Milan Christmas Meeting 2023

QED at NNLO – now what & what next?

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Paul Scherrer Institut

Milano, 21st December 2023

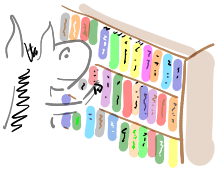
- higher-order predictions and comparison with precision experiments...
- ...but not at the LHC! $\rightarrow$ focus on low-energy QED scattering processes
- theoretical background for lepton experiments
- all this in

McMule

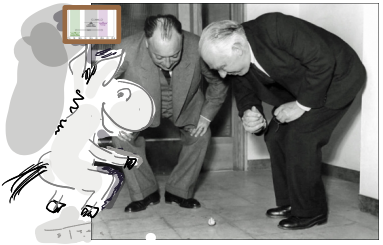
Monte Carlo for MUons and other LEptons
`https://mule-tools.gitlab.io/`

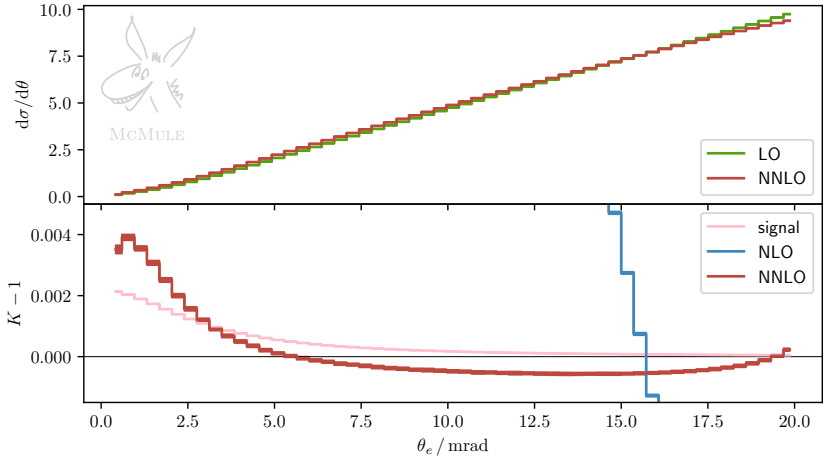
◇ fully-differential Monte Carlo integrator, not an event generator (wip)

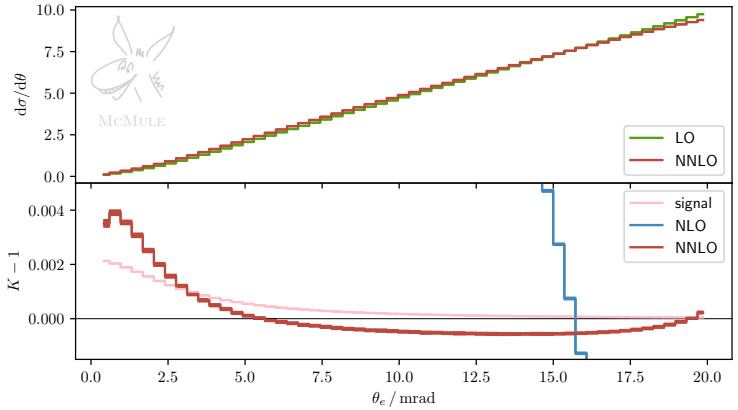
quick recap about last year



- (fairly) easy observable → experimental precision is greatly pushed
- theorists disagree, with the experiment *and* among each other
- new ideas require theory under control at 10 ppm
- we reached NNLO precision in (massive!) QED [Broggio et al. 22]




10.0
7.5
5.0
2.5
0.0
$\mathrm{d}\sigma/\mathrm{d}\theta$
McMule
LO
NNLO
0.004
0.002
0.000
$K-1$
signal
NLO
NNLO
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
θ_e / mrad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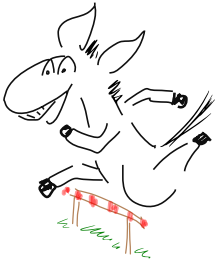
- what next?
  - ◇ $e\mu \to e\mu$ @N3LO
  - ◇ $ee \to \mu\mu\gamma$ @NNLO (+crossing)
  - ◇ YFS resummation/parton shower
- now what?
  - ◇ can someone else benefit from a $2 \to 2$ NNLO calculation?

what next?



- a solution to IR divergences at all orders

  $\rightarrow$ FKS$^{\ell}$ (error: 0)

- a solution to (some) massive multi-loop amplitudes

  $\rightarrow$ massification (error: $\alpha^2\, 10^{-3}$ vs full)

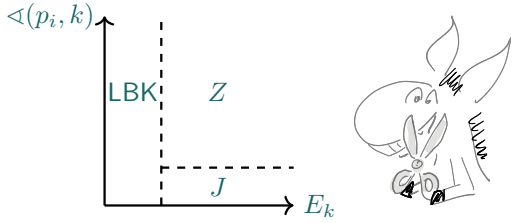
- a solution to instabilities due to lepton masses in real corrections

  $\rightarrow$ next-to-soft stabilisation (error: $\alpha^2\, 10^{-2}$ vs quad)

  $\rightarrow$ jettification (error: $\alpha^3\, 10^{-2}$ expected)

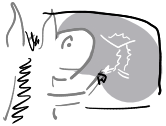
- VVV :: $ee \to \gamma^*$ known [Fael et al. 22]
- RVV :: $ee \to \gamma^*\gamma$ is the bottleneck
  - ◇ known for $m_e = 0$ [Garland et al. 02, Badger et al. 23] but need $m_e \neq 0$

    $m^2 \ll Q^2$ [Engel et al. 18] — $k \cdot p_i \sim m^2 \ll Q^2$ [???] — $E_k \ll Q,\, m_e$ [Engel 23]



- RRV :: $ee \to \gamma^*\gamma\gamma$ with tools + multi-γ LBK [Engel] for NTS stabilisation
- RRR :: $ee \to \gamma^*\gamma\gamma\gamma$ trivial but dangerous

- NLO :: can *easily* be done with full mass dependence
- NNLO :: with (reasonable!) approximations
  - ◇ two-loop $\to$ apply e and μ massification to e.g. [Badger et al. 23]
  - ◇ one-loop $\to$ doable with full mass dependence + NTS stabilisation
  - ◇ tricky integration for tree-level phase space
- subset with one-photon exchange ($\to$ doable with full mass dependence) can give better estimate of massification error
- massification is okay here as the process is highly required by radiative return experiments, with well-separated tagged photons



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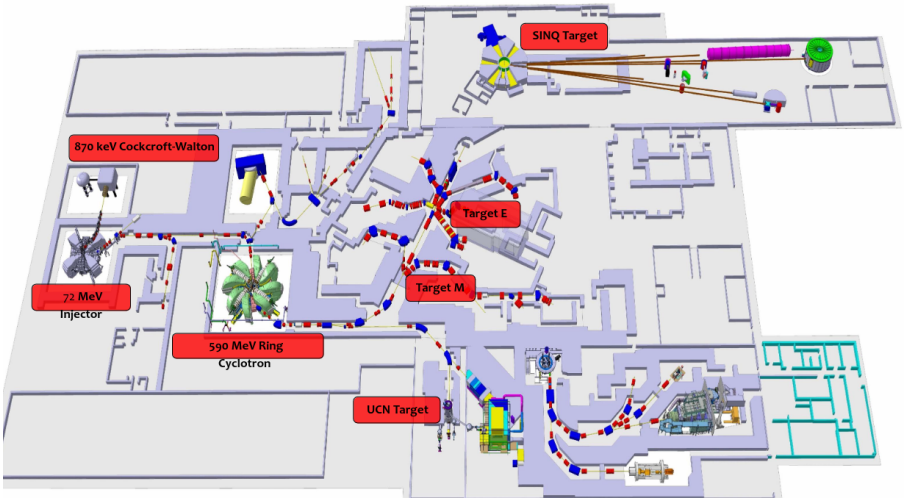
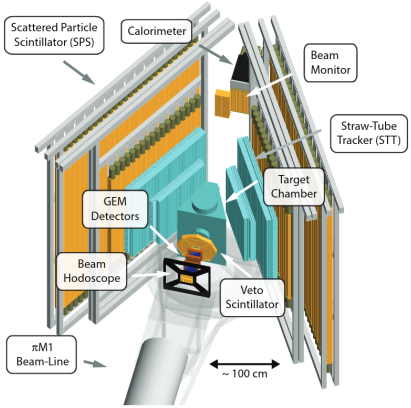

SINQ Target
870 keV Cockcroft-Walton
Target E
Target M
72 MeV
Injector
590 MeV Ring
Cyclotron
UCN Target

MUon Scattering Experiment @ Paul Scherrer Institut



McMuse [Engel et al. 23]

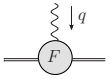
lepton-proton scattering

$$\ell\, p \to \ell\, p$$

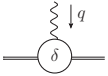
lepton-proton scattering (*known subset*)

$$\ell\, p^{1\gamma} \to \ell\, p^{1\gamma} \qquad \text{“single-dipole”}$$

$$\ell\, \mu \to \ell\, \mu \qquad \text{“point-like”}$$



$$= e\gamma^\mu F_1(Q^2,\Lambda) + \frac{i\sigma^{\mu\nu}q_\nu}{2m_p}F_2(Q^2,\Lambda)$$

$=$  $+$ 

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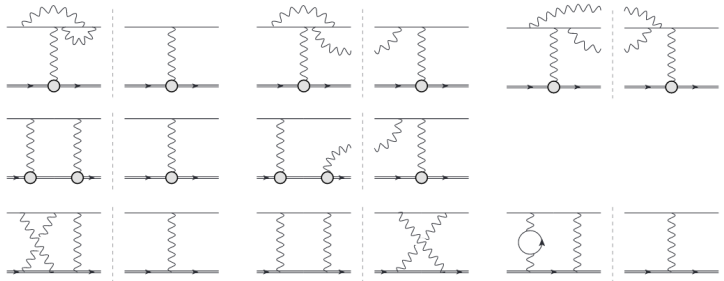
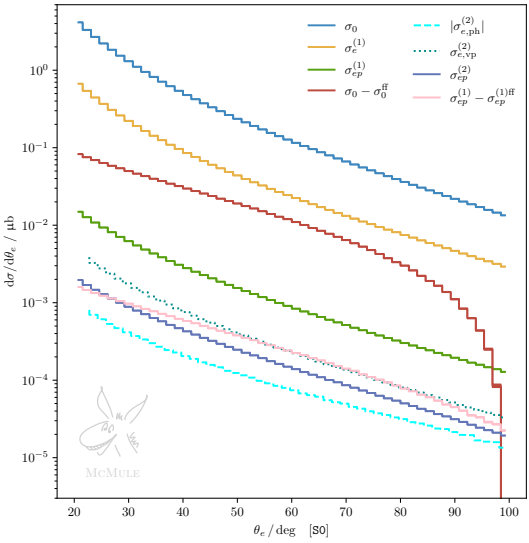
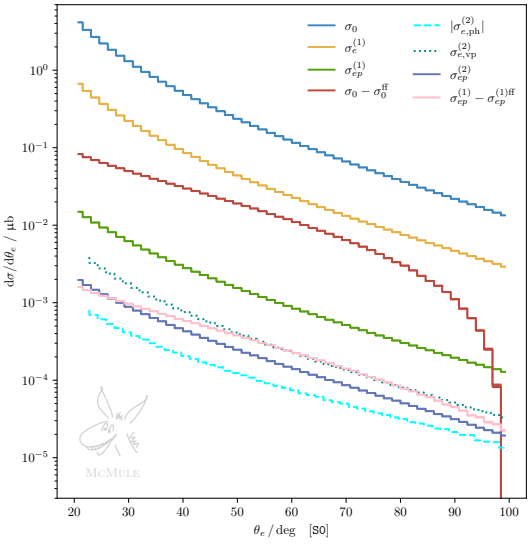


differential results





◇ NLO QED $\gtrsim$ LO hadronic

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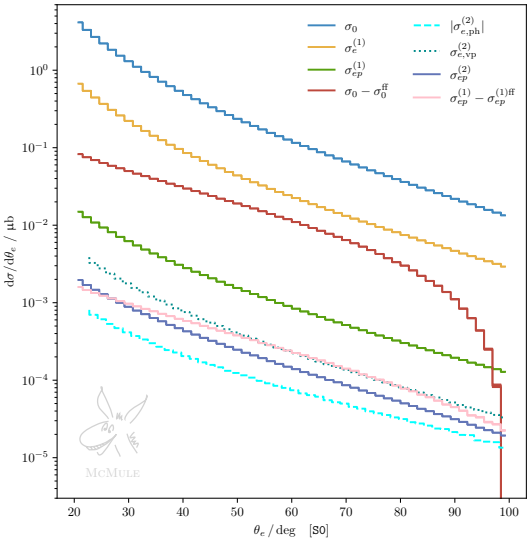

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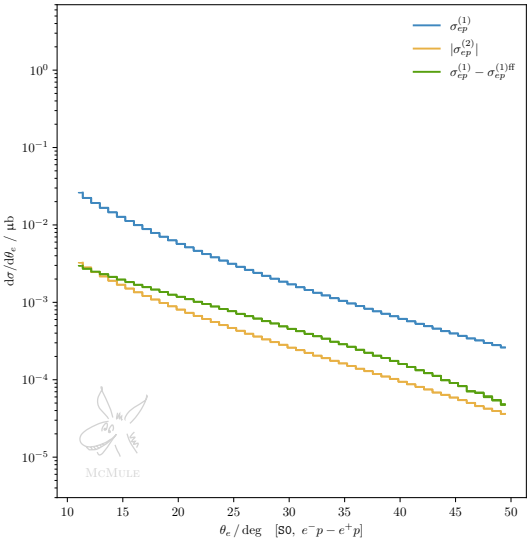

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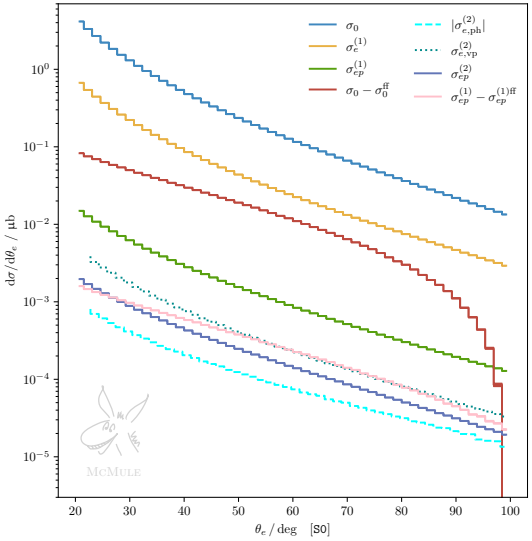
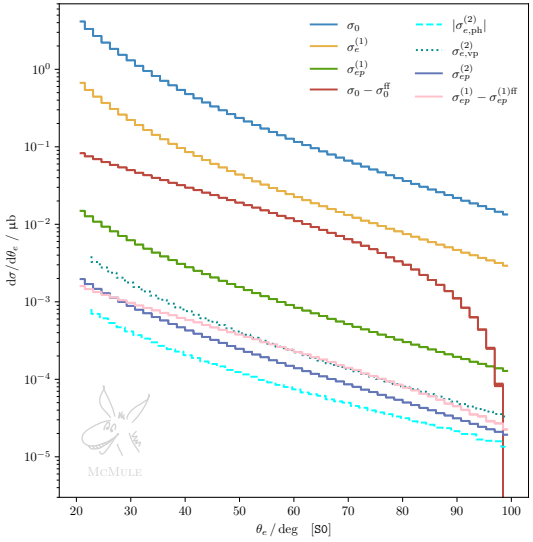
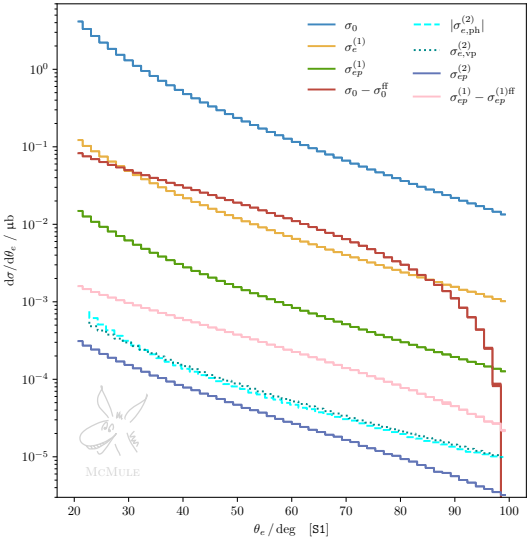


a closer look at MUSE



– forward calorimeter ($\sphericalangle \sim 100$ mrad)



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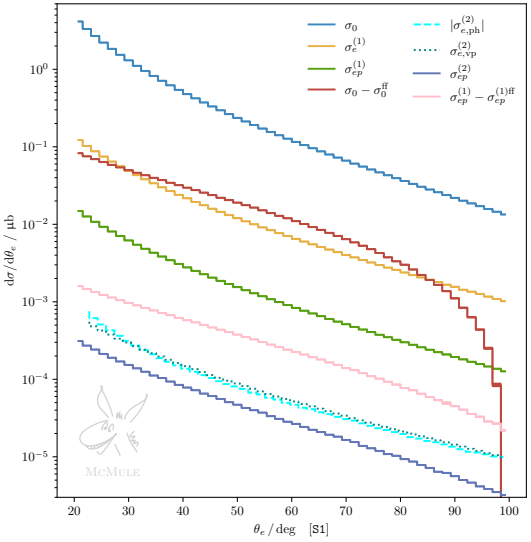
– remove events inside w/ $E_\gamma^{\text{tot}} > 0.4p$

(@MUSE $p = 210$ MeV)



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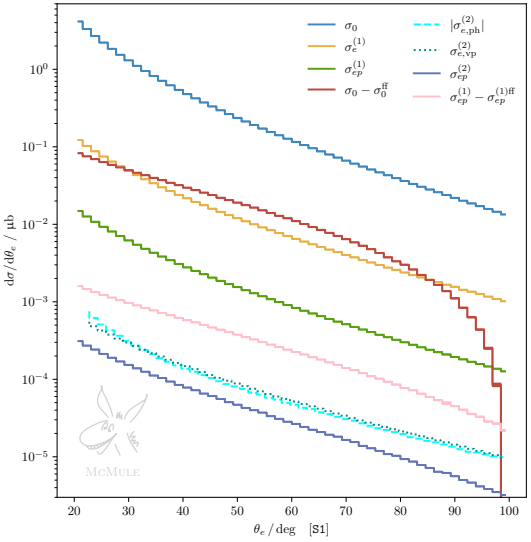

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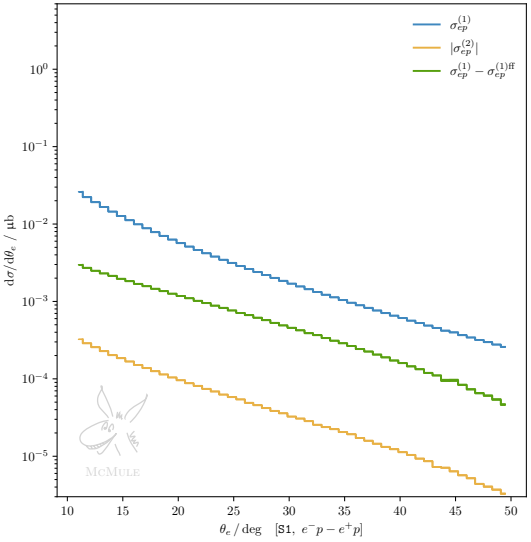

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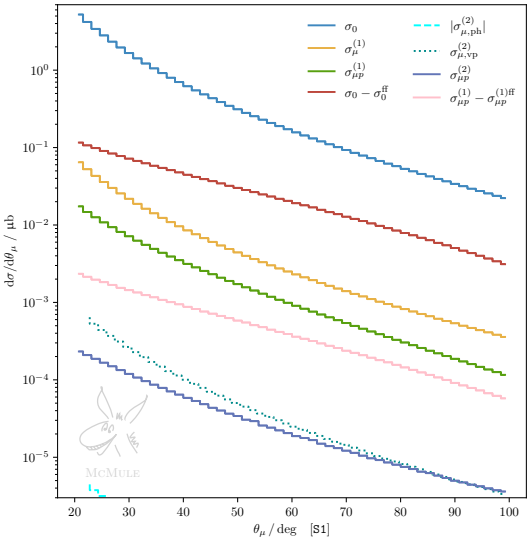
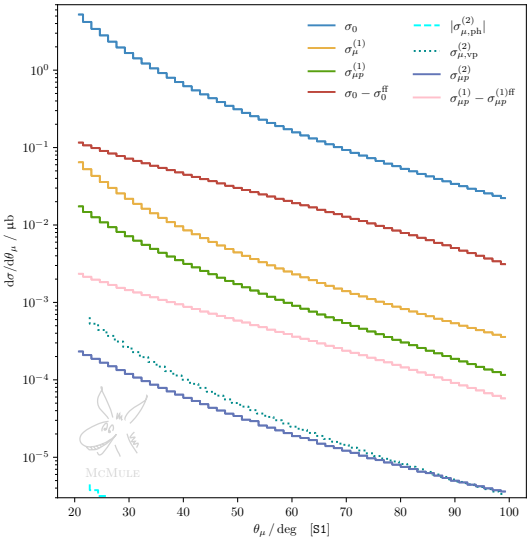


muons are available at MUSE



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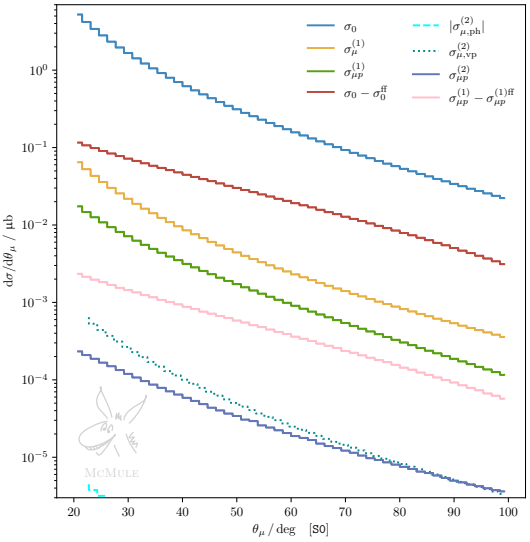

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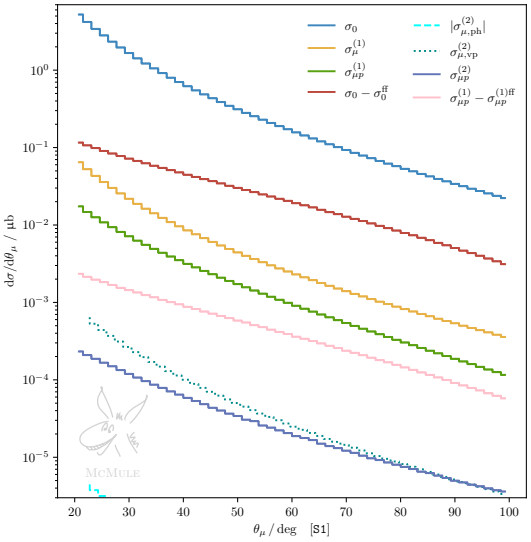

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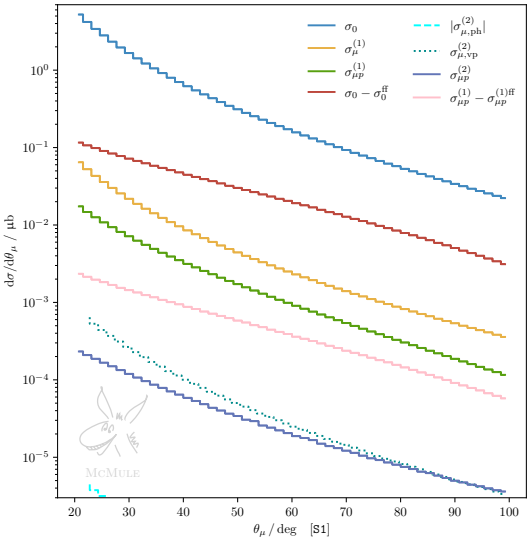

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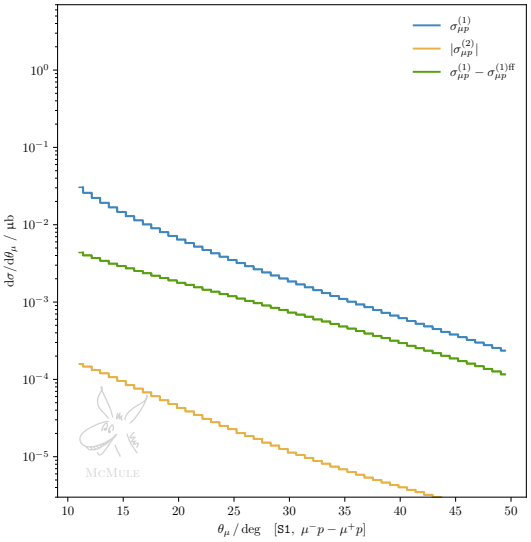

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