Towards efficient N3LO predictions:
Power Corrections for 0-jettiness Subtractions at N3LO

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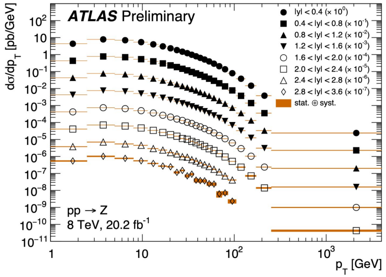
Based on:

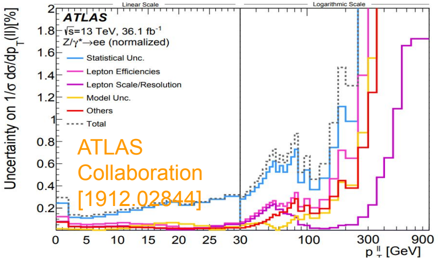
"N3LO Power Corrections for 0-jettiness Subtractions With Fiducial Cuts"
GV
[to appear]

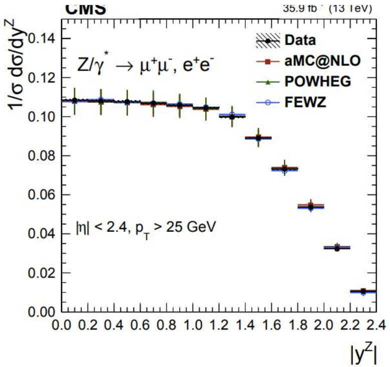
Testing the SM at Percent Level Accuracy

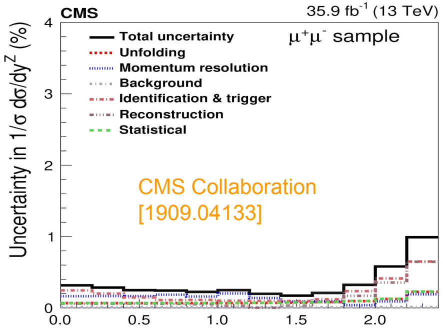
Astonishing level of precision in experimental measurements of key benchmark processes.

Example: normalized differential distributions in Drell-Yan measured with few **per-mille** level accuracy



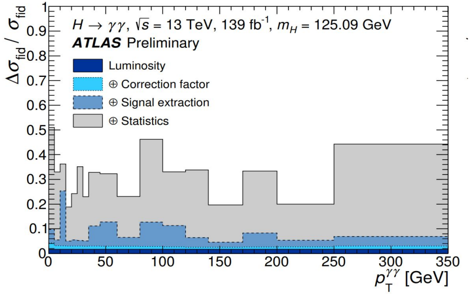




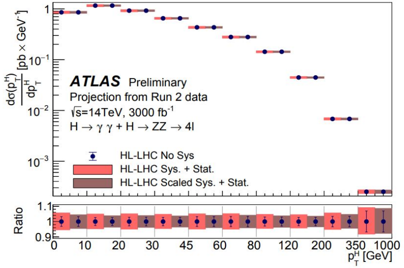


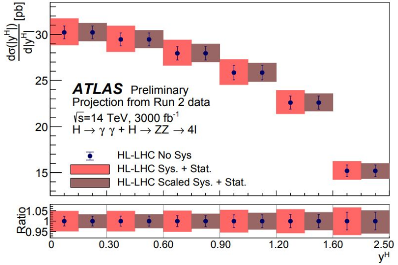
...and plethora of very precise differential distributions from LEP, future EIC measurements, possible future colliders, etc...

Higgs measurements at the moment are limited by statistics

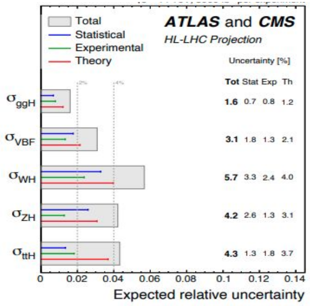


...but statistics **will improve dramatically** with HL LHC...





With **percent level measurement** of Higgs distributions, theory errors are projected to be a major limiting factor for Higgs precision program



Standard Model Phenomenology at percent level

We should aim at comparable precision from the theory side!

$$\hat{\sigma}_{ab\to X} = \underbrace{\sigma_0}_{\text{LO}} + \underbrace{\alpha_s\sigma_1}_{\text{NLO}} + \underbrace{\alpha_s^2\sigma_2}_{\text{NNLO}} + \underbrace{\alpha_s^3\sigma_3}_{\text{N}^3\text{LO}} + \ldots$$

N3LO corrections (or at least *good* estimates of them) will be **necessary for percent level** phenomenology

CAVEAT!

Often times convergence turns out to be slower than naive estimate

=> **N3LO gives few <u>percent</u> (not per-mille) shift**

	Q [GeV]	$\delta\sigma^{\text{N}^3\text{LO}}$	$\delta\sigma^{\text{NNLO}}$
$gg \to$ Higgs	m_H	3.5%	30%
$b\bar{b} \to$ Higgs	m_H	-2.3%	2.1%
NCDY	30	-4.8%	−0.34%
	100	-2.1%	−2.3%
CCDY(W^+)	30	-4.7%	−0.1%
	150	-2.0%	−0.1%
CCDY(W^-)	30	-5.0%	−0.1%
	150	-2.1%	−0.6%

n3loxs [Baglio, Duhr, Mistlberger, Szafron '22]

"The Path Forward to N3LO"
Snowmass Whitepaper
[Caola, Chen, Duhr, Liu, Mistlberger, Petriello, GV, Weinzierl]

Predictions for Differential Cross Sections at higher order

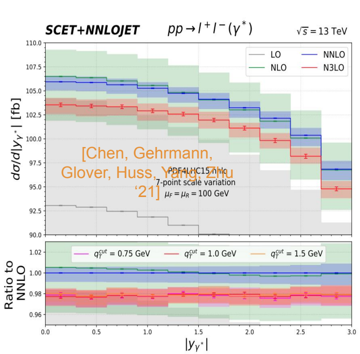
- Cross sections require **integration over phase space**

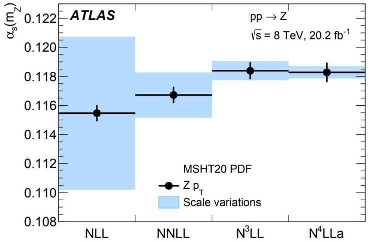
$$\sigma = f_1 \circ f_2 \circ \int d\Phi |M|^2$$

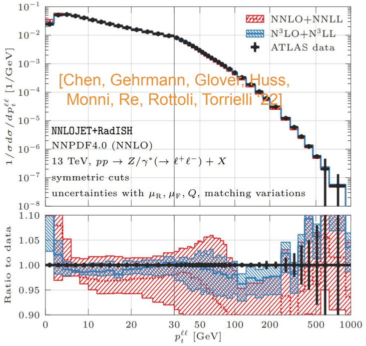
- Complexity of infrared singularities grows with loop order
- Extremely challenging to systematize their treatment order by order
- Use EFT methods to systematize study of collinear and soft radiation at the cross section level
- Used to derived first results at N3LO using q_T subtraction

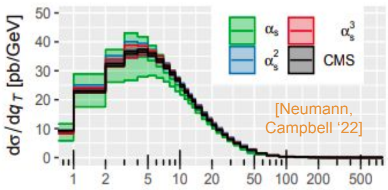
Precision Standard Model Phenomenology at N3LO

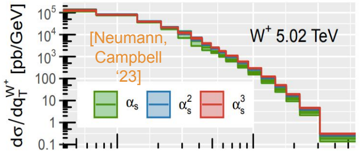
- **N3LO TMDPDF** were **last missing ingredient** for q_T slicing at N3LO
- Enabled N3LO predictions for differential and fiducial Drell-Yan and Higgs production
- Marked the advent of a new level of accuracy for the precision program at the LHC

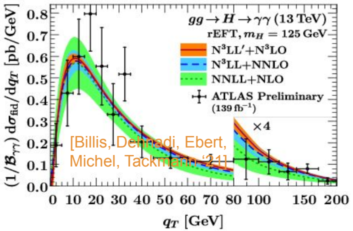


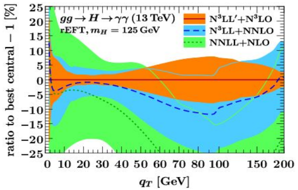












And many more:
[Ju, Schönherr '21]
[Camarda, Cieri, Ferrera '21]
[Re, Rottoli, Torrielli '21]
…

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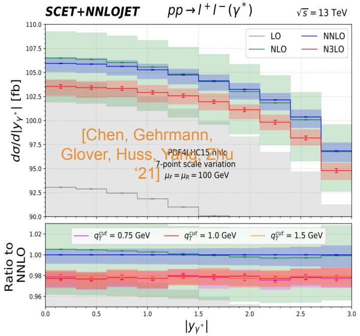

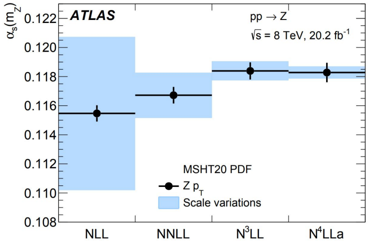

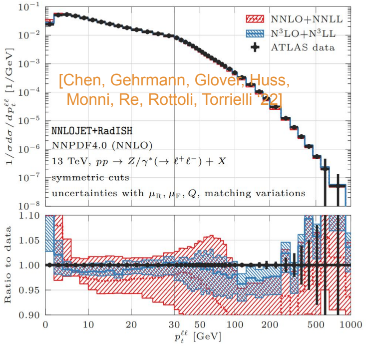

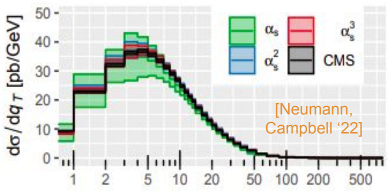

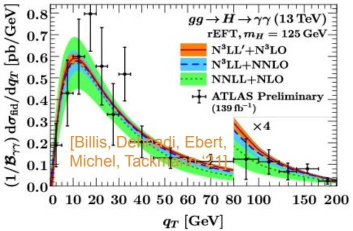

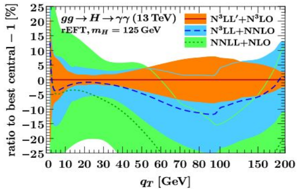

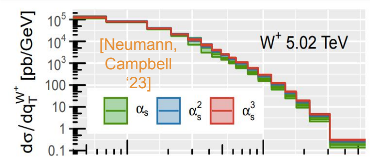

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However…

- Numerical (slicing) error of these methods very difficult to control at this order
- Extreme push of NNLO+j predictions well into the IR needed (NNLOjet pushed to q_T = 0.5 GeV)
- Calculations take **O(10 million) CPU hours**
- Almost any change will require to run everything from scratch
- Other results use O(100k) CPU hours and stop at 5 GeV … this requires wild extrapolation to 0 to obtain finite results.
- Going forward, these facts pose issues for the practical usability of these predictions

In short, starting to think about how to move from

making N3LO predictions **possible**,

to

making N3LO predictions (more) **efficient**, **stable**, and **usable**

(at least for some color singlet processes…which may also turn out to be a necessary stepping stone to make other processes possible at N3LO)

Differential color singlet production at N3LO

- To do this, let's start by looking at how to get N3LO predictions for color singlet

Projection to Born

[Cacciari,Dreyer,Karlberg,Salam,Zanderighi '15]

$$\frac{\mathrm{d}\sigma_F^{\mathrm{N}^k\mathrm{LO}}}{\mathrm{d}\mathcal{O}} = \left(\frac{\mathrm{d}\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{(k-1)}\mathrm{LO}}}{\mathrm{d}\mathcal{O}} - \frac{\mathrm{d}\sigma_{F+\mathrm{jet}}^{\mathrm{N}^{(k-1)}\mathrm{LO}}}{\mathrm{d}\widetilde{\mathcal{O}}}\right) + \frac{\mathrm{d}\sigma_F^{\mathrm{N}^k\mathrm{LO}}}{\mathrm{d}\widetilde{\mathcal{O}}}$$

Locally subtracted real emissions — **Integrated counterterm**

- **PRO:** Counterterm is the full ME
  => Great numerical efficiency
- **Cons:** Integrated counterterm is very hard to obtain (analytic differential distribution at N3LO in full kinematics)

q_T or 0-jettiness subtraction

q_T Subtraction: [Catani, Grazzini '07]

N-Jettiness: [Boughezal, Focke, Liu, Petriello '15] [Gaunt, Stahlhofen, Tackmann, Walsh '15]

$$\sigma(X) = \int_0^{q_{T\,\mathrm{cut}}} \mathrm{d}q_T \frac{\mathrm{d}\sigma^{\mathrm{sing}}(X)}{\mathrm{d}q_T} + \int_{q_{T\,\mathrm{cut}}} \mathrm{d}q_T \frac{\mathrm{d}\sigma(X)}{\mathrm{d}q_T} + \Delta\sigma(X, q_{T\,\mathrm{cut}})$$

Below the cut region — **Above the cut region** — **Residual**

- **PRO:** analytic ingredients from EFT at leading power
- **Cons:** numerically challenging

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Below the cut region — **Above the cut region** — **Residual**

- **PRO:** analy... EFT at leading...

- **Cons:** n... challenging

Today I will focus on this

Improving non-local subtraction methods: General Setup

$$\sigma = \int_0^{\tau_{\rm cut}} d\tau \frac{d\sigma}{d\tau} + \int_{\tau_{\rm cut}}^{\tau_{\max}} d\tau \frac{d\sigma}{d\tau}$$
$$= \int_0^{\tau_{\rm cut}} d\tau \left[\frac{d\sigma^{(0)}}{d\tau} + \sum_{i>0} \frac{d\sigma^{(i)}}{d\tau}\right] + \int_{\tau_{\rm cut}}^{\tau_{\max}} d\tau \frac{d\sigma}{d\tau}$$
$$= \sigma_{\rm sub}(\tau_{\rm cut}) + \Delta\sigma(\tau_{\rm cut}) + \int_{\tau_{\rm cut}}^{\tau_{\max}} d\tau \frac{d\sigma}{d\tau} .$$

Above the cut result

- Resolved extra radiation
- No events in Born configuration
- Lower number of loops
- Calculate numerically with lower order subtraction schemes

Below the cut region:

- Singular distribution
- Contains most complicated cancellation of IR div.
- Control it analytically via factorization theorems

Residual/slicing error:

- Non singular terms from below the cut
- Reducing this requires pushing cut parameter to very small values
- Can be improved analytically by calculation next to leading power distribution

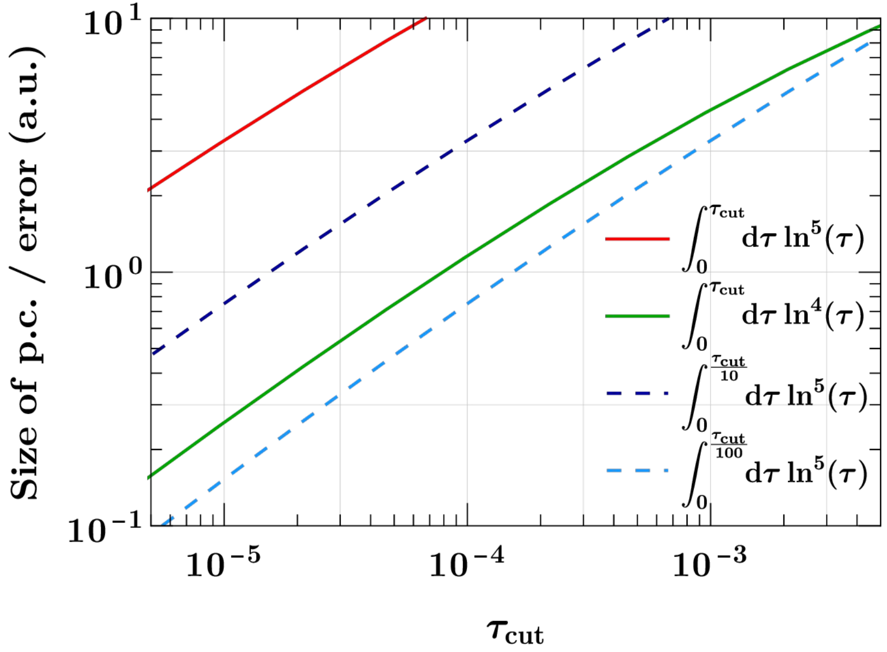
$$\sigma_{\rm sub}(\tau_{\rm cut}) = \int_0^{\tau_{\rm cut}} d\tau \frac{d\sigma^{\rm sub}}{d\tau} ,$$

$$\Delta\sigma(\tau_{\rm cut}) = \int_0^{\tau_{\rm cut}} d\tau \left[\frac{d\sigma}{d\tau} - \frac{d\sigma^{\rm sub}}{d\tau}\right]$$

Improving non-local subtraction methods: Power corrections

$$\Delta\sigma(\tau_{\rm cut}) = \int_0^{\tau_{\rm cut}} \mathrm{d}\tau \left[\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} - \frac{\mathrm{d}\sigma^{\rm sub}}{\mathrm{d}\tau}\right]$$

- At N3LO power corrections start with **5th power of log**
- Taking $\tau_{\rm cut}$ small reduces single power, but increases size of log => very slow convergence
- Each order in the log equivalent to ~ a 50 fold reduction in $\tau_{\rm cut}$



$$\Delta\sigma^{N3LO}(\tau_{\rm cut}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\rm cut}} \mathrm{d}\tau \left(c_{3,5}^{\rm NLP}\ln^5\tau + c_{3,4}^{\rm NLP}\ln^4\tau + c_{3,3}^{\rm NLP}\ln^3\tau + \dots\right)$$

A word on linear vs quadratic power corrections

$$\text{0-jettiness}: \quad \Delta\sigma^{N3LO}(\tau_{\rm cut}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\rm cut}} d\tau \left(c_{3,5}^{\rm NLP} \ln^5 \tau + c_{3,4}^{\rm NLP} \ln^4 \tau + c_{3,3}^{\rm NLP} \ln^3 \tau + \dots\right)$$

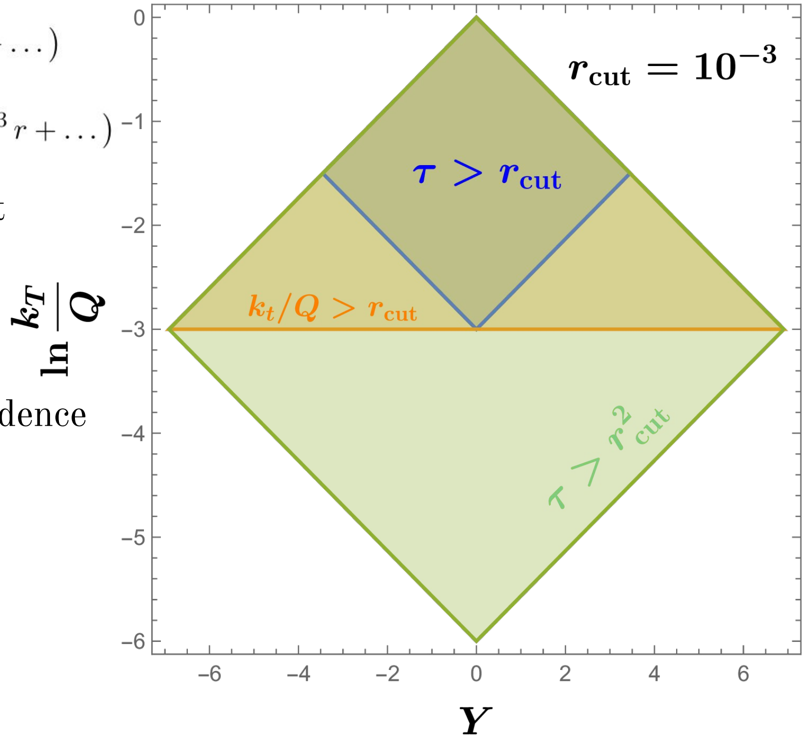
$$q_T: \quad \Delta\sigma^{N3LO}(q_{T\rm cut}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{q^2_{T\rm cut}/Q^2} dr \left(d_{3,5}^{\rm NLP} \ln^5 r + d_{3,4}^{\rm NLP} \ln^4 r + d_{3,3}^{\rm NLP} \ln^3 r + \dots\right)$$

- Scaling in q_T of the slicing param. may lead to the impression that q_T subtraction has *quadratic* power corrections, while jettiness has *linear* power corrections.

- But it all comes down to how one decides to treat the angle dependence

$$\tau = \frac{q_T}{Q} e^{-|Y|} \sim \begin{cases} \frac{q_T}{Q} & \text{soft emissions} \\ \frac{q_T^2}{Q^2} & \text{collinear emissions} \end{cases}$$

- In practice, key point is what is more challenging numerically for the above the cut code:
  - 0-jettiness: better suppression of collinear emissions
  - q_T : better suppression of wide angle soft emissions



Note: fiducial p.c. generating *linear* terms in q_T, go as $\sqrt{\tau_{\rm cut}}$ in the case of 0-jettiness

Improving non-local subtraction methods: Power corrections

$$\Delta\sigma(\tau_{\mathrm{cut}}) = \int_0^{\tau_{\mathrm{cut}}} \mathrm{d}\tau \left[\frac{\mathrm{d}\sigma}{\mathrm{d}\tau} - \frac{\mathrm{d}\sigma^{\mathrm{sub}}}{\mathrm{d}\tau}\right]$$

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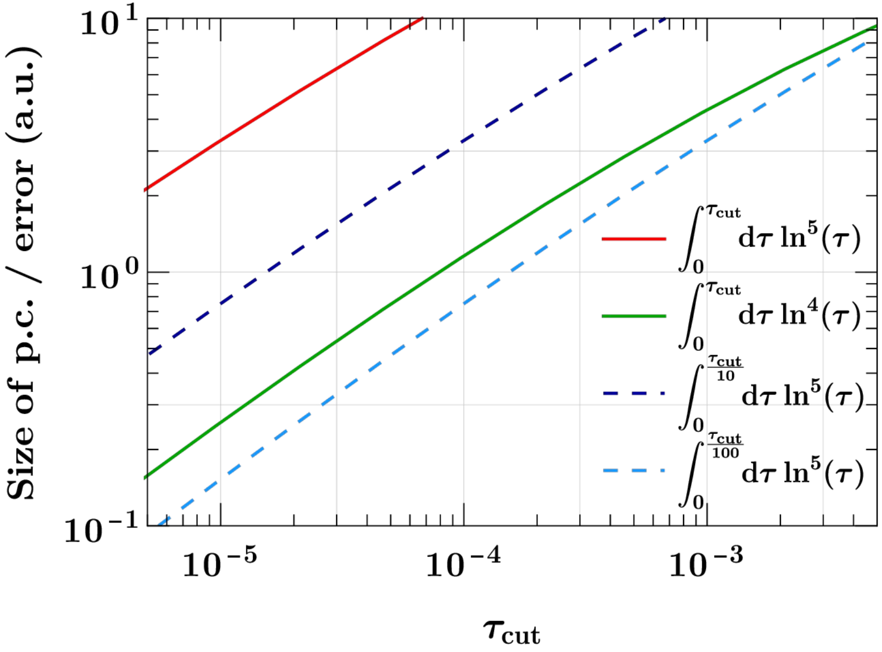

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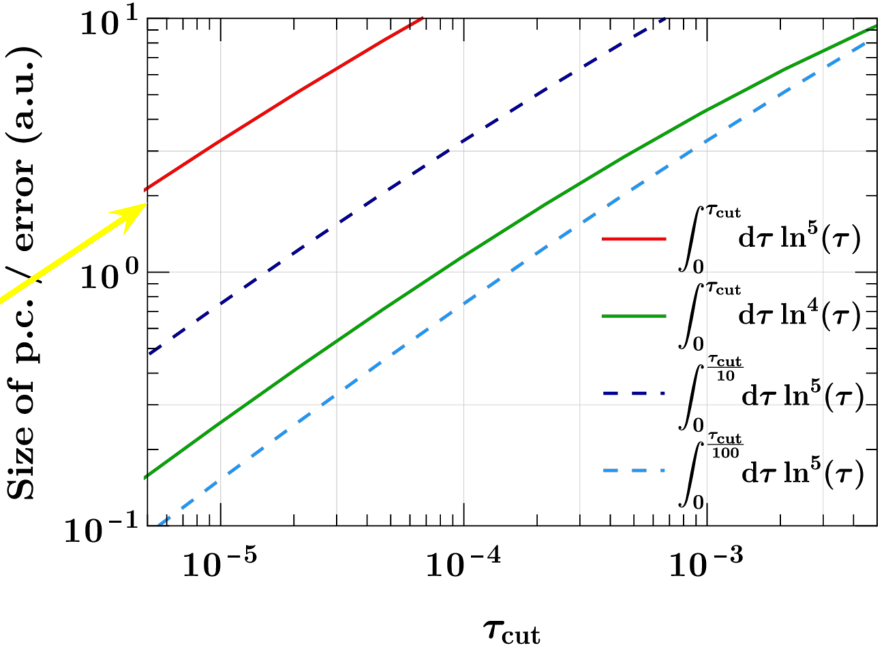
- At ... po...
- Tak... incre... ence
- Each ... to ~ a 50 fold reduct...

Very straightforward way of improving slicing:

Obtain the leading logarithmic term at NLP analytically



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0-Jettiness Power Corrections at N3LO

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- For 0-jettiness, use consistency relations to relate full LL to RVV correction in collinear limit.
  [Moult, Rothen, Stewart, Tackmann, Zhu '16] [Moult, Stewart, **GV**, Zhu '19]

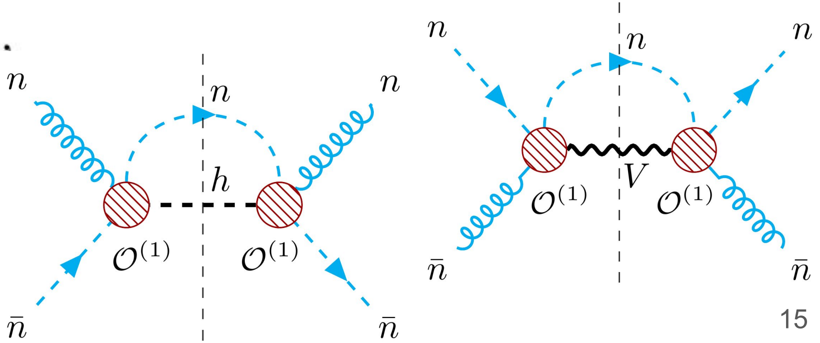
- Focus on Drell-Yan and Higgs production. Single collinear emission fully differential in rapidity:

$$\frac{d\sigma_n^{\rm NLP}}{dQ^2 dY d\mathcal{T}} \sim \int_{x_a}^1 \frac{dz_a}{z_a} \frac{(Q^2\tau)^{-\epsilon}}{(1-z_a)^\epsilon} \Big\{ \underbrace{\tau A^{(0)}(\tau, z_a, \epsilon)}_{\text{LP Matrix Element}} \underbrace{\left[-f_a\left(\frac{x_a}{z_a}\right) f_b(x_b) + f_a\left(\frac{x_a}{z_a}\right) x_b f_b'(x_b) \right]}_{\text{NLP Phase Space}}$$

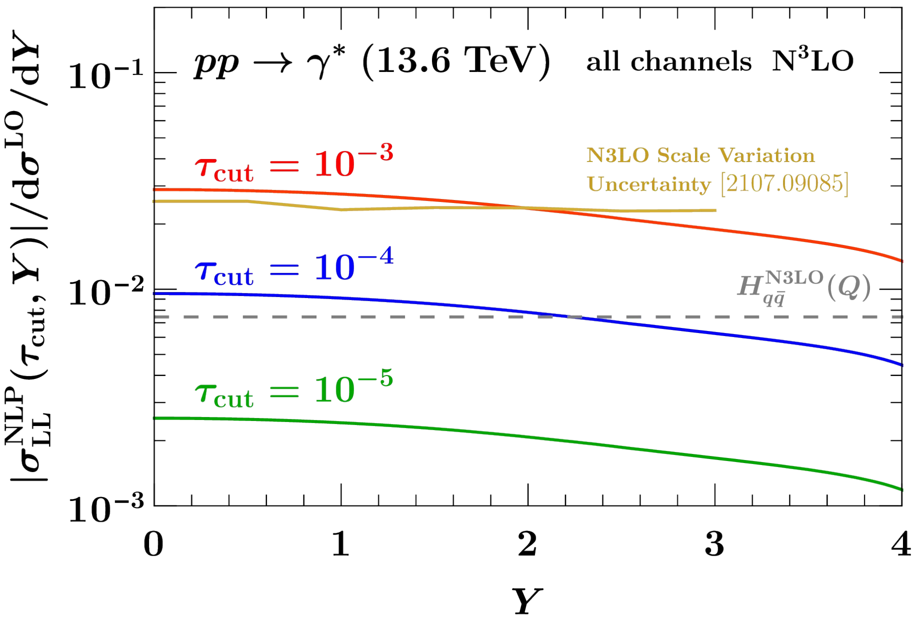
$$+ \underbrace{f_a\left(\frac{x_a}{z_a}\right) f_b(x_b)}_{\text{LP Phase Space}} \underbrace{A^{(2)}(\tau, z_a, \epsilon)}_{\text{NLP Matrix Element}} \Big\} .$$

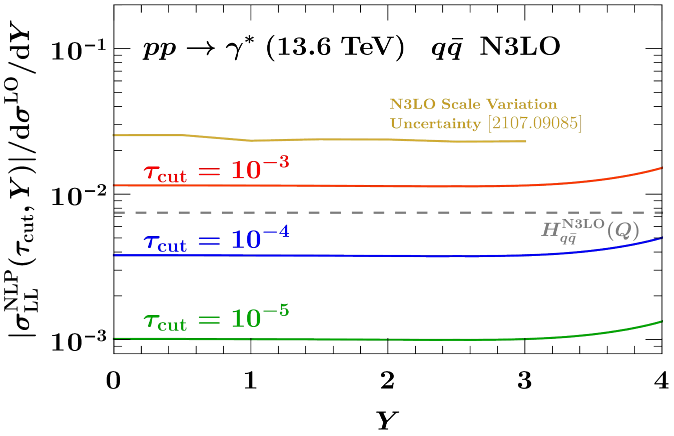
[Ebert, Moult, Stewart, Tackmann, **GV**, Zhu '18]

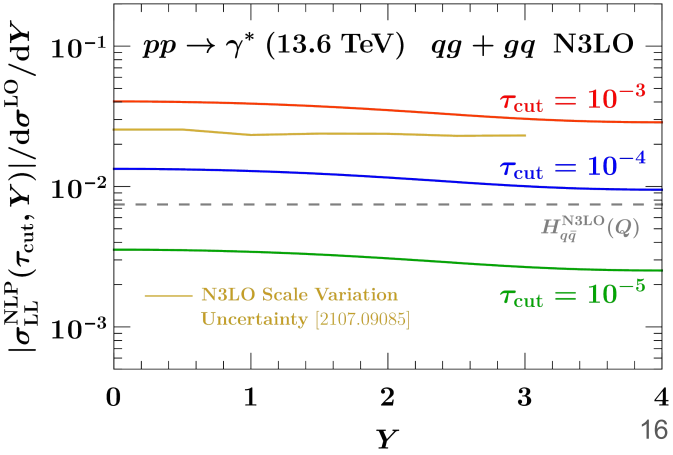
- LL contributions also from off-diagonal $qg + gq$ channels via subleading power hard scattering operators and Lagrangian insertions



0-Jettiness Power Corrections at N3LO: Results for DY





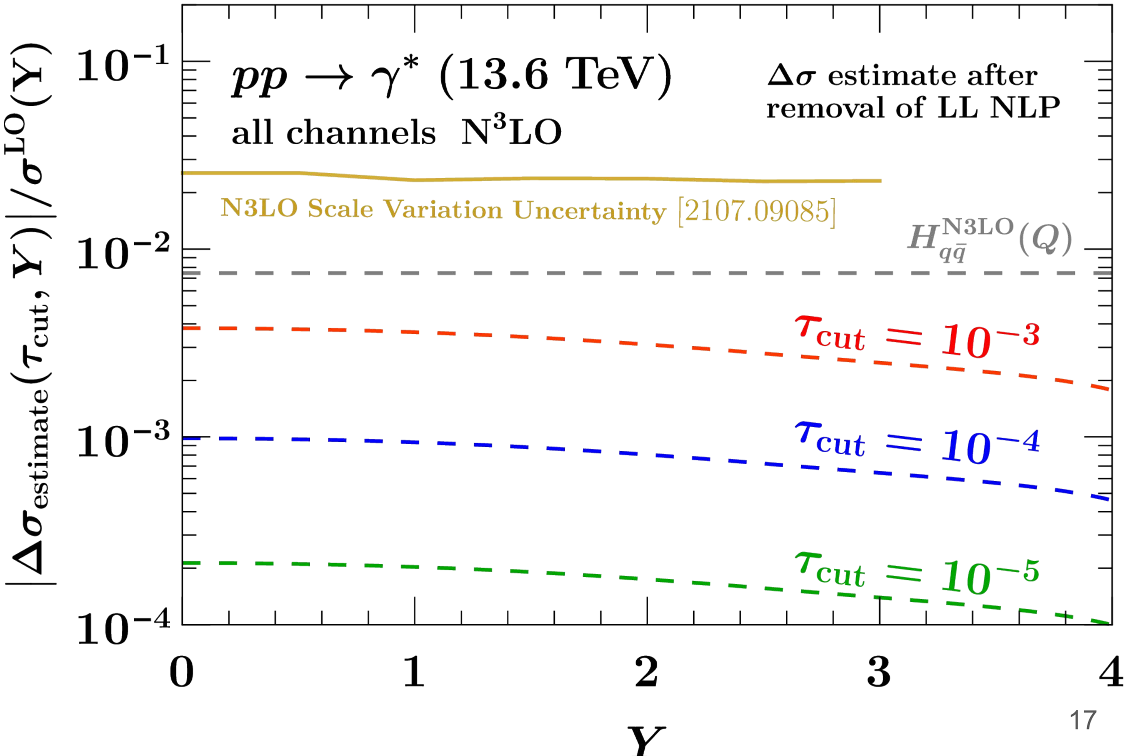


- By the size of LL NLP: 0-jettiness with standard setup (only LP in subtraction term) would require $\tau_{\rm cut} \sim 10^{-5}$ or even smaller.
- Off-diagonal channel has large power corrections (in line with empirical observation in q_T slicing at N3LO)

0-Jettiness P.C. at N3LO: Estimate of residual error for DY

$$\Delta\sigma^{N3LO}(\tau_{\text{cut}}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\text{cut}}} d\tau \left(c_{3,5}^{\text{NLP}} \ln^5 \tau + c_{3,4}^{\text{NLP}} \ln^4 \tau + c_{3,3}^{\text{NLP}} \ln^3 \tau + \ldots \right)$$

- Leading logarithm calculated
  Can remove it from error and add to analytic subtraction term
- Estimate residual slicing error removing LL NLP
- Assume same size as LL coefficient
  (in line with what seen at previous orders)
  for subleading logs and powers
- Slicing error significantly reduced.
  O(x50) larger cut allowed.
- May save millions of CPU hours and allow for better convergence studies



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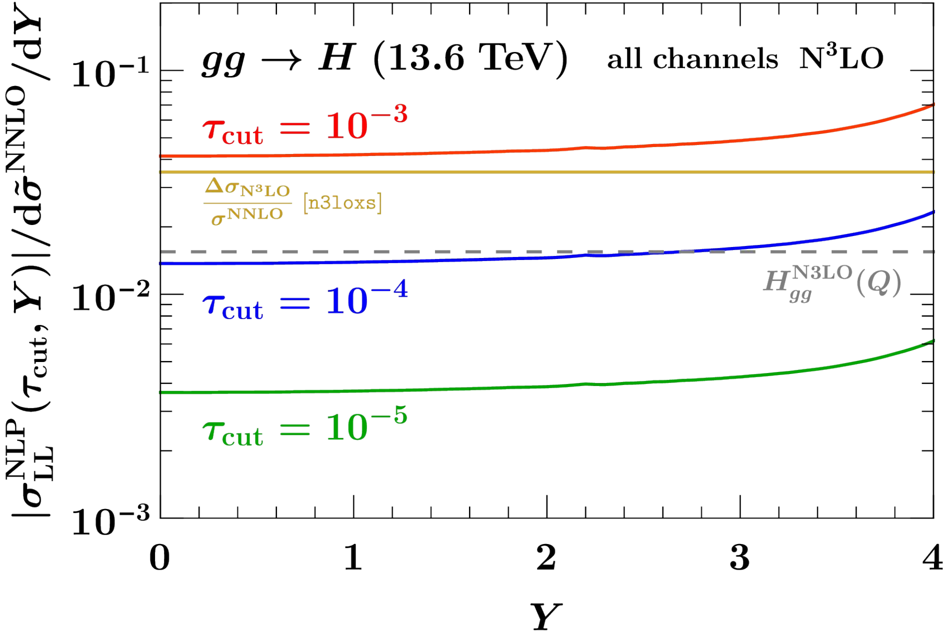
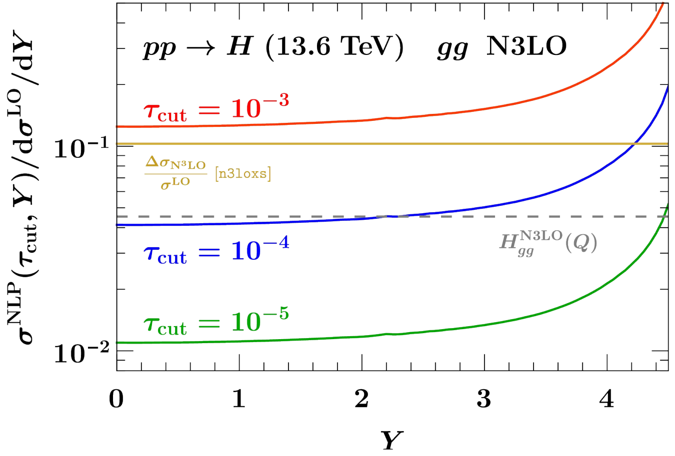
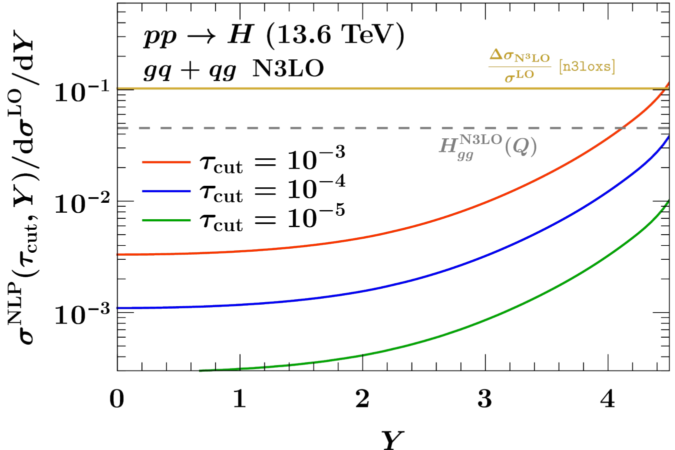




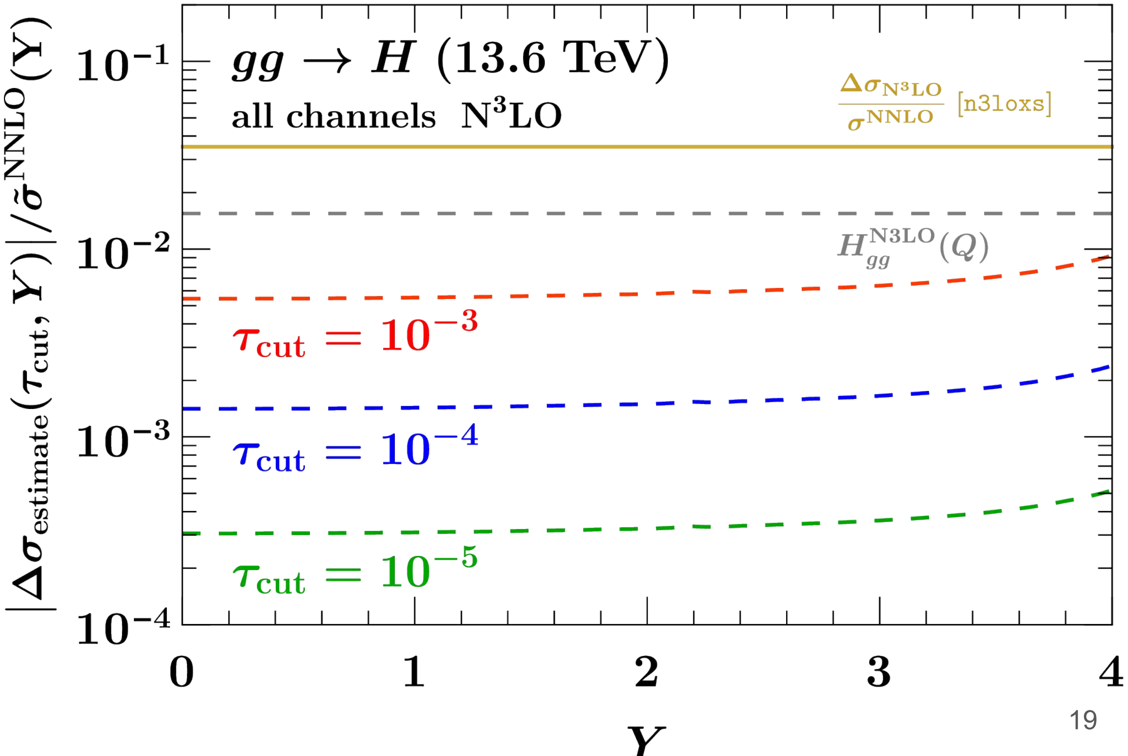


- Similar story for the case of Higgs production in gluon fusion
- Here the off-diagonal channel is negligible, as it is often the case with the Higgs

0-Jettiness P.C. at N3LO: Estimate of residual error for Higgs

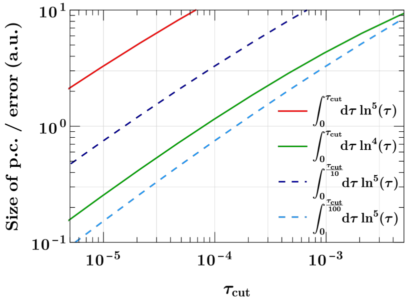
$$\Delta\sigma^{N3LO}(\tau_{\rm cut}) \sim \left(\frac{\alpha_s}{4\pi}\right)^3 \int_0^{\tau_{\rm cut}} d\tau \left(c_{3,5}^{\rm NLP} \ln^5 \tau + c_{3,4}^{\rm NLP} \ln^4 \tau + c_{3,3}^{\rm NLP} \ln^3 \tau + \dots\right)$$

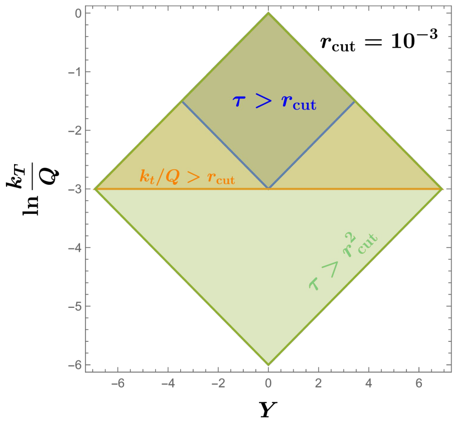
- Play the same game for estimating residual slicing error after the inclusion of LL NLP in the subtraction term:
- Assume same size as LL coefficient (in line with what seen at previous orders) for subleading logs and powers
- Slicing error significantly reduced. O(x50) larger cut allowed.
- May become not so far from running time of P2B method



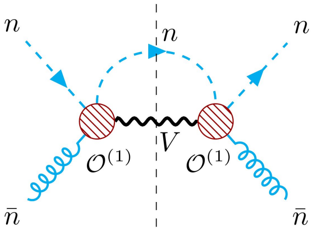
Conclusion

- Discussed challenges of N3LO calculations and slicing methods

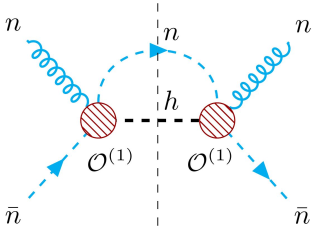


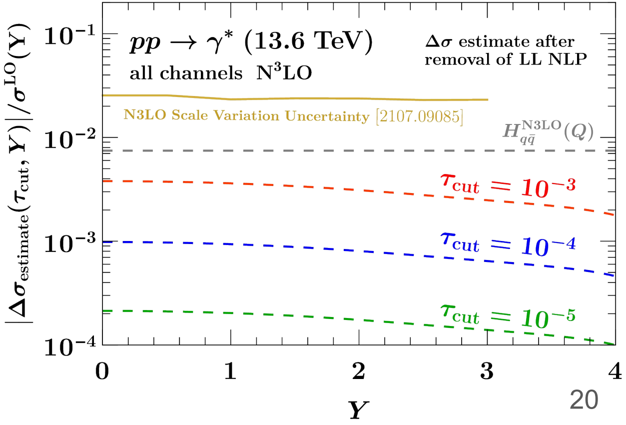


- Presented the calculation of the LL NLP term at N3LO for 0-jettiness fully differential in the Born kinematics



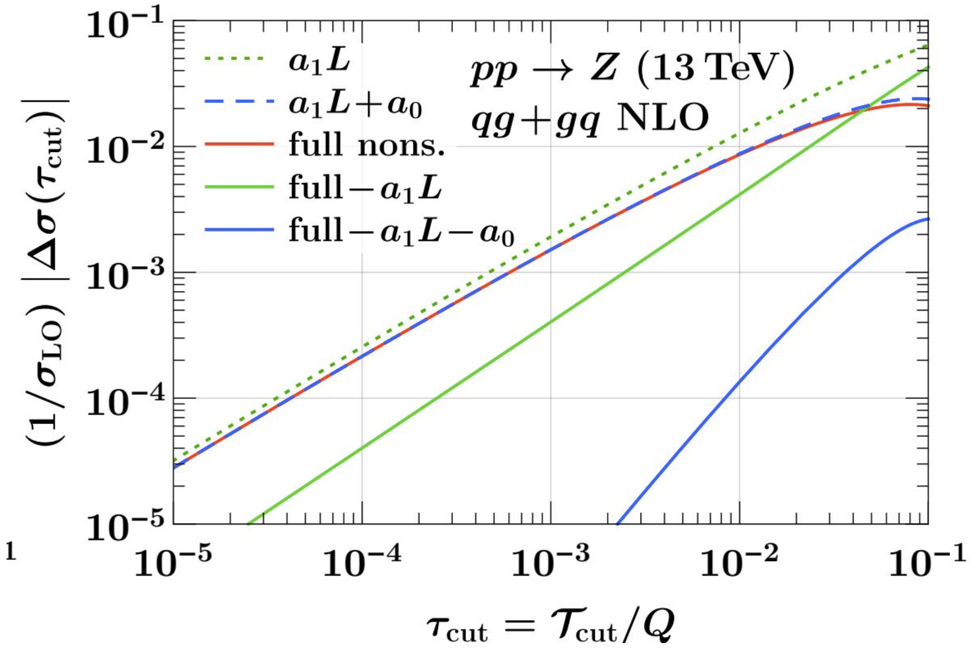
- Illustrated impact on slicing error for Drell-Yan and Higgs production

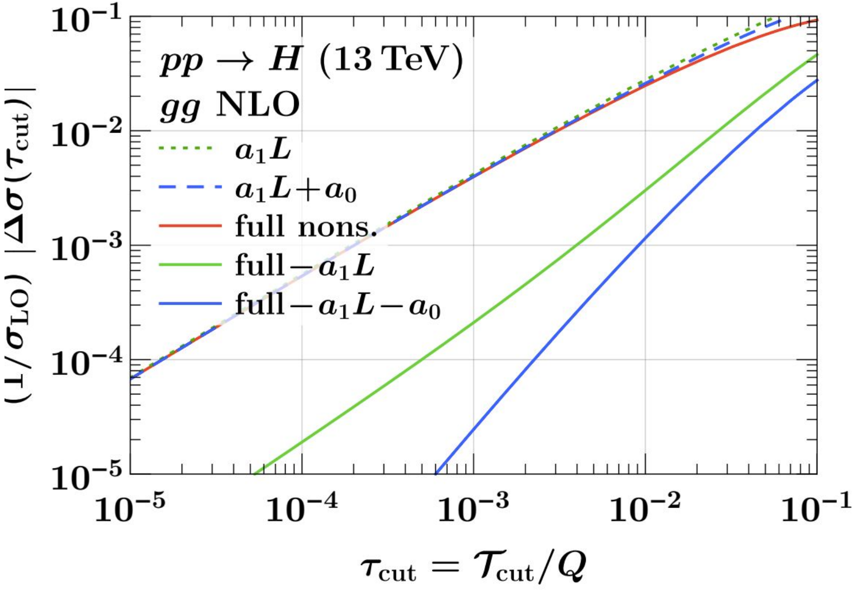




Backup

Log behaviour at NLP NLO

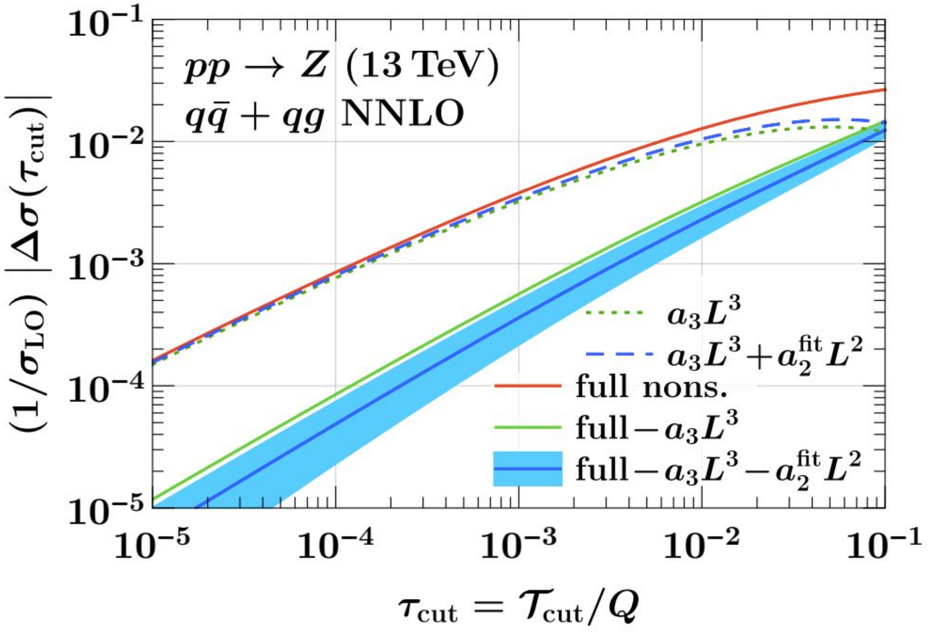




[1807.10764]

Log behaviour at NLP NNLO

[1612.00450]



[1710.03227]