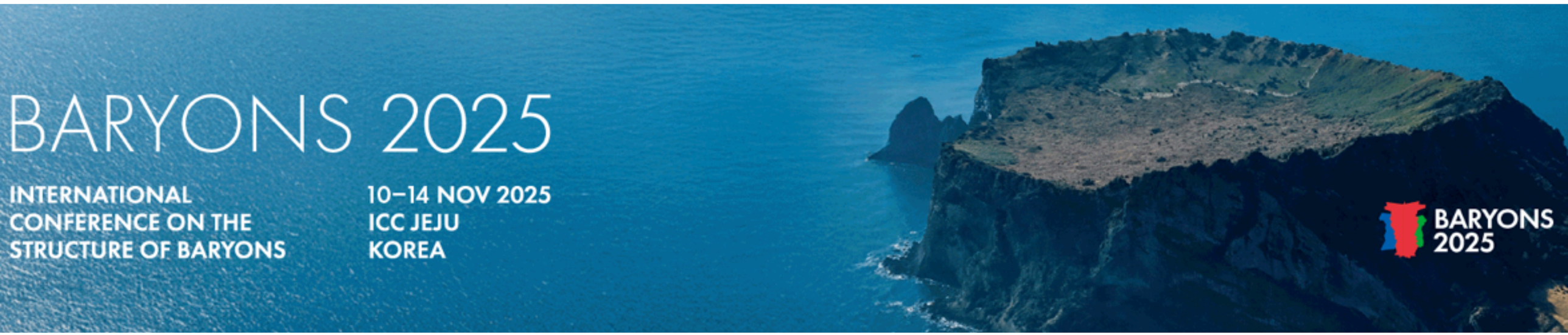


# Multipole structure of nucleon tensor form factors in the chiral quark-soliton model

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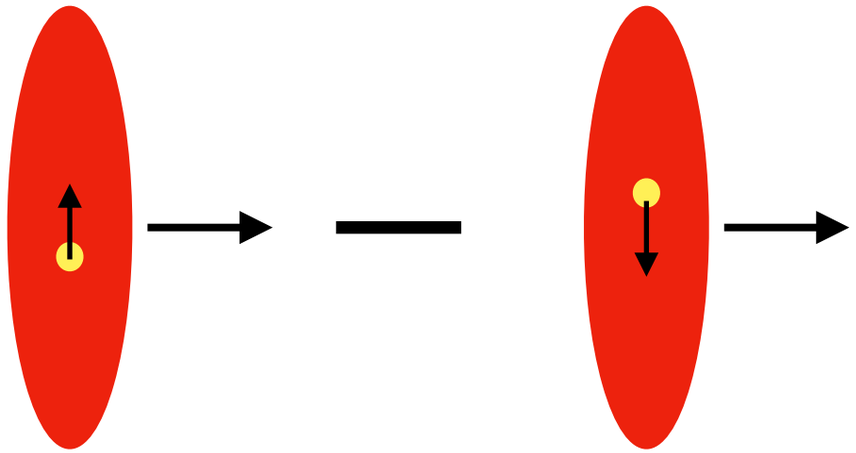
IN COLLABORATION WITH June-Young Kim, Ho-Yeon Won
and Hyun-Chul Kim

# MOTIVATION

- Transversity

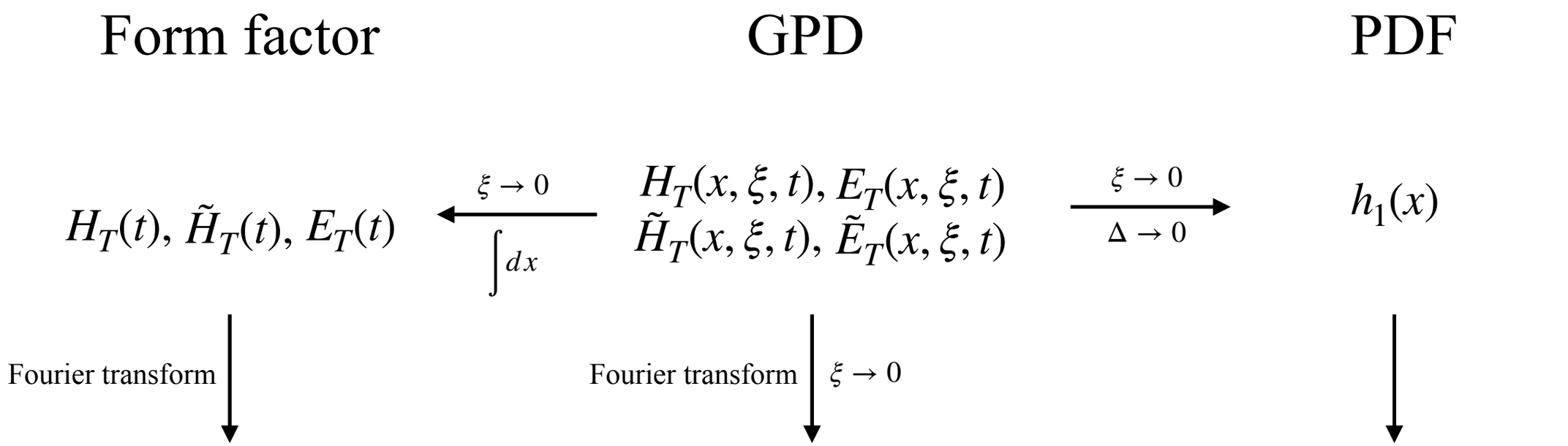


$$q^{\uparrow}(x) - q^{\downarrow}(x) = h_1^q(x)$$

- Tensor charge and transversity distribution

$$\int_{-1}^{1} dx\, h_1^q(x) = g_T^q$$

# MOTIVATION




[2] D Brommel, Pion Structure from the Lattice, Ph, D. Thesis, Regensburg U. 2007.

# Multipole structure

- Tensor current matrix elements

$$\mathcal{M}^q[i\sigma^{\mu\nu}] = \langle N(p', S_3')|\bar{\psi}_q(0) i\sigma^{\mu\nu}\psi_q(0)|N(p', S_3')\rangle$$
$$= \bar{u}\left[H_T^q i\sigma^{\mu\nu} + \tilde{H}_T^q \frac{P^\mu\Delta^\nu - \Delta^\mu P^\nu}{M_N^2} + E_T^q \frac{\gamma^\mu\Delta^\nu - \Delta^\mu\gamma^\nu}{2M_N}\right] u$$

- Matrix elements in Breit frame

$$\mathcal{M}^q[i\sigma^{0k}] = Y_1^k(t)\delta_{S_3' S_3}\sqrt{-t}\left[H_T^q(t) + E_T^q(t) + \frac{2(P^0)^2}{M_N^2}\tilde{H}_T^q(t)\right]$$

$$\mathcal{M}^q[i\sigma^{0k}] = i\epsilon^{ijk}\sigma_{S_3'}^k Y_0(t) 2M_N \left[\left(\frac{P^0}{3M_N} + \frac{2}{3}\right) H_T^q(t) + \frac{t}{6M_N^2} E_T^q(t)\right]$$
$$- i\epsilon^{ijk}\sigma^m Y_2^{km}(t)\frac{t}{2M_N}\left[\frac{M_N}{P^0 + M_N} H_T^q(t) + E_T^q(t)\right]$$

$$Y_0 = 1, \quad Y_1^i = \frac{\Delta^i}{|\boldsymbol{\Delta}|}, \quad Y_2^{ij} = \frac{\Delta^i\Delta^j}{|\boldsymbol{\Delta}|^2} - \frac{1}{3}\delta^{ij}$$

# Multipole structure

- Tensor current matrix elements

$$\mathcal{M}^q[i\sigma^{\mu\nu}] = \langle N(p', S_3')|\bar{\psi}_q(0) i\sigma^{\mu\nu}\psi_q(0)|N(p', S_3')\rangle$$
$$= \bar{u}\left[H_T^q i\sigma^{\mu\nu} + \tilde{H}_T^q \frac{P^\mu\Delta^\nu - \Delta^\mu P^\nu}{M_N^2} + E_T^q \frac{\gamma^\mu\Delta^\nu - \Delta^\mu\gamma^\nu}{2M_N}\right] u$$

- Matrix elements in 2-D Drell-Yan West Frame

$$\mathcal{M}^q[i\sigma^{+j}] = 2P^+\Bigg[i\epsilon^{3jm}\sigma^m\, X_0\left\{H_T^q - \frac{t}{4M_N^2}\tilde{H}_T^q\right\}$$
$$+ X_1^j \frac{\sqrt{-t}}{2M_N}\left\{E_T^q + 2\tilde{H}_T^q\right\}$$
$$+ i\epsilon^{3jl}\sigma^m\, X_2^{lm}\frac{t}{4M_N^2}\left\{2\tilde{H}_T^q\right\}\Bigg].$$

$$X_0 = 1, \quad X_1^i = \frac{\Delta_\perp^i}{|\boldsymbol{\Delta}_\perp|}, \quad X_2^{ij} = \frac{\Delta_\perp^i\Delta_\perp^j}{|\boldsymbol{\Delta}_\perp|^2} - \frac{1}{2}\delta^{ij}.$$

# Multipole structure

- Tensor current matrix elements

$$\mathcal{M}^q[i\sigma^{\mu\nu}] = \langle N(p', S_3')|\bar{\psi}_q(0) i\sigma^{\mu\nu}\psi_q(0)|N(p', S_3')\rangle$$
$$= \bar{u}\left[H_T^q i\sigma^{\mu\nu} + \tilde{H}_T^q \frac{P^\mu \Delta^\nu - \Delta^\mu P^\nu}{M_N^2} + E_T^q \frac{\gamma^\mu \Delta^\nu - \Delta^\mu \gamma^\nu}{2M_N}\right] u$$

- Multipole form factor

$$F^{u-d}_{\mathrm{mf},0}(t) = H_T^{u-d}(t) + \frac{t}{6M_N^2} E_T^{u-d}(t) \qquad F^{u+d}_{\mathrm{mf},0}(t) = H_T^{u+d}(t) + \frac{t}{6M_N^2} E_T^{u+d}(t)$$
$$F^{u+d}_{\mathrm{mf},1}(t) = E_T^{u+d}(t) + 2\tilde{H}_T^{u+d}(t) \qquad F^{u-d}_{\mathrm{mf},1}(t) = H_T^{u-d}(t) + E_T^{u-d}(t) + 2\tilde{H}_T^{u-d}(t)$$
$$F^{u-d}_{\mathrm{mf},2}(t) = E_T^{u-d}(t) \qquad F^{u+d}_{\mathrm{mf},2}(t) = E_T^{u+d}(t)$$

- Tensor moments

$$H_T^q(0) = g_T^q, \quad E_T^q(0) + 2\tilde{H}_T^q(0) = \kappa_T^q, \quad 2\tilde{H}_T(0) = Q_T^q$$

[4] C. V. Christov, A. Blotz, H.-C. Kim, P. Pobylitsa, T. Watabe, T. Meissner, E. Ruiz Arriola, and K. Goeke, Baryons as nontopological chiral solitons, Prog. Part. Nucl. Phys. 37, 91 (1996)

# Chiral quark soliton model

- Partition function

$$\mathcal{Z}_{\rm eff} = \int D\psi D\psi^\dagger DU \exp\left[\int \psi^\dagger (i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m})\psi\right] = \int DU \exp\left(-S_{\rm eff}[U]\right),$$

$$S_{\rm eff}[U] := -N_c {\rm Tr}\log\left(i\partial\!\!\!/ + iMU^{\gamma_5} + i\hat{m}\right).$$

- Two point correlation function

$$\lim_{T\to\infty} \Pi(T) = \langle 0|J_N(\vec{0},T/2)J_N^\dagger(\vec{0},-T/2)|0\rangle \sim e^{-M_N T}.$$

$$J_N(x) = \frac{1}{N_c!}\epsilon_{a_1\ldots a_{N_c}} \Gamma^{f_1\ldots f_{N_c}}_{(TT_3)(JJ_3)} \psi_{f_1 a_1}(x)\ldots\psi_{f_{N_c} a_{N_c}}(x).$$

$$\Pi(T) = \Gamma^{f_i\ldots f_{N_c}} \left(\Gamma^{g_i\ldots g_{N_c}}\right)^* \frac{1}{Z_{\rm eff}} \int DU \prod_{i=1}^{N_c} G(\vec{0},T/2;\vec{0},-T/2|U)_{f_i g_i} e^{-S_{\rm eff}[U]}$$

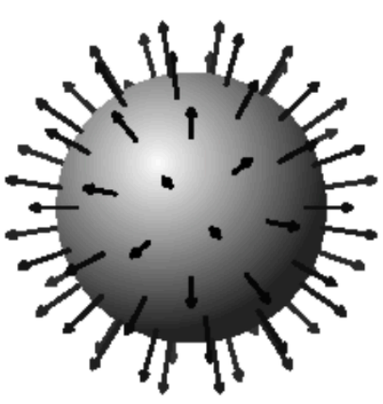
[4] C. V. Christov, A. Blotz, H.-C. Kim, P. Pobylitsa, T. Watabe, T. Meissner, E. Ruiz Arriola, and K. Goeke, Baryons as nontopological chiral solitons, Prog. Part. Nucl. Phys. 37, 91 (1996)

# Chiral quark soliton model

- Saddle point solution

$$\left.\frac{\delta S_{\rm eff}[U]}{\delta U}\right|_{U=U_{\rm cl}} = 0$$

- Hedgehog ansatz



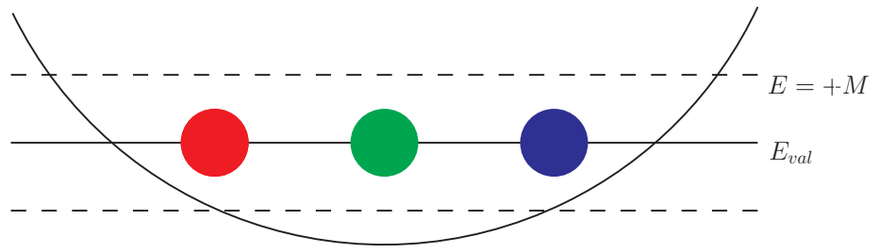
hedgehog

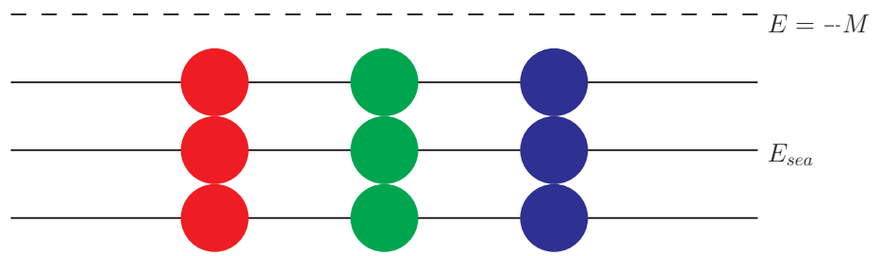
$$U = \exp[i\vec{n}\cdot\vec{\tau}P(r)]$$

$$U^{\gamma_5} = \exp[i\gamma_5\vec{n}\cdot\vec{\tau}P(r)]$$

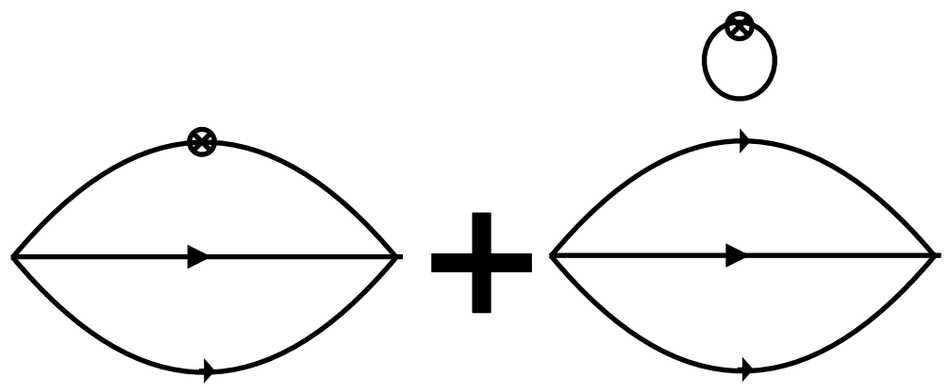
$$= \frac{1+\gamma_5}{2}U + \frac{1-\gamma_5}{2}U^\dagger$$

- Classical nucleon




$$\langle N(p',S_3')|\bar{\psi}_q(0)i\sigma^{\mu\nu}\psi_q(0)|N(p',S_3')\rangle =$$

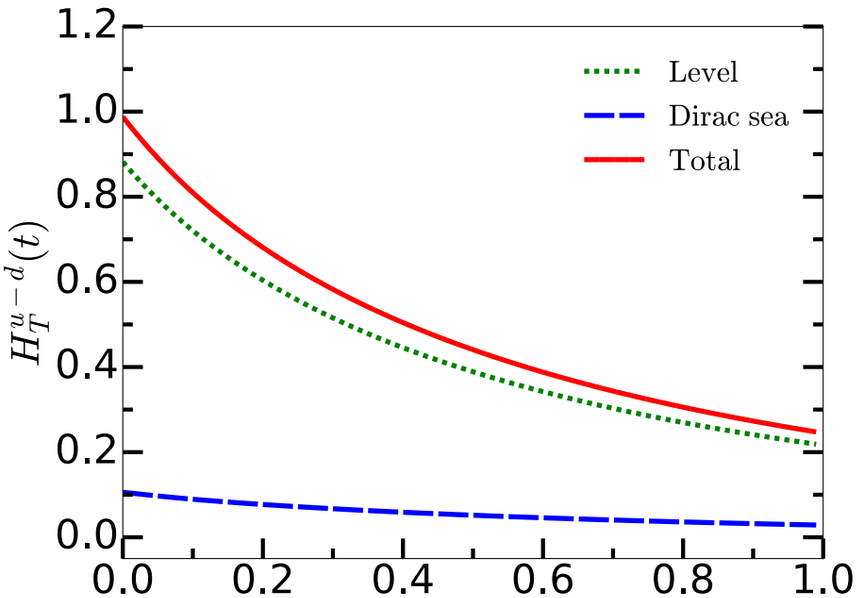


Valence(Level) Sea

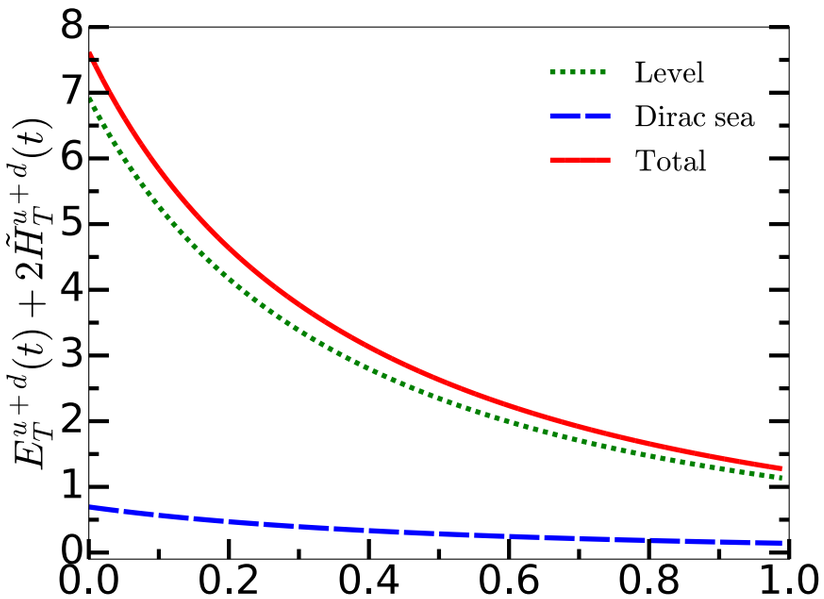
[4] C. V. Christov, A. Blotz, H.-C. Kim, P. Pobylitsa, T. Watabe, T. Meissner, E. Ruiz Arriola, and K. Goeke, Baryons as nontopological chiral solitons, Prog. Part. Nucl. Phys. 37, 91 (1996)

# Results

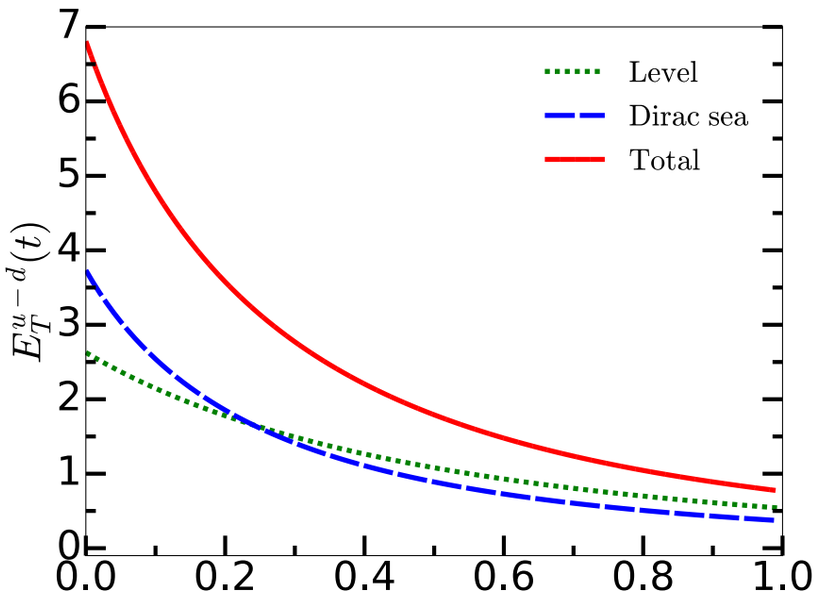
- t dependence




Monopole form factor




Dipole form factor

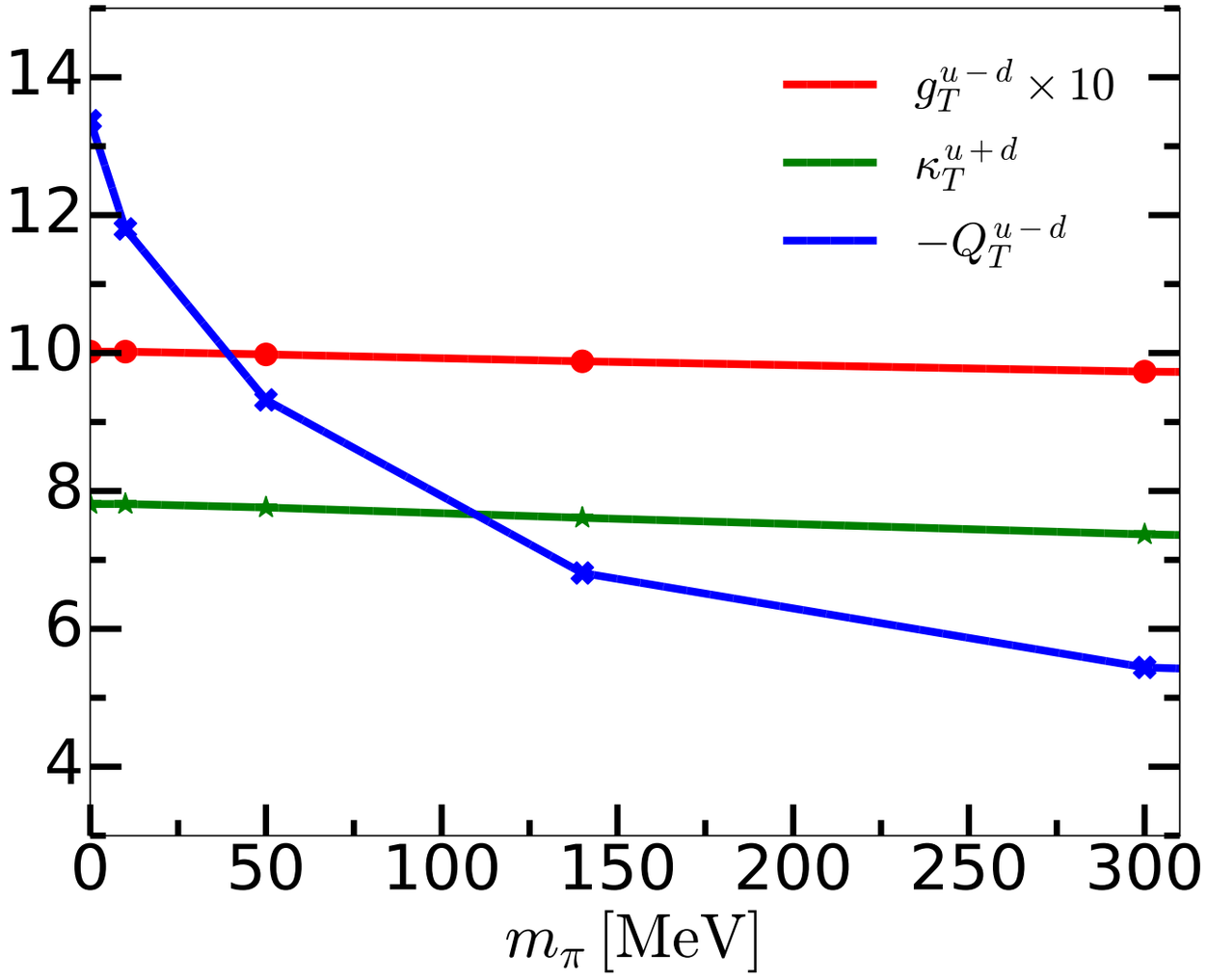



Quadrupole form factor

| | Level | Dirac Sea | Total | Non-relativistic limit |
|---|---|---|---|---|
| $g_T^{u-d}$ | 0.88 | 0.11 | 0.99 | 1 |
| $\kappa_T^{u+d}$ | 6.92 | 0.69 | 7.61 | 0 |
| $-Q_T^{u-d}$ | 3.08 | 3.94 | 7.02 | 0 |

# Results

- Pion mass dependence




| $m_\pi$ | $g_T^{u-d}$[lev] | $g_T^{u-d}$[sea] | $g_T^{u-d}$[tot] |
|---|---|---|---|
| 0 | 0.885 | 0.117 | 1.002 |
| 10 | 0.885 | 0.117 | 1.002 |
| 50 | 0.884 | 0.114 | 0.998 |
| 140 | 0.882 | 0.106 | 0.988 |
| 300 | 0.882 | 0.091 | 0.973 |
| 600 | 0.897 | 0.065 | 0.962 |
| $m_\pi$ | $\kappa_T^{u+d}$[lev] | $\kappa_T^{u+d}$[sea] | $\kappa_T^{u+d}$[tot] |
| 0 | 6.87 | 0.94 | 7.81 |
| 10 | 6.87 | 0.94 | 7.81 |
| 50 | 6.88 | 0.88 | 7.76 |
| 140 | 6.92 | 0.69 | 7.61 |
| 300 | 6.86 | 0.51 | 7.37 |
| 600 | 6.66 | 0.43 | 7.09 |
| $m_\pi$ | $-Q_T^{u-d}$[lev] | $-Q_T^{u-d}$[sea] | $-Q_T^{u-d}$[tot] |
| 0 | 3.01 | 9.58 | 12.59 |
| 10 | 3.01 | 8.40 | 11.42 |
| 50 | 3.03 | 6.21 | 9.24 |
| 140 | 3.08 | 3.94 | 7.02 |
| 300 | 3.01 | 2.73 | 5.74 |
| 600 | 2.80 | 2.50 | 5.30 |

# Summary & outlook

- In this work, we estimate the tensor charge($g_T^{u-d} = 0.99$), anomalous tensor magnetic moment($\kappa_T^{u+d} = 7.61$) and tensor quadrupole moment($Q_T = -7.02$).

- $\left(\kappa_T^{u+d}\right)_{\text{NR}} \approx 0,\ \left(Q_T^{u-d}\right)_{\text{NR}} \approx 0, \left(Q_T^{u-d}\right)_{\text{sea}} \approx \frac{1}{2} Q_T^{u-d}$

- By varying the pion mass and observing the trends in the tensor moments, we found ($Q_T^{u-d}$) shows a tendency to be significantly enhanced in the chiral limit.

- We expect to extract information to explain the transversely polarized spin distribution using tensor form factors.

# Thank you very much