BARYONS 2025

ϕ and Ω Production via Recombination of Parton Showers in Heavy Ion Collisions

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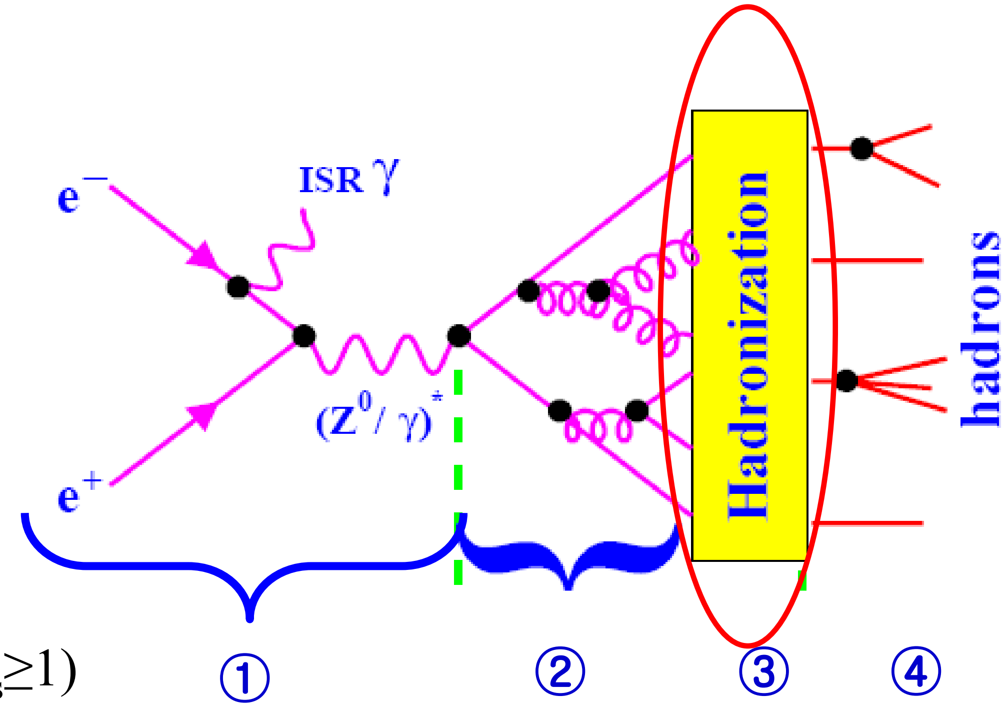
Motivation

"QCD event:": $e^+e^- \longrightarrow (Z^0/\gamma)^* \longrightarrow q\bar{q} \longrightarrow \text{hadrons}$

① Electroweak "hard interaction"

② QCD Perturbative Phase ($\alpha_s < 1$)
Fixed order

- α_s^2 (two loop : NNLO)
- α_s^3(some quantities) or "parton shower"
- NNLA ; Next-to-Leading Logarithm Approximation; Summations to all orders

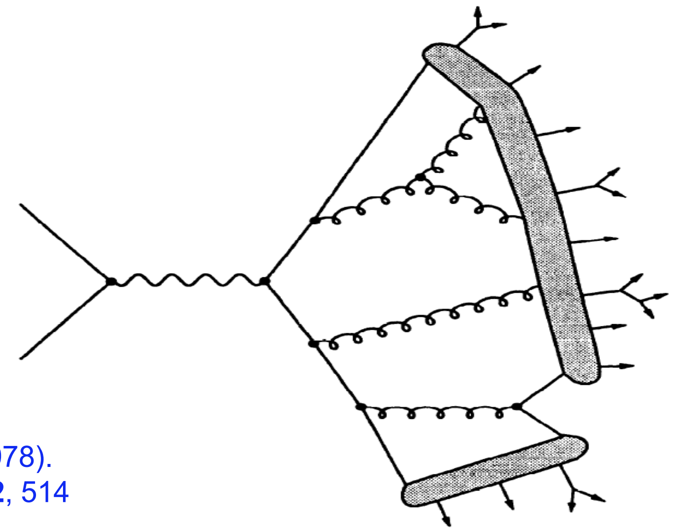


③ QCD Non-perturbative Phase ($\alpha_s \geq 1$)

④ Resonance Decay (unstable heavy hadrons decay into pions, kaons, and protons)

Hadronization (Fragmentation)

❑ Fragmentation Models;



- Independent Fragmentation

$$\frac{dN}{d^2 p_{\text{had}}} = \sum_{\text{jet}} \int dz \frac{dN}{d^2 p_{\text{jet}}} \frac{D_{\text{had/jet}}(z, Q^2)}{z^2}$$

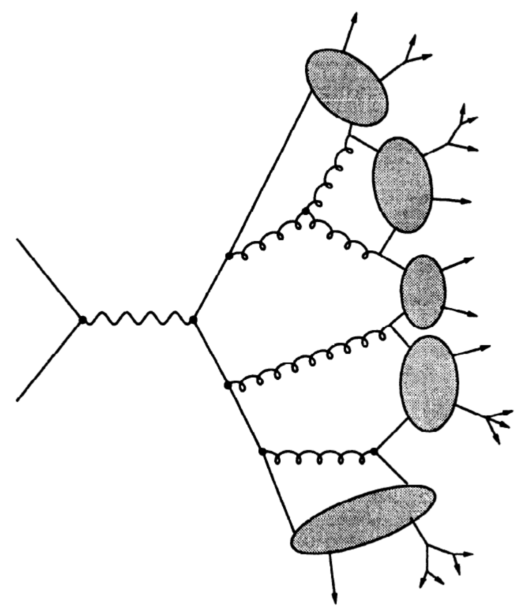
$$z = \frac{p_h}{p_j}$$

Field and Feynman, Nucl. Phys. B **136**, 1 (1978).
Kniehl, Kramer and Potter, Nucl. Phys. B **582**, 514 (2000)

- String Fragmentation (PYTHIA)

Andersson, Gustafson and Soderberg, Nucl. Phys. B **264**, 29 (1986)

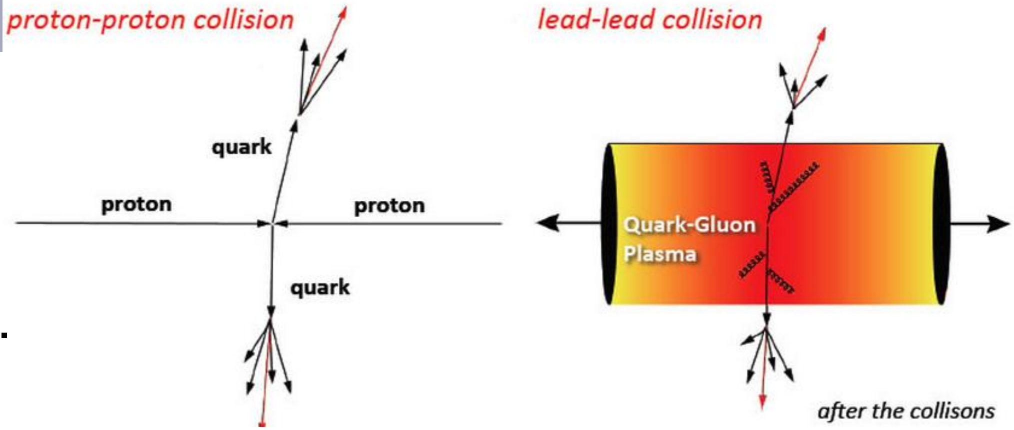
$$f(z) \propto z^{-1}(1-z)^a \exp\left(-\frac{bm_\perp^2}{z}\right)$$



- Cluster Fragmentation (HERWIG)

Marchesini, Webber, Abbiendi, Knowles, Seymour and Stanco, Comput. Phys. Commun. **67**, 465 (1992).

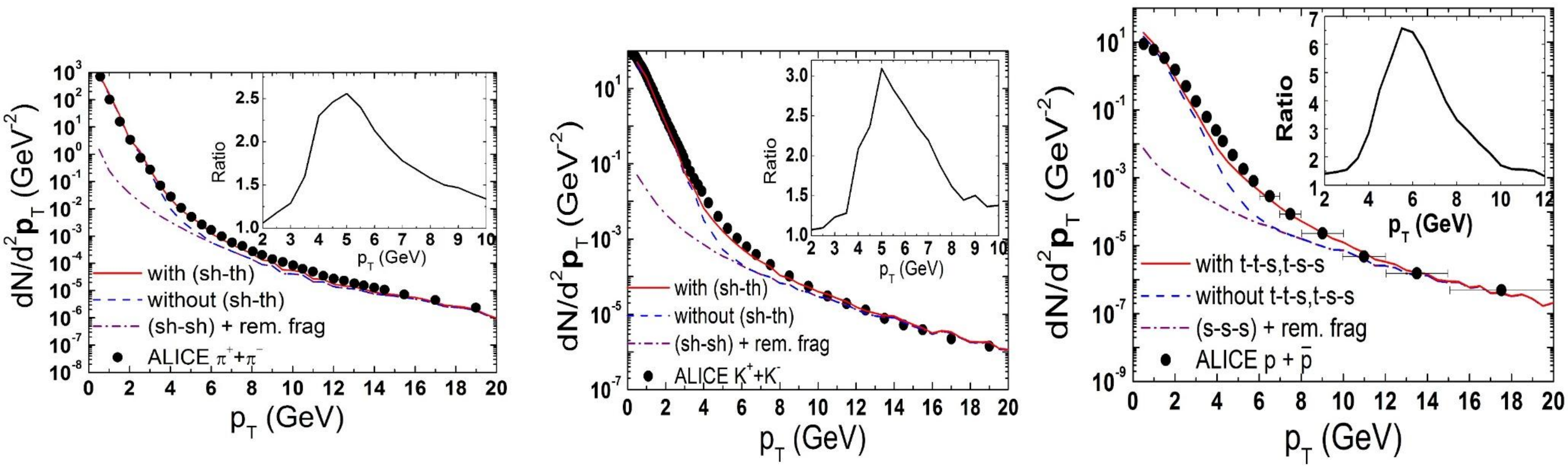
- Hadronization from QGP may be different from that of partons in elementary collisions (in vaccum).



- The fragmentation model could not describe hadron p_T spectra in low and intermediate momentum region which are very crucial for studying medium-effects in hadronization.

- Recombination Models
  - Greco, Ko, and Levai (Phys. Rev. C 68,034904, Phys. Rev. Lett. 90, 202302)
    recombination of minijets and thermal quarks in QGP
  - FMNB (Phys. Rev. Lett. 90, 202303)
    Hadrons at $p_T < 5$ GeV are formed by recombination of partons from the dense parton phase created in central collisions.
  - Hwa and Yang (Phys. Rev. C 67, 034902, Phys. Rev. C 70, 024905)
    Use SPD obtained from FF <-experimental results.
  - KCH, Fries, and Ko. (Phys. Rev. C 93, 045207); Jet fragmentation via recombination of parton showers.

- Fries, KCH, and Ko: Jet hadronization via recombination of parton showers in vacuum and medium, NPA 956, 601-604
Works for pion, kaons, and protons are suggested.



- ϕ and Ω production for RHIC (Au+Au @200GeV) are studied in a dynamical quark coalescence (**Chen and Ko, PRC 73, 044903**) (AMPT -> dynamic quark coalescence)

Our work

Generating Parton in Central Pb+Pb Collisions at $\sqrt{s} = 5.02$ TeV

- Partons are classified into two components: **thermal partons in QGP** and **shower partons inside jets**
  - Thermal partons are dominant in low momentum region; exponential law; generated by Blast Wave Model
  - Jets are dominant in high momentum region; power law. Jet partons are generated by HIJING (the Heavy Ion Jet INteraction Generator).
  - Energetic jets will be branch into shower partons. Meanwhile, they experience energy loss and jet quenching. This simulation is done by a showering subroutine of Q-PYTHIA, a Monte Carlo medium-modified shower parton generator.
  - Gluons are forcibly decayed into $u\bar{u}, d\bar{d},$ or $s\bar{s}$ isotropically in their rest frame.
- We have done this work in an event-by-event basis.

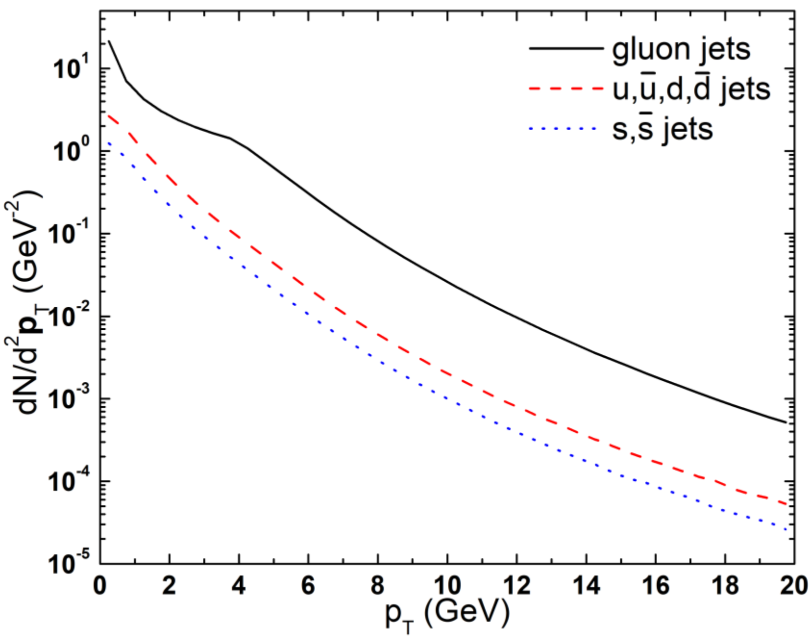
Thermal Partons in QGP

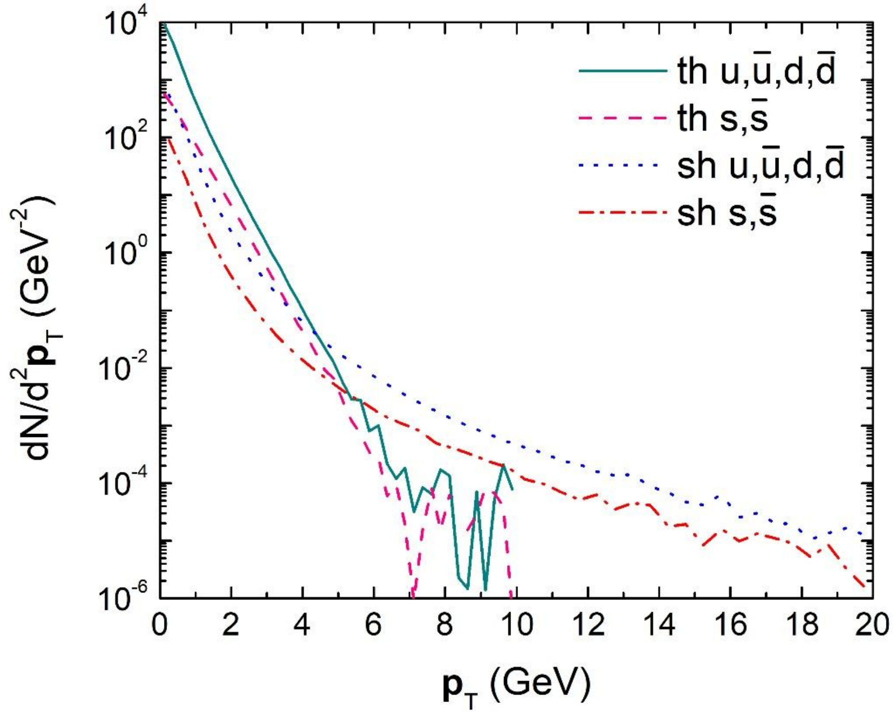
- Use blast wave model in central Pb-Pb collisions (0-10 %) at $\sqrt{s}$=5.02 TeV.
- Distribution of thermal partons at low momenta is parameterized as
$$\frac{dN_{q,\bar{q}}}{d^2\mathbf{r}_T d^2\mathbf{p}_T} = \frac{g_{q,\bar{q}}\tau m_T}{(2\pi)^3}\exp\left(-\frac{\gamma_T(m_T-\mathbf{p}_T\cdot\mathbf{v}_T)\mp\mu_b}{T}\right),$$
where $v_T = \beta\left(\frac{\boldsymbol{r}_T}{R_\perp}\right)$
- Isotropic in transverse plane and boost invariant longitudinally in $|y| < 0.5$.
$z = \tau_c \sinh y$, $t = \tau_c \cosh y$; $p_z = m_T \sinh\eta$, $E = m_T \cosh\eta$
Bjorken correlation $\eta = y$.
- Parameters: $R_\perp = 14.1$ fm, $\tau_c = 8$ fm/c, the volume $V \approx 5000$ fm^3 , $\beta = 0.7\ c$, and $T = 175$ MeV,
Minissale, Plumari, Sun, and Greco, Eur. Phys. J. C(2024) 84:228

Shower Partons in Jets

- HIJING Monte Carlo generator generates jet partons produced at central Pb-Pb collisions at $\sqrt{s} = 5.02$ TeV available at LHC.

- The obtained jets are branched into shower partons by a subroutine of Q-PYTHIA. In this process, shower partons experience medium modification.
  Armesto, Conqueiro, and Salgado, *Eur.Phys.J.C* 63 (2009) 679-690
  - Q-Pythia provides medium-modified splitting:
    $$P^{tot}(z) = P^{vac}(z) + \Delta P^{med}(z)$$
    Wang, Guo, NPA 696 (2001),788, Armesto, Cnqueiro, Salgado, Xiang, JHEP 0802 (2008), 048, Ovanesyan and Vitev, J. High Energy Phys. 06 (2011) 080.
  - Based BDMPS **(Baier-Dokshitzer-Mueller-Peigné-Schiff)**
    $$\frac{dN}{dzdk_{\perp}^{2}} = \frac{\alpha_s}{2\pi k_{\perp}^{2}}[P(z) + \Delta P^{med}(z)]$$





For the space-time information of shower partons, they are assumed to be distributed uniformly on the transverse plane z=0 at time t =0.
After $t = 0$, shower partons move with their velocities $\mathbf{v} = \mathbf{p}/E$ during their lifetimes $\tau = \frac{\gamma}{Q}$ in the lab frame, where $\gamma = 1/\sqrt{1 - v^2}$ and Q: virtuality of the parton.
Propagate all shower partons until their proper times reach τ_c

Coalescence model

- Coalescence formulas to form meson and baryon are given

$$\frac{dN_M}{d^3\mathbf{P}_M} = g_M \int d^3\mathbf{x}_1 d^3\mathbf{p}_1 d^3\mathbf{x}_2 d^3\mathbf{p}_2 f_q(\mathbf{x}_1,\mathbf{p}_1) f_{\bar{q}}(\mathbf{x}_2,\mathbf{p}_2)$$
$$\times W_M(\mathbf{y},\mathbf{k})\delta(\mathbf{P}_M - \mathbf{p}_1 - \mathbf{p}_2)$$ and

$$\frac{dN_B}{d^3\mathbf{P}_B} = g_B \int d^3\mathbf{x}_1 d^3\mathbf{p}_1 d^3\mathbf{x}_2 d^3\mathbf{p}_2 d^3\mathbf{x}_3 d^3\mathbf{p}_3 f_{q_1}(\mathbf{x}_1,\mathbf{p}_1) f_{q_2}(\mathbf{x}_2,\mathbf{p}_2)$$
$$\times f_{q_3}(\mathbf{x}_3,\mathbf{p}_3) W_B(\mathbf{y},\mathbf{k};\mathbf{y}_2,\mathbf{k}_2)\delta(\mathbf{P}_B - \mathbf{p}_1 - \mathbf{p}_2 - \mathbf{p}_3),$$ where

$$\mathbf{y} = \mathbf{x}_1 - \mathbf{x}_2,\ \mathbf{k} = \frac{\mathbf{P}_1 - \mathbf{P}_2}{2};\ \mathbf{y}_2 = \frac{\mathbf{x}_1 + \mathbf{x}_2}{2} - \mathbf{x}_3,\ \mathbf{k}_2 = \frac{\mathbf{p}_1 + \mathbf{p}_2 - 2\mathbf{p}_3}{3}$$

Han, Fries, Ko, PRC **93**, 045207(2016)

- The width parameters for ϕ and Ω are $\sigma_\phi = 0.46$ fm and σ_Ω=0.495 fm which are related to harmonic oscillator frequency and reduced constituent masses. Cho, et al. PRC **84**, 064910 (2011)

- $g_M = \frac{1}{12}$ and $g_B = \frac{1}{216}$; color spin degeneracy factors.

Hadronization of Partons

- We used the Gaussian Wigner functions for meson and baryon in their ground states

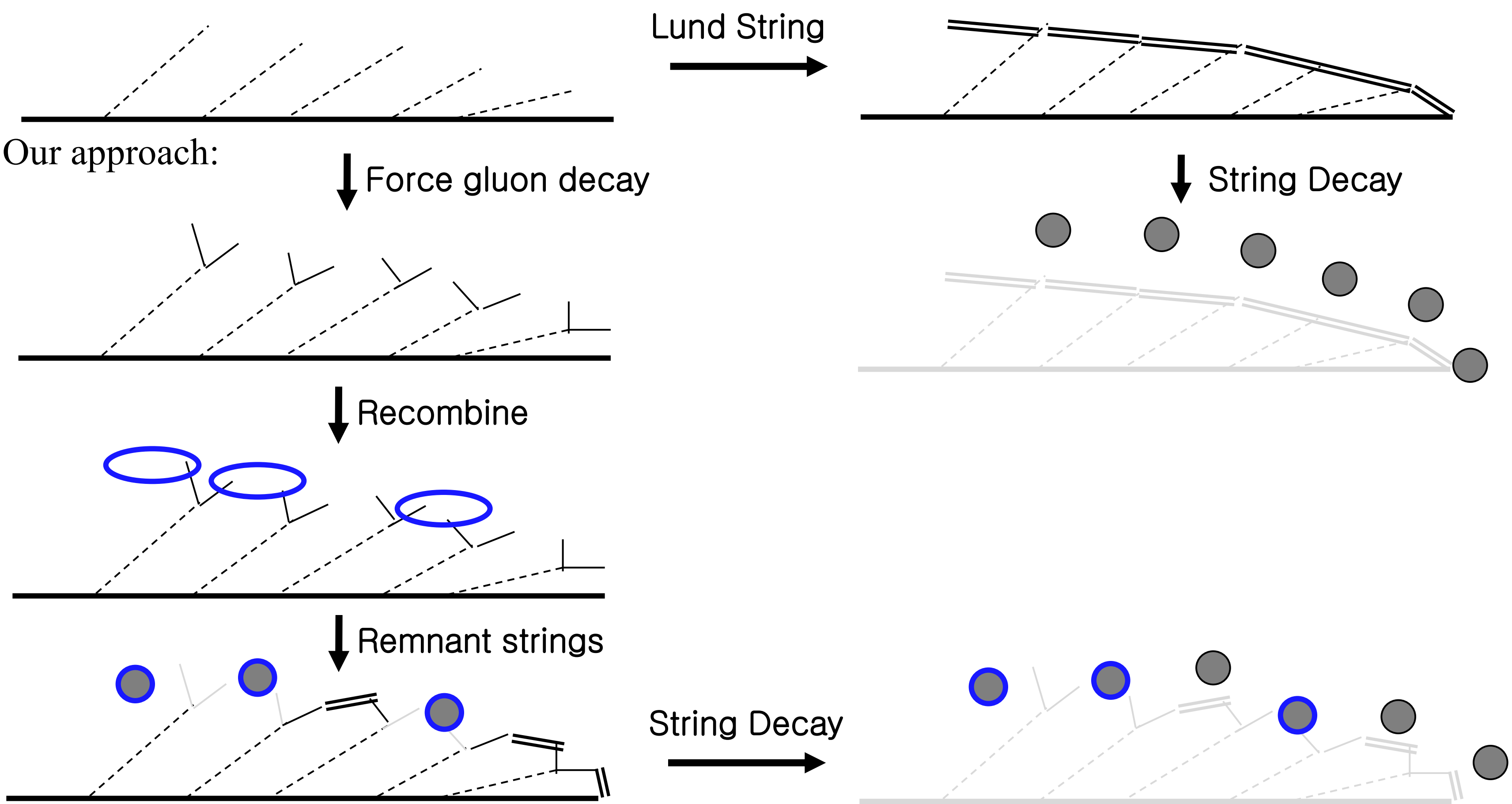
$$W_M(y,k) = 8\exp\left(-\frac{y^2}{\sigma_M^2} - k^2\sigma_M^2\right),$$

$$W_B(y_1,y_2;k_1,k_2) = 64\exp\left(-\frac{y_1^2-y_2^2}{\sigma_B^2} - (k_1^2+k_2^2)\sigma_B^2\right)$$

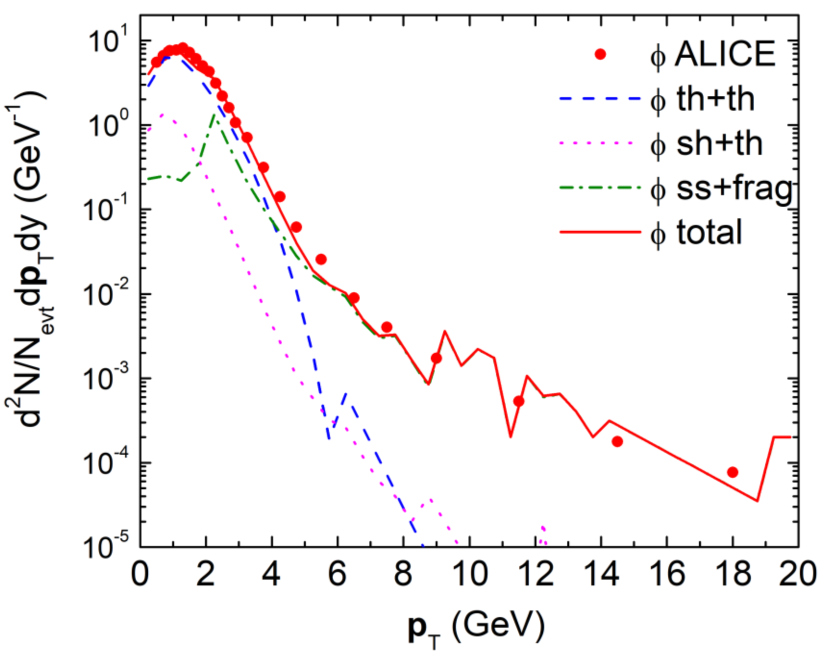
- Treat three contributions separately: thermal-thermal recombination, thermal-shower recombination, shower-shower recombination, and remnant shower fragmentation by using PYEXEC, a subroutine of PYTHIA.

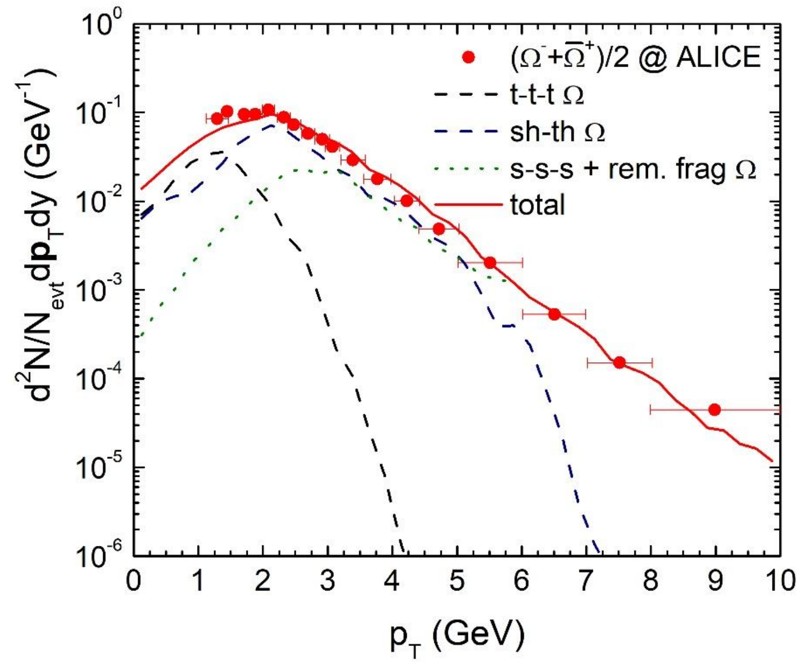
Recombination + Remnant Strings

- Extract Q-PYTHIA parton showers evolved to a scale Q_0.
- Standard PYTHIA Lund string fragmentation:
- Our approach:



p_T Spectra of ϕ and Ω





ALICE, Phys. Rev. C **106**, 034907

ALICE, PoS(EPS-HEP2017)168

Conclusion and Discussions

- Studied ϕ meson and Ω baryon production in Pb–Pb collisions at $\sqrt{s_{NN}}$ = 5.02 TeV.
- Used a hybrid hadronization model combining
  - Thermal + Thermal,
  - Shower + Thermal, and
  - Shower + Shower recombination,
    along with remnant fragmentation via PYTHIA.
- Thermal partons were generated from the blast-wave model, and shower partons originated from HIJING jets evolved by Q-PYTHIA.
- The resulting p_T spectra of ϕ and Ω agree well with LHC experimental data for central collisions.
- Results indicate that recombination of parton showers still plays a key role in strange hadron production in the QGP medium.

Thank you for your attention.

Backup Slides

PHSP information of Shower partons

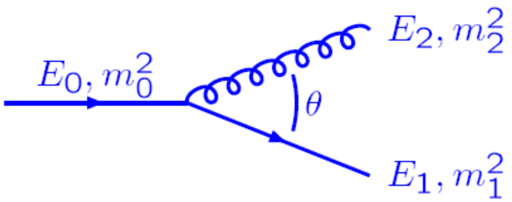
- Perturbative shower evolution in vacuum down to Q_0= 1 GeV. Below Q_0 , PYTHIA sets it to 0.
- Virtualities of gluons are randomly chosen between $2m_{u.d}$ and $m_{\max} > 2m_s$, and gluons are forced to decay isotropically into a $q\bar{q}$ in the gluon rest frame. ($m_{\max}$=1.25 GeV, $m_{u.d}$=0.33 GeV, $m_s = 0.5$ GeV)
- Above the strangeness threshold, the ratio of light to strange quarks is

$$\frac{\Gamma(g^* \to u\bar{u}, d\bar{d})}{\Gamma(g^* \to s\bar{s})} = 2\frac{m^2 + 2m_{u,d}^2}{m^2 + m_s^2}\sqrt{\frac{m^2 - 4m_{u,d}^2}{m^2 - 4m_s^2}}$$

- From energy-time uncertainty relation, lifetimes of partons are taken as 1/Q in their rest frame. (Q : virtualities of partons)
- The time is boosted into the lab frame and partons move with velocity $\frac{\mathbf{p}}{E}$ during this time.

Evolution of final state shower

- It is controlled by evolution equations



$$\mathrm{d}\mathcal{P}_{a\to bc} = \frac{\alpha_s}{2\pi}\frac{\mathrm{d}Q^2}{Q^2} P_{a\to bc}(z)\,\mathrm{d}z \cdot (\mathrm{Sudakov})$$

  - DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) equation

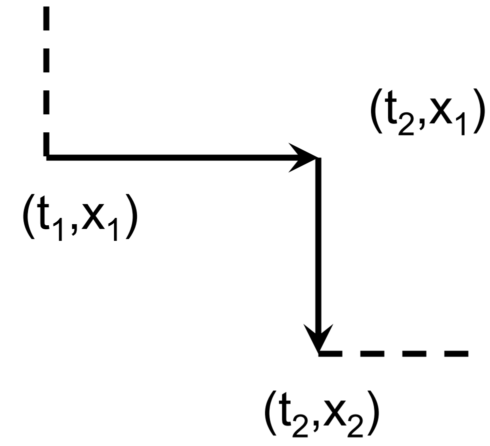
$$P_{q\to qg} = \frac{4}{3}\frac{1+z^2}{1-z}$$

$$P_{g\to gg} = 3\frac{(1-z(1-z))^2}{z(1-z)}$$

$$P_{g\to q\overline{q}} = \frac{n_f}{2}(z^2+(1-z)^2) \quad (n_f = \text{no. of quark flavours})$$

  - decreasing E, m^2, θ

Monte Carlo Method



- MC branching algorithm : given virtual mass scale and momentum fraction (t_1,x_1) after some step of the evolution, or as initial conditions, it generates values (t_2,x_2) after the next step.
- Since probability of evolving from t_1 to t_2 w/out branching is $\Delta(t_2)/\Delta(t_1)$, t_2 can be generated with the correct distribution by solving
  $\Delta(t_2)/\Delta(t_1) = R$
  where R is random number (uniform on [0,1]) and

$$\Delta(t) \equiv \exp\left[-\int_{t_0}^{t} \frac{dt'}{t'} \int dz \frac{\alpha_s}{2\pi} \hat{P}(z)\right].$$

The recombination model

$$\langle \mathbf{r}_1, \mathbf{r}_2 | i \rangle = \phi_1(\mathbf{r}_1)\phi_2(\mathbf{r}_2) \qquad \langle \mathbf{r}_1, \mathbf{r}_2 | f \rangle = \frac{1}{\sqrt{V}} e^{i\mathbf{K}_M \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2} \Phi_M(\mathbf{r}_1 - \mathbf{r}_2)$$

$\mathbf{r}_1$, $\mathbf{r}_2$: coordinates of a system with quark and antiquark.
$\Phi_M(\mathbf{r}_1 - \mathbf{r}_2)$: internal wave function of meson

Probability of the recombination of a quark and antiquark pair into a meson is calculated by the squared overlap amplitude based on the sudden approximation

$$\begin{aligned}
\mathcal{P} &= \sum_{\mathbf{K}_M} |\langle f | i \rangle|^2 = \sum_{\mathbf{K}_M} \int d^3\mathbf{r}_1 d^3\mathbf{r}_2 d^3\mathbf{r}_1' d^3\mathbf{r}_2' \langle f | \mathbf{r}_1, \mathbf{r}_2 \rangle \langle \mathbf{r}_1, \mathbf{r}_2 | i \rangle \langle i | \mathbf{r}_1', \mathbf{r}_2' \rangle \langle \mathbf{r}_1', \mathbf{r}_2' | f \rangle \\
&= V \int \frac{d^3\mathbf{K}_M}{(2\pi)^3} d^3\mathbf{r}_1 d^3\mathbf{r}_2 d^3\mathbf{r}_1' d^3\mathbf{r}_2' \frac{1}{V} e^{-i\mathbf{K}_M \cdot (\mathbf{r}_1 + \mathbf{r}_2)/2} \Phi^*(\mathbf{r}_1 - \mathbf{r}_2) \phi_1(\mathbf{r}_1) \phi_2(\mathbf{r}_2) \\
&\times \phi_1^*(\mathbf{r}_1') \phi^*(\mathbf{r}_2') e^{i\mathbf{K}_M \cdot (\mathbf{r}_1' + \mathbf{r}_2')/2} \Phi_M(\mathbf{r}_1' - \mathbf{r}_2')
\end{aligned}$$

where I used $\sum_{\mathbf{P}} = V \int \frac{d^3\mathbf{P}}{(2\pi)^3}$

$$\begin{aligned}
\mathcal{P} = \int & \frac{d^3\mathbf{K}_M}{(2\pi)^3} d^3\mathbf{x}_1 d^3\mathbf{x}_2 d^3\mathbf{y}_1 d^3\mathbf{y}_2 e^{-i\mathbf{K}_M \cdot (\mathbf{y}_1 + \mathbf{y}_2)/2} \Phi_M^* \left(\mathbf{x}_1 + \frac{\mathbf{y}_1}{2} - \mathbf{x}_2 - \frac{\mathbf{y}_2}{2} \right) \\
& \times \phi_1 \left(\mathbf{x}_1 + \frac{\mathbf{y}_1}{2} \right) \phi_2 \left(\mathbf{x}_2 + \frac{\mathbf{y}_2}{2} \right) \phi_1^* \left(\mathbf{x}_1 - \frac{\mathbf{y}_1}{2} \right) \phi_2^* \left(\mathbf{x}_2 - \frac{\mathbf{y}_2}{2} \right) \\
& \times \Phi_M \left(\mathbf{x}_1 - \frac{\mathbf{y}_1}{2} - \mathbf{x}_2 + \frac{\mathbf{y}_2}{2} \right)
\end{aligned}$$

definition of new coordinates

$$\mathbf{x}_1 = \frac{\mathbf{r}_1 + \mathbf{r}_1'}{2}; \quad \mathbf{x}_2 = \frac{\mathbf{r}_2 + \mathbf{r}_2'}{2}$$
$$\mathbf{y}_1 = \mathbf{r}_1 - \mathbf{r}_1'; \quad \mathbf{y}_2 = \mathbf{r}_2 - \mathbf{r}_2',$$

Taking the Wigner transformation,

$$f_i(\mathbf{x}_i, \mathbf{k}_i) = \int d^3\mathbf{y}_i \phi_i^* \left(\mathbf{x}_i - \frac{\mathbf{y}_1}{2}\right) \phi_i \left(\mathbf{x}_i + \frac{\mathbf{y}_i}{2}\right) e^{-i\mathbf{k}_i \cdot \mathbf{y}_i}, \qquad (i = 1, 2)$$

we have $\phi_i^* \left(\mathbf{x}_i - \frac{\mathbf{y}_i}{2}\right) \phi_i \left(\mathbf{x}_i + \frac{\mathbf{y}_i}{2}\right) = \int \frac{d^3\mathbf{k}_i}{(2\pi)^3} f_i(\mathbf{x}_i, \mathbf{k}_i) e^{i\mathbf{k}_i \cdot \mathbf{y}_i}$

Similarly, we obtain

$$\Phi_M^* \left((\mathbf{x}_1 - \mathbf{x}_2) + \frac{\mathbf{y}_1 - \mathbf{y}_2}{2}\right) \Phi_M \left((\mathbf{x}_1 - \mathbf{x}_2) - \frac{\mathbf{y}_1 - \mathbf{y}_2}{2}\right) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} W_M(\mathbf{x}_1 - \mathbf{x}_2, \mathbf{k}) e^{-i\mathbf{k}\cdot(\mathbf{y}_1 - \mathbf{y}_2)}$$

where $W_M(\mathbf{x}_1 - \mathbf{x}_2, \mathbf{k})$ is the Wigner function of the meson.

$$\begin{aligned}
\mathcal{P} &= \int \frac{d^3\mathbf{K}_M}{(2\pi)^3} d^3\mathbf{x}_1 d^3\mathbf{x}_2 d^3\mathbf{y}_1 d^3\mathbf{y}_2 \frac{d^3\mathbf{k}_1}{(2\pi)^3} \frac{d^3\mathbf{k}_2}{(2\pi)^3} \frac{d^3\mathbf{k}}{(2\pi)^3} e^{-i\mathbf{K}_M \cdot \frac{\mathbf{y}_1 + \mathbf{y}_2}{2}} \\
&\times f_1(\mathbf{x}_1, \mathbf{k}_1) e^{i\mathbf{k}_1 \cdot \mathbf{y}_1} f_2(\mathbf{x}_2, \mathbf{k}_2) e^{i\mathbf{k}_2 \cdot \mathbf{y}_2} W_M(\mathbf{x}_1 - \mathbf{x}_2, \mathbf{k}) e^{-i\mathbf{k}\cdot(\mathbf{y}_1 - \mathbf{y}_2)} \\
&= \int \frac{d^3\mathbf{K}_M}{(2\pi)^3} d^3\mathbf{x}_1 d^3\mathbf{x}_2 \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2 d^3\mathbf{k}}{(2\pi)^3} \delta^3\left(-\frac{\mathbf{K}_M}{2} + \mathbf{k}_1 - \mathbf{k}\right) \delta^3\left(-\frac{\mathbf{K}_M}{2} + \mathbf{k}_2 + \mathbf{k}\right) \\
&\times f_1(\mathbf{x}_1, \mathbf{k}_1) f_2(\mathbf{x}_2, \mathbf{k}_2) W_M(\mathbf{x}_1 - \mathbf{x}_2, \mathbf{k}) \\
&= \int \frac{d^3\mathbf{K}_M}{(2\pi)^3} d^3\mathbf{x}_1 d^3\mathbf{x}_2 \frac{d^3\mathbf{k}_1 d^3\mathbf{k}_2}{(2\pi)^3} f_1(\mathbf{x}_1, \mathbf{k}_1) f_2(\mathbf{x}_2, \mathbf{k}_2) \\
&\times W_M\left(\mathbf{x}_1 - \mathbf{x}_2, \frac{\mathbf{k}_1 - \mathbf{k}_2}{2}\right) \delta^3(\mathbf{k}_1 + \mathbf{k}_2 - \mathbf{K}_M)
\end{aligned}$$

As a result, probability distribution of a quark and antiquark pair to form a meson can written as

$$\frac{d\mathcal{P}}{d^3\mathbf{K}_M} = \int \frac{d^3\mathbf{x}_1 d^3\mathbf{k}_1}{(2\pi)^3} \frac{d^3\mathbf{x}_2 d^3\mathbf{k}_2}{(2\pi)^3} f_1(\mathbf{x}_1, \mathbf{k}_1) f_2(\mathbf{x}_2, \mathbf{k}_2) W_M(\mathbf{y}, \mathbf{k})\, \delta^3(\mathbf{K}_M - \mathbf{k}_1 - \mathbf{k}_2)$$

where $\mathbf{y} = \mathbf{x}_1 - \mathbf{x}_2, \quad \mathbf{k} = \dfrac{\mathbf{k}_1 - \mathbf{k}_2}{2}$

Phase distribution functions of quarks and antiquarks in parton showers $f_q(\mathbf{x}, \mathbf{k})$ and $f_{\bar{q}}(\mathbf{x}, \mathbf{k})$ are normalized as $N_{q,\bar{q}} = \int d^3x d^3p f_{q,\bar{q}}(\mathbf{x}, \mathbf{p})$;

For the formation of mesons from quark-antiquark and baryons from three quarks,

we have

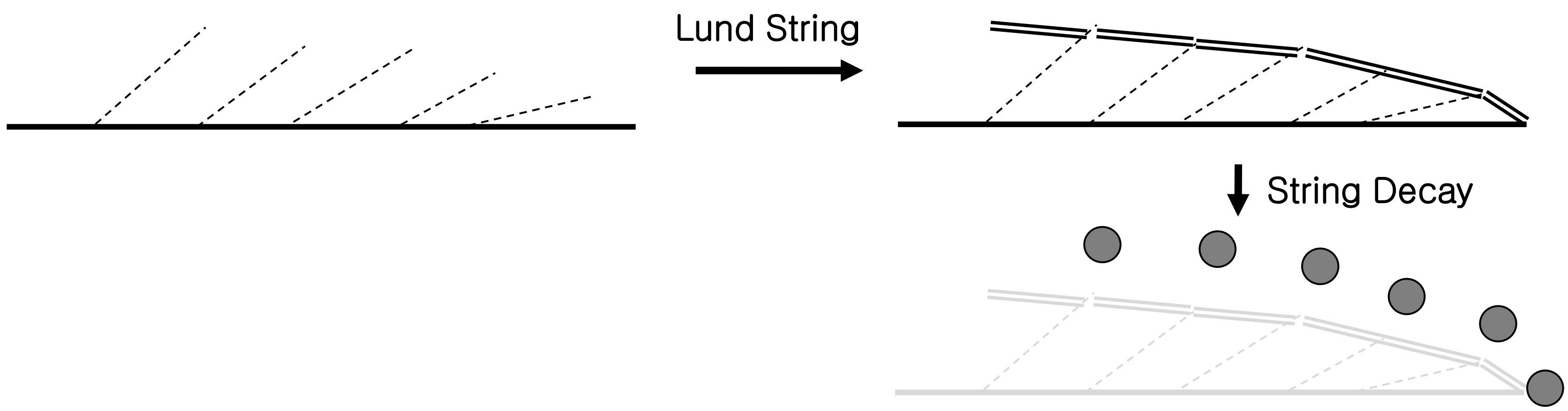
$$\frac{dN_M}{d^3\mathbf{K}_M} = g_M \int d^3\mathbf{x}_1 d^3\mathbf{k}_1 d^3\mathbf{x}_2 d^3\mathbf{k}_2 f_q(\mathbf{x}_1, \mathbf{k}_1) f_{\bar{q}}(\mathbf{x}_2, \mathbf{k}_2) W_M(\mathbf{y}, \mathbf{k}) \delta^{(3)}(\mathbf{K}_M - \mathbf{k}_1 - \mathbf{k}_2)$$

$$\frac{dN_B}{d^3\mathbf{K}_B} = g_B \int d^3\mathbf{x}_1 d^3\mathbf{k}_1 d^3\mathbf{x}_2 d^3\mathbf{k}_2 d^3\mathbf{x}_3 d^3\mathbf{k}_3 f_{q_1}(\mathbf{x}_1, \mathbf{k}_1) f_{q_2}(\mathbf{x}_2, \mathbf{k}_2) \times f_{q_3}(\mathbf{x}_3, \mathbf{k}_3) W_B(\mathbf{y}_1, \mathbf{k}_1; \mathbf{y}_2, \mathbf{k}_2) \delta^{(3)}(\mathbf{K}_B - \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3),$$

$W_M(\mathbf{y}, \mathbf{k})$ and $W_B(\mathbf{y}_1, \mathbf{k}_1; \mathbf{y}_2, \mathbf{k}_2)$ are the Wigner functions of meson and baryon in terms of relative coordinates and relative momenta.

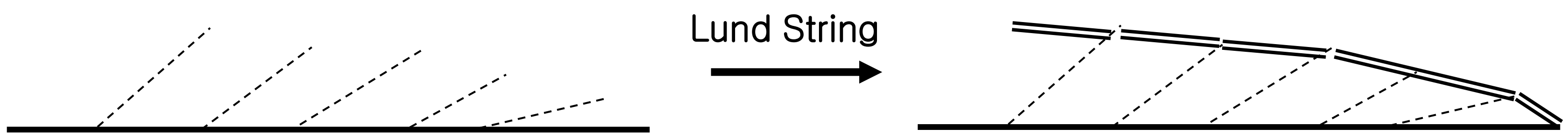
String Fragmentation

- Extract PYTHIA parton showers evolved to a scale Q_0.
- Standard PYTHIA Lund string fragmentation:



Recombination + Remnant Strings

- Extract PYTHIA parton showers evolved to a scale Q_0.
- Standard PYTHIA Lund string fragmentation:



- Our approach: