Lattice Parton Collaboration

中國科學院高能物理研究所
Institute of High Energy Physics
Chinese Academy of Sciences

BARYONS 2025

Advances in **Lattice QCD** for Determining Light-cone Distribution Amplitudes (**LCDA**) : From **Mesons** to **Baryons**

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(Lattice Parton Collaboration)

Nov. 11 @ Baryon 2025

OUTLINE

01 **Motivation**

Baryon; LCDA; Review Meson & Baryon LCDA

02 **Baryon LCDA on Lattice**

LaMET; Quasi DA; Matching

03 **Process and Improvements**

Simulation improvement; Renormalization; Limited FT

04 **Summary and Outlook**

LCDAs of Lambda & Proton

Motivation About Meson & Baryon

- Meson — ubiquitous messengers

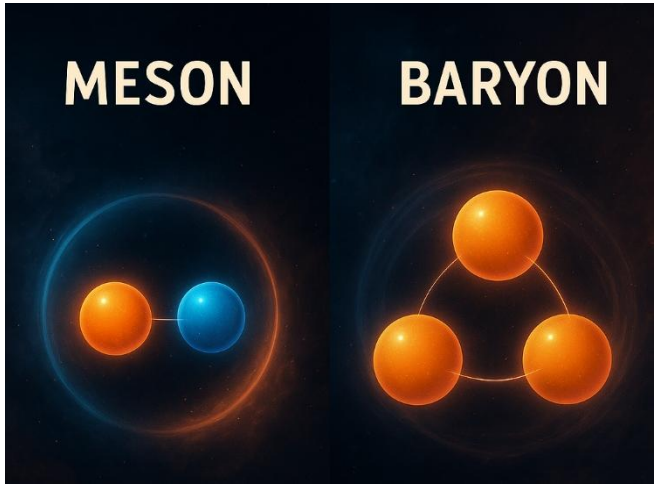
  Light Meson $SU(3)_{flavor}$ $8 \otimes 1$

  Heavy Meson $SU(4)_{flavor}$ …

- Baryon — cornerstone particles

  Octet baryons

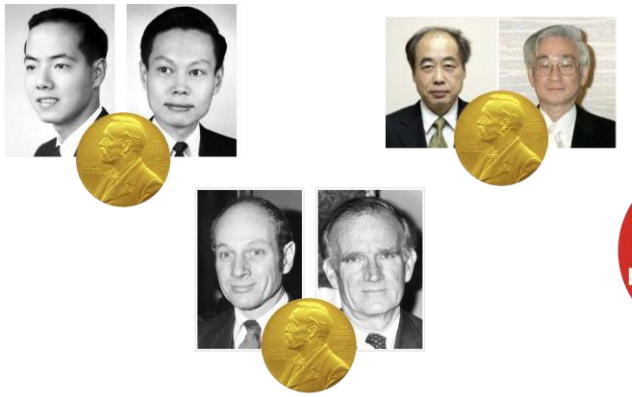
  Decuplet baryons



Motivation About Meson & Baryon

Meson and Baryon in Flavor Physics

- 1956, Parity violation in weak interaction
- 1964, Observation of CP violation in **Kaon meson**
- 1973, Kobayashi-Maskawa mechanism
- 2004, Observation of direct CPV in **B meson**
- 2019, Observation of direct CPV in **D meson**
- 2025, Observation of direct CPV in **Baryon $\Lambda_b^0 \to p$**

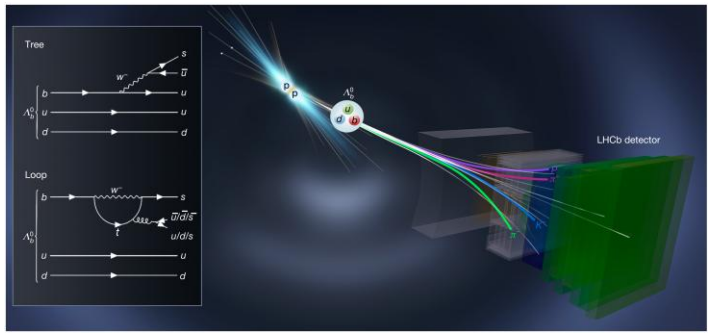


physicsworld TOP 10 BREAKTHROUGH 2019

Observation of charge–parity symmetry breaking in baryon decays

LHCb Collaboration

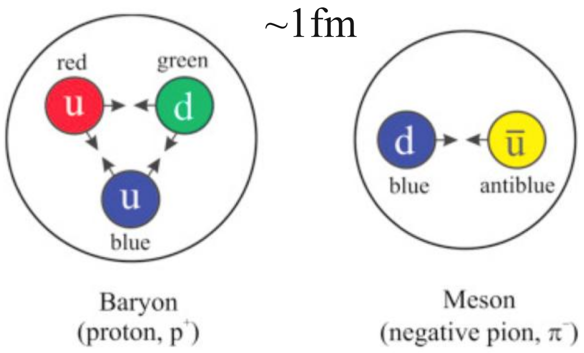
Nature **643**, 1223–1228 (2025) | Cite this article

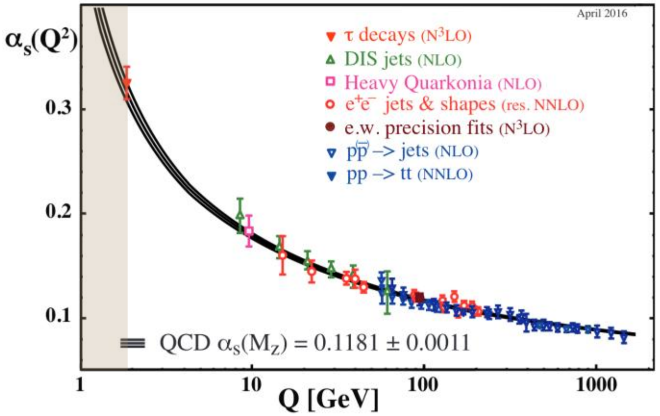


LHCb, Nature 643 8074 (2025)； Theoretical: *J.J.Han, et.al. PRL 134 221801(2025)*

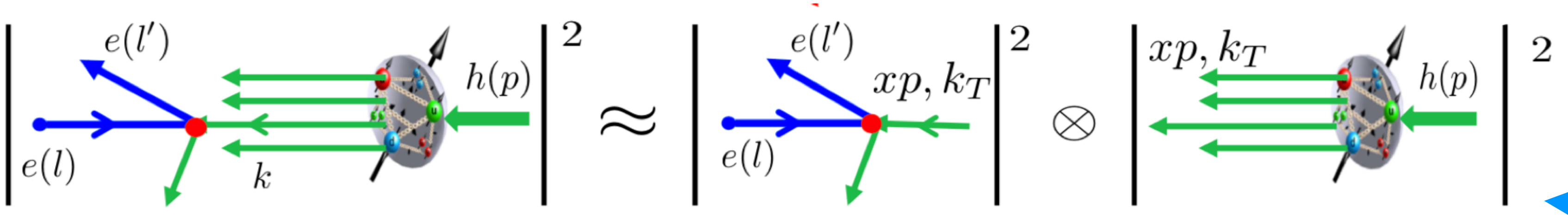
Motivation About LCDA

• Color confinement and Asymptotic freedom of QCD





• QCD Factorization



QCD Cross section

Perturbative
(Hard kernel)

Non-perturbative
(Parton structure in hadron)

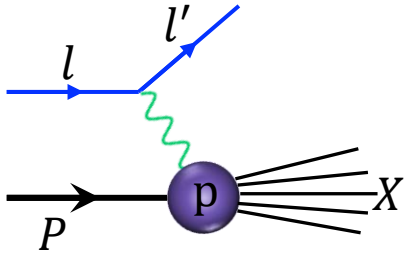
Longitudinal distribution of parton density in hadrons (PDF, DA)

Motivation About LCDA

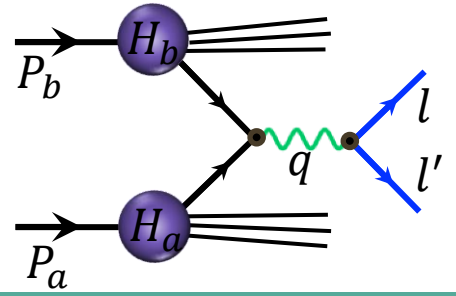
- One try to probe internal structure of nucleons → **Inclusive beam-target collision**

Defined PDFs

DIS

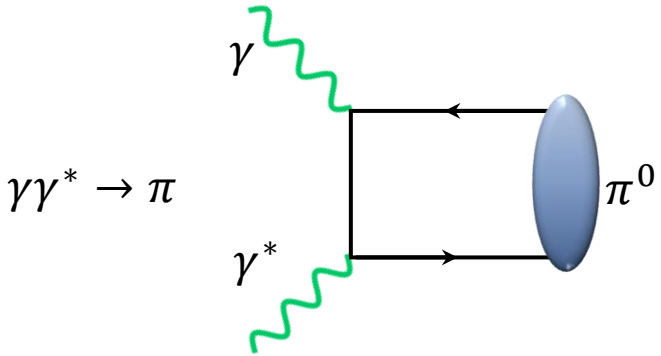


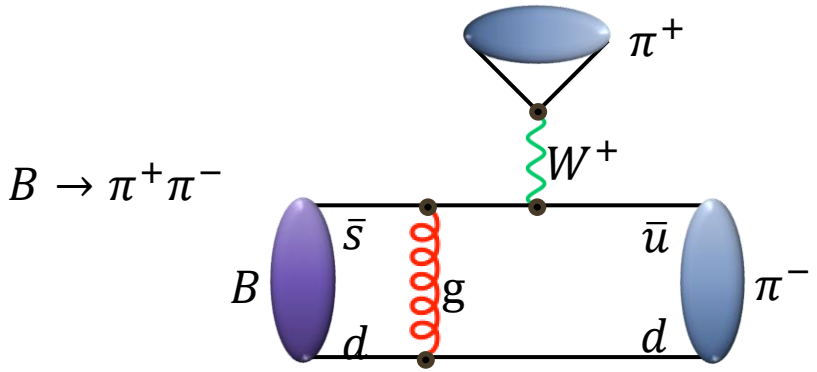
Drell-Yan

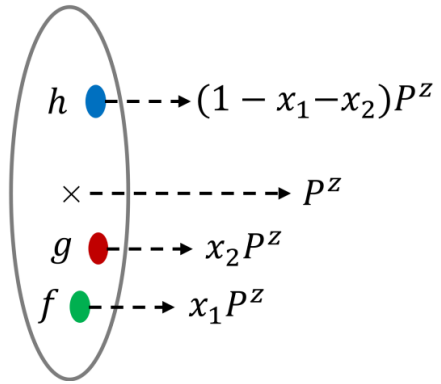


- One try to obtain richer QCD dynamical information → **Exclusive scattering**

Defined LCDAs







Motivation About LCDA

Light Meson LCDAs: (1977 - now)

- Asymptotic form

  Chernyak, Zhitnitsky, 1977; Lepage, Brodsky, 1979; ……
- QCD Sum rules

  Chernyak, Zhitnitsky, 1982; Braun, Filyanov, 1989; ……
- Dyson-Schwinger Equation

  Chang, Cloet, et.al, 2013; Gao, Chang, et.al, 2014; ……
- Global Fits

  Cheng, et.al, 2020; Hua, Li, Lu, Wang, Xing, 2021; ……
- Models

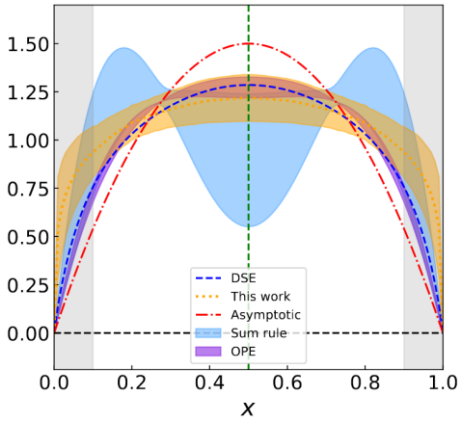
  Arriola, Broniowski, 2002; Zhong, Zhu, et.al, 2021; ……
- **Lattice QCD with OPE**

  Braun, et al., 2016; RQCD collaboration, 2019, 2020; ……
- **Lattice QCD with LaMET**

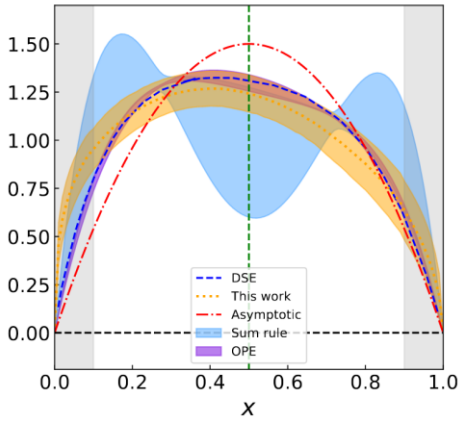
  LP3, 2019; LPC, 2021, 2022; ……
- Quantum Computing Li , et al., 2023

x-dependent LCDAs

$\phi_\pi(x)$



$\phi_K(x)$

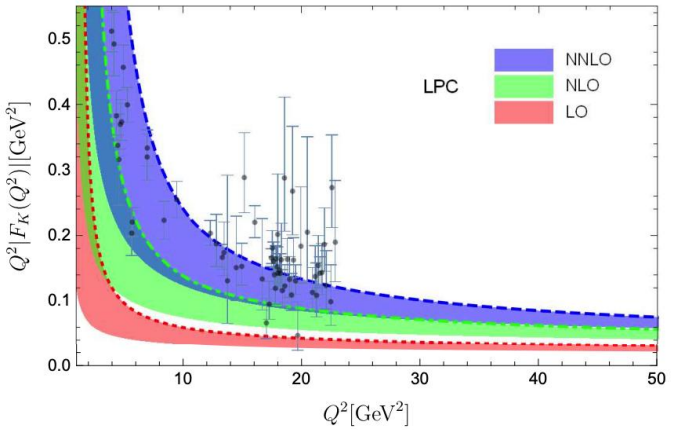


LPC, PRL 127 062002(2021)

Pion Dirac form factors:

Application:

L.B.Chen et.al
PRL 132 201901(2024),
PLB 870 139886(2025)



What's Next ?

Motivation About Baryon LCDA

Light Baryon LCDAs: (1983 - now)

- Asymptotic form

  Chernyak, Zhitnitsky, 1983 ; ……
- QCD Sum rules

  King, et.al, 1987; Stefanis, et.al, 1993; Braun, et.al, 2006; ……
- Models parametrization

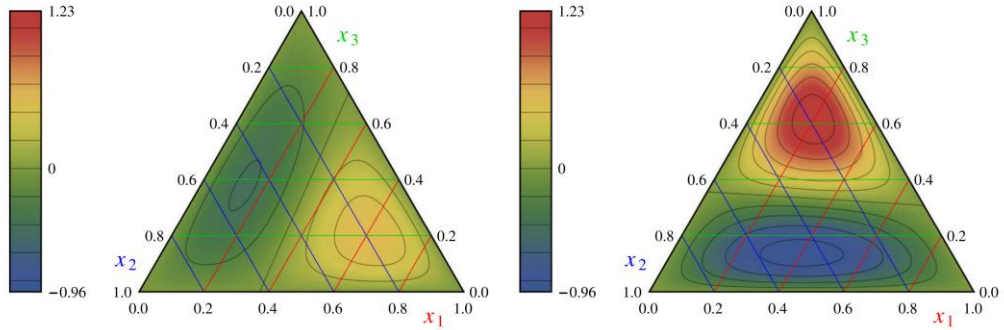
  Bell, Feldmann, Wang, Matthew 2013; ……
- **Lattice QCD with OPE**

  QCDSF, 2008, 2009; RQCD, 2016, 2019(2025) ……
- **Lattice QCD with LaMET**

  LPC, 2024,2025; ……

PRD 111, 034510, arXiv:2508.08971

Leading moments

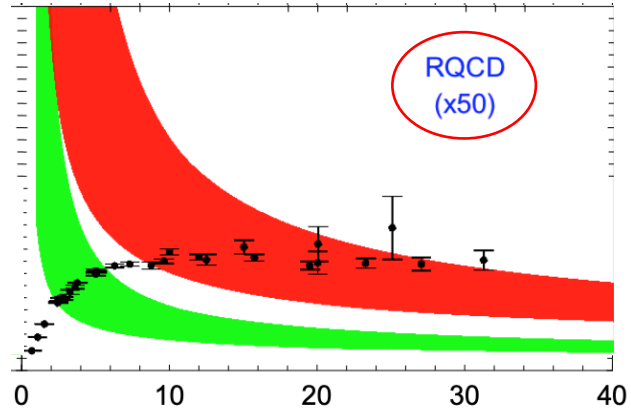


RQCD, Eur. Phys. J. A (2019)

Application:

J.J.Han, et.al.
PRL 134 221801(2025)
L.B.Chen, et.al.
PRL 135 131903(2025)

Nucleon Dirac form factors:



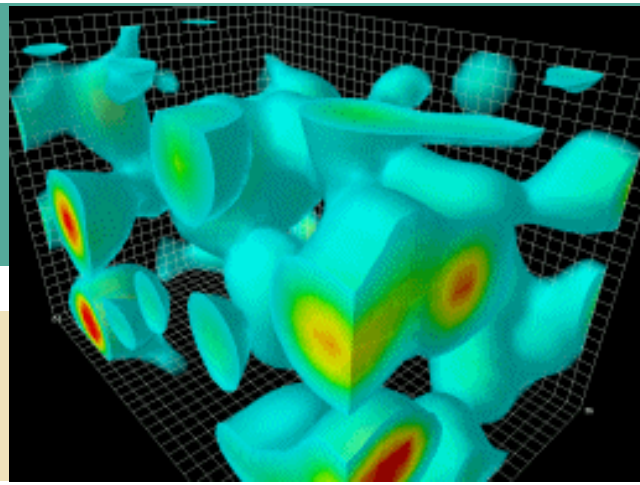
Soft contribution dominates in baryon decay,
leading moments far from enough

More is Different

Motivation Moments & LaMET

Lattice QCD **formulate a Feynman path integral on discrete 4D Euclidean grid.**

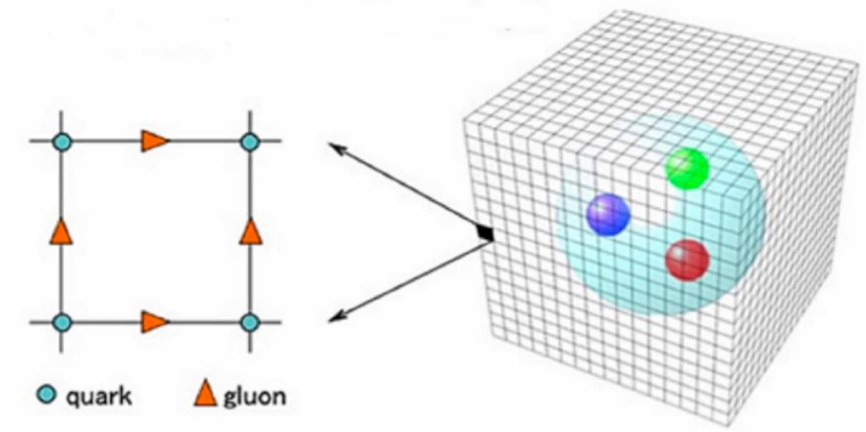
Numerical simulations based on a QCD Lagrangian with discrete from:



$$\mathcal{L} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi - \frac{1}{4}F^{a}_{\mu\nu}F^{a,\mu\nu} \quad \xrightarrow[t = i\tau]{} \quad \mathcal{L}^{E} = \bar{\psi}\left(i\gamma^{\mu}D_{\mu} + m\right)\psi + \frac{1}{4}F^{a}_{\mu\nu}F^{a,\mu\nu}$$

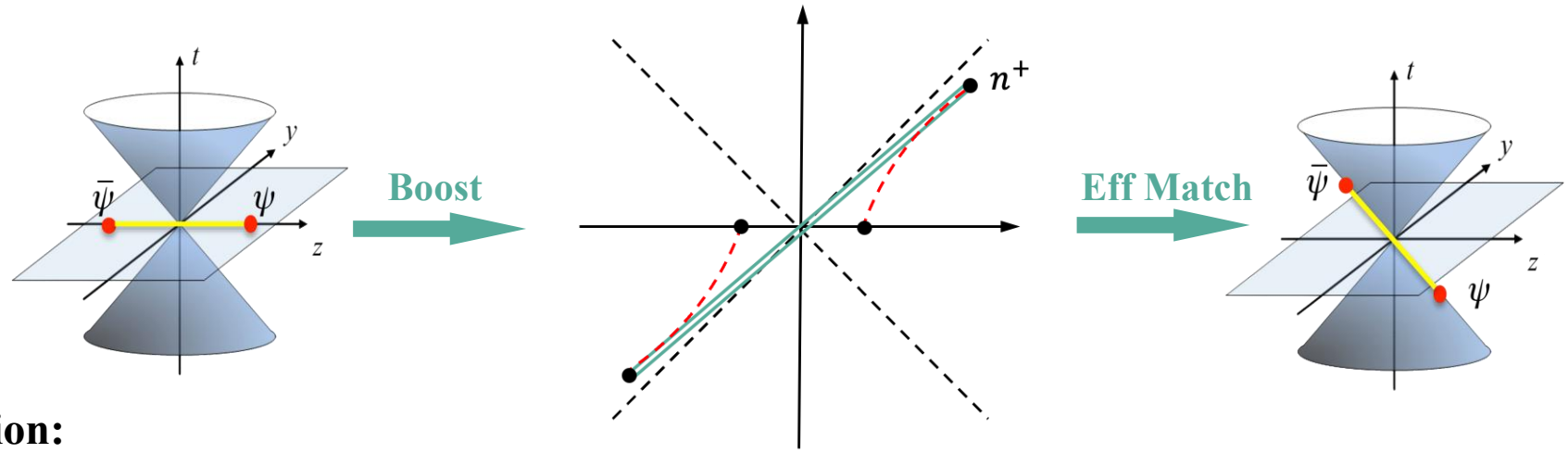
$$S_E^{\text{latt}} = -\sum_{\square}\frac{6}{g^2}\text{Retr}_N\left(U_{\square,\mu\nu}\right) - \sum_{q}\bar{q}\left(D_{\mu}^{\text{lat}}\gamma_{\mu} + am_q\right)q$$

- Applicable to non-perturbation calculations ☺
- Euclidean time separation
- Not match to **light-cone** correlation ☹



Motivation Moments & LaMET

Large momentum effective theory (LaMET) ☺ : the light-cone non-local operator correlated to Euclidean non-local operator with a large momentum boost



LaMET expansion:

LCDA — Matching kernel — quasi-DA

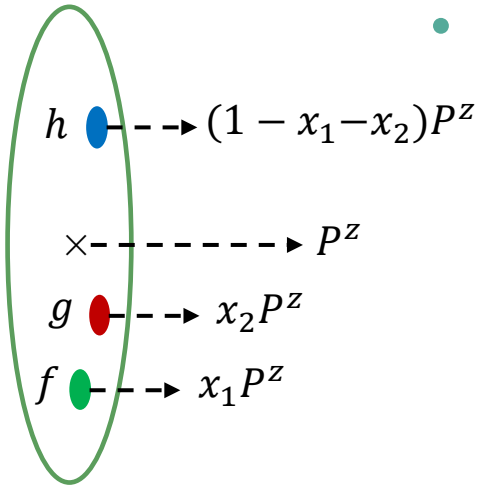
$$\phi(x_1,x_2;\mu)=\int \mathrm{d}y_1\mathrm{d}y_2\ \mathcal{C}(x_1,x_2;y_1,y_2;P^z,\mu)\ \tilde{\phi}(y_1,y_2;P^z,\mu)$$

Higher power corrections

$$+\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(x_1P^z)^2},\frac{\Lambda^2_{\rm QCD}}{(x_2P^z)^2},\frac{\Lambda^2_{\rm QCD}}{[(1-x_1-x_2)P^z]^2}\right)$$

- Light meson: LP3, 2019; LPC, 2021, 2022; ANL,2024; ……
- Heavy meson: LPC, 2024,2025; ……
- Light baryon: LPC, 2025; ……

Baryon LCDA on Lattice



- **Definition of Baryon LCDAs:**

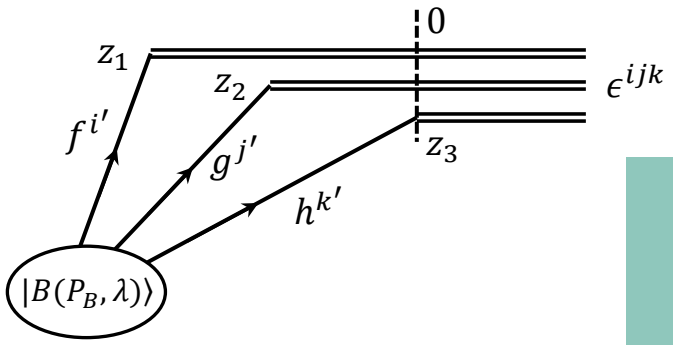
$$M_L\left(\xi_1,\xi_2;P\right)=\langle 0|\epsilon^{ijk}f_\alpha^{i'}\left(\xi_1 n\right)W_{i'i}\left(\xi_1,\xi_0\right)g_\beta^{j'}\left(\xi_2 n\right)W_{j'j}\left(\xi_2,\xi_0\right)h_\gamma^{k}\left(\xi_0 n\right)|B(P,\lambda)\rangle$$

$$\Phi\left(x_1,x_2,\mu\right)=\int\frac{\mathrm{d}P^+\xi_1}{2\pi}\frac{\mathrm{d}P^+\xi_2}{2\pi}\mathrm{e}^{\mathrm{i}x_1P^+\xi_1+\mathrm{i}x_2P^+\xi_2}\frac{M_L\left(\xi_1,\xi_2;P,\mu\right)}{M_L(0,0;P,\mu)}$$

V.L.C & I.R.Z NPB 24652 (1984)
G.R.Farrar et.al. NPB 311585 (1989)

- **Corresponding quasi-DAs on Euclidean lattice:**

C.Han et.al. JHEP 12044 (2023)
LPC, PRD 111, 034510 (2025)



$$f_M(p_1,p_2)\tilde{\Phi}^0(x_1,x_2)=\int\frac{p_{z1}dz_1}{2\pi}\frac{p_{z2}dz_2}{2\pi}e^{-i(x_1p_{z1}z_1+x_2p_{z2}z_2)}\langle 0|\hat{O}\left(z_1,z_2,\tilde{\Gamma}\right)|P^z\rangle$$

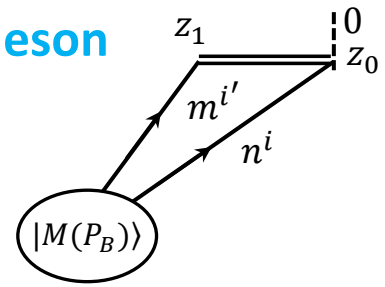
$$\hat{O}_\gamma(\vec{x},t;z_1,z_2)=\epsilon^{ijk}W^{ii'}(\infty,\vec{x}+z_1n_z)f_\alpha^{i'}(\vec{x}+z_1n_z,t)$$
$$\times\tilde{\Gamma}_{\alpha\beta}W^{jj'}(\infty,\vec{x}+z_2n_z,t)g_\beta^{j'}(\vec{x}+z_2n_z)\times W^{kk'}(\infty,\vec{x})h_\gamma^{k'}(\vec{x},t)$$

Octet	n	p	Λ
fgh	d d u	u u d	u d s

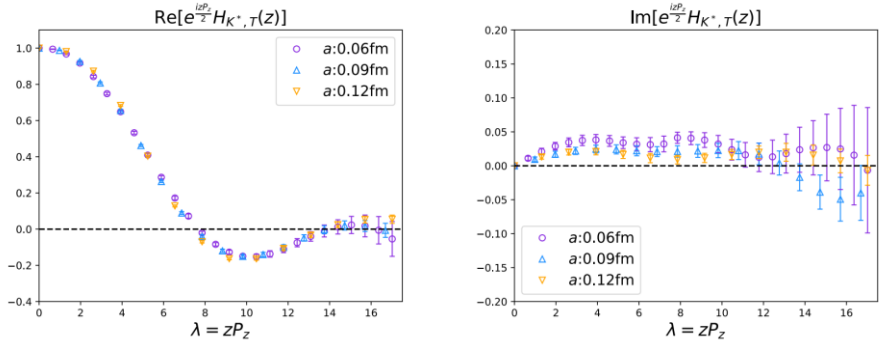
Challenge from Meson to Baryon

1D ⟶ **2D**

➢ **Definition of Meson Quasi-DAs:**

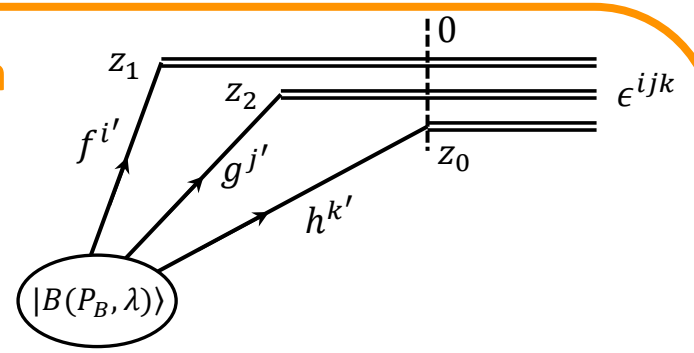


$$\hat{O}(\vec{x},t;z) = m_\alpha^{i'}(\vec{x}+zn_z,t)\tilde{\Gamma}_{\alpha\beta}W^{i'i}(\vec{x}+zn_z,\vec{x},t)n_\beta^i(\vec{x},t)$$



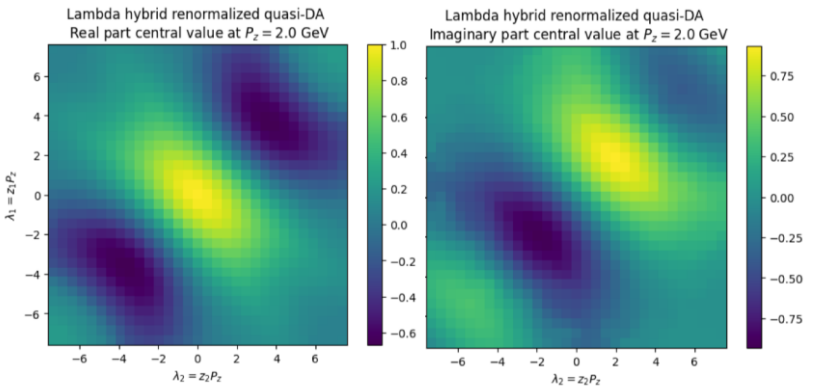
LPC, PRL 127, 062002 (2021)

➢ **Definition of Baryon Quasi-DAs:**



1 Direction (longitudinal)
but **2 Dimention !**

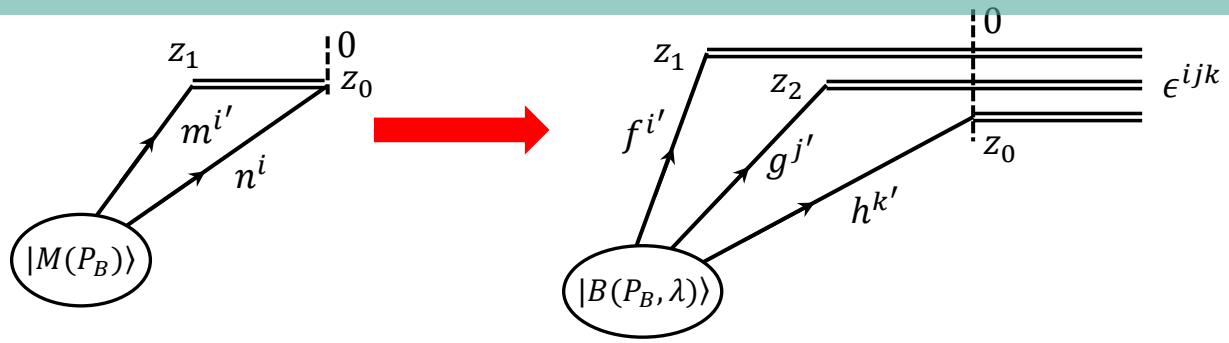
$$\hat{O}_\gamma(\vec{x},t;z_1,z_2) = \epsilon^{ijk}W^{ii'}(\infty,\vec{x}+z_1n_z)f_\alpha^{i'}(\vec{x}+z_1n_z,t)$$
$$\times\tilde{\Gamma}_{\alpha\beta}W^{jj'}(\infty,\vec{x}+z_2n_z,t)g_\beta^{j'}(\vec{x}+z_2n_z)\times W^{kk'}(\infty,\vec{x})h_\gamma^{k'}(\vec{x},t)$$

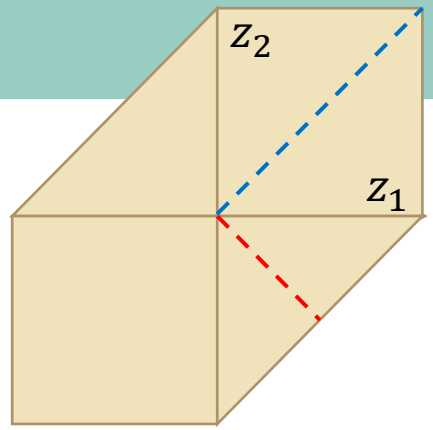


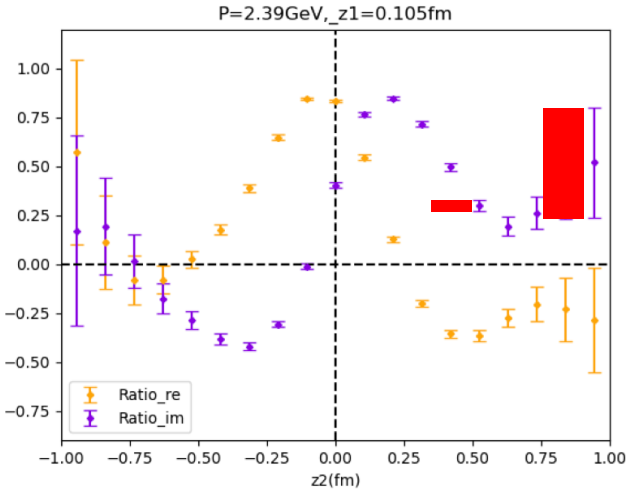
LPC, arXiv:2508.08971 (2025)

Challenge from Meson to Baryon

Challenge in Signal





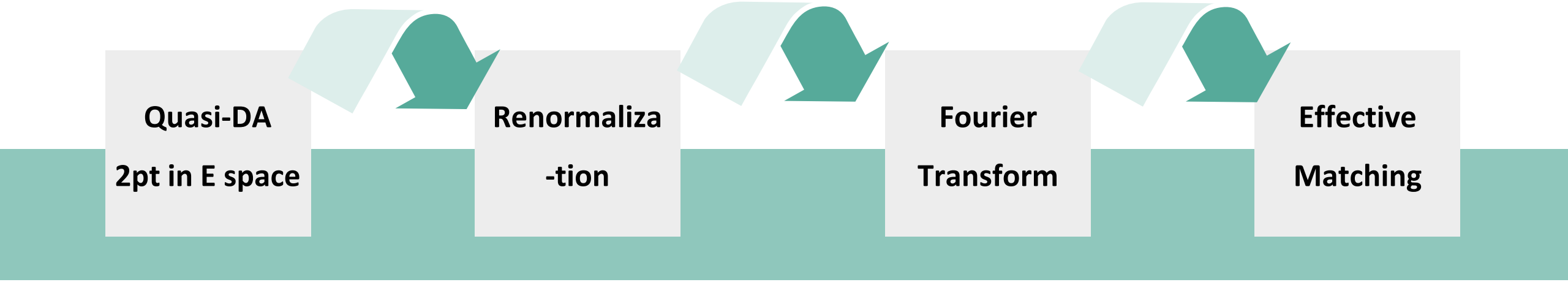


- **From effective Wilson length → Signal to Noise Ratio**
  - Signal to Noise Ratio * 8 -> Statistics * 64
  - Totally 64*10*2
- **Dynamic enhance: SNR 200% ↑ (eff 400%) !**
- **Coding in Pyquda: Computing efficiency 800% ↑ !**

Thanks for X.Y.Jiang, arXiv: 2411.08461

Baryon LCDA on Lattice LaMET

- **Determining Baryon LCDA on Lattice with LaMET**



- **Improvement**

Lattice setup Dynamic enhance	Self renorm Hybrid scheme	Statistical Approach	High dimension Matching

- **Based on CLQCD Ensembles**
  - Three lattice spacing for $a \to 0$ limit
  - Three momentum for $P_z \to \infty$ limit

Ensemble	Volume	Lattice spacing	π mass	measurement	P^z
C24P29	$24^3 \times 72$	0.105 fm	293 MeV	864*4*9	0, 1.96, 2.45, 2.94 GeV
F32P30	$32^3 \times 96$	0.078 fm	303 MeV	777*4*8	0, 2.00, 2.49, 2.99 GeV
G36P29	$36^3 \times 108$	0.069 fm	295MeV	656*6*8	0, 2.00, 2.50, 3.00 GeV
H48P32	$48^3 \times 144$	0.052 fm	317 MeV	550*6*9	0, 1.98, 2.48, 2.98 GeV
C32P23	$32^3 \times 64$	0.105 fm	228 MeV	954*4*8	0, 1.84, 2.21, 2.57, 2.94 GeV
F32P21	$32^3 \times 64$	0.078 fm	210 MeV	459*4*8	0, 2.00, 2.49, 2.99 GeV
C48P14	$48^3 \times 96$	0.105 fm	136 MeV	302*4*16	0, 1.96, 2.45, 2.94 GeV

Kinematically-enhance:

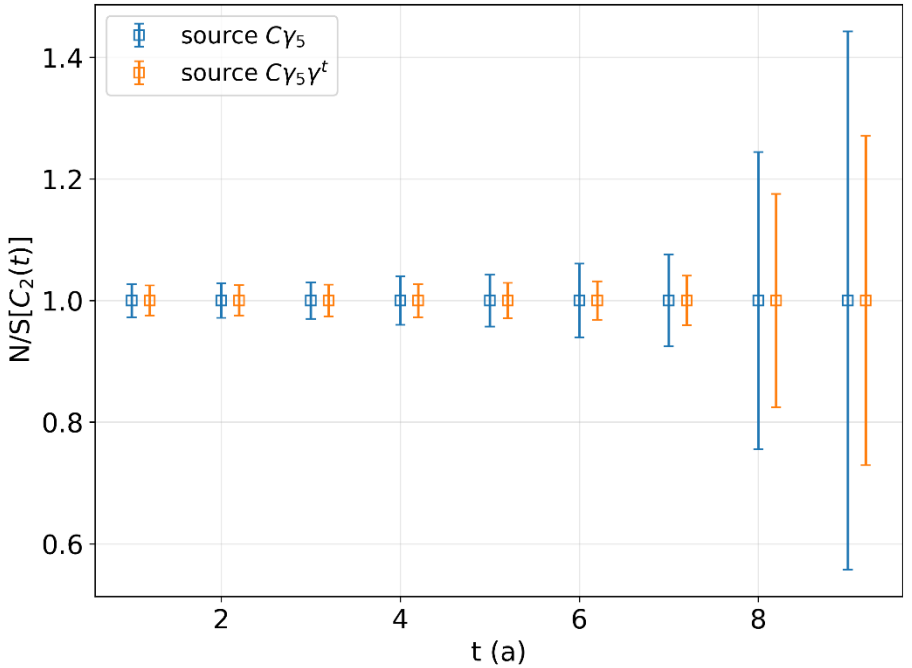
$$C_2^{\pi}(z,\vec{P},t)=\int d^3y e^{-i\vec{P}\cdot\vec{y}}\langle 0|\underbrace{\bar{\psi}_1(\vec{y},t)\Gamma_1 U(\vec{y},\vec{y}-z\hat{z})\psi_2(\vec{y}-z\hat{z},t)}_{Sink}\underbrace{\bar{\psi}_2(0,0)\Gamma_2\psi_1(0,0)}_{Src}|0\rangle$$ *RZ et.al. PRD 112 L051502(2025)*

Decomposition in light-cone limit :

PLB 545,345 (2002), EPJC 33, 75(2004)

$$N_{\gamma_5}=\epsilon^{ijk}(f_i^T\ C\gamma_5\ g_j)h_k$$
$$N_{\gamma_5\gamma_\mu}=\epsilon^{ijk}(f_i^T\ C\gamma_5\gamma_\mu\ g_j)h_k$$

Can be further extended in baryon case.

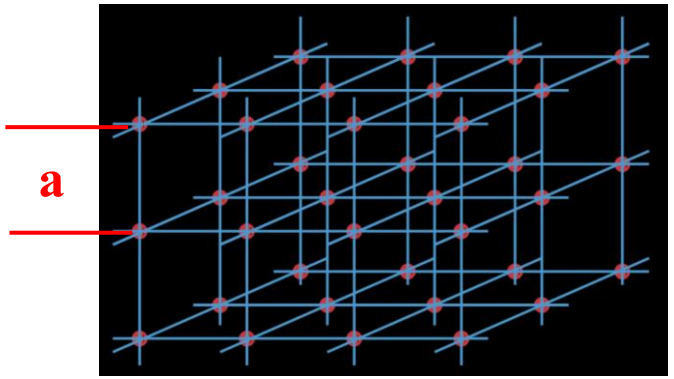


1 step towards precision calculation

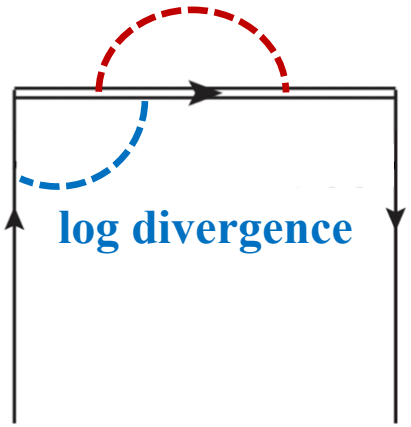
—— See Mu Hua's report

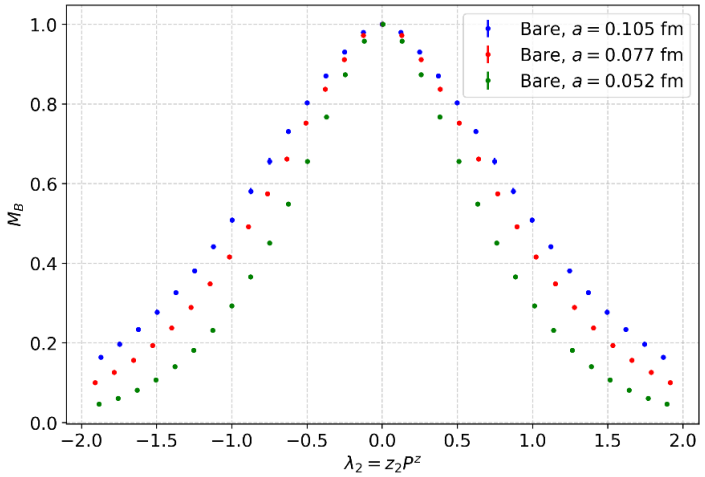
- **Renormalization on Lattice**

$$\frac{1}{a}\ cut \quad \rightarrow \quad \overline{\text{MS}}$$



- **Divergences on Lattice**





- **RI/MOM Scheme**

  Alexandrou et.al, NPB 2017, Stewart, Zhao, PRD 2018
- **Ratio Scheme**

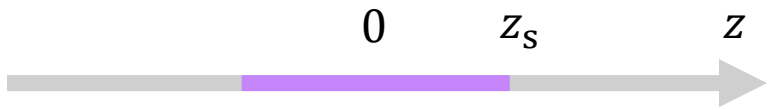
  Radyushkin et.al, PRD 2017
- **Hybrid Scheme and Self Renormalization**

  Ji et.al, NPB 2021, Huo et.al, NPB 2021

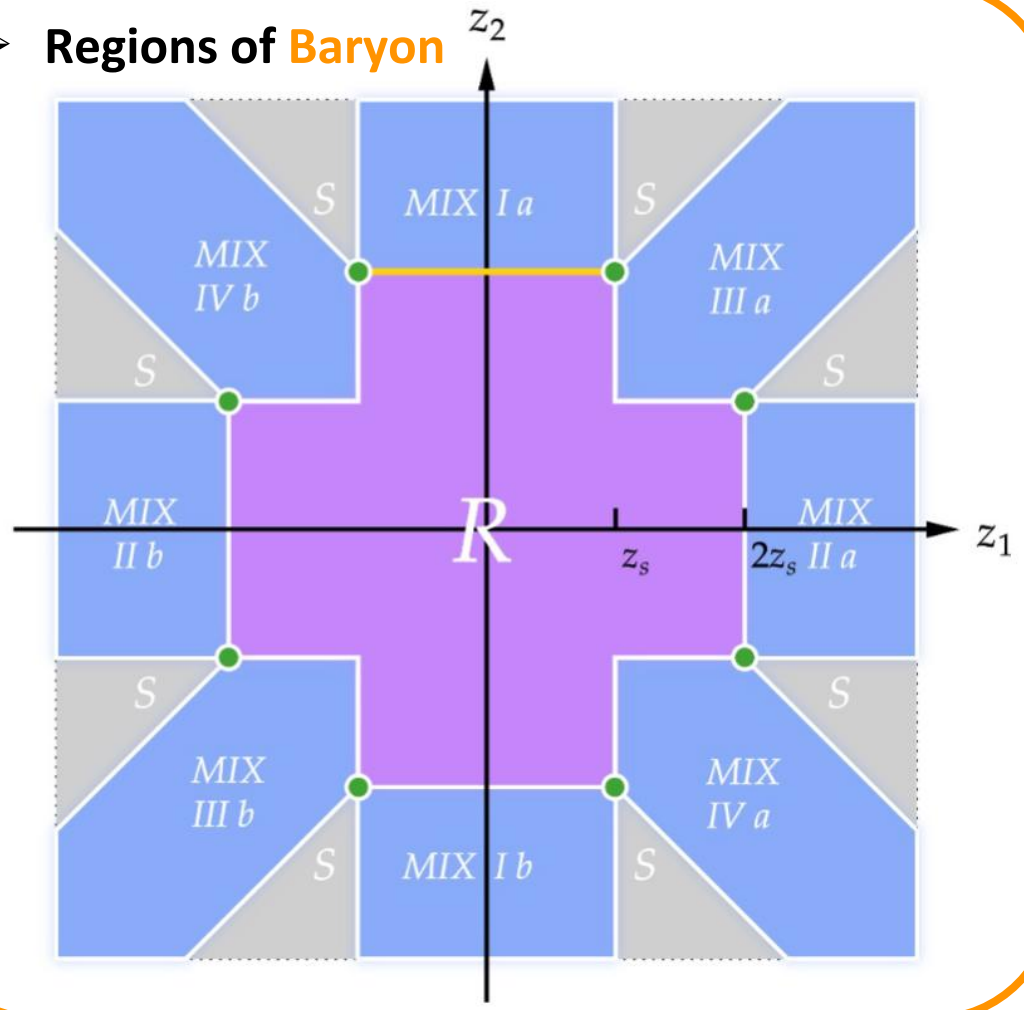
  —— The best solution

1D → 2D

➢ Regions of Meson



➢ Regions of Baryon



- **Self & Hybrid renormalization**

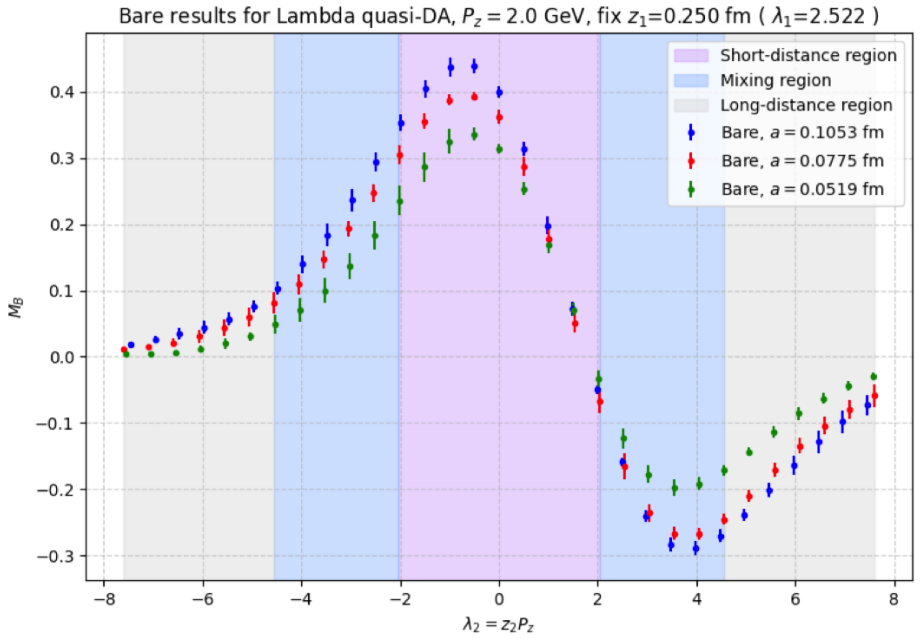
The best renormalization scheme for the nonlocal operators that is self-consistent with the lattice data.

— for parton physics on LQCD

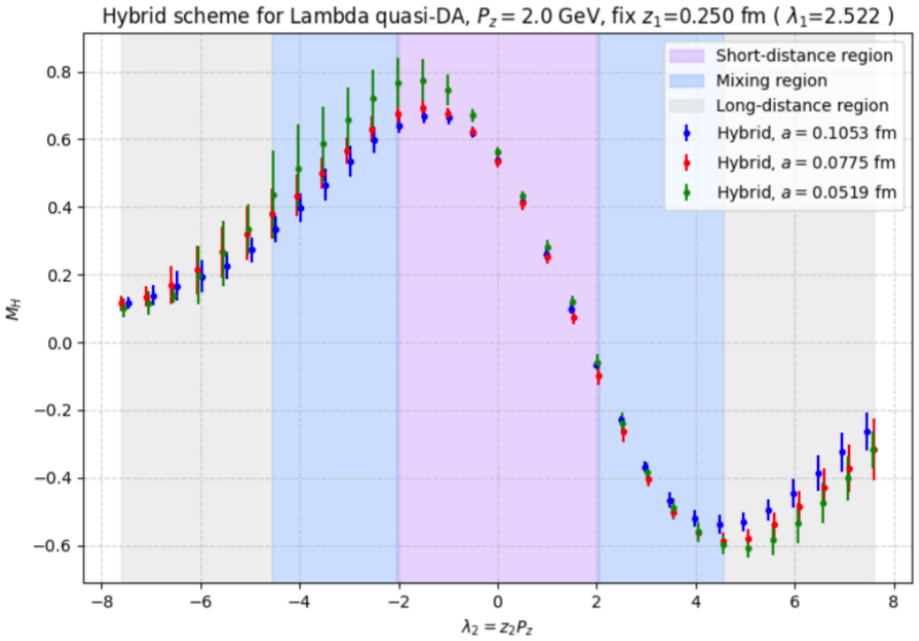
LPC, arXiv:2508.08971 (2025)

- **Hybrid (based on self renormalization) scheme for baryon quasi-DA**

Bare quasi-DA



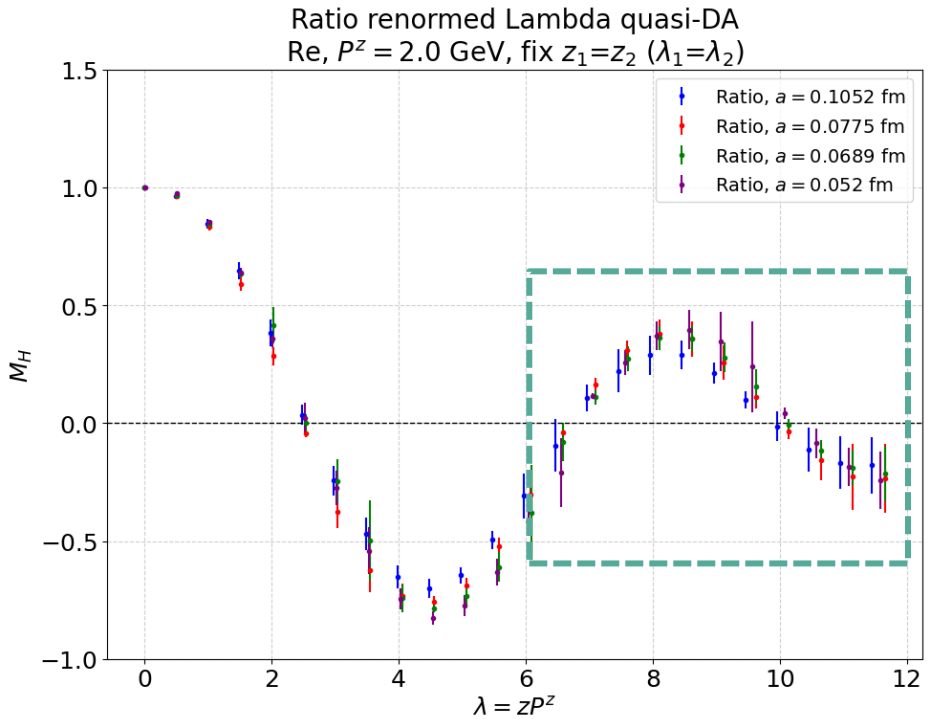
Hybrid renormalized quasi-DA



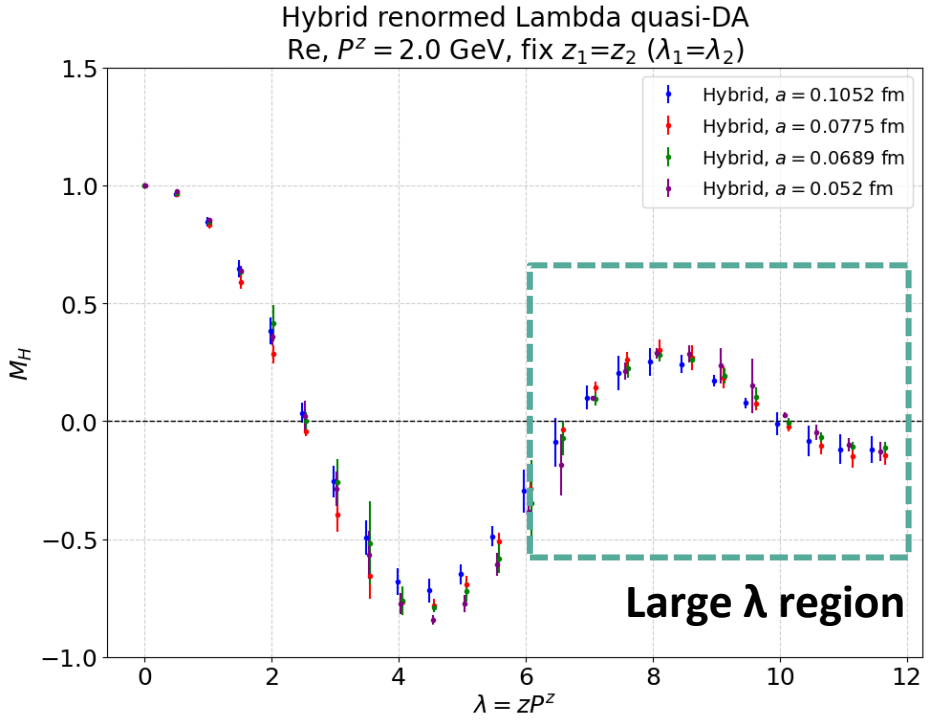
—— See Mu Hua's report

- **Hybrid (based on self renormalization) scheme for baryon quasi-DA**

Ratio renormalization scheme



Hybrid renormalization scheme



SNR enhance

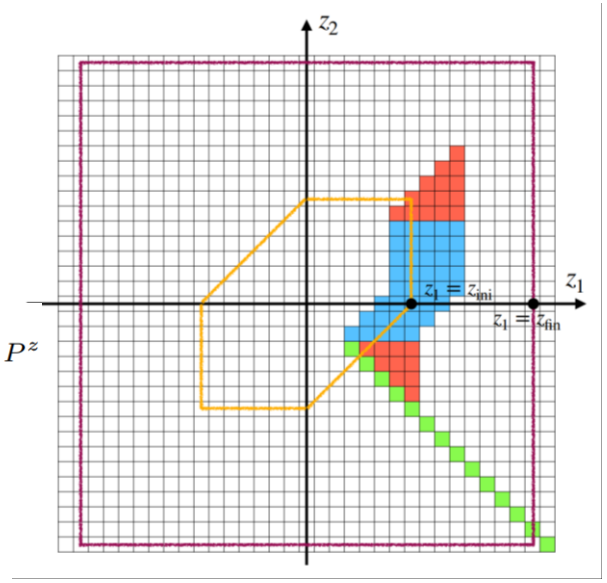
Quasi from coordinate to momentum space: Limited discrete Fourier transform

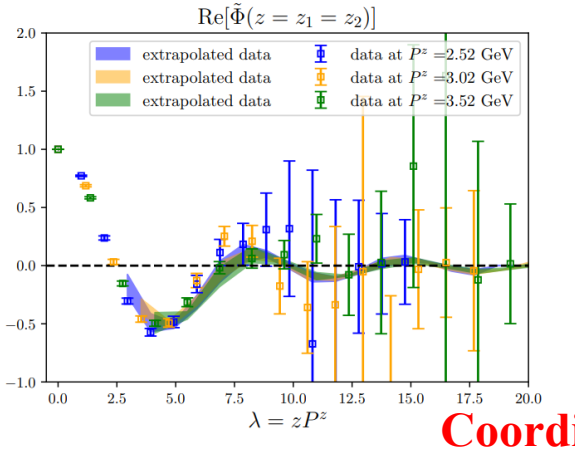
$$g(\lambda) = \int dx e^{-i\lambda x} f(x) \rightarrow f(x) = \frac{1}{2\pi} \sum_{\lambda_{\min}}^{\lambda_{max}} \Delta\lambda e^{i\lambda x} g(\lambda)$$

LPC NPB 964 115311(2021);—hybrid asymptotic ***Extrapolation***

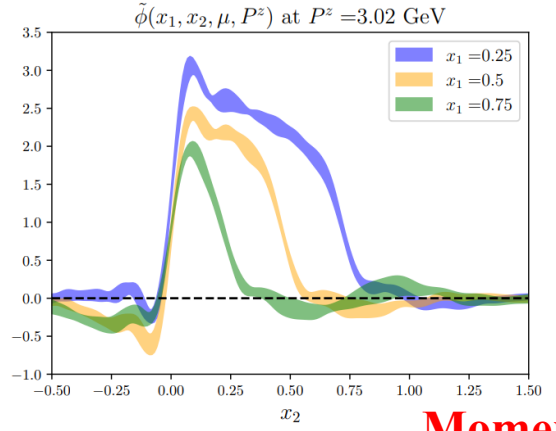
- **2D Extrapolation for Baryon Coordinate space Quasi-DAs:**

$$\tilde{\Phi}(z_1, z_2; d_1, d_2) = \int_0^1 dx_1 \int_0^1 dx_2 \, e^{ix_1 z_1 P^z} e^{ix_2 z_2 P^z} \times C_0 x_1^{d_1} x_2^{d_1} (1 - x_1 - x_2)^{d_2}.$$





Coordinate space



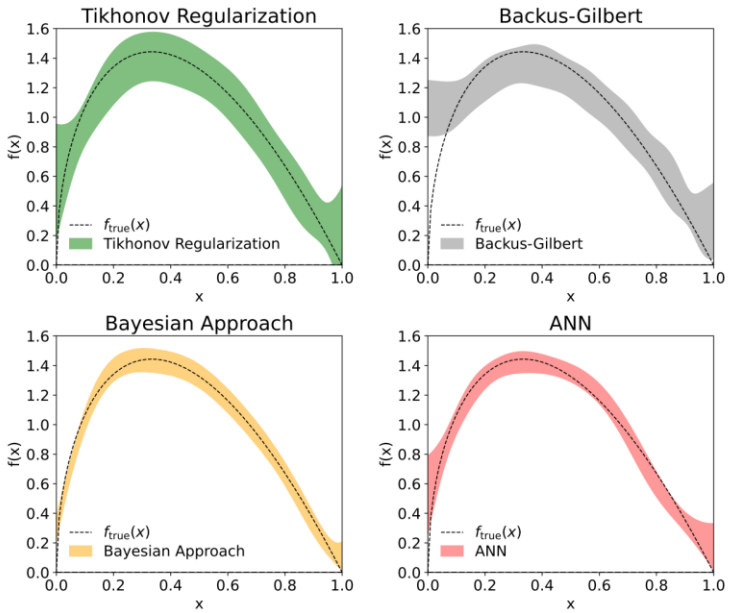
Momentum space

Limited discrete Fourier transform: May be circumvented ?

- Tikhonov regularization
- Backus-Gilbert
- Bayesian approach

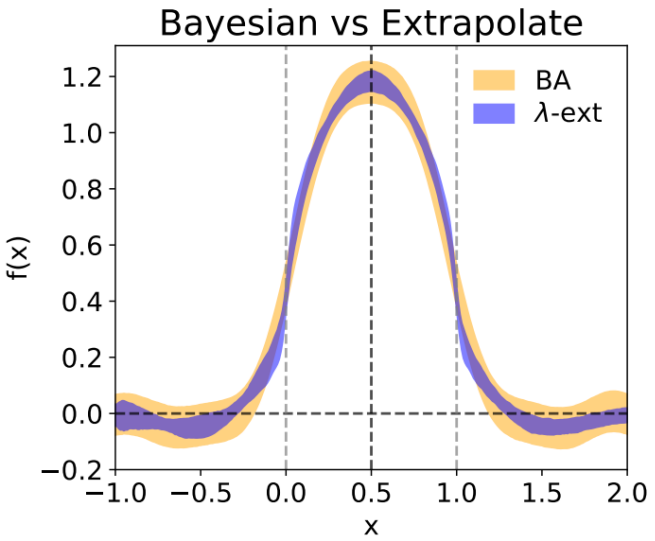
Regulator method and ANN

➢ Artificial neurons network



Still on going

- a support plan for extrapolation
- a safer error estimate
- Maybe more …



Ling, et.al. arXiv:2511.03593(2025)

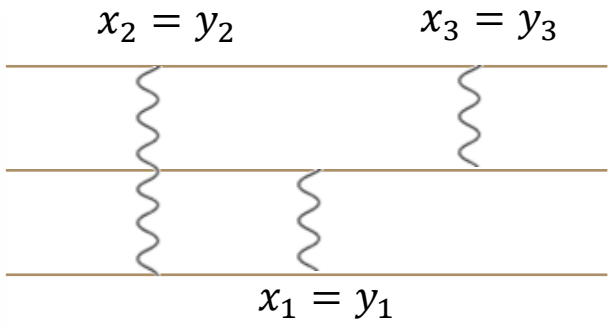
- **LaMET matching for baryon LCDA**

LPC, PRD 111, 034510 (2025)

- **Meson:**
$$\tilde{\phi}(x) = \int_0^1 dy C(x,y)\phi(y) + \mathcal{O}\left(\frac{1}{(xP^z)^2}, \frac{1}{[(1-x)P^z]^2}\right)$$
- **Baryon:**
$$\tilde{\phi}(x_1,x_2) = \int_0^1 dy_1 \int_0^{1-y_1} dy_2 C(x_1,x_2,y_1,y_2)\phi(y_1,y_2) + \mathcal{O}\left(\frac{1}{(x_1P^z)^2}, \frac{1}{(x_2P^z)^2}, \frac{1}{[(1-x_1-x_2)P^z]^2}\right)$$

- **Plus function → Double plus function**

$$\begin{aligned} C(x_1,x_2,y_1,y_2,\mu) &= \delta(x_1-y_1)\,\delta(x_2-y_2) \\ &+\frac{\alpha_s C_F}{2\pi}\left[\left(\frac{1}{4}C_2(x_1,x_2,y_1,y_2) - \frac{7}{8}\frac{-1}{|x1-y1|}\right)\delta(x_2-y_2)\right. \\ &+\left(\frac{1}{4}C_2(x_2,x_1,y_2,y_1) - \frac{7}{8}\frac{-1}{|x2-y2|}\right)\delta(x_1-y_1) \\ &\left.+\left(\frac{1}{4}C_3(x_1,x_2,y_1,y_2) + \frac{1}{4}C_3(x_2,x_1,y_2,y_1) - \frac{3}{4}\frac{-2}{|x_1-y_1-x_2+y_2|}\right)\delta(x_1+x_2-y_1-y_2)\right]_{\oplus}, \end{aligned}$$

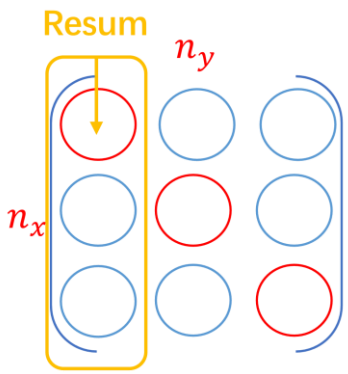


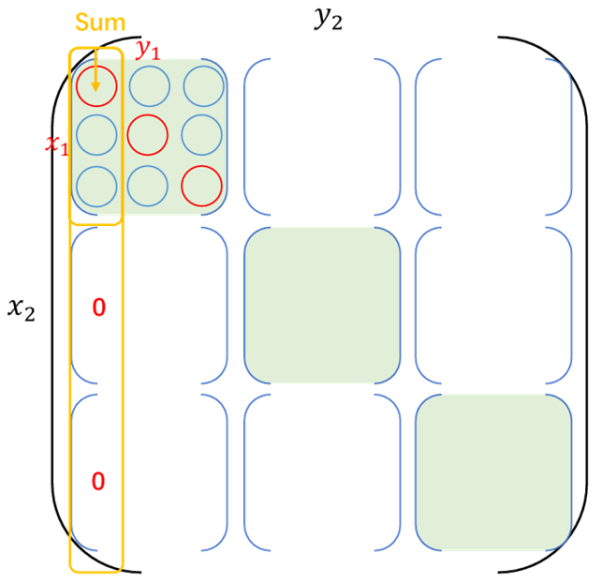
- **High dimension matching for baryon LCDA**

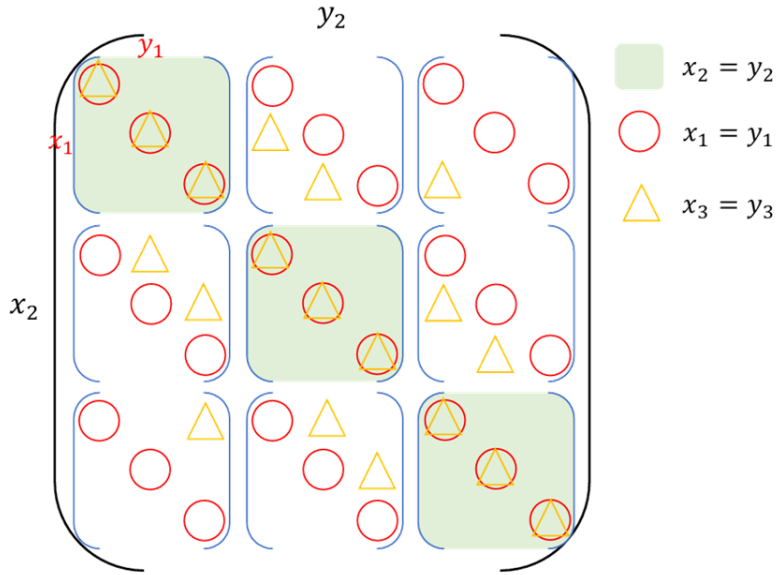
LPC, PRD 111, 034510 (2025)

- **Meson:** $$\tilde{\phi}(x) = \int_0^1 dy C(x,y)\phi(y) + \mathcal{O}\left(\frac{1}{(xP^z)^2}, \frac{1}{[(1-x)P^z]^2}\right)$$
- **Baryon:** $$\tilde{\phi}(x_1,x_2) = \int_0^1 dy_1 \int_0^{1-y_1} dy_2 C(x_1,x_2,y_1,y_2)\phi(y_1,y_2) + \mathcal{O}\left(\frac{1}{(x_1P^z)^2}, \frac{1}{(x_2P^z)^2}, \frac{1}{[(1-x_1-x_2)P^z]^2}\right)$$

- **1D ⊗ 1D matrix → 2D ⊗ 2D tensor**







Summary & Outlook

- **Determining Baryon LCDA on Lattice with LaMET**

Smearing tech | Ratio scheme | 2D Extrapolation | 2D ⊗ 2D matching

LPC, PRD 111, 034510 (2025)

Multi lattice spacing
Dynamic enhance

Self renormalization
Hybrid scheme

Statistical
Approach

LPC, arXiv:2508.08971 (2025)

Physical Pion mass
Larger measurements

Physical limit
Extrapolation

Coming soon!

Summary & Outlook

Thanks for Your Attention !

Backup

Backup

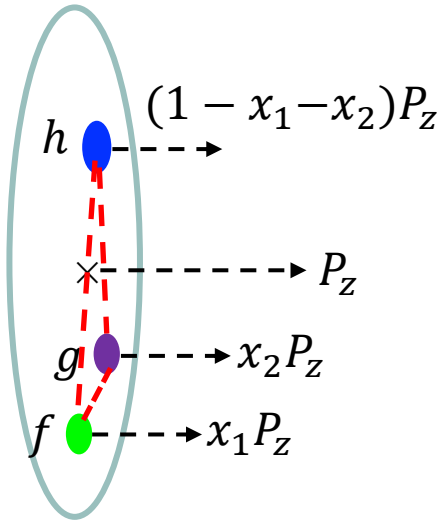
Backup of outlook

- We have extend the numerical computation on Lattice from light meson to light baryon in the LaMET framework.

- To calculate all leading twist structure Proton and Lambda LCDA, we improve:
  - ☐ lattice simulation $a \to 0$
  - ☐ simulation enhance
  - ☐ matching scheme
  - ☐ strategies for limited FT
  - ☐ renormalization scheme
  - ☒ Twist analysis …

- Please stay tuned our results for all leading twists LCDA of Proton and Lambda
- High twists will be the next

Baryon LCDA on Lattice



- **Definition of baryon LCDA:**

$$M_L\left(\xi_1,\xi_2;P\right)=\langle 0|\epsilon^{ijk}f_\alpha^{i'}\left(\xi_1 n\right)W_{i'i}\left(\xi_1,\xi_0\right)g_\beta^{j'}\left(\xi_2 n\right)W_{j'j}\left(\xi_2,\xi_0\right)h_\gamma^{k}\left(\xi_0 n\right)|B(P,\lambda)\rangle$$

$$\Phi\left(x_1,x_2,\mu\right)=\int\frac{\mathrm{d}P^+\xi_1}{2\pi}\frac{\mathrm{d}P^+\xi_2}{2\pi}\mathrm{e}^{\mathrm{i}x_1P^+\xi_1+\mathrm{i}x_2P^+\xi_2}\frac{M_L\left(\xi_1,\xi_2;P,\mu\right)}{M_L(0,0;P,\mu)}$$

C.Han et.al. JHEP 07019 (2024);
V.L.C & I.R.Z NPB 24652(1984); G.R.Farrar et.al. NPB 311585(1989)

- **Leading twist octet baryon LCDA:**

Octet	n	p	Λ
fgh	d d u	u u d	u d s

$$\begin{aligned}&\langle 0|f_\alpha(z_1n)g_\beta\left(z_2n\right)h_\gamma\left(z_3n\right)|B\left(P_B,\lambda\right)\rangle\\&=\frac{1}{4}f_V\left[\left(P_BC\right)_{\alpha\beta}\left(\gamma_5u_B\right)_\gamma V^B\left(z_in\cdot P_B\right)+\left(P_B\gamma_5C\right)_{\alpha\beta}\left(u_B\right)_\gamma A^B\left(z_in\cdot P_B\right)\right]\\&+\frac{1}{4}f_T(i\sigma_{\mu\nu}P_B^\nu C)_{\alpha\beta}(\gamma^\mu\gamma_5u_B)_\gamma T^B\left(z_in\cdot P_B\right),\end{aligned}$$

Motivation Moments vs LaMET

Operator Product Expansion (OPE) **VS** **Large Momentum Effective theory(LaMET)**

Limited for only few moments:

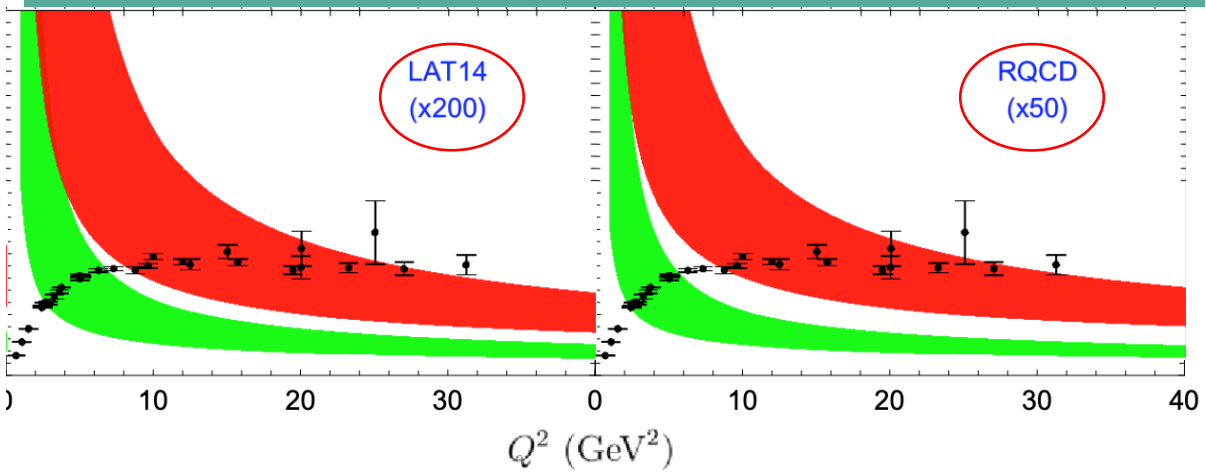
- First two moments for light mesons
- First moments for light baryons
- Inverse problem from moments to LCDA

High twist/moments are more important in Baryon

J.J.Han, et.al. EPJC 82 8 (2022), 686 PRL 134 221801(2025)

L.B.Chen, et.al. PRL 135 131903 (2025)

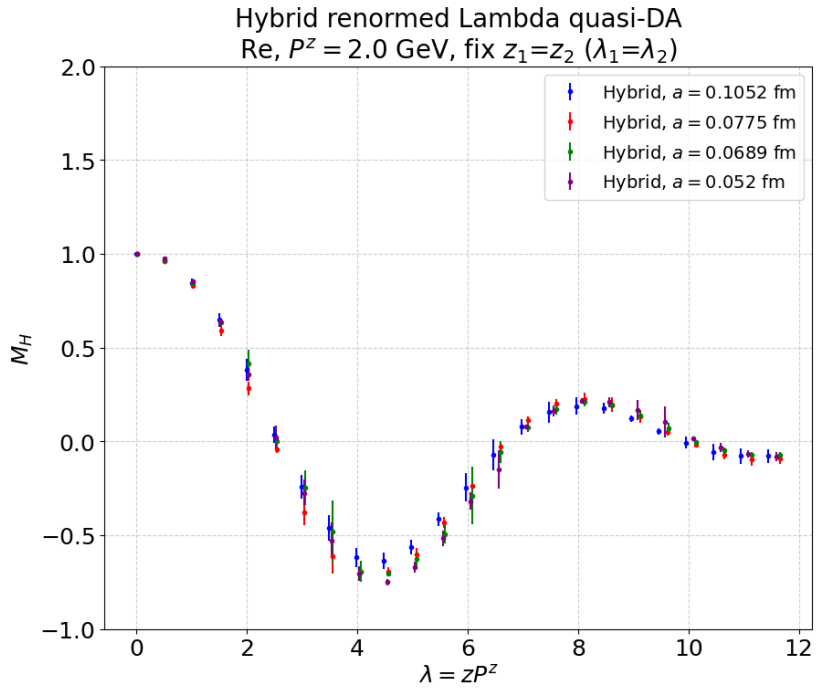
Nucleon Dirac form factors:

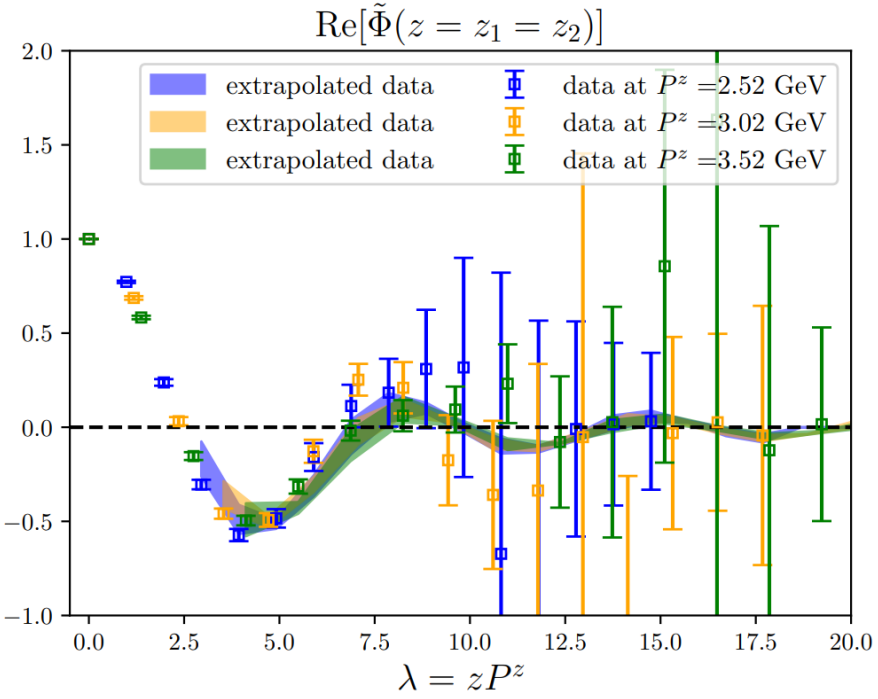


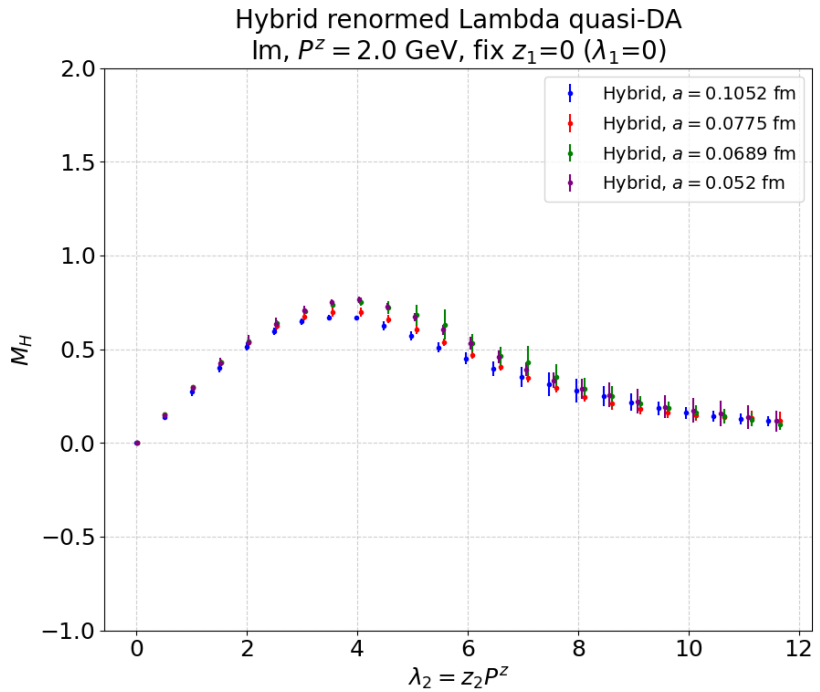
Soft contribution dominates in baryon decay , leading twist far from enough

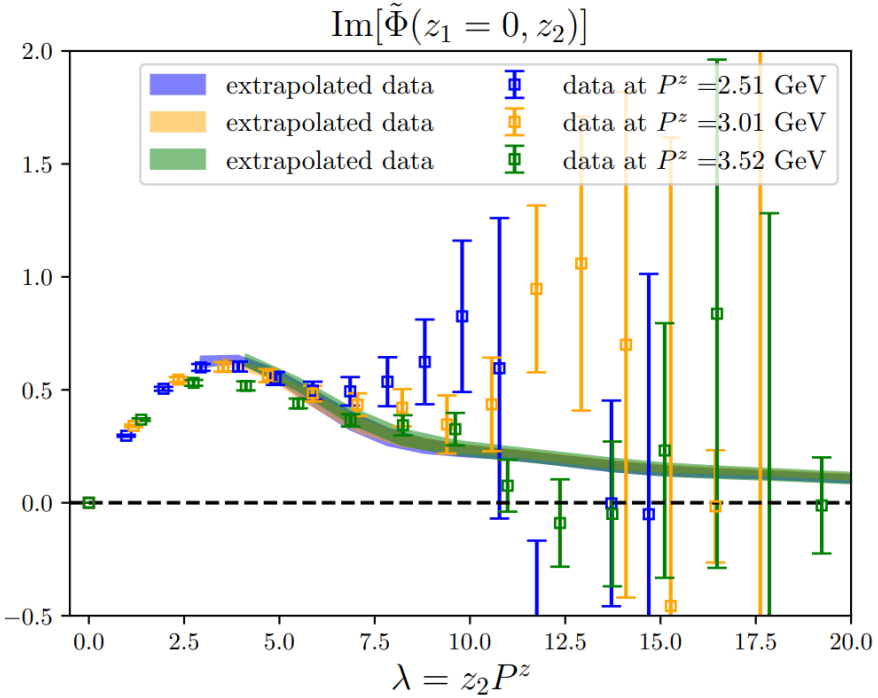
Signal improve

Renormed Quasi-DA









LPC, PRD 111, 034510 (2025)