THE THREE LOOP AMPLITUDE FOR V+jet PRODUCTION

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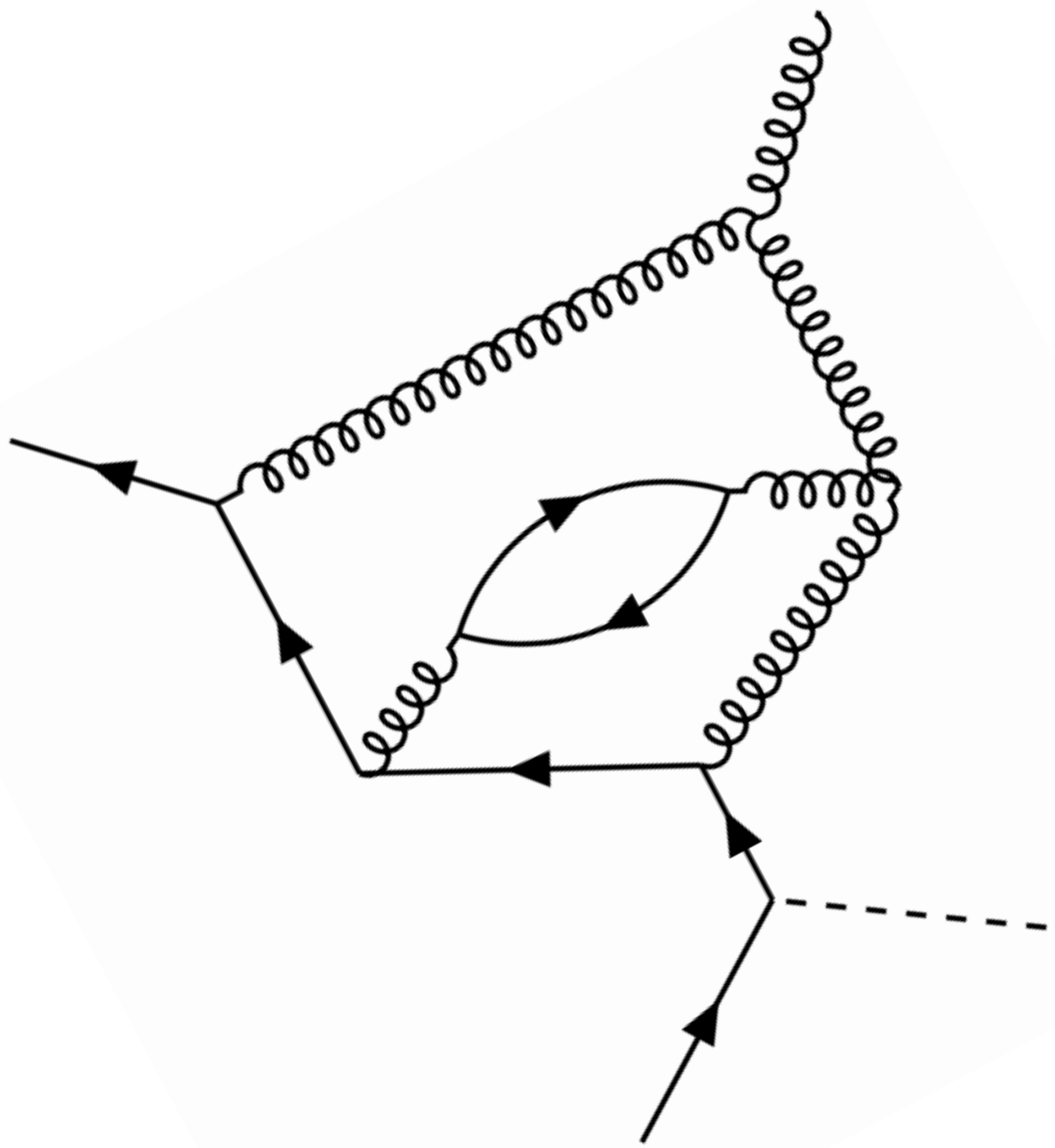
Summary

1. Introduction
   - Do we need N3LO for V+jet production?
   - State of the Art in theory prediction
2. The Calculation
   - Tensor decomposition
   - The workflow
   - Results
3. What's next?
   - Full amplitude, non planar 3L master integrals, …



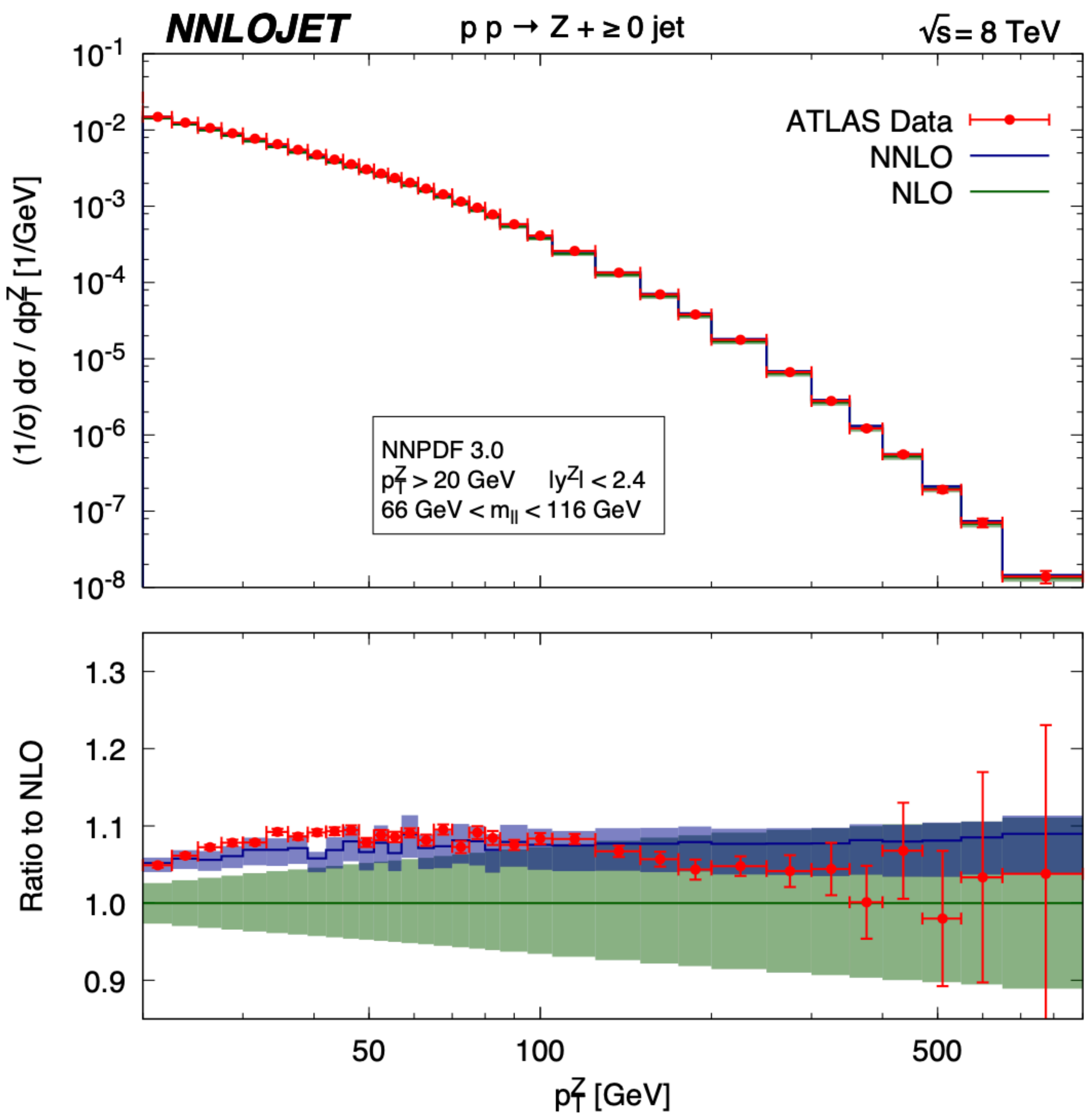
Do we need N3LO for V+jet?

- Determination of α_s and constraint on PDFs

- Measures of differential distribution for Z production extremely accurate $\mathcal{O}(<1\%)$ [CMS, ATLAS]: high rate and clear leptonic signature

- NNLO for V+jet obtained via antenna subtraction [Gehrmann - De Ridder, Gehrmann, Glover, et al., 2016], N-jettiness [Boughezal, Liu, Petriello, 2016]

  - Z - p_T distribution, leading jet distribution, Angular coefficients → details of gauge boson and its polarisation states



State of the Art in theory predictions

$$d\sigma_{pp\to X} = \sum_{i,j} \int\int dx_1 dx_2\, f_{p_1,i}(x_1, Q_F) f_{p_2,j}(x_2, Q_F) \times d\hat{\sigma}_{ij\to X}(x_1 P_1, x_2 P_2, Q_F) + \mathcal{O}\left((\Lambda_{QCD}/Q)^p\right)$$

- Among the ingredients to the $\mathcal{O}(1\%)$ accuracy is the precise determination of partonic cross section
- Perturbative expansion → Multiloop and higher multiplicity amplitudes
  → Suitable subtraction procedure (Antenna, Slicing, …)

On the amplitudes side

- 4 point massless master integrals [Henn et al. '20]
- 4 points 3L massless amps. ($gggg$, $qqgg$, $4q$, $qq\gamma\gamma$, …) [Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi, '21-'22]

➡ 4 point with one external mass @3L is one of the next things to do

State of the Art in theory predictions: new results!

Perturbative expansion for V + jet amplitude: the N3LO

$$d\sigma_{q\bar{q}\to Vj} = d\sigma^{LO}_{q\bar{q}\to Vj} + \left(\frac{\alpha_s}{2\pi}\right) d\sigma^{NLO}_{q\bar{q}\to Vj} + \left(\frac{\alpha_s}{2\pi}\right)^2 d\sigma^{NNLO}_{q\bar{q}\to Vj} + \mathcal{O}(\alpha_s^3)$$

✓ Tree Level V + 4j

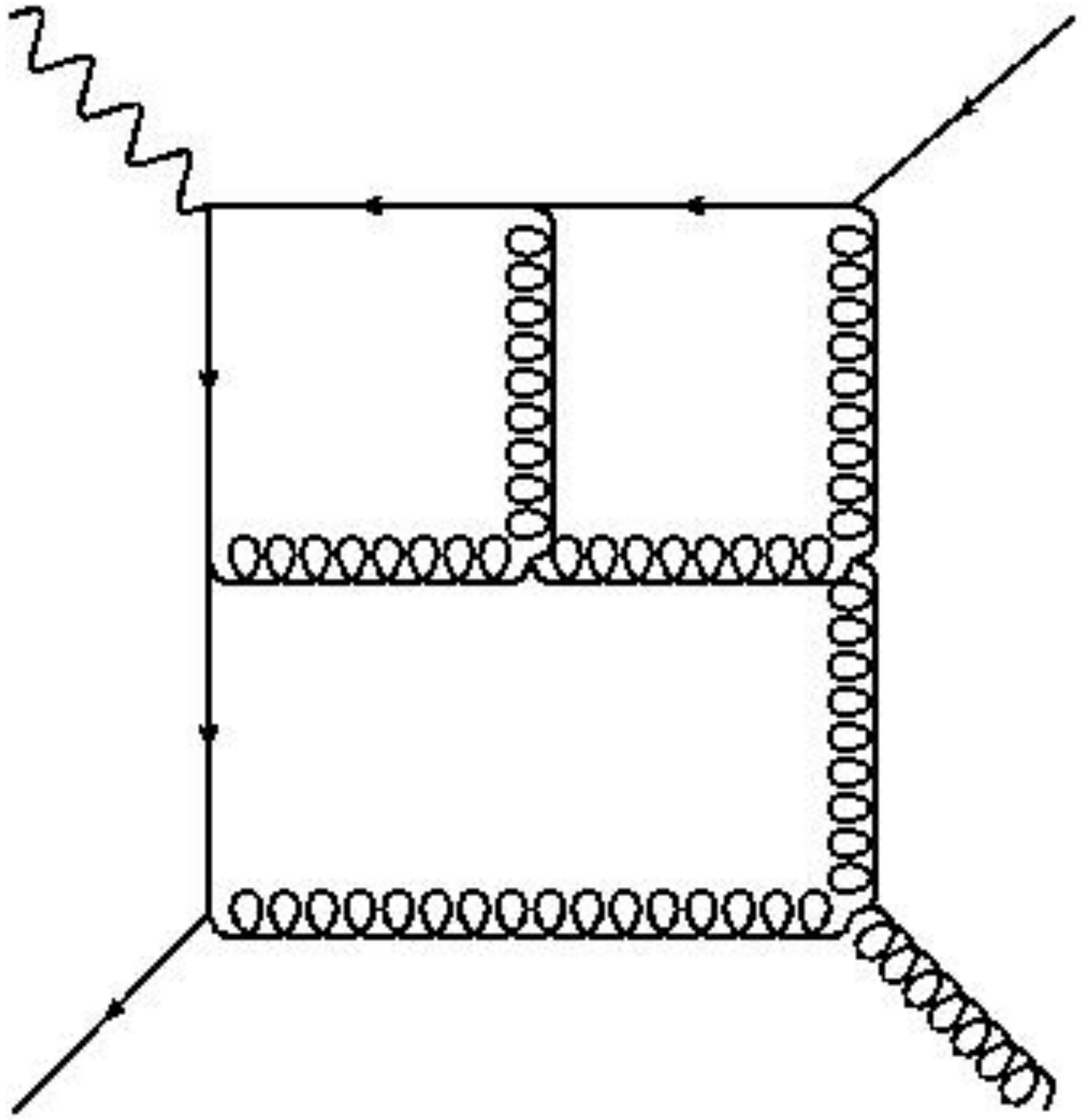
✓ 1- Loop V + 3j

- 2- Loop V + 2j, all integrals [Abreu, Chicherin, Ita, Page, Sotnikov, Tschernow, Zoia, 2023], and leading color amps. [Abreu, Cordero, Ita, Klinkert, Page, Sotnikov, 2021]

★ 3- Loop V + j is work in progress, RESULTS!

1. 2- Loop V + j at transcendental weight six, including axial-vector coupling, done!

2. 3- Loop V + j in leading color approximation, done!



Computing amplitudes: tensor decomposition

- We are interested in helicity amplitudes
  - We work with 4-dim external states ('t Hooft-Veltman scheme) and dim-reg for the loops
  - Write a decomposition based on external particles and symmetries

Use gauge symmetry, gluons transversality and kinematics to constrain the tensor structures → $\mathscr{A} = \sum_{i=1}^{N} F_i T_i$

- Define projectors to form factors $P_j = \sum_{i=1}^{N} c^i_{(j)} T_i^\dagger$, such that $\sum_{pol} P_j \mathscr{A} = F_j$, see [Peraro, Tancredi, 2021]
- 24 tensor structures for $Zggg$ and 12 for $Zgq\bar{q}$
- Helicity amplitude projectors as linear combinations of the form factor projectors

Computational setup

1. Diagrams generated with QGRAF [Nogueira, 1993]

2. Feynman rules, colour algebra and projection onto form factors is performed in FORM [Vermaseren, 1999]

- The resulting integrand is a combination of scalar integrals

$$F = \sum_{l=1}^{N_{int}} r_l(s_{ij}, d)\, Int_l$$

3. Integral **reduction** performed with KIRA [Klappert, Lange, Maierhöfer, Usovitsch, 2021] and REDUZE [Studerus, von Matteuffel, 2012]

At 3L non negligible **analytic** and **computational** complexity.

4. Insert reductions and **solution**, UV renorm., IR subtr., analytic continuation

The 2 loop amplitudes at $\mathcal{O}(\epsilon^2)$: master integrals and UV renormalisation

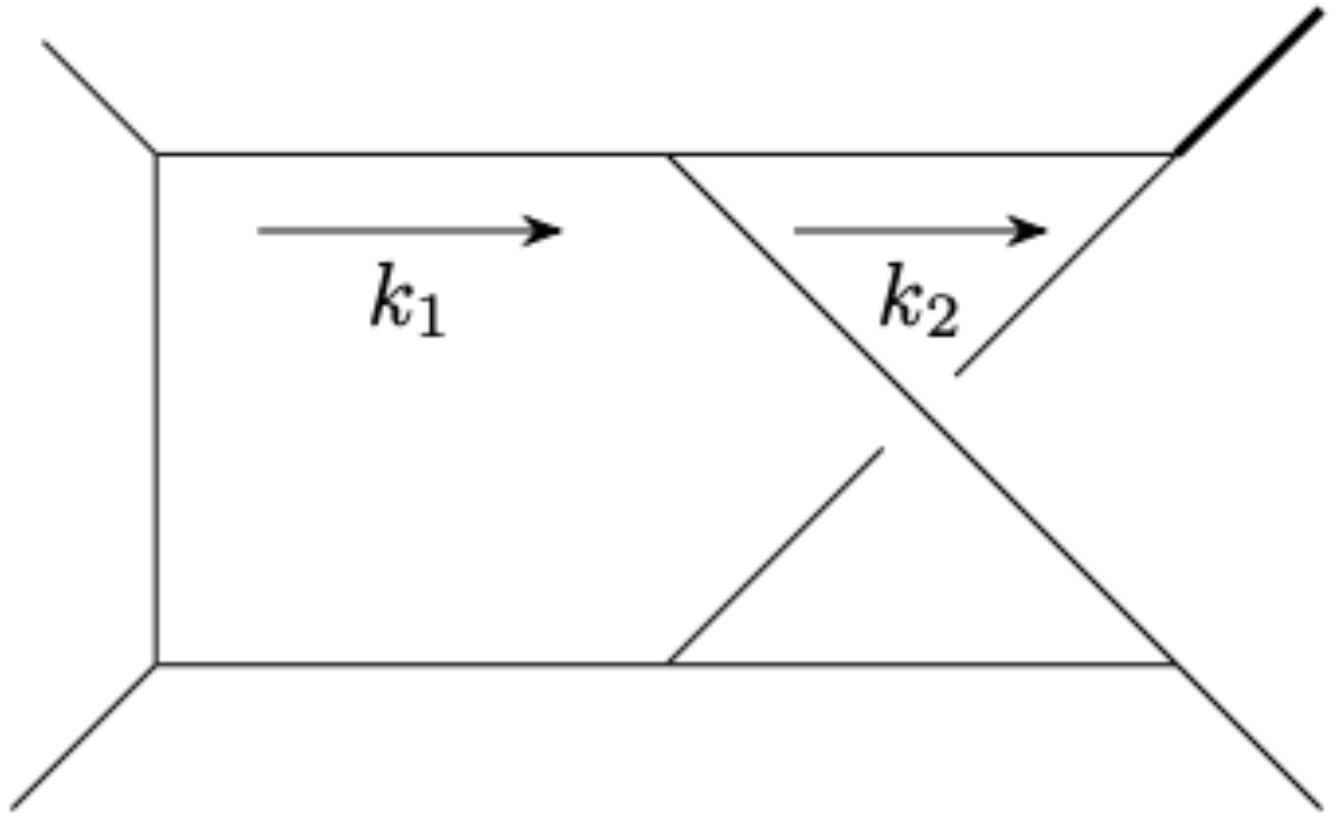
- 4 point with one external mass @2L [Gehrmann, Remiddi 2001]

  ✓ We provide a canonical basis [Henn, 2012], using the package DLogBasis [Henn, Mistlberger, 2020]

  ✓ Solution in terms of GHPLs to $\mathcal{O}(\epsilon^2)$ using PolyLogTools (not at weight 6!) [Duhr, Dulat, 2019]
  Alphabet $\{y, z, 1-y, 1-z, y+z, 1-y-z\}$, where $y = s_{13}/m_V^2,\ z = s_{23}/m_V^2$.

- The axial-vector amplitude needs an **extra finite renormalisation** when working in the Larin scheme, $\gamma^\mu\gamma^5 \to \frac{1}{6}\epsilon^{\mu\nu\rho\sigma}\gamma_\nu\gamma_\rho\gamma_\sigma$, in order to restore the axial Ward Identities [Larin, 1993]

  - We checked the renormalisation against the axial anomaly relation



$$\partial_\mu \langle J_5^\mu(x)\mathcal{O}(x_1,\ldots,x_n)\rangle = \frac{\alpha_s}{8\pi}\langle G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}(x)\mathcal{O}(x_1,\ldots,x_n)\rangle$$

The 3 loop leading color for $Vgq\bar{q}$

The 3 loop leading color for $Vgq\bar{q}$

$$\mathscr{A}^{(3)} = N^3\,\Omega_1^{(3)} + N\,\Omega_2^{(3)} + \frac{1}{N}\Omega_3^{(3)} + \frac{1}{N^3}\Omega_4^{(3)} + N_f\,N^2\Omega_5^{(3)} + N_f\Omega_6^{(3)} + \frac{N_f}{N^2}\Omega_7^{(3)}$$

$$+N_f^2 N\,\Omega_8^{(3)} + \frac{N_f^2}{N}\Omega_9^{(3)} + N_f^3\Omega_{10}^{(3)} + N_{f,V}N^2\Omega_{11}^{(3)} + N_{f,V}\Omega_{12}^{(3)}$$

$$+\frac{N_{f,V}}{N^2}\Omega_{13}^{(3)} + N_f N_{f,V} N\Omega_{14}^{(3)} + \frac{N_f N_{f,V}}{N}\Omega_{15}^{(3)}, + N_f^2 N_{f,V}\Omega_{16}^{(3)}$$

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- Since $N_F \approx N_C$, we consider $N_F^2 N_C, N_C^2 N_F, N_C^3$ $\longrightarrow$ only planar contributions!
- Planar topologies at 3 loop computed [Canko, Syrrakos, '22]
- Alphabet the same as at two loop $\{y, z, 1-y, 1-z, y+z, 1-y-z\}$

The 3 loop leading color for $Vgq\bar{q}$

- Unreduced contribution $F = \sum_{l=1}^{N_{int}} r_l(s_{ij}, d)\, Int_l$ with $N_{Int} \approx 100k$

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KIRA

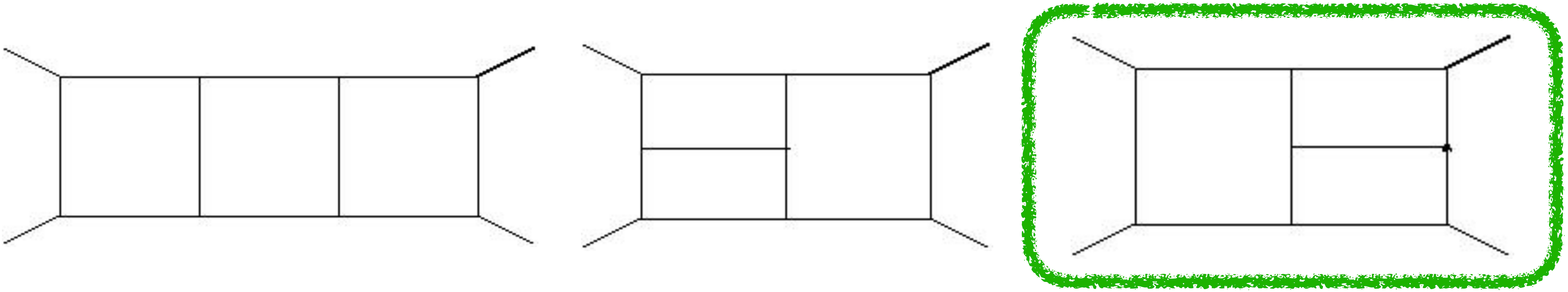
Reduction to masters of family with 10+5 (ISP) = 15 propagators with ints up to rank 5: expensive!

- Reduction in terms of 291 master integral (mod crossings)
- The insertion of the reduction and master solutions both analytically and with finite field reconstruction using FiniteFlow [Peraro, 2019]
  - We reconstructed only the coefficients of the appearing GHPLs

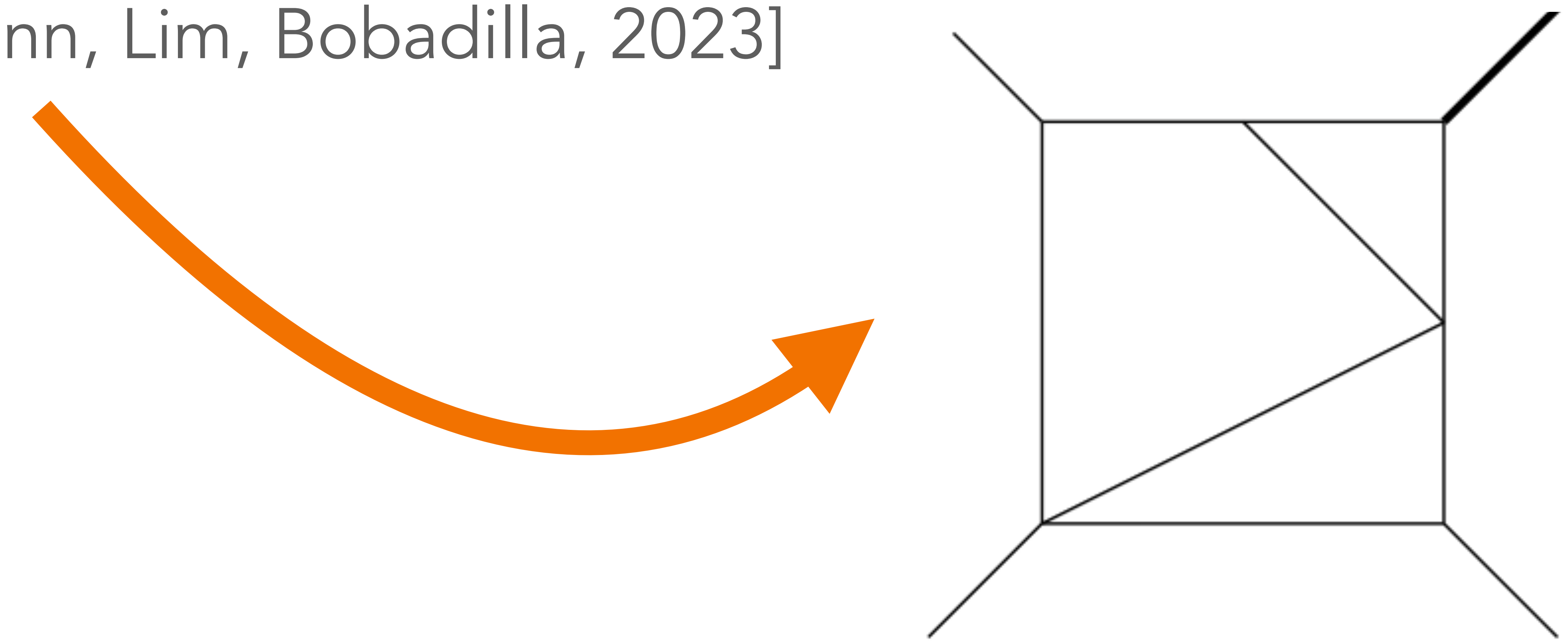
The 3 loop leading color for $Vgq\bar{q}$: the adjacency condition

- It was noticed in $\mathcal{N} = 4$ super-YM that the letters $\{1-x, 1-y, 1-z\}$ never appear next to each other in the symbol [Dixon, McLeod, Wilhelm, 2021] $\longrightarrow$ adjacency conjecture

- In the case considered, it holds up to two loop at the level of the integrals



- Together with the evaluation of the first non-planar topologies, the adjacency conjecture was found to be violated in one of the planar topologies at 3L [Henn, Lim, Bobadilla, 2023]



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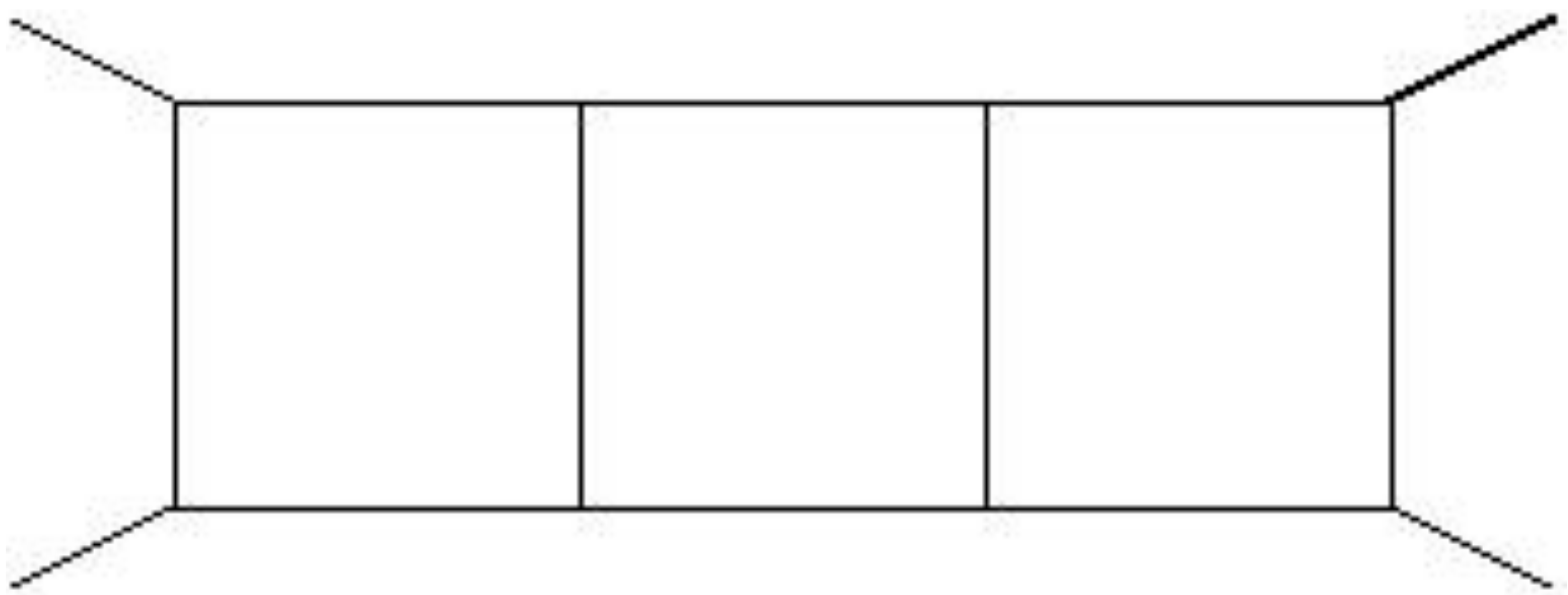
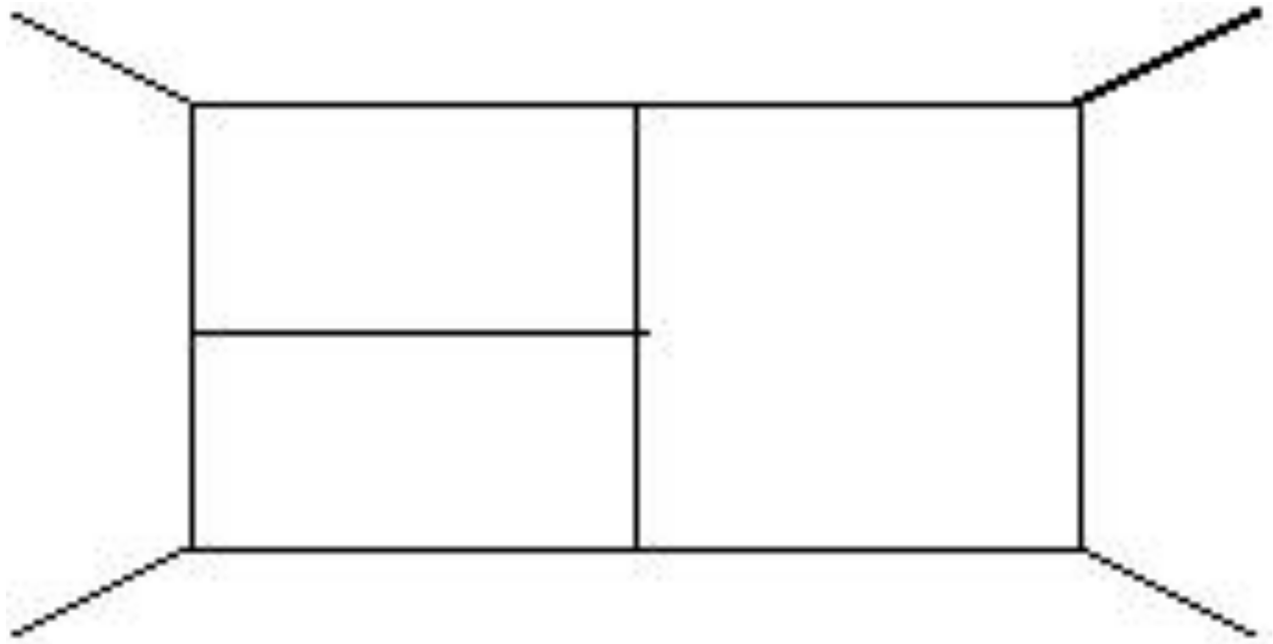
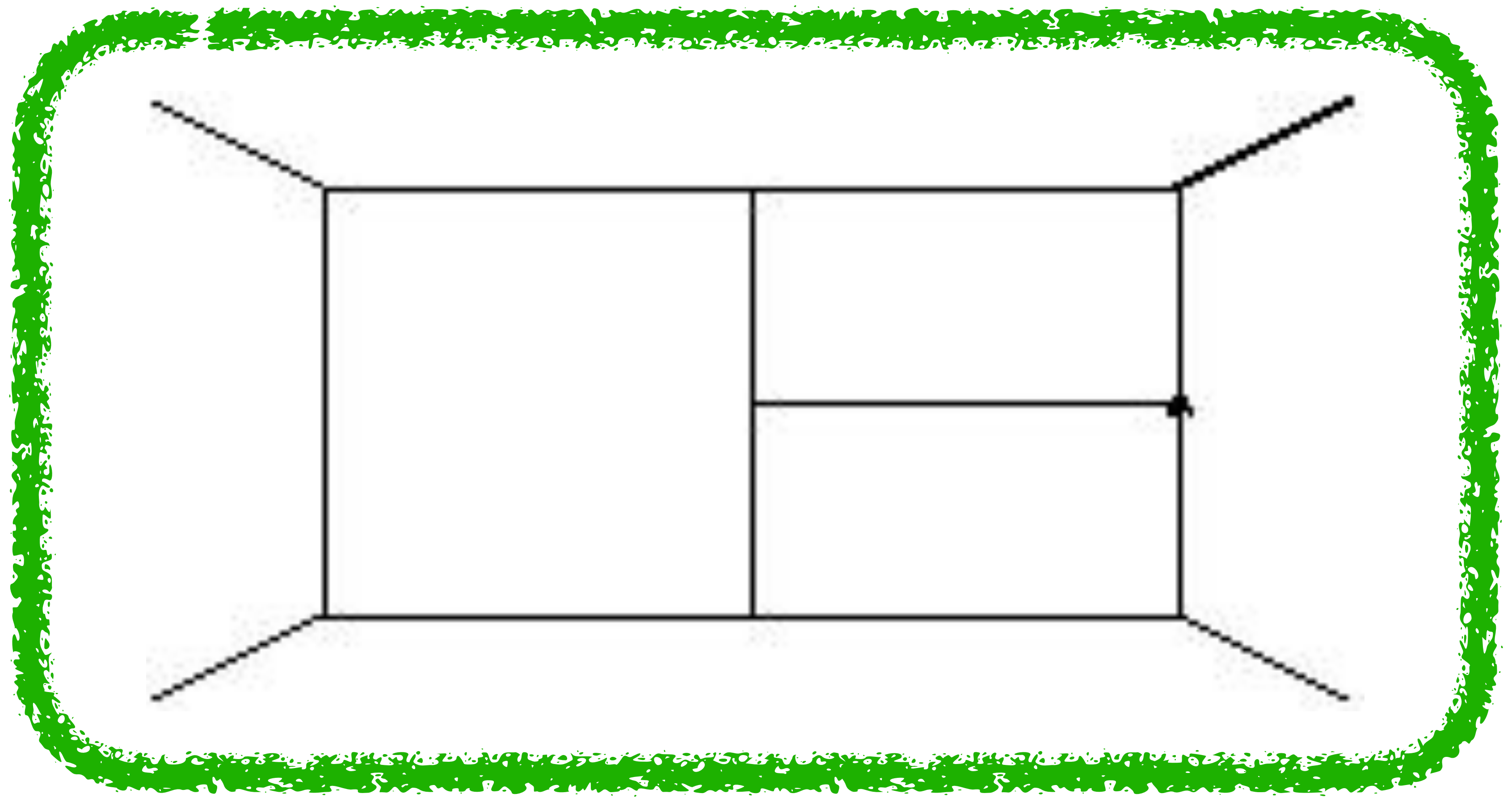

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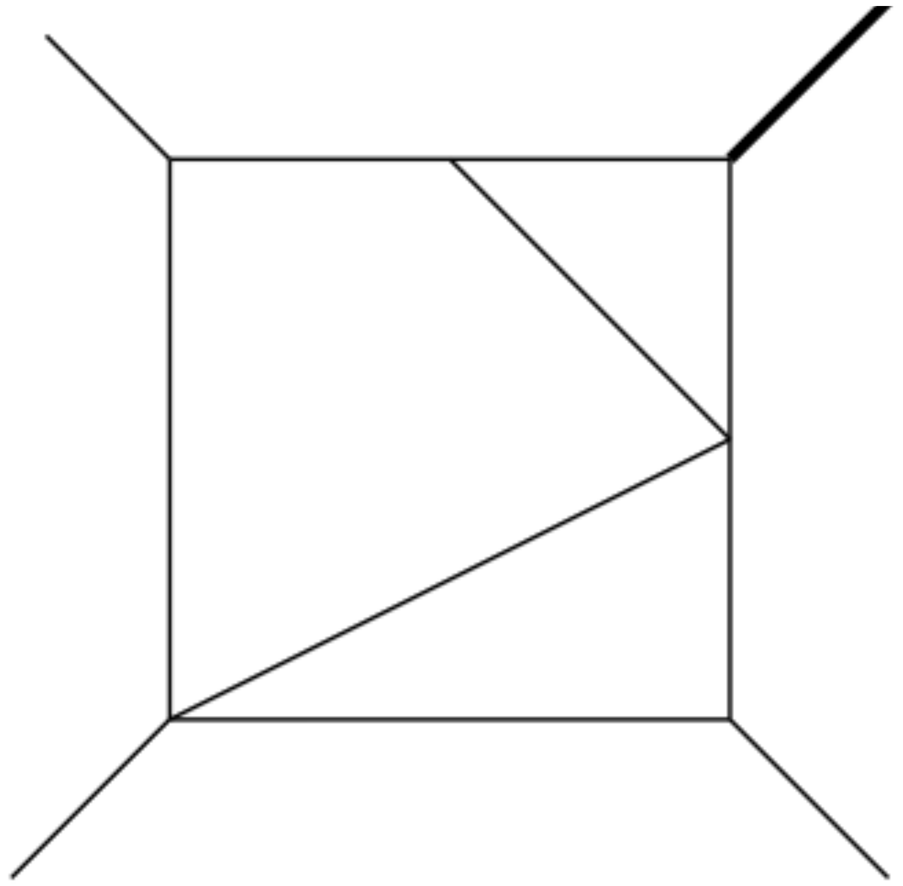
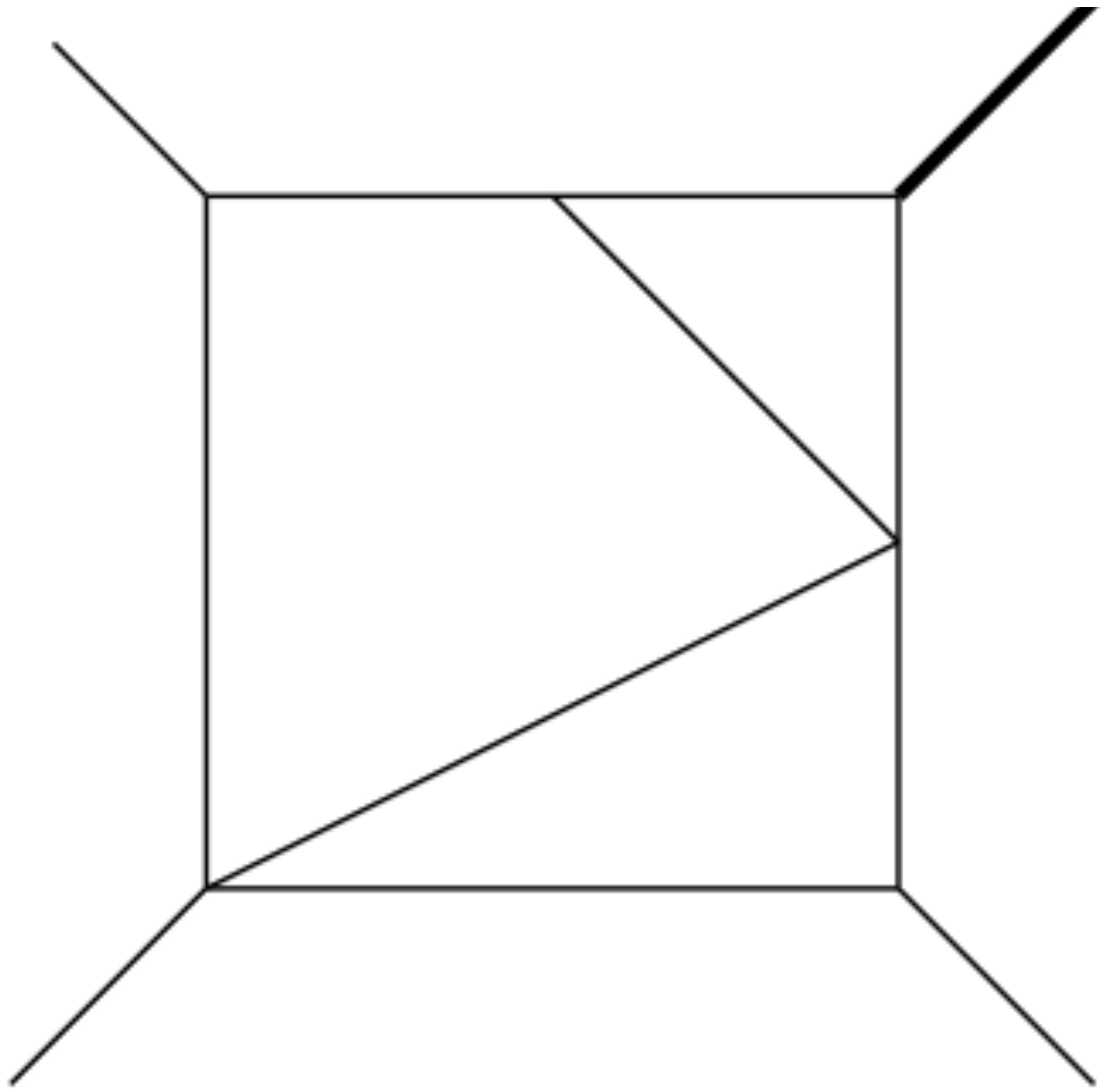
? $\cdots \otimes 1-x \otimes 1-y \otimes \ldots$?



The 3 loop leading color for $Vgq\bar{q}$: the adjacency condition

- $d\vec{I} = \epsilon \sum_i A_i \; d\log\alpha_i \; \vec{I}$ where $\alpha_i \in \{y, z, 1-y, 1-z, y+z, \cdots\}$
  - In term of the diff equation matrices, the condition which is violated is: $A_i \cdot A_j = 0$ whenever $\alpha_i, \alpha_j \in \{1-x, 1-y, 1-z\}$



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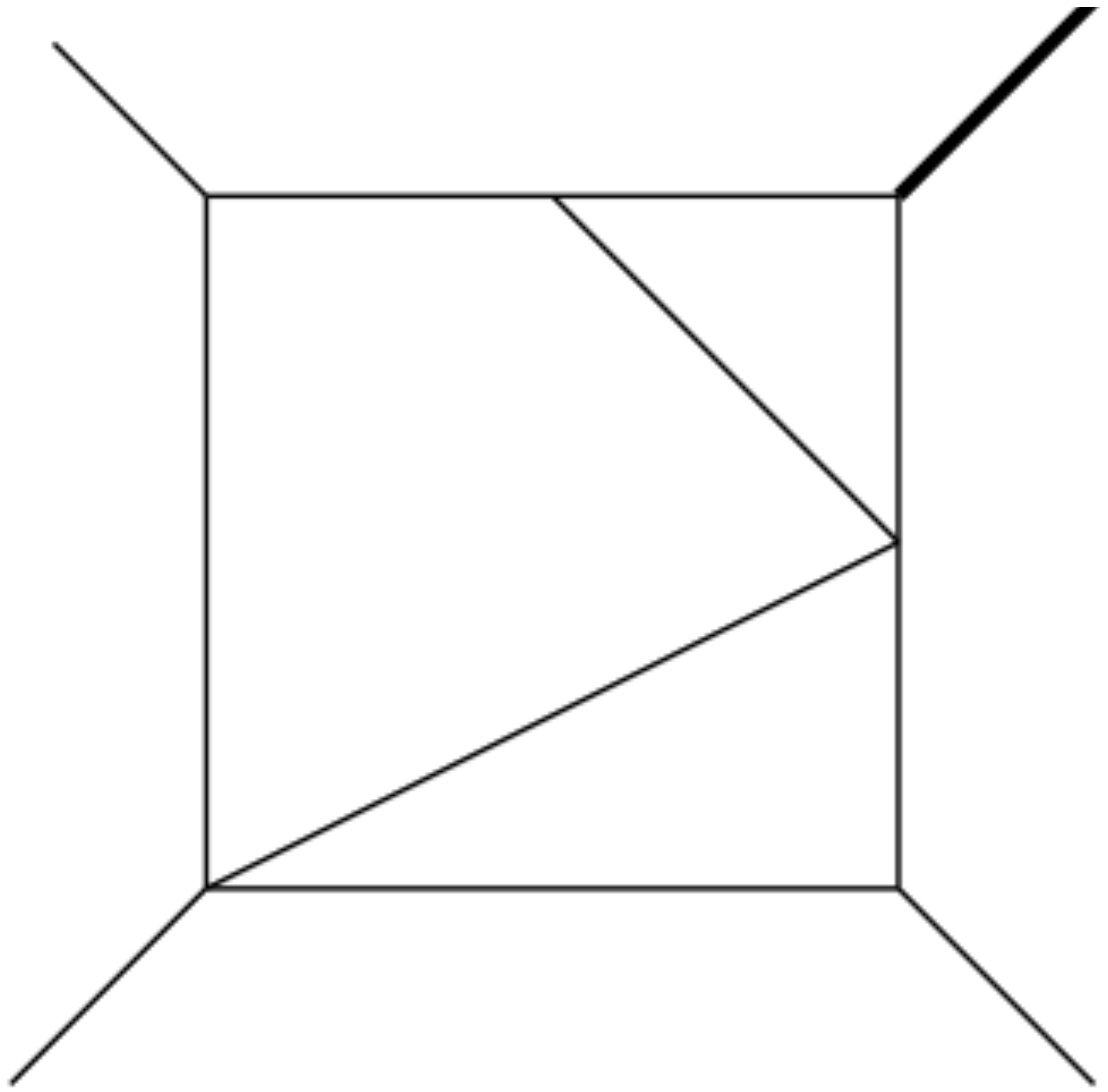

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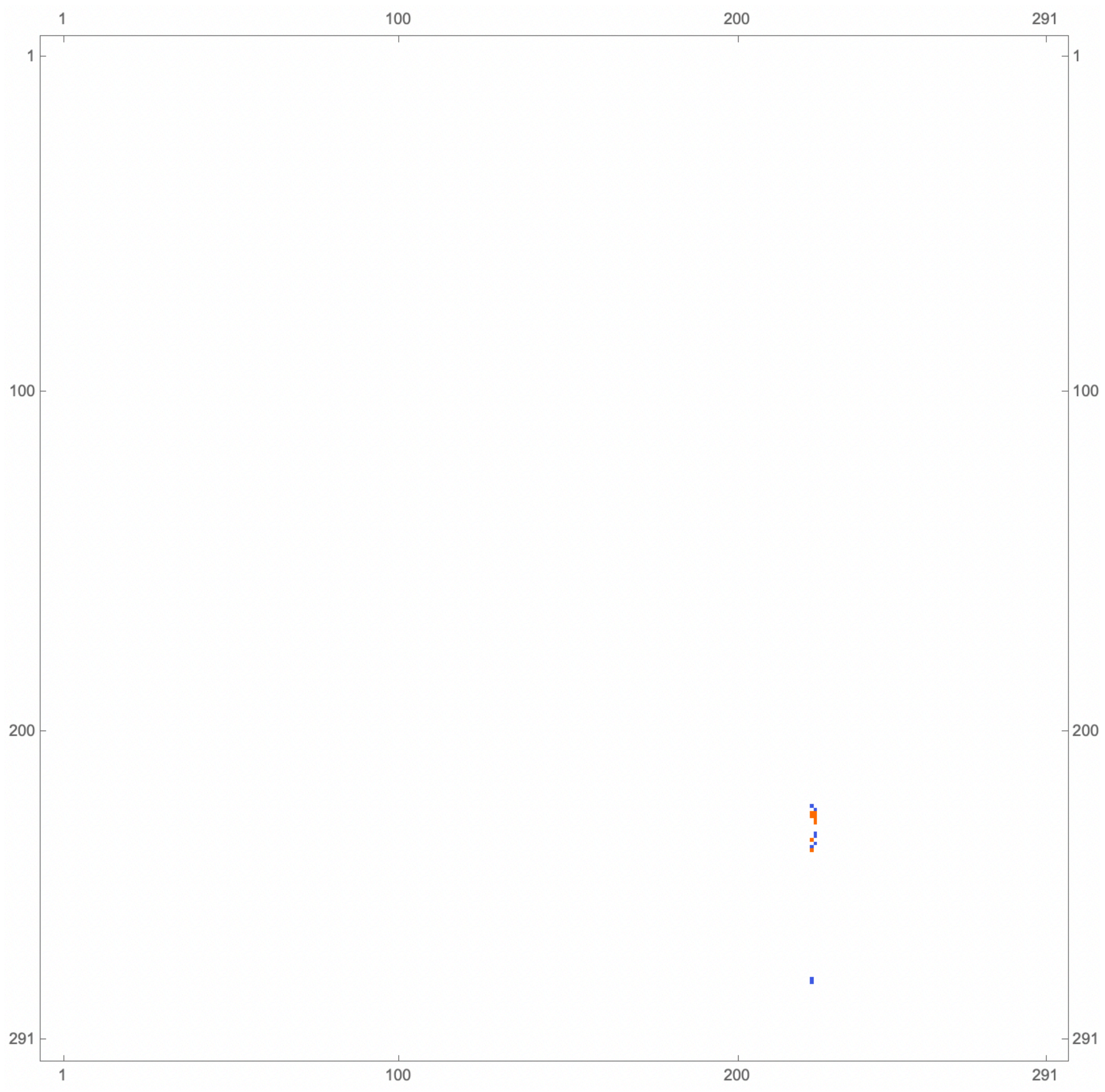

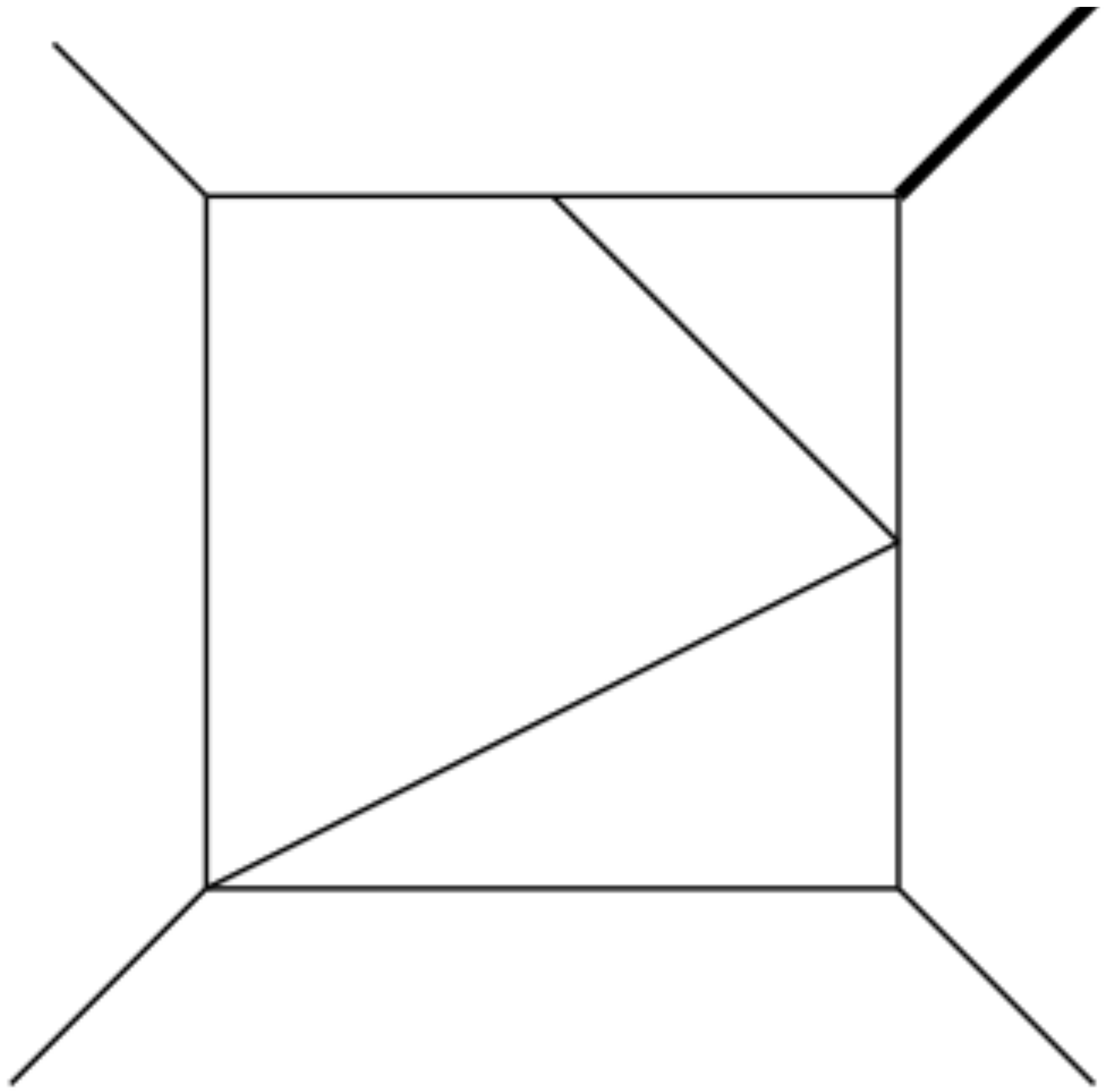

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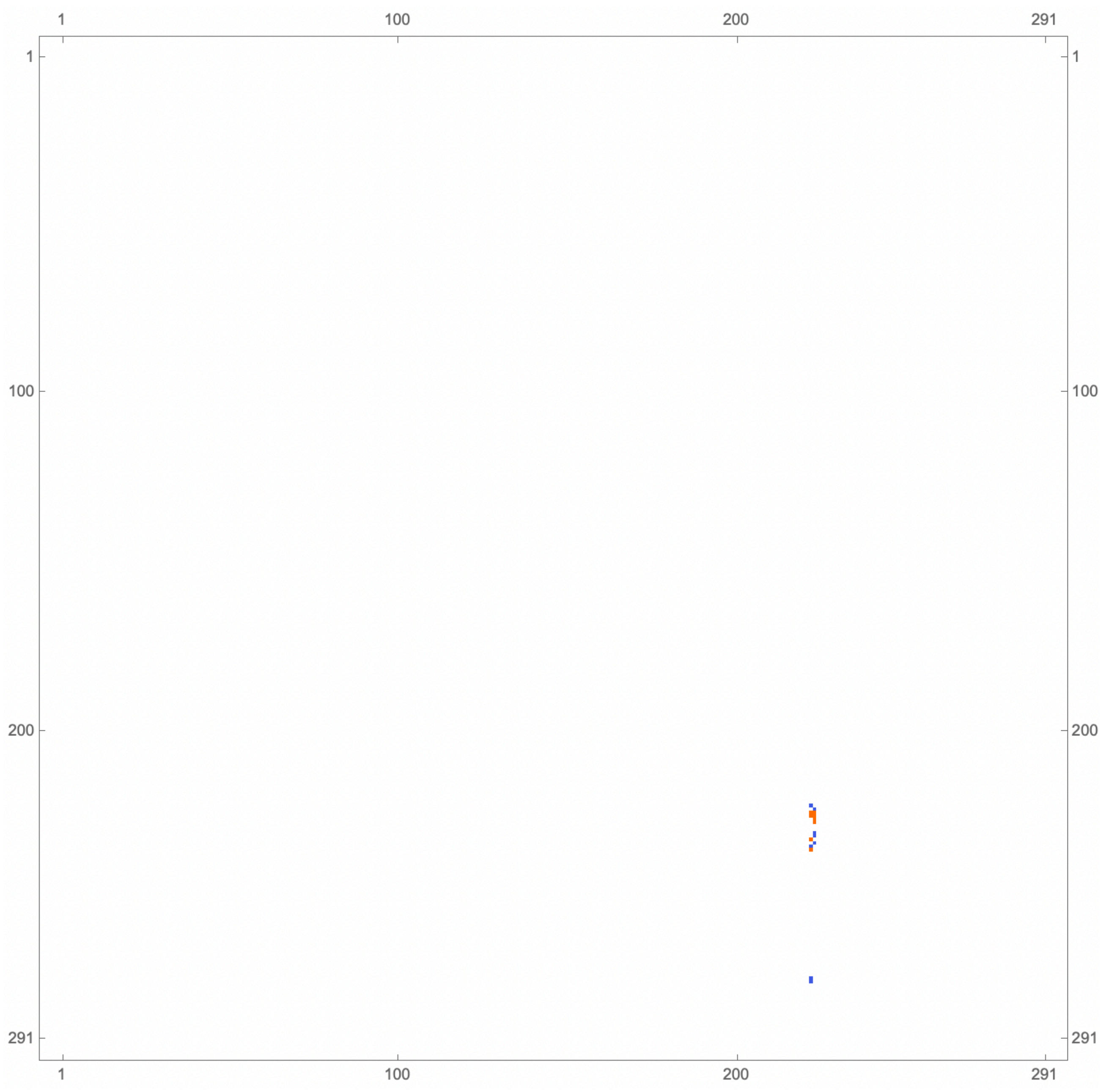

$$\cdots \otimes 1-x \otimes 1-y \otimes \cdots$$

- Surprisingly, the conjectured is **satisfied** at the level of the amplitude
- Because of the propagation of the conjecture-violating symbols in the upper sectors, it is not easy to trace the cancellation

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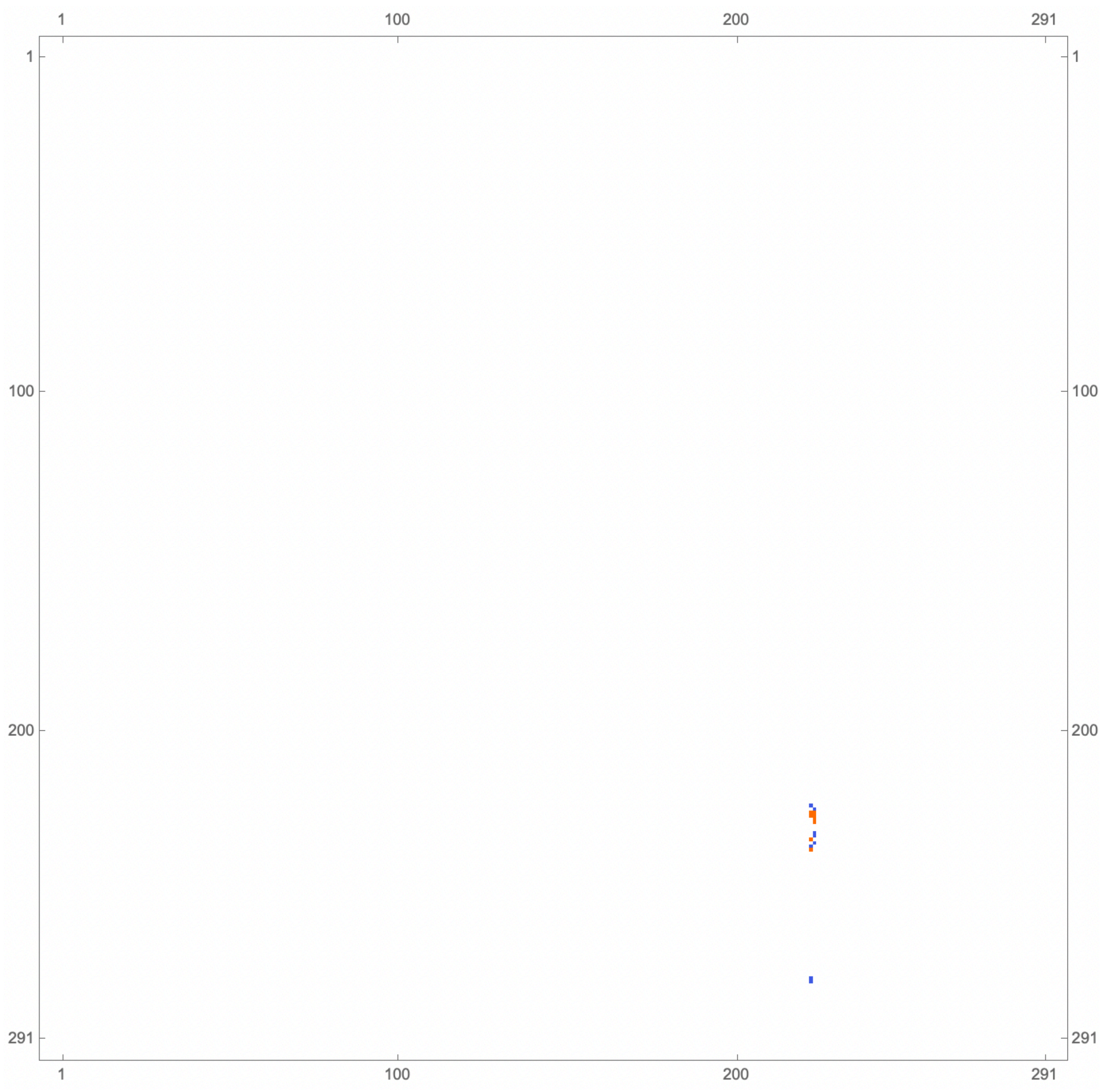

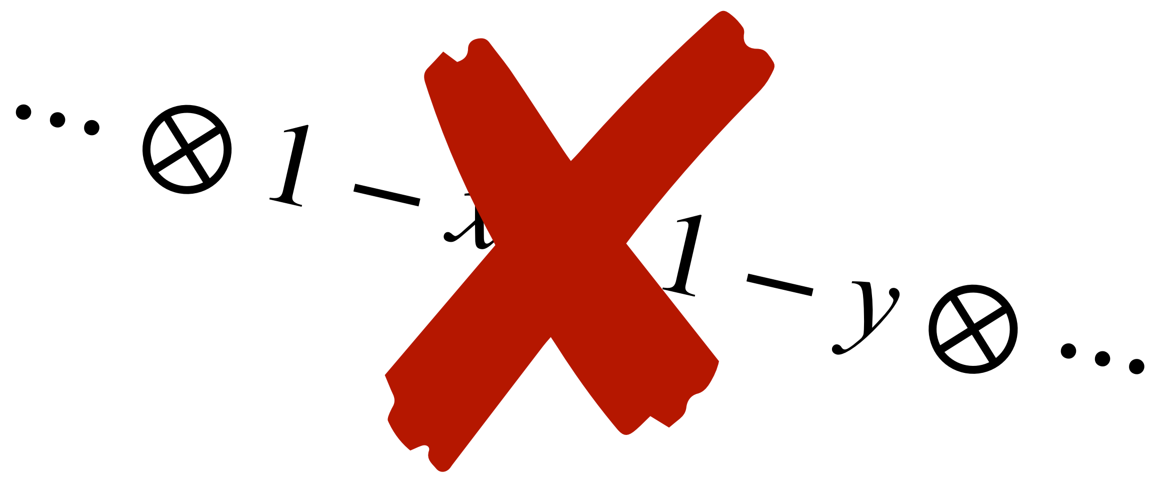

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Results

- Canonical basis for the for 2L and extension up to $\mathcal{O}(\epsilon^2)$
- Solution in terms GHPLs with simple alphabet $\{y, z, 1-y, 1-z, y+z, 1-y-z\}$, fast numerical evaluation
- Completed the missing vector-axial calculation and 3L leading color for $V + jet$
- UV renormalised and IR subtracted helicity amplitudes analytically continued to all physical regions: $V \to gq\bar{q},\ q\bar{q} \to Vg,\ qg \to Vq,\ \bar{q}g \to V\bar{q}$
- Results have been check numerically at 1L with OpenLoops2 [Buccioni et al., 2019], and against available literature at 2L

The Work in Progress: non planar ints and the Higgs amplitude

In collaboration with Johannes Henn, Jungwon Lim, William J. Torres Bobadilla

★ *TODO list: preliminary news*

- $H + jet$ Feynman diagram representation is combinatorially more complex
  - *5334* diagrams for $Zgq\bar{q}$ @3L **vs** *35827* diagrams for $Hggg$ @3L
  - Moderate swell of intermediate expressions ~$6\,Gb$ for $H + jet$ leading color, ~2 mln integrals, with non planar integrals up to rank 6
- 5 non planar top sectors contributing to the leading color of $H + j$
  - The canonical basis for them has a richer alphabet: quadratic letters and a square root
  - Ansatz approach and different representations (Baikov, Baikov loop-by-loop, 4-dim param.) to find *dlog* candidates

Thank You!