The Colourful Antenna Subtraction Method (maybe)

Universität Zürich[UZH]

Swiss National Science Foundation

Matteo Marcoli

Milan Christmas Meeting

21/12/2023

Durham University

IP3

THE ROYAL SOCIETY

Newton International Fellowship

All-gluons case: 10.1007/JHEP10(2022)099 with X. Chen, T. Gehrmann, N. Glover and A. Huss

General case: hep-ph/2310.19757 with T. Gehrmann and N. Glover

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INTRODUCTION

Fixed-order calculations in QCD

$$\sigma = \sigma_0 + \left(\frac{\alpha_s}{2\pi}\right)\sigma_1 + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma_2 + \left(\frac{\alpha_s}{2\pi}\right)^3 \sigma_3 + \ldots$$

Leading Order (LO)

**Next-to-
Leading Order (NLO)**

**Next-to-Next-to-
Leading Order (NNLO)**

**Next-to- ... -to-
Leading Order (N3LO)**

Fixed-order calculations in QCD

$$\sigma = \sigma_0 + \left(\frac{\alpha_s}{2\pi}\right)\sigma_1 + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma_2 + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma_3 + \ldots$$

Leading Order (LO)

Next-to-Leading Order (NLO)

Next-to-Next-to-Leading Order (NNLO)

Next-to- … -to-Leading Order (N3LO)

General techniques:

- Dipole subtraction; [Catani,Seymour '96]
- FKS subtraction; [Frixione,Kunszt,Signer '96]

Matching to parton showers:

- MC@NLO; [Frixione,Webber '02]
- POWHEG; [Nason '04] [Frixione,Nason,Oleari,Re '07]

Several multi-purpose frameworks implementing NLO+PS calculations;

Fixed-order calculations in QCD

$$\sigma = \sigma_0 + \left(\frac{\alpha_s}{2\pi}\right)\sigma_1 + \left(\frac{\alpha_s}{2\pi}\right)^2\sigma_2 + \left(\frac{\alpha_s}{2\pi}\right)^3\sigma_3 + \ldots$$

Leading Order (LO)

Next-to-Leading Order (NLO)

Next-to-Next-to-Leading Order (NNLO)

Next-to- … -to-Leading Order (N3LO)

Several proposed/implemented approaches:

- Antenna subtraction; [Gehrmann,Gehrmann-De Ridder,Glover '05] [Currie,Glover,Wells '13]
- qT-slicing; [Catani,Grazzini '07]
- Sector-improved residue subtraction; [Czakon '10] [Czakon,Heymes '14]
- N-jettiness slicing; [Gaunt,Stahlhofen,Tackmann,Walsh '14]
- CoLouRFulNNLO; [Del Duca,Duhr,Kardos,Somogyi,Szor,Trocsanyi,Tulipant '16]
- Local analytic sector subtraction; [Magnea,Maina,Pelliccioli,Signorile-Signorile,Torrielli,Uccirati '17]
- Nested soft-collinear subtraction; [Caola,Melnikov,Rontsch '17]
- Projection-to-Born; [Cacciari,Dreyer,Karlberg,Salam,Zanderighi '18]

Still not the same level of generality and automation as at NLO

Towards fully general NNLO subtraction methods

- Antenna subtraction; **this talk**

[Chen,Gehrmann,Glover,Huss,MM `22] [Gehrmann,Glover,MM `23]

- Local analytic sector subtraction; **Chiara's talk**

[Bertolotti,Magnea,Pelliccioli,Ratti,Signorile-Signorile,Torrielli,Uccirati `22]

- Nested soft-collinear subtraction; **Davide's talk**

[Devoto,Melnikov,Röntsch,Signorile-Signorile,Tagliabue `23]

- process-independent;
- arbitrary multiplicity;
- improved understanding of IR cancellation (analytically);
- automation;

$2 \to 3$ NNLO phenomenology with **STRIPPER** (sector-improved residue subtraction) and **MATRIX** (qT-slicing)

COLOURFUL ANTENNA SUBTRACTION

Local subtraction at NLO

Partonic cross section at NLO:

mass factorization

$$\int_n \equiv$$ Integration over an n-particle phase space

$$d\sigma_{NLO} = \int_n (d\sigma^V + d\sigma^{MF}) + \int_{n+1} d\sigma^R$$

infrared divergent   infrared divergent

The sum is finite, but V and R need to be computed **separately** (numerical phase space integration).

Subtraction at NLO:

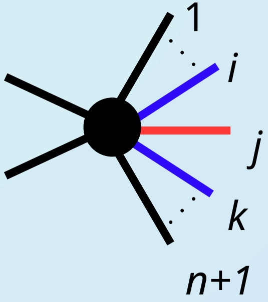
virtual subtraction term   real subtraction term

$$d\sigma_{NLO} = \int_n [d\sigma^V - d\sigma^T] + \int_{n+1} [d\sigma^R - d\sigma^S]$$

with $d\sigma^T = -\int_1 d\sigma^S - d\sigma^{MF}$ to recover the original result.

Antenna subtraction at NLO

Real

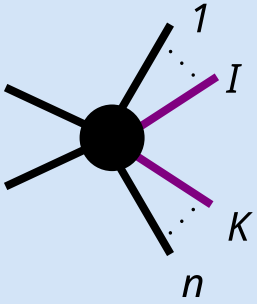


parton j soft or collinear to i or k

factorization properties of QCD

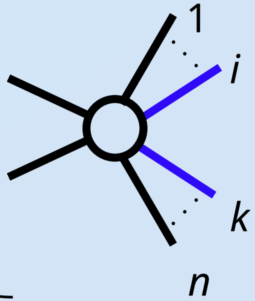
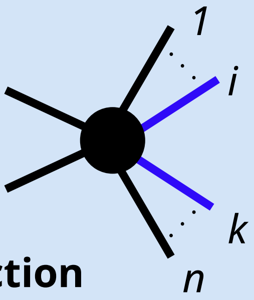
3-parton tree antenna

$$\underbrace{X_3^0(i,j,k)}_{\text{Antenna function}} \cdot$$



Analytical integration over PS of the unresolved radiation

Virtual

Poles $\left[\right.$  $\left.\right]$ = Poles $\left[\; \underbrace{\mathcal{X}_3^0(s_{ik})}_{\text{Integrated antenna function}} \cdot \right.$  $\left.\right]$

[Gehrmann-De Ridder,Gehrmann,Glover '05] [Currie,Glover,Wells '13]

Antenna functions at NLO

Different antenna functions for different partonic configurations:

i	j	k	$X_3^0(i,j,k)$
q	g	$\bar{q}$	A_3^0
q	g	g	D_3^0
q	q'	$\bar{q}'$	E_3^0
g	g	g	F_3^0
g	q	$\bar{q}$	G_3^0

$$X_3^0(i,j,k) \xrightarrow{j\to 0} S_{ijk}^0 = \frac{2s_{ik}}{s_{ij}s_{jk}}$$

$$X_3^0(i,j,k) \xrightarrow{i\|j} \frac{1}{s_{ij}} P_{ij\to I}^0(z)$$

[Gehrmann,Gehrmann-De Ridder,Glover '05]

Local subtraction at NNLO

Partonic cross section at NNLO:

$$d\sigma_{NNLO} = \int_n (d\sigma^{VV} + d\sigma^{MF,2}) + \int_{n+1} (d\sigma^{RV} + d\sigma^{MF,1}) + \int_{n+2} d\sigma^{RR}$$

infrared divergent — infrared divergent — infrared divergent

Subtraction at NNLO:

$$d\sigma_{NNLO} = \int_n [d\sigma^{VV} - d\sigma^{U}] + \int_{n+1} [d\sigma^{RV} - d\sigma^{T}] + \int_{n+2} [d\sigma^{RR} - d\sigma^{S}]$$

double-virtual subtraction term

real-virtual subtraction term

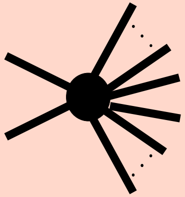
double-real subtraction term

$$d\sigma^{S} = d\sigma^{S,1} + d\sigma^{S,2} \qquad d\sigma^{T} = d\sigma^{VS} - \int_1 d\sigma^{S,1} - d\sigma^{MF,1} \qquad d\sigma^{U} = -\int_1 d\sigma^{VS} - \int_2 d\sigma^{S,2} - d\sigma^{MF,2}$$

Antenna subtraction at NNLO

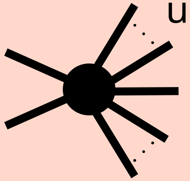
[Gehrmann-De Ridder,Gehrmann,Glover '05] [Currie,Glover,Wells '13]

RR:



n+2

$\longrightarrow$ $X_3^0 \cdot$



n+1

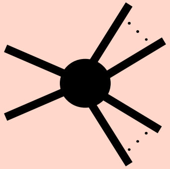
single unresolved

$+$

4-parton tree antenna

$X_4^0 \cdot$

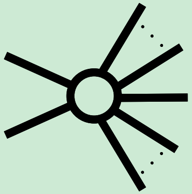
$X_3^0 X_3^0 \cdot$



n

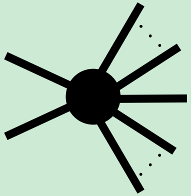
double unresolved

RV:



n+1

$\longrightarrow$ $\mathcal{X}_3^0 \cdot$

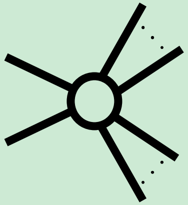


n+1

removes ε-poles

$+$ $X_3^0 \cdot$

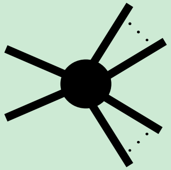
tree x loop



n

$+$ $X_3^1 \cdot$

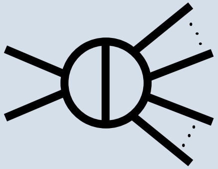
loop x tree



n

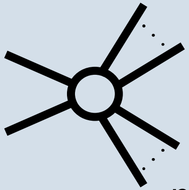
3-parton 1-loop antenna

VV:



n

$\longrightarrow$ $\mathcal{X}_3^0 \cdot$



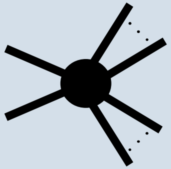
n

$+$

$\mathcal{X}_4^0 \cdot$

$\mathcal{X}_3^1 \cdot$

$\mathcal{X}_3^0 \otimes \mathcal{X}_3^0 \cdot$



n

Antenna functions at NNLO

Different antenna functions for different partonic configurations:

i	j₁	j₂	k	$X_4^0(i,j_1,j_2,k)$
q	g	$\bar{q}$	$\bar{q}$	$A_4^0,\ \tilde{A}_4^0$
q	q'	$\bar{q}'$	$\bar{q}$	B_4^0
q	q	$\bar{q}$	$\bar{q}$	C_4^0
q	g	g	g	D_4^0
q	q'	$\bar{q}'$	g	$E_4^0,\ \tilde{E}_4^0$
g	g	g	g	F_4^0
g	q	$\bar{q}$	g	$G_4^0,\ \tilde{G}_4^0$
q	$\bar{q}$	q'	$\bar{q}'$	H_4^0

i	j	k	$X_3^1(i,j,k)$
q	g	$\bar{q}$	$A_3^1,\ \hat{A}_3^1,\ A_3^1$
q	g	g	$D_3^1,\ \hat{D}_3^1,\ D_3^1$
q	q'	$\bar{q}'$	$E_3^1,\ \hat{E}_3^1,\ E_3^1$
g	g	g	$F_3^1,\ \hat{F}_3^1$
g	q	$\bar{q}$	$G_3^1,\ \hat{G}_3^1,\ G_3^1$

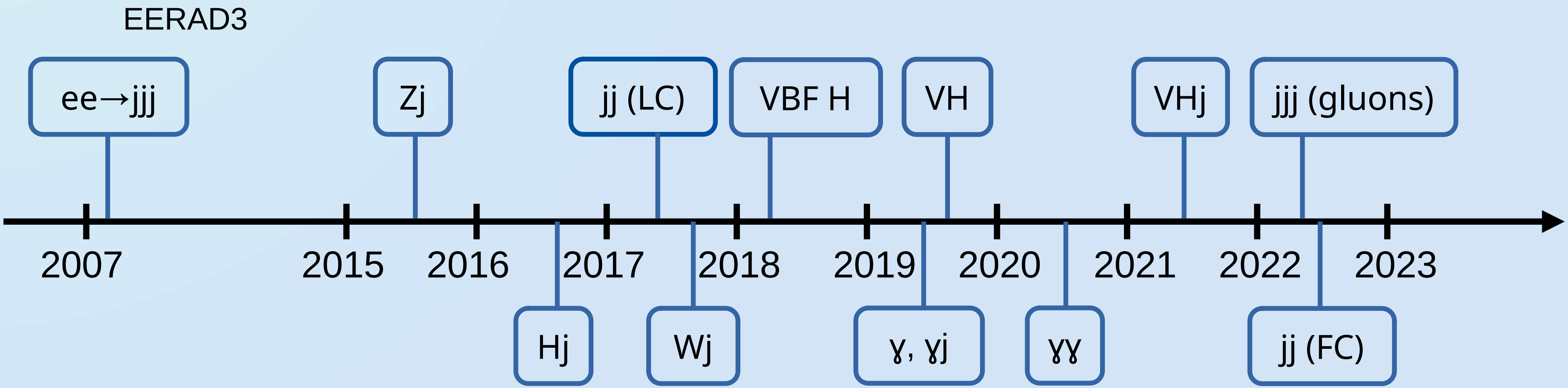
$\widetilde{X}_n^\ell$: subleading-colour

$\widehat{X}_n^\ell$: fermionic loop

[Gehrmann,Gehrmann-De Ridder,Glover '05]

History of antenna subtraction

Successfully applied to a variety of processes in the past decade within ***NNLOJET***:



[Braun-White, Chen, Cruz-Martinez, Fox, Garcia-Rodriguez, Gauld, Gehrmann, Gehrmann-De Ridder, Glover, Hoefer, Huss, Jaquier, Majer, MM, Mo, Morgan, Schuermann, Stagnitto, Pires, Walker, Withehead]

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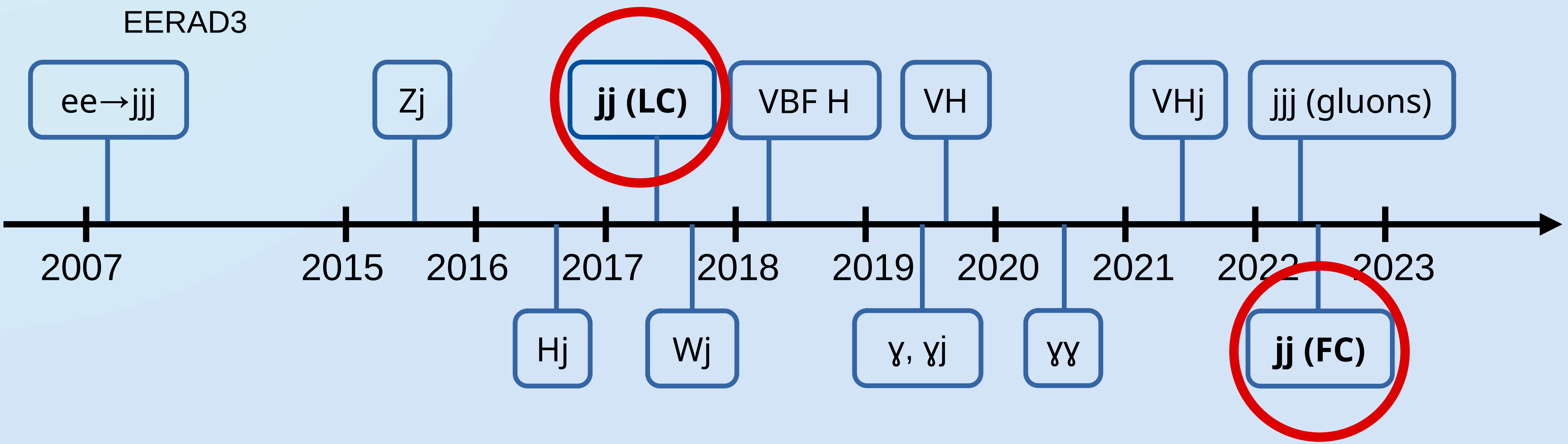

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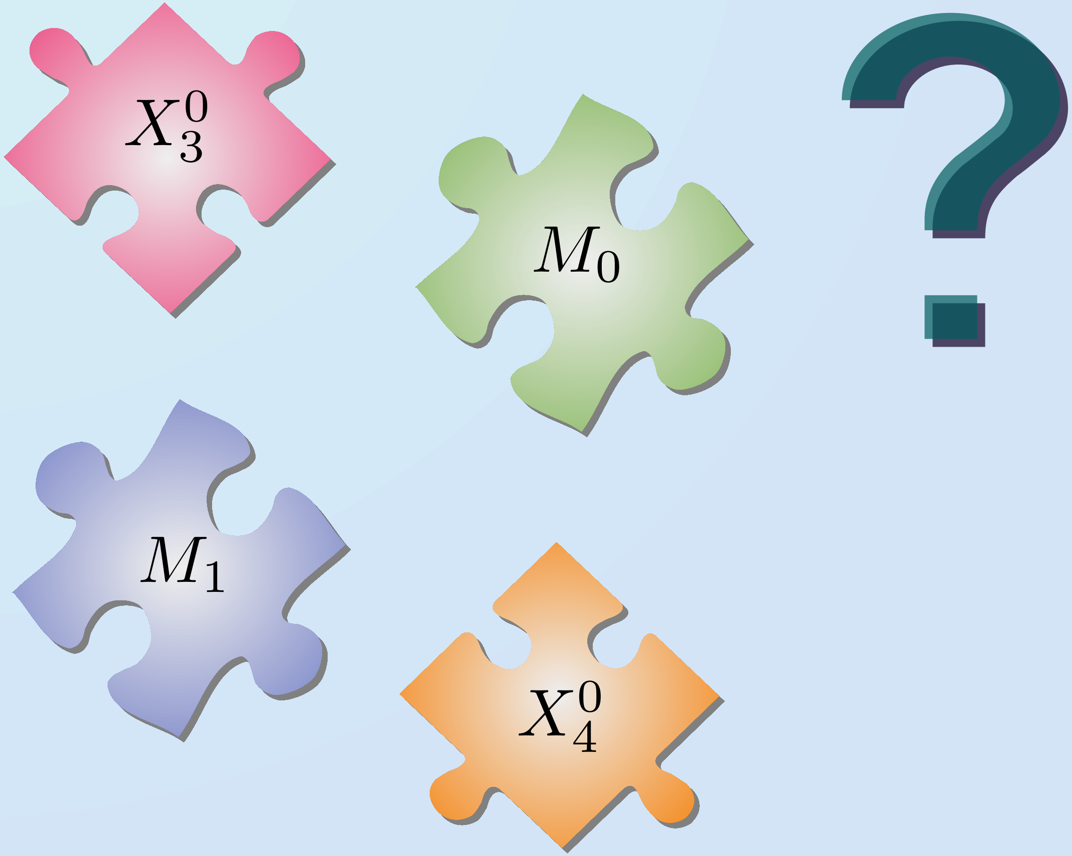
How to assemble the subtraction terms?

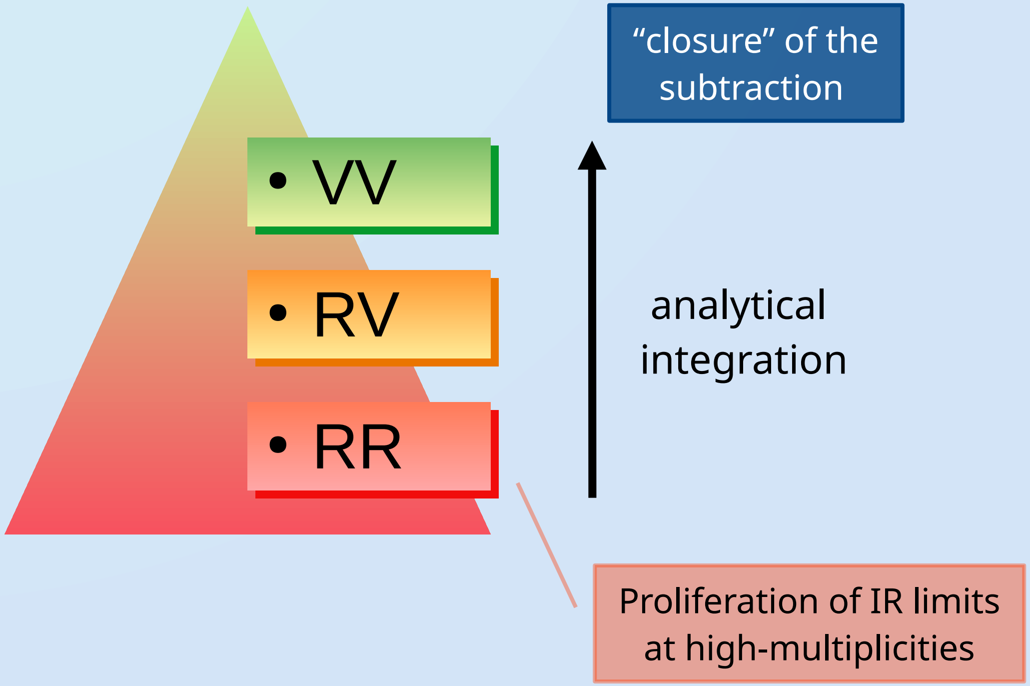


- No **systematic** approach: a lot of work/time for each process;
- **Poor scaling** with the number of external partons n_p ;
- Highly non-trivial treatment beyond **leading colour** for $n_p \geq 4$;

How to assemble the subtraction terms?

Complexity

Traditional approach



How to assemble the subtraction terms?

Complexity

Traditional approach

- VV
- RV
- RR

analytical integration

"closure" of the subtraction

Proliferation of IR limits at high-multiplicities

New approach

- VV
- RV
- RR

Predictable in full generality

"unintegration" or insertion of unresolved partons

"closure" of the subtraction

IR singularities of virtual amplitudes in colour space

[Catani '98] [Bern, De Freitas, Dixon '03] [Becher, Neubert '09]

IR singularity structure at **one-loop**:

$$|A^1_{n+2}\rangle = \boldsymbol{I}^{(1)}|A^0_{n+2}\rangle + \text{finite terms}$$

one-loop tree-level

$$\boldsymbol{I}^{(1)} = \sum_{(i,j)} (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)\, \mathcal{I}^{(1)}_{ij}(\epsilon)$$

Colour-charge dipoles

Colour-stripped singular functions

IR singularity structure at **two-loop**:

one-loop tree-level

$$|A^2_{n+2}\rangle = \boldsymbol{I}^{(1)}|A^1_{n+2}\rangle + \boldsymbol{I}^{(2)}|A^0_{n+2}\rangle$$

two-loop

$$+\,\text{finite terms}$$

$$\boldsymbol{I}^{(2)}(\epsilon,\mu_r^2) = -\frac{1}{2}\sum_{(i,j)}\sum_{(k,l)} (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)(\boldsymbol{T}_k \cdot \boldsymbol{T}_l)\, \mathcal{I}^{(1)}_{ij}(\epsilon)\mathcal{I}^{(1)}_{kl}(\epsilon) - \frac{\beta_0}{\epsilon}\sum_{(i,j)} (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)\, \mathcal{I}^{(1)}_{ij}(\epsilon) + \sum_{(i,j)} (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)\, \mathcal{I}^{(2)}_{ij}(\epsilon)$$

$\boldsymbol{T}_i$ SU(3) generator in the representation of parton *i*

$$\mathcal{I}^{(2)}_{ij}(\epsilon,\mu_r^2) = e^{-\epsilon\gamma_E}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon} + K\right)\mathcal{I}^{(1)}_{ij}(2\epsilon) - \mathcal{H}^{(2)}_{ij}(\epsilon)$$

Integrated dipoles

$$\mathcal{J}^{(1)} = \sum_{(i,j)\geq 3} (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)\, \mathcal{J}_2^{(1)}(i,j) + \sum_{i\neq 1,2} (\boldsymbol{T}_1 \cdot \boldsymbol{T}_i)\, \mathcal{J}_2^{(1)}(1,i) + \sum_{i\neq 1,2} (\boldsymbol{T}_2 \cdot \boldsymbol{T}_i)\, \mathcal{J}_2^{(1)}(2,i) + (\boldsymbol{T}_1 \cdot \boldsymbol{T}_2)\, \mathcal{J}_2^{(1)}(1,2)$$

$$\mathcal{J}^{(2)} = N_c \sum_{(i,j)\geq 3} (\boldsymbol{T}_i \cdot \boldsymbol{T}_j)\, \mathcal{J}_2^{(2)}(i,j) + N_c \sum_{i\neq 1,2} (\boldsymbol{T}_1 \cdot \boldsymbol{T}_i)\, \mathcal{J}_2^{(2)}(1,i) + N_c \sum_{i\neq 1,2} (\boldsymbol{T}_2 \cdot \boldsymbol{T}_i)\, \mathcal{J}_2^{(2)}(2,i) + N_c(\boldsymbol{T}_1 \cdot \boldsymbol{T}_2)\, \mathcal{J}_2^{(2)}(1,2)$$

both hard radiators in the final state | one hard radiator in the initial state | both hard radiators in the initial state

Colour decomposition: $$\mathcal{J}_2^{(\ell)} = J_2^{(\ell)} + \frac{1}{N_c^2}\tilde{J}_2^{(\ell)} + \frac{N_f}{N_c}\hat{J}_2^{(\ell)} + \frac{N_f}{N_c^3}\hat{\tilde{J}}_2^{(\ell)} + \frac{N_f^2}{N_c^2}\hat{\hat{J}}_2^{(\ell)}$$

Colour-stripped integrated dipoles

$$J_2^{(1)} = c_{\mathcal{X}_3^0}\mathcal{X}_3^0 + c_{\Gamma^{(1)}}\Gamma^{(1)}$$

$$J_2^{(2)} = c_{\mathcal{X}_4^0}\mathcal{X}_4^0 + c_{\mathcal{X}_3^1}\mathcal{X}_3^1 + c_{\mathcal{X}_3^0\mathcal{X}_3^0}\mathcal{X}_3^0\mathcal{X}_3^0 + c_{\beta_0}\frac{\beta_0}{\epsilon}\mathcal{X}_3^0\left(\frac{|s|}{\mu_r^2}\right)^{-\epsilon} + c_{\Gamma^{(2)}}\overline{\Gamma}^{(2)}$$

Key property: written in terms of **integrated antenna functions** and MF kernels (for IS radiation)

IR singularities of virtual amplitudes

$$Poles\left[\mathcal{J}_2^{(1)}(i,j)\right] = Poles\left[\mathrm{Re}\left(\mathcal{I}_{ij}^{(1)}(\epsilon)\right)\right]$$

$$Poles\left[N_c\mathcal{J}_2^{(2)}(i,j) - \frac{\beta_0}{\epsilon}\mathcal{J}_2^{(1)}(i,j)\right] = Poles\left[\mathrm{Re}\left(\mathcal{I}_{ij}^{(2)}(\epsilon) - \frac{\beta_0}{\epsilon}\mathcal{I}_{ij}^{(1)}(\epsilon)\right)\right]$$

Equivalent to Catani's operators*

* Careful with quark-gluon dipoles

$$Poles\left(\mathrm{d}\hat{\sigma}^V\right) = \mathcal{N}_V \int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\mathrm{d}\Phi_n\, J_n^n(\Phi_n)\, Poles\left[2\,\langle A_{n+2}^0|\boldsymbol{\mathcal{J}}^{(1)}|A_{n+2}^0\rangle\right]$$

$$Poles\left(\mathrm{d}\hat{\sigma}^{VV}\right) = \mathcal{N}_{VV} \int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\mathrm{d}\Phi_n J_n^n(\Phi_n) Poles\Bigg\{ \times 2\Big[\langle A_{n+2}^0|\boldsymbol{\mathcal{J}}^{(1)}|A_{n+2}^1\rangle + \langle A_{n+2}^1|\boldsymbol{\mathcal{J}}^{(1)}|A_{n+2}^0\rangle - \frac{b_0 N_c}{\epsilon}\langle A_{n+2}^0|\boldsymbol{\mathcal{J}}^{(1)}|A_{n+2}^0\rangle - \langle A_{n+2}^0|\boldsymbol{\mathcal{J}}^{(1)}\otimes\boldsymbol{\mathcal{J}}^{(1)}|A_{n+2}^0\rangle + \langle A_{n+2}^0|\boldsymbol{\mathcal{J}}^{(2)}|A_{n+2}^0\rangle\Big]\Bigg\}$$

IR structure of one- and two-loop MEs in terms of integrated antenna functions

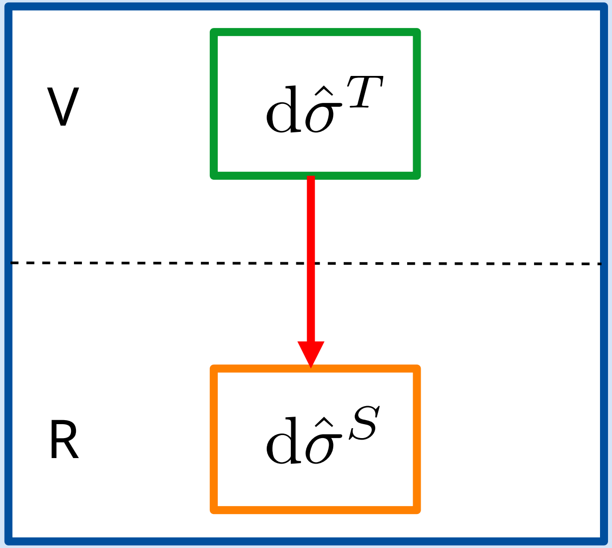
Subtraction at NLO

[Chen, Gehrmann, Glover, Huss, MM '22] [Gehrmann, Glover, MM '23]

- Use integrated dipoles to construct the virtual subtraction term;

$$\mathrm{d}\sigma^{T} = \mathcal{N}_V \int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2} \mathrm{d}\Phi_n \, J_n^n(\Phi_n) \, \left[2 \langle A_{n+2}^0 | \boldsymbol{\mathcal{J}}^{(1)} | A_{n+2}^0 \rangle\right]$$

Guaranteed to remove the virtual poles



Subtraction at NLO

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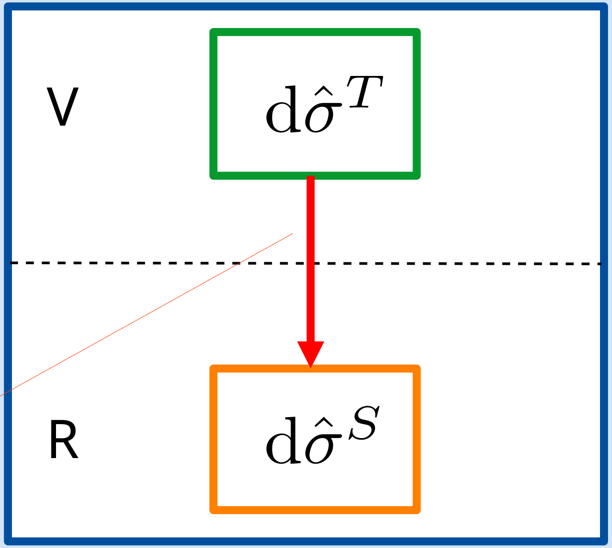
$$\mathrm{d}\sigma^T = \mathcal{N}_V \int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2} \mathrm{d}\Phi_n \, J_n^n(\Phi_n) \, \left[2 \langle A_{n+2}^0 | \boldsymbol{\mathcal{J}}^{(1)} | A_{n+2}^0 \rangle \right]$$

Guaranteed to remove the virtual poles



- Infer the real subtraction term through the **insertion of an unresolved parton**;

$$\mathrm{d}\sigma^S = -\mathcal{I}ns \left[\mathrm{d}\sigma^T \right]$$

Inverse operation with respect to:

$$\mathrm{d}\sigma^T = -\int_1 \mathrm{d}\sigma^S - \mathrm{d}\sigma^{MF}$$

Unintegration: insertion of an unresolved parton

Algorithmic procedure:

Remove the splitting kernels from integrated dipole

↓

Convert integrated antenna functions to unintegrated ones

↓

Apply momentum mapping

↓

Overall adjustments

One-to-one correspondence between integrated and unintegrated antenna functions

virtuals: n-particle PS

reals: (n+1)-particle PS

$$\mathcal{X}_3^0(s_{ij})\, A_{n+2}^0(.,i,.,j,.) \leftrightarrow X_3^0(i,u,j) A_{n+2}^0(.,\widetilde{iu},.,\widetilde{uj},.)$$

unresolved parton

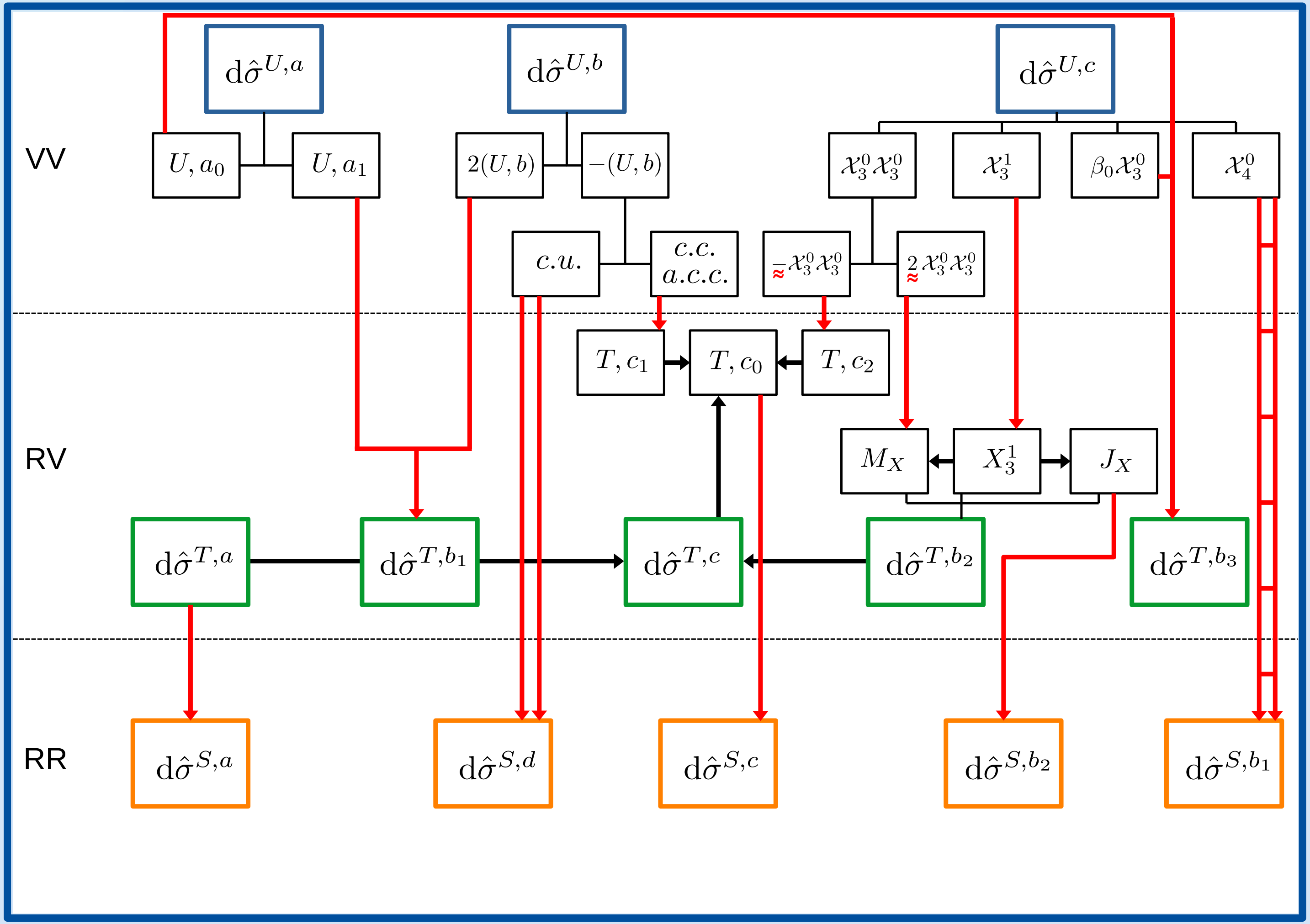
momentum mapping

[Chen, Gehrmann, Glover, Huss, MM '22] [Gehrmann, Glover, MM '23]

Subtraction at NNLO

[Chen, Gehrmann, Glover, Huss, MM ‘22] [Gehrmann, Glover, MM ‘23]

- Double virtual subtraction term from **integrated dipoles**;
- **First insertion** of an unresolved parton
  +
  generation of **new structures** for the real virtual subtraction term;
- **Second insertion** of an unresolved parton for the double-real subtraction term.



Double virtual subtraction term

Straightforward use of integrated dipoles in colour space:

$$\mathrm{d}\sigma^U = \mathcal{N}_{VV} \int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\mathrm{d}\Phi_n J_n^n(\Phi_n)$$

$$\times 2\Big\{\langle A^0_{n+2}|\boldsymbol{\mathcal{J}}^{(1)}|A^1_{n+2}\rangle + \langle A^1_{n+2}|\boldsymbol{\mathcal{J}}^{(1)}|A^0_{n+2}\rangle - \frac{\beta_0}{\epsilon}\langle A^0_{n+2}|\boldsymbol{\mathcal{J}}^{(1)}|A^0_{n+2}\rangle$$

$\mathrm{d}\sigma^{U,a_0}$

single one-loop insertion at one-loop

$\mathrm{d}\sigma^{U,a_1}$

$$-\langle A^0_{n+2}|\boldsymbol{\mathcal{J}}^{(1)} \otimes \boldsymbol{\mathcal{J}}^{(1)}|A^0_{n+2}\rangle + \langle A^0_{n+2}|\boldsymbol{\mathcal{J}}^{(2)}|A^0_{n+2}\rangle\Big\}$$

double one-loop insertion

two-loop insertion

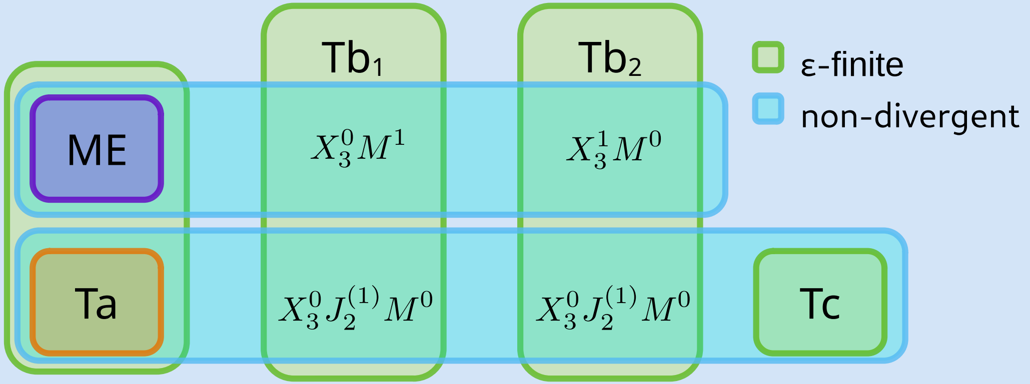
$\mathrm{d}\sigma^{U,b}$

$\mathrm{d}\sigma^{U,c}$

Real virtual subtraction term

The real-virtual subtraction term has to:

- remove explicit IR poles;
- Subtract soft and collinear behaviour;



removal of explicit poles
$$\mathrm{d}\sigma^{T,a} = \mathcal{N}_V \int \frac{\mathrm{d}x_1}{x_1}\frac{\mathrm{d}x_2}{x_2}\mathrm{d}\Phi_n\, J_n^n(\Phi_n)\, \left[2\left\langle A_{n+3}^0 \middle| \boldsymbol{\mathcal{J}}^{(1)} \middle| A_{n+3}^0 \right\rangle\right]$$

(tree-level divergence) x (one-loop ME)
$$\mathrm{d}\sigma^{T,b_1} = -\mathcal{I}ns\left[\mathrm{d}\sigma^{U,a_1}\right] - 2\mathcal{I}ns\left[\mathrm{d}\sigma^{U,b}\right] - \mathrm{d}\sigma^{MF,1,b}$$

(one-loop divergence) x (tree-level ME)
$$\mathrm{d}\sigma^{T,b_2} = -\mathcal{I}ns\left[\mathrm{d}\sigma^{U,c,\mathcal{X}_3^1}\right] + \mathrm{d}\sigma^{T,b_2,J_X} + \mathrm{d}\sigma^{T,b_2,M_X}$$

compensates for oversubtraction
$$\mathrm{d}\sigma^{T,c} = \frac{1}{2}\left[\sigma^{T,c,\mathrm{prel.}} + \sigma^{T,c,\mathcal{S}} + \sigma^{T,c,\mathrm{extra}}\right]$$

Double real subtraction term

single-unresolved $$\mathrm{d}\sigma^{S,a} = -\mathcal{I}ns\left[\mathrm{d}\sigma^{T,a}\right]$$

colour-connected double-unresolved $$\mathrm{d}\sigma^{S,b_1} = -\mathcal{I}ns_2\left[\mathrm{d}\sigma^{U,c,\mathcal{X}_4^0}\right]$$

simultaneous double insertion of two colour-connected partons:

$$\mathcal{X}_4^0(s_{ij})\, A_{n+2}^0(.,i,.,j,.)$$
$$\updownarrow$$
$$X_4^0(i,u_1,u_2,j)\, A_{n+2}^0(.,\widetilde{iu_1u_2},.,\widetilde{u_1u_2j},.)$$

removes single-uresolved from Sb1 $$\mathrm{d}\sigma^{S,b_2} = -\mathcal{I}ns\left[\mathrm{d}\sigma^{T,b_2,J_X}\right]$$

almost colour-connected double-unresolved $$\mathrm{d}\sigma^{S,c} = -\mathcal{I}ns\left[\mathrm{d}\sigma^{T,c_0}\right]$$

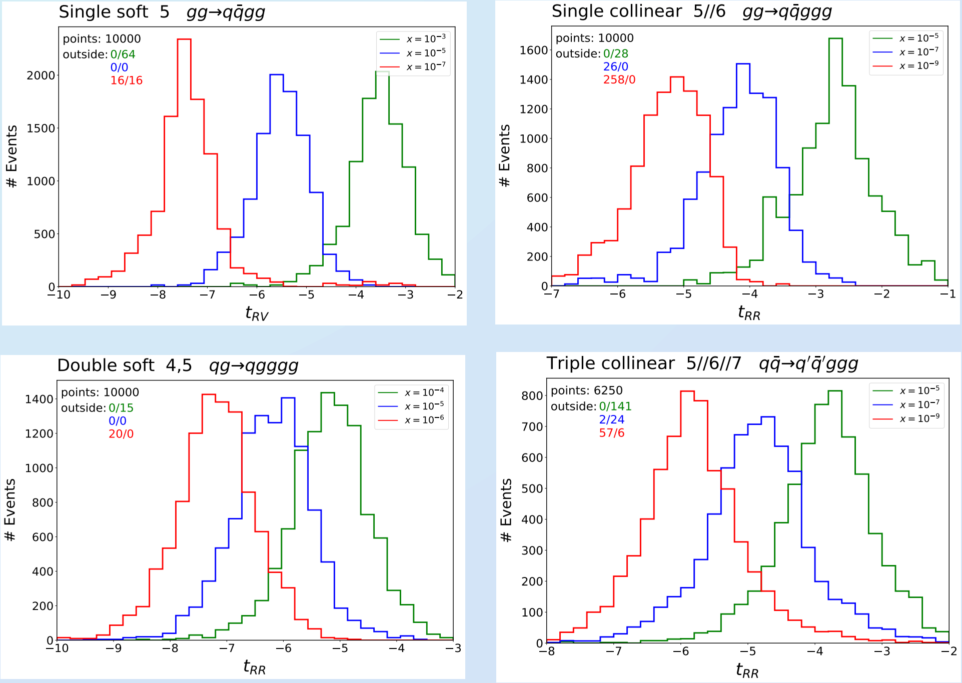
colour-unconnected double-unresolved $$\mathrm{d}\sigma^{S,d} = +\mathcal{I}ns\left[\mathcal{I}ns\left[\mathrm{d}\sigma^{U,b,c.u.}\right]\right]$$

iterated single insertion

Application to hadronic three-jet production

Construction and validation of the subtraction terms:

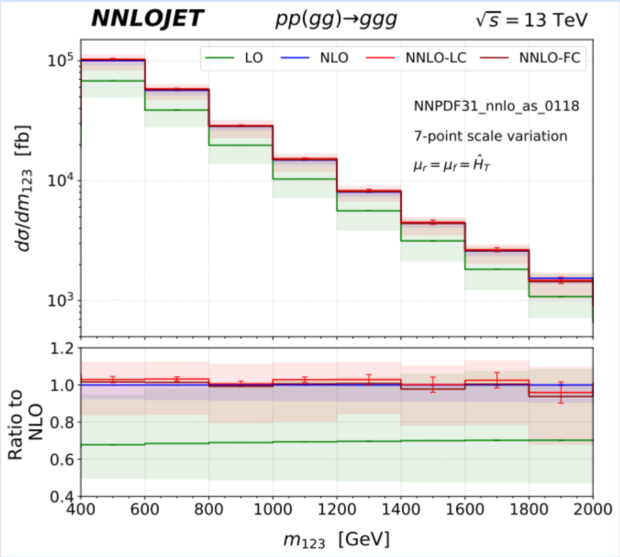
- Cancellation of ε-poles;
- Correct pointwise subtraction;



[Gehrmann, Glover, MM '23]

Gluonic subprocess **gg→ggg**:

- full complexity of 2→3 processes;
- computationally demanding;
- good convergence across all distributions;



[Chen, Gehrmann, Glover, Huss, MM '22]

CONCLUSIONS AND OUTLOOK

I presented the **colouful antenna subtraction method**: a general approach for NNLO calculations in massless QCD.

It is designed to naturally address **high-multiplicity processes** and it is particularly prone to be **fully automated**.

We contructed and validated the NNLO subtraction terms for **hadronic three-jet production: pp→jjj**.

The natural conclusion of the project is the calculation of the full NNLO correction.

Applications to **high-multiplicity phenomenology**:

- pp→γγj
- pp→Vjj
- e^+e^-→jjjj

Future extension to **other classes of antenna functions**:

- idealized antenna functions
- identified final states
- massive fermions
- polarized beams

[Braun-White,Glover,Preuss '23]
[Fox,Glover '23]

[Gehrmann,Schuermann '22]
[Gehrmann,Stagnitto '22]

[Gehrmann-De Ridder,Ritzmann '09]

Thank you for your attention
and Merry Christmas!

BACKUP SLIDES

NNLO corrections for 2→3 processes

Five-point two-loop amplitudes:

- massless legs:
  - pp→ɣɣɣ
  - pp→ɣɣj
  - pp→jjj
  - pp→ɣjj
- one massive leg:
  - pp→Hjj
  - pp→Vjj
  - pp→Wɣj

[Abreu, Agarwal, Badger, Brønnum-Hansen, Buccioni, Chawdry, Chicherin, Cordero, Czakon, De Laurentis, Devoto, Dormans, Gambuti, Gehrmann, Hartanto, Henn, Ita, Klinkert, Kris, Lo Presti, Mitev, Mitov, Moodie, Page, Peraro, Poncelet, Sotnikov, Tancredi, von Manteuffel, Zoia, 2018-2023]

Cross section calculations @NNLO:

- **STRIPPER**:
  - pp→ɣɣɣ [Chawdry,Czakon,Mitov,Poncelet \`20]
  - pp→ɣɣj [Chawdry,Czakon,Mitov,Poncelet \`21]
  - pp→jjj [Czakon,Mitov,Poncelet \`21] [Alvarez,Cantero,Czakon, Lorente,Mitov,Poncelet \`23]
  - pp→Wbb [Hartanto,Poncelet,Popescu,Zoia \`22]
  - pp→ɣjj [Badger,Czakon,Hartanto,Mitov, Moodie,Peraro,Poncelet,Zoia \`23]
- **NNLOJET**:
  - gg→ggg [Chen,Gehrmann,Glover,Huss,MM \`22]
- **MATRIX**:
  - pp→ɣɣɣ [Kallweit,Sotnikov,Wiesemann \`20]
  - pp→Htt [Catani,Devoto,Grazzini,Kallweit, Mazzitelli,Savoini \`22]
  - pp→Wbb [Buonocore,Devoto,Kallweit,Mazzitelli, Rottoli,Savoini \`22]
  - pp→Wtt [Buonocore,Devoto,Grazzini,Kallweit, Mazzitelli,Rottoli,Savoini \`23]

Mass factorization kernels

$$\Gamma^{(1)}_{ab}(x) = -\frac{1}{\epsilon} P^0_{ab}(x)$$

$$\overline{\Gamma}^{(2)}_{ab}(x) = -\frac{1}{2\epsilon}\left(P^1_{ab}(x) + \frac{\beta_0}{\epsilon} P^0_{ab}(x)\right)$$

$P^{(0)}_{ab}(x), \quad P^{(1)}_{ab}(x)$ LO and NLO Altarelli-Parisi splitting kernels

$$\beta_0 = \frac{11}{6} N_c - \frac{1}{3} N_f$$

Quark-gluon dipoles

Needed to remove **spurious singularities** present in integrated quark-gluon antenna functions

$$Poles\left[N_c\left(\mathcal{J}_2^{(2)}(q,g)+\mathcal{J}_2^{(2)}(g,\bar{q})-2\overline{\mathcal{J}}_2^{(2)}(q,\bar{q})-\frac{\beta_0}{\epsilon}\left(\mathcal{J}_2^{(1)}(q,g)+\mathcal{J}_2^{(1)}(g,\bar{q})\right)\right]=$$

$$Poles\left[\mathrm{Re}\left(\mathcal{I}_{qg}^{(2)}(\epsilon)+\mathcal{I}_{g\bar{q}}^{(2)}(\epsilon)-\frac{\beta_0}{\epsilon}\left(\mathcal{I}_{qg}^{(1)}(\epsilon)+\mathcal{I}_{g\bar{q}}^{(1)}(\epsilon)\right)\right)\right]$$

Quark-gluon antenna function are obtained from the decay of a **neutralino**. The colour structure of these **supersymmetric** matrix elements differs from the one in QCD

$$\chi\to\tilde{g}g$$

Hadronic three-jet production

α_s, PDF determination

$$pp \to jjj$$

Five coloured particles at LO

Cutting-edge calculation in QCD, the most challenging in massless QCD given the available MEs.

[Czakon,Mitov,Poncelet '21] [Alvarez,Catnero,Czakon,Lorente,Mitov,Poncelet '23]

Matrix elements:

100 million CPU hours !

- 5-parton two-loop: dedicated **C++ libraries**; [Abreu,Cordero,Ita,Page,Sotnikov '21] [De Laurentis, Ita, Klinkert, Sotnikov '23] [Agarwal,Buccioni,Devoto,Gambuti, von Manteuffel,Tancredi '23] full-colour
- 6-parton one-loop: **OpenLoops**, crucial IR stability**;**
- 7-parton tree-level: **OpenLoops**; [Buccioni,Lang,Lindert,Maierhöfer, Pozzorini,Zhang,Zoller '19]
- 5-, 6-parton tree-level, 5-parton one-loop (subtraction terms): hard coded in ***NNLOJET***;

Validation of the subtraction terms

- **VV**:
  - exact cancellation of **explicit poles** (symbolic);
- **RV**:
  - exact cancellation of **explicit poles** (symbolic);
  - cancellation of **divergent behaviour** (numerical);
- **RR**:
  - cancellation of **divergent behaviour** (numerical);

- mild IR depth
- standard IR depth
- severe IR depth

We generate 10000 events in a given IR limit and we compute:

$$t = \log_{10}(|1 - \mathrm{ME}/\mathrm{sub}|)$$

which measures the number of digits of ME-sub agreement.