Electroweak logarithms in OpenLoops

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Artwork by C.Z.

Introduction

- In the energy range above the **EW** scale ($\sqrt{s} \gg M_W$), *Sudakov logs* represent the leading contribution of **EW** radiative corrections

- *Sudakov logarithms* from **N^{n}LO** **EW** corrections

$$\alpha^n \log^k \frac{|r_{kl}|}{m_i^2}, \qquad 1 \leq k \leq 2n$$

- At **NLO**

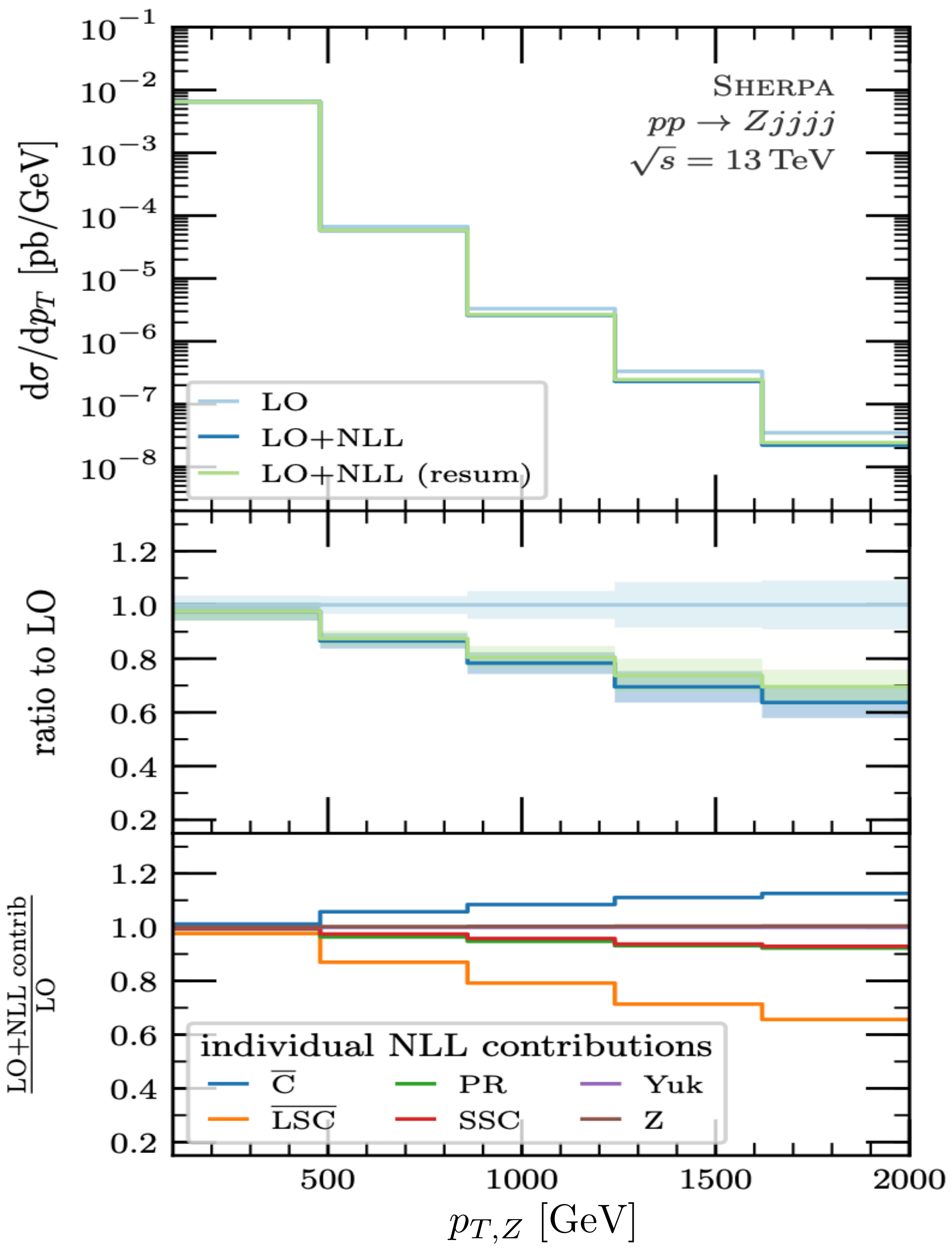
Double logs: $$L(|r_{kl}|, m_i^2) = \frac{\alpha}{4\pi} \log^2 \frac{|r_{kl}|}{m_i^2}$$

Single logs: $$l(|r_{kl}|, m_i^2) = \frac{\alpha}{4\pi} \log \frac{|r_{kl}|}{m_i^2}$$

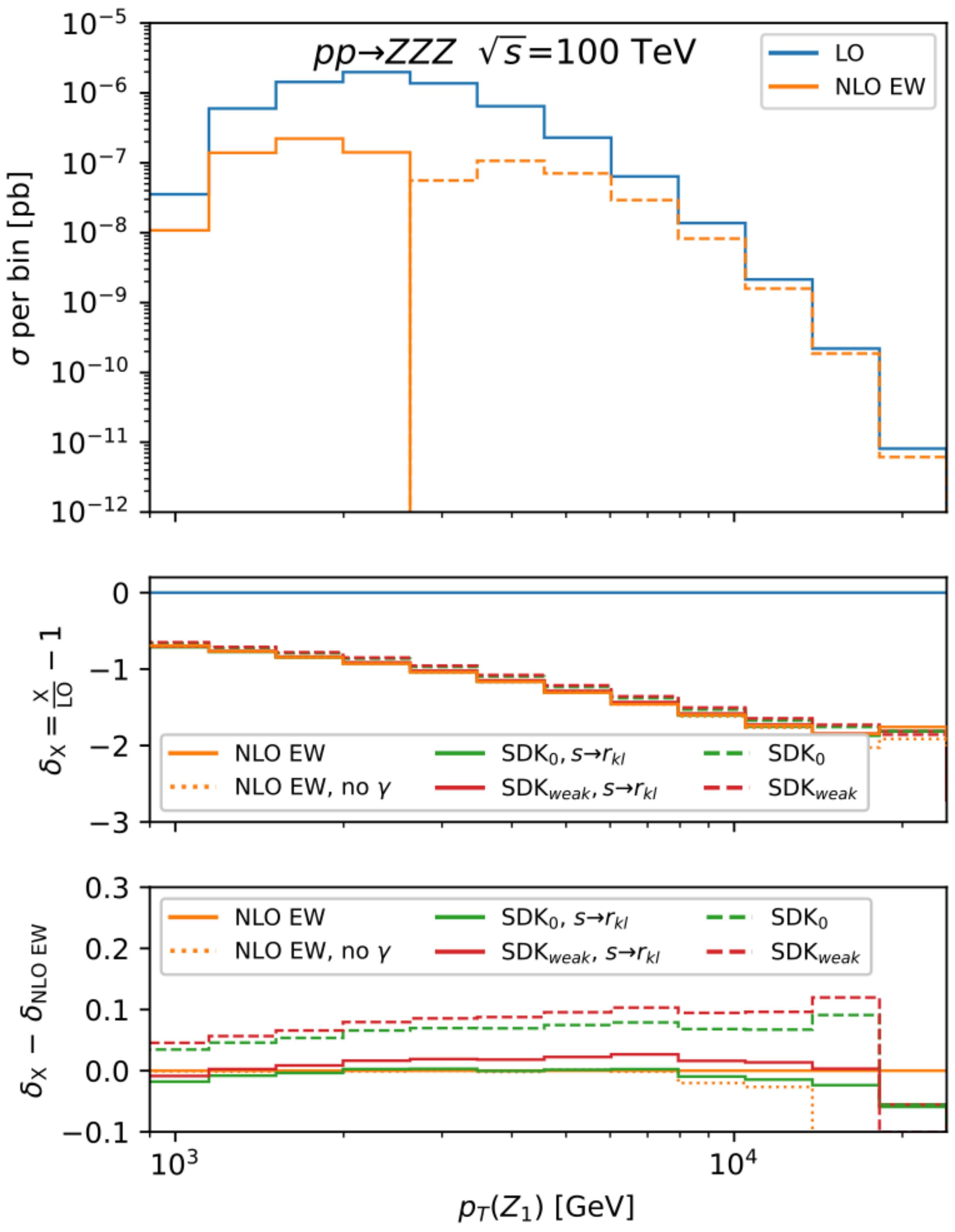
$$r_{kl} \equiv (p_k + p_l)^2$$

Introduction

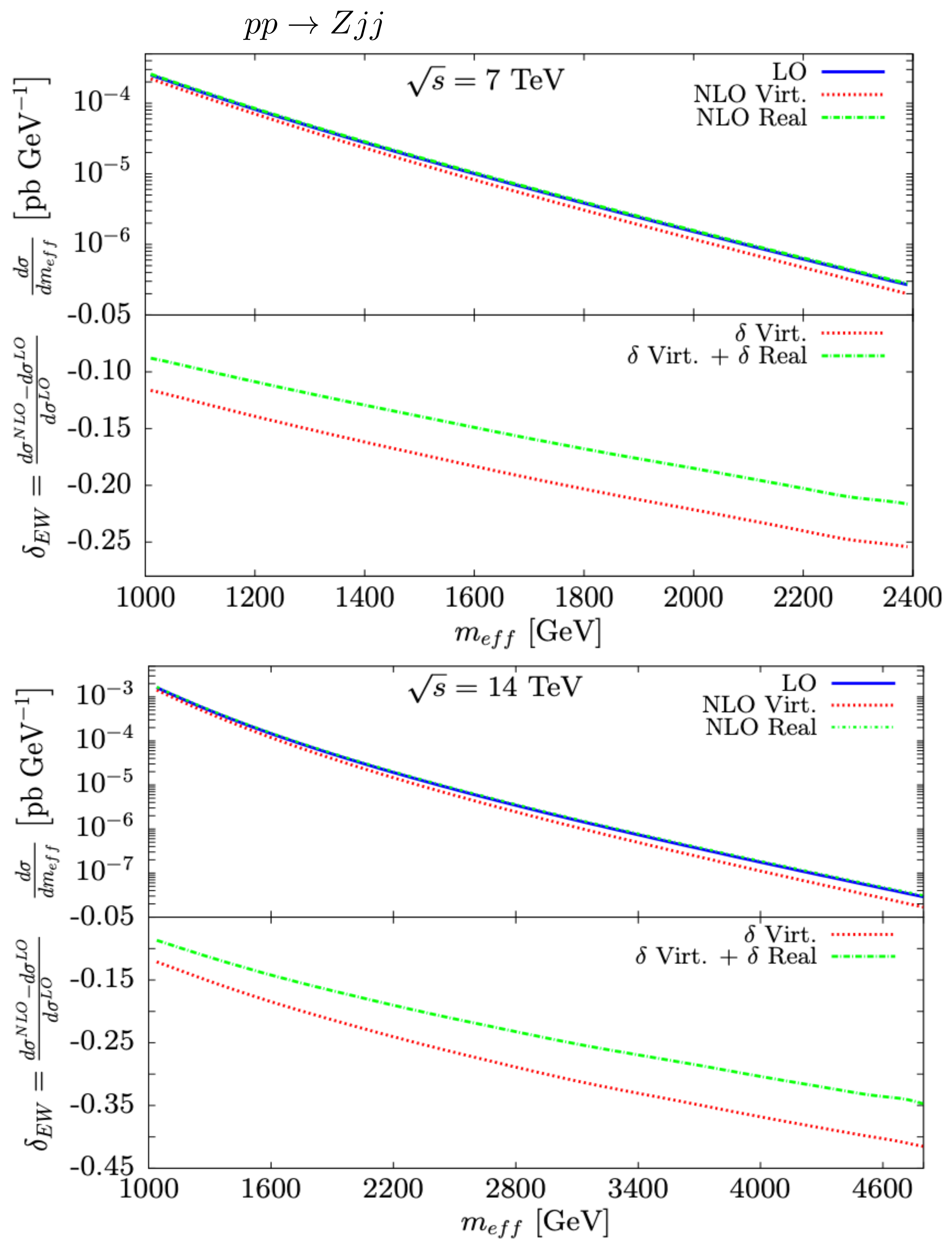
- Without clear signs of NP as resonances, small deviations in tails of kinematic distributions are under scrutiny
- **NLO EW** corrections and their *Sudakov approximation* are crucial as they can provide several tens % effects in tails



[Bothmann, Napoletano 2006.14635; 2020]



[Pagani, Zaro 2110.03714; 2021]



[Chiesa *et al.* 1305.6837; 2013]

Framework: notation & conventions

- Convention: all incoming particles, i.e. $n \to 0$ process

$$\varphi_{i_1}(p_1)\ldots\varphi_{i_n}(p_n) \to 0$$

- DP algorithm based on logarithmic approximation (LA):

⟶ Hierarchy scales

$$\mu^2 = s \sim r_{kl} \equiv (p_k + p_l)^2 \gg m_{\mathrm{t}}^2, M_{\mathrm{H}}^2 > M_{\mathrm{Z,W}}^2 \gg m_f^2 \gg \lambda^2, \qquad \forall k,l$$

⟶ Not mass-suppressed Born matrix element, i.e. $\mathcal{M}_0^{\varphi_{i_1}\ldots\varphi_{i_n}} \sim E^d$

⟶ At one-loop keep only leading and universal double & single logarithmic corrections

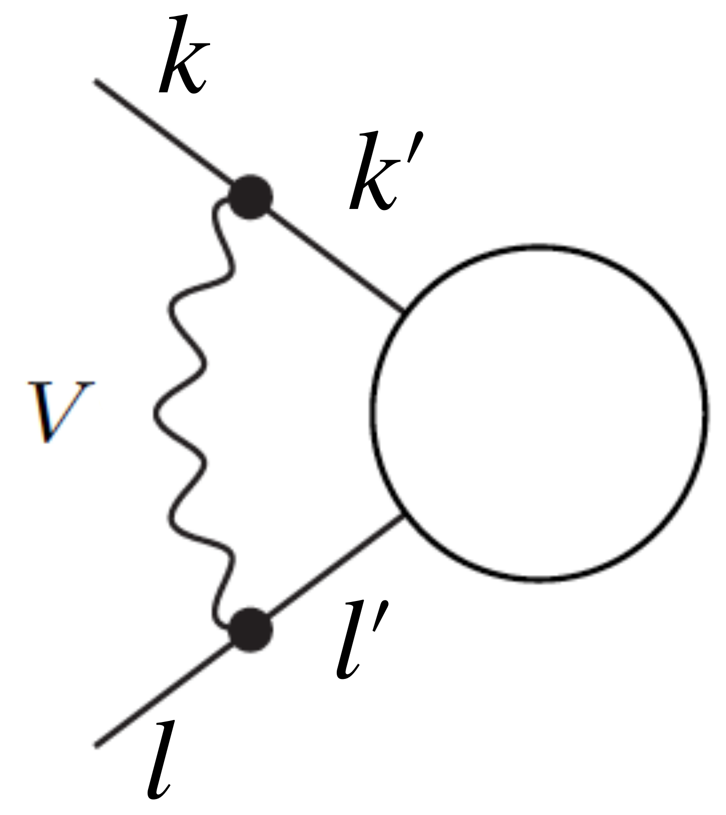
$$\delta^{\mathrm{DL}}\mathcal{M}^{\varphi_{i_1}\ldots\varphi_{i_n}} \sim E^d L \qquad\qquad \delta^{\mathrm{SL}}\mathcal{M}^{\varphi_{i_1}\ldots\varphi_{i_n}} \sim E^d l$$

neglecting constant $(\sim \alpha E^d)$ and mass suppressed $(\sim M^n E^{d-n} L)$ contributions

[Denner and Pozzorini 0010201; *2001*]

Double Logs (DL)

- DL originate from triangle diagrams where two external legs exchange a ***soft and collinear* (SC)** gauge boson V



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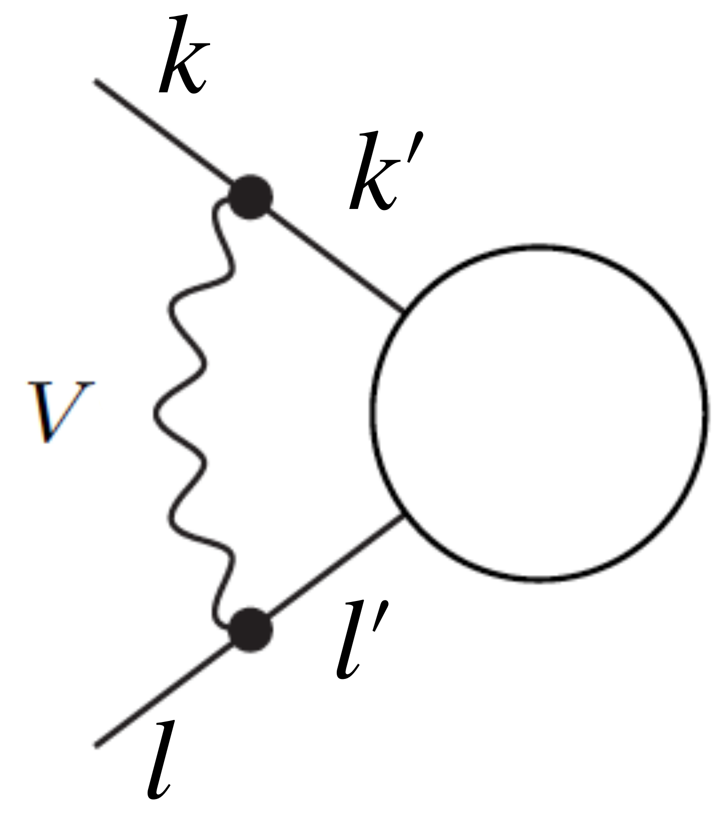

- DL originate from triangle diagrams where two external legs exchange a ***soft and collinear* (SC)** gauge boson V

- In the *Eikonal approximation*[1], the loop integral reduces to the scalar three-point function C_0, which ***factorises***

$$\delta^{\mathrm{DL}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \overset{\mathrm{LA}}{=} \sum_k \sum_{l<k} \sum_V \sum_{k',l'} \frac{\alpha}{4\pi} I^V_{kk'} I^{\bar{V}}_{ll'} \underbrace{\left[\log^2 \frac{|r_{kl}|}{M_V^2} - 2i\pi\Theta(r_{kl}) \log \frac{|r_{kl}|}{M_V^2}\right]}_{\propto\, C_0|_{\mathrm{LA}}} \mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i'_l}\cdots\varphi_{i_n}}$$

with $r_{kl} = (p_k + p_l)^2$

[1]NB: external longitudinal gauge bosons require GBET

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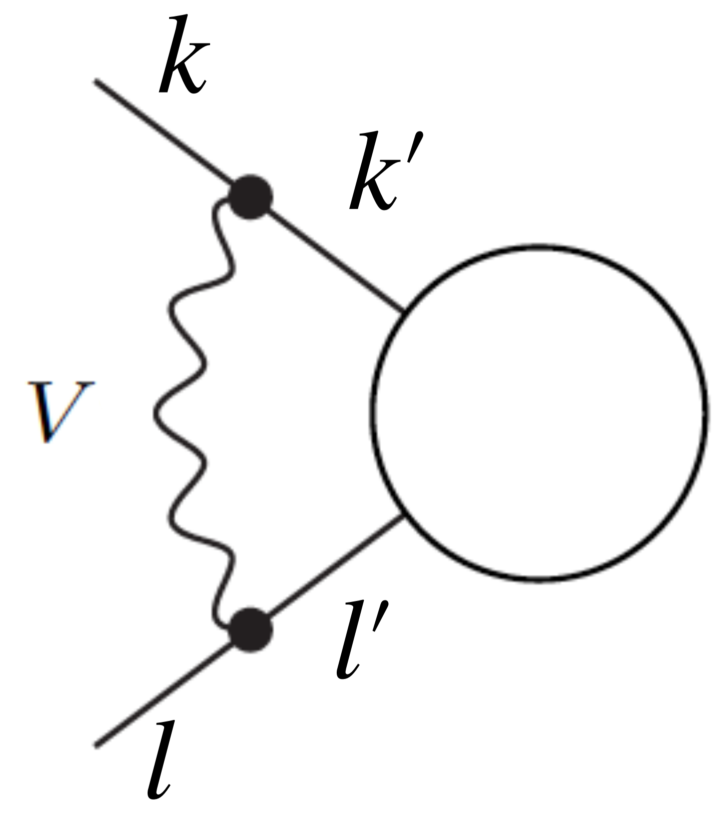

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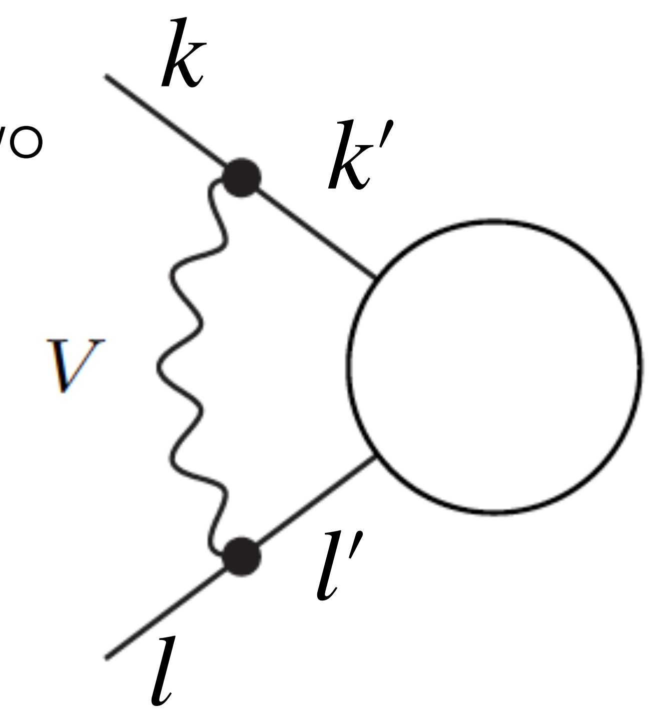
with $r_{kl} = (p_k + p_l)^2$

- Consequence of C_0 ***factorisation***: DL are ***universal***, i.e. process independent

[1]NB: external longitudinal gauge bosons require GBET

Double Logs: LSC, SSC, S-SSC

DL originate when two external legs exchange a ***soft and collinear*** **(SC)** gauge boson V



- DL can be split into

→ **Leading *Soft-Collinear*** (**LSC**): angular independent, single sum over external legs

$$\delta^{\mathrm{LSC}}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} = \sum_k \sum_{k'} \sum_V \delta^{\mathrm{LSC,V}}_{kk'} \mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i'_k}\dots\varphi_{i_n}}, \qquad \delta^{\mathrm{LSC,\ V}}_{kk'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \boxed{\log^2\left(\frac{s}{M_V^2}\right)}$$

→ **Subleading *Soft-Collinear*** (**SSC**) and **Sub SSC**: angular dependent, double sum over external legs

$$\delta^{\mathrm{(S-)SSC}}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} = \sum_k \sum_{l<k} \sum_{k',l'} \sum_V \delta^{\mathrm{(S-)SSC,V}}_{kk'll'} \mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i'_k}\dots\varphi_{i'_l}\dots\varphi_{i_n}}$$

$$\delta^{\mathrm{SSC,\ V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \boxed{\log\left(\frac{s}{M_V^2}\right) \log\left(\frac{|r_{kl}|}{s}\right)} \qquad \delta^{\mathrm{S\text{-}SSC,\ V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \boxed{\log^2\left(\frac{|r_{kl}|}{s}\right)}$$

→ Formally not part of LA and omitted in original DP, but needed for reliable estimates as firstly pointed out in [Pagani, Zaro 2110.03714; *2021*]

LA: $s \sim r_{kl} \equiv (p_k + p_l)^2 \gg M^2_{\mathrm{Z,W}} \quad \forall k, l$

[Denner and Pozzorini 0010201; *2001*]

Single Logs (SL)

- SL have a triple origin

Single Logs (SL): PR

- SL have a triple origin

→ **PR**: UV renormalisation of **EW** dimensionless parameters

$$g_{i,0} = g_i + \delta g_i = (1 + \delta Z_{g_i})\, g_i$$

$$\mu^2_{i,0} = \mu^2_i + \delta\mu^2_i$$

$$\varphi_{i,0} = \left(1 + \frac{1}{2}\delta Z_{\varphi_i\varphi_j}\right)\varphi_j$$

$$g_{i,0} = g_i + \delta g_i = (1 + \delta Z_{g_i})\, g_i$$

Renormalisation of masses and couplings with mass dimensions brings only mass-suppressed corrections

Single Logs (SL): PR & WFR

- SL have a triple origin

→ **PR**: UV renormalisation of **EW** dimensionless parameters

$$g_{i,0} = g_i + \delta g_i = (1 + \delta Z_{g_i})\, g_i$$

→ **WF**: wave-function renormalisation of external fields

$$\varphi_{i,0} = \left(1 + \frac{1}{2}\delta Z_{\varphi_i \varphi_j}\right) \varphi_j$$

yields to the ***factorised*** correction

$$\delta^{\mathrm{WF}} \mathcal{M}^{\varphi_{i_1} \dots \varphi_{i_n}} \stackrel{\mathrm{LA}}{=} \sum_k \sum_{k'} \delta^{\mathrm{WF}}_{kk'} \mathcal{M}_0^{\varphi_{i_1} \dots \varphi_{i'_k} \dots \varphi_{i_n}}, \qquad \delta^{\mathrm{WF}}_{kk'} = \frac{1}{2}\delta Z_{kk'}$$

$$\mu^2_{i,0} = \mu^2_i + \delta\mu^2_i$$

$$\varphi_{i,0} = \left(1 + \frac{1}{2}\delta Z_{\varphi_i \varphi_j}\right) \varphi_j$$

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Renormalisation of masses and couplings with mass dimensions brings only mass-suppressed corrections

Single Logs (SL): Coll

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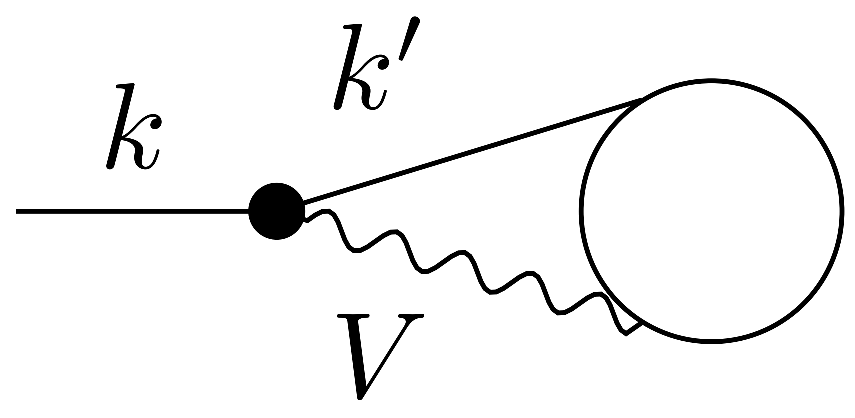
→ **Coll**: external leg emission of a collinear gauge boson



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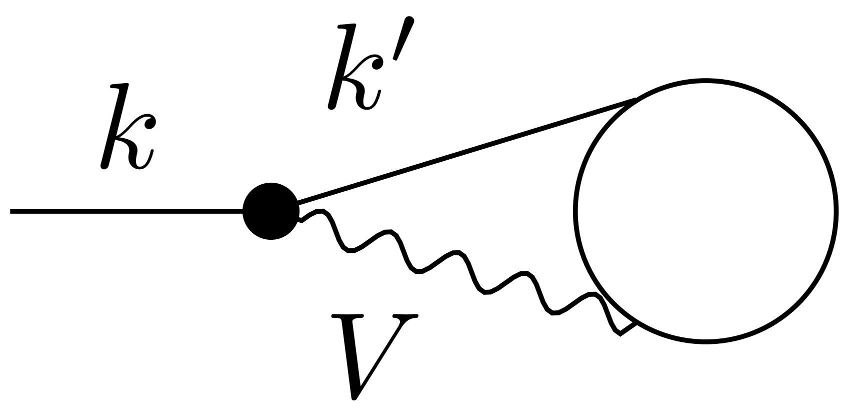
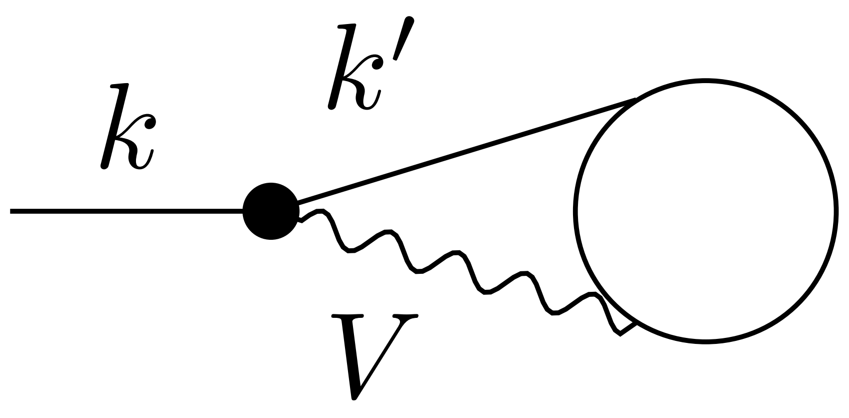


Its evaluation in *Collinear approximation* leads to the ***factorised*** contribution

$$\delta^{\text{coll}}\mathcal{M}^{\varphi_{i_1}\cdots\varphi_{i_n}} \overset{\text{LA}}{=} \sum_{k}\sum_{k'} \delta^{\text{coll}}_{kk'}\mathcal{M}_0^{\varphi_{i_1}\cdots\varphi_{i'_k}\cdots\varphi_{i_n}}, \qquad \delta^{\text{coll}}_{kk'} \sim \frac{\alpha}{4\pi}\sum_V I^V I^{\bar{V}} \log\left(\frac{s}{M_V^2}\right)$$

Single Logs (SL): Coll

- SL have a triple origin



→ **Coll**: external leg emission of a collinear gauge boson

Its evaluation in *Collinear approximation* leads to the ***factorised*** contribution

$$\delta^{\mathrm{coll}}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} \stackrel{\mathrm{LA}}{=} \sum_{k}\sum_{k'}\delta^{\mathrm{coll}}_{kk'}\mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i'_k}\dots\varphi_{i_n}}, \qquad \delta^{\mathrm{coll}}_{kk'} \sim \frac{\alpha}{4\pi}\sum_{V} I^{V} I^{\bar{V}} \log\left(\frac{s}{M_V^2}\right)$$

→ **C**: Full gauge-invariant SL correction associated to external fields:

$$\delta^{\mathrm{C}}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} \stackrel{\mathrm{LA}}{=} \sum_{k}\sum_{k'}\delta^{\mathrm{C}}_{kk'}\mathcal{M}_0^{\varphi_{i_1}\dots\varphi_{i'_k}\dots\varphi_{i_n}}, \qquad \delta^{\mathrm{C}}_{kk'} = \left(\delta^{\mathrm{coll}}_{kk'} + \delta^{\mathrm{WF}}_{kk'}\right)\Big|_{\mu^2=s}$$

Implementation in OpenLoops: why

- **EW** *Sudakov logarithms* at one-loop already implemented in
  - ALPGEN: Chiesa *et al*, 1305.6837; ***2013***
  - Sherpa: Bothmann, Napoletano 2006.14635; ***2020***
  - MadGraph: Pagani, Zaro 2110.03714; ***2021***

However:

  - Even if automated, one-loop computations can be very complicated (e.g. high multiplicity processes)
  - No **NNLO/two-loop** level automation available
  - **EW** *Sudakov logs* have nice properties: ***factorisation***, being the leading contribution of radiative corrections

- OpenLoops (OL): automated tool for the calculation of tree and one-loop amplitudes [Buccioni *et al*, 1907.13071; ***2019***]

- Goal of the implementation: evaluate **NLO EW** *Sudakov* corrections via tree amplitudes (w/o loop computations $\rightarrow 20-30$ times faster) and make them available to any MC with OL interface

Results: $pp \to ZZ$

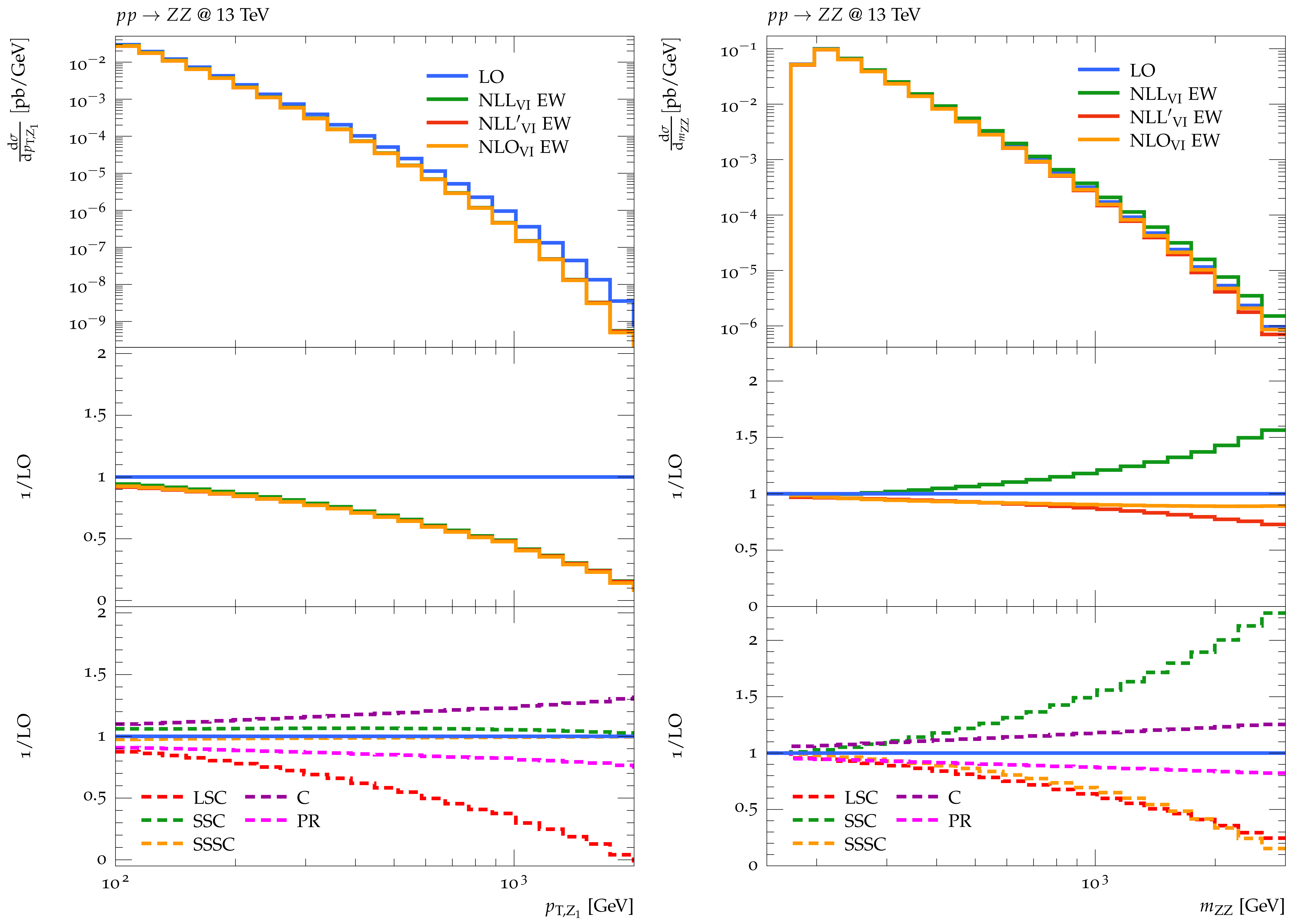
NLL EW: [Accomando *et al,* 0409247; 2004]

Full **NLO EW**: [Bierweiler *et al,* 1305.5402; 2013]

Full **NLO**: [Baglio *et al,* 1307.4331; 2016]

NNLO QCD+NLO EW: [Grazzini *et al,* 1912.00068; 2020]

NLO EW vs **NLL EW**: [Bothmann *et al,* 2111.13453; 2021]



$$\text{NLL}_{\text{VI}}\ \text{EW} = (1 + \text{LSC} + \text{SSC} + \text{C} + \text{PR} + \mathbf{I})\text{LO}$$

$$\text{NLL'}_{\text{VI}}\ \text{EW} = (1 + \text{LSC} + \text{SSC} + \text{SSSC} + \text{C} + \text{PR} + \mathbf{I})\text{LO}$$

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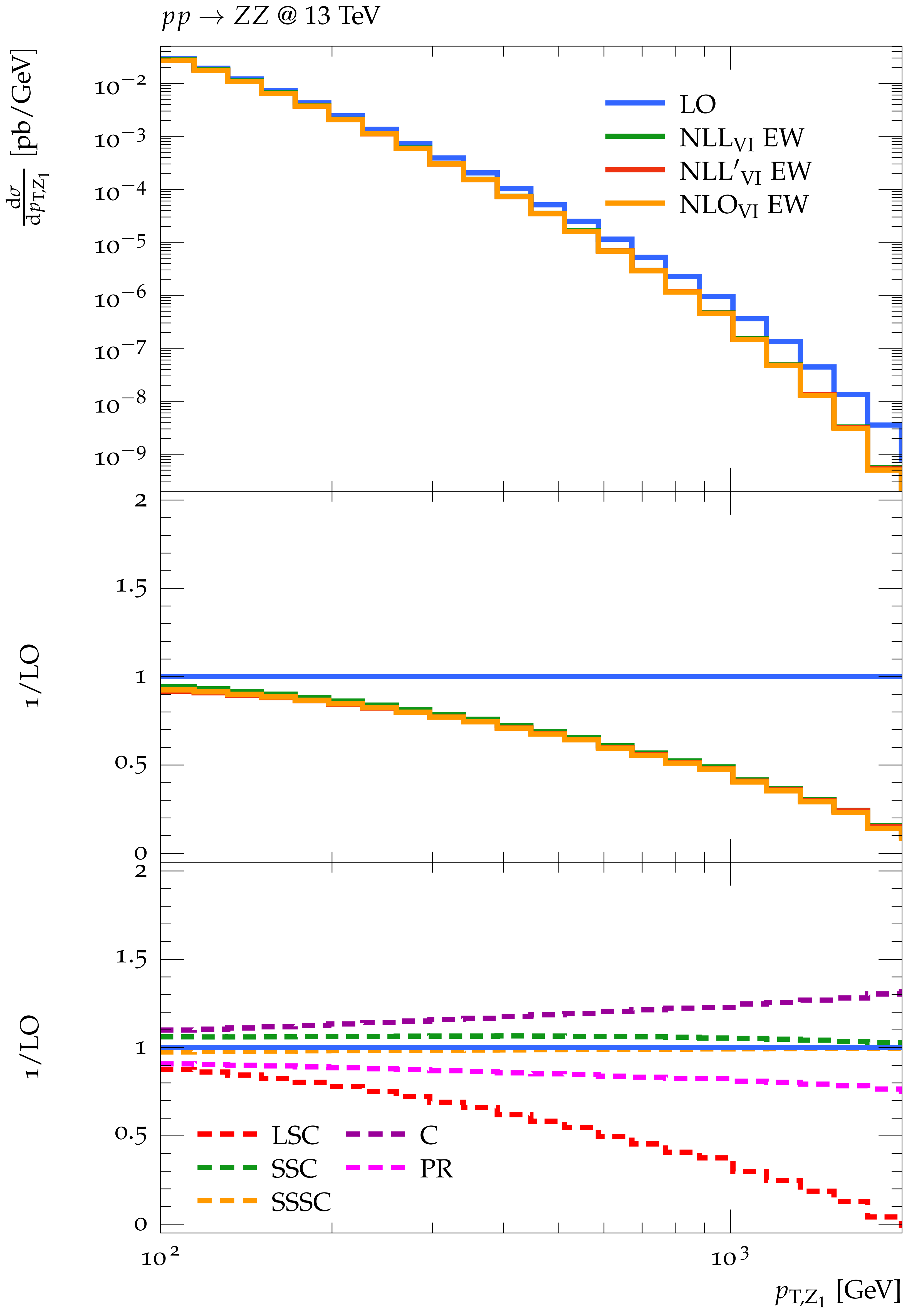
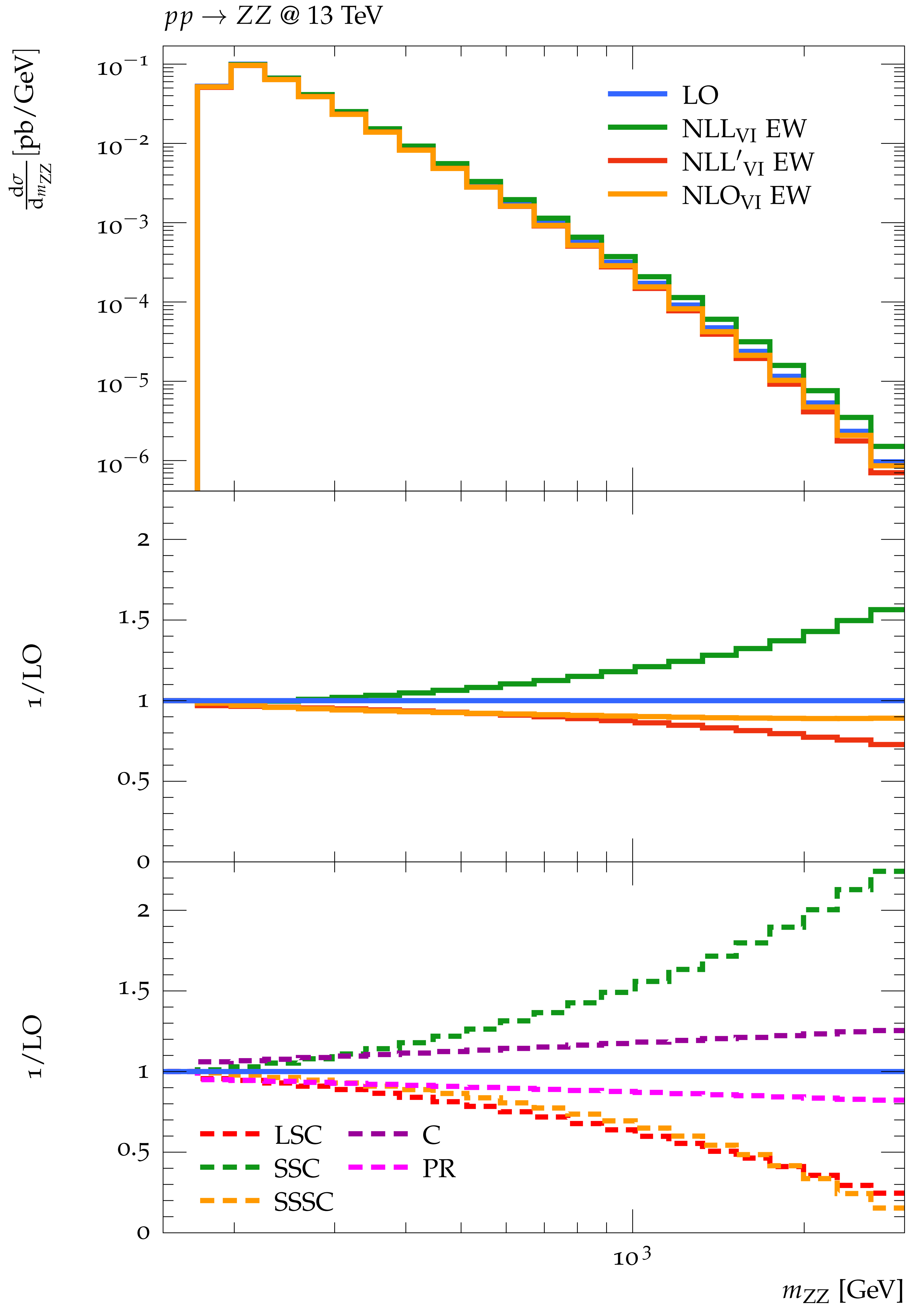




SSC and **SSSC** become very sizeable for PS regions where <u>*Sudakov*</u> condition

$$s \sim r_{kl} \equiv (p_k + p_l)^2 \gg M^2_{\mathrm{Z,W}} \quad \forall k, l$$

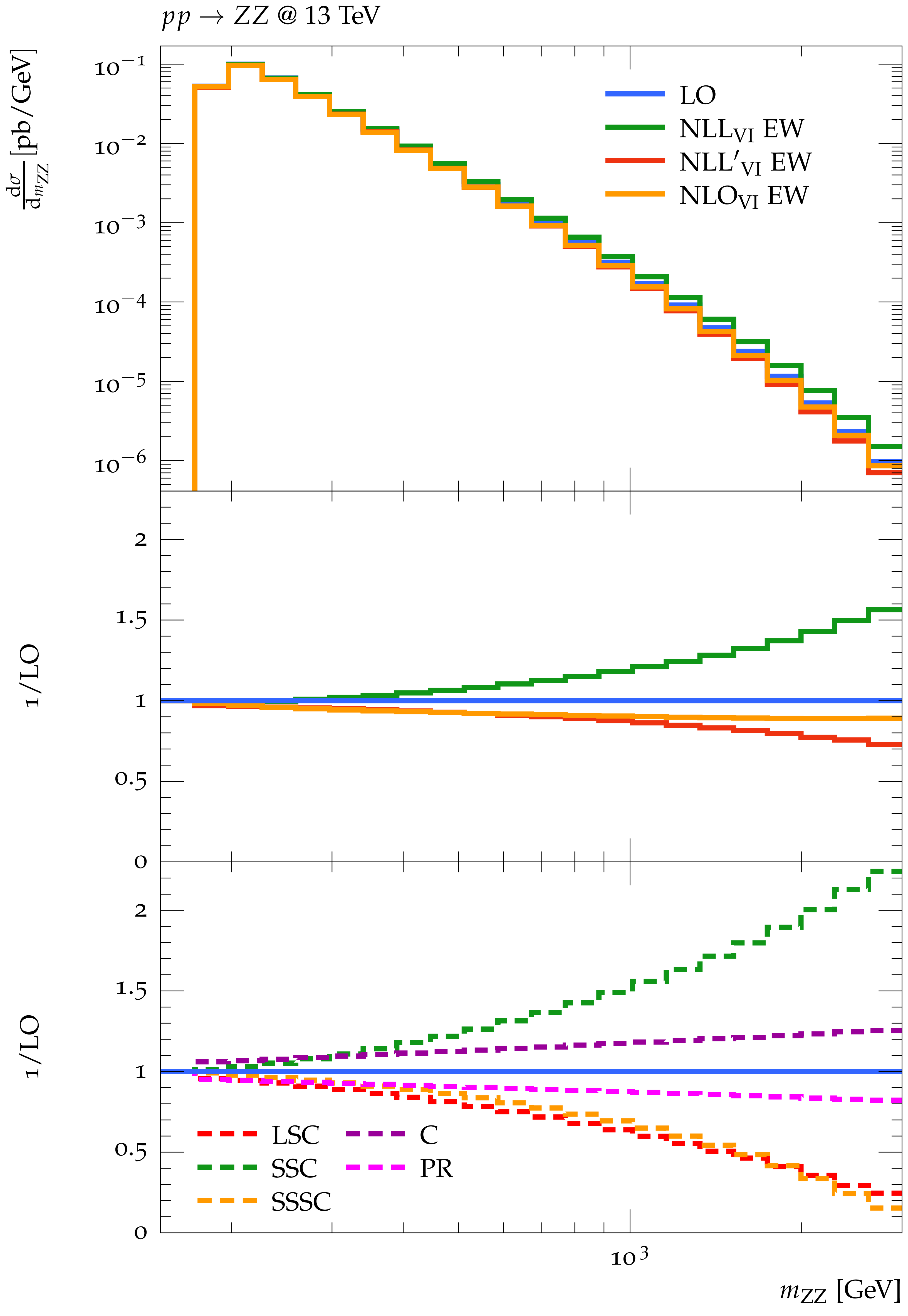
is violated, with hierarchy among invariants

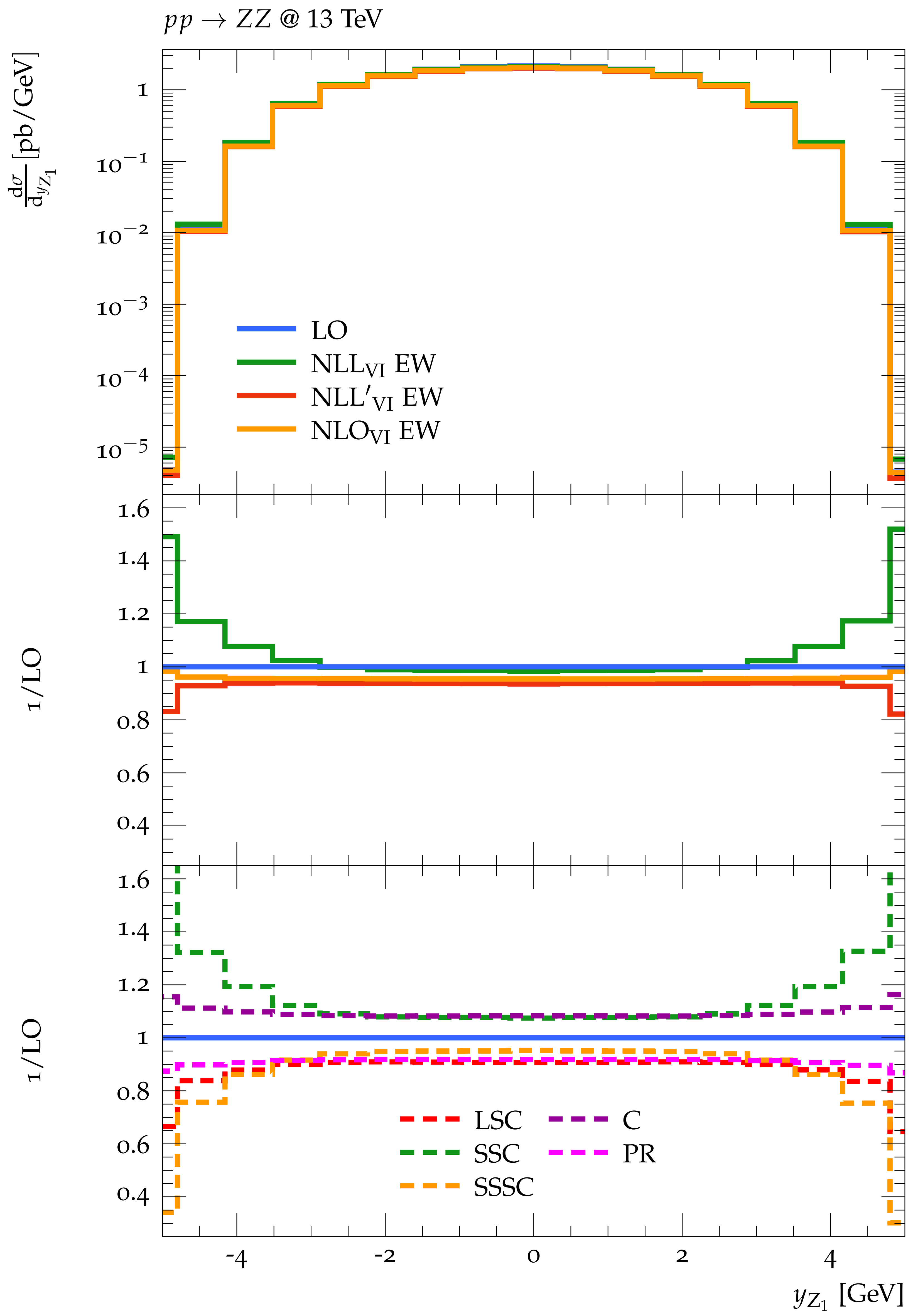
$$s \sim r_{kl} \equiv (p_k + p_l)^2 \gg r_{k'l'} \equiv (p_{k'} + p_{l'})^2 \gg M^2_{\mathrm{Z,W}}$$

$$\delta^{\mathrm{SSC,\ V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log\left(\frac{s}{M_V^2}\right) \log\left(\frac{|r_{kl}|}{s}\right)$$

$$\delta^{\mathrm{S\text{-}SSC,\ V}}_{kk'll'} \sim \frac{\alpha}{4\pi} I^V I^{\bar{V}} \log^2\left(\frac{|r_{kl}|}{s}\right)$$

Results: $pp \to ZZ$

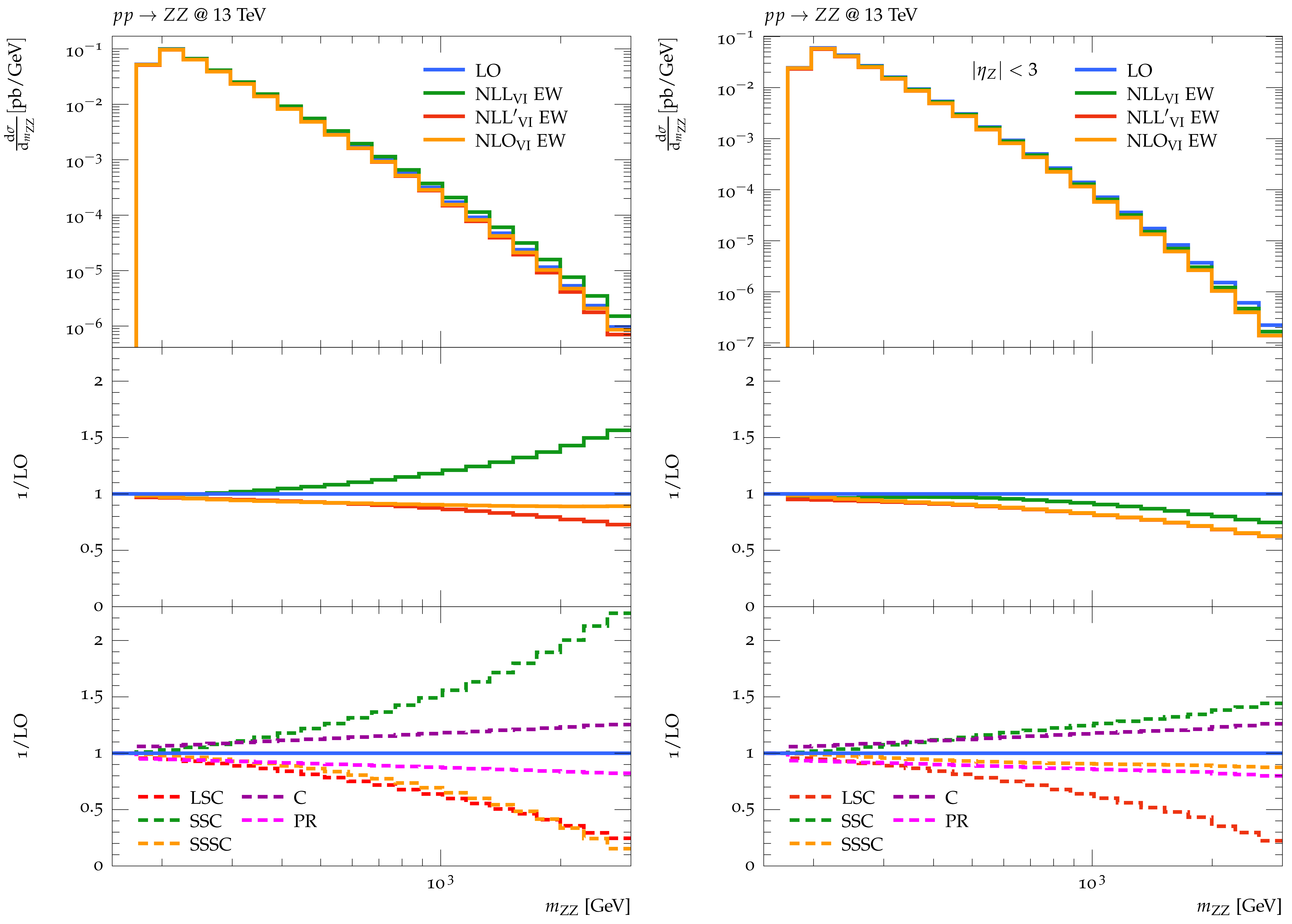




Two important considerations come from the rapidity distribution:

- The inclusion of **SSSC** allows for a better *Sudakov* approximation, in particular for $|y_Z| < 3$
- For very forward configurations, i.e. outside the central region $|y_Z| < 3$, **SSC** and **SSSC** rapidly grow

Results: $pp \to ZZ$



Pseudo-rapidity cut $|\eta_Z| < 3$ avoids very forward configurations which introduce large ratios of invariants; done in any realistic analysis

Again, the inclusion of **SSSC** provides better predictions. However, no full control on it! (Non-universal) **SSSC**-like terms arise also from LA of 4-point functions

The difference $\mathrm{NLL'} - \mathrm{NLL}$ estimates $\mathcal{O}(\alpha)$ effects beyond LA

Results: $pp \to W^+W^-$

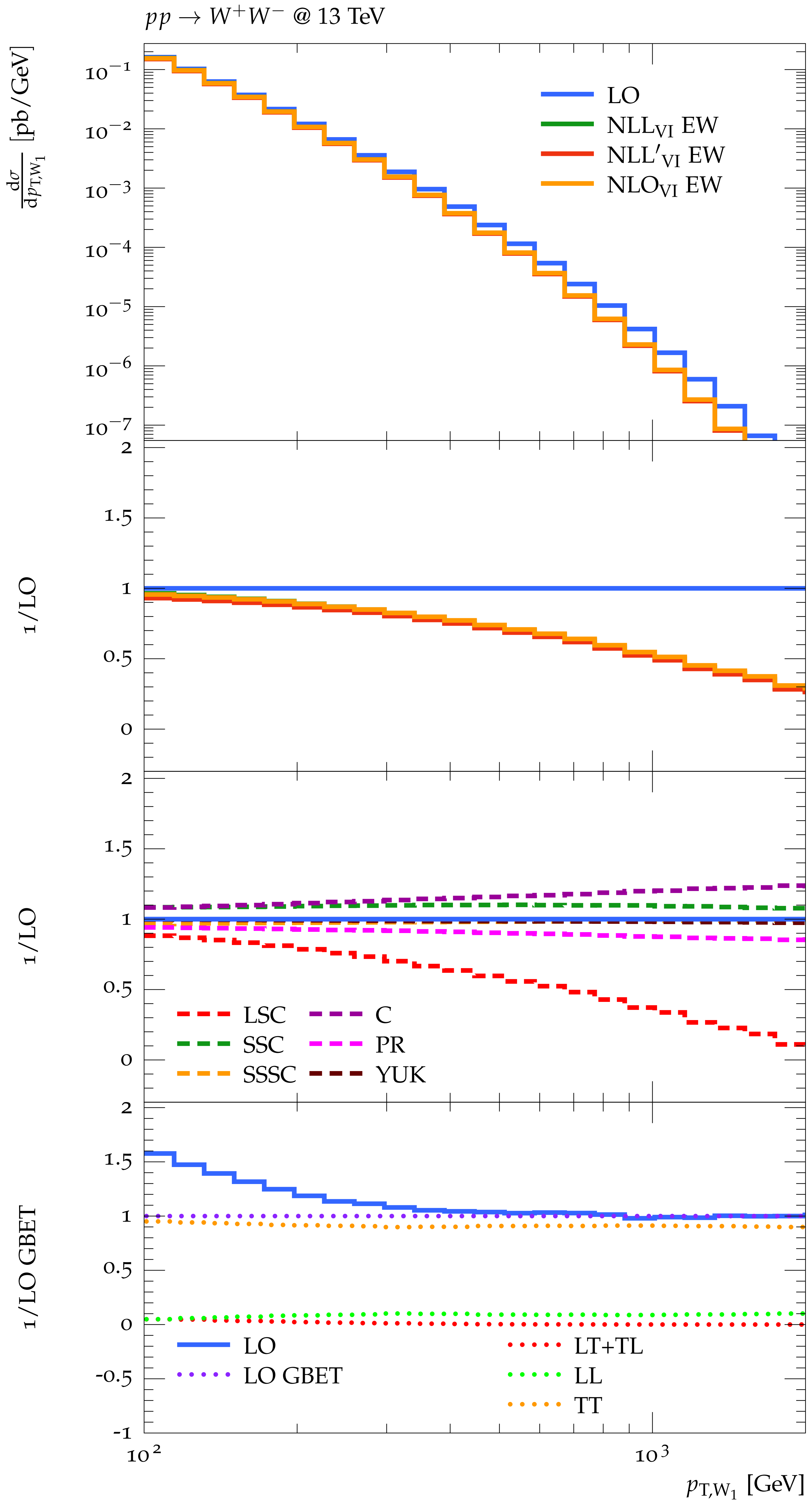
NLL EW: [Accomando et al, 0409247; 2004]

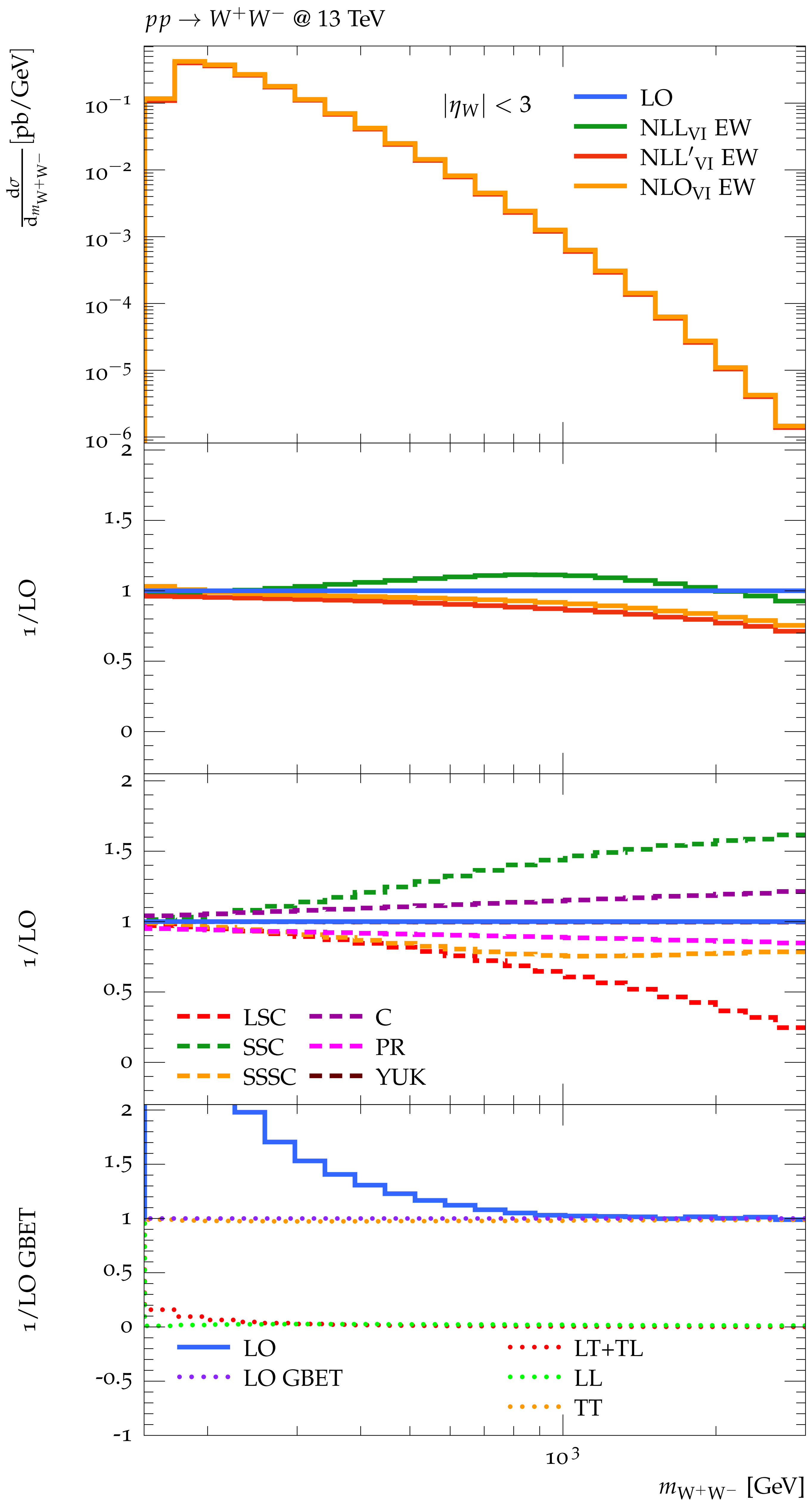
Full **NLO EW**: [Bierweile *et al*, 1208.3147; 2012]

Full **NLO**: [Baglio *et al*, 1307.4331; 2016]

Mixed NLO QCD - EW: [Bräuer *et al*, 2005.12128; 2020]

NNLO QCD+NLO EW: [Grazzini *et al*, 1912.00068; 2020]





Here **LT** and **TL** polarisation configurations are mass-suppressed while mixed **TT** and **LL** are not.

However, **LT** and **TL** are several orders of magnitude smaller than both **TT** and **LL**.

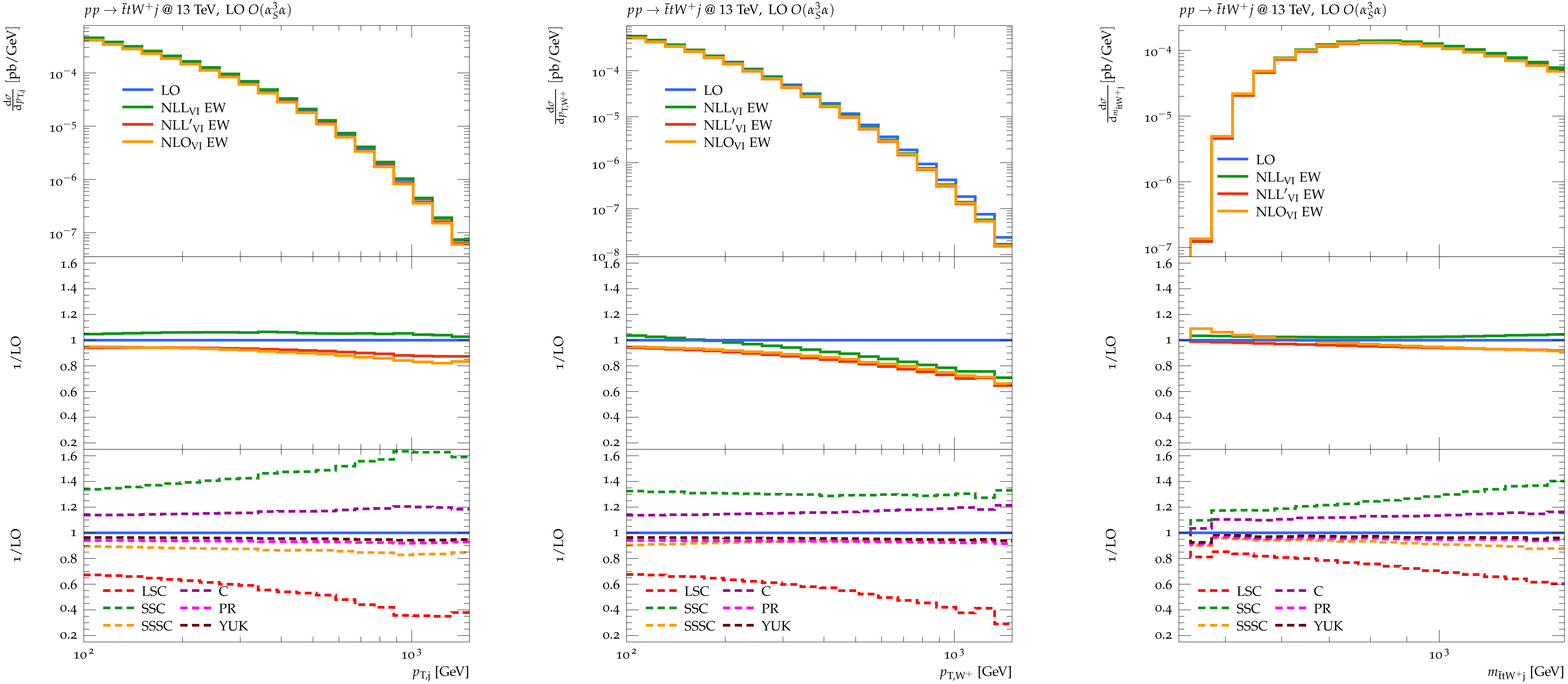
Within this setup, *Sudakov* approximation can be directly employed for these observables

Results: $pp \to t\bar{t}W^+j$

Multijet merging @ **NLO**: [Frederix & Tsinikos, 2108.07826; 2021]

NNLO QCD to $t\bar{t}W$: [Buonocore *et al*, 2306.16311; 2023]

NLO EW vs **NLL EW**: [Lindert & L.M., 2312.07927; 2023]

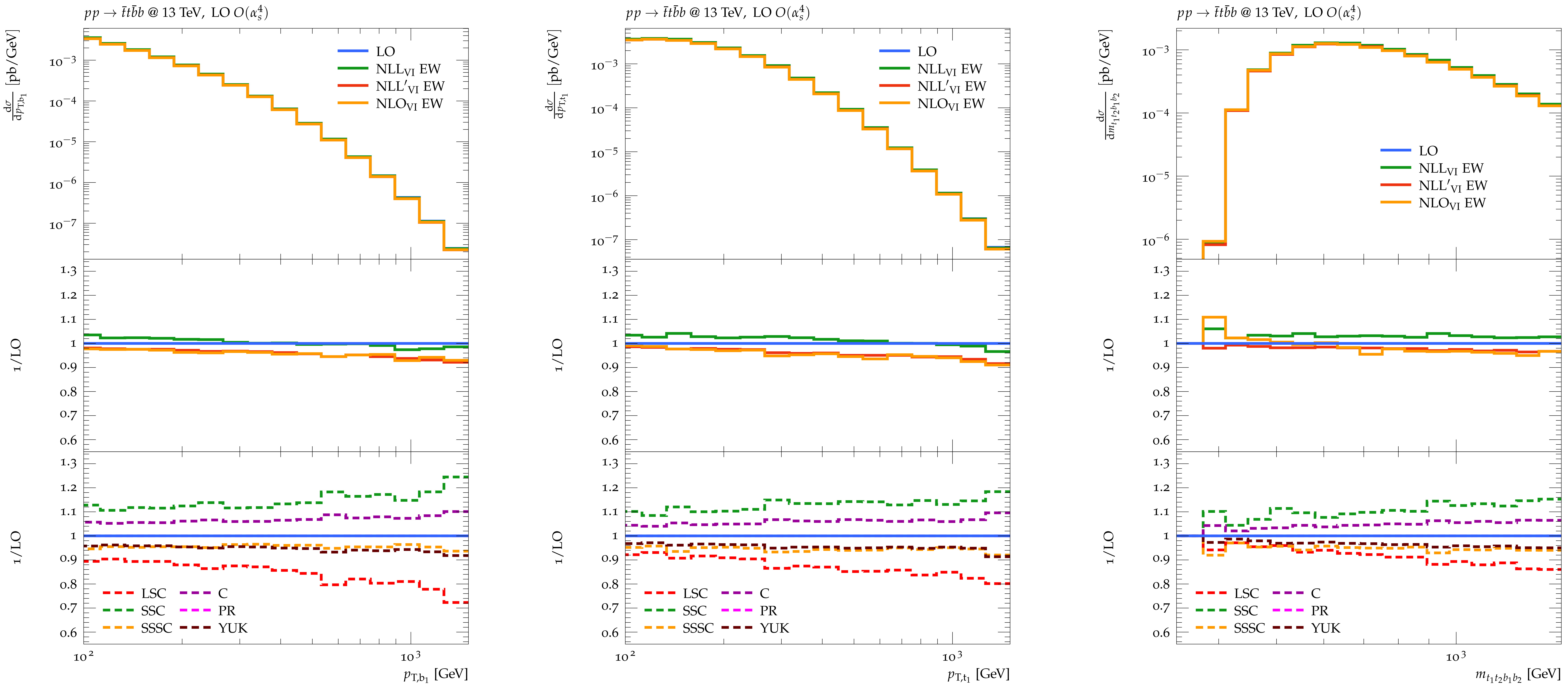


- $pp \to t\bar{t} + X$ are important backgrounds in Higgs analyses and/or BSM searches, but also for tests of EWSB
- Algorithm easily applicable to high multiplicity processes

Results: $pp \to t\bar{t}b\bar{b}$

NLO QCD: [Bredenstein *et al*, 0905.0110; 2009]

Status: [CMS collaboration, 2309.144422; 2023]



NLO EW never computed before and expected to be small. We explicitly checked and verified it, observing $\sim 6-7\,\%$ @ $p_T \approx 1$ TeV

Still a preliminary analysis! A more detailed study of **NLO EW** corrections will follow

Conclusions and outlook

- In the **EW** sector, radiative corrections at high energies are dominated by *Sudakov logarithms* which significantly enhance tails of kinematic distributions (> 10%)

- Exploiting the ***universality*** of *Sudakov logs* we developed an effective CT vertex approach for the DP algorithm and implemented it in OpenLoops

Reduction of one-loop **EW** corrections to a tree-level problem with percent level of agreement

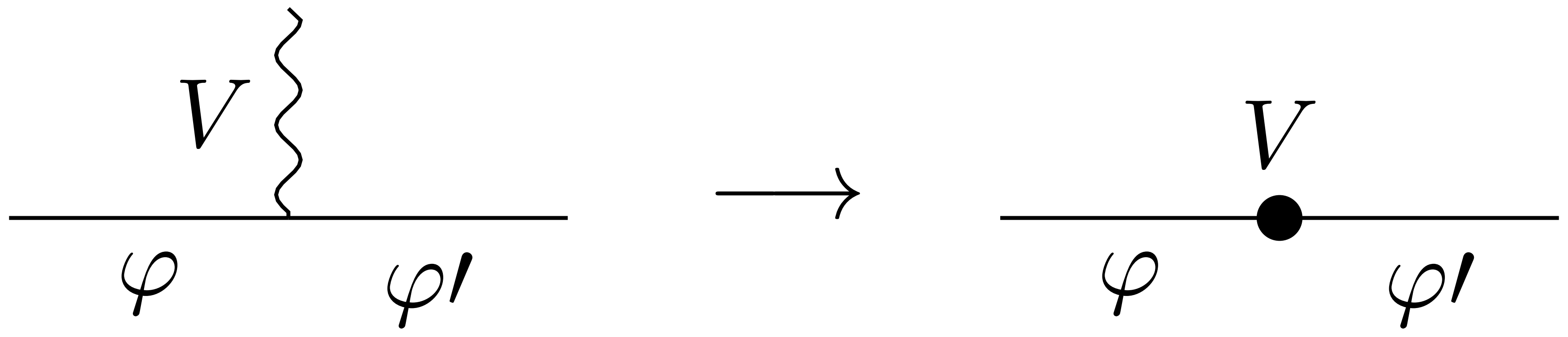
- Additional aspects of the implementation:
  - ▶ Model independent (applicable to both **SM** and **BSM** scenarios)
  - ▶ Direct employment in PS Event Generators with OL interface
  - ▶ Can be used together with differential QED radiation at **NLO** (both mass and dim reg are available)
  - ▶ Support **EW** corrections for resonant processes

- Outlook:
  - ▶ Resummation for preservation of PT
  - ▶ Dressing **NLL** **EW** *Sudakov logs* with **QCD** loops, i.e. **mixed** **QCD**-**EW** corrections
  - ▶ Suitable for **NNLO/two-loop** extension

Backup

Implementation in OpenLoops: DL & C

- Representation of Denner-Pozzorini algorithm via effective CT vertices

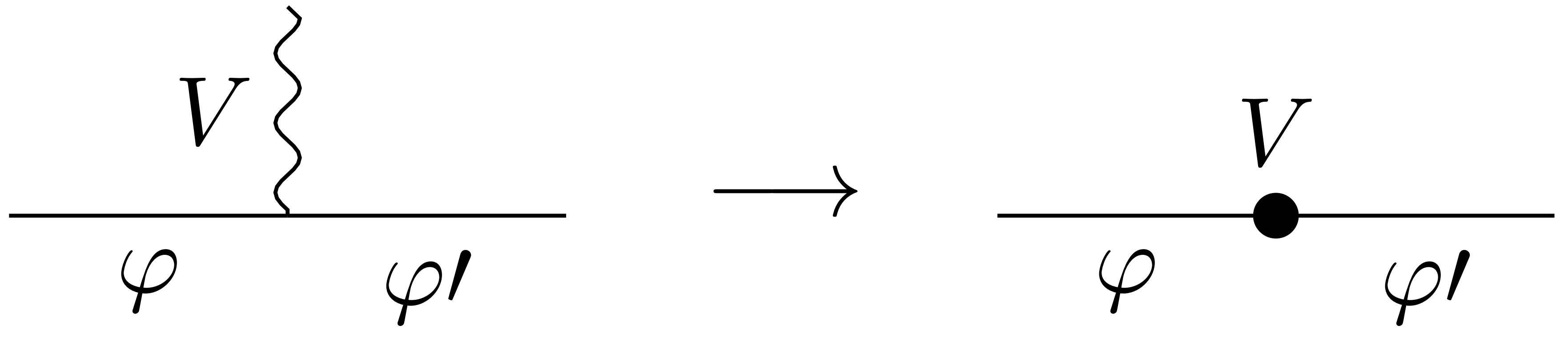


$$= ieI^{V}_{\varphi\varphi'}K^{V}_{\mathrm{ew}}$$

reducing one-loop amplitudes to tree-level ones via double CT insertions

Implementation in OpenLoops: DL & C

- Representation of Denner-Pozzorini algorithm via effective CT vertices



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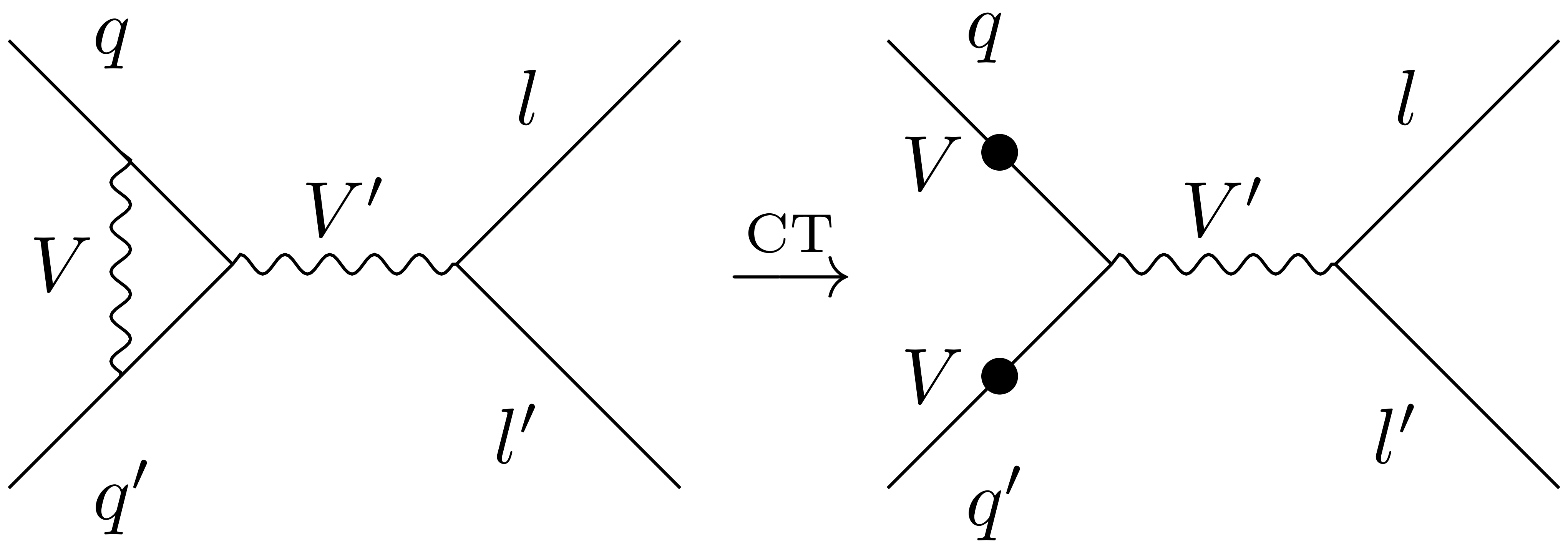
reducing one-loop amplitudes to tree-level ones via double CT insertions

Eg.: Drell-Yann



$$\Rightarrow \mathcal{M} \sim e^2 \sum_{V=A,Z,W^\pm} I^V_q I^V_{q'} \left(K^V_{\mathrm{ew}}\right)^2 \mathcal{M}_0$$

$$\sim \delta^{\mathrm{DL,\ SL}}$$

Implementation in OpenLoops: PR

- Effective CT vertices are suitable for evaluation of ***soft-collinear*** and ***collinear*** *Sudakov* corrections

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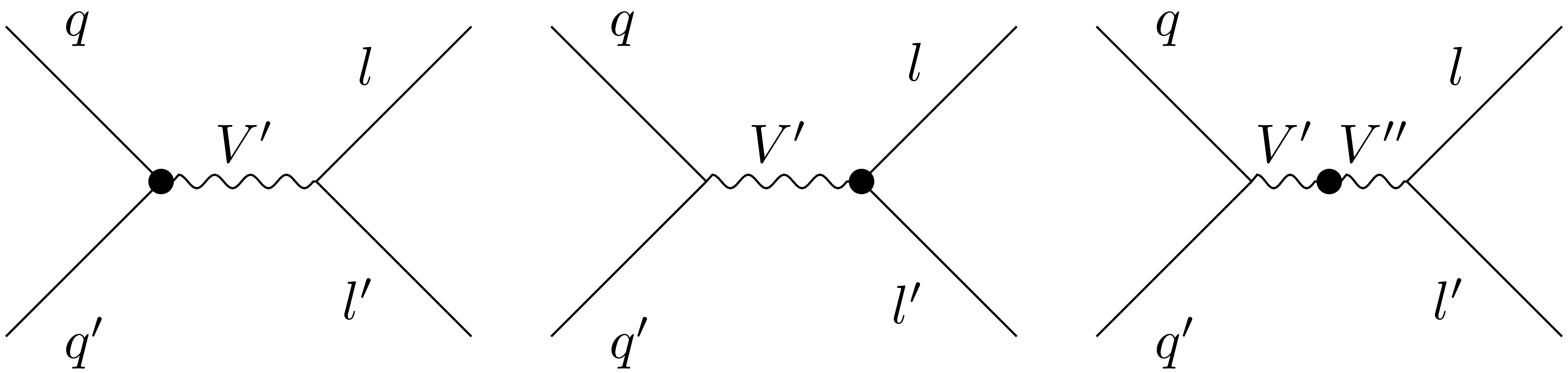
- Single logs coming from **PR** contributions can be evaluated via generation of standard UV counterterms, e.g.



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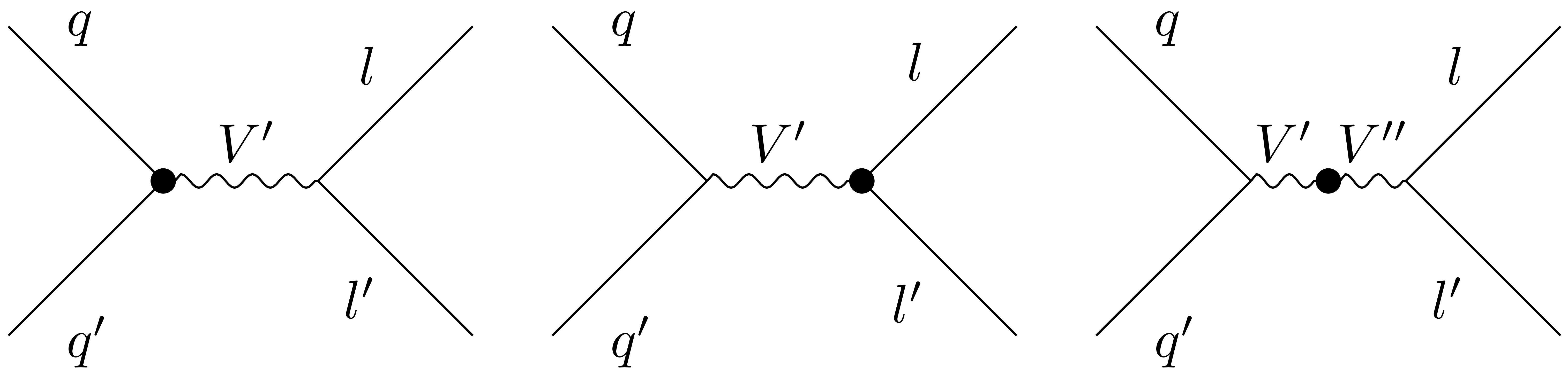
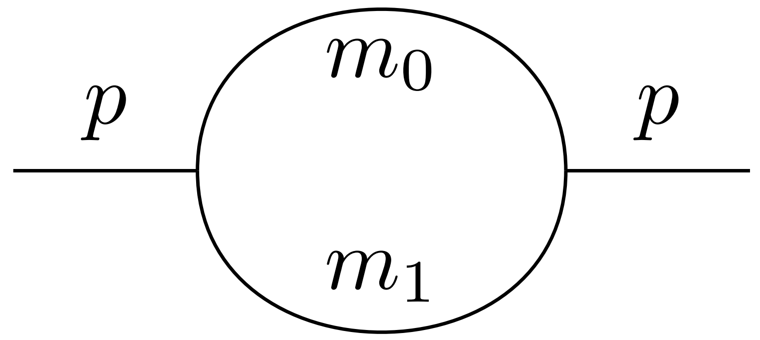


setting all the **WFRCs** to zero

- Alternative way: set $\delta_{kk'}^{\mathrm{WF}}$ to zero and evaluate **WF** + **PR** via standard UV counterterms

Single Logs: PR

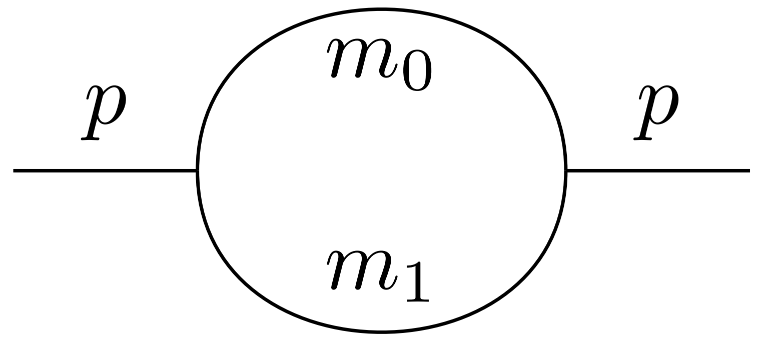
- Generic two-point function



$$\frac{\mathrm{i}}{(4\pi)^2} B_{\{0,\mu,\mu\nu\}}\left(p, m_0, m_1\right) := \mu^{4-D} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{\{1, q_\mu, q_\mu q_\nu\}}{\left(q^2 - m_0^2 + \mathrm{i}\varepsilon\right)\left[(q+p)^2 - m_1^2 + \mathrm{i}\varepsilon\right]}$$

Single Logs: PR

- Generic two-point function

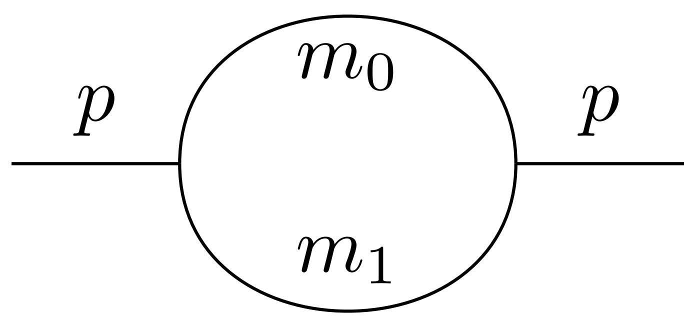


$$\frac{\mathrm{i}}{(4\pi)^2} B_{\{0,\mu,\mu\nu\}}\left(p, m_0, m_1\right) := \mu^{4-D} \int \frac{\mathrm{d}^D q}{(2\pi)^D} \frac{\{1, q_\mu, q_\mu q_\nu\}}{\left(q^2 - m_0^2 + \mathrm{i}\varepsilon\right)\left[(q+p)^2 - m_1^2 + \mathrm{i}\varepsilon\right]}$$

- In LA $\mu^2 = s \gg p^2, m_0^2, m_1^2 \Rightarrow$ four possible hierarchy of masses

$$\begin{aligned}
&(a) \quad m_i^2 \ll p^2 \text{ and } p^2 - m_{1-i}^2 \ll p^2 \quad \text{for } i = 0 \text{ or } \quad i = 1, \\
&(b) \quad \text{not} \quad (a) \text{ and } m_i^2 \not\gg p^2 \text{ for } i = 0, 1, \\
&(c) \quad m_0^2 = m_1^2 \gg p^2 \\
&(d) \quad m_i^2 \gg p^2 \not\gg m_{1-i}^2 \text{ for } i = 0 \text{ or } i = 1
\end{aligned}$$

Single Logs: PR



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- Results for two point functions and their derivatives

$$B_0\left(p, m_0, m_1\right) \stackrel{\mathrm{LA}}{=} \log \frac{\mu^2}{M^2},$$

$$B_1\left(p^2, m_0, m_1\right) \stackrel{\mathrm{LA}}{=} -\frac{1}{2} \log \frac{\mu^2}{M^2},$$

$$\frac{1}{p^2} B_{00}\left(p^2, m_0, m_1\right) \stackrel{\mathrm{LA}}{=} \frac{3m_0^2 + 3m_1^2 - p^2}{12p^2} \log \frac{\mu^2}{M^2},$$

$$\frac{1}{p^2} g^{\mu\nu} B_{\mu\nu}\left(p^2, m_0, m_1\right) \stackrel{\mathrm{LA}}{=} \frac{m_0^2 + m_1^2}{p^2} \log \frac{\mu^2}{M^2}$$

$$p^2 B_0'\left(p, m_0, m_1\right) \stackrel{\mathrm{LA}}{=} \frac{1}{2} \log \frac{m_{1-i}^2}{m_i^2} = \frac{1}{2} \log \frac{p^2}{\lambda^2},$$

$$p^2 B_1'\left(p^2, m_0, m_1\right) + \frac{1}{2} p^2 B_0'\left(p, m_0, m_1\right) \stackrel{\mathrm{LA}}{=} -\frac{1}{4} \log \frac{m_0^2}{m_1^2}$$

Implementation in OpenLoops: resonances

- DP algorithm:
  - ▶ At $\sqrt{s} \gg M_W$, **NLO EW** radiative corrections are DL and SL
  - ▶ These corrections are ***universal***, i.e. are associated to *external* states only

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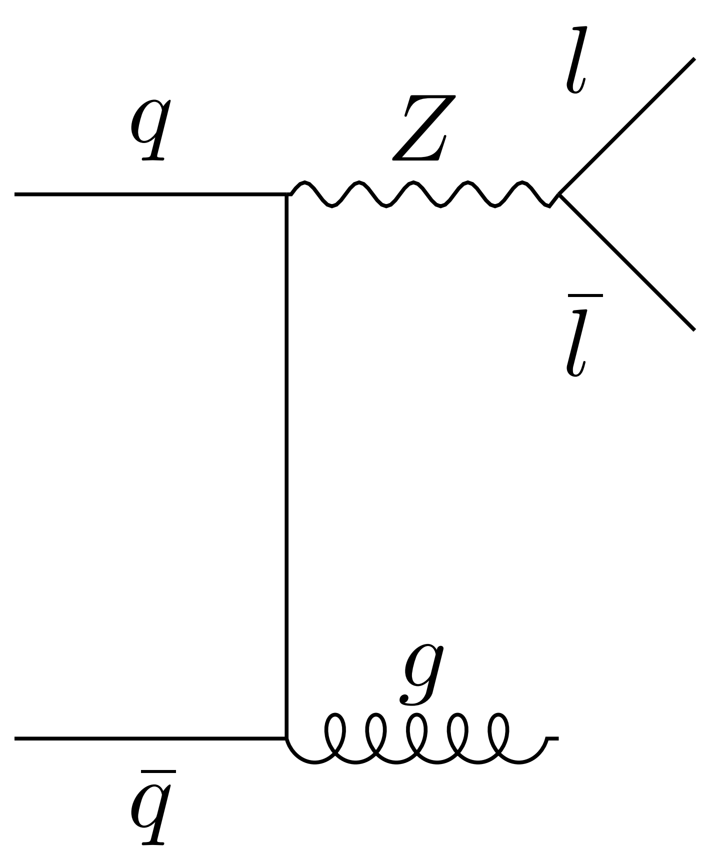

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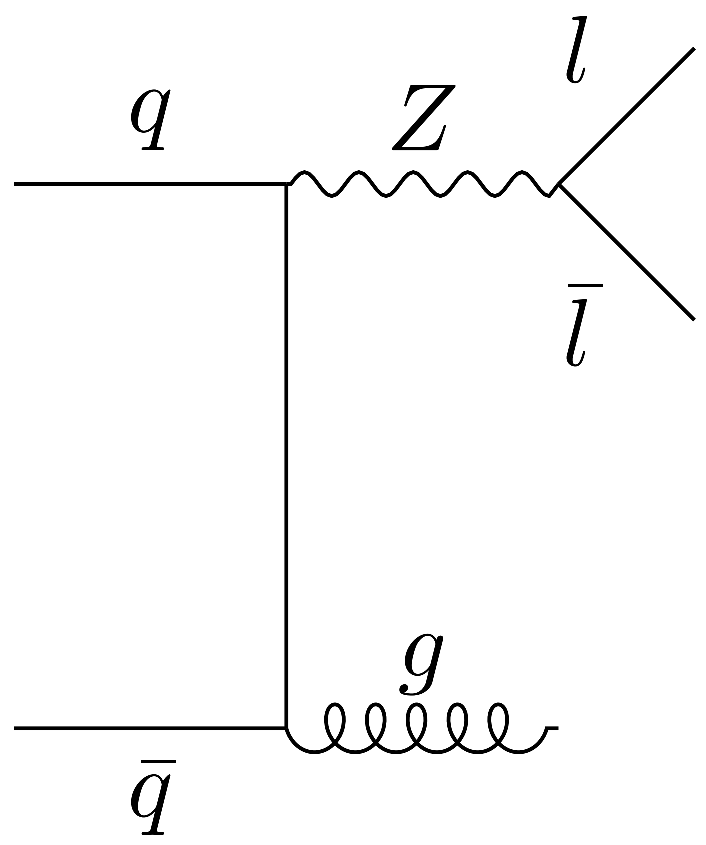

LSC, C: $\delta_{kk}^{\mathrm{LSC,C}}$, $k \in \{q, \bar{q}, l, \bar{l}\}$

SSC, S-SSC: $\delta_{kl}^{\mathrm{(S\text{-})SSC}}$, $k \neq l$ and $k, l \in \{q, \bar{q}, l, \bar{l}\}$

PR: CTs for $Z\bar{q}q, Z\bar{l}l$ vertices

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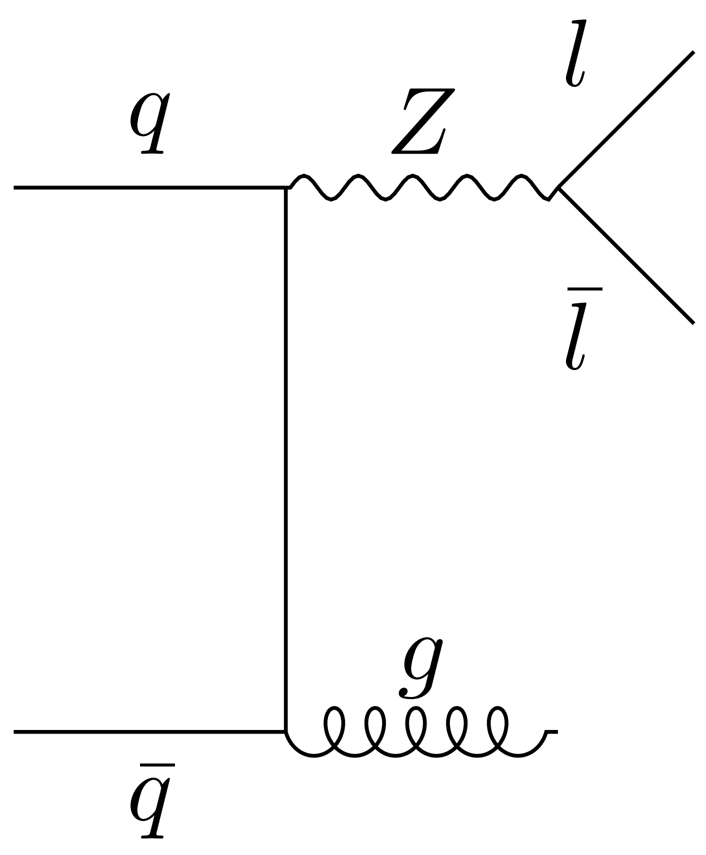

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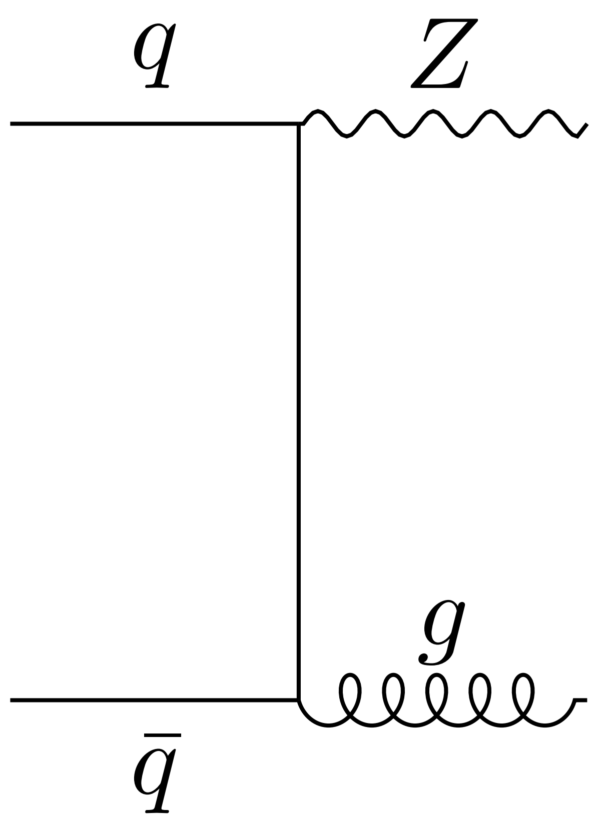
- In the kinematic region where the Z boson is nearly on shell



LSC, **C**: $\delta_{kk}^{\mathrm{LSC,C}}$, $k \in \{q, \bar{q}, Z\}$

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Implementation in OpenLoops: resonances

- Solution: evaluation of *Sudakov* corrections associated to both Z and $\{l, \bar{l}\}$ with different weights $P_i(k_i)$

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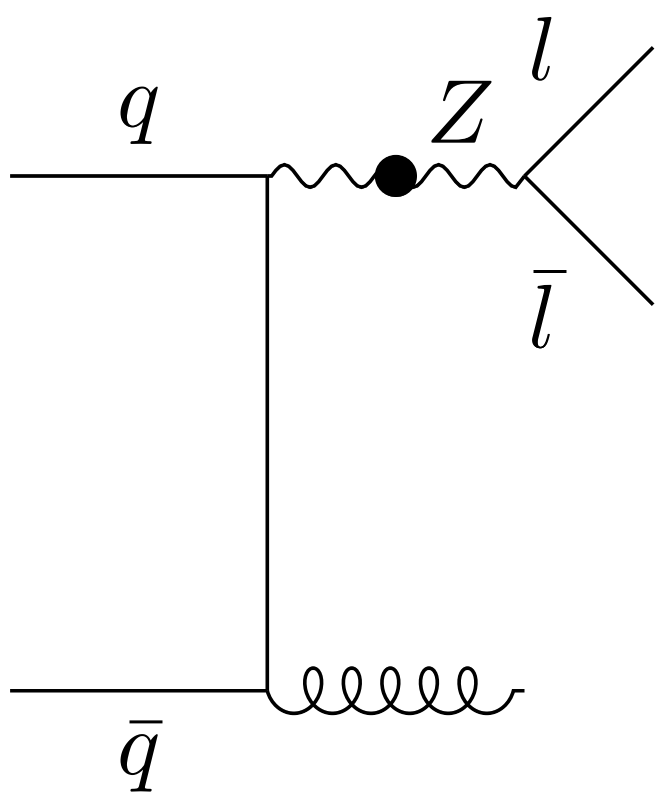
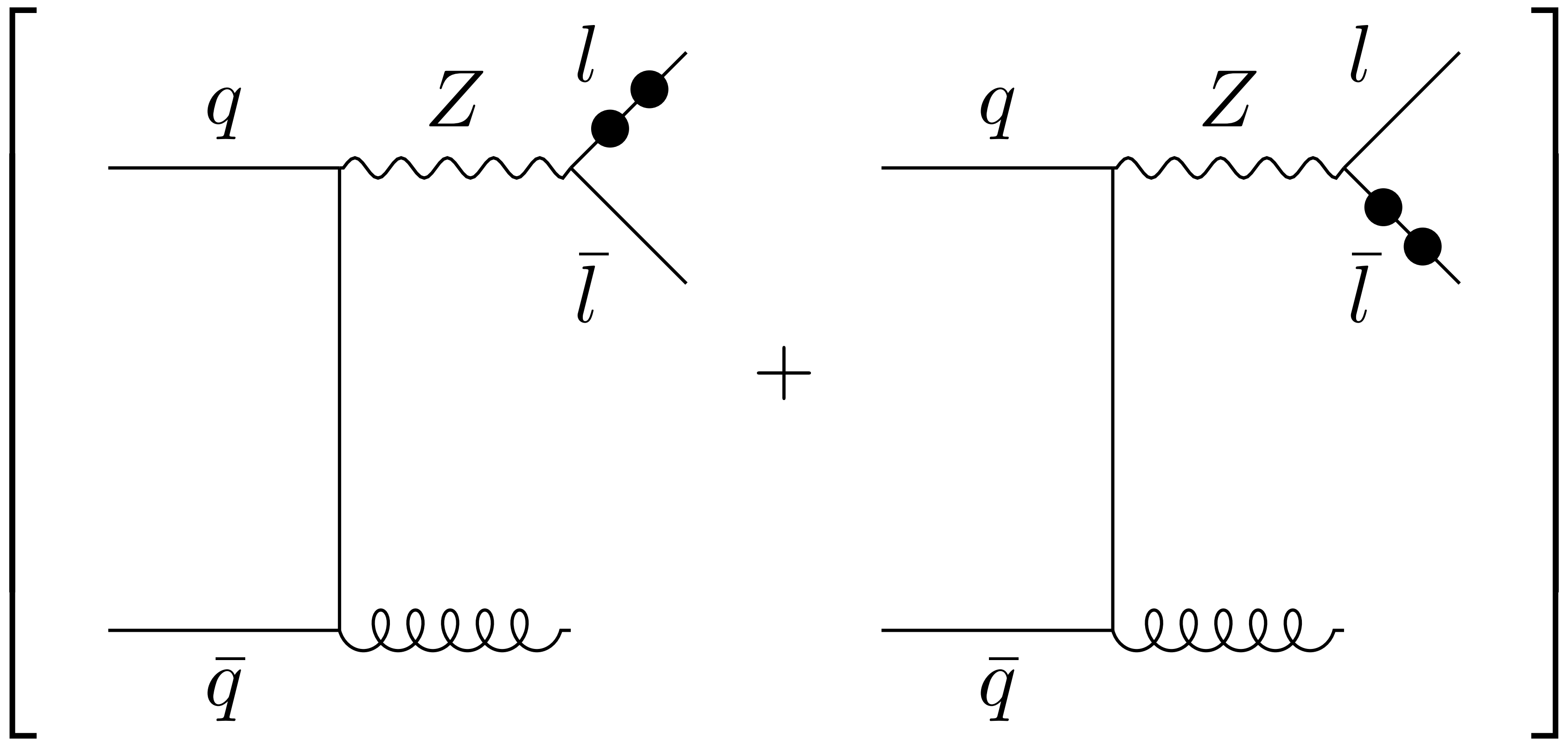


$\times P_Z(k)$



$\times P_{\{l,\bar{l}\}}(k) = 1 - P_Z(k)$

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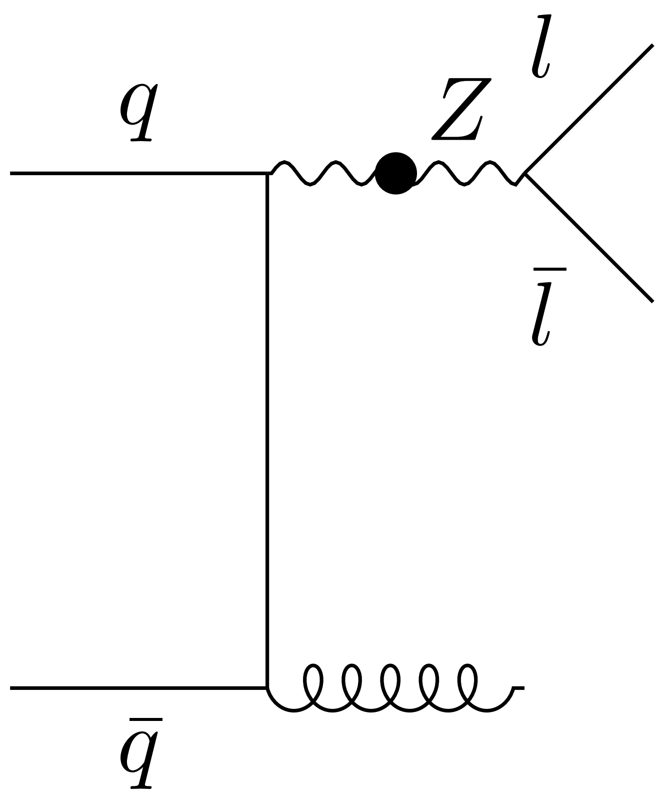
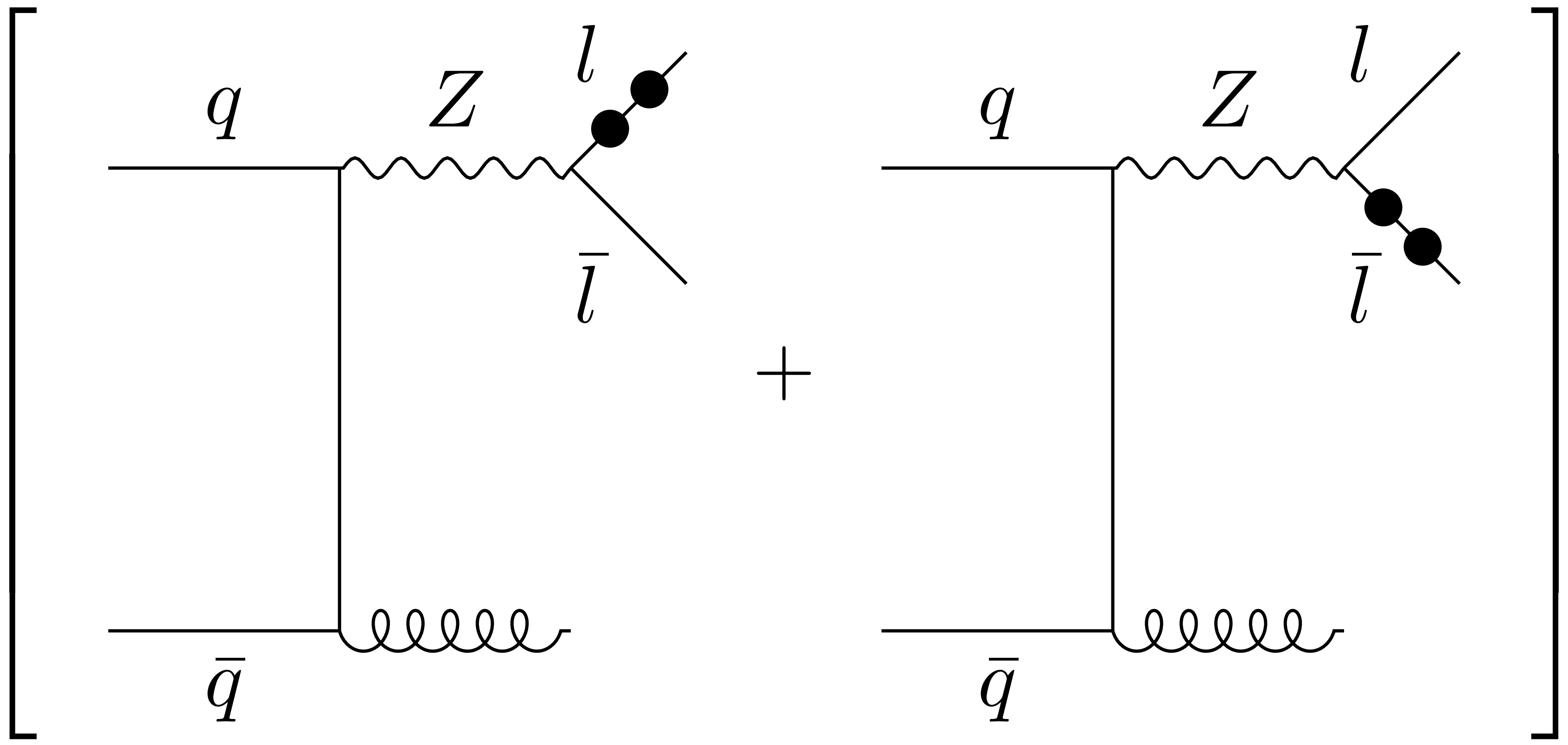


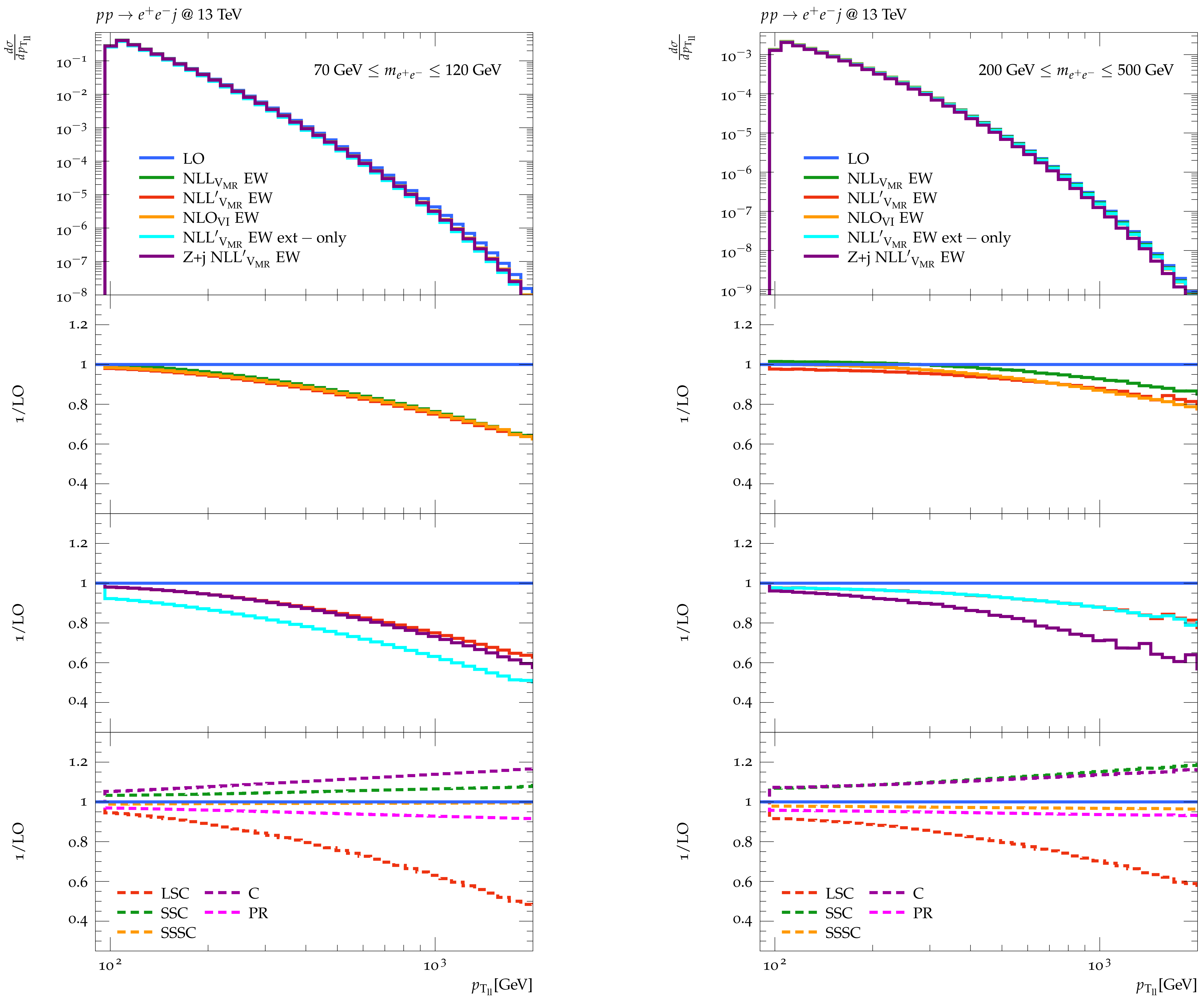
$\times P_Z(k)$

$$P_{X_i}(k_i) = \left| \frac{\mu^2_{X_i} - M^2_{X_i}\Gamma^2_{X_i}}{(k_i^2 - \mu^2_{X_i})^2 + \mu^2_{X_i}} \right| = \begin{cases} 1 & \text{if } k_i^2 \to M^2_{X_i} \\ 0 & \text{if } k_i^2 \to \infty \end{cases}$$



$\times P_{\{l,\bar{l}\}}(k) = 1 - P_Z(k)$

Results: $pp \to e^+e^-j$



External insertions approach fails in reproducing the full NLO prediction for the $m_{e^+e^-}$ range "capturing" the resonance

Issue naturally solved with internal insertions controlled by projectors

Automatic recover of standard algorithm when far from the resonance

Implementation in OpenLoops: projectors

- Explicit expression of the projectors for unstable particles X

$$P_{X_i}(k_i) = \left| \frac{\mu_{X_i}^2 - M_{X_i}^2 \Gamma_{X_i}^2}{(k_i^2 - \mu_{X_i}^2)^2 + \mu_{X_i}^2} \right| = \begin{cases} 1 & \text{if } k_i^2 \to M_{X_i}^2 \\ 0 & \text{if } k_i^2 \to \infty \end{cases}$$

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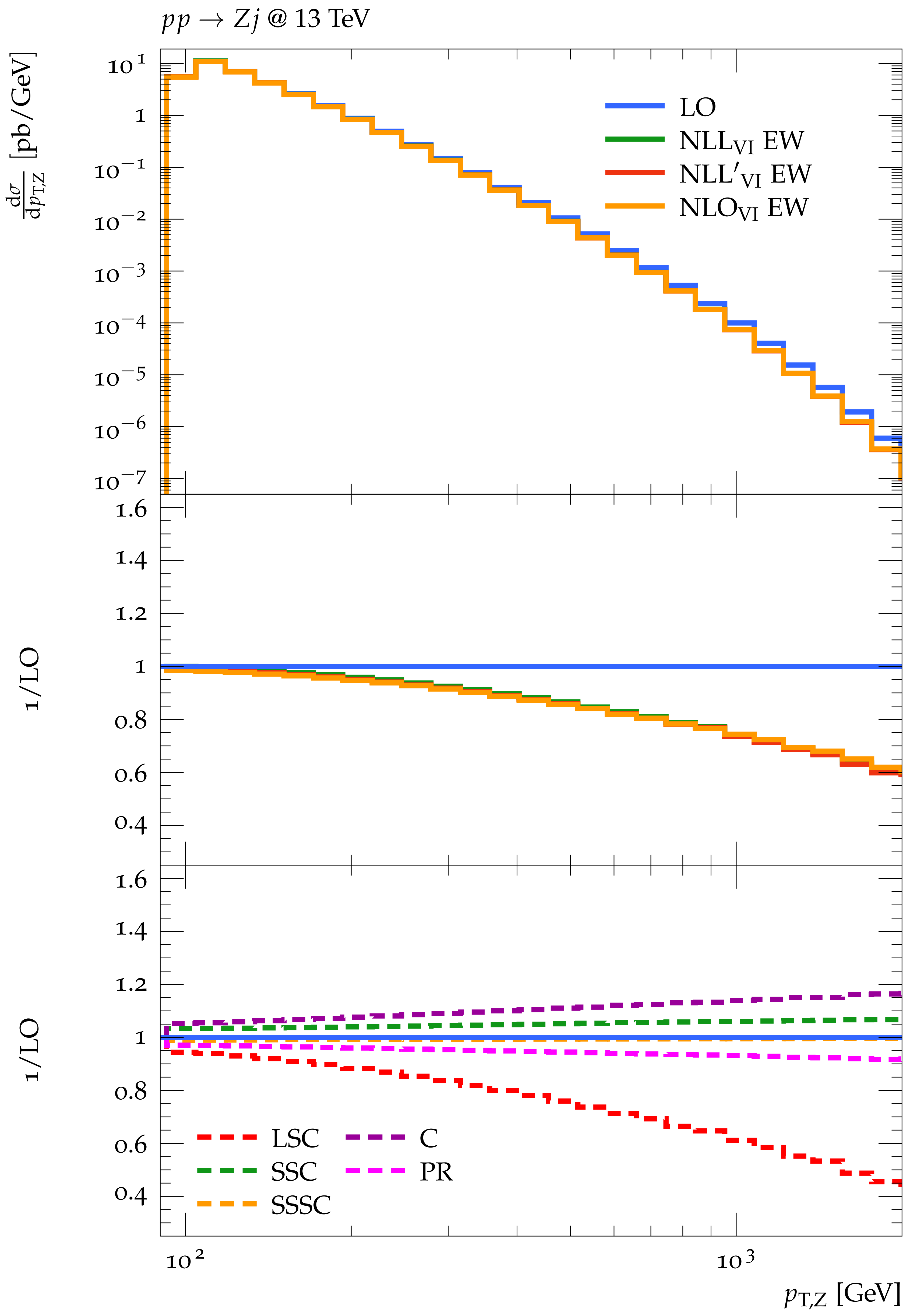
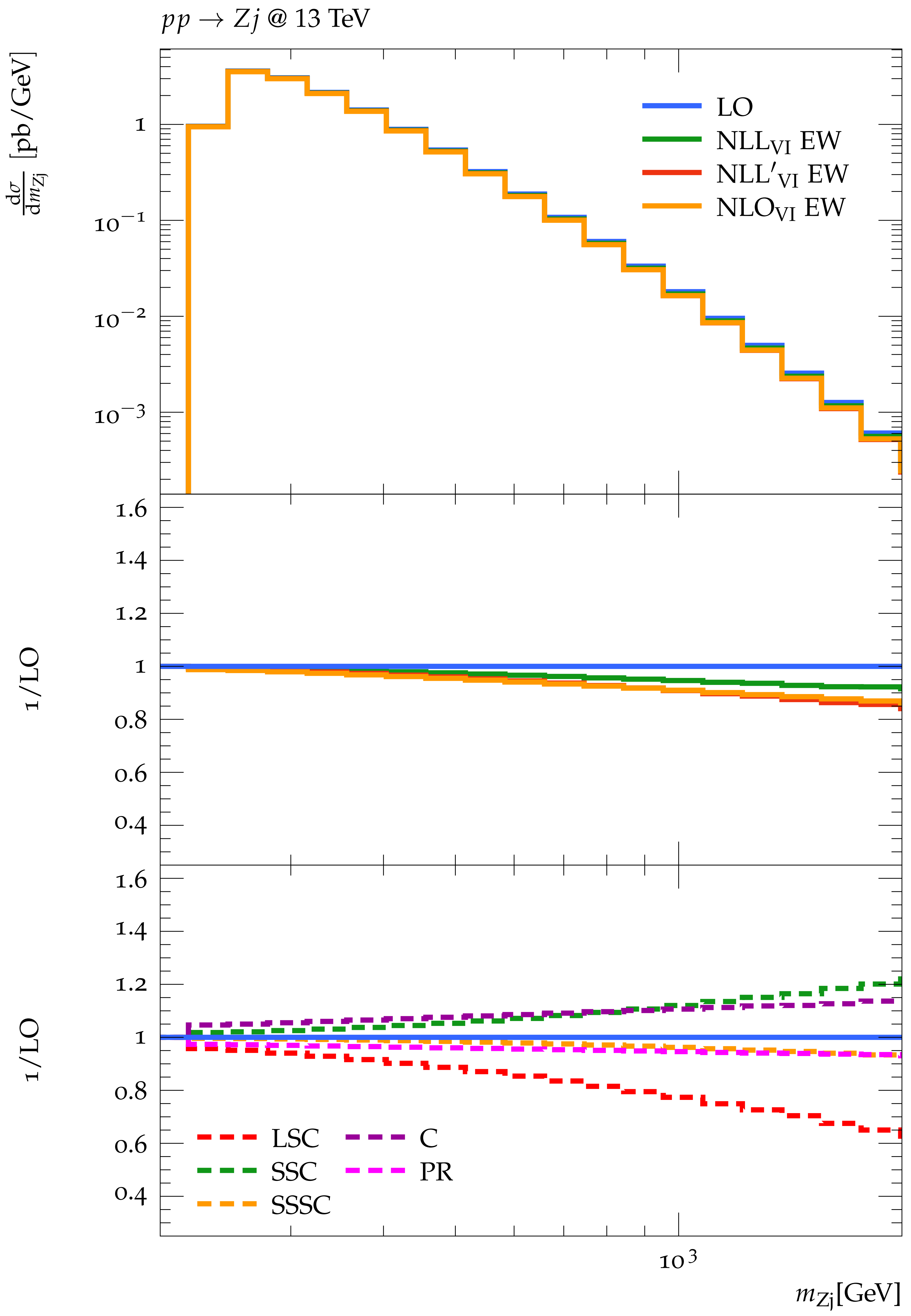
- Unitarity is violated but it can be restored:
  - Evaluation of $P_{X_i}(k_i)$ for a given psp
  - Generation of random number $0 \leq a \leq 1$
  - Choice $P_{X_i} = \begin{cases} 1 & \text{if } P_{X_i} \geq a \\ 0 & \text{if } P_{X_i} \leq a \end{cases}$

Additional results

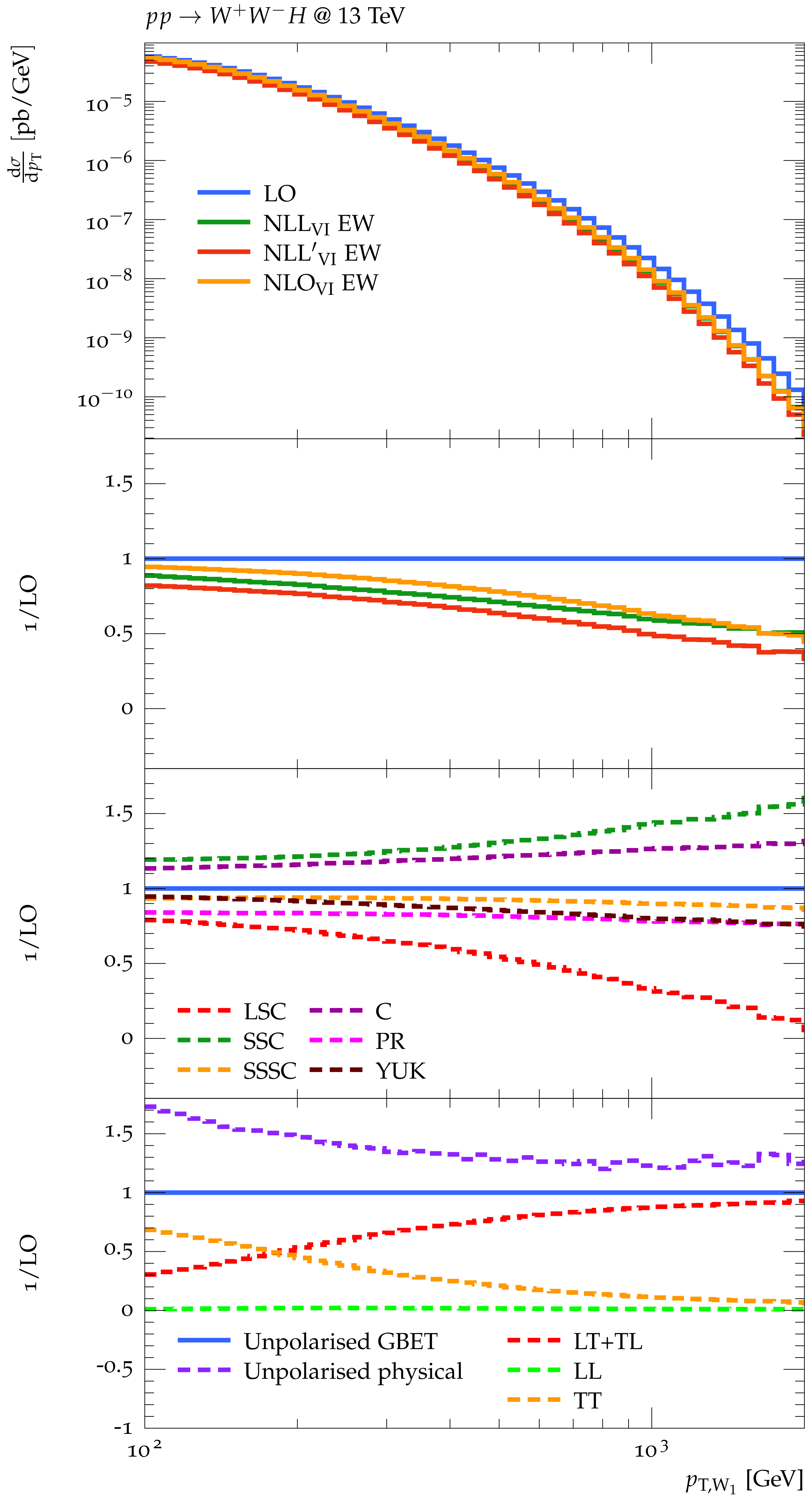
Results: $pp \to Zj$

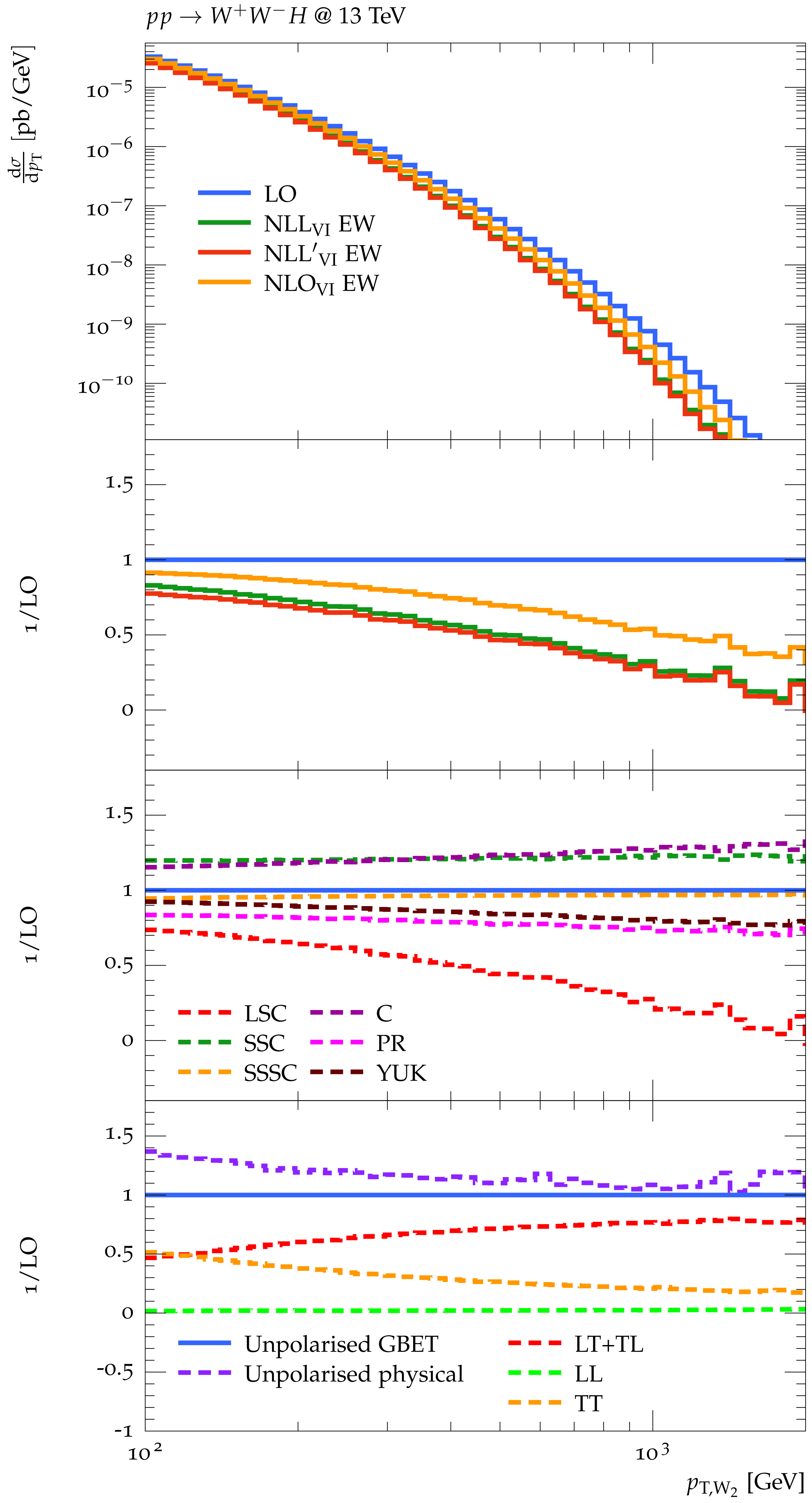




Results: $pp \to W^+W^-H$

NLO QCD: [Mao et al, 0903.2885; 2009]
Full **NLO**: [Alwall *et al,* 1405.0301; 2014]
NLO QCD PS: [Baglio, 1609.05907; 2016]





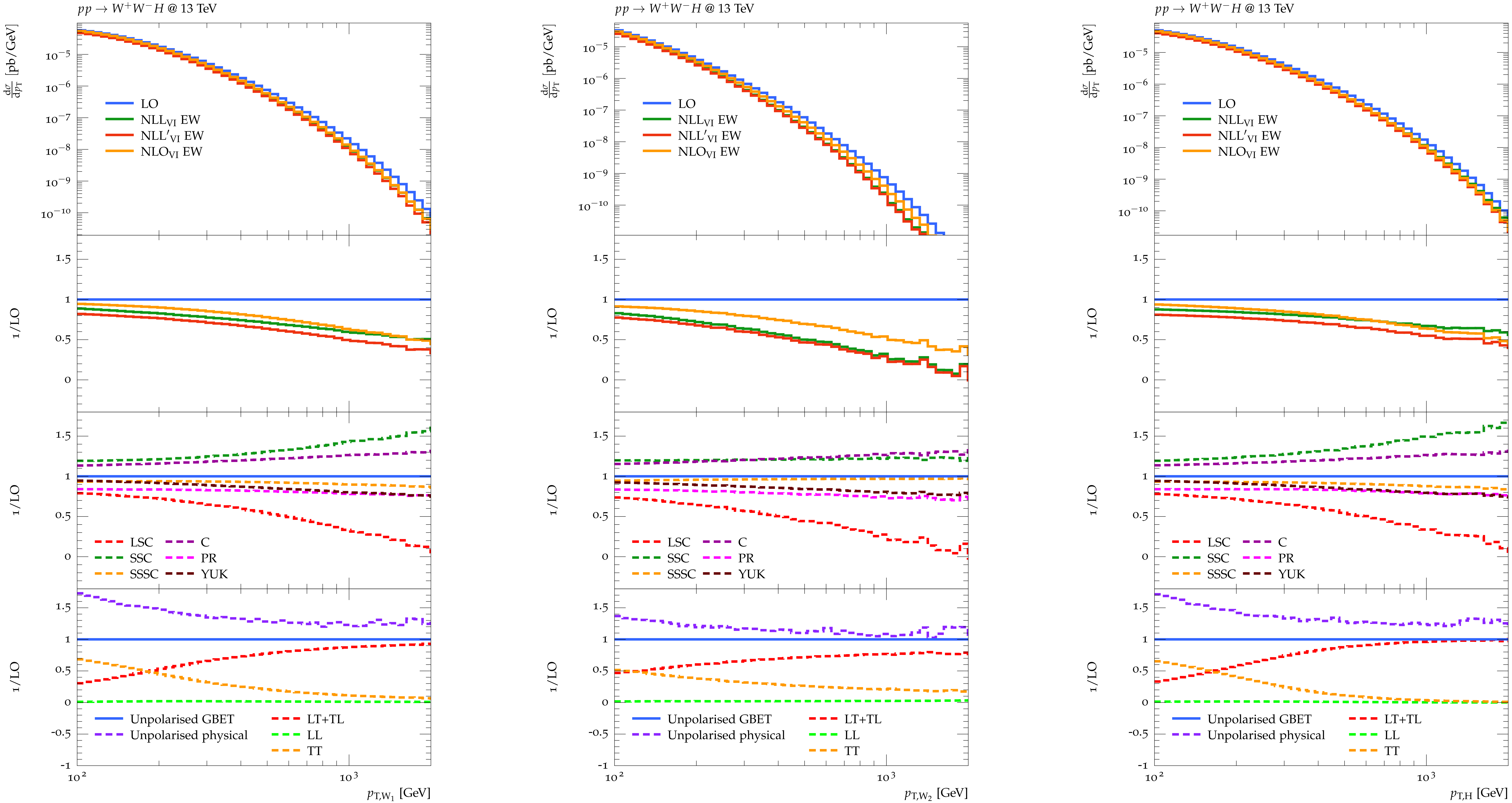
Here **TT** and **LL** polarisation configurations are mass-suppressed while mixed **LT** and **TL** are not

Small but sizeable contribution to the LO coming from **TT**. In the tails

- $p_{\mathrm{T,W_1}}$: $\sim 5\,\%$
- $p_{\mathrm{T,W_2}}$: $\sim 15\,\%$

Within this setup, *Sudakov* approximation cannot be directly employed for these observables

Results: $pp \to W^+W^-H$



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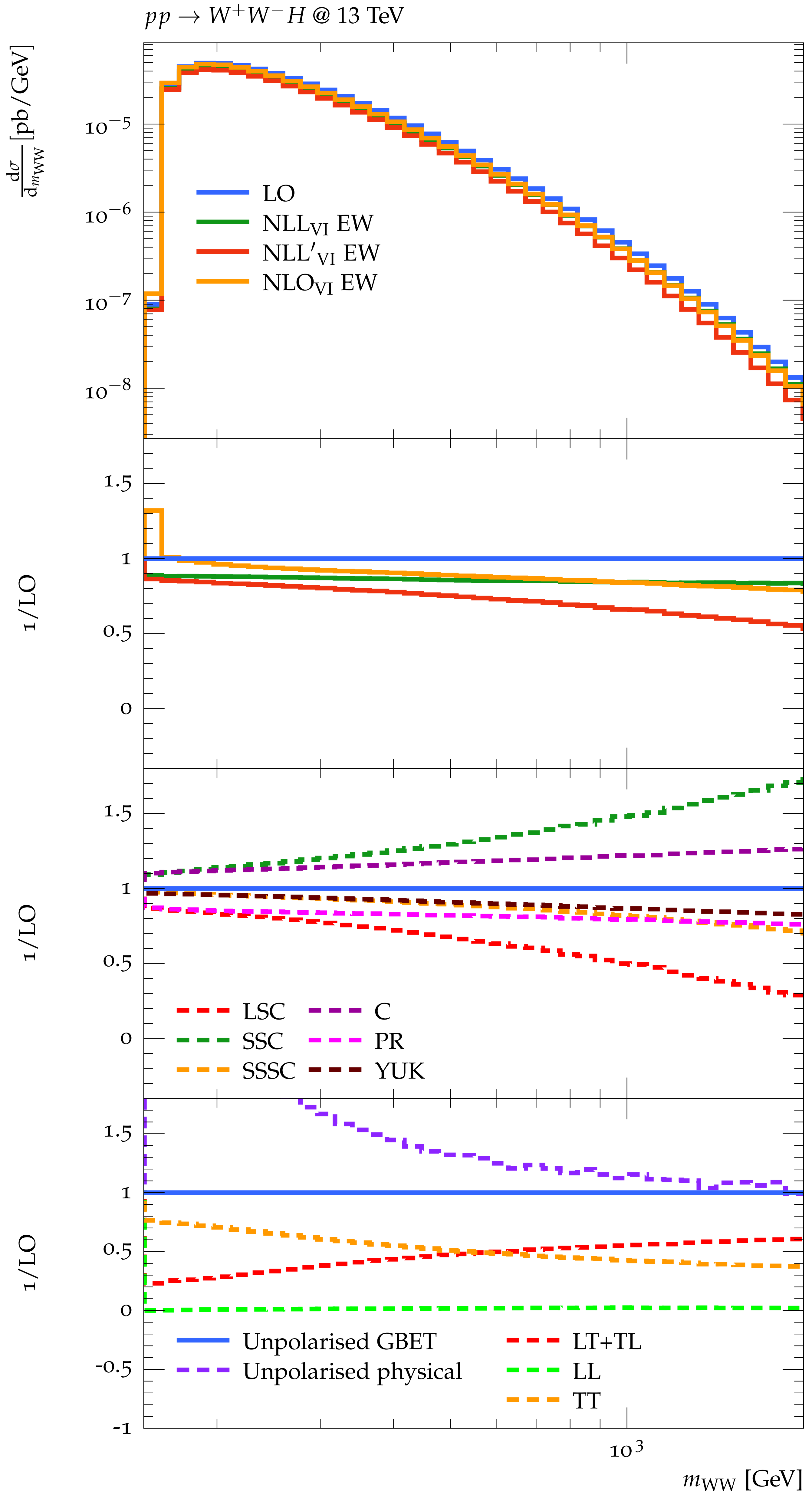
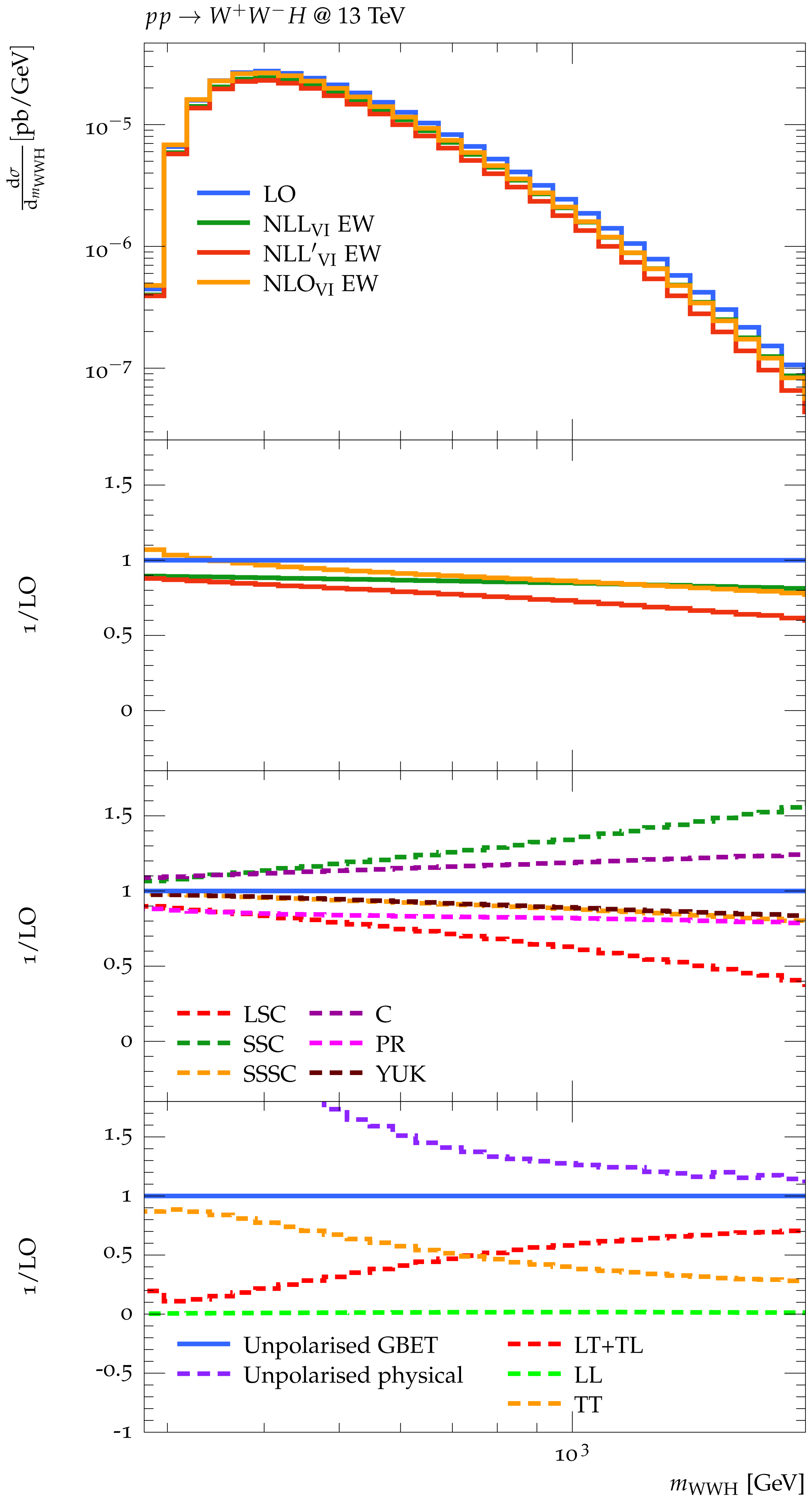


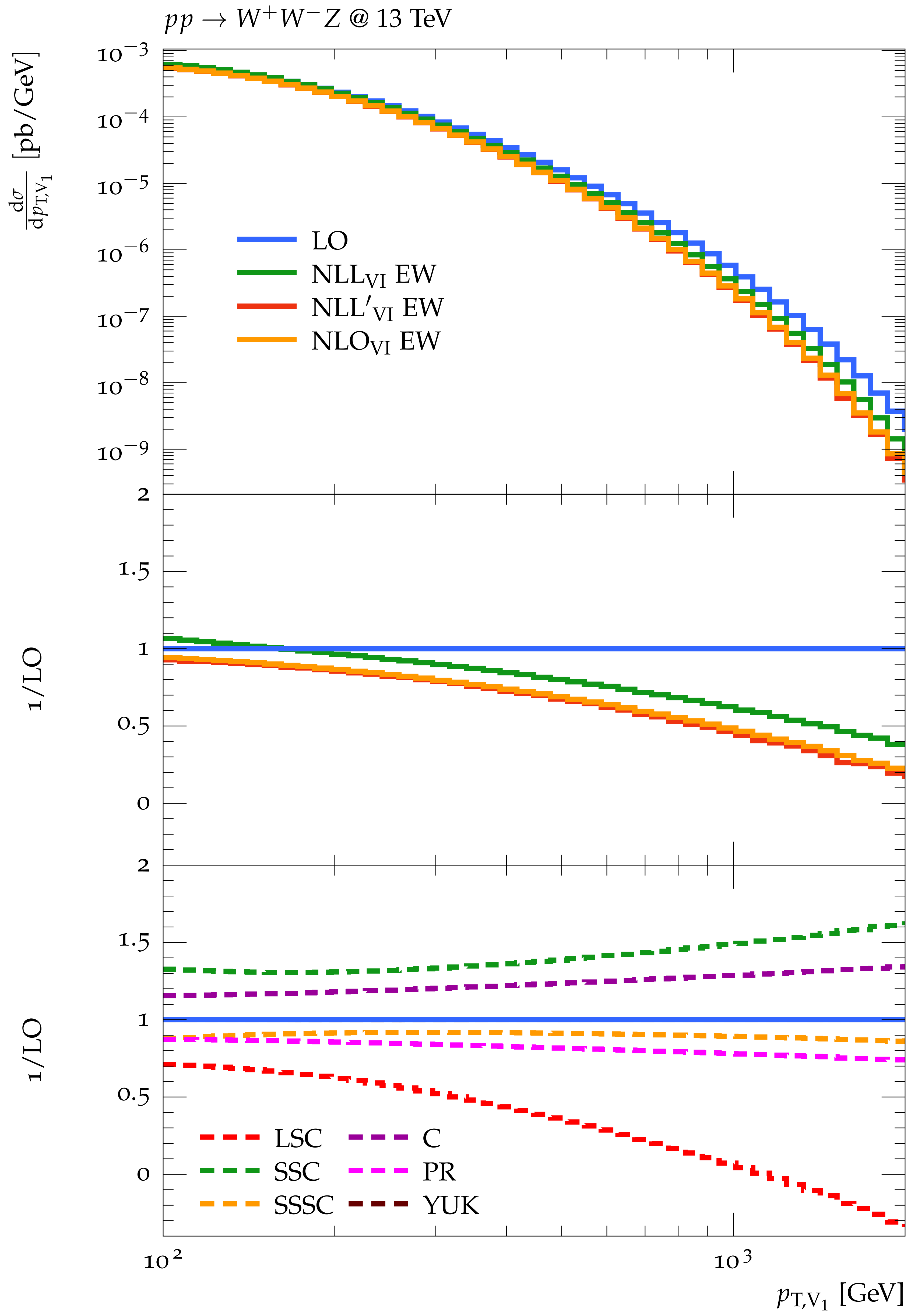


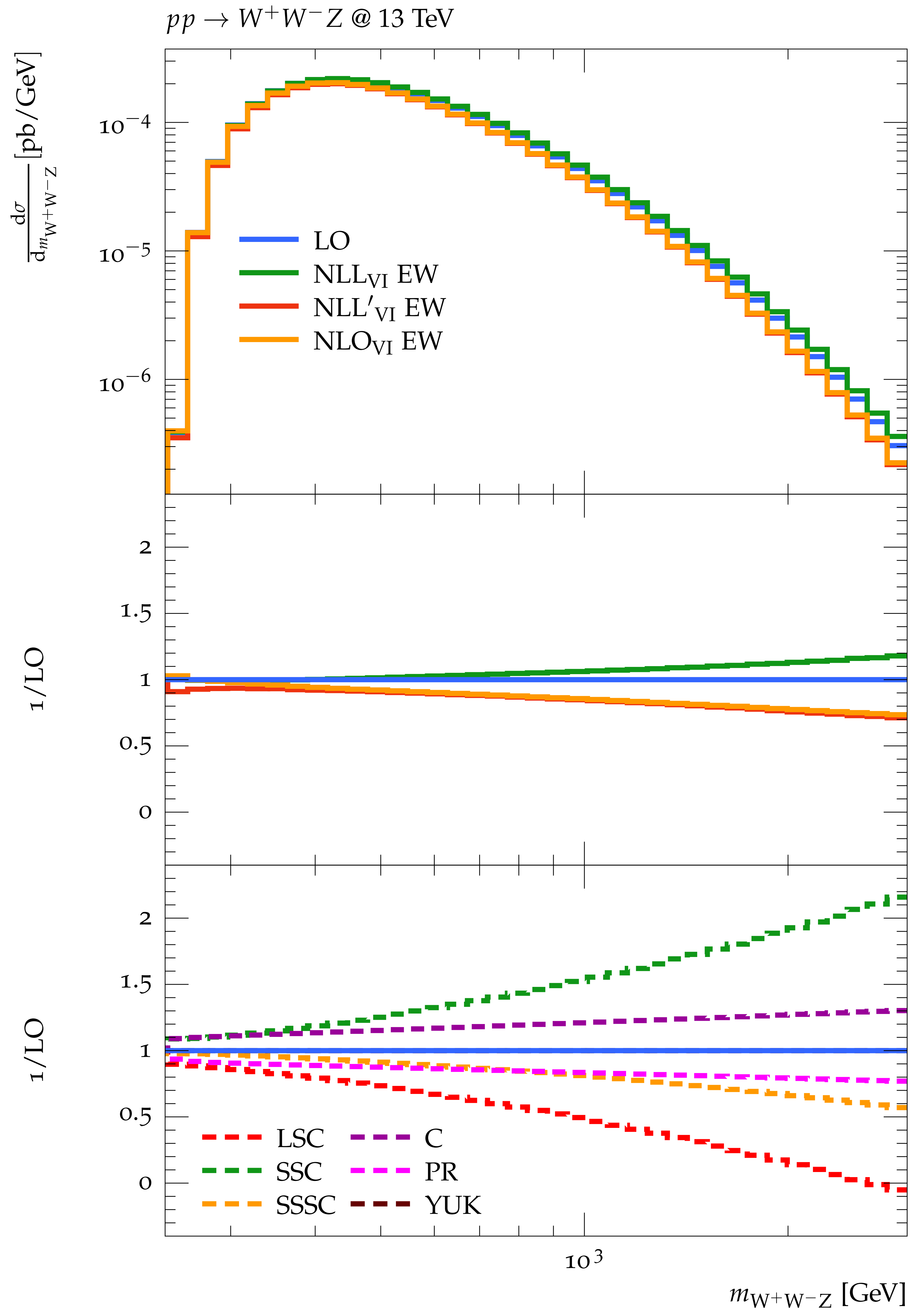
Too big contribution to the LO coming from the mass-suppressed **TT** fraction, around $30 - 40\,\%$

Significantly higher energies are required to further suppress **TT** and apply the *Sudakov* approximation

Less appealing solution: systematically derive and implement all mass-suppressed corrections

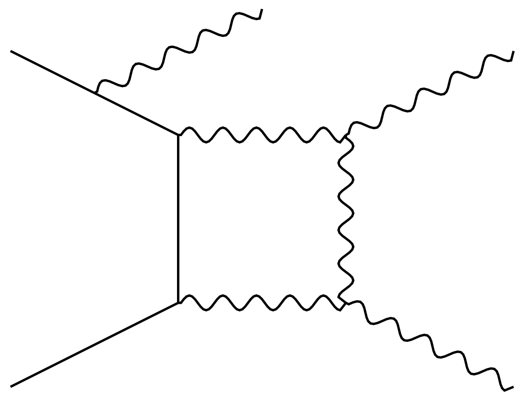
Results: $pp \to W^+W^-Z$





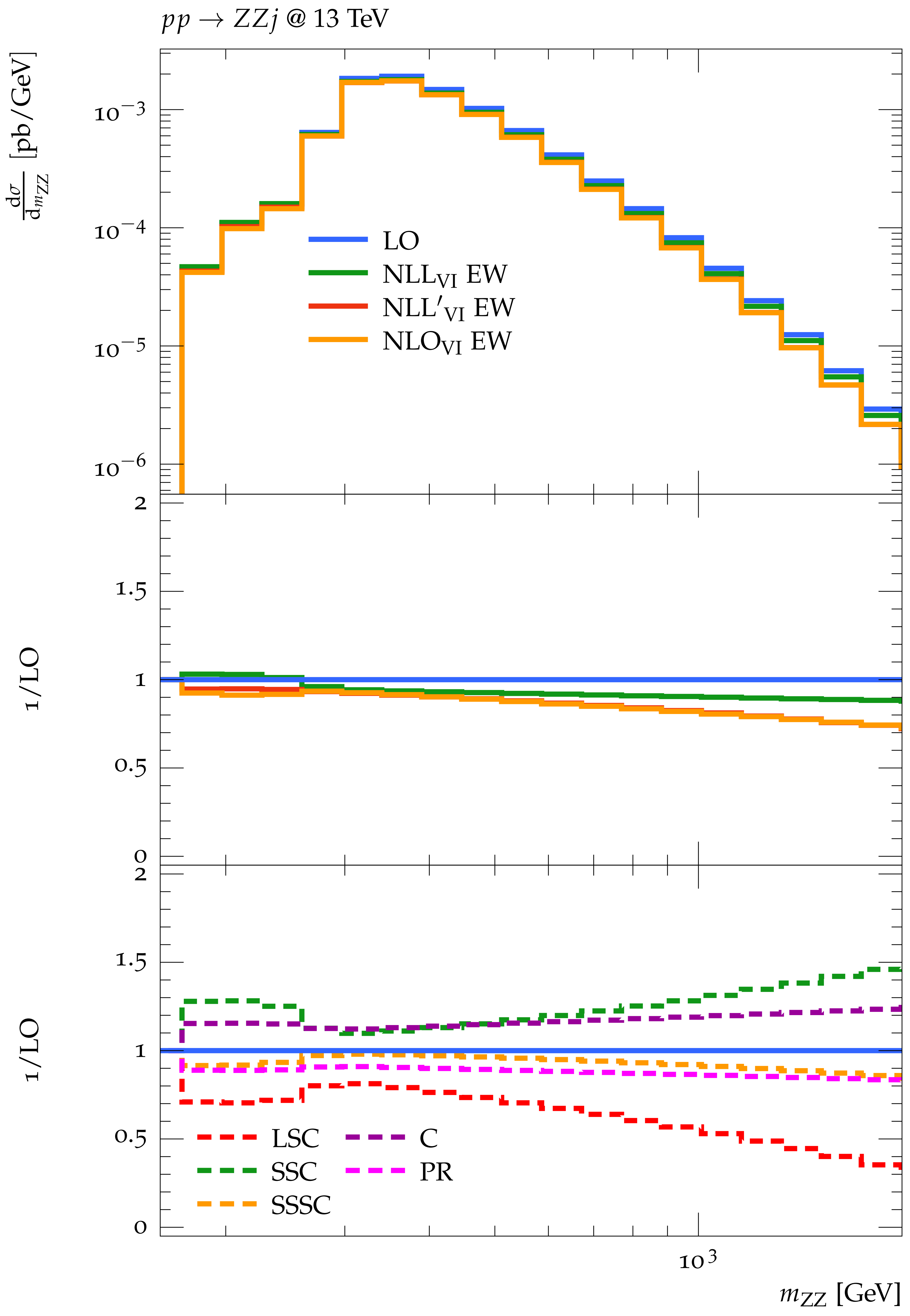
The inclusion of **SSSC** provides better predictions, but there is no full control on it!

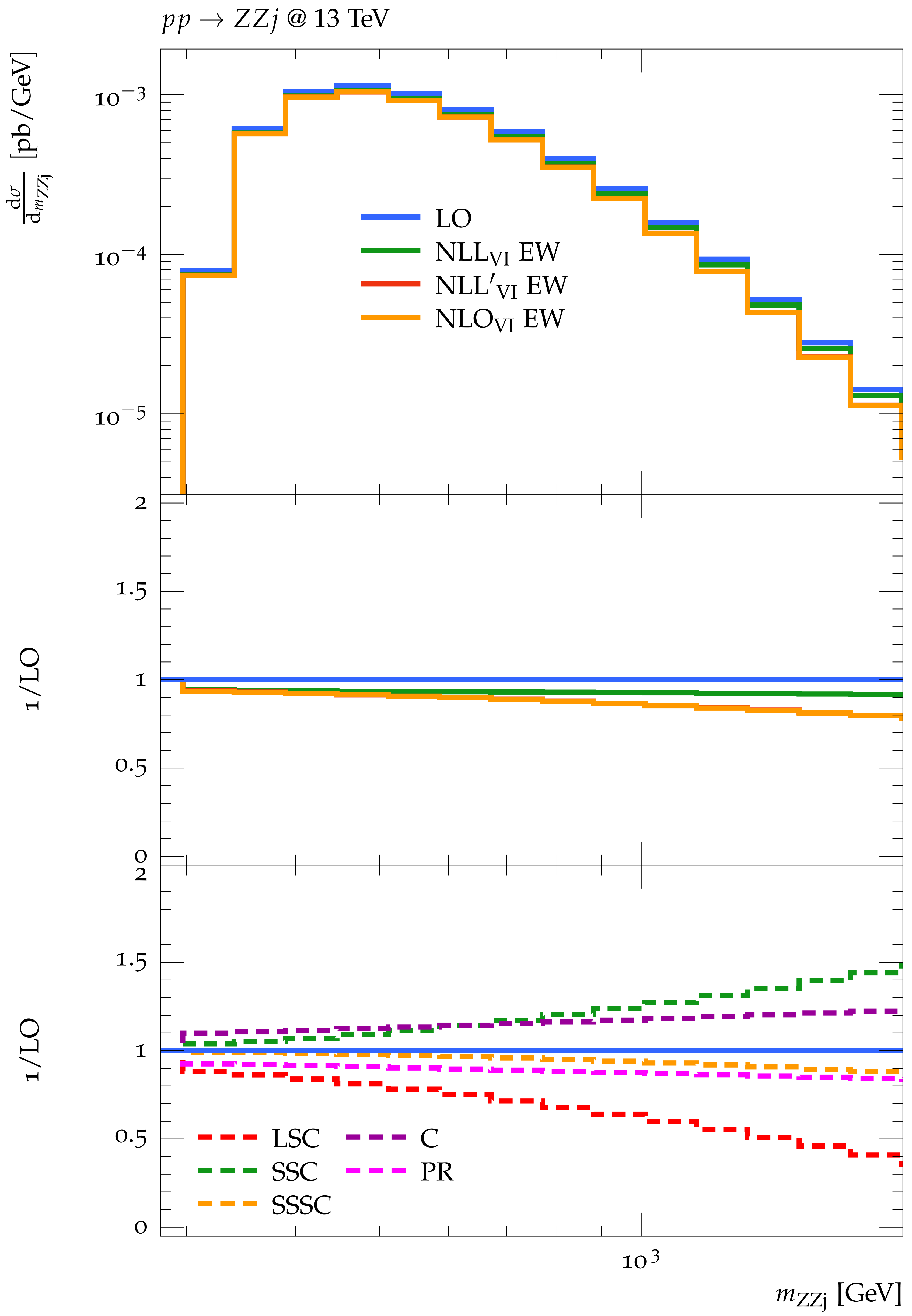
(Non-universal) **SSSC**-like terms arise
also from box diagrams



$$\sim D_0 \sim \log\left(\frac{|r_{kl}|}{s}\right)$$

Results: $pp \to ZZj$





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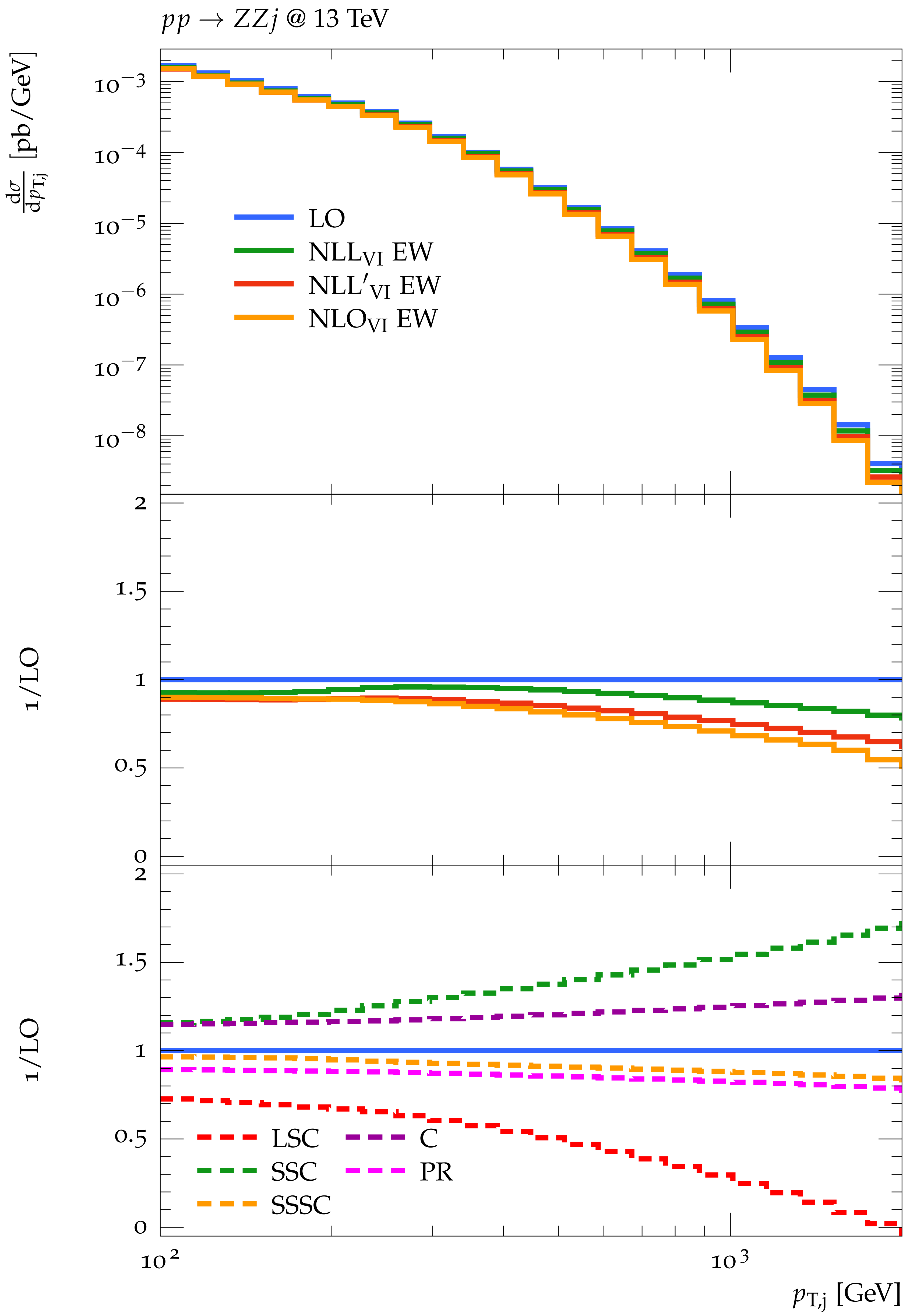
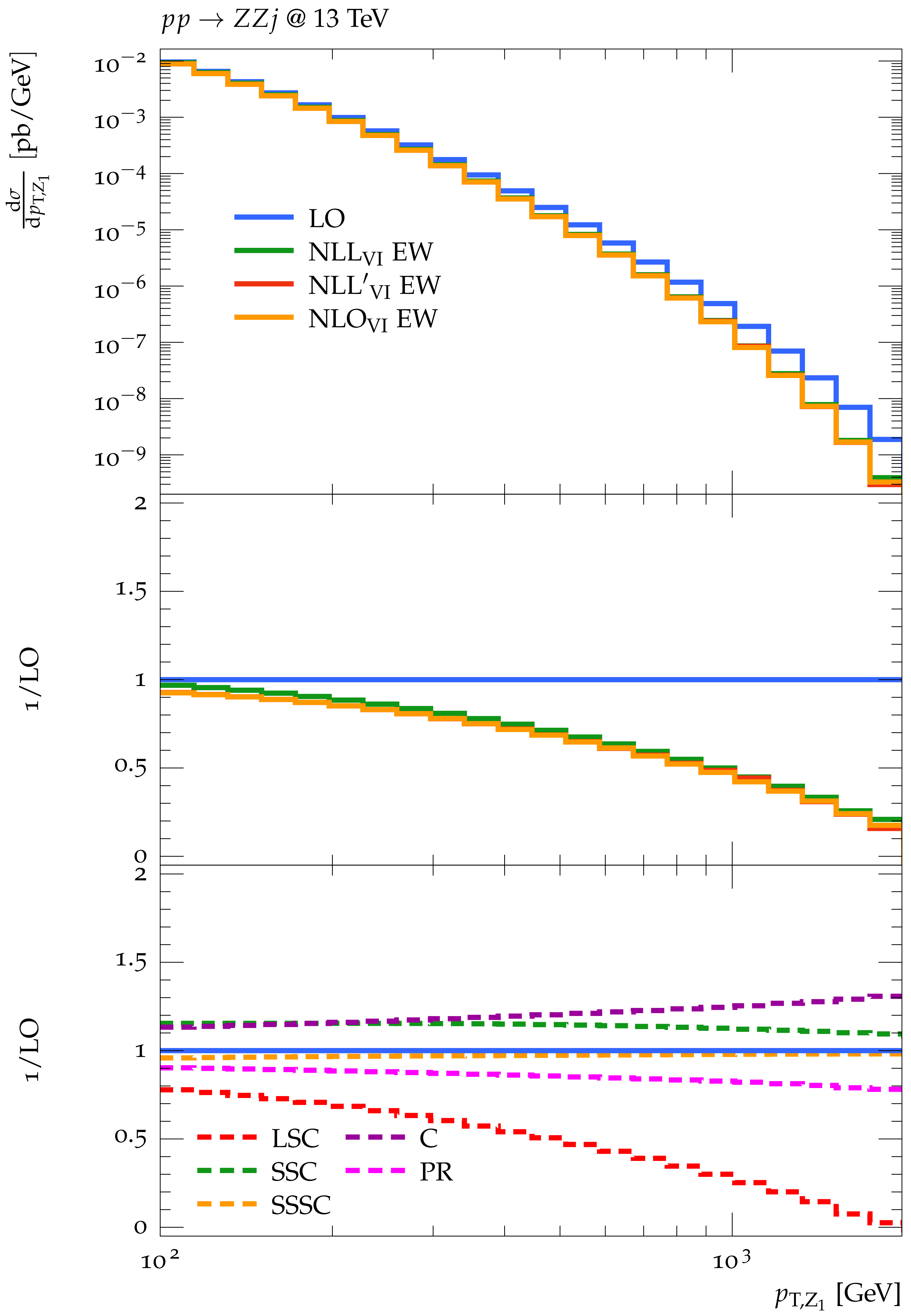
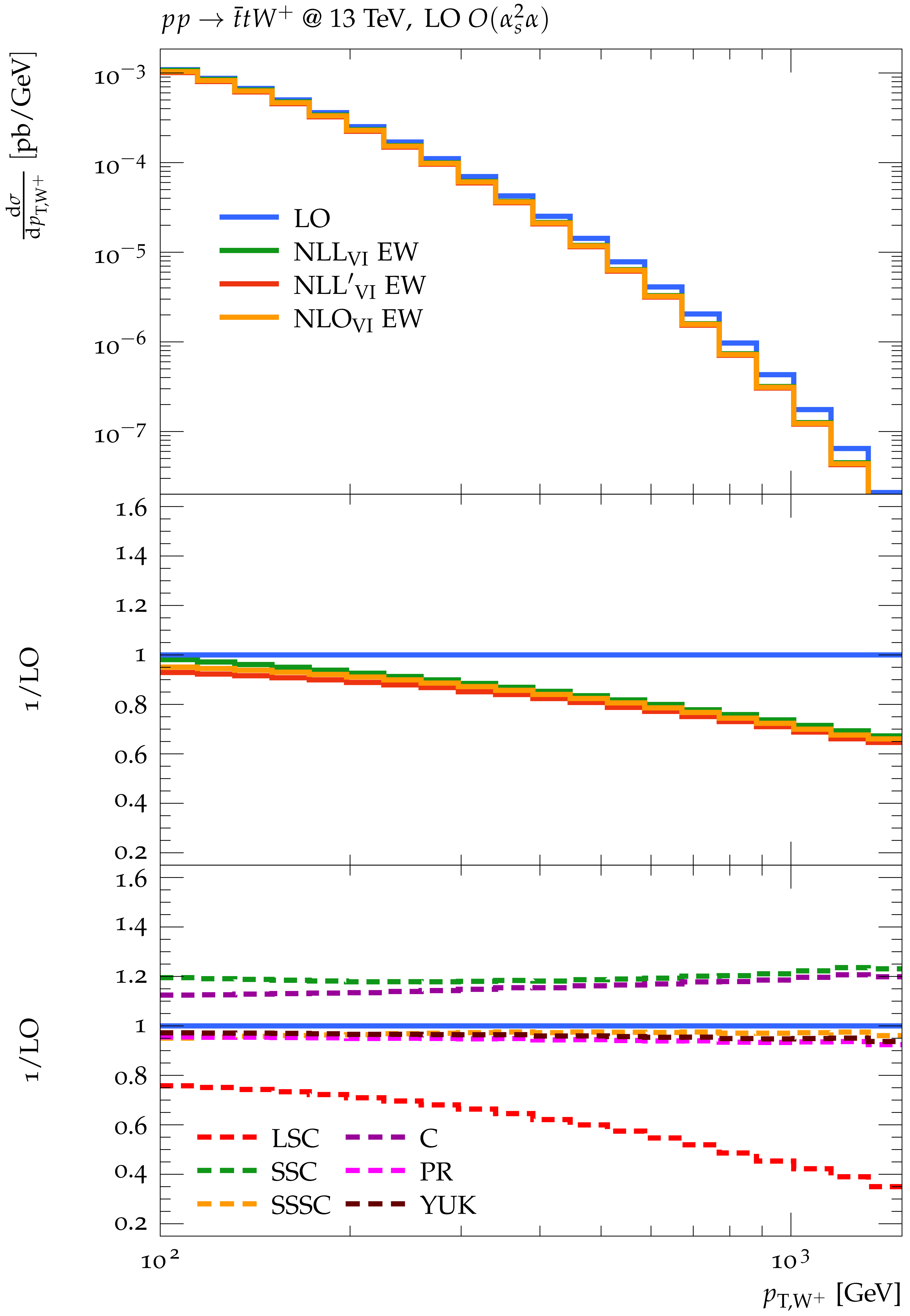
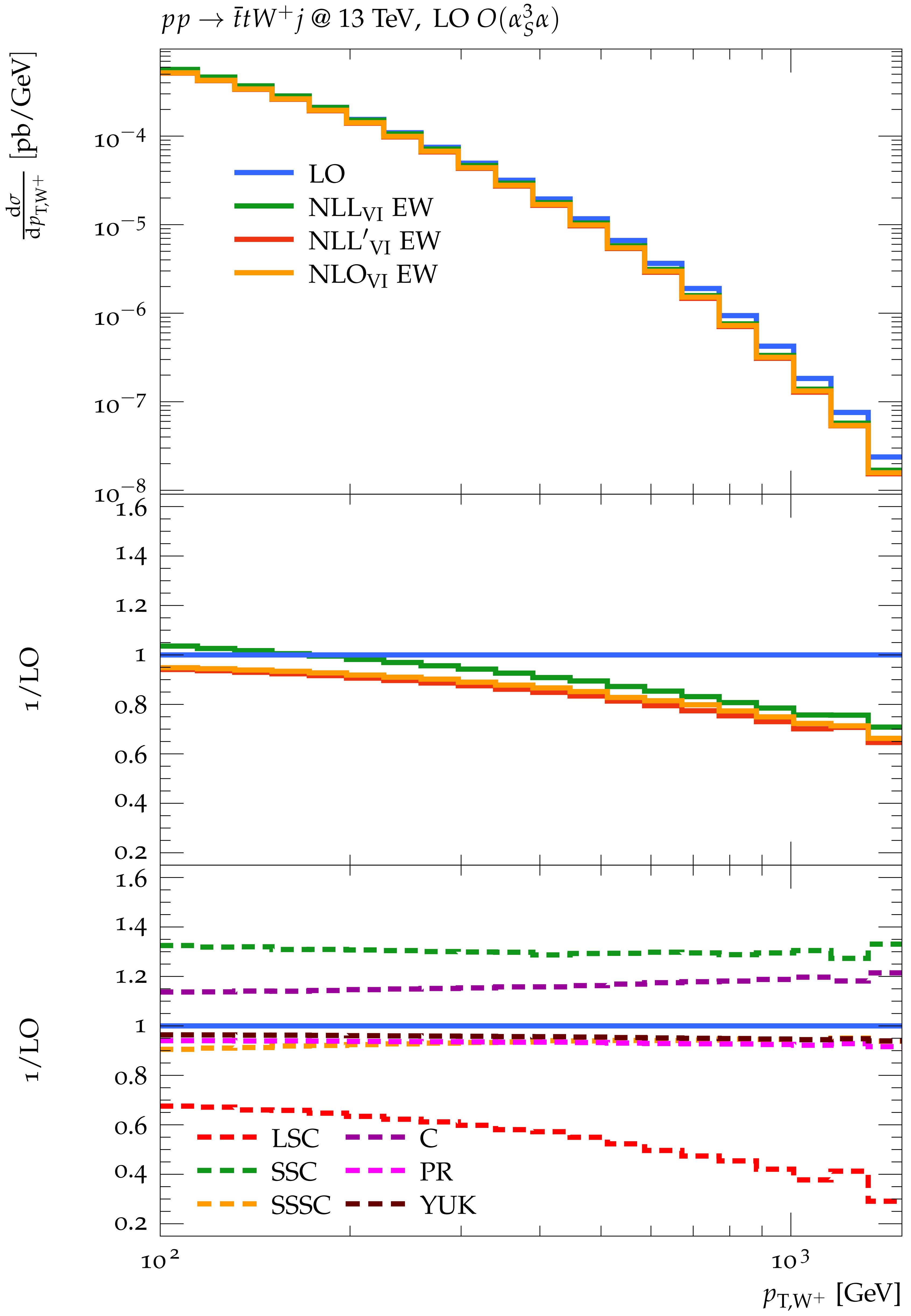


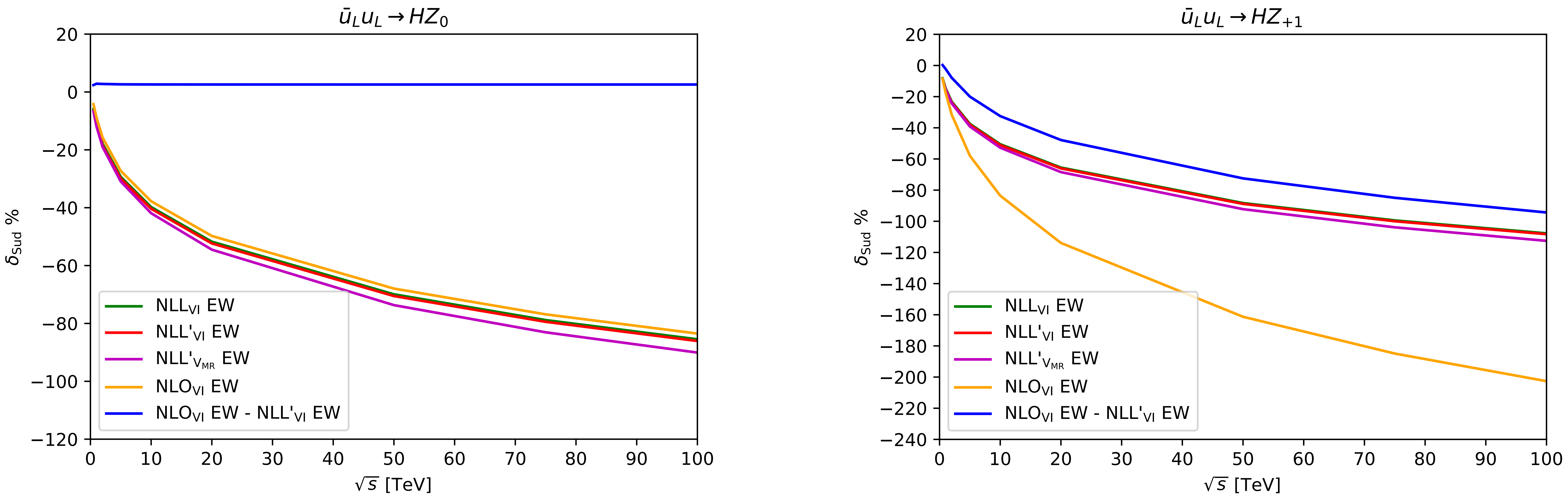


Results: $pp \to ttW^+$ & $pp \to ttW^+j$





Amplitude-level validation: $\sqrt{s}$ scan



- In *Sudakov* approximation: keep only double and singular logarithmic corrections

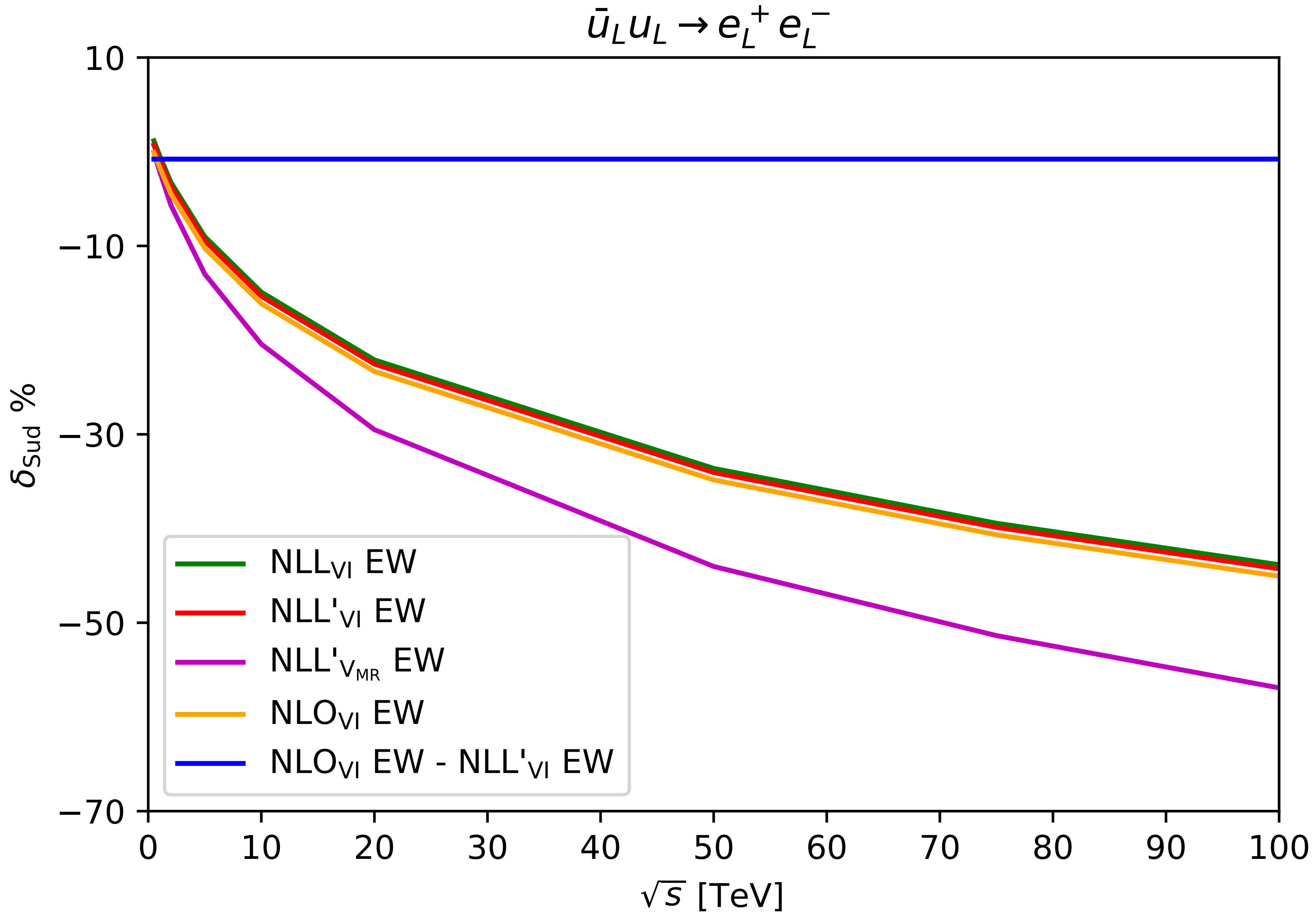
$$\delta^{\text{DL}}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} \sim E^d L \qquad \delta^{\text{SL}}\mathcal{M}^{\varphi_{i_1}\dots\varphi_{i_n}} \sim E^d l$$

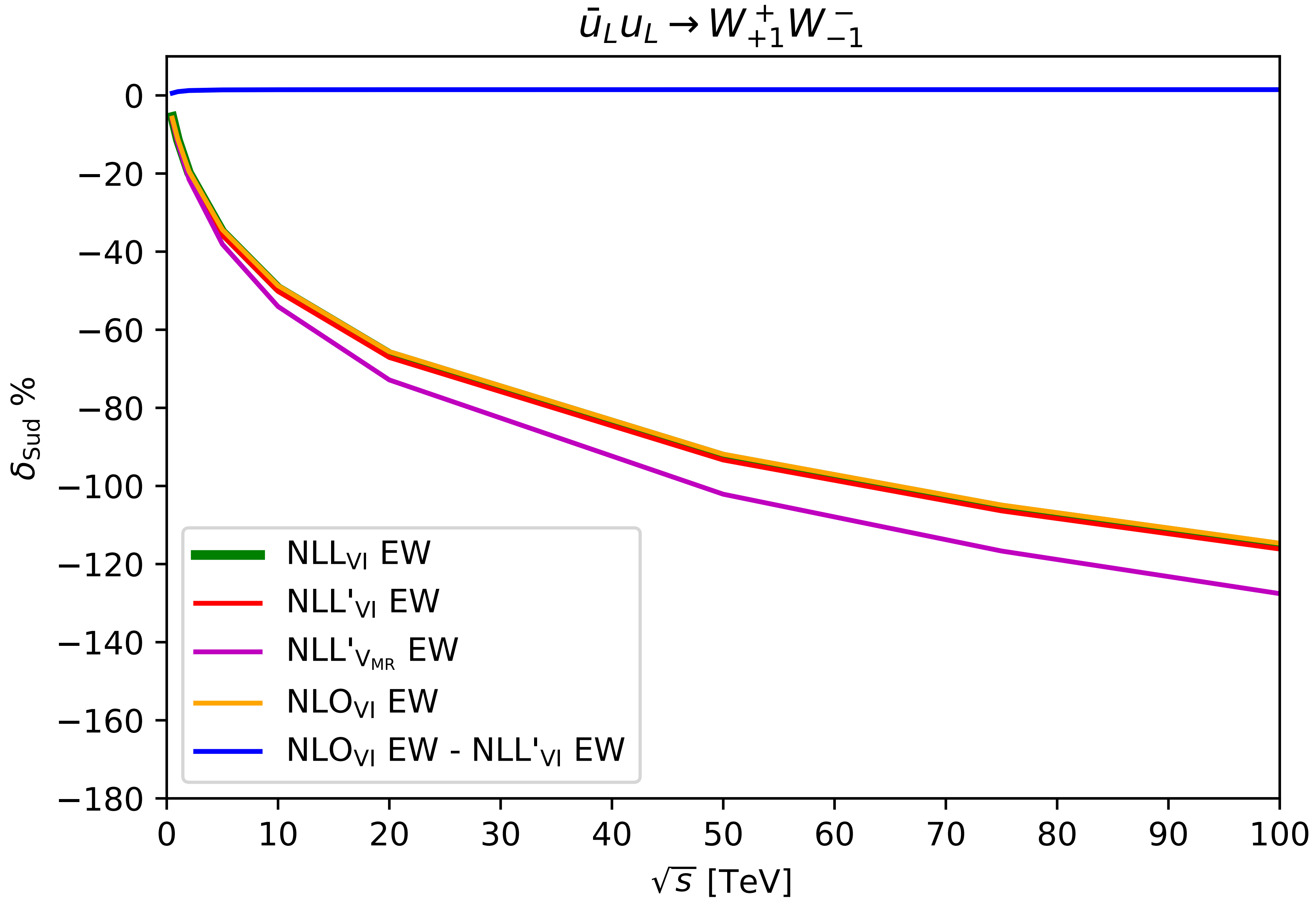
neglecting constant $(\sim \alpha E^d)$ and mass suppressed $(\sim M^n E^{d-n} L)$ terms

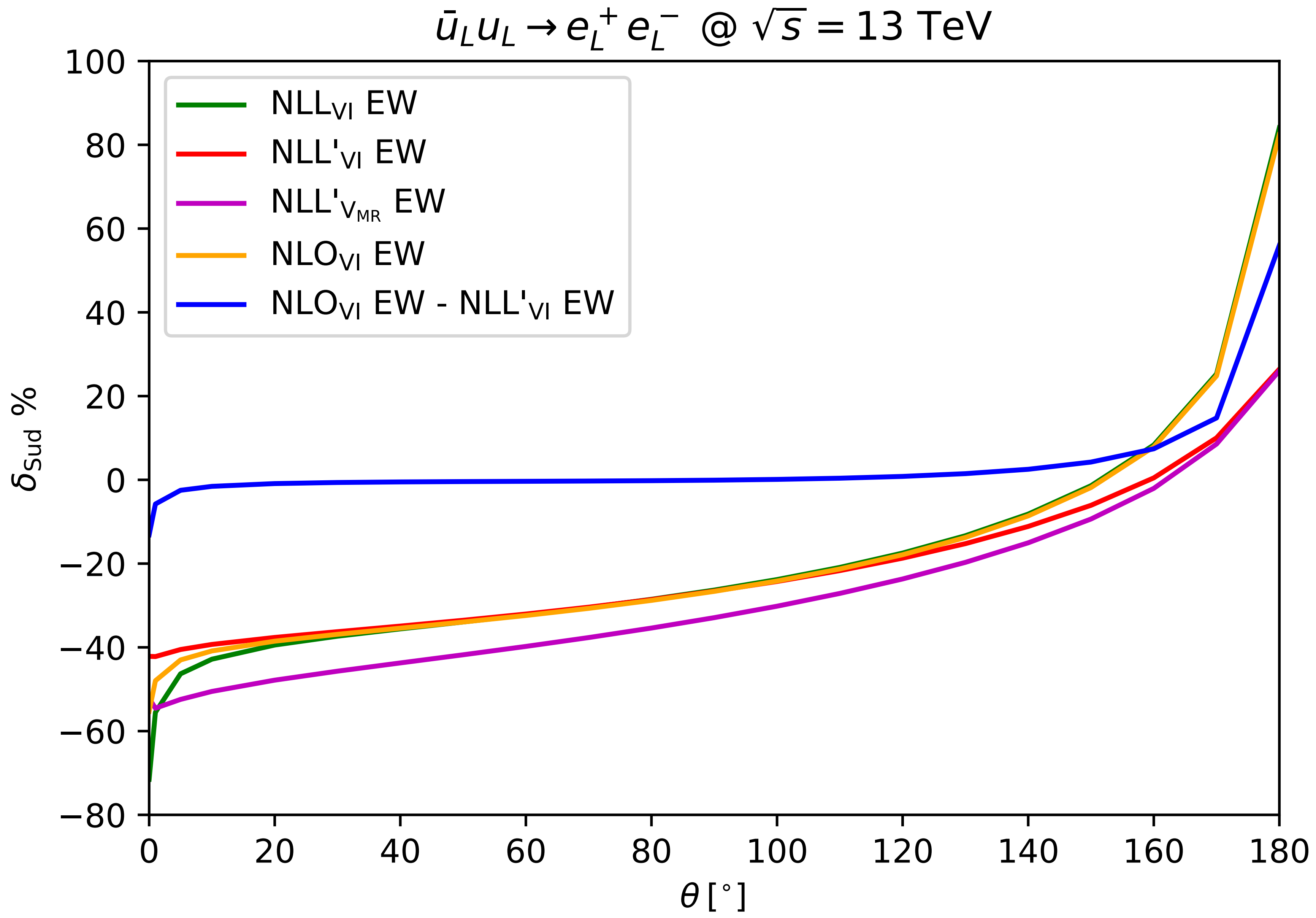
- In the high energy limit and for non mass-suppressed [2] matrix elements we expect $\text{NLO}_{\text{VI}}\ \text{EW} - \text{NLL'}_{\text{VI}}\ \text{EW} \propto\ const$

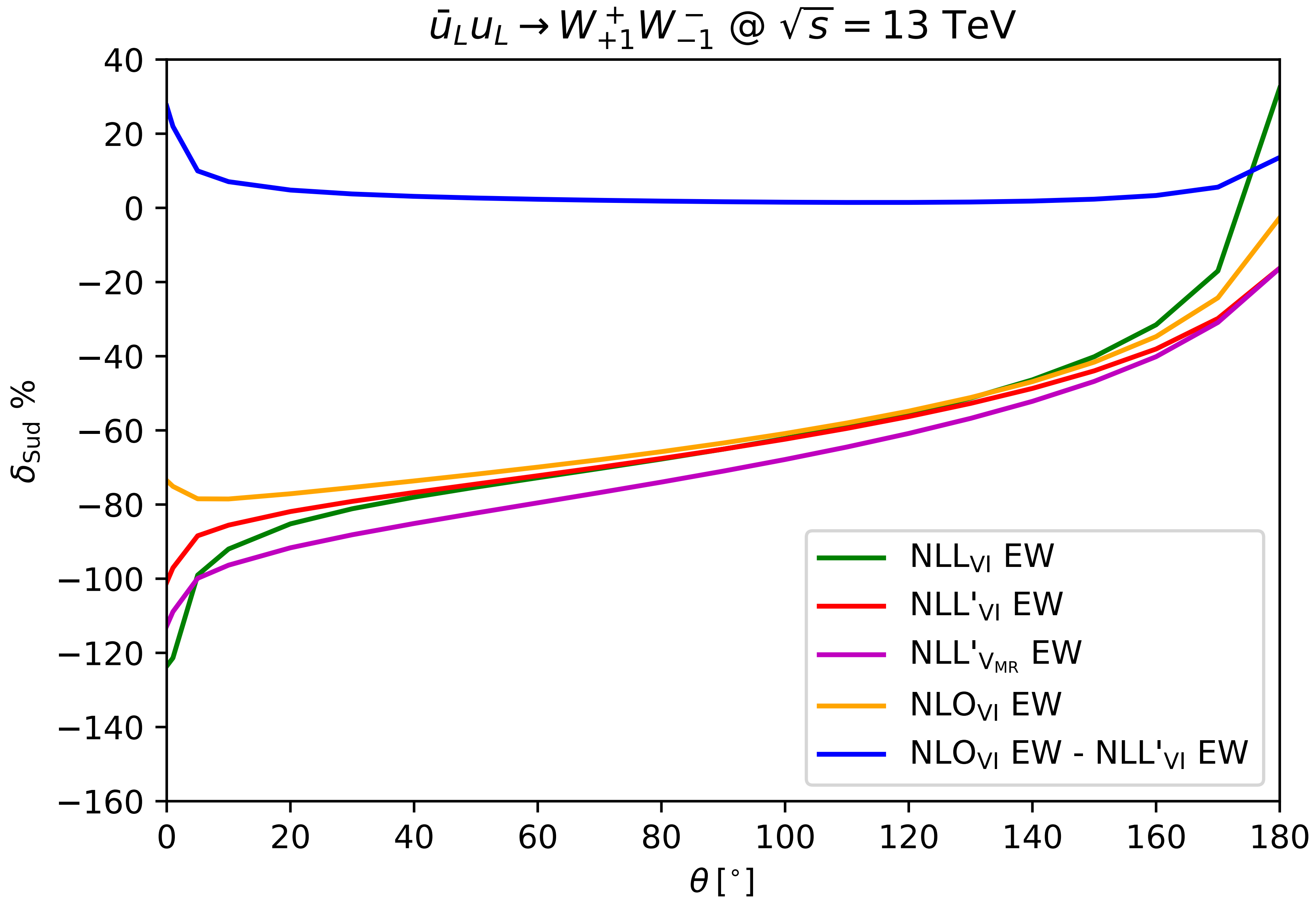
[2]NB: non mass-suppressed configurations scale like $\sim \sqrt{s}^{4-n}$

Amplitude-level validation: $\sqrt{s}$ and θ scans









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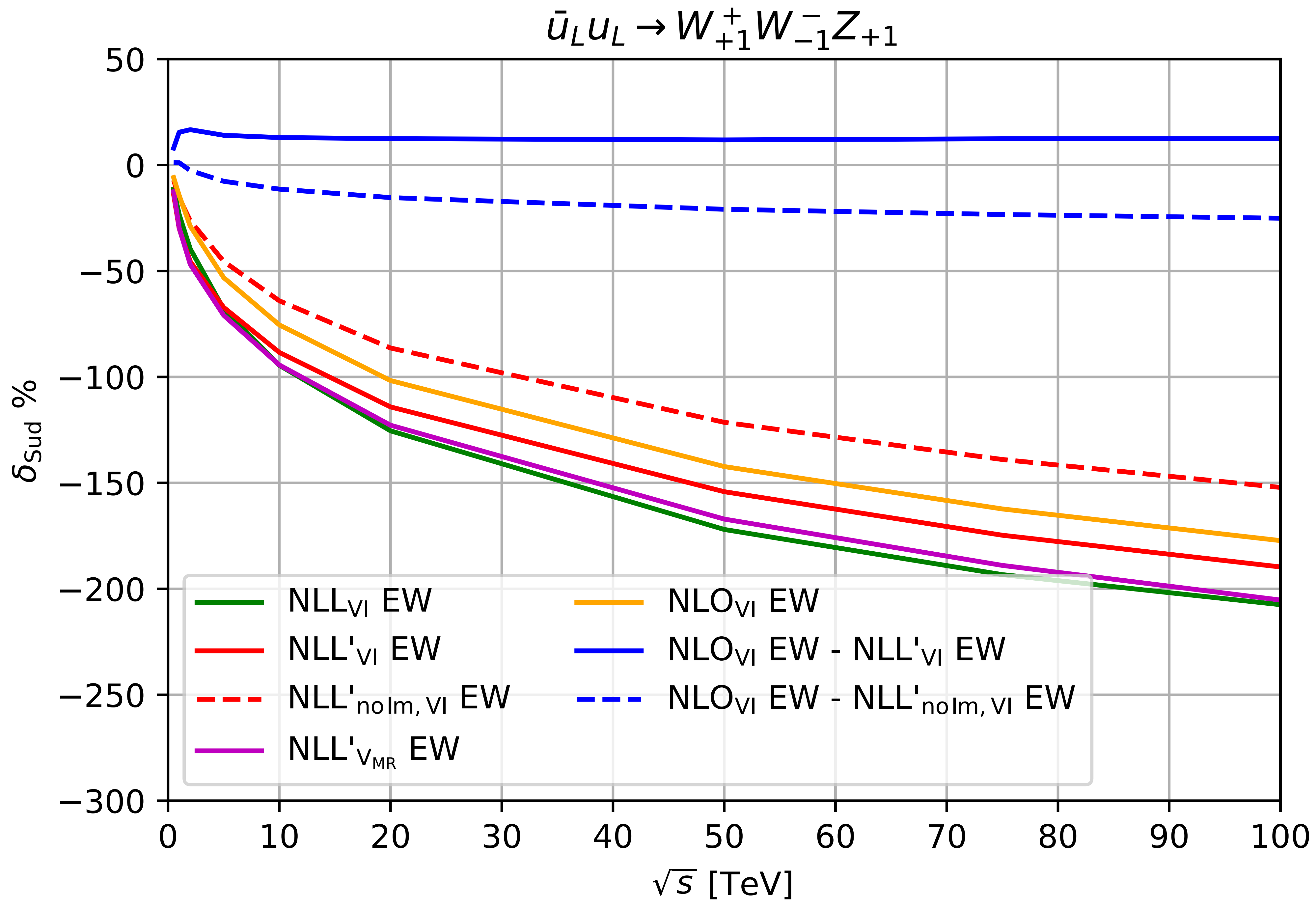

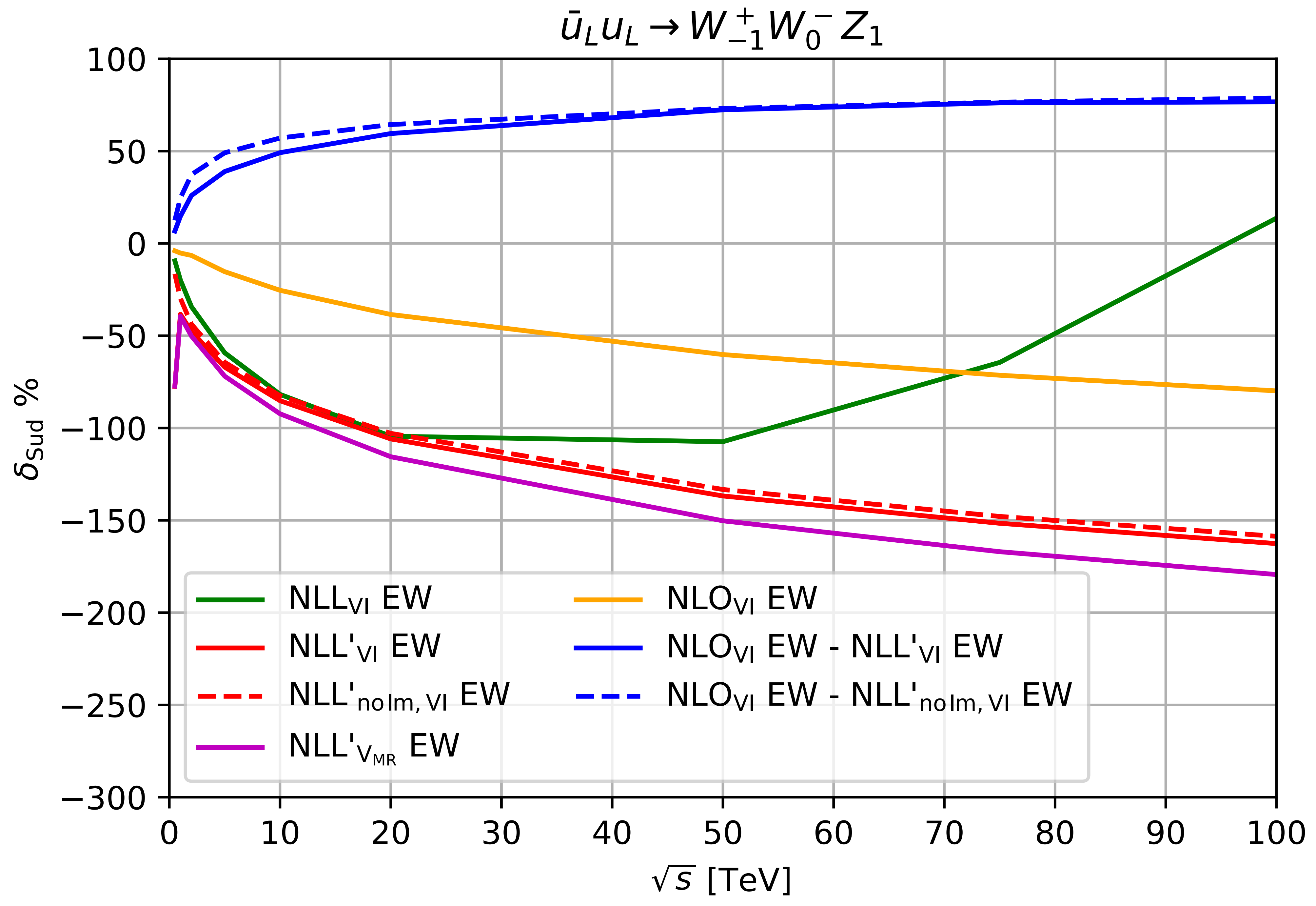

- In the high energy limit and for non mass-suppressed matrix elements we expect $\mathrm{NLO_{VI}\ \ EW - NLL'_{VI}\ \ EW} \propto \ const$

- Inclusion of the phase in DL from the LA of C_0, i.e.

$$C_0|_{\rm LA} \propto \left[\log^2 \frac{|r_{kl}|}{M_V^2} - 2i\pi\Theta(r_{kl}) \log \frac{|r_{kl}|}{M_V^2}\right]$$

is crucial in $2 \to n$ processes with $n \geq 3$: without phase $\mathrm{NLO_{VI}\ \ EW - NLL'_{VI}\ \ EW}$ shows a logarithmic dependence. This has been firstly noticed in [Pagani, Zaro 2110.03714; 2021]