

Baryons 2025

# Electromagnetic form factors of nucleons in the effective chiral theory

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# Introduction

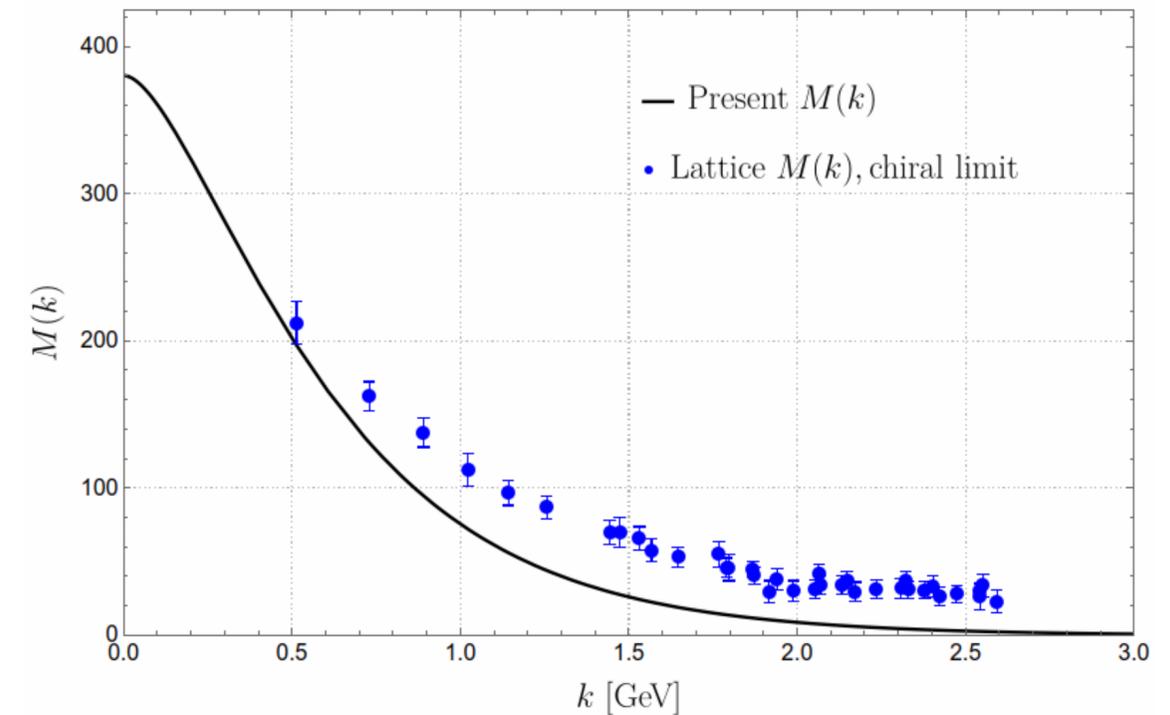
## Chiral quark-soliton model from the instanton vacuum

*“If both quark and pion momenta are smaller than  $1/\bar{\rho}$ , the dynamical mass  $M(k)$  can be approximated by  $M = M(0)$ .”*

C. V. Christov, A. Blotz, H.-C. Kim, P. Pobylitsa, T. Watabe, T. Meissner, E. Ruiz Arriola, and K. Goeke, Prog. Part. Nucl. Phys. 37, 91 (1996)



Chiral quark-soliton model



G. Burgio, M. Schrock, H. Reinhardt, and M. Quandt, Phys. Rev. D 86, 014506 (2012)

***What if we revive the momentum dependence of quark mass?***

# Introduction

## Momentum dependent dynamical quark mass

$$M(k) = M_0 F^2(k\bar{\rho}), \quad F(k\bar{\rho}) = 2t \left[ I_0(t)K_1(t) - I_1(t)K_0(t) - \frac{1}{t}I_1(t)K_1(t) \right]_{t=\frac{1}{2}k\bar{\rho}}$$

$M(0) = M_0 \approx 350 \text{ MeV}$

Additional features from reviving  $M(k)$  :

- Natural regulator : other regularizations(PV, proper-time, ...) are **not needed**.
- Nonlocal interaction : momentum dependence of  $M(k)$  **breaks the gauge invariance**.
- No free parameter : the value of  $M(0)$  is given from the instanton vacuum.

# Effective chiral theory

## Theoretical framework

Effective low-energy QCD partition function in the Euclidean space

$$\mathcal{Z} = \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\psi^\dagger \exp[-S_{\text{E}\chi\text{T}}]$$

$$S_{\text{E}\chi\text{T}} = - \int d^4x \psi^\dagger \left( i\gamma_\mu \partial_\mu + im + i\overleftarrow{F}(i\partial)M_0U\gamma_5\overrightarrow{F}(i\partial) \right) \psi \quad \xRightarrow[F(k) \rightarrow 1]{} \quad S_{\chi\text{QSM}} = - \int d^4x \psi^\dagger \left( i\gamma_\mu \partial_\mu + im + iM_0U\gamma_5 \right) \psi$$

$m = 5.5 \text{ MeV}$  and  $M_0 = 386 \text{ MeV}$  : no free parameters!

K. Goeke, M. M. Musakhanov, and M. Siddikov, Phys. Rev. D 76, 076007 (2007)

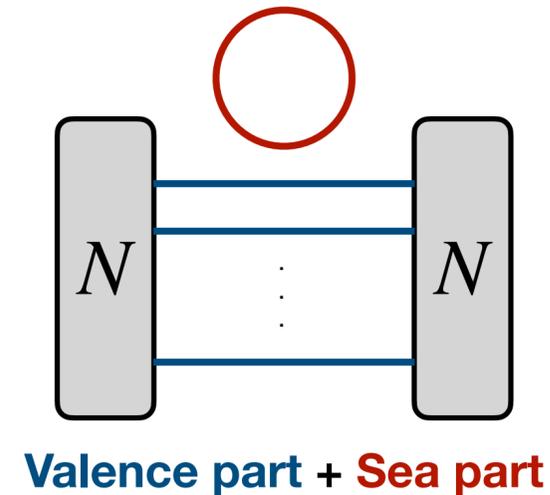
# Effective chiral theory

## Saddle-point approximation

In the large  $N_c$  limit,  $\int \mathcal{D}U$  can be evaluated using the saddle-point approximation.

$$\left. \frac{\delta}{\delta U} (N_c E_{\text{val}} + E_{\text{sea}}) \right|_{U=U_c} = 0 \implies M_{\text{cl}} = N_c E_{\text{val}}[U_c] + E_{\text{sea}}[U_c]$$

$$\exp[-S_{\text{E}\chi\text{T}}] \xrightarrow{T \rightarrow \infty} \exp[-(N_c E_{\text{val}} + E_{\text{sea}})T]$$



Hedgehog Ansatz  $U_c = \exp[i(\hat{n} \cdot \vec{\tau})P(r)], \quad U^{\gamma_5} = \frac{1 + \gamma_5}{2}U + \frac{1 - \gamma_5}{2}U^\dagger$

# Effective chiral theory

## Zero mode quantization

Saddle-point approximation : neglecting mesonic quantum fluctuations  $U \rightarrow U_c$

Zero mode quantization :  $U^{\gamma_5}(\mathbf{x}, t) = R(t) U_c^{\gamma_5}(\mathbf{x} - \mathbf{Z}(t)) R^\dagger(t)$ ,  $\int \mathcal{D}U \longrightarrow \int \mathcal{D}R \mathcal{D}\mathbf{Z}$

$$\exp[-S_{E\chi T}] \xrightarrow{T \rightarrow \infty} \exp\left[-(N_c E_{\text{val}} + E_{\text{sea}})T - \frac{I}{2} \int dt \Omega^a(t) \Omega^a(t)\right]$$

# Effective chiral theory

## Saddle-point approximation and zero mode quantization

In the large $\frac{\delta}{\delta U}(N_c E_\chi)$	$E_{\text{val}}$ [MeV]	$N_c E_{\text{val}}$ [MeV]	$E_{\text{sea}}$ [MeV]	$M_{\text{cl}}$ [MeV]
This work ( $M_0=386$ MeV)	168.2	504.7	534.8	1039
Chiral limit ( $M_0=359$ MeV)	152.6	457.7	529.7	987.4
$\chi$ QSM ( $M_0=386$ MeV)	223.3	669.8	588.0	1258
$\chi$ QSM ( $M_0=420$ MeV)	203.5	610.6	645.8	1256

Zero mode qu $\exp[-S_{E\chi T}]_{T \rightarrow 0}$	$I_{\text{val}}$ [fm]	$I_{\text{sea}}$ [fm]	$I$ [fm]	$M_{\Delta-N}$ [MeV]
This work ( $M_0=386$ MeV)	1.0547	0.1244	1.1791	251.04
Chiral limit ( $M_0=359$ MeV)	1.0847	0.3147	1.3995	211.50
$\chi$ QSM ( $M_0=386$ MeV)	0.9487	0.2484	1.1971	247.26
$\chi$ QSM ( $M_0=420$ MeV)	0.7752	0.2553	1.0306	287.20

# Effective chiral theory

## Saddle-point approximation and zero mode quantization

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# Effective chiral theory

## Conserved electromagnetic current

$$\langle N(p', S') | j_\mu(0) | N(p, S) \rangle = \bar{u}(p', S') \left( \gamma_\mu F_1(q^2) + i\sigma_{\mu\nu} \frac{q^\nu}{2M_N} F_2(q^2) \right) u(p, S)$$

$j_\mu = \psi^\dagger \gamma_\mu \hat{Q} \psi$  is not conserved due to  $M(k)$ .

$$\hat{Q} = \begin{pmatrix} \frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{pmatrix} = \frac{1}{2} \left( \frac{1}{3} \mathbf{I} + \tau^3 \right) : \text{Quark charge matrix}$$

$$F(i\partial) \rightarrow F(i\partial - \hat{Q}V)$$

M. M. Musakhanov and H.-C. Kim, Phys. Lett. B 572, 181 (2003)



$$j_\mu = \frac{\delta S[V]}{\delta V_\mu} \Big|_{V=0} \text{ is conserved.}$$

$$j_\mu = \psi^\dagger \left( \underbrace{\gamma_\mu \hat{Q}}_{\text{Local}} + \underbrace{i \overleftarrow{F}_\mu \hat{Q} M_0 U^{\gamma_5} \overrightarrow{F} + i \overleftarrow{F} M_0 U^{\gamma_5} \hat{Q} \overrightarrow{F}_\mu}_{\text{Nonlocal}} \right) \psi = \psi^\dagger \Gamma_\mu[\hat{Q}; U^{\gamma_5}] \psi$$

$$F_\mu(k) = \frac{\partial F(k)}{\partial k_\mu}$$

# Electromagnetic form factors

## Electric form factors

$$G_{E,M}^p(q^2) = \frac{1}{2} \left( G_{E,M}^{T=0}(q^2) + G_{E,M}^{T=1}(q^2) \right) \quad G_{E,M}^n(q^2) = \frac{1}{2} \left( G_{E,M}^{T=0}(q^2) - G_{E,M}^{T=1}(q^2) \right)$$

$$G_E^{T=0}(q^2) = \frac{N_c}{3} \int d^3\mathbf{Z} j_0(|\vec{q}||\mathbf{Z}|) \left[ z_{\text{val}} \langle \text{val} | \mathbf{Z} \rangle \left( 1 + i\overleftarrow{F}_4 \gamma_4 U_c^{\gamma_5} \vec{F} + i\overleftarrow{F}_4 \gamma_4 U_c^{\gamma_5} \vec{F}_4 \right) \langle \mathbf{Z} | \text{val} \rangle + \int \frac{d\omega}{2\pi i} \sum_{n_\omega} \frac{\langle n_\omega | \mathbf{Z} \rangle \left( 1 + i\overleftarrow{F}_4 \gamma_4 U_c^{\gamma_5} \vec{F} + i\overleftarrow{F}_4 \gamma_4 U_c^{\gamma_5} \vec{F}_4 \right) \langle \mathbf{Z} | n_\omega \rangle}{\omega + iE_n(\omega)} \right]$$

$G_E^{T=0}(0) = 1$  : electric charge is conserved with nonlocal effect.

$$G_E^{T=1}(q^2) = \frac{N_c}{6I} \int d^3\mathbf{Z} j_0(|\vec{q}||\mathbf{Z}|) \left[ z_{\text{val}} \sum_n \frac{\langle \text{val} | \mathbf{Z} \rangle t^a \langle \mathbf{Z} | n \rangle \langle n | t^a | \text{val} \rangle}{E_n - E_{\text{val}}} - \frac{1}{2} z_{\text{val}} \langle \text{val} | \mathbf{Z} \rangle T^{aa} \langle \mathbf{Z} | \text{val} \rangle \right. \\ \left. + \frac{1}{2} \int \frac{d\omega}{2\pi} \sum_{n,m} \frac{\langle n | \mathbf{Z} \rangle t^a \langle \mathbf{Z} | m \rangle \langle m | t^a | n \rangle}{(\omega + iE_n(\omega))(\omega + iE_m(\omega))} - \frac{1}{2} \int \frac{d\omega}{2\pi i} \sum_n \frac{\langle n | \mathbf{Z} \rangle T^{aa} \langle \mathbf{Z} | n \rangle}{\omega + iE_n(\omega)} \right]$$

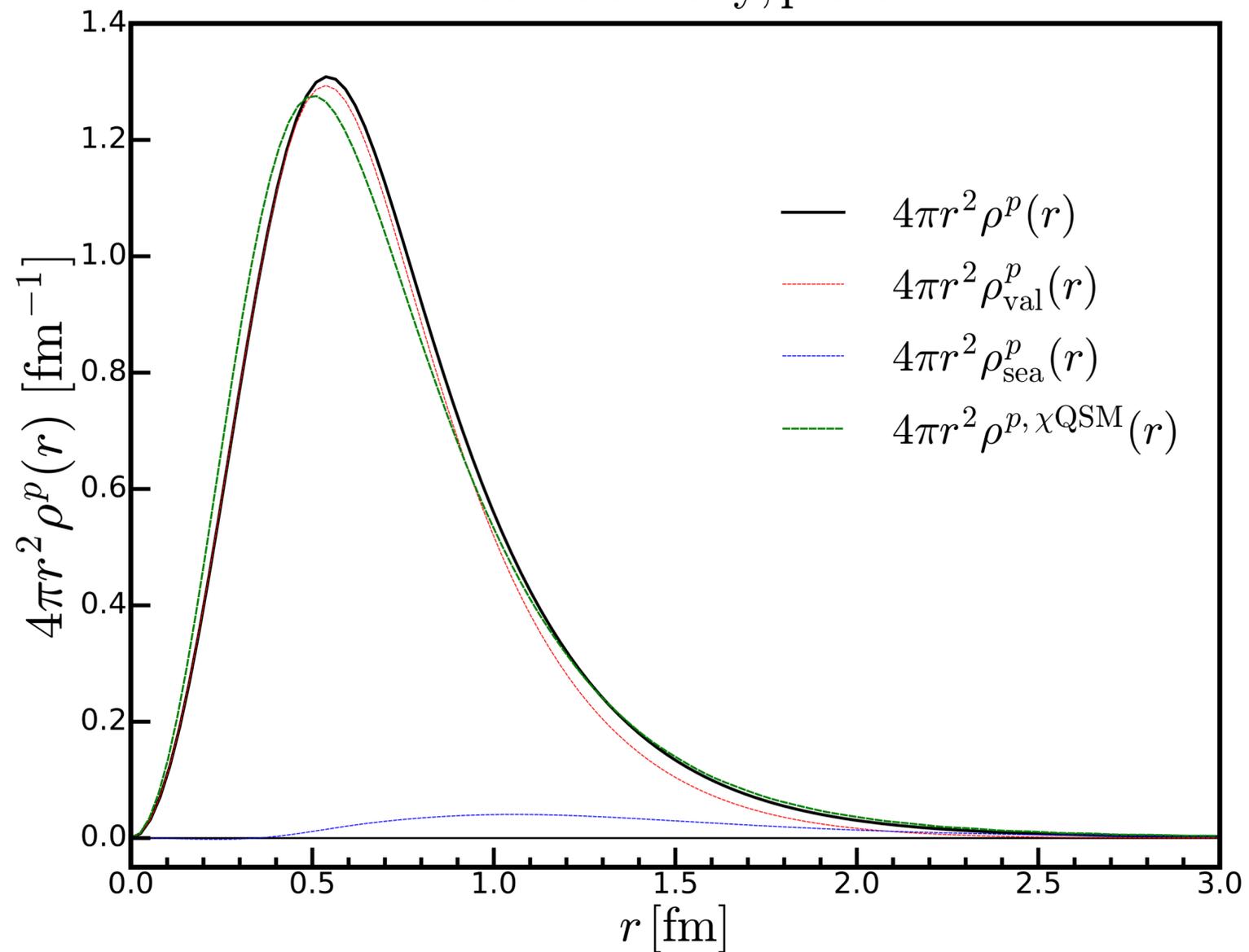
$$t^a = \tau^a + i\overleftarrow{F}_4 \gamma_4 \tau^a U_c^{\gamma_5} \vec{F} + i\overleftarrow{F}_4 \gamma_4 U_c^{\gamma_5} \tau^a \vec{F}_4, \quad T^{aa} = \overleftarrow{F}_4 \gamma_4 U_c^{\gamma_5} \tau^a \tau^a \vec{F}_{44} + 2\overleftarrow{F}_4 \gamma_4 \tau^a U_c^{\gamma_5} \tau^a \vec{F}_4 + \overleftarrow{F}_{44} \gamma_4 \tau^a \tau^a U_c^{\gamma_5} \vec{F}$$

$G_E^{T=1}(0) = 1$  : iso-vector electric form factor is properly normalized.

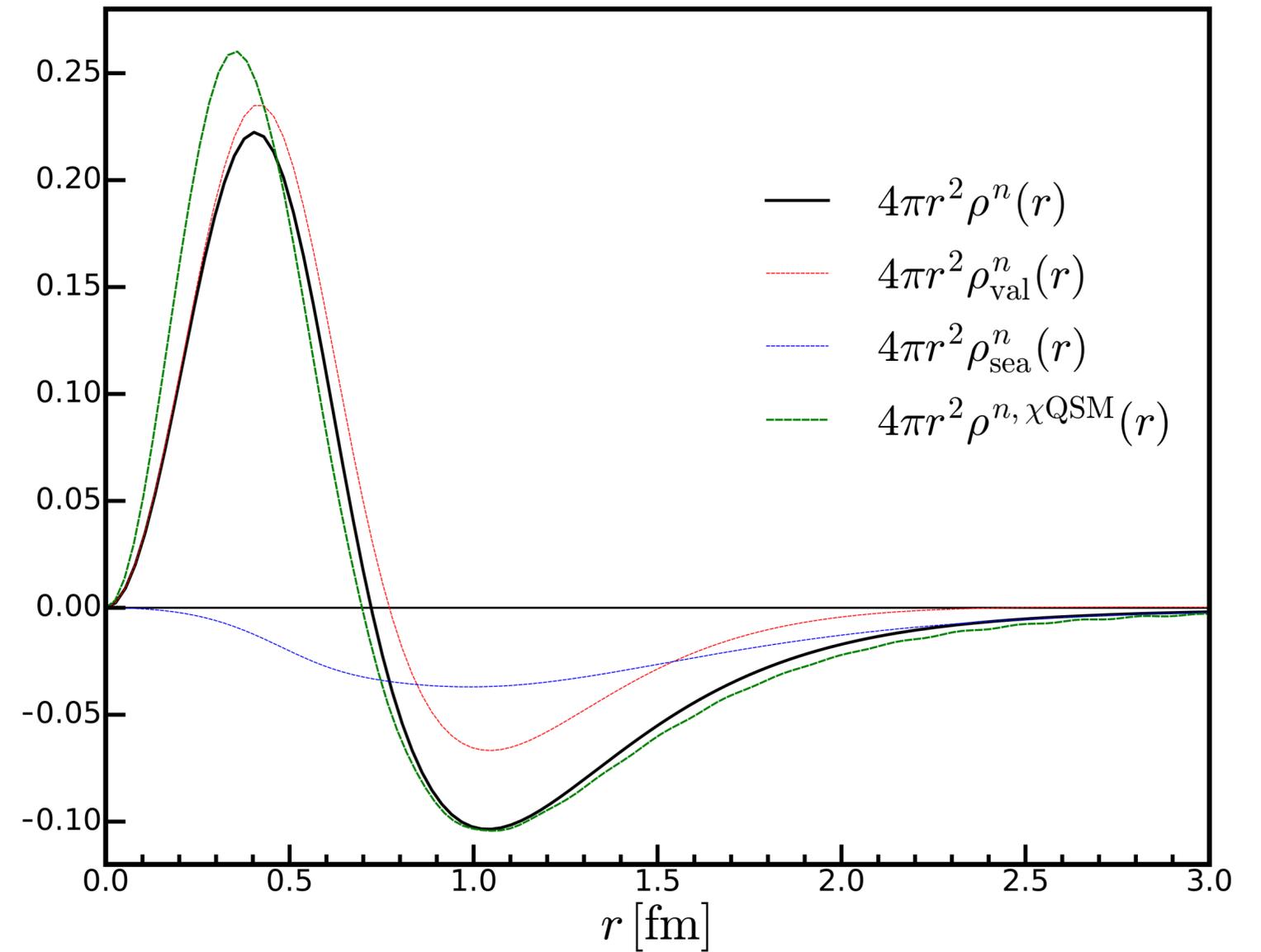
# Electromagnetic form factors

## Electric densities, results(Preliminary)

Electric density, proton



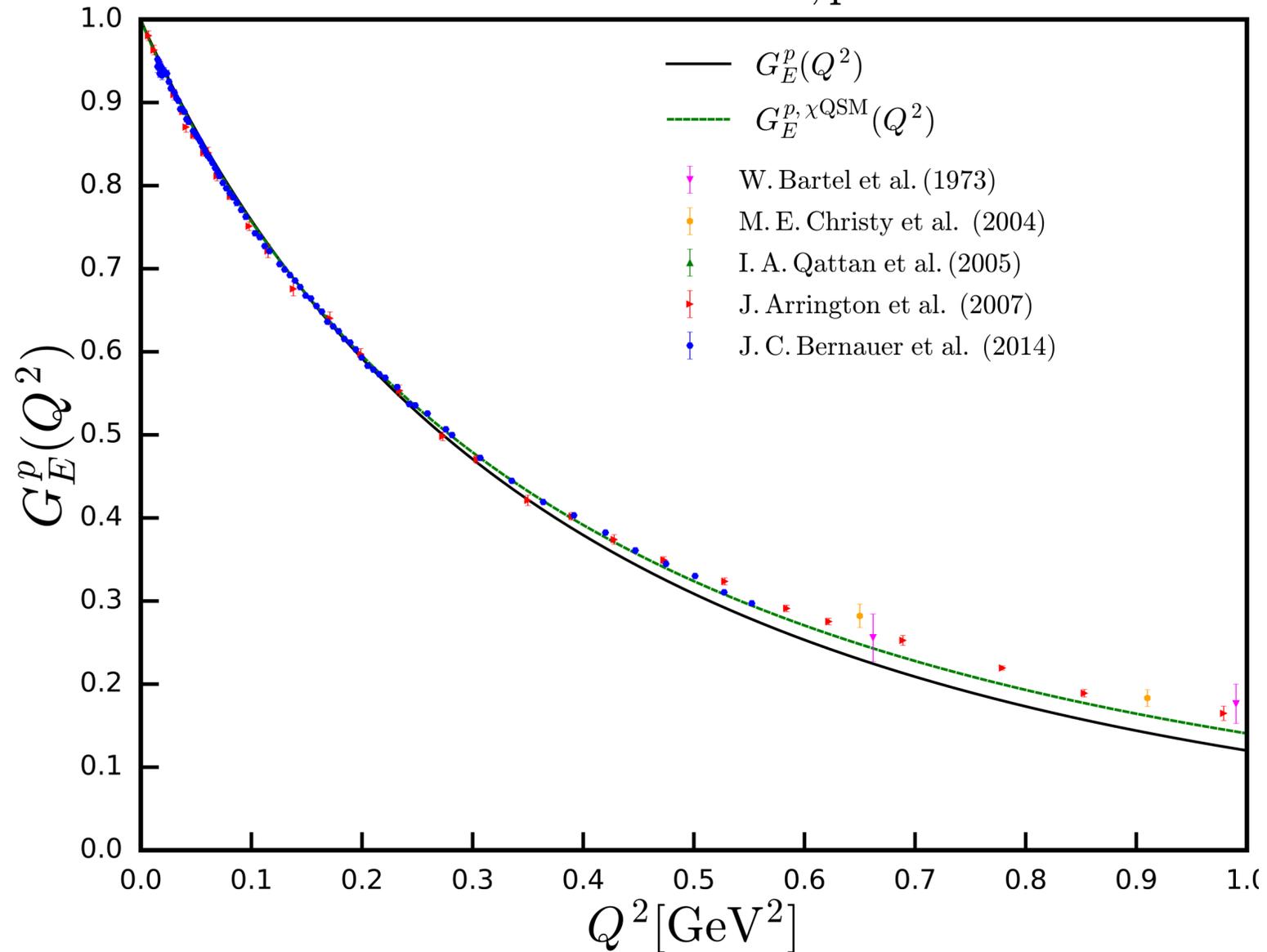
Electric density, neutron



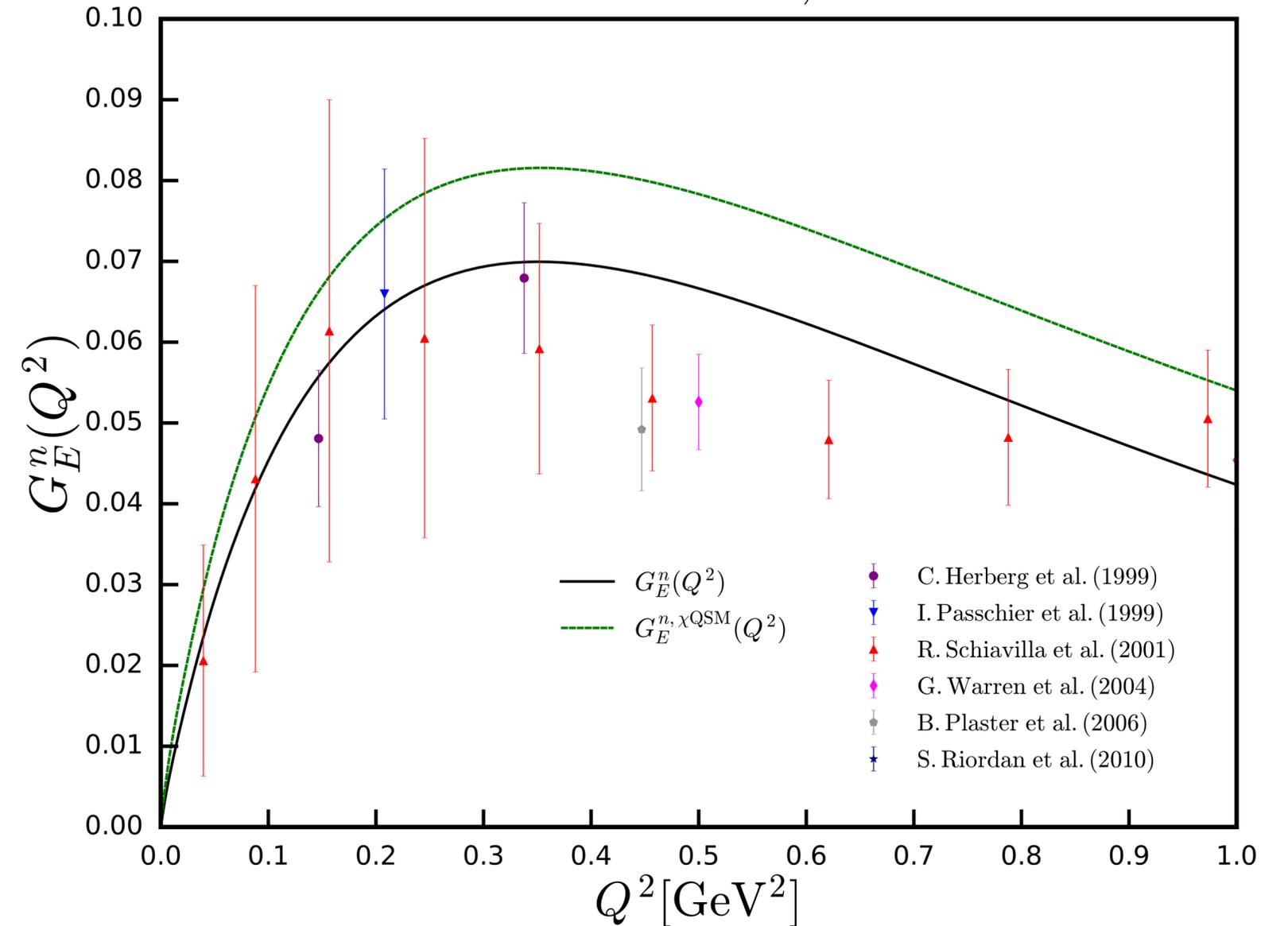
# Electromagnetic form factors

## Electric form factors, results(Preliminary)

Electric form factor, proton



Electric form factor, neutron



# Electromagnetic form factors

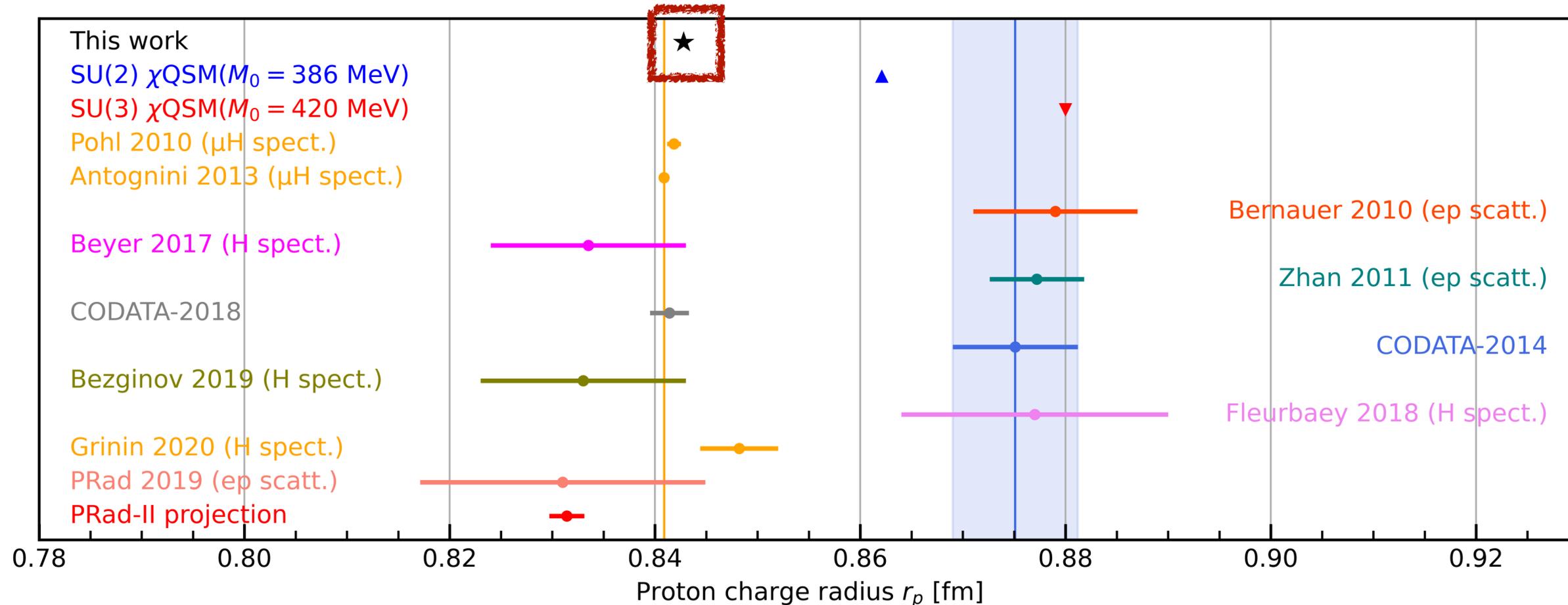
## Charge radii(Preliminary)

$$\langle r_E^2 \rangle = -6 \left. \frac{dG_E(q^2)}{dq^2} \right|_{q^2=0}$$

	This work	$\chi$ QSM	Experiments
$r_{T=0}^2$ [fm <sup>2</sup> ]	0.5510	0.5234	$0.597 \pm 0.008$
$r_{T=1}^2$ [fm <sup>2</sup> ]	0.8695	0.9629	$0.817 \pm 0.008$
$r_p$ [fm]	0.8428	0.8621	$0.8409 \pm 0.0004$
$r_n^2$ [fm <sup>2</sup> ]	-0.1593	-0.2197	$-0.110 \pm 0.008$

PDG

H. Atac, M. Constantinou, Z.-E. Meziani, M. Paolone and N. Sparveris, Nature Comm. 12, 1759 (2021)



# Electromagnetic form factors

## Magnetic form factors

Electric form factors

$$1 + i\overleftarrow{F}_4\gamma_4 U_c^{\gamma_5}\vec{F} + i\overleftarrow{F}\gamma_4 U_c^{\gamma_5}\vec{F}_4$$

$$\tau^a + i\overleftarrow{F}_4\gamma_4\tau^a U_c^{\gamma_5}\vec{F} + i\overleftarrow{F}\gamma_4 U_c^{\gamma_5}\tau^a\vec{F}_4$$

magnetic form factors

$$\gamma_5(\hat{n} \times \vec{\sigma})_z - \overleftarrow{F}_4\gamma_4\hat{L}_z U_c^{\gamma_5}\vec{F} - \overleftarrow{F}\gamma_4 U_c^{\gamma_5}\hat{L}_z\vec{F}_4$$

$$\gamma_5(\hat{n} \times \vec{\sigma})_z\tau_z - \overleftarrow{F}_4\gamma_4\hat{L}_z\tau_z U_c^{\gamma_5}\vec{F} - \overleftarrow{F}\gamma_4 U_c^{\gamma_5}\tau_z\hat{L}_z\vec{F}_4$$

Local contributions

Nonlocal contributions

appear due to  $M(k)$

# Summary

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- We are investigating **electromagnetic form factors of nucleons** within the effective chiral theory derived from the QCD instanton vacuum.
- To respect charge conservation, the nonlocality of the dynamical mass  $M(k)$  **is not negligible.**
- Predictions for electric observables (form factors, charge radii, etc.) are consistent with experiment.
- Calculations of the magnetic form factors and related physical quantities are in progress.

**Thank you!**

# Introduction

## Chiral quark-soliton model from the instanton vacuum

The dynamical quark mass  $M(k)$  is parametrically small :

$$\bar{\rho}/\bar{R} \approx 1/3 \implies M\bar{\rho} \approx \mathcal{O}(\bar{\rho}^2 \bar{R}^2) \ll 1$$

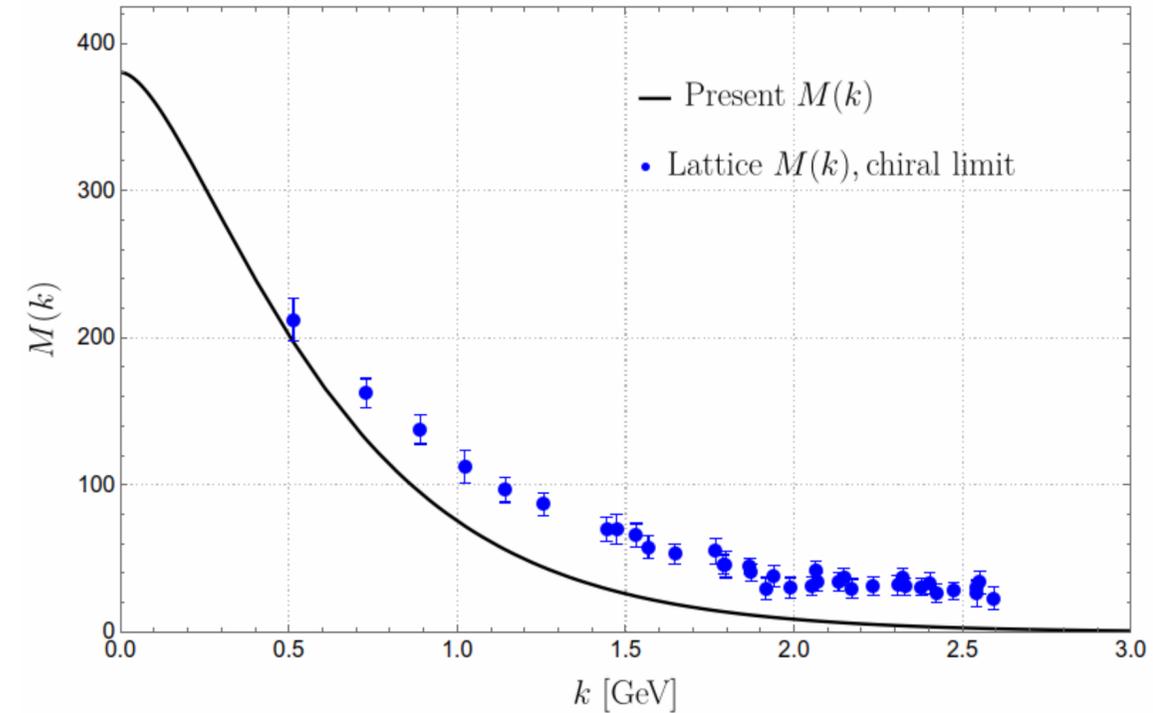
In the range of quark momenta  $k \gg 1/\bar{\rho}$ ,  $M(k)$  vanishes.

*“If both quark and pion momenta are smaller than  $1/\bar{\rho}$ , the dynamical mass  $M(k)$  can be approximated by  $M = M(0)$ .”*

C. V. Christov, A. Blotz, H.-C. Kim, P. Pobylitsa, T. Watabe, T. Meissner, E. Ruiz Arriola, and K. Goeke, Prog. Part. Nucl. Phys. 37, 91 (1996)



Chiral quark-soliton model



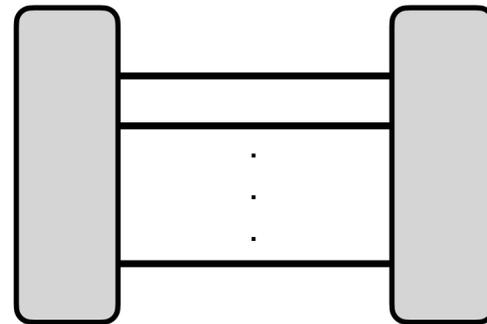
G. Burgio, M. Schrock, H. Reinhardt, and M. Quandt, Phys. Rev. D 86, 014506 (2012)

# Additional slide

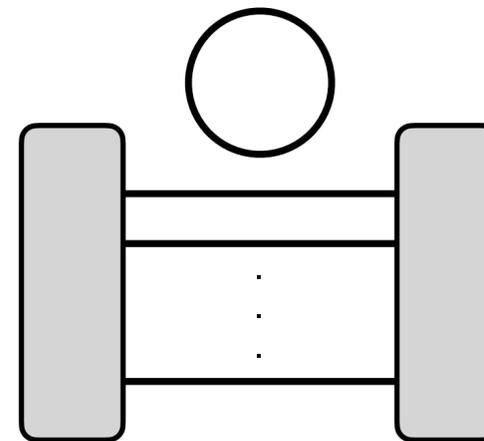
$$J_N(x) = \frac{1}{N_c!} \varepsilon^{\alpha_1 \dots \alpha_{N_c}} \Gamma_{(TT_3)(JJ_3)}^{f_1 \dots f_{N_c}} \psi_{\alpha_1 f_1}(x) \dots \psi_{\alpha_{N_c} f_{N_c}}(x)$$

$$\Pi_N(T) = \langle 0 | J_N(0, T/2) J_N^\dagger(0, -T/2) | 0 \rangle \sim \int \mathcal{D}U \prod_{i=1}^{N_c} \langle 0, T/2 | \frac{1}{D[U]} | 0, -T/2 \rangle e^{-S_{\text{eff}}}$$

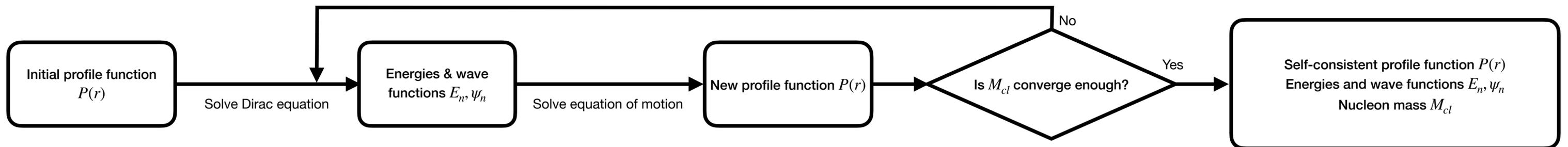
$$\xrightarrow{T \rightarrow \infty} \exp \left[ - (N_c E_{\text{val}} + E_{\text{sea}}) T \right]$$



Valence part



Valence part + Sea part



# Effective chiral theory

## Zero mode quantization

Zero mode quantization :  $U^{\gamma_5}(\mathbf{x}, t) = R(t) U_c^{\gamma_5}(\mathbf{x} - \mathbf{Z}(t)) R^\dagger(t), \quad \int \mathcal{D}U \longrightarrow \int \mathcal{D}R \mathcal{D}\mathbf{Z}$

: Translational quantization    
  : Rotational quantization

$$\Pi_N(T) \xrightarrow{T \rightarrow \infty} \exp \left[ - (N_c E_{\text{val}} + E_{\text{sea}}) T - \frac{I^{ab}}{2} \int dt \Omega^a(t) \Omega^b(t) \right] \quad I = I^{aa} = 1.179 \text{ fm}$$

# Electromagnetic form factors

## Electric form factors

$$G_E^{T=0}(q^2) = \frac{N_c}{3} \int d^3\mathbf{Z} j_0(|\vec{q}||\mathbf{Z}|) \left[ z_{\text{val}} \langle \text{val} | \mathbf{Z} \rangle \left( 1 + i\overleftarrow{F}_{4\gamma_4} U_c^{\gamma_5} \vec{F} + i\overleftarrow{F}_{\gamma_4} U_c^{\gamma_5} \vec{F}_4 \right) \langle \mathbf{Z} | \text{val} \rangle \Big|_{\omega=-iE_{\text{val}}(\omega_v)} + \int \frac{d\omega}{2\pi i} \sum_{n_\omega} \frac{\langle n_\omega | \mathbf{Z} \rangle \left( 1 + i\overleftarrow{F}_{4\gamma_4} U_c^{\gamma_5} \vec{F} + i\overleftarrow{F}_{\gamma_4} U_c^{\gamma_5} \vec{F}_4 \right) \langle \mathbf{Z} | n_\omega \rangle}{\omega + iE_n(\omega)} \right]$$

$$\sim \mathcal{O}(N_c^0)$$

$$G_E^{T=0}(0) = 1 : \text{electric charge is conserved with nonlocal effect.}$$

$$G_E^{T=1}(q^2) = \frac{N_c}{6I} \int d^3\mathbf{Z} j_0(|\vec{q}||\mathbf{Z}|) \left[ z_{\text{val}} \sum_n \frac{\langle \text{val} | \mathbf{Z} \rangle t^a \langle \mathbf{Z} | n \rangle \langle n | t^a | \text{val} \rangle}{E_n - E_{\text{val}}} \Big|_{\omega=-iE_{\text{val}}(\omega_v)} - \frac{1}{2} z_{\text{val}} \langle \text{val} | \mathbf{Z} \rangle T^{aa} \langle \mathbf{Z} | \text{val} \rangle \Big|_{\omega=-iE_{\text{val}}(\omega_v)} \right]$$

$$\sim \mathcal{O}(N_c^{-1})$$

$$t^a = \tau^a + i\overleftarrow{F}_{4\gamma_4} \tau^a U_c^{\gamma_5} \vec{F} + i\overleftarrow{F}_{\gamma_4} U_c^{\gamma_5} \tau^a \vec{F}_4 \quad \leftarrow \quad + \frac{1}{2} \int \frac{d\omega}{2\pi} \sum_{n,m} \frac{\langle n | \mathbf{Z} \rangle t^a \langle \mathbf{Z} | m \rangle \langle m | t^a | n \rangle}{(\omega + iE_n(\omega))(\omega + iE_m(\omega))} - \frac{1}{2} \int \frac{d\omega}{2\pi i} \sum_n \frac{\langle n | \mathbf{Z} \rangle T^{aa} \langle \mathbf{Z} | n \rangle}{\omega + iE_n(\omega)}$$

$$T^{aa} = \overleftarrow{F}_{\gamma_4} U_c^{\gamma_5} \tau^a \tau^a \overleftarrow{F}_{44} + 2\overleftarrow{F}_{4\gamma_4} \tau^a U_c^{\gamma_5} \tau^a \overleftarrow{F}_4 + \overleftarrow{F}_{44} \gamma_4 \tau^a \tau^a U_c^{\gamma_5} \overleftarrow{F}$$