

CP violation in $D - \bar{D}$ mixing: Standard Model versus Current Bounds

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Introduction

CPV in charm provides a unique probe of New Physics (NP)

- sensitive to NP in the up sector
- SM charm physics is CP conserving to first approximation (2 generation dominance)

Nevertheless, the statement "any signal for CPV would be NP" needs sharpening due to continuing improvement in experimental bounds:

- In the SM, CPV in mixing enters at $O(V_{cb}V_{ub}/V_{cs}V_{us}) \sim 10^{-3}$
 - how large can SM indirect CPV really be?
- In the SM, direct CPV enters at $O([V_{cb}V_{ub}/V_{cs}V_{us}] \alpha_s/\pi) \sim 10^{-4}$ in singly Cabibbo suppressed decays (SCS)
 - how large can SM direct CPV really be?

A bit more detail on CPV in mixing (more later):

- transition amplitudes between the strong interaction meson eigenstates \bar{D}^0, D^0

$$\langle D^0 | H | \bar{D}^0 \rangle = M_{12} - \frac{i}{2} \Gamma_{12}, \quad \langle \bar{D}^0 | H | D^0 \rangle = M_{12}^* - \frac{i}{2} \Gamma_{12}^*$$

- The mixing parameters [$\Gamma = (\Gamma_1 + \Gamma_2)/2 = \text{average decay width}$]

$$x_{12} \equiv 2|M_{12}|/\Gamma, \quad y_{12} \equiv |\Gamma_{12}|/\Gamma, \quad \phi_{12} \equiv \arg(M_{12}/\Gamma_{12})$$

ϕ_{12} is a CP violating weak phase, responsible for CP violation in mixing

- Relations to CP conserving observables $|\Delta m|, |\Delta\Gamma|$:

$$|x| \equiv \frac{|\Delta m|}{\Gamma} = x_{12} [1 + O(\sin^2 \phi_{12})], \quad |y| \equiv \frac{|\Delta\Gamma|}{2\Gamma} = y_{12} [1 + O(\sin^2 \phi_{12})]$$

- relation to CP violation in pure mixing (CPVMIX): semileptonic CP asymmetry

$$a_{\text{SL}} \equiv \frac{\Gamma(D^0(t) \rightarrow \ell^- X) - \Gamma(\overline{D}^0(t) \rightarrow \ell^+ X)}{\Gamma(D^0(t) \rightarrow \ell^- X) + \Gamma(\overline{D}^0(t) \rightarrow \ell^+ X)} = \frac{2 x_{12} y_{12} \sin \phi_{12}}{x_{12}^2 + y_{12}^2} [1 + O(\sin \phi_{12})]$$

- CP violation in the interference of decays with and without mixing (CPVINT): time-dependent CP asymmetries

Example: SCS decays to CP eigenstates, $D^0 \rightarrow K^+ K^-, \pi^+ \pi^-$

$$\Gamma(D^0(t) \rightarrow f) \propto \exp[-\hat{\Gamma}_{D^0 \rightarrow f} t], \quad \Gamma(\overline{D}^0(t) \rightarrow f) \propto \exp[-\hat{\Gamma}_{\overline{D}^0 \rightarrow f} t]$$

The CP asymmetry: $\Delta Y_f \equiv (\hat{\Gamma}_{\overline{D}^0 \rightarrow f} - \hat{\Gamma}_{D^0 \rightarrow f}) / 2\Gamma_D$

$$\Delta Y_f = -x_{12} \sin \phi_{12} [1 + O(\sin \phi_{12})]$$

+ possible contributions from new weak phases in decay

- to understand how large indirect CPV (CPVMIX & CPVINT) can be in the SM **must improve on** “ $\sin \phi_{12}$ enters at $O(V_{cb} V_{ub} / V_{cs} V_{us})$ ”

Outline

- A model-independent upper bound on $\sin \phi_{12}$ in the SM - with Yuval Grossman and Zoltan Ligeti
 - the bound is proportional to an $SU(3)_F$ breaking parameter
 - this parameter can be bounded experimentally in the future
- Updated bounds on $\sin \phi_{12}$ from experiment - thanks to Rolf Andreasson, Mike Sokoloff for the fits
 - makes essential use of mode-independent relations between CPVMIX and CPVINT - Grossman, Nir, Perez; AK, Sokoloff
 - includes the recent CDF $D^0 \rightarrow \pi^+ \pi^-$ and $D^0 \rightarrow K^+ K^-$ time-integrated CP asymmetries

A model-independent bound on $\sin \phi_{12}$ in the SM

Three types of D decay

- Cabibbo Favored (CF)

$$c \rightarrow s\bar{d}u \quad (D \rightarrow K^- \pi^+)$$

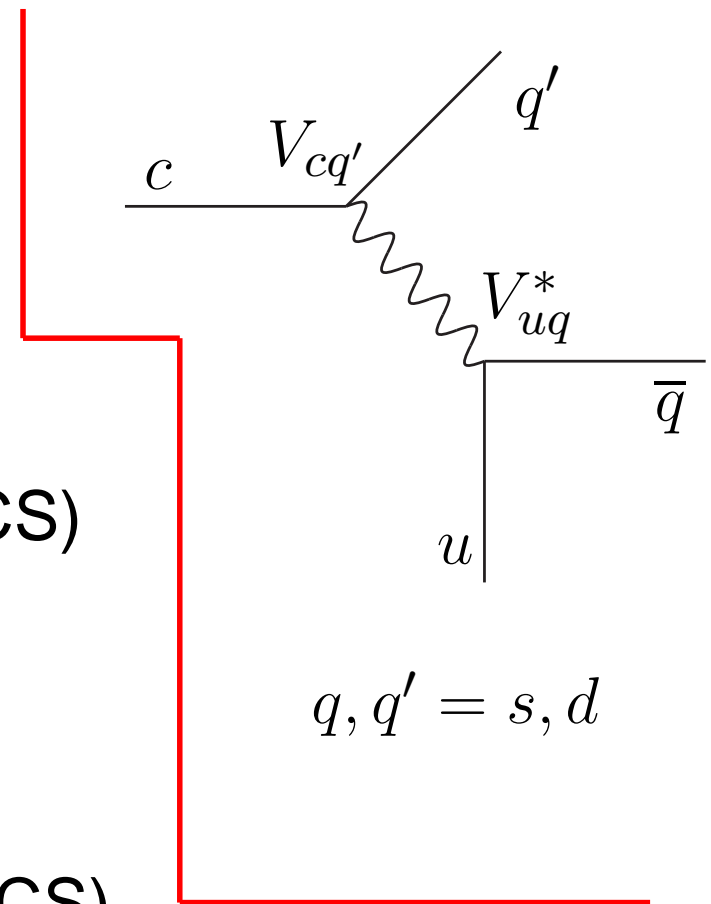
- Singly Cabibbo Suppressed (SCS)

$$c \rightarrow s\bar{s}u \quad (D \rightarrow K^- K^+)$$

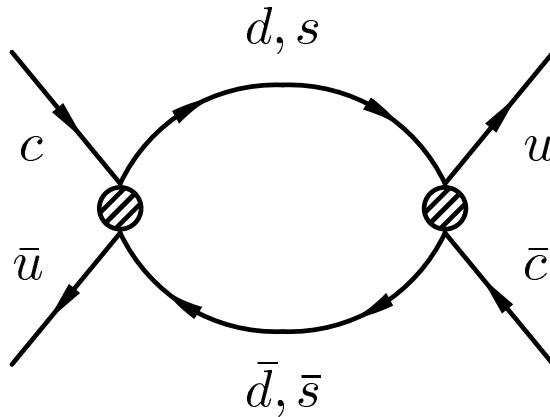
$$c \rightarrow d\bar{d}u \quad (D \rightarrow \pi^- \pi^+)$$

- Doubly Cabibbo Suppressed (DCS)

$$c \rightarrow d\bar{s}u \quad (D \rightarrow \pi^- K^+)$$



$$\Gamma_{12} = - \left(\lambda_s^2 \Gamma_{ss} + 2\lambda_s \lambda_d \Gamma_{sd} + \lambda_d^2 \Gamma_{dd} \right), \quad \text{where } \lambda_p = V_{cp} V_{up}^*$$



Γ_{xy} in the OPE picture

Γ_{ss} : via SCS operators $c \rightarrow s\bar{s}u$

Γ_{dd} : via SCS operators $c \rightarrow d\bar{d}u$

Γ_{sd} : via CF & DCS operators $c \rightarrow s\bar{d}u$, $c \rightarrow d\bar{s}u$

● from a sum over decays to common exclusive final states [Falk et al.](#):

$$\lambda_s^2 \Gamma_{ss} = \Gamma \sum_n \eta_{\text{CP}}(n) \cos \delta_n \sqrt{\mathcal{B}(D^0 \rightarrow n) \mathcal{B}(D^0 \rightarrow \bar{n})}, \dots$$

δ_n = strong phase difference between $\mathcal{A}(D^0 \rightarrow n)$ and $\mathcal{A}(\bar{D}^0 \rightarrow n)$; $\eta_{\text{CP}} = \pm 1$

Derivation of the SM bound

- using CKM unitarity, can write $\Gamma_{12} = \Gamma_{12}^0 + \delta \Gamma_{12}^0$ (responsible for ϕ_{12})

$$\Gamma_{12}^0 = -\lambda_s^2 (\Gamma_{ss} - 2\Gamma_{sd} + \Gamma_{dd}), \quad \delta \Gamma_{12} = 2\lambda_s \lambda_b (\Gamma_{sd} - \Gamma_{dd}) - \lambda_b^2 \Gamma_{dd}$$

- $\phi_{12} = \arg(M_{12}/\Gamma_{12}) \approx -\text{Im}(\delta \Gamma_{12}/\Gamma_{12}^0) \Rightarrow$

$$\phi_{12} = 2 |\lambda_b \lambda_s| \sin \gamma \frac{\Gamma_{sd}}{\Gamma_{12}^0} \left(\frac{\Gamma_{sd} - \Gamma_{dd}}{\Gamma_{sd}} + \left| \frac{\lambda_b}{\lambda_s} \right| \cos \gamma \frac{\Gamma_{dd}}{\Gamma_{sd}} \right)$$

- taking $|y| = |\Gamma_{12}^0|/\Gamma$ (can ignore CPV here) \Rightarrow

$$|\phi_{12}| = 2 \left| \frac{\lambda_b \lambda_s \sin \gamma}{y} \right| \times \left| \frac{\Gamma_{sd}}{\Gamma} \right| \times \left| \frac{\Gamma_{sd} - \Gamma_{dd}}{\Gamma_{sd}} + \left| \frac{\lambda_b}{\lambda_s} \right| \cos \gamma \frac{\Gamma_{dd}}{\Gamma_{sd}} \right|$$

with experimental inputs for y , CKM obtain

$$|\phi_{12}| = 0.008 \times \left| \frac{\Gamma_{sd}}{\Gamma} \right| \times \left| \frac{\Gamma_{sd} - \Gamma_{dd}}{\Gamma_{sd}} + 2.5 \times 10^{-4} \frac{\Gamma_{dd}}{\Gamma_{sd}} \right|$$

Proof that $|\Gamma_{sd}/\Gamma| < 1$ up to small $SU(3)_F$ breaking

- the two physical decay widths are $\Gamma_{1,2} = \Gamma \pm |\Gamma_{12}|$

$$\Gamma_{1,2} > 0 \Rightarrow |\Gamma_{12}|/\Gamma < 1$$

- consider hypothetical $D^0 - \bar{D}^0$ system with no SCS decays, and with arbitrary “CKM” suppression $\tilde{\lambda}^2$ (not λ^2), of the SM DCS decay amplitudes or operators:

$$|\Gamma_{12}| = \tilde{\lambda}^2 2\Gamma_{sd}^{SM}, \quad \Gamma = \Gamma_{CF}^{SM} + \tilde{\lambda}^4 (\Gamma_{DCS}^{SM}/\lambda^4)$$

- Data supports small $SU(3)_F$ breaking in DCS vs. CF:
 $\Gamma_{DCS}/\lambda^4 = \Gamma_{CF}(1 + \epsilon_\Gamma)$, and small ϵ_Γ

$$\frac{|\Gamma_{12}|}{\Gamma} < 1 \Rightarrow \frac{\tilde{\lambda}^2 2|\Gamma_{sd}|}{\Gamma_{CF}(1 + \tilde{\lambda}^4[1 + \epsilon_\Gamma])} < 1$$

- tightest upper bound on Γ_{sd} realized at $\tilde{\lambda}^2 \approx 1$,

$$\frac{|\Gamma_{sd}|}{\Gamma} < 1 + \epsilon_\Gamma \quad (\text{SM})$$

- Introduce additional $SU(3)_F$ breaking parameters:

$$\epsilon_d \equiv \frac{\Gamma_{dd} - \Gamma_{sd}}{\Gamma_{sd}}, \quad \epsilon_s \equiv \frac{\Gamma_{ss} - \Gamma_{sd}}{\Gamma_{sd}}$$

- The bounds for CKM, y central values:

$$|\phi_{12}| < 2 \left| \frac{\lambda_b \lambda_s \sin \gamma}{y} \right| \times |\epsilon_d| (1 + \epsilon_\Gamma/2) = 0.008 |\epsilon_d| (1 + \epsilon_\Gamma)$$

$$|\phi_{12}| < 2 \left| \frac{\lambda_b \lambda_s \sin \gamma}{y} \right| \times |\epsilon_s| (1 + \epsilon_\Gamma/2) + 2 \left| \frac{\lambda_b}{\lambda_s} \sin \gamma \right| = 0.008 |\epsilon_s| (1 + \epsilon_\Gamma/2) + 0.00$$

- Expectation for ϵ_Γ , or **how close is Γ_{DCS}/Γ_{CF} to $\tan^4 \theta_c = 2.9 \times 10^{-3}$?**

- time-dependent $D \rightarrow K\pi \Rightarrow \Gamma(K^+\pi^-)/\Gamma(K^-\pi^+) = (3.3 \pm 0.1) \times 10^{-3}$
- time-integrated measurements yield ratio up to $O(10 - 20)\%$ corrections from interference of CF/DCS amplitudes:
 - $\Gamma(K^+\pi^-\pi^0)/\Gamma(K^-\pi^+\pi^0) \approx (2.20 \pm 0.10) \times 10^{-3}$
 - $\Gamma(K^+\pi^+\pi^-\pi^-)/\Gamma(K^-\pi^-\pi^+\pi^+) \approx (3.24_{-0.22}^{+0.25}) \times 10^{-3}$
- data \Rightarrow canonical $SU(3)_F$ breaking, $\epsilon_\Gamma \approx (10 - 30)\%$
makes sense - no large phase space effects

The bound, continued:

● Take $|\epsilon_s|, |\epsilon_d| < 1$

$$\Rightarrow |\phi_{12}| < 0.01$$

● Violation of this bound would require both $|\epsilon_s|, |\epsilon_d| > 1$. How could this happen?

● Would require

$$\text{sign}(\Gamma_{ss}) = -\text{sign}(\Gamma_{dd}), \quad \text{and}$$

$$\frac{\Gamma_{dd}}{\Gamma_{sd}} > 2 \quad \text{and} \quad \frac{\Gamma_{ss}}{\Gamma_{sd}} < 0 \quad \text{or vice versa}$$

In this case still expect $|\epsilon_s|, |\epsilon_d| = O(1)$, and $|\phi_{12}| \lesssim 0.01$

● ultimately, will be able to constrain $|\epsilon_s|, |\epsilon_d|$ by considering sums over exclusive state in Γ_{xy}

● An OPE analysis yields $\phi_{12} \ll 0.01$ for dim-6,7 operators [Borowski et al](#), however the authors have suggested that higher dimensional operators may yield $\phi_{12} \sim 10^{-2}$

**Updated bounds on $\sin \phi_{12}$ from
experiment** thanks to Rolf Andreasson and Mike Sokoloff for fits

Updated bounds on CPV in SCS $D^0 \rightarrow K^+ K^-$, $\pi^+ \pi^-$ decays

The time-integrated CP asymmetry

$$a_f \equiv \frac{\Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow f)}{\Gamma(D^0 \rightarrow f) + \Gamma(\bar{D}^0 \rightarrow f)}$$

Expanding to leading order in subleading amplitudes, mass difference, width difference

at the B -factories: Grossman, A.K., Nir

$$a_f = a_f^{\text{dir}} + a^{\text{ind}}, \quad a^{\text{ind}} = a^m + a^i$$

at CDF (due to cut on proper decay time):

$$a_f = a_f^{\text{dir}} + 2.40 a^{\text{ind}} \quad (\pi^+ \pi^-); \quad a_f = a_f^{\text{dir}} + 2.65 a^{\text{ind}} \quad (K^+ K^-)$$

a^{dir} is direct CP violation

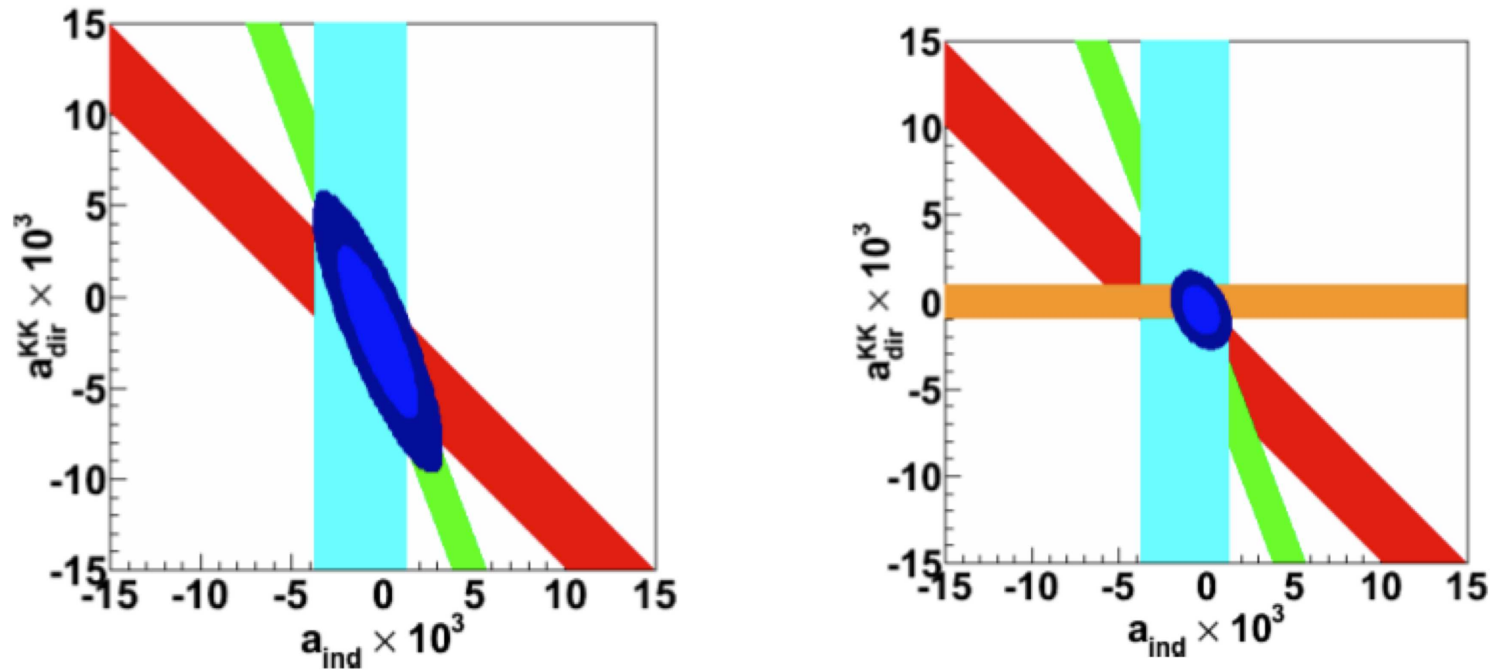
a^m : CP violation in mixing CPVMIX

a^i : CP violation in the interference of decays with and without mixing CPVINT

the total indirect CP asymmetry a^{ind} is universal - independent of final state. Note $a^{\text{ind}} = \Delta Y$ (the time-dependent CP asymmetry)

Separating indirect and direct CP violation

Combine the Belle, BaBar, and CDF $KK, \pi\pi$ time-integrated measurements a_f , with the Belle/BaBar time-dependent measurement $\Delta Y = a^{\text{ind}}$



left: ΔY (aqua), a_f BaBar/Belle (red); a_f CDF (green).

right: $|a_{KK, \pi\pi}^{\text{dir}}| < 0.2\%$ for models with negligible new weak phases in decay, e.g., SM

- without imposing any theoretical constraints on a^{dir} obtain

$$a^{\text{ind}} = -0.026 \pm 0.14\%; \quad \text{compared to } \Delta Y = 0.123 \pm 0.248\%$$

$$a^{\text{dir}}(\pi\pi) = 0.24 \pm 0.36\%, \quad a^{\text{dir}}(KK) = 0.19 \pm 0.31\%$$

- from an analysis of $a^{\text{dir}}(KK, \pi\pi)$ in the SM

- at leading power: naive factorization + $O(\alpha_s)$ corrections $a^{\text{dir}} = O(10^{-4})$
- power corrections, e.g., annihilation, FSI, can enhance a^{dir} by $O(10)$
- therefore, expect $|a^{\text{dir}}| < 0.2\%$ in the SM and in models with no new weak phases in decay
- theoretical uncertainty \Rightarrow the window for NP in a^{dir} is rapidly closing

- adding this constraint to the time-integrated and time-dependent measurements

$$\Rightarrow a^{\text{ind}} = (-0.023 \pm 0.09)\%!$$

in models with no new weak phases in decay. The small error in this case is due to the CDF measurements

Updated experimental bounds on $\sin \phi_{12}$

- The mixing observables

$$\phi, |q/p|, x = \frac{m_2 - m_1}{\Gamma}, y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma}$$

ϕ is the phase difference between mixing, decay amps. For example,

$$a_{\text{SL}} = 2(|q/p| - 1)$$

$$a^m(KK, \pi\pi) = -\frac{y}{2} \cos \phi \left(\left| \frac{q}{p} \right| - \left| \frac{p}{q} \right| \right), \quad a^i(KK, \pi\pi) = \frac{x}{2} \sin \phi \left(\left| \frac{q}{p} \right| + \left| \frac{p}{q} \right| \right)$$

- in absence of new weak phases in decay, $\phi_{12} \neq 0$ is the only source of CPV. Therefore, CPVMIX (a^m) and CPVINT (a^i) are related (Grossman, Nir, Perez; A.K. and M. Sokoloff)
- obtain relations between theory parameters $\phi_{12}, x_{12}, y_{12}$ and the mixing observables $\phi, |q/p|, x, y$. For example,

$$\tan 2\phi = -\frac{\sin 2\phi_{12}}{\cos 2\phi_{12} + y_{12}^2/x_{12}^2}$$

- given current bounds on direct CP asymmetries, this relation also holds to good approximation when allowing for new weak phases in decay

Strategy

- HFAG has fit the observables $\phi, |q/p|, x, y$ to the $D - \bar{D}$ mixing and CPV data
- they have not included the time-integrated data for $K^+ K^-, \pi^+ \pi^-$ (pre-FPCP)
- have obtained their fit with error matrix, for ΔY subtracted **thanks to Alan Schwartz for providing this info**
- have added the new average for $a^{\text{ind}} = \Delta Y$, without and with the a^{dir} constraint, as an additional independent observable
- using the relations between $\phi_{12}, x_{12}, y_{12}$ and the mixing observables $\phi, |q/p|, x, y$ and a^{ind} , we have fit for $\phi_{12}, x_{12}, y_{12}$, using this error matrix

Results for ϕ_{12}^{exp}

- Without imposing $\alpha^{\text{dir}}(KK, \pi\pi)$ constraint, obtain

$$\phi_{12} = 0.03 \pm 0.11 \text{ [rad]}$$

for no new weak phases in CF/DCS decays,

$$\phi_{12} = 0.07 \pm 0.14 \text{ [rad]}$$

allowing for new weak phases in CF/DCS decays.

- Imposing $\alpha^{\text{dir}}(KK, \pi\pi) < 0.002$, corresponding to models with no new weak phases in SCS decays, and for no new weak phases in CF/DCS decays, obtain

$$\phi_{12} = 0.03 \pm 0.09 \text{ [rad]}$$

This applies to a wide class of models which do not have new weak phases in SCS, CF, and DCS decays

- used parabolic errors. robust treatment with non-parabolic errors to be carried out by HFAG, taking into account the model-independent relations and time-integrated CPV data

Conclusion

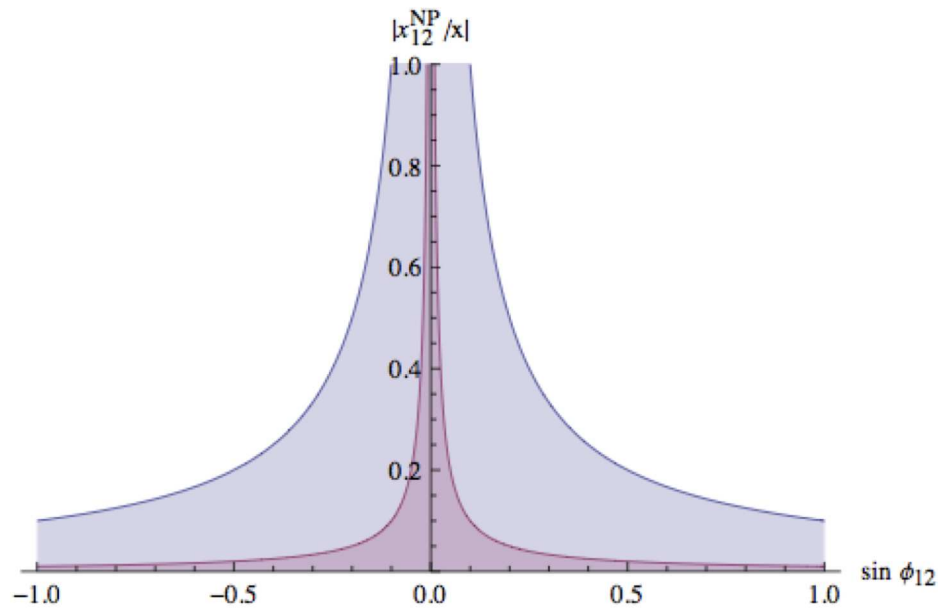
- in the SM $\phi_{12} \sim 0.01$
 - indirect CP asymmetries are $O(x_{12} \sin \phi_{12}) < 10^{-4}$ in the SM
- updated fit to HFAG outputs + $a^{\text{ind}}(KK, \pi\pi)$ yields

$$\phi_{12} \sim \pm 0.10 \quad \text{at } 1\sigma$$

so plenty of room for NP

- writing $M_{12} = M_{12}^{\text{SM}} + M_{12}^{\text{NP}}$ the bounds imply (taking M^{SM} real, $\Gamma_{12} = \Gamma_{12}^{\text{SM}}$)

$$\left| \frac{\text{Im}(M_{12}^{\text{NP}})}{M_{12}} \right| = |\sin \phi_{12}| \lesssim 0.10$$



- all indirect CP asymmetries (time-dependent, time-integrated, SCS, CF/DCS) are $\propto x_{12} \sin \phi_{12}$
- allowing $x_{12}^{\text{NP}} \sim x_{12}^{\text{exp}}$ and taking into account the current situation

$$x_{12} \sin \phi_{12} \lesssim 10^{-3}$$

can represent the allowed region of x_{12}^{NP} vs. $\sin \phi_{12}^{\text{NP}}$ as above [Gedalia et al.](#), thanks to [G. Perez](#) for updating the plot,

- the dark region corresponds to the SM bound $x_{12} \sin \phi_{12} \lesssim 10^{-4}$, and is the region in which sensitivity to NP would be lost