Long-Range Near-Side Ridge Structure in Small Systems:

Momentum Kick Model with Multiplicity Dependence

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Introduction

I.   Research topic

  ↳ To describe **_long-range near-side ridge structures_** in small system.

■ Introduction

I.   Research topic

   ↳ To describe ***long-range near-side ridge structures*** in small system.

   o   What is a long-range near-side structure?

   ↳ An enhanced yield in region $2 < |\Delta\eta| < 4$ and $|\Delta\phi| \approx 0$
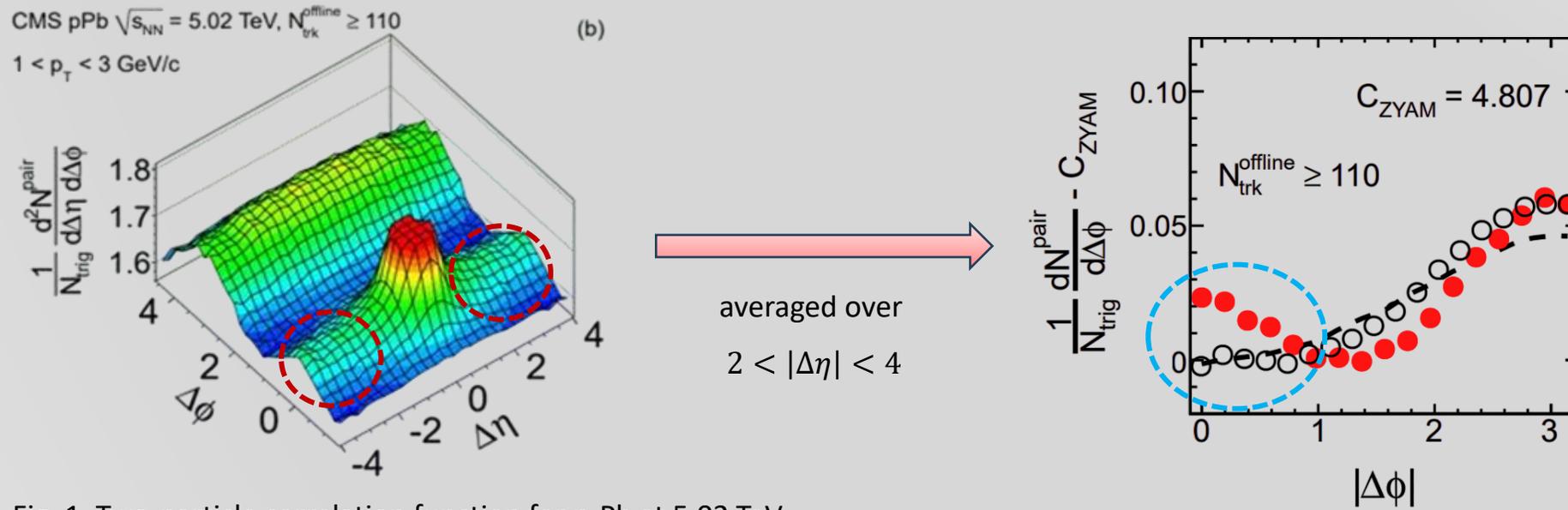

averaged over

$2 < |\Delta\eta| < 4$

Fig. 1. Two-particle correlation function for p-Pb at 5.02 TeV

Introduction

○   Observation : the ridge structure in high-multiplicity p-p and p-Pb.

Low multiplicity

High multiplicity

Low multiplicity

High multiplicity

CMS pp $\sqrt{s}$ = 13 TeV
$10 \leq N_{offline}^{trk} < 20$
$1 \leq p_T^{trig},\ p_T^{assoc} < 3$ GeV/c

CMS pp $\sqrt{s}$ = 13 TeV
$105 \leq N_{offline}^{trk} < 150$
$1 \leq p_T^{trig},\ p_T^{assoc} < 3$ GeV/c

CMS pPb $\sqrt{s_{NN}}$ =5.02 TeV
$N_{offline}^{trk} < 35$
$1 \leq p_T < 3$ GeV/c

CMS pPb $\sqrt{s_{NN}}$ =5.02 TeV
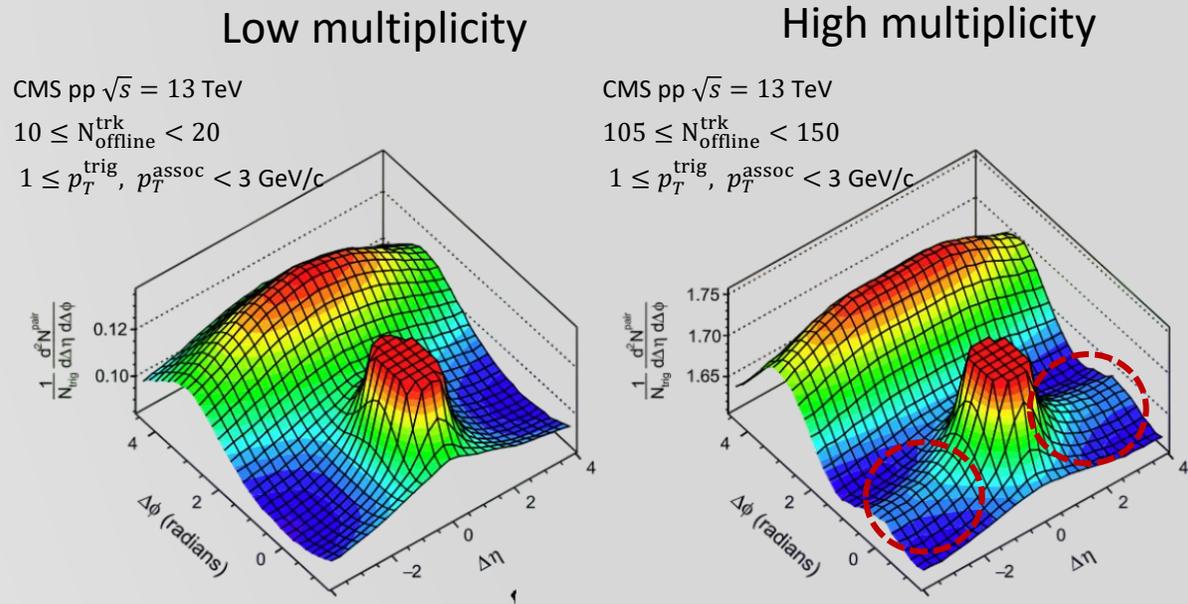$N_{offline}^{trk} \geq 110$
$1 \leq p_T < 3$ GeV/c

Fig. 2. Ridge structure in **p-p at 13 TeV**.

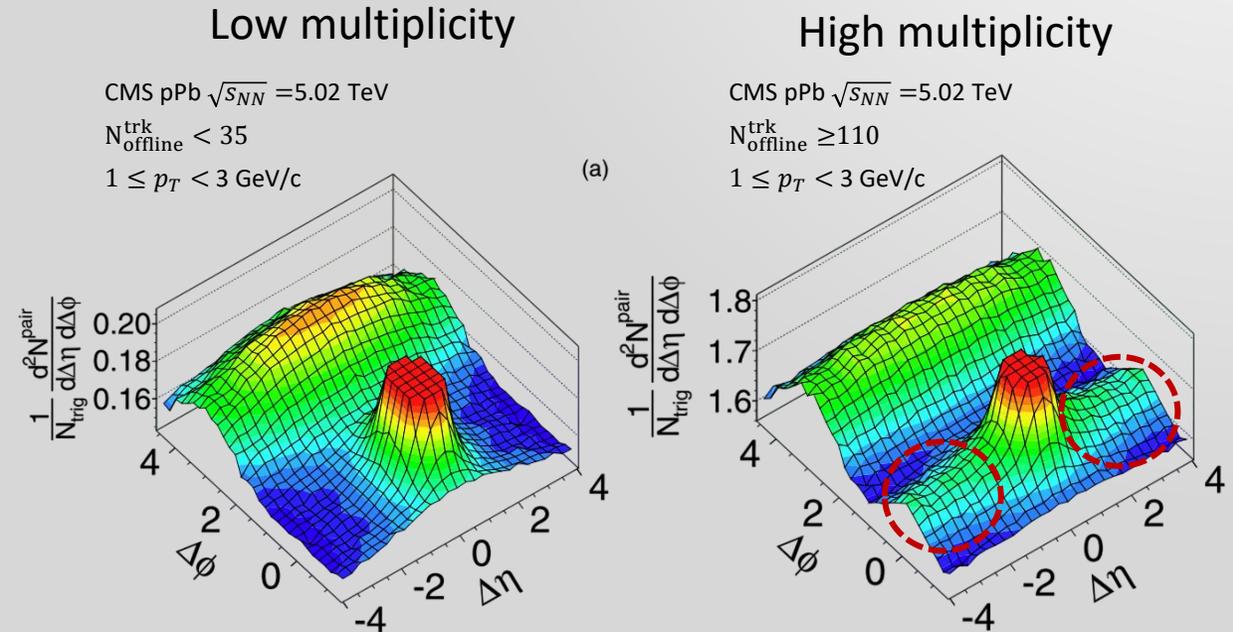(Left) Low-multiplicity (Right) high-multiplicity

PLB 765 (2017) 193-220

Fig. 3. Ridge structure in **p-Pb at 5.02 TeV**.

(Left) Low-multiplicity (Right) high-multiplicity

PLB 718 (2013) 795-814

II. Motivation

↳ Long range region($2 < |\Delta\eta| < 4$) → **the early-stage dynamics** after collisions.

o **Explained**

The ridge observed in **heavy-ion collisions** is understood as a consequence of **QGP collective flow**.

o **Problem**

In **small systems (p–p and p–Pb)**, the existence of a QGP is **uncertain**, and the origin of the ridge remains **unclear**.

o **Goal**

Investigate the **ridge structure in p–Pb collisions at $\sqrt{s_{NN}} = 5.02$ TeV**, extending previous studies in **p–p collisions at 7 TeV and 13 TeV**. (PRC 84 (2011)024901, NPA 1047 (2024) 122875)

▪ Theory

I. Momentum kick model with multiplicity dependence (MKMwM)

↳ A jet kicks nearby medium partons that then spread over the long-range near-side ,forming the ridge structure.

Fig. 4. Geometric representation of yield fragmentation in the MKM.

- **Theory**
  - ○ Formalism

Ridge yield

$:=$ the ensemble average of ***the final momentum distribution of the kicked partons***

$$\left[\frac{1}{N_{\text{trig}}}\frac{dN_{ch}}{p_{T_f}dp_{T_f}d\Delta\eta d\Delta\phi}\right]_{\text{ridge}} = \left\langle \frac{2}{3}\cdot\sum_{n}^{N}f_{R_n}\cdot\left[\frac{dF_n}{p_{T_f}dp_{T_f}d\Delta\eta d\Delta\phi}\right]\right\rangle$$

$$\simeq \frac{2}{3}f_R\cdot\langle N_k\rangle\cdot\left[\frac{dF}{p_{T_f}dp_{T_f}d\Delta\eta d\Delta\phi}\right]$$

f_R : the average attenuation factor

$\langle N_k\rangle$: the mean number of ***kicked partons*** $\Leftarrow$ multiplicity dependence part

$\left[\dfrac{dF}{p_{T_f}dp_{T_f}d\Delta\eta d\Delta\phi}\right]$: the ***final momentum distribution*** of medium partons. $\Leftarrow$ momentum kick part

Theory

a. Multiplicity dependence part $\langle N_k \rangle$

- The **Glauber model** quantifies multiplicity (N_{ch}) and N_k as a function of the impact parameter b.

$$N_k(\boldsymbol{b}) = \int_0^{l_{max}} \frac{dl}{2t} \sigma_{MP} \, N_{MP}(\boldsymbol{b}), \qquad t \approx t_0 + l,$$

- where N_{MP} : *medium parton density* , σ_{MP} : *jet-medium scattering cross-section, l* : *jet's trajectory*.

- Model parameters $—\kappa', \; \sigma_{MP}, \; \zeta, \, t_0—$ are indirectly determined. [details in back-up]
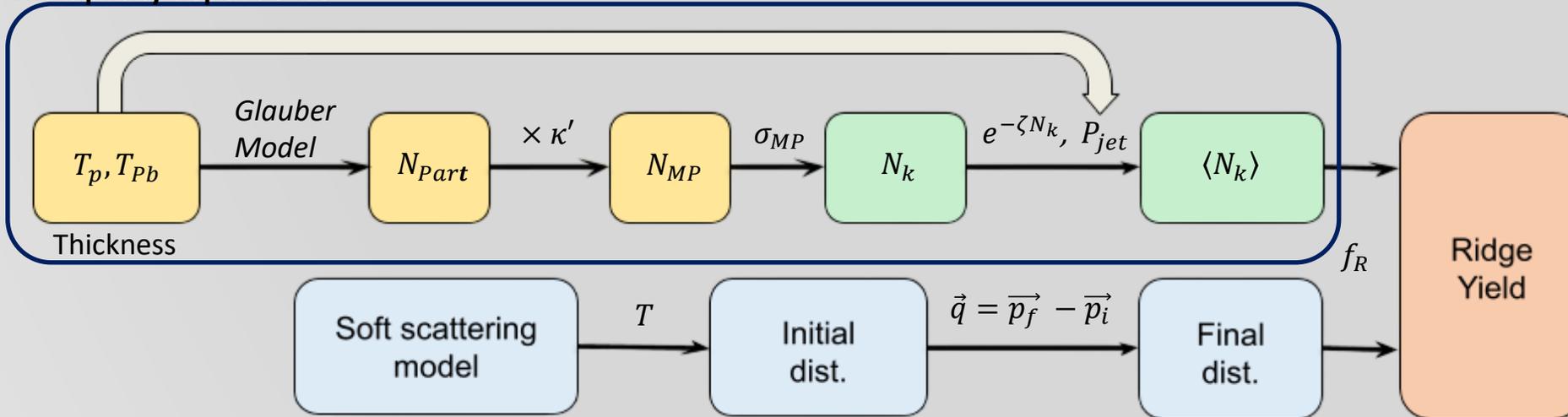
Multiplicity dependence

Fig. 5. Flow diagram of ridge yield in MKMwM

a.   Multiplicity dependence part $\langle N_k \rangle$

- $N_{MP} \simeq \kappa' \ N_{part} = \kappa' \left[T_p(J_0) \left(1 - \left[1 - T_{Pb}(J_0 - b)\sigma_{NN}^{inel} \right]^{208} \right) + 208 \cdot T_p(J_0)T_{Pb}(J_0 - b)\sigma_{NN}^{inel} \right],$

- $\kappa' = \frac{\langle N_{MP} \rangle}{\langle N_{part} \rangle} = \frac{3}{2} \cdot \frac{\langle N_{ch} \rangle}{\langle N_{part} \rangle}.$

- $\langle N_k \rangle(b, \phi_s) = \frac{\int N_k \ e^{-\zeta N_k} \ P_{jet}(J_0, b)dJ_0}{\int e^{-\zeta N_k} \ P_{jet}(J_0, b)dJ_0}$

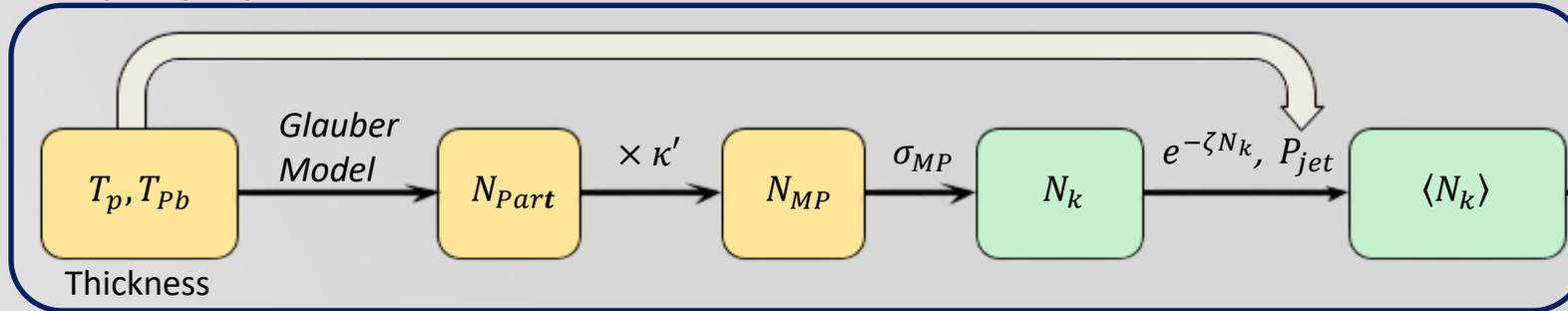- $N_{ch}(b) = \frac{2}{3} \int N_{MP}(b, J_0) \ dJ_0$

Multiplicity dependence

Fig. 5. Flow diagram of ridge yield in MKMwM
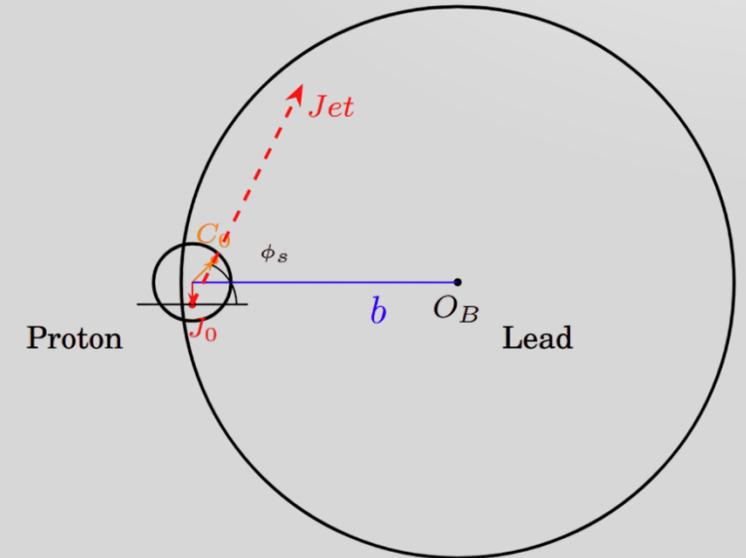

Fig. 6. Collision geometry of p-Pb collisions.

b. Momentum kick part

- The **soft scattering model** phenomenologically introduces the **initial momentum distribution** of medium partons.

- The key parameters —**medium temperature T and momentum transfer $\vec{q}$**— characterize the interaction strength between the jet and the medium.

- Finally, the parameters $(T, \vec{q}, f_R)$ are **extracted by fitting** the model to the **CMS $\Delta\phi$ correlation data**.
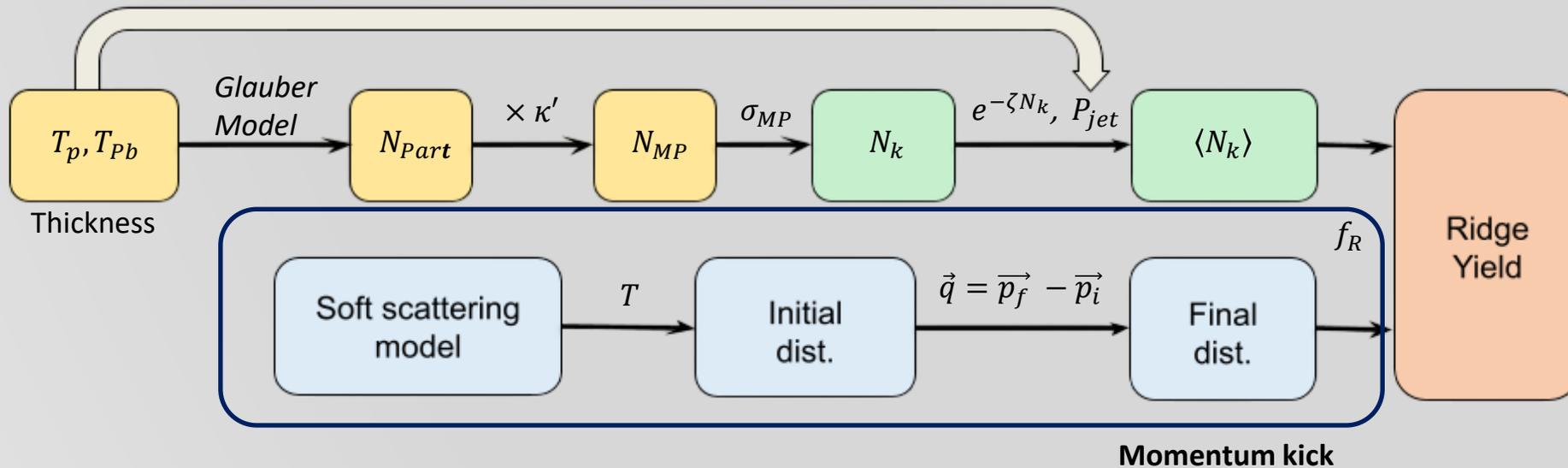
Fig. 5. Flow diagram of ridge yield in MKMwM

■ Theory

b. Momentum kick part

- The initial dist. : $\dfrac{dF}{p_{T_i} dp_{T_i} dy_i d\phi_i} = A_{ridge} (1 - x)^a \dfrac{e^{-m_{T_i}/T}}{\sqrt{m_d^2 + p_{T_i}^2}}$,

- The finial dist. : $\dfrac{dF}{p_{T_f} dp_{T_f} d\eta_f d\phi_f} = \left[\dfrac{dF}{p_{T_i} dp_{T_i} dy_i d\phi_i} \dfrac{E_f}{E_i} \right]_{\overrightarrow{p_i}=\overrightarrow{p_f} - \vec{q}} \cdot \sqrt{1 - \dfrac{m_\pi^2}{\left(m_\pi^2 + p_{T_f}^2 \right) \cosh^2 y_f}}$,

- $\left[\dfrac{1}{N_{trig}} \dfrac{dN_{ch}}{p_{T_f} dp_{T_f} d\Delta\eta d\Delta\phi} \right]_{ridge} \simeq \dfrac{2}{3} f_R \cdot \langle N_k \rangle \cdot \left[\dfrac{dF}{p_{T_f} dp_{T_f} d\Delta\eta d\Delta\phi} \right]$
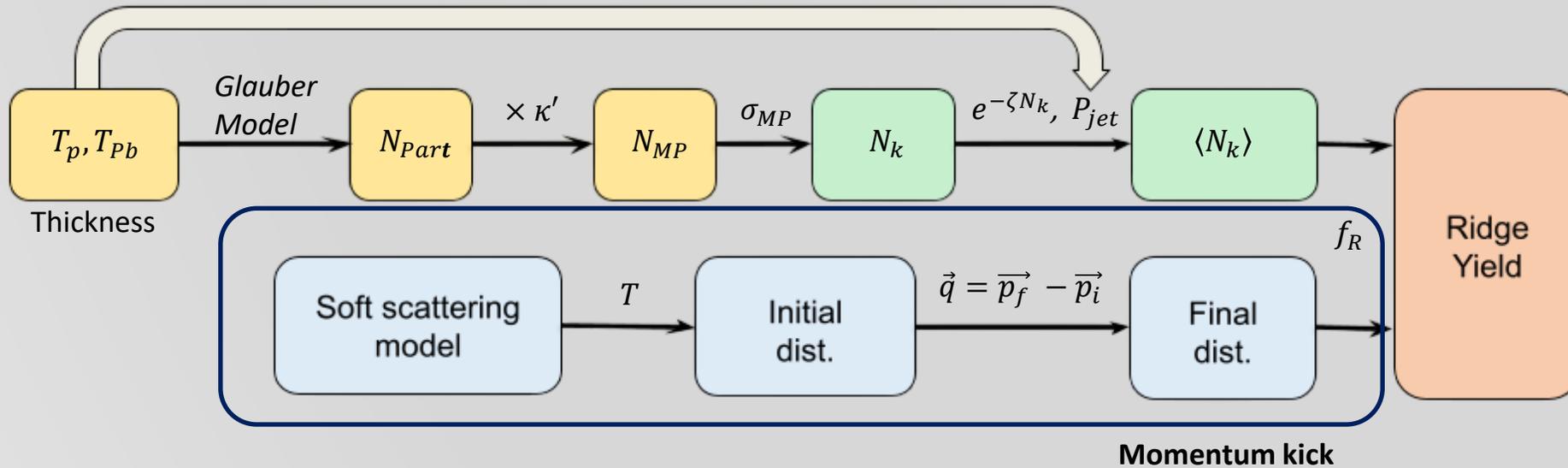

Fig. 5. Flow diagram of ridge yield in MKMwM

▪ Result and Analysis

a.   Multiplicity dependence of $\langle N_k \rangle$

In p-Pb collisions,

- At $b \approx 7$ fm, $\langle N_k \rangle \approx 3.$
- $\langle N_k \rangle \geq 3$ for $N_{ch} \geq 100$.

In p-p collisions,

- $\langle N_k \rangle = 1 \sim 3$ for $N_{ch} \geq 100$

- Different collision geometry
  - ⇒ Different kick effect
  - ⇒ stronger ridge formation in p–Pb than in p–p.
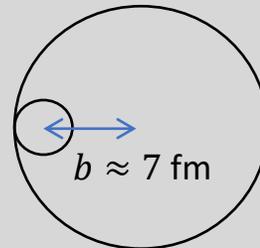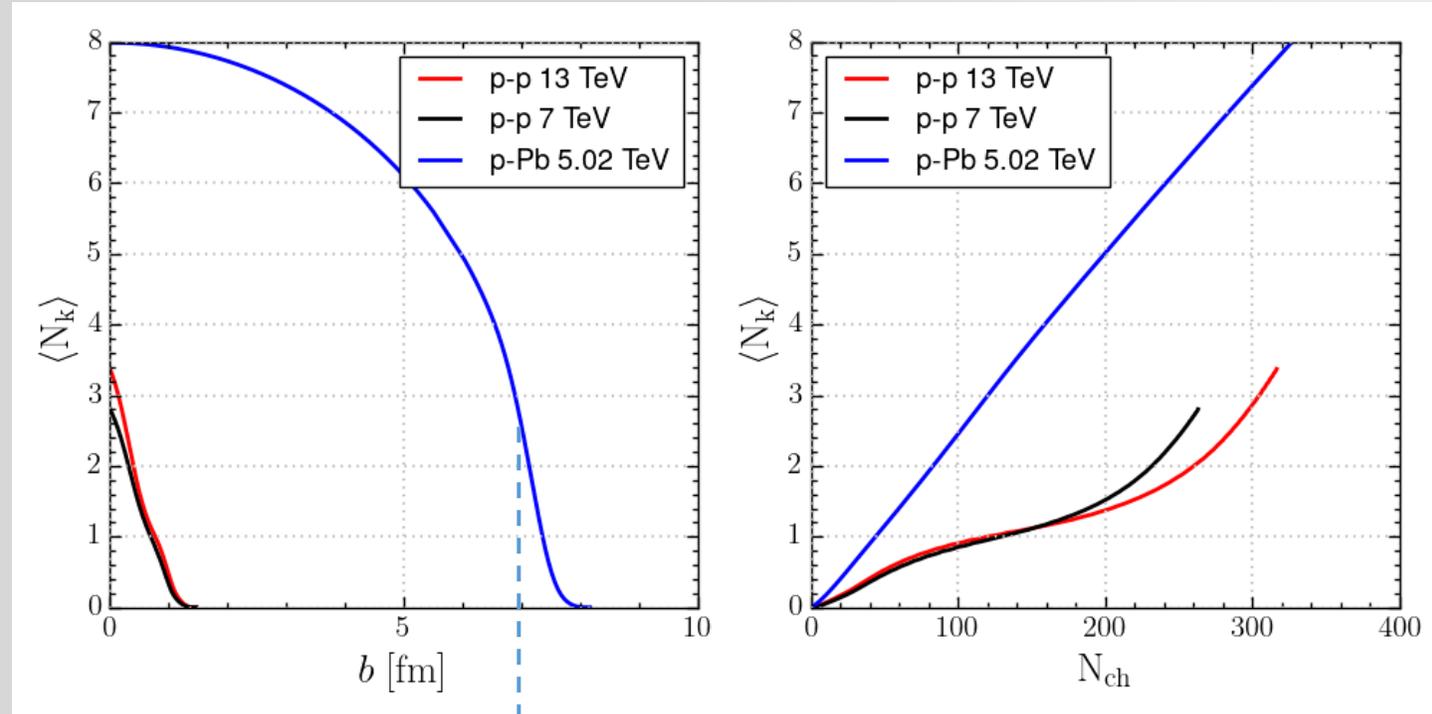

Fig. 6. Multiplicity dependence of $\langle N_k \rangle$

(left) $\langle N_k \rangle$ vs b ,   (right) $\langle N_k \rangle$ vs N_{ch}

b. CMS $\Delta\phi$ distribution $(90 \leq N_{ch} < 110, N_{ch} \geq 110)$

Column : same p_T bin.
Row : same N_{ch} interval.

- The best-fit parameters are
  $q = 1.0$ GeV/c, $T = 0.74$ GeV

- In $1 < p_T < 2$ GeV/c ,
  the ridge structure becomes
  prominent on the near side.

- For $N_{ch} \geq 110,$
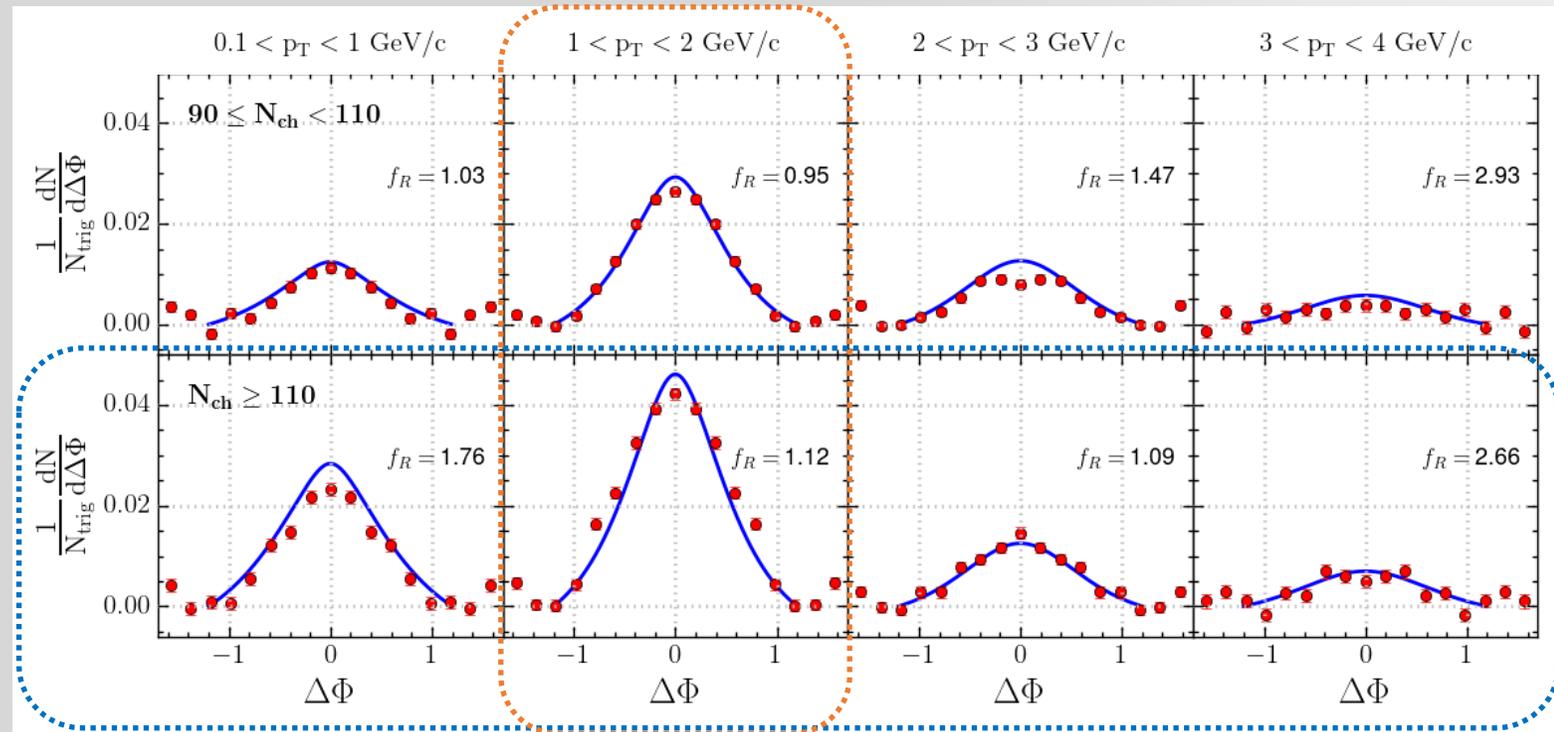 *the ridge structure is noticeable.*

Fig. 8. Comparison of the MKMwM calculations (blue solid lines) with CMS p–Pb data (red circles) at 5.02 TeV. The associated yield per trigger is shown as a function of Δφ for different p_T and N_{ch}.

Result and Analysis

c. CMS p_T & N_{ch} distribution.

Averaged over $|\Delta\phi| \leq 1.2$ for
(a) high multiplicity, (b) intermediate- p_T

- The **associated yield peaks**
  around $1 < p_T < 2$ **GeV/c.**

- The **associated yield increases**
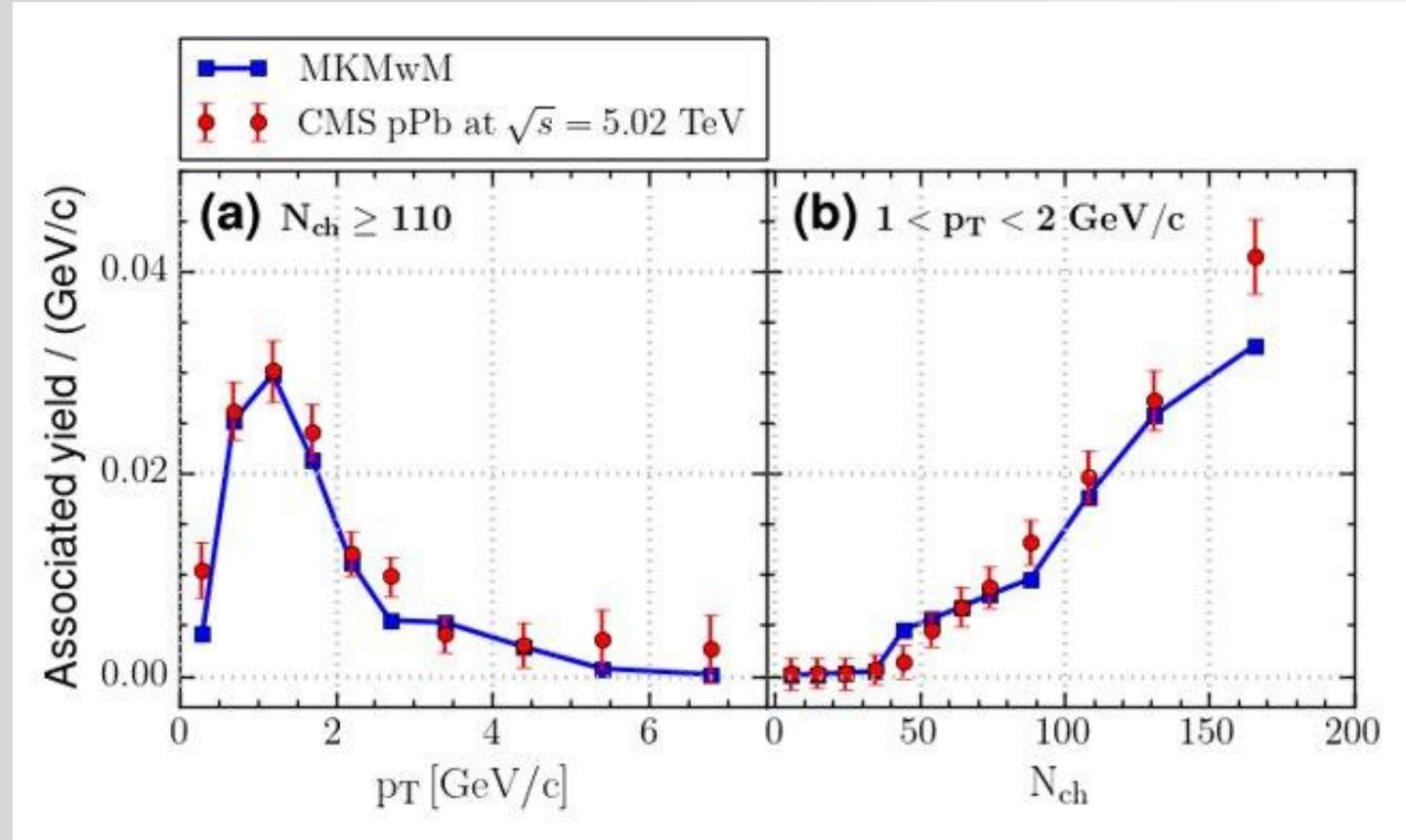 approximately **linearly with** N_{ch}**.**

Fig. 9. Comparison of the MKMwM calculations (blue solid lines) with CMS p–Pb data (red circles)

at 5.02 TeV. (a) the averaged near-side ridge yield as a function of p_T for $N_{ch} \geq 110$. (b) the ridge yield as a function of N_{ch} in $1 < p_T < 2$ GeV/c.

■ Summary

➢ The **MKMwM** describes ridge formation through *jet–medium momentum transfer* and *multiplicity weighting*.

➢ The MKMwM successfully reproduces the CMS ridge data:
  ✓ The best-fit parameters are $q = 1.0$ GeV/c, $T = 0.74$ GeV
  ✓ The **ridge structure emerges** in the range $1 < p_T < 2$ GeV/c and $35 < N_{ch} < 90$.
  ✓ In the **region** $1 < p_T < 2$ GeV/c , ridge features become cleary visible.
  ✓ For $N_{ch} \geq 110$, the ridge structure are pronounced.
  ✓ The associated yield shows **a peak** around $1 < p_T < 2$ GeV/c and **scales linearly with** N_{ch}.

➢ **Future work**: application to **p-Pb collisions at** $\sqrt{s_{NN}}$ =8.16 TeV.

Thank you!

Back-up

■ Theory

I.   Momentum kick model with multiplicity dependence (MKMwM)

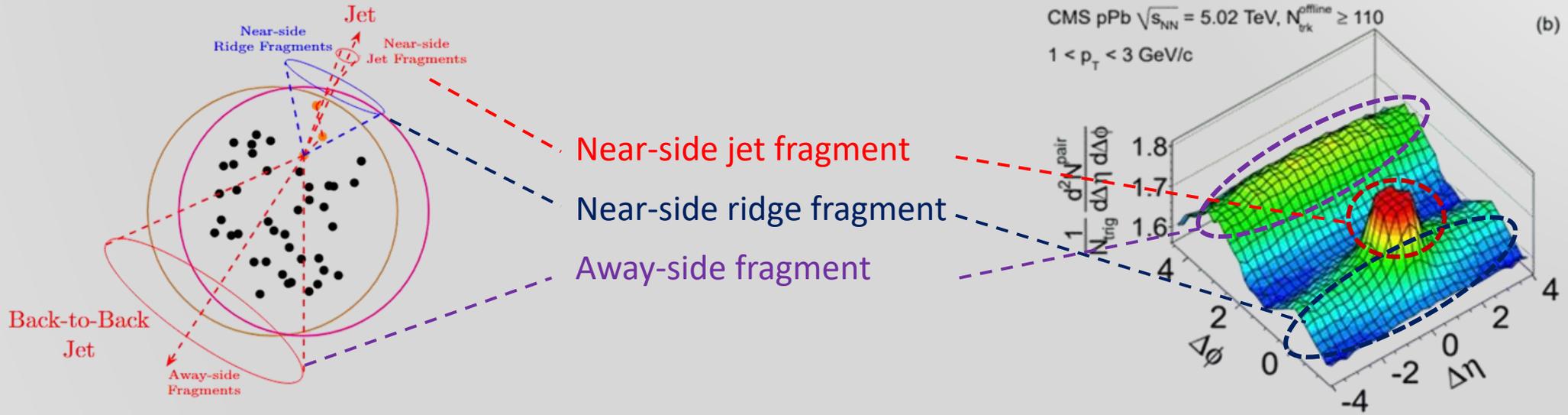↳ A jet kicks nearby medium partons

Fig. 4. Geometric representation of yield fragmentation in the MKM.

- **Theory**

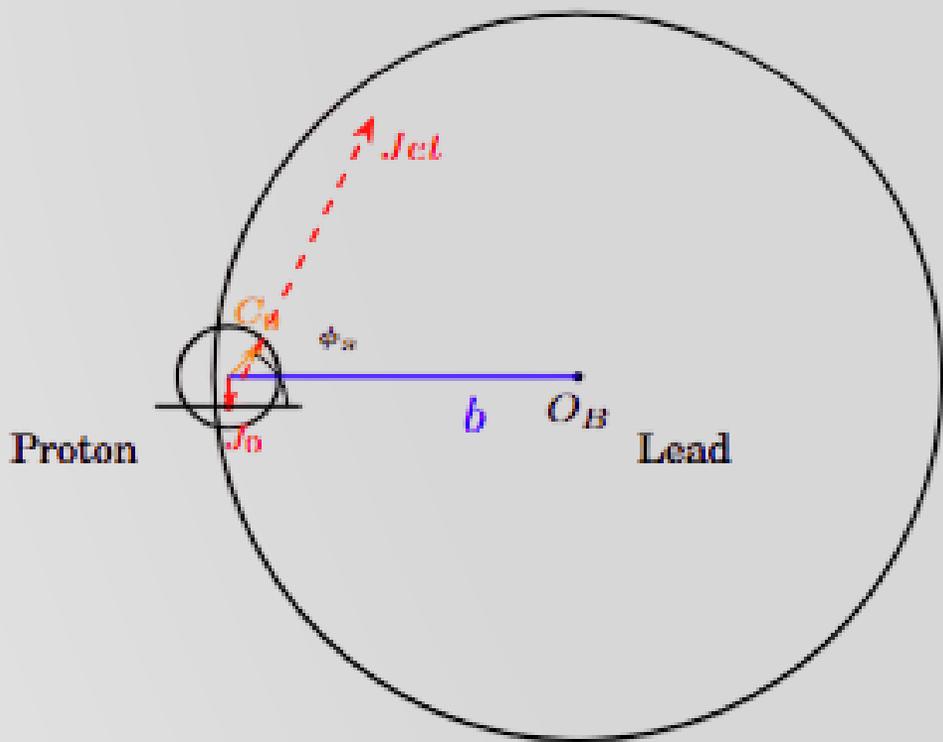a. Multiplicity dependence part $\langle N_k \rangle$

Fig. Schematic representation of proton and lead

on the transverse plane in high energy collisions.

1. A jet is produced at a point J_0 in the overlapping.

2. Medium partons are created after t_0.

3. The jet travels through l w.r.t ϕ_s in the reaction plane .

4. The jet collides with nearby partons.

5. The medium parton density (N_{MP}) traverses the jet-medium cross section (σ_{MP}).

$$N_k(\boldsymbol{b}) = \int_0^{l_{max}} \frac{dl}{2t} \sigma_{MP} \, N_{MP}(\boldsymbol{b}), \qquad t \approx t_0 + l.$$

a. Multiplicity dependence part $\langle N_k \rangle$

- Glauber model quantifies

$$N_{MP} \simeq \kappa'\, N_{part} = \kappa' \left[T_p(J_0)\left(1 - \left[1 - T_{Pb}(J_0 - b)\sigma_{NN}^{inel}\right]^{208}\right) + 208 \cdot T_p(J_0)T_{Pb}(J_0 - b)\sigma_{NN}^{inel} \right]$$

$$\kappa' = \frac{\langle N_{MP} \rangle}{\langle N_{part} \rangle} = \frac{3}{2} \cdot \frac{\langle N_{ch} \rangle}{\langle N_{part} \rangle}.$$

$$N_{ch}(b) = \frac{2}{3} \int N_{MP}(b, J_0)\, dJ_0$$
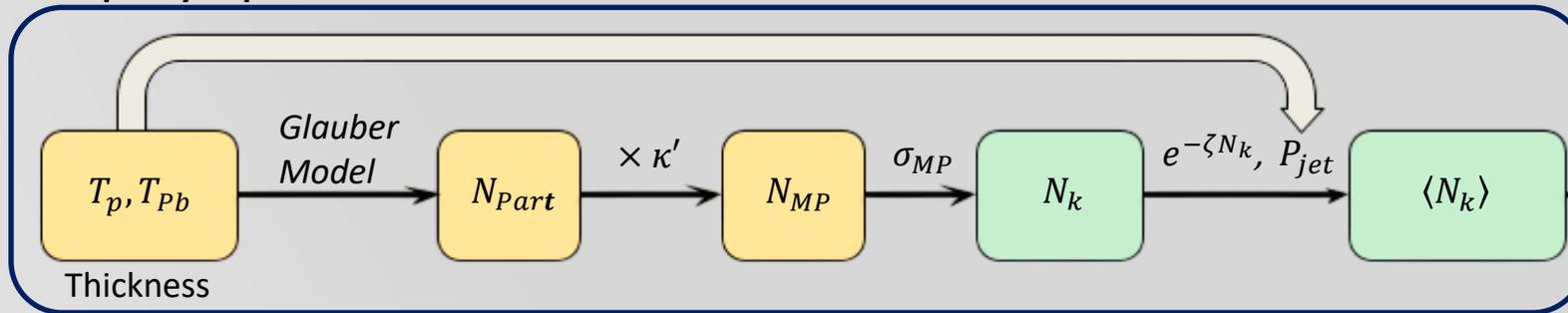
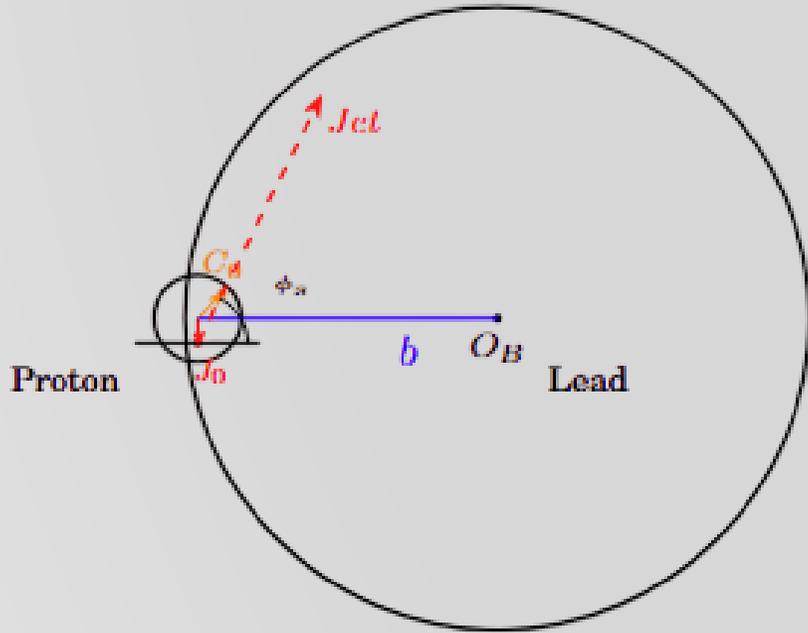**Multiplicity dependence**

Fig. 5. Flow diagram of ridge yield in MKMwM

- # Theory

To consider jet energy loss due to collisions and other effects,

1.   Jet production probability

$$P_{jet} = T_p(s)T_{Pb}(b - s) \text{ with normalization.}$$

2.   This attenuation for jet energy loss $e^{-\zeta N_k}$.

3.   In this ensemble,

$$\langle N_k \rangle(b, \phi_s) = \frac{\int N_k \, e^{-\zeta N_k} \, P_{jet}(J_0, b) \, dJ_0}{\int e^{-\zeta N_k} \, P_{jet}(J_0, b) \, dJ_0}$$

We use the averaged $\langle N_k \rangle$ over ϕ_s

Fig. Schematic representation of proton and lead on the transverse plane in high energy collisions.

Result

o   Parameter fitting result

Physical Parameter	$p-p$ ($\sqrt{s} = 7$ TeV)	$p-p$ ($\sqrt{s} = 13$ TeV)	$p-Pb$ ($\sqrt{s} = 5.02$ TeV)
MKM			
q (GeV/c)	2.0	1.2	1.0
f_R	1.0	$0.44 \sim 1.89$	$0.64 \sim 2.66$
T (GeV)	0.70	0.77	0.74
m_d (GeV)	1.0	1.0	1.0
a	0.5	0.5	0.5
Multiplicity (Centrality) Dependence			
R_A & R_B (fm)	0.80	0.74	0.79 & 7.37
t_0 (fm/c)	0.43	0.39	0.43
κ'	367	236	30.7
σ_{MP} (mb)	1.4	1.4	1.4
ζ	0.20	0.20	0.007

▪ Result and analysis

a. Multiplicity dependence parameters

- The radius of the proton and lead is determined the inelastic cross section measured as $\sigma_{in} = 2.09$ b at LHC.

$$R_p + R_{Pb} = \sqrt{\frac{\sigma_{in}}{\pi}} = 8.16 \text{ fm.}$$

- $R_p = 0.79$ fm is adopted from CMS result due to a comparable energy scale.

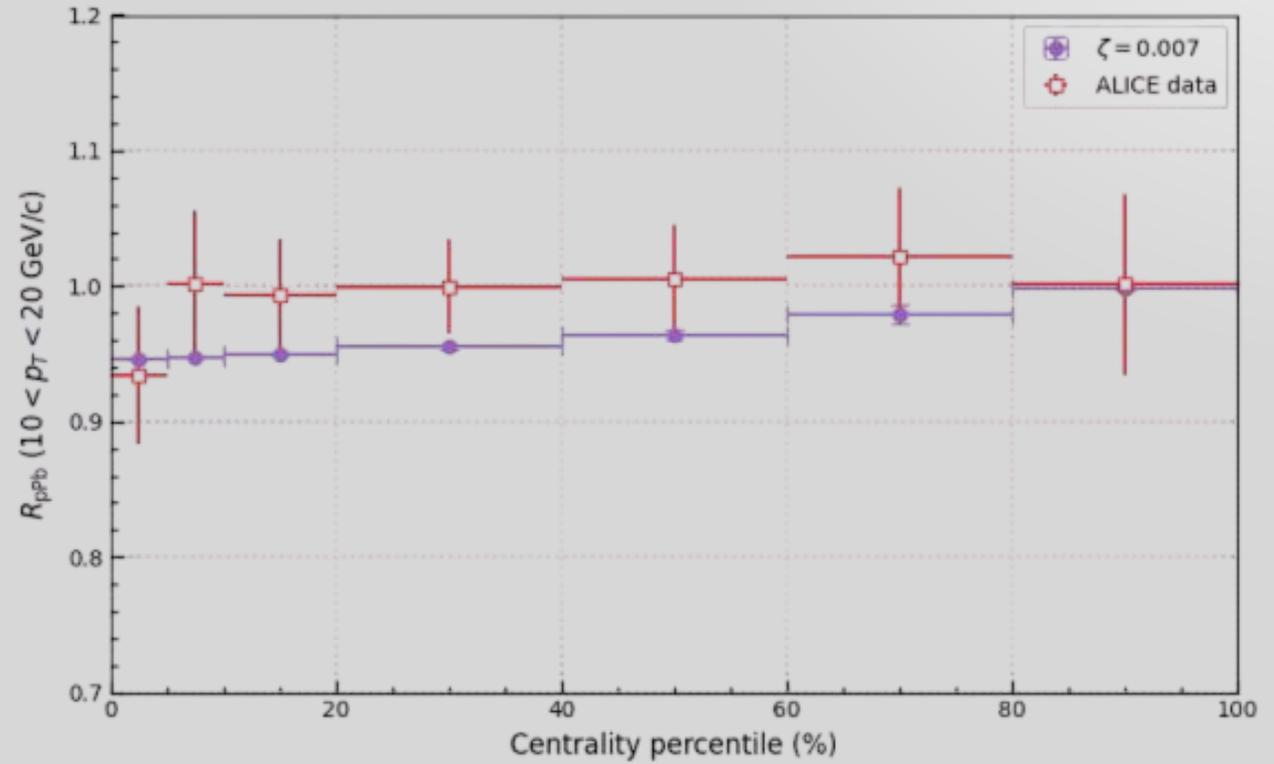- t_0 is not a directly measurable quantity, but is related to p_T via the uncertainty relation:

$$t_0 \simeq 0.43 \text{ fm/c}$$

$$\kappa' = \frac{3}{2} \cdot \frac{\langle N_{ch} \rangle}{\langle N_{part} \rangle} = \frac{3}{2} \cdot \frac{161}{7.87} = 30.7$$

■ Result and analysis

a. Multiplicity dependence parameters

- σ_{MP} & ζ are determined by fitting jet quenching measure $Y_{pPb} = \frac{N_{trig}}{N_{bin}} = \int ds\, e^{-\zeta N_k} P_{jet}$ to R_{pPb}.

- σ_{MP} = 1.4 mb is fixed, which is estimated in the previous study .

- ζ = 0.007.

Result and Analysis

b.   CMS $\Delta\phi$ distribution ($N_{ch} < 35$, $35 \leq N_{ch} < 90$)

Column : same p_T bin.
Row   : same N_{ch} interval.

- The best-fit parameters are
  $q = 1.0$ GeV/c, $T = 0.74$ GeV

- In ($1 < p_T < 2$ GeV/c , $35 \leq N_{ch} < 90$),
  ridge structure begins to emerge on the near side.

- The difference around $|\Delta\phi| \approx 1.5$
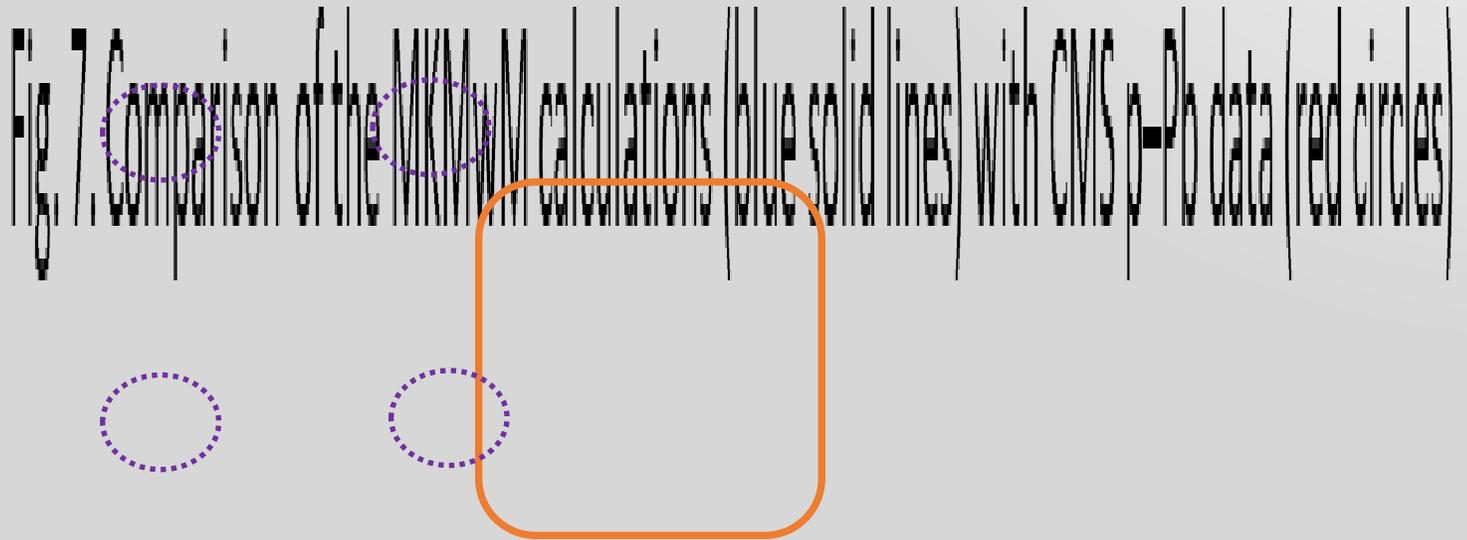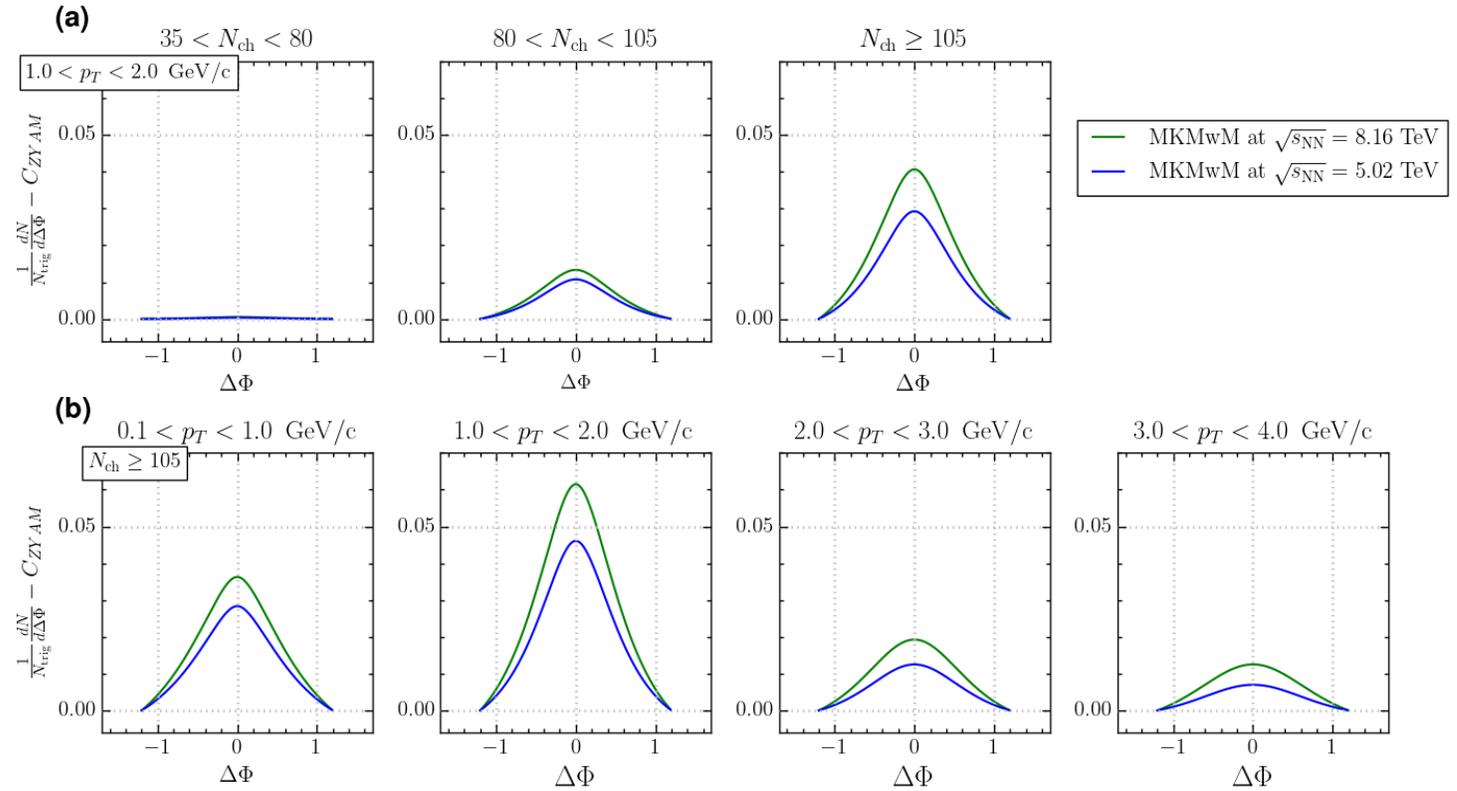 originates from *the away side ridge contribution.*

Fig. 7. Comparison of the MKMwM calculations (blue solid lines) with CMS p–Pb data (red circles) at 5.02 TeV. The associated yield per trigger is shown as a function of $\Delta\varphi$ for different p_T and N_{ch}.

MKMwM prediction to pPb collisions at 8.16 TeV

T	q
0.84 GeV	1.0 GeV