The EMFFs of the hyperon in the time like region

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Outline

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- **Background**
- **Theoretical framework**
- **Fit results**
- **Summary**

Background

- Electromagnetic form factor (EMFF)
- Spacelike region ($q^2 < 0$):
  structure about baryon
  baryon's inner electric and magnetic distribution
  measurement by $e^- B \longrightarrow e^- B$ (Fig. 1. left)
- Timelike region ($q^2 > 0$):
  polarized electric charges distribution (Sci.Bull. 68 (2023) 2729-2733)
  measurement by $e^- e^+ \longrightarrow B \bar{B}$ (Fig. 1. right)

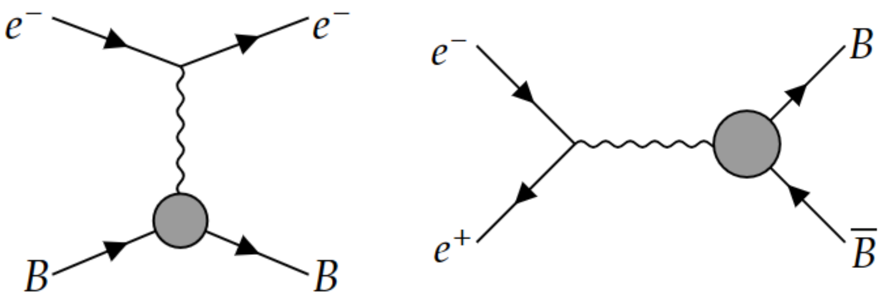


Fig. 1.

- First measurement baryon EMFFs proton,   in 1973, by ADONE73
- First measurement Lambda EMFFs in 1983 (DM2), Later, BaBar (2007) also measured this process.High-precision measurements of ($e^+e^- \to \Lambda\bar{\Lambda}$) by BESIII (2018, 2021, 2023).

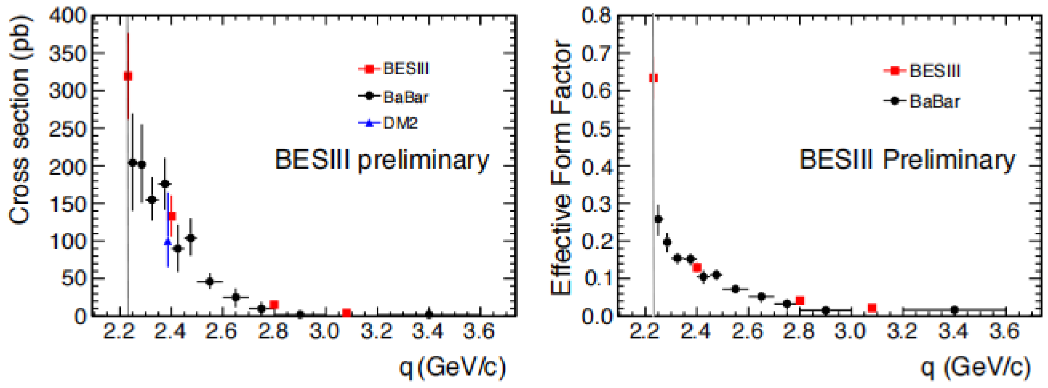


Fig. 2.

Theoretical framework

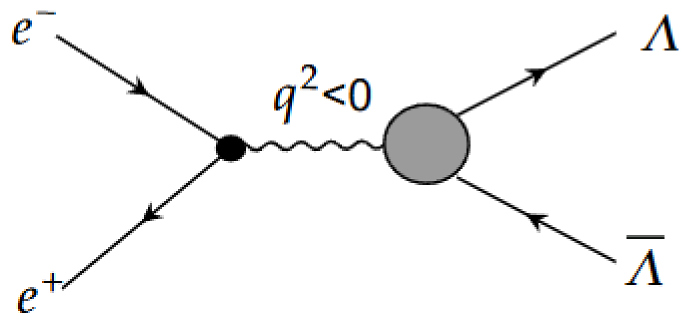


Fig. 3.

Theoretical study in Lambda's EMFFs by $e^+e^- \to \Lambda\bar{\Lambda}$

the cross section and differential cross section can be parameterized by electric form factor(G_E) and magnetic form factor(G_M). (eq. (1))

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2\beta}{4s} C(s)\left[|G_M^Y(s)|^2 (1+\cos^2\theta) + \frac{4M_Y^2}{s}|G_E^Y(s)|^2 \sin^2\theta \right]$$

$$\sigma_{e^+e^- \to \bar{Y}Y} = \frac{4\pi\alpha^2\beta}{3s} C(s)\left[|G_M(s)|^2 + \frac{2M_Y^2}{s}|G_E(s)|^2 \right]$$

$$|G_{\rm eff}(s)| = \sqrt{\frac{\sigma_{e^+e^- \to \bar{Y}Y}(s)}{\frac{4\pi\alpha^2\beta}{3s}C(s)\left[1+\frac{2M_Y^2}{s}\right]}} \quad (1)$$

- $\alpha = 1/137$
- Y： hyperon
- G_E: electric form factor
- G_M: magnetic form factor
- C(s): S-wave Sommerfeld-Gamow factor
- $C(y) = y/(1-e^{-y})$ with $y = \pi\alpha M_N/k_N$
- β: phase space factor, $\beta = k_N/k_e$
- G_{eff}: effective form factor

Theoretical framework

$$F_{LL'}^{\bar{Y}Y,e^+e^-} = -\frac{4\alpha}{9} f_L^{\bar{Y}Y} f_{L'}^{e^+e^-} \quad (2)$$

with

$$f_0^{\bar{Y}Y} = G_M + \frac{M_N}{\sqrt{s}} G_E, \qquad f_0^{e^+e^-} = 1 + \frac{m_e}{\sqrt{s}}$$

$$f_2^{\bar{Y}Y} = \frac{1}{\sqrt{2}}\left(G_M - \frac{2M_N}{\sqrt{s}} G_E\right), \qquad f_2^{e^+e^-} = \frac{1}{\sqrt{2}}\left(1 - \frac{2m_e}{\sqrt{s}}\right)$$

- eq. (2): Factorization of the partial wave amplitude
- eq. (3): Relation between cross section and partial wave amplitude

$$\sigma_{e^+e^-\to\Lambda\bar{\Lambda}} = \frac{3\pi\beta}{s} C(s)\left(|F_{00}|^2 + |F_{20}|^2 + |F_{02}|^2 + |F_{22}|^2\right) \quad (3)$$

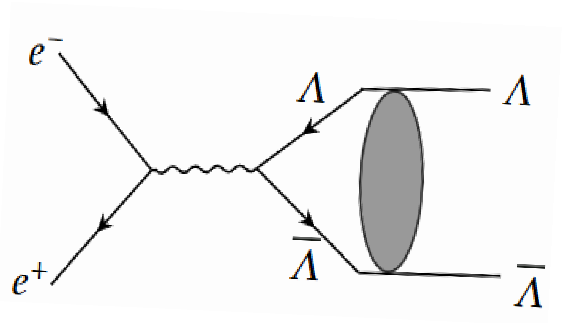


Fig. 4.

- The energy region under study is near the threshold
- Final state interaction(FSI) should be considered!
- complete feynman diagram is Fig. 4.
- Total angular momentum is conservation, we consider [3S1-3D1] coupled partial wave in final lambda system

Theoretical framework

Lagrangian and potential about FSI is derived from chiral effective field theory (χEFT) up to next-to-leading order (NLO). we think the potential consists of three part (eq. (4))

$$V^{total} = V^{contact} + V^{annilation} + V^{meson\ exchange} \tag{4}$$

Relevant feynman diagrams Fig.5

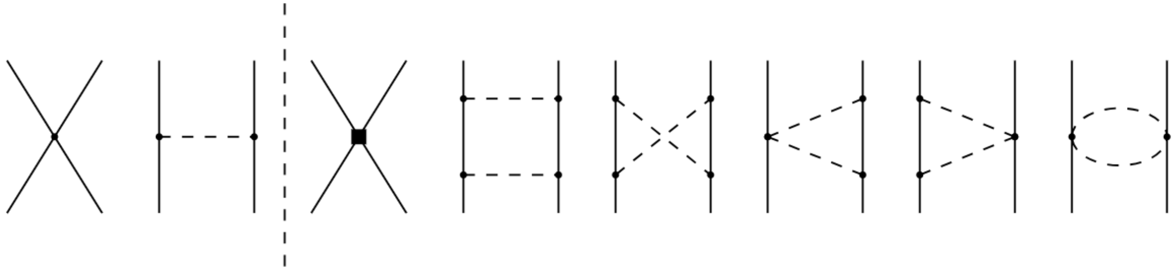


Fig. 5.

The contact term and annihilation part parameterized as follow. C is low-energy constants (LECs)

$$\begin{aligned}
V({}^3S_1) &= \tilde{C}_{3S_1} + C_{3S_1}(p^2 + p'^2) \\
V({}^3D_1 - {}^3S_1) &= C_{\epsilon_1} p'^2 \\
V({}^3S_1 - {}^3D_1) &= C_{\epsilon_1} p^2 \\
V({}^3D_1) &= 0
\end{aligned} \tag{5}$$

$$\begin{aligned}
V^{\mathrm{ann}}({}^3S_1) &= -i\left(\tilde{C}^a_{3S_1} + C^a_{3S_1} p^2\right)\left(\tilde{C}^a_{3S_1} + C^a_{3S_1} p'^2\right) \\
V^{\mathrm{ann}}({}^3D_1 - {}^3S_1) &= -iC^a_{\epsilon_1} p'^2 \left(\tilde{C}^a_{3S_1} + C^a_{3S_1} p^2\right) \\
V^{\mathrm{ann}}({}^3S_1 - {}^3D_1) &= -iC^a_{\epsilon_1} p^2 \left(\tilde{C}^a_{3S_1} + C^a_{3S_1} p'^2\right) \\
V^{\mathrm{ann}}({}^3D_1) &= -i\left[\left(C^a_{3S_1}\right)^2 + \left(C^a_{3D_1}\right)^2\right] p^2 p'^2
\end{aligned} \tag{6}$$

Theoretical framework

the meson exchange process: pseudoscalar-meson exchange lagrangian is derived from chiral effective field theory (χEFT)

$$\mathcal{L}_1 = -\frac{\sqrt{2}}{2f_0} tr\left(D\bar{B}\gamma^\mu\gamma_5\{\partial_\mu P, B\} + F\bar{B}\gamma^\mu\gamma_5[\partial_\mu P, B]\right) \qquad \mathcal{L}_2 = \frac{1}{4f_0^2} tr\left(i\bar{B}\gamma^\mu[[P,\partial_\mu P], B]\right) \qquad (7)$$

with

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix} \qquad P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

D and F: coupling constans
f_0: goldstone boson decay constant

Calculated the amplitude in old-fashioned time-ordered perturbation theory,we have

$$W_I = -\int dx^3 [\mathcal{L}_I(x)]_{x^0=0}$$

For example: Considering one meson exchange processes.

Amplitude: $\langle \Lambda\bar{\Lambda} \,|\, WGW \,|\, \Lambda\bar{\Lambda} \rangle$ (8)

Propagator: $G = 1/\left(z - H_0 + i0^+\right)$ (9)

Theoretical framework

Derive potential from amplitude: $\langle \Lambda\bar{\Lambda} | \, WGW \, | \Lambda\bar{\Lambda} \rangle = (2\pi)^3 \delta^3 (p_1 + p_2 - p_1' - p_2') V$ (9)

Defined the standard one meson exchange potential as $I_{B_1B_2 \to B_3B_4}$ called the isospin factor

$$V^{OBE}_{B_1B_2 \to B_3B_4} = -f_{B_1B_3P} f_{B_2B_4P} \frac{(\sigma_1 \cdot q)(\sigma_2 \cdot q)}{q^2 + m_p^2} \mathcal{I}_{B_1B_2 \to B_3B_4} \quad (10)$$

Other standard meson exchange process potential refer JHEP 08 (2024) 208

Table 1:
isospin factor table

Isospin	η
$I = 0$	1

Diagram	Isospin	Intermediate baryons	$\pi\pi$	$\eta\eta$	Intermediate baryons	$K\bar{K}$
Planar Box	$I = 0$	$\bar{\Lambda}\Lambda$	0	1	$\bar{N}N$	2
		$\bar{\Sigma}\Sigma$	3	0	$\bar{\Xi}\Xi$	2
Cross Box	$I = 0$	$\bar{\Lambda}\Lambda$	0	1	$\bar{\Xi}N, \bar{N}\Xi$	2
		$\bar{\Sigma}\Sigma$	3	0		

Theoretical framework

We can classify the potential we derived as a central potential (1 VC), a spin–spin potential ($\sigma_1 \cdot \sigma_2$ VS) and a tensor-type potential ($\sigma_1 \cdot q\sigma_2 \cdot q$ VT)

Transfer helicity potential $|JM;\lambda_1\lambda_2>$ into partial wave representation $|LSJ>$

$$
\begin{aligned}
&\langle j\,0\,j|V|j\,0\,j\rangle \\
&= 2\pi\int_{-1}^{1} dz\left\{V_C - 3V_\sigma + p'^2p^2\left(z^2-1\right)V_{\sigma L} - q^2V_{\sigma q} - k^2V_{\sigma k}\right\}P_j(z), \\
&\langle j\,1\,j|V|j\,1\,j\rangle \\
&= 2\pi\int_{-1}^{1} dz\left\{\left[V_c + V_\sigma + 2p'pzV_{SL} - p'^2p^2\left(1+3z^2\right)V_{\sigma L} + 4k^2V_{\sigma q} + \tfrac{1}{4}q^2V_{\sigma k}\right]P_j(z)\right. \\
&\quad \left. + \left[-p'pV_{SL} + 2p'^2p^2zV_{\sigma L} - 2p'p\left(V_{\sigma q} - \tfrac{1}{4}V_{\sigma k}\right)\right] \times \left(P_{j-1}(z) + P_{j+1}(z)\right)\right\} \\
&\langle j\pm 1,\ 1\,j|V|j\pm 1,\ 1\,j\rangle \\
&= 2\pi\int_{-1}^{1} dz\left\{p'p\left[-V_{SL} \pm \tfrac{2}{2j+1}\left(-p'pzV_{\sigma L} + V_{\sigma q} - \tfrac{1}{4}V_{\sigma k}\right)\right]P_j(z) + [V_C + V_\sigma + p'pz\right. \\
&\quad \left. V_{SL} + p'^2p^2\left(1-z^2\right)V_{\sigma L} \pm \tfrac{1}{2j+1}\left(2p'^2p^2V_{\sigma L} - \left(p'^2+p^2\right)\left(V_{\sigma q} + \tfrac{1}{4}V_{\sigma k}\right)\right)\right]P_{j\pm 1}(z)\Big\} \\
&\langle j\pm 1,\ 1\,j|V|j\mp 1,\ 1\,j\rangle \\
&= \frac{\sqrt{j(j+1)}}{2j+1}2\pi\int_{-1}^{1} dz\left\{-p'p(4V_{\sigma q} - V_{\sigma k})P_j(z) + \left[\mp\tfrac{2p'^2p^2}{2j+1}V_{\sigma L} + p'^2\left(2V_{\sigma q} + \tfrac{1}{2}V_{\sigma k}\right)\right]\right. \\
&\quad \left. P_{j\mp 1}(z) + \left[\pm\tfrac{2p'^2p^2}{2j+1}V_{\sigma L} + p^2\left(2V_{\sigma q} + \tfrac{1}{2}V_{\sigma k}\right)P_{j\pm 1}(z)\right]\right\}
\end{aligned}
\tag{11}
$$

Theoretical framework

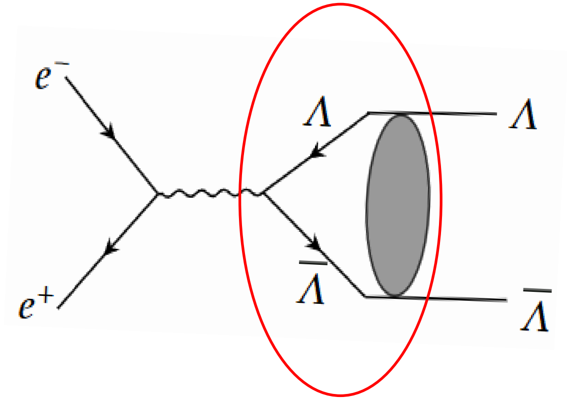
Derive FSI T amplitude by solving Lippmann-Schwinger (LS) equation.

$$T^{\rho''\rho'}_{\nu''\nu'}(p'',p';E_k) = V^{\rho''\rho'}_{\nu''\nu'}(p'',p') + \sum_{\rho\nu}\int \frac{\mathrm{d}pp^2}{(2\pi)^3} V^{\rho''\rho}_{\nu''\nu}(p'',p)\frac{1}{2E_k-2E_p+i0^+}T^{\rho\rho'}_{\nu\nu'}(p,p';E_k) \tag{12}$$

Potential needs to cut off by a regular function before it putted into LS equation $f(\Lambda) = exp\left[-\left(p^6+p'^6\right)/\Lambda^6\right]$ (13)

Derive $\Lambda\bar{\Lambda}\gamma$ vertex amplitude from solve distorted wave Born approximation (DWBA)

$$f^{\rho''\rho'}_{L'}(k;E_k) = f^{\rho''\rho',0}_{L'}(k) + \sum_{L}\int_0^\infty \frac{\mathrm{d}pp^2}{(2\pi)^3} f^{\rho''\rho,0}_{L}(p)\frac{1}{2E_k-2E_p+i0^+}T^{\rho\rho'}_{L'L}(p,k;E_k) \tag{14}$$



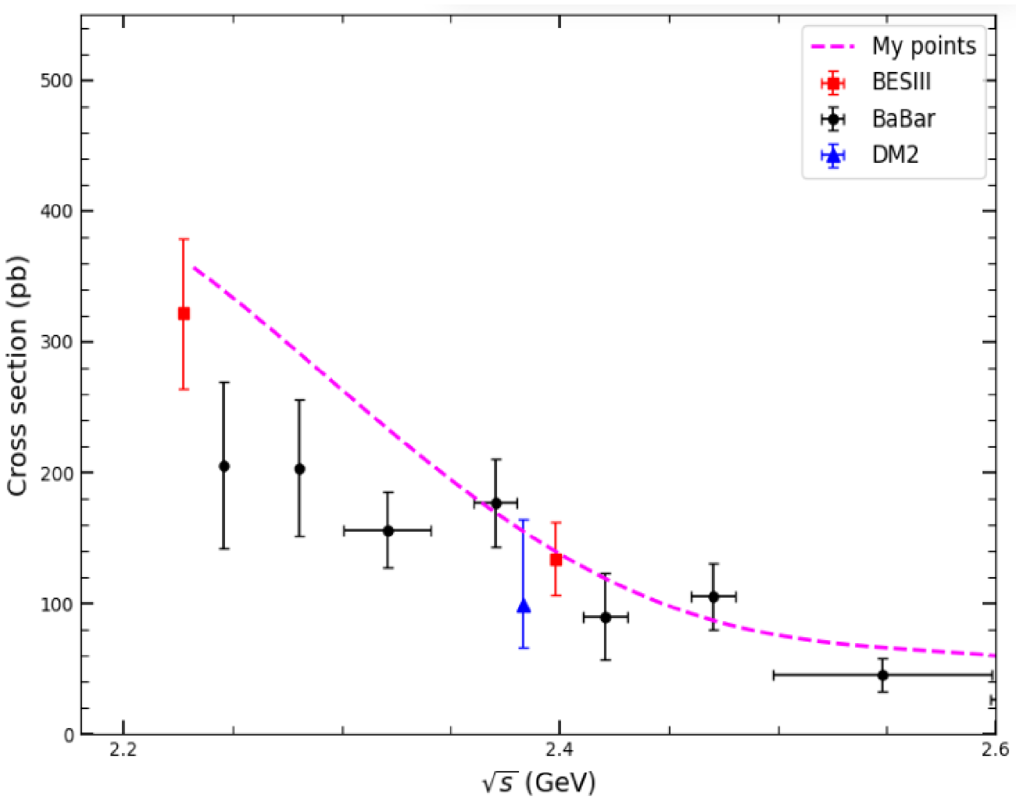
with

$$f_0^{\bar{Y}Y,0}(p) = G_M^{\bar{Y}Y,0} + \frac{M_Y}{2\sqrt{M_Y^2+p^2}}G_E^{\bar{Y}Y,0} \qquad f_2^{\bar{Y}Y,0}(p) = \frac{1}{\sqrt{2}}\left(G_M^{\bar{Y}Y,0} - \frac{M_Y}{2\sqrt{M_Y^2+p^2}}G_E^{\bar{Y}Y,0}\right) \tag{15}$$

G_E^0, G_M^0 is bare electromagnetic form factor

Fit results

Results about LECs and bare electromagnetic form factor fit to cross section data from threshold to 2.6GeV.



Summary

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Summary

- Constructed the $\Lambda\bar{\Lambda}$ potential within SU(3) Chiral effective field theory up to next-to-leading order
- Derived the amplitude about $\Lambda\bar{\Lambda}$ by solve LS equation
- Derived the amplitude about $e^+e^- \to \Lambda\bar{\Lambda}$ in distorted wave born approximation
- Fit experiment data, calculated the cross section

Conclusion

- our theory calculation is consistent with experiment result

In the future

- Consider affection abous various cutoff Λ to cross section and effective form factor
- Consider particle channel couple $N\bar{N} - \Lambda\bar{\Lambda}$ in solving LS equation
- Looking forward more experiment data to improve our fit result and calculate effective form factor

Thanks for your attention !