Diagrammatic Analysis of Charmless Three-Body B Decays

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Abstract

The diagrammatic method has been used in the past to extract, among other things the weak phase γ using observables in $B \to K\pi$ decays. We generalize the diagrammatic decomposition method to more-than-two-body decays, and systematically apply this method to the charmless decays of B mesons to three pseudoscalars. The method of contractions is used to demonstrate the existence of relations between tree and electroweak-penguin diagrams in $B \to PPP$ decays. Together with the use of diagrammatic decomposition and Dalitz-plot analyses, these relations can be used to cleanly extract the weak phase Gamma from $B \to K\pi\pi$ decays.

Accounting for Symmetry

- Isospin (SU(3)) symmetry requires treating pions (and kaons) as identical particles
- Bose-Einstein symmetry requires symmetrical wave functions
- To account for this, diagrams will come in **different versions**
- Each version corresponds to a representation of permutation group S2 (for isospin) or S3 (for SU(3))
- Comparing predictions to experiment will thus require extracting the components of amplitudes with the correct symmetries
- **This can be done with Dalitz-plot amplitude analyses**

Applying Diagrams to $B \to PPP$

The dominant contribution to $B \to PPP$ decays comes from the following diagrams :

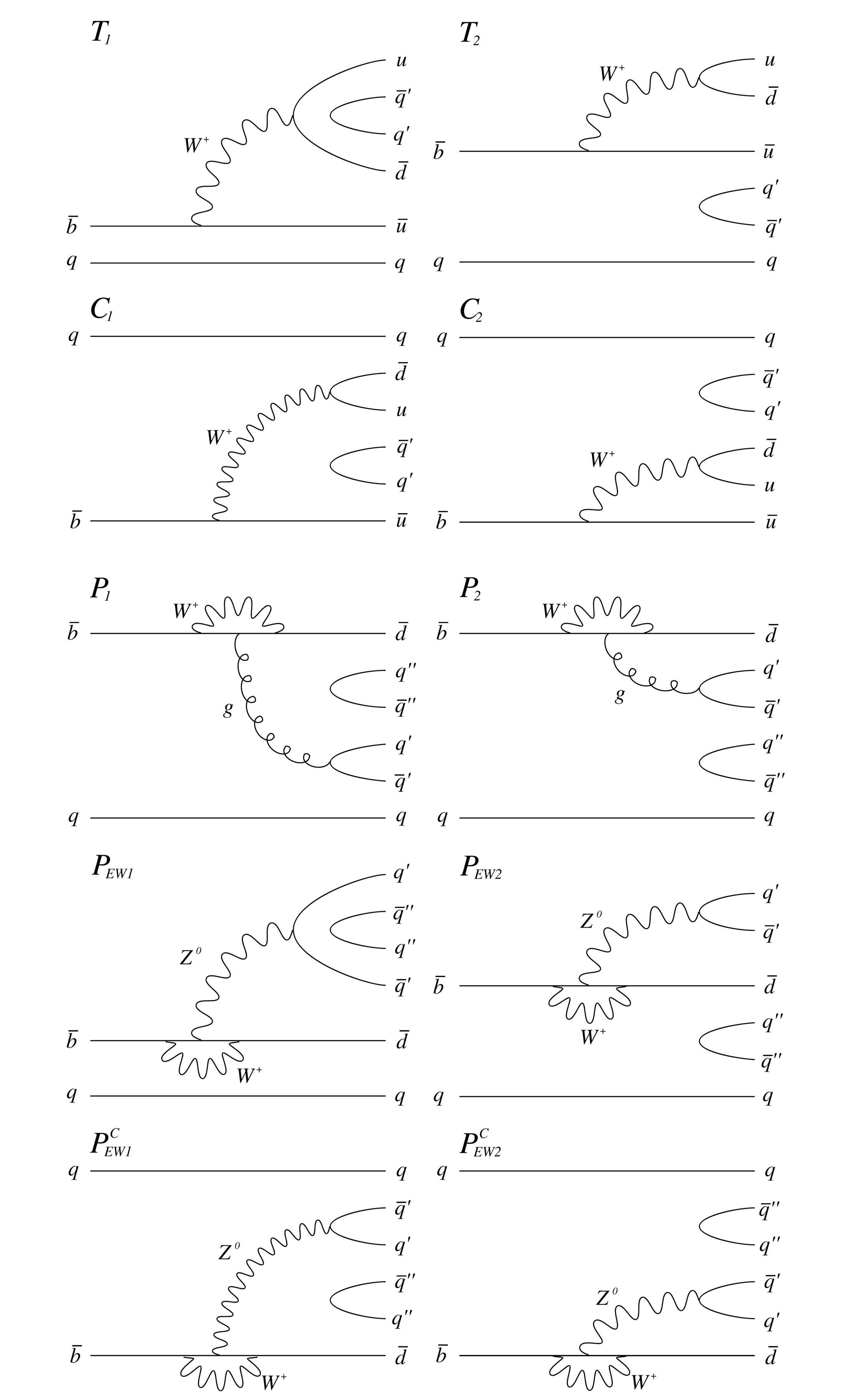


FIGURE: Diagrams for a $B \to \pi\pi\pi$ decay

- Other diagrams actively involving the spectator quark are expected to be suppressed by a factor of f_B/m_B
- Isospin symmetry establishes the equivalence of diagrams within a given type of decay. The different types are :
  - $B \to K\pi\pi$
  - $B \to KK\bar{K}$
  - $B \to K\bar{K}\pi$
  - $B \to \pi\pi\pi$

An Important Example : $B \to (K\pi\pi)_{sym}$

$$
\begin{aligned}
\sqrt{2}A(B^+ \to K^0\pi^+\pi^0)_{sym} &= -T'_a e^{i\gamma} - T'_b e^{i\gamma} + P'_{EW,a} + P'_{EW,b} \ , \\
A(B^0_d \to K^0\pi^+\pi^-)_{sym} &= -T'_a e^{i\gamma} - P'_a e^{i\gamma} + P'_b \ , \\
\sqrt{2}A(B^0_d \to K^0\pi^0\pi^0)_{sym} &= -T'_b e^{i\gamma} + P'_a e^{i\gamma} - P'_b + P'_{EW,a} + P'_{EW,b} \ , \\
A(B^+ \to K^+\pi^+\pi^-)_{sym} &= -P'_a e^{i\gamma} + P'_b - P'_{EW,a} \ , \\
\sqrt{2}A(B^+ \to K^+\pi^0\pi^0)_{sym} &= T'_a e^{i\gamma} + T'_b e^{i\gamma} + P'_a e^{i\gamma} - P'_b - P'_{EW,b} \ , \\
\sqrt{2}A(B^0_d \to K^+\pi^0\pi^-)_{sym} &= T'_a e^{i\gamma} + T'_b e^{i\gamma} - P'_{EW,a} - P'_{EW,b} \ ,
\end{aligned}
$$

Extracting Symmetrical Amplitudes

- Dalitz-plots can be used to extract the amplitude components with the desired symmetries
- Isobar model : $\mathcal{M} = \sum a_j F_j(x,y)$
- a_j coefficients can be fitted to the measured Dalitz-plot
- $f(x,y)_{sym} = \frac{1}{\sqrt{2}}(f(x,y) + f(y,x))$
- Representations of S3 can also be extracted

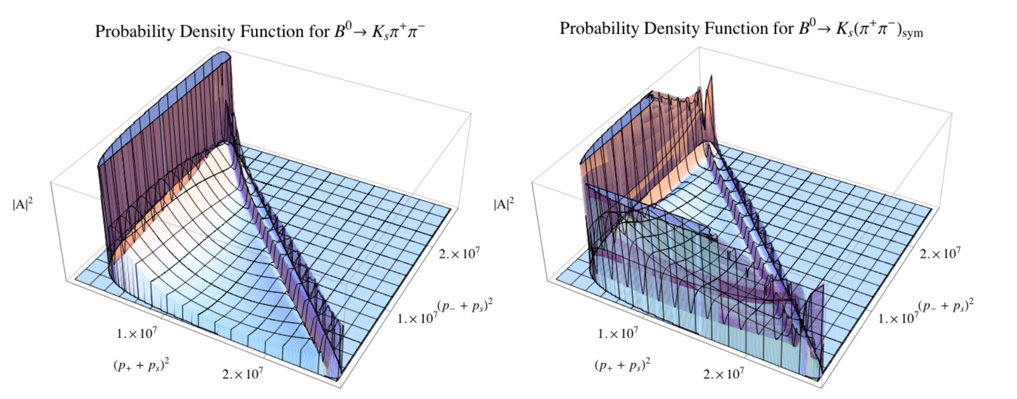


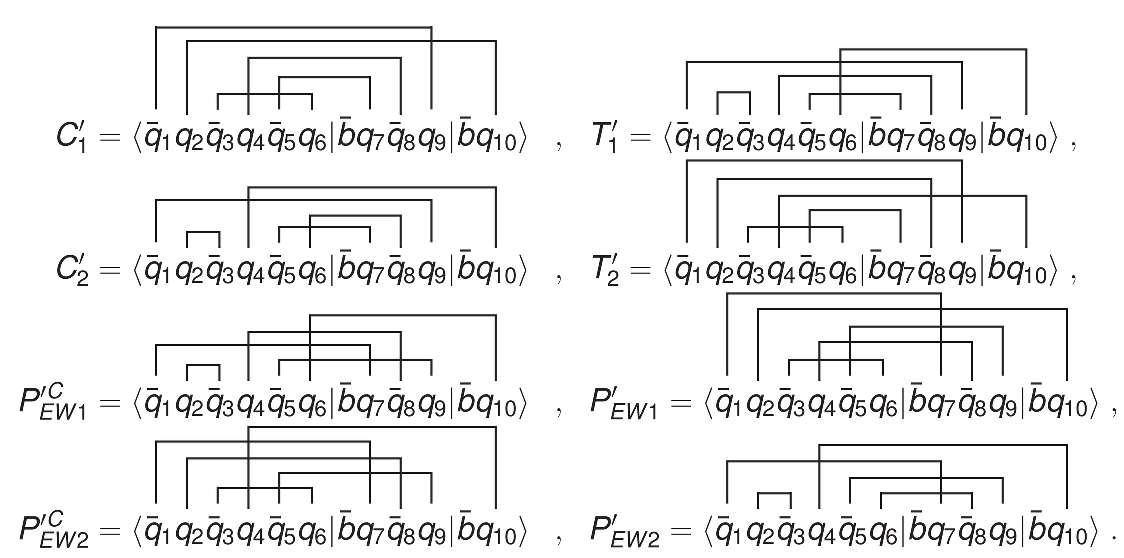
FIGURE: Probability Density Functions for $B^0_d \to K_s\pi^+\pi^-$ and for the symmetric part of the amplitude

Contractions and EWP-Tree Relations

- The effective weak hamiltonian :

$$
H_{eff} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} \left(\sum_{p=u,c} \lambda_p^{(q)} (c_1(\mu) O_1^p(\mu) + c_2(\mu) O_2^p(\mu)) - \lambda_t^{(q)} \sum_{i=3}^{10} c_i(\mu) O_i(\mu) \right)
$$

- Matrix elements of four-quark operators can be expressed in terms of contractions
- Contractions can be related to diagrams :



- Under SU(3) symmetry, this allows us to obtain relations between the **fully symmetric** diagrams :

$$
P'_{EW1} = -\frac{3}{2}\frac{|\lambda_t^{(s)}|}{|\lambda_u^{(s)}|}\frac{c_9 + c_{10}}{c_1 + c_2} T'_1 \ , \qquad P'_{EW2} = -\frac{3}{2}\frac{|\lambda_t^{(s)}|}{|\lambda_u^{(s)}|}\frac{c_9 + c_{10}}{c_1 + c_2} T'_2 \ ,
$$

$$
P'^C_{EW1} = -\frac{3}{2}\frac{|\lambda_t^{(s)}|}{|\lambda_u^{(s)}|}\frac{c_9 + c_{10}}{c_1 + c_2} C'_1 \ , \qquad P'^C_{EW2} = -\frac{3}{2}\frac{|\lambda_t^{(s)}|}{|\lambda_u^{(s)}|}\frac{c_9 + c_{10}}{c_1 + c_2} C'_2 \ .
$$

- SU(3)-breaking errors are $O(30\%)$, but since trees and EWPs are sub-leading effects, the net error is expected to be $O(5\%)$

Extracting γ from $B \to K\pi\pi$

- The $B \to K\pi\pi$ sector contains **11** (independent, measurable) observables :
  - $\Gamma(B^+ \to K^+\pi^+\pi^-)$, $\Gamma(B^+ \to K^+\pi^0\pi^0)$, $\Gamma(B^0 \to K^+\pi^0\pi^-)$, $\Gamma(B^0 \to K^0\pi^+\pi^-)$, $\Gamma(B^0 \to K^0\pi^0\pi^0)$
  - The five direct CP asymmetries
  - The indirect CP asymmetry of $B^0 \to K^0\pi^+\pi^-$
- Using the fully symmetric amplitudes and the EWP-tree relations, the diagrammatic decomposition gives **10** theoretical parameters
- γ **can thus be extracted** by fitting the parameters to the observables

New Relations Between Amplitudes

- Neglecting annihilation and exchange diagrams reveals new relations between amplitudes
- These relations are invisible to the exact Wigner-Eckart analysis
- These are further tests of the SM which can be made once the symmetric and antisymmetric scenarios are experimentally distinguished

References

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- M. Imbeault, N. Rey-Le Lorier and D. London, arXiv :1011.4973
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