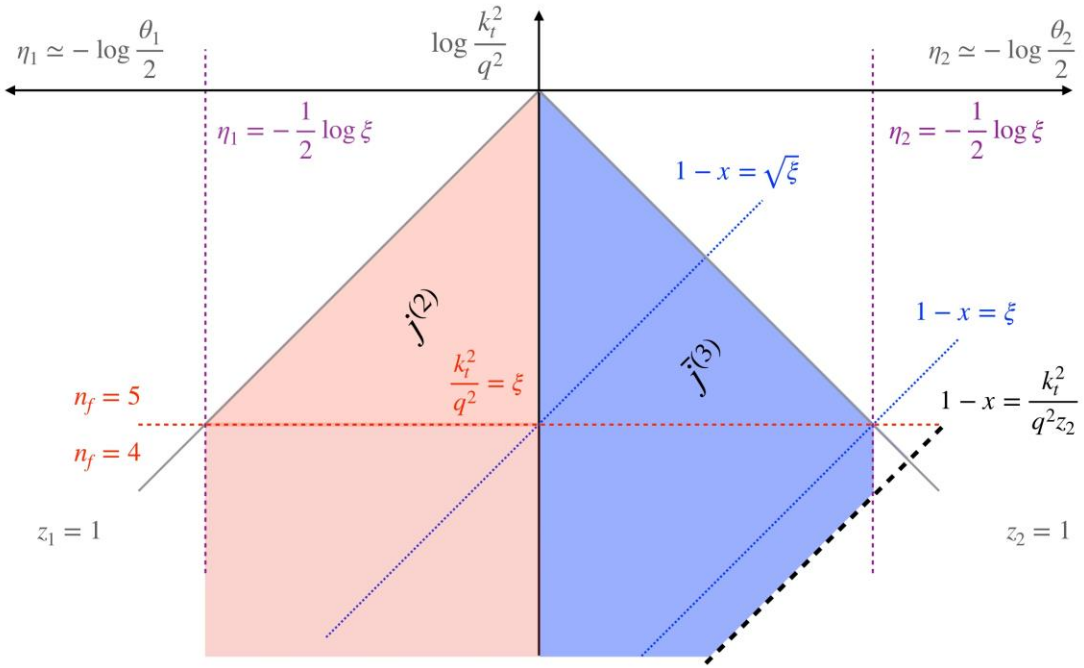
- Mass effects starts to become relevant also in the coefficient function.



# Region 3: $\xi > 1 - x$

- When $1 - x < \xi$ we recover the massive calculation with the mass logs exponentiated.

- At $\mathcal{O}(\alpha_S)$: $\bar{J}^{(3)} = \frac{\alpha_S C_F}{2\pi} \log^2 \xi + \cdots$



# Resummed formula

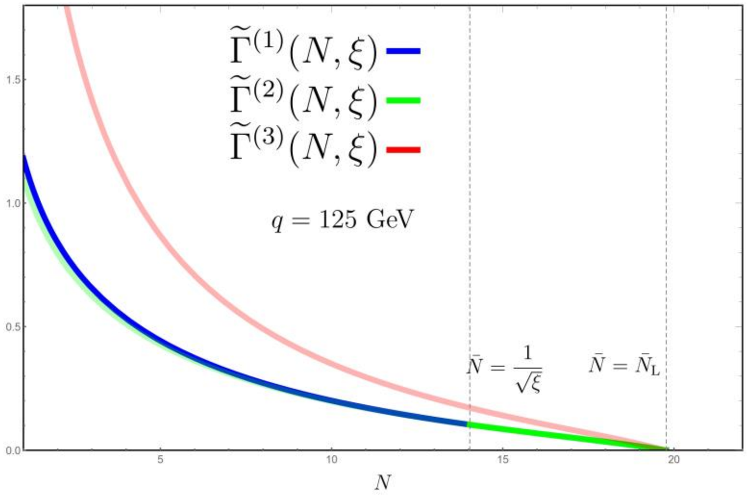
The full resumed calculation is given by:

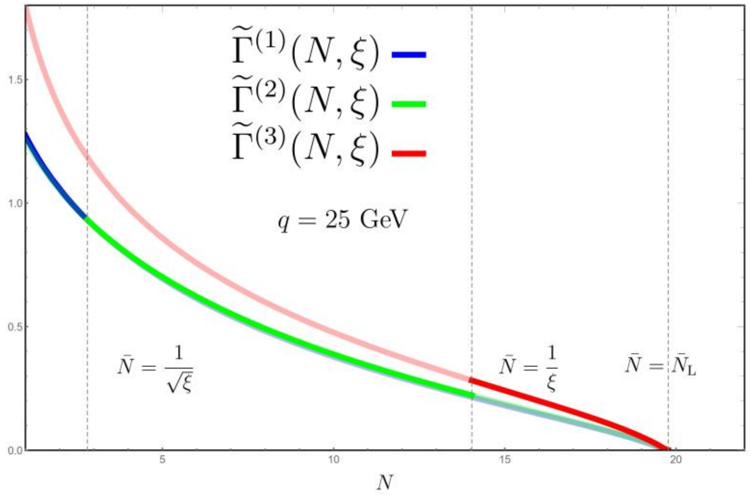
$$\frac{1}{\Gamma_0}\frac{\mathrm{d}\Gamma}{\mathrm{d}x} = \int_{c-i\infty}^{c+i\infty}\frac{\mathrm{d}N}{2\pi i}\,x^{-N}\begin{cases}\widetilde{\Gamma}^{(1)}(N,\xi), & \text{if } 1-x>\sqrt{\xi},\\ \widetilde{\Gamma}^{(2)}(N,\xi), & \text{if } \xi<1-x<\sqrt{\xi},\\ \widetilde{\Gamma}^{(3)}(N,\xi), & \text{if } 1-x<\xi,\end{cases}$$

- The matching conditions for $\tilde{\Gamma}^{(1)}, \tilde{\Gamma}^{(3)}$ are determined by comparing our result with the massless and massive scheme calculations.

- Arbitrariness for the overall constant of $\tilde{\Gamma}^{(2)}$ (see also U. Aglietti- G. Ferrera).

# Plots & Results




- Plot for real values of $N$, solid lines refers to the corresponding region in momentum space.
- Landau singularity at $N \simeq 20$ shadows the role of $\tilde{\Gamma}^{(3)}$ .
- Work in progress for DIS, for which we have a wider $Q^2$ range.

# Heavy flavor jet substructure: $\boldsymbol{\theta_g}$ and $\mathbf{z_g}$ distributions

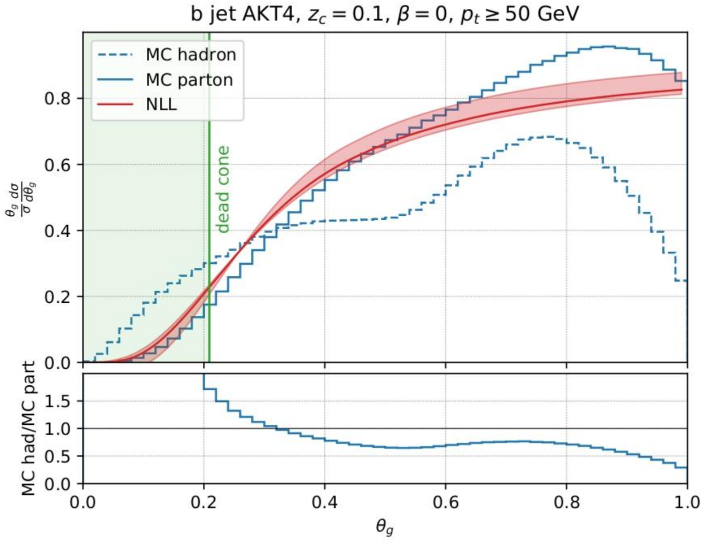
- We can use the formalism developed to compute jet substructure observables.

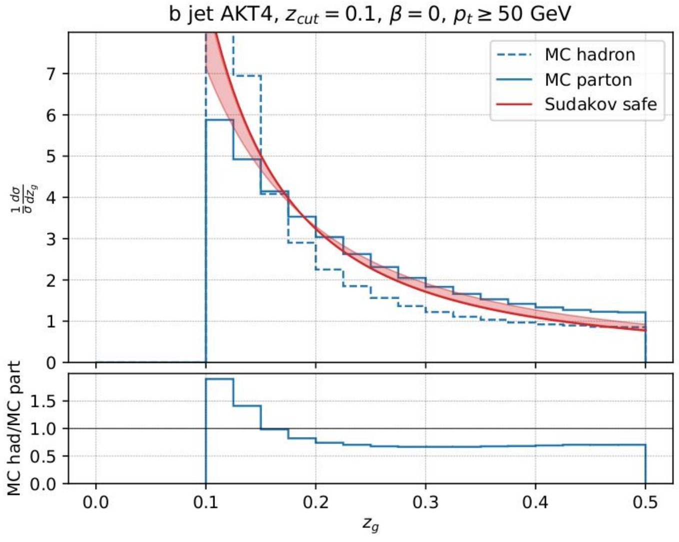
- We analyze heavy-flavour initiated jets groomed with the Soft-Drop procedure (A. Larkoski, S. Marzani, G. Soyez, J. Thaler). For details see backup slides.

➡ $\theta_g$: angular opening of the groomed jet (access to the dead-cone)
$z_g$: allows us to probe the heavy quark splitting function

Recent measurement by ALICE of the SD observables on c-jets.

# Results for $\boldsymbol{\theta_g}$ and $\boldsymbol{z_g}$




- Good agreement with Monte Carlo simulation @ $p_T = 50\ GeV$ and $\beta = 0$
- Simulations done with Herwig (Leading order matrix element+ Parton Shower).

# Conclusions and Outlook

- The merging of the massive and massless calculation is far from trivial because of the fact that the massless and soft limit do not commute.

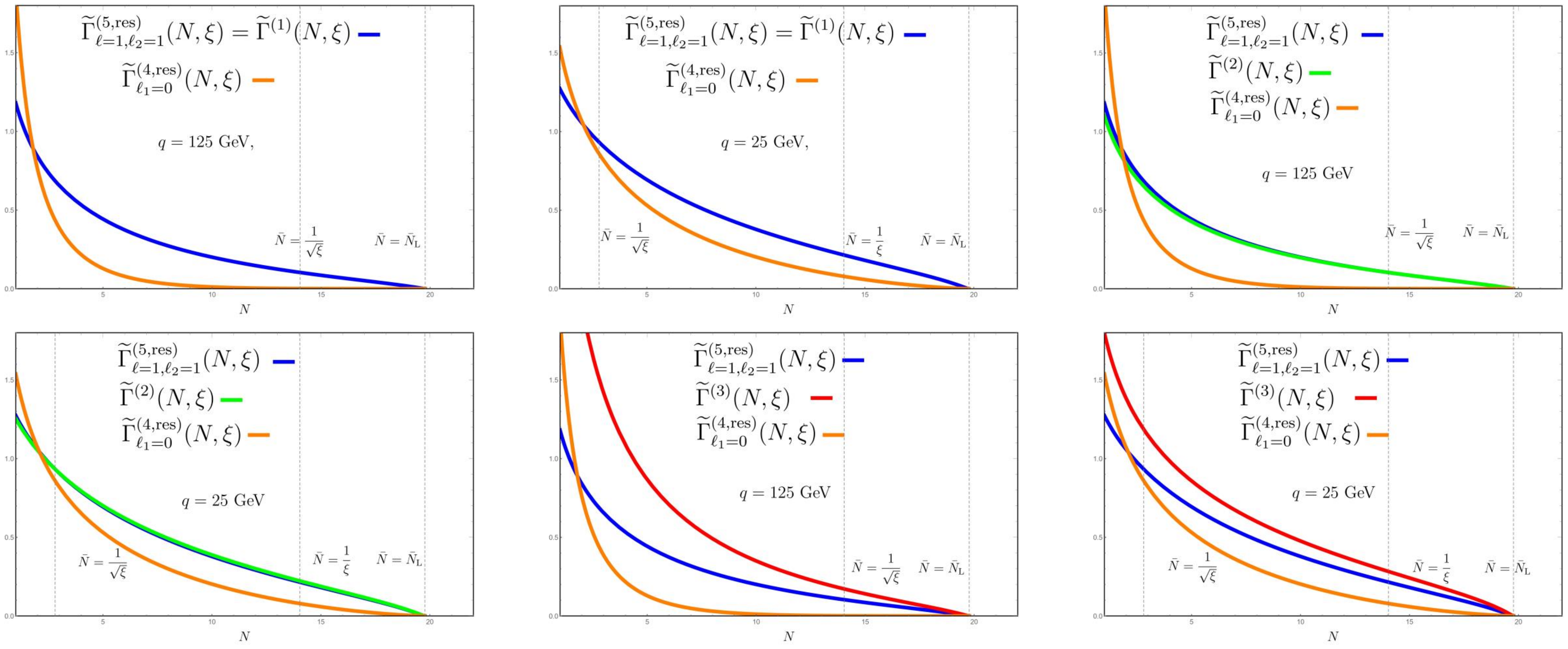
- We build a joint resummation in such a way that if we are in the regime in which if $1-x<\xi$ we recover the massive scheme resummation and if $1-x>\xi$ we have the resummed expression obtained by M.Cacciari, S. Catani at NLL accuracy.

- Our formalism can be applied for jet substructure calculations with heavy flavors (S. Caletti, A.G.,S.Marzani).

- Studies for DIS processes (in preparation).

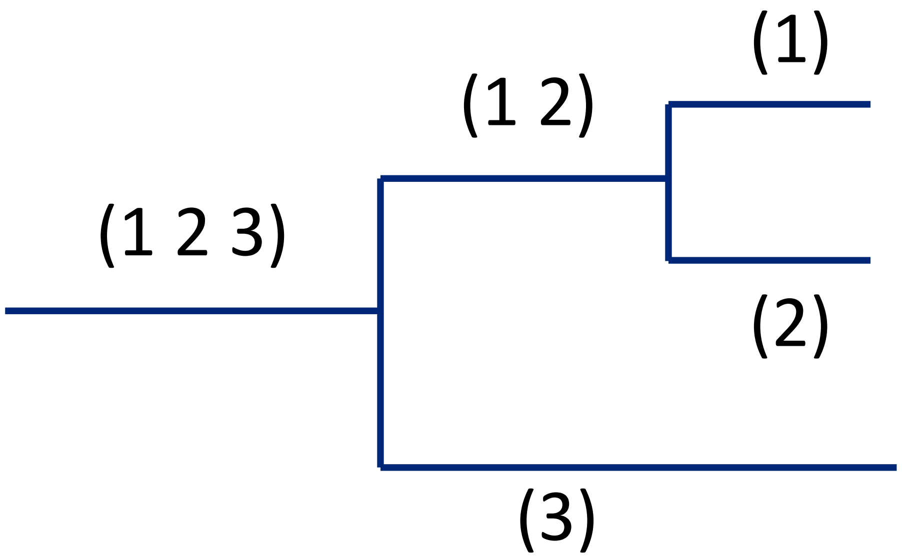
Thanks for your attention!!

# Backup: Plots for $q = 125, 25\ GeV$



# Backup: The Soft Drop Procedure

The SD algorithm removes consistently soft emission at large angle




$$\frac{\min\left(p_{t(12)}, p_{t(3)}\right)}{p_{t(12)} + p_{t(3)}} > z_{\mathrm{cut}} \left(\frac{\Delta_{(12)(3)}}{R_0}\right)^{\beta},$$

$$\Delta_{(12)(3)} = \sqrt{(y_{(12)} - y_{(3)})^2 + (\phi_{(12)} - \phi_{(3)})^2}.$$

After the declustering procedure, the jet constituents are re-clustered  according to C/A.

# Backup: Definition of the observables

The first branching that passes the soft-drop procedure defines the groomed jet radius $\theta_g$ and the groomed fraction of momentum $z_g$

$$\theta_g = \frac{\Delta_{(12)(3)}}{R_0}, \qquad z_g = \frac{\min\left(p_{t(12)}, p_{t(3)}\right)}{p_{t(12)} + p_{t(3)}}.$$

- The $\theta_g$ distribution is IRC safe both for massive and massless particles.
- The $z_g$ distribution is IRC safe for massless particles only for $\beta < 0$. Conversely, it is always IRC safe for massive particles.

# Backup: plot for c and light jet @ $p_T = 50\ GeV$