Lattice Parton Collaboration

SHANGHAI JIAO TONG UNIVERSITY

BARYONS 2025

x-dependent Baryon Light-cone DA from Lattice QCD

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(Lattice Parton Collaboration)

Nov. 11 @ BARYONS 2025

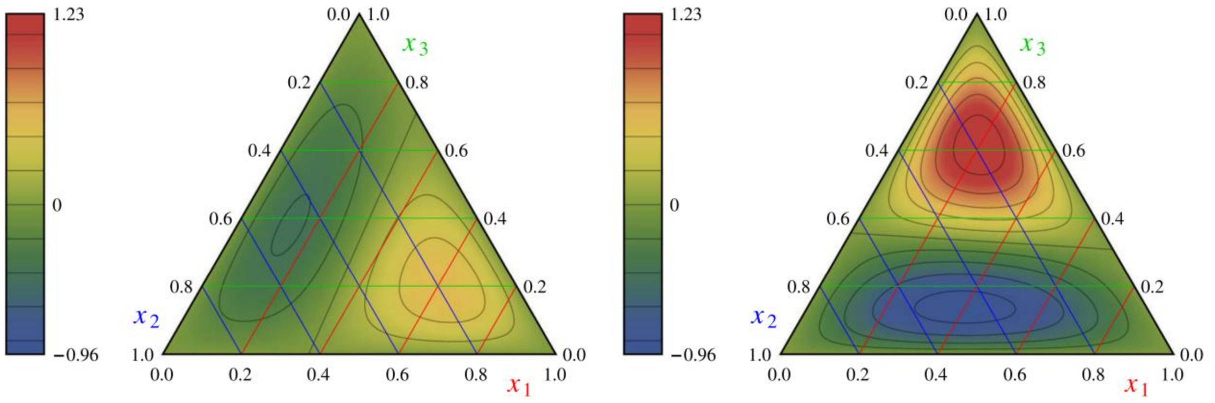
OUTLINE

How to access LCDAs ?

Research methods: (1983 - now)

- **Asymptotic form**
  - *Chernyak, Zhitnitsky, 1983; ……*
- **QCD Sum Rules**
  - *King, Sachrajda, 1987;*
  - *Stefanis, Bergmann, 1993;*
  - *Braun, et al, 2000, 2006; ……*
- **Models parametrization**
  - *Bell, et al, 2013; ……*

RQCD, Eur. Phys. J. A (2019) 55;

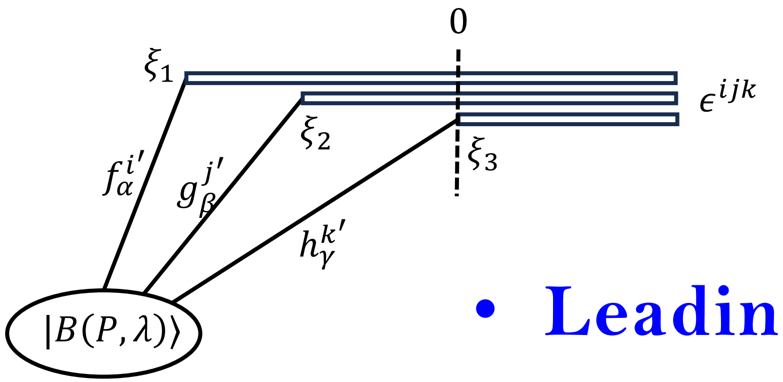


- **Moments from Lattice QCD**
  - *QCDSF, 2008, 2009;*
  - *RQCD, 2016, 2019, 2025; ……*
- **x-dependence from Lattice QCD**
  - *LPC, PRD 111, 034510 (2025);*
  - *LPC, arXiv: 2508.08971; ……*

Definition of LCDAs

- **Operator definition in coordinate space:**

$$H_{\alpha\beta\gamma}(\xi_1,\xi_2,\xi_3) = \epsilon^{ijk}\langle 0| f_\alpha^{i'}\boxed{(\xi_1 n)}W^{i'i}(\xi_1 n,\xi_0 n) \times g_\beta^{j'}\boxed{(\xi_2 n)}W^{j'j}(\xi_2 n,\xi_0 n)$$
$$\times\, h_\gamma^{k'}(\xi_3 n)W^{k'k}(\xi_3 n,\xi_0 n)|B(P,\lambda)\rangle$$



- **Leading twist for octet baryon:**

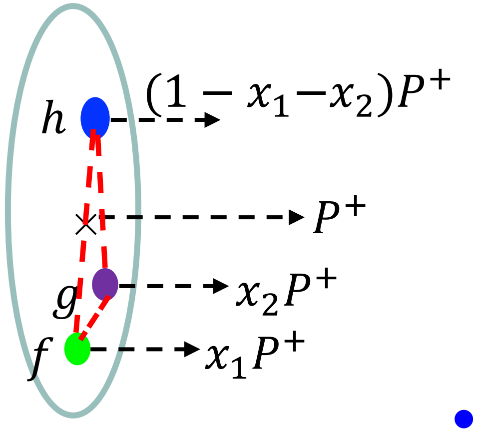
$$\langle 0|f_\alpha(\xi_1 n)g_\beta(\xi_2 n)h_\gamma(\xi_3 n)|B(P,\lambda)\rangle$$
$$= \frac{1}{4}f_V\left[(\not{P}C)_{\alpha\beta}(\gamma_5 u_B)_\gamma \boldsymbol{\Phi_V}(\xi_i n\cdot P) + (\not{P}\gamma_5 C)_{\alpha\beta}(u_B)_\gamma \boldsymbol{\Phi_A}(\xi_i n\cdot P)\right]$$
$$+\frac{1}{4}f_T(\mathrm{i}\sigma_{\mu\nu}P^\nu C)_{\alpha\beta}(\gamma^\mu\gamma_5 u_B)_\gamma \boldsymbol{\Phi_T}(\xi_i n\cdot P)$$

Octet	n	p	Λ
fgh	d d u	u u d	u d s

V.L.C & I.R.Z NPB 246 52(1984)

Definition of LCDAs

- **Definition of baryon LCDA:**



$$\phi(x_1,x_2,\mu)=\int\frac{P^+\mathrm{d}\xi_1^-}{2\pi}\frac{P^+\mathrm{d}\xi_2^-}{2\pi}\,\mathrm{e}^{\mathrm{i}(x_1\xi_1^-+x_2\xi_2^-)P^+}\,\Phi(\xi_1,\xi_2,\mu)$$

- **Leading twist for octet baryon LCDA**

$$-f_V\Phi_V(\xi_i n\cdot P)P^+\gamma_5 u_B=\langle 0|f^T(\xi_1 n)(C\not{n})g(\xi_2 n)\;h(0)|B\rangle$$

PRD 111, 034510 (2025) —— Λ: $$f_V\Phi_A(\xi_i n\cdot P)P^+ u_B=\langle 0|f^T(\xi_1 n)(C\gamma_5\not{n})g(\xi_2 n)\;h(0)|B\rangle$$

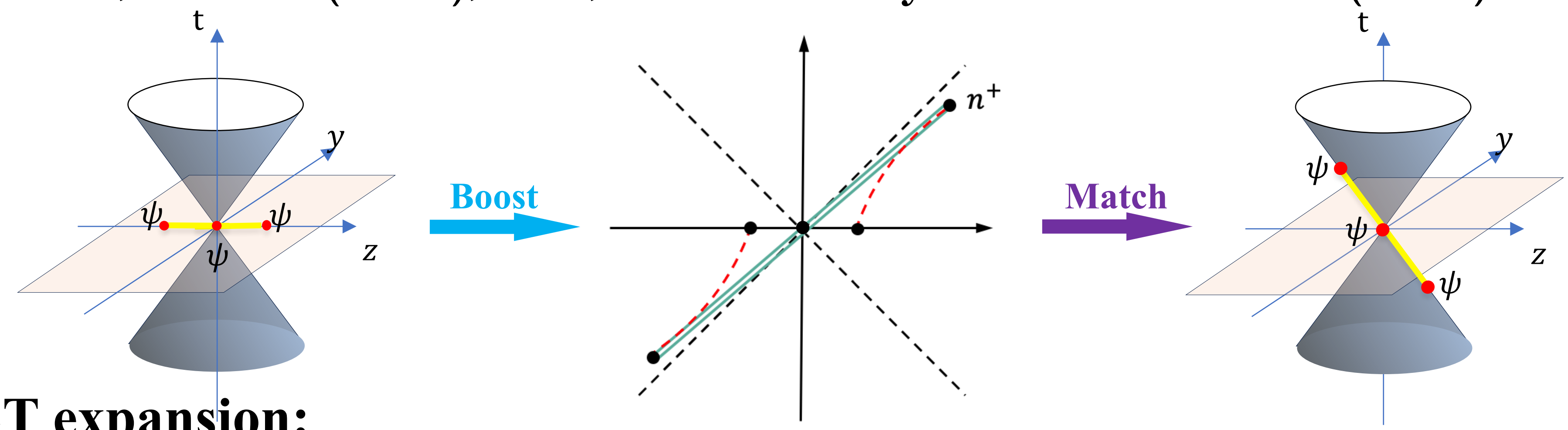
$$2f_T\Phi_T(\xi_i n\cdot P)P^+\gamma_5 u_B=\langle 0|f^T(\xi_1 n)(\mathrm{i}C\sigma_{\mu\nu}n^\nu)g(\xi_2 n)\;\gamma^\mu h(0)|B\rangle$$

V.L.C & I.R.Z NPB 246 52(1984);
G.R.Farrar et.al. NPB 311 585(1989);
C.Han et.al. JHEP 07 019 (2024);

How to calculate LCDAs ?

Large-momentum Effective Theory

X.Ji, PRL 110, 262002 (2013); X.Ji, Sci.China Phys.Mech.Astron. 57 (2014)



LaMET expansion:

LCDA — Matching kernel — quasi-DA

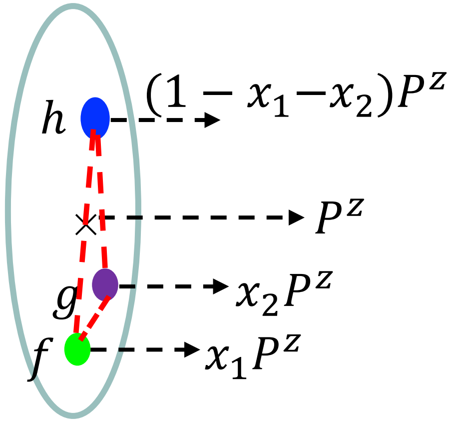
$$\phi(x_1,x_2;\mu) = \int \mathrm{d}y_1\mathrm{d}y_2\ \mathcal{C}(x_1,x_2;y_1,y_2;P^z,\mu)\boxed{\tilde{\phi}(y_1,y_2;P^z,\mu)}$$

$$+\,\mathcal{O}\left(\frac{\Lambda^2_{\rm QCD}}{(x_1P^z)^2},\frac{\Lambda^2_{\rm QCD}}{(x_2P^z)^2},\frac{\Lambda^2_{\rm QCD}}{[(1-x_1-x_2)P^z]^2}\right)$$

power corrections

Definition of LCDAs

- **Definition of baryon quasi-DA:**



$$\tilde{\phi}(x_1, x_2; P^z, \mu) = \int \frac{P^z \mathrm{d}z_1}{2\pi} \frac{P^z \mathrm{d}z_2}{2\pi} \mathrm{e}^{-\mathrm{i}(x_1 z_1 + x_2 z_2)P^z} \tilde{\Phi}(z_1, z_2; P^z, \mu)$$

- **Matrix elements of quasi-DA:**

$$-f_V \tilde{\Phi}_V(z_i, P^z) P^\nu \gamma_5 u_B = \langle 0 | f^T(z_1)(C\gamma^\nu) g(z_2)\ h(0) | B \rangle$$

PRD 111, 034510 (2025) — Λ: $$f_V \tilde{\Phi}_A(z_i, P^z) P^\nu u_B = \langle 0 | f^T(z_1)(C\gamma_5\gamma^\nu) g(z_2)\ h(0) | B \rangle$$

$$2f_T \tilde{\Phi}_T(z_i, P^z) P^\nu \gamma_5 u_B = \langle 0 | f^T(z_1) \left(\frac{1}{2} C[\gamma^\nu, \gamma^\mu] \right) g(z_2)\ \gamma^\mu h(0) | B \rangle$$

V.L.C & I.R.Z NPB 246 52(1984);
G.R.Farrar et.al. NPB 311 585(1989);
C.Han et.al. JHEP 07 019 (2024);

Lattice simulation

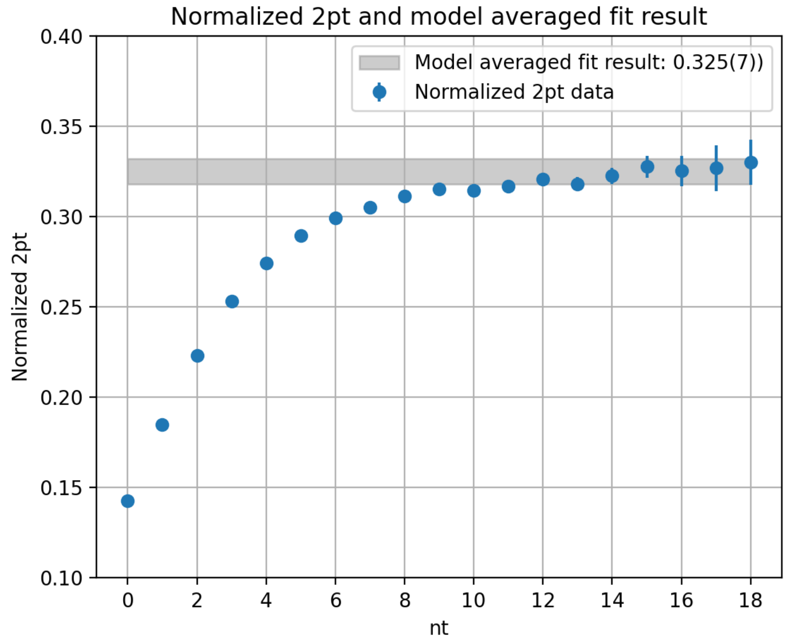
Nonlocal 2-point function related to baryon quasi-DA:

$$C_2(z_1, z_2; t, P^z) = \int \mathrm{d}^3x\, \mathrm{e}^{-\mathrm{i}\vec{x}\cdot\vec{P}} \langle 0| \mathcal{O}^{\gamma'}_{\mathrm{Sink}}(\vec{x}, t; z_1, z_2) \overline{\mathcal{O}}^{\gamma}_{\mathrm{Src}}(0, 0; 0, 0) T_{\gamma'\gamma} |0\rangle$$

- **Extract & norm quasi-DA from 2-point function**

$$\frac{C_2(z_1, z_2; P^z, t)}{C_2(0, 0; P^z, t)} \xrightarrow{t\to\infty} \frac{\widetilde{\Phi}(z_1, z_2; P^z)}{\widetilde{\Phi}(0, 0; P^z)}$$

$$C_2^{\mathrm{Norm}}(z_1, z_2; P^z, t) = \underbrace{\widetilde{\Phi}(z_1, z_2; P^z)}_{\text{ground-state}} \Big(1 + \underbrace{\Delta A(z_1, z_2; P^z)\mathrm{e}^{-\Delta E t}}_{\text{excited-states}}\Big)$$



Lattice simulation

Nonlocal 2-point function related to baryon quasi-DA:

$$C_2(z_1, z_2; t, P^z) = \int \mathrm{d}^3x\ \mathrm{e}^{-\mathrm{i}\vec{x}\cdot\vec{P}} \langle 0| \underbrace{\mathcal{O}^{\gamma'}_{\text{Sink}}(\vec{x}, t; z_1, z_2)}_{\text{Determined by DA}} \underbrace{\overline{\mathcal{O}}^{\gamma}_{\text{Src}}(0,0;0,0) T_{\gamma'\gamma}}_{\text{Up to our choice !}} |0\rangle$$

- **Interpolators and projection operators**

□ Sink-side Interpolators

$$\mathcal{O}_{\text{Sink},V} = f^T(z_1)(C\gamma^t)g(z_2)\ h(0)$$
$$\mathcal{O}_{\text{Sink},A} = f^T(z_1)(C\gamma_5\gamma^t)g(z_2)\ h(0)$$
$$\mathcal{O}_{\text{Sink},T} = f^T(z_1)(C[\gamma^t, \gamma^y]/2)g(z_2)\ \gamma_y\ h(0)$$

□ Projector

$$T = I, \gamma^t, \gamma^z$$

?

□ Source-side Interpolators

$$\mathcal{O}^P_{\text{Src}} = (u^T\ C\gamma_5\ d)u$$

?

$$\mathcal{O}^\Lambda_{\text{Src}} = \frac{1}{\sqrt{6}}\Big(2(u^T\ C\gamma_5\ d)s + (u^T\ C\gamma_5\ s)d + (s^T\ C\gamma_5\ d)u\Big)$$

Lattice simulation: Improvements

Improvement from interpolator: Kinematically-enhancement

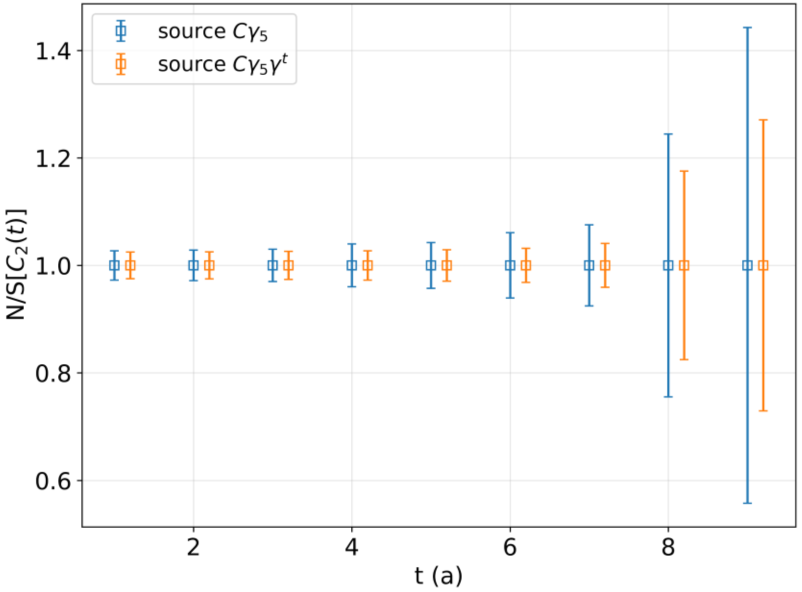
- Standard interpolator: $N_{\gamma_5} = \epsilon^{ijk}(f_i^T \; C\gamma_5 \; g_j)h_k$ for a static baryon states

$$\langle 0|N_{\gamma_5}|N(P)\rangle = \lambda u(P)$$

- Enhanced interpolator : $N_{\gamma_5\gamma_\mu} = \epsilon^{ijk}(f_i^T \; C\gamma_5\gamma_\mu \; g_j)h_k$ better overlap with boosted states

$$\langle 0|N_{\gamma_5\gamma_\mu}|N(P)\rangle = \alpha P_\mu u(P) + \beta\gamma_\mu u(P)$$

$$\Longrightarrow \quad \mathrm{SNR}(C_{2\mathrm{pt}}(t\to\infty)) \propto \boxed{\frac{P_\mu}{M_0}} \mathrm{e}^{-E' t}$$



Improvement from projector

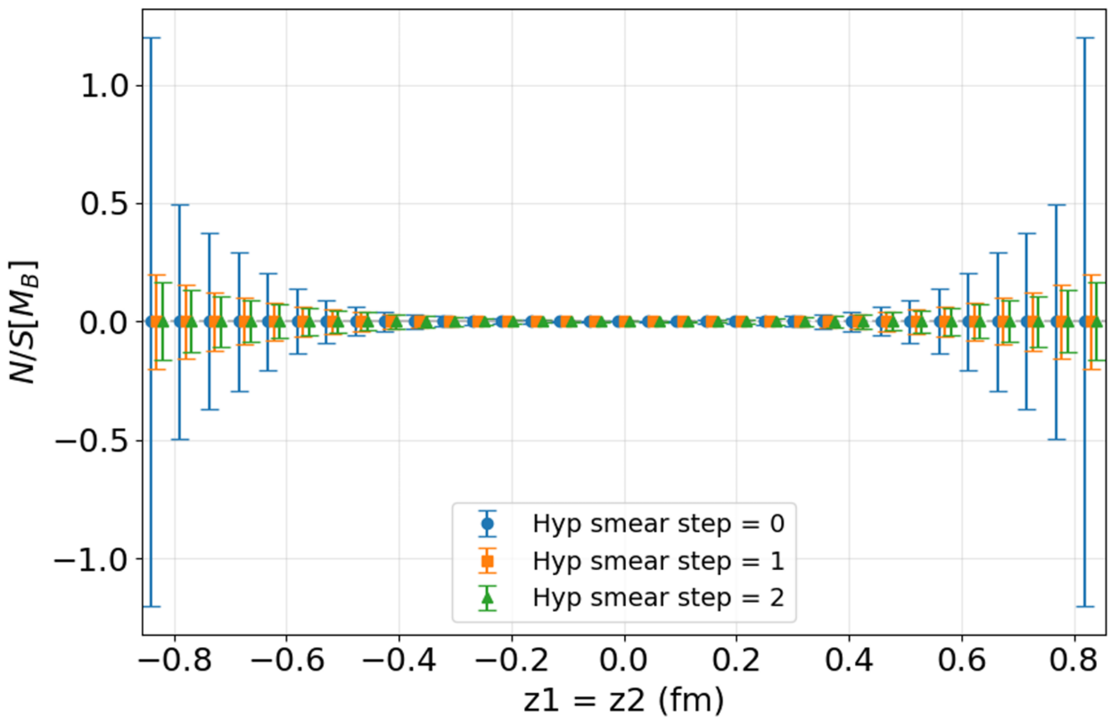
$$T = \gamma_t + \gamma_z$$ for leading twist contributions

R.Zhang et.al, arXiv:2501.00729

Lattice simulation: Improvements

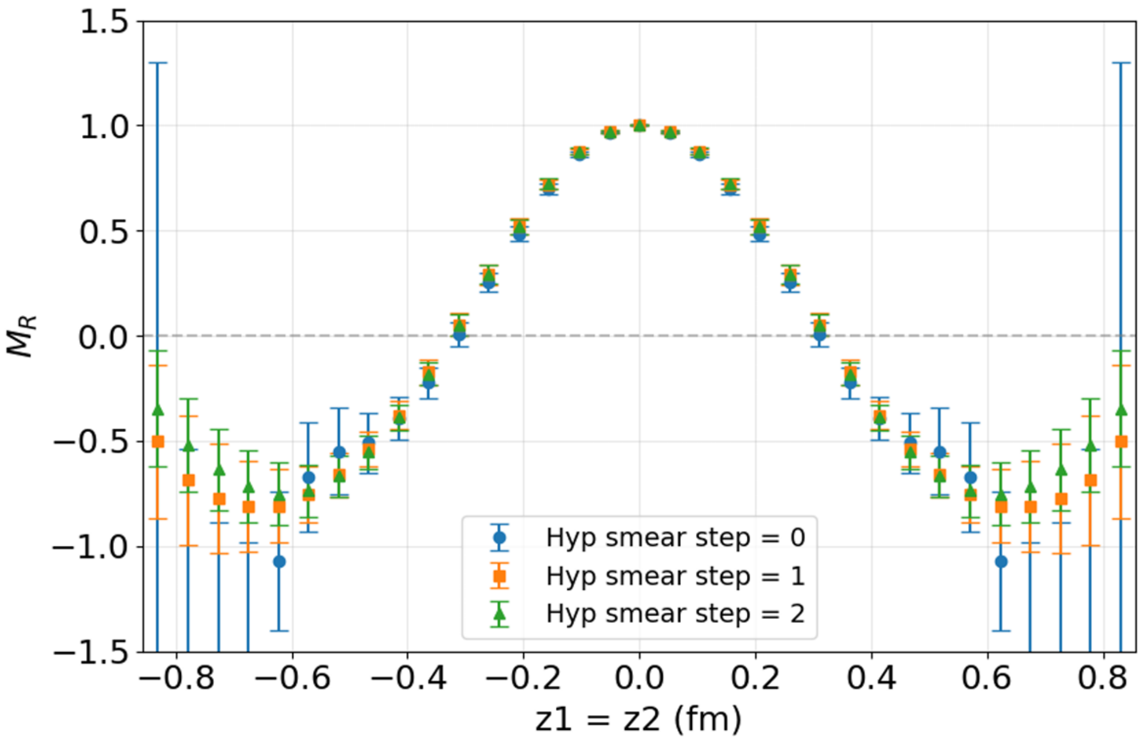
Improvement from gauge link: HYP smearing

NtS ratio of bare quasi-DA



Great signal improvement !

Ratio scheme quasi-DA



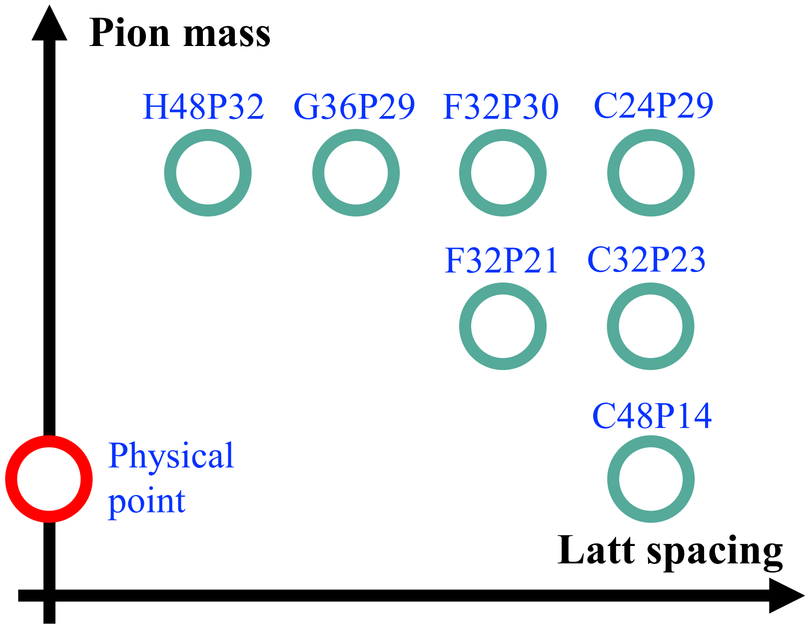
Does not affect the renormalized result

Lattice simulation

CLQCD Ensembles

- 4 lattice spacings for **continuum** limit & renormalization
- 3 pion masses for **physical mass** limit
- multi momentums for $P_z \to \infty$ limit

Good precision !



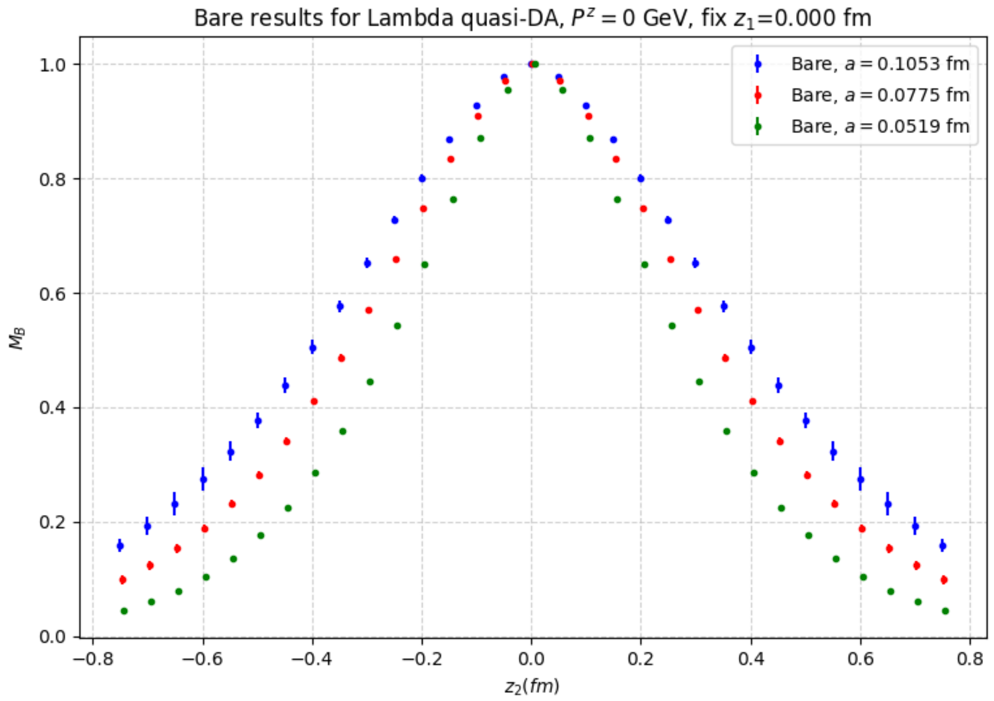
Ensemble	Lattice spacing	π mass	measurements	Pz/GeV
C24P29	0.105 fm	292 MeV	864 *4 src *9 it0	1.47, 1.96, 2.45, 2.94
C32P23	0.105 fm	228 MeV	954 *4 src *8 it0	1.47, 1.84, 2.21, 2.57, 2.94
C48P14	0.105 fm	136 MeV	302 *4 src *16 it0	1.47, 1.96, 2.45, 2.94
F32P30	0.078 fm	300 MeV	777 *4 src *8 it0	1.50, 2.00, 2.49, 2.99
F32P21	0.078 fm	210 MeV	459 *4 src *8 it0	1.50, 2.00, 2.49, 2.99
G36P29	0.069 fm	297 MeV	656 *6 src *8 it0	1.50, 2.00, 2.50, 3.00
H48P32	0.052 fm	317 MeV	550 *6 src *9 it0	1.49, 1.98, 2.48, 2.98

Bare quasi-DA results: Linear divergence

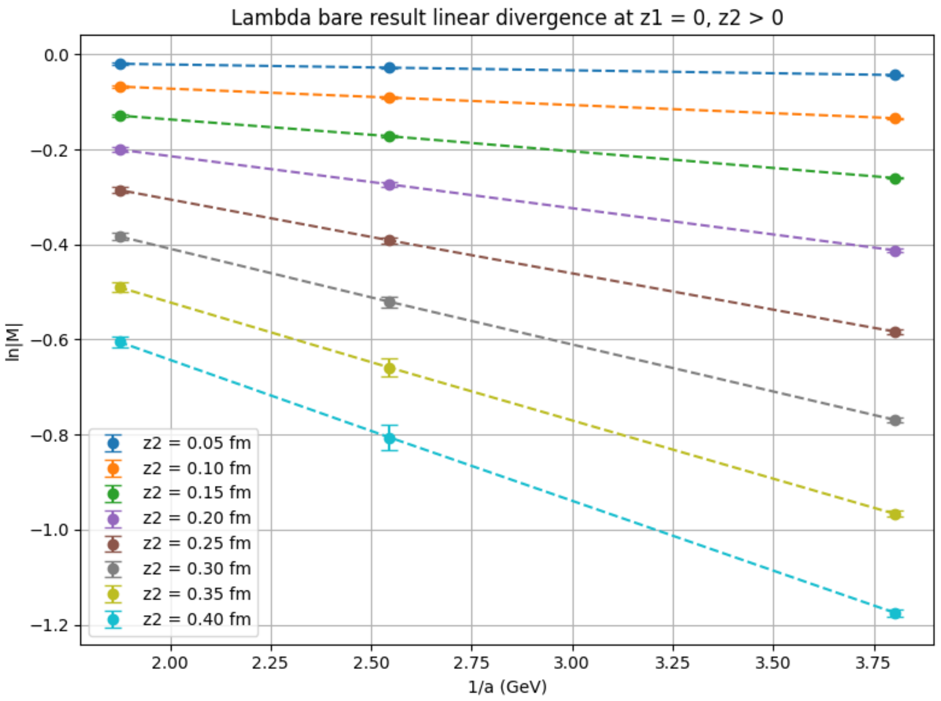
Bare quasi-DA on different lattice spacing

Lambda A term

P^z=0 GeV



log scale



$\frac{1}{a}$ linear behavior

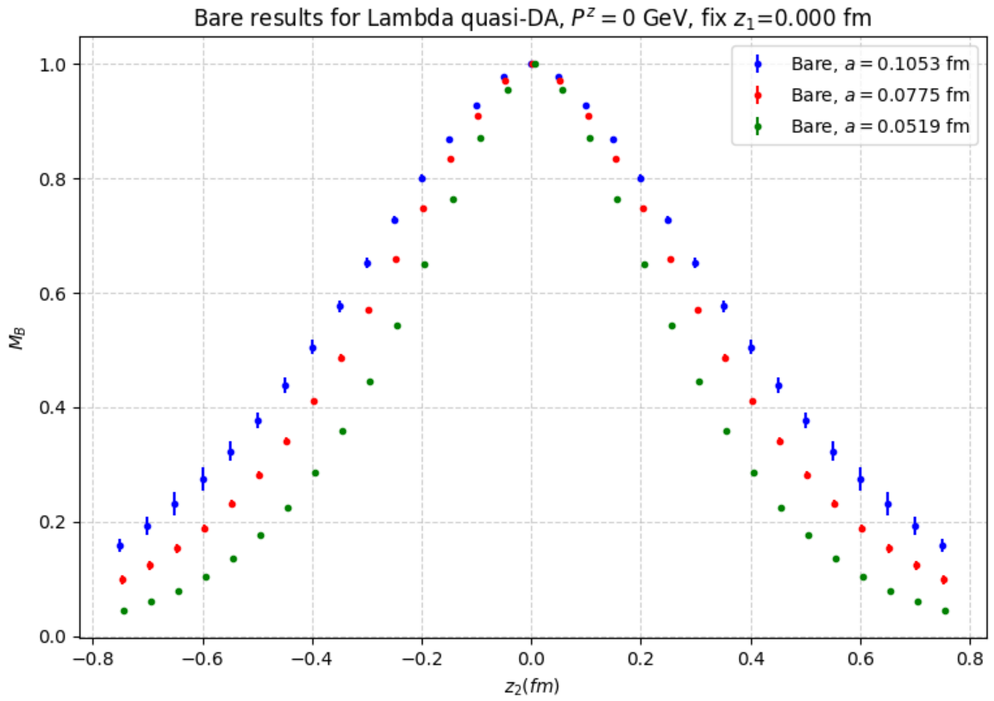
Bare quasi-DA results: Linear divergence

Linear divergence of Normed Bare quasi-DA

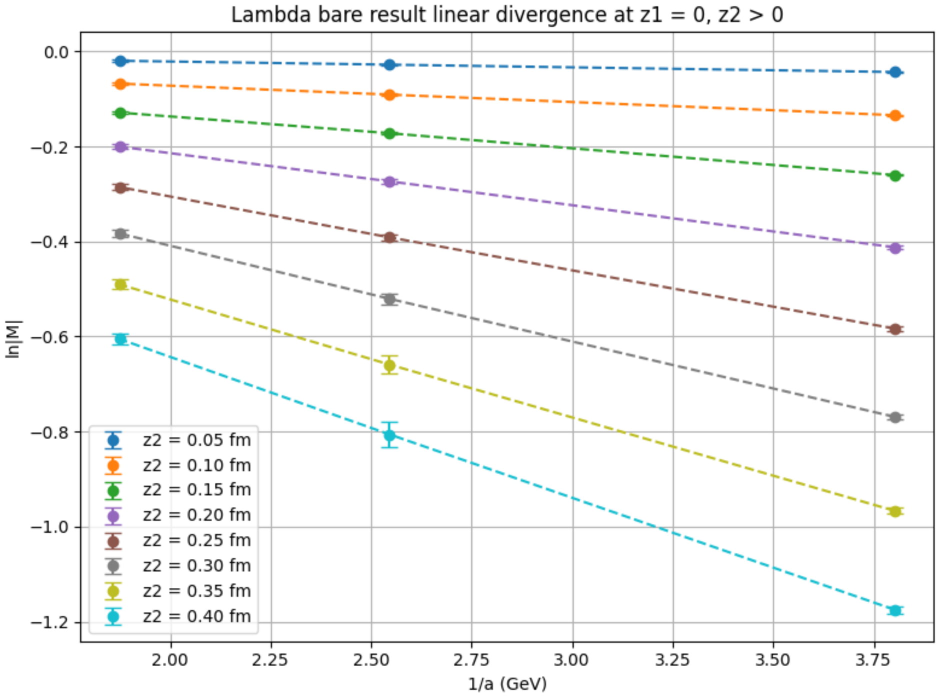
$$\tilde{z} = \begin{cases} |z_1 - z_2| & z_1 z_2 < 0 \\ \max(z_1, z_2) & z_1 z_2 \geq 0 \end{cases}$$

$$M(z_1, z_2; P^z, a) \propto \exp\left[\left(\boxed{\frac{k}{a\ln(a\Lambda_{\rm QCD})}} + m_0\right)\tilde{z}\right] m(z_1, z_2, P^z)$$

P^z=0 GeV



log scale



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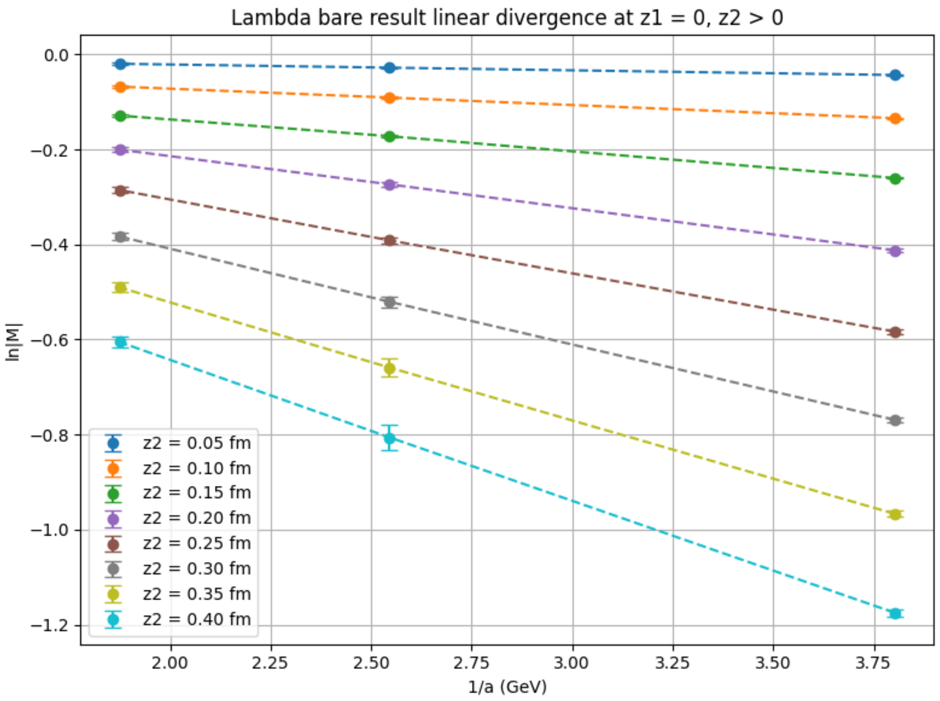
$$M(z_1, z_2; P^z, a) \propto \exp\left[\left(\boxed{\frac{k}{a\ln(a\Lambda_{\mathrm{QCD}})}} + m_0\right)\tilde{z}\right] m(z_1, z_2, P^z)$$

log scale



We need proper renormalization scheme to eliminate the linear divergences !

Hybrid Renormalization

Renormalization on Lattice

Scheme conversion: $\frac{1}{a}$ cutoff $\rightarrow$ $\overline{\mathrm{MS}}$

Schemes on Lattice

- **RI/MOM Scheme**

  Alexandrou et.al, NPB 2017;

  Stewart, Zhao, PRD 2018; ……

- **Ratio Scheme**

  Radyushkin et.al, PRD 2017; ……

- **Self Renormalization Scheme**

  Ji et.al, NPB 2021; Huo et.al, NPB 2021

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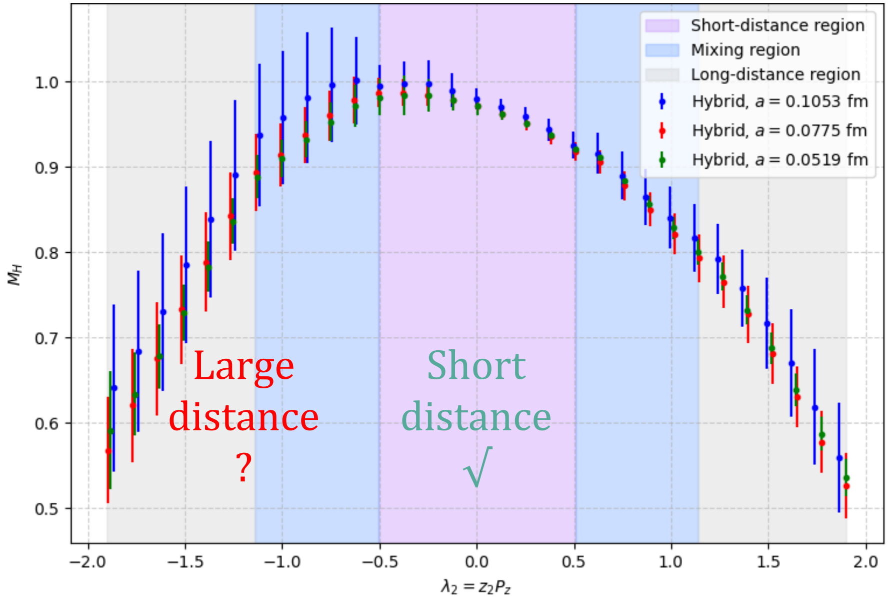
Ratio



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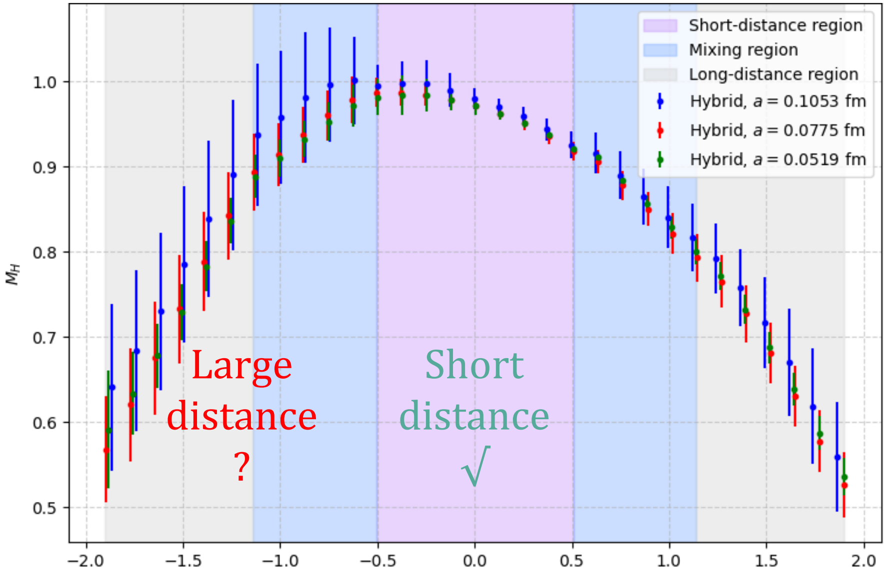
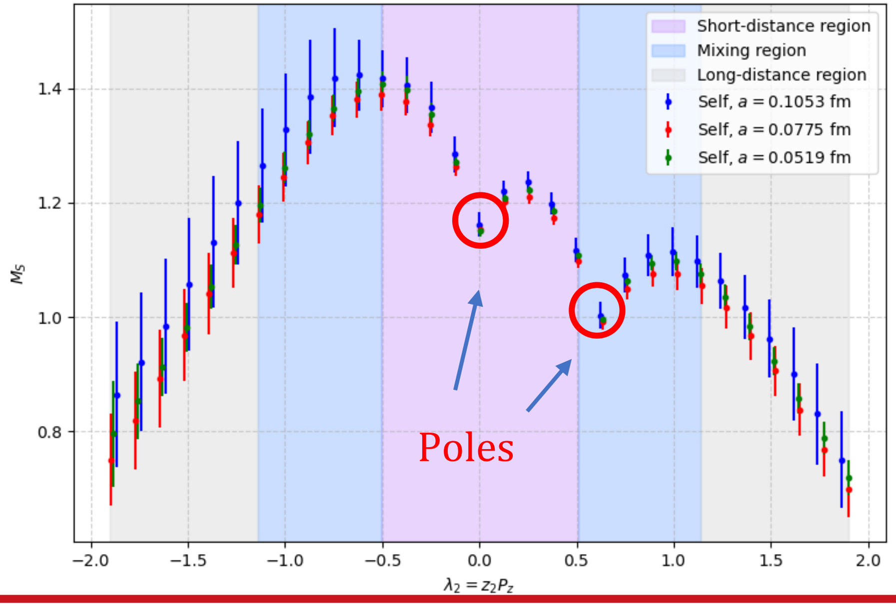
Ratio



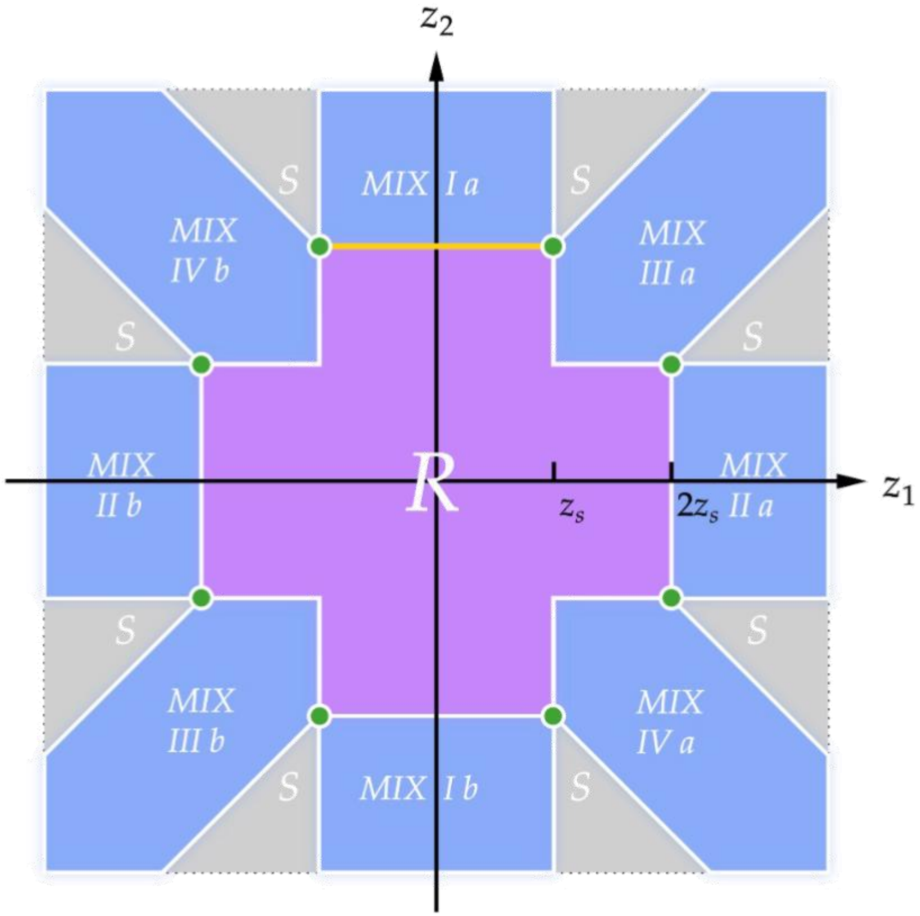
Self ren.



Hybrid Renormalization

2-Dimensional Hybrid Renormalization

—— Best choice for all z_1-z_2 regions & LaMET matching



- **1) Short-distance regions: Ratio**

$$\frac{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, 0, \mu)}$$

$(|z_1|\ \&\ |z_2|\ \&\ |z_1 - z_2| < 0.3\ \mathrm{fm})$

- **2) Long-distance regions: Self-renormalization**

$$\frac{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\mathrm{MS}}}(\mathrm{sign}(z_1)z_s, \mathrm{sign}(z_2)2z_s, 0, 0, \mu)}$$

$(|z_1|\ \&\ |z_2|\ \&\ |z_1 - z_2| > 0.3\ \mathrm{fm})$

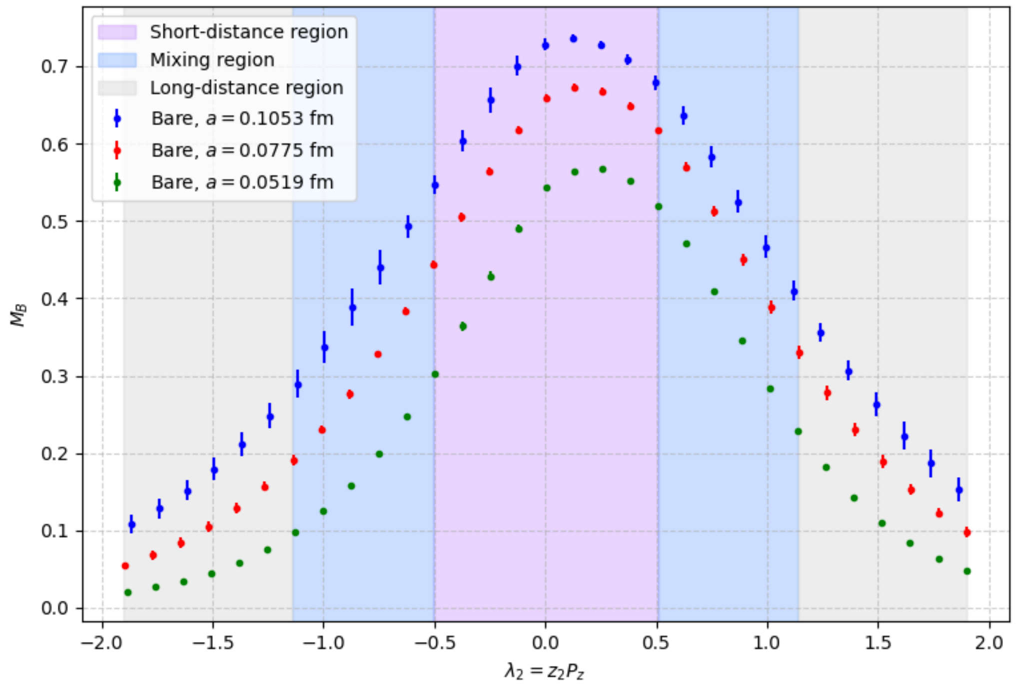
- **3) Mixing regions**

$$\frac{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\mathrm{MS}}}(z_1, \mathrm{sign}(z_2)2z_s, 0, 0, \mu)} \cdot \theta(z_s - |z_1|)\theta(|z_2| - 2z_s)$$

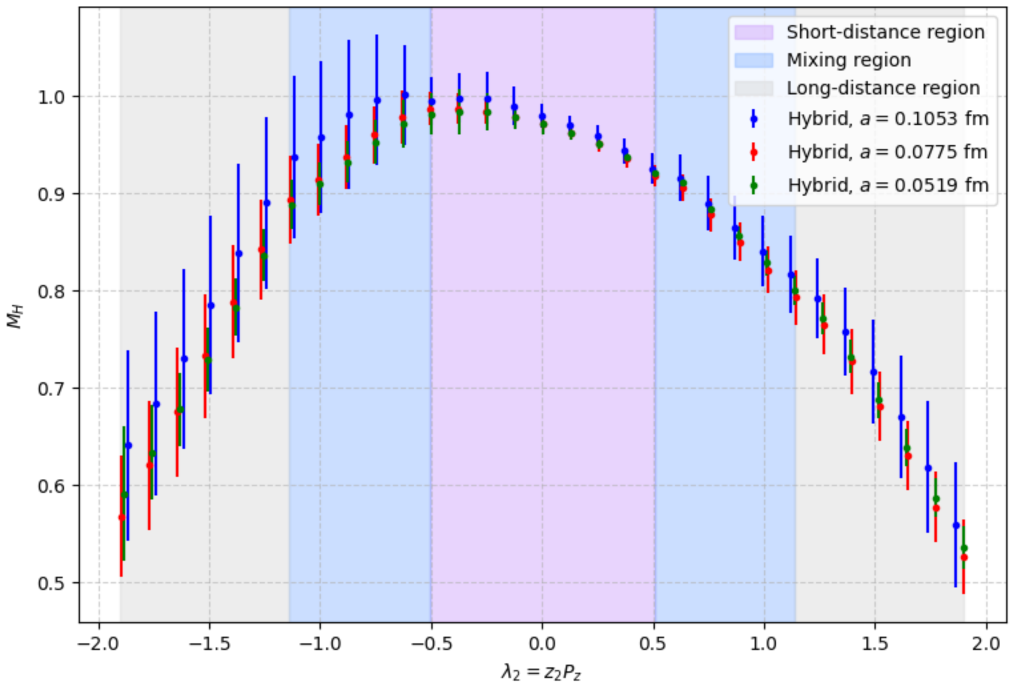
LPC, arXiv: 2508.08971

Hybrid Renormalization

Bare Matrix Element



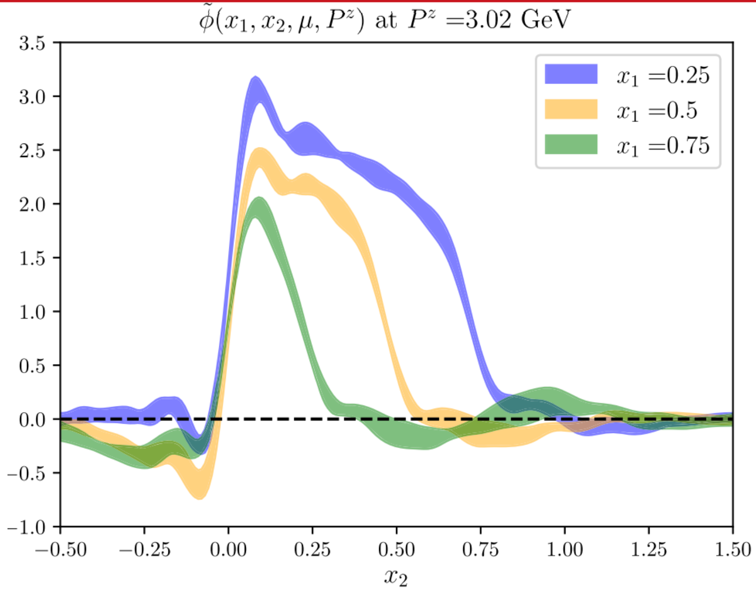
Hybrid Renormalization result



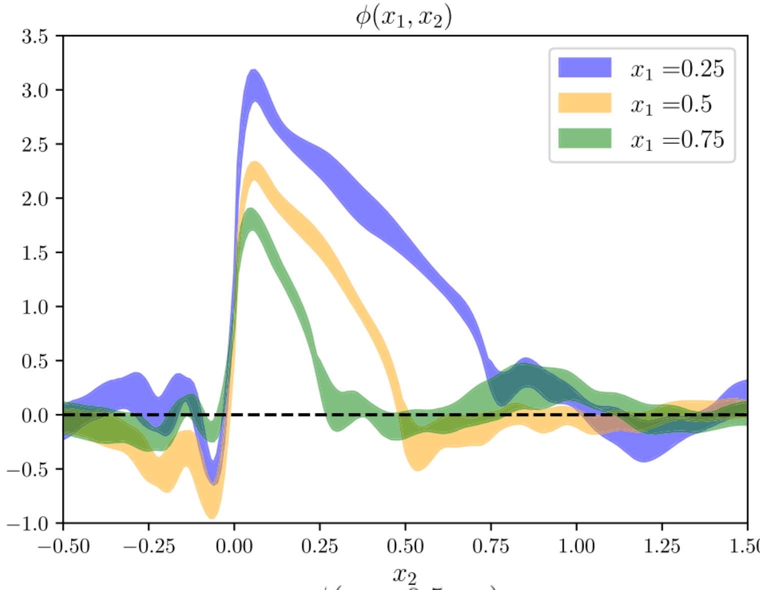
- Satisfying elimination of linear divergence
- Well defined at short, long & mixing regions
- Smooth between different regions

LPC, arXiv: 2508.08971

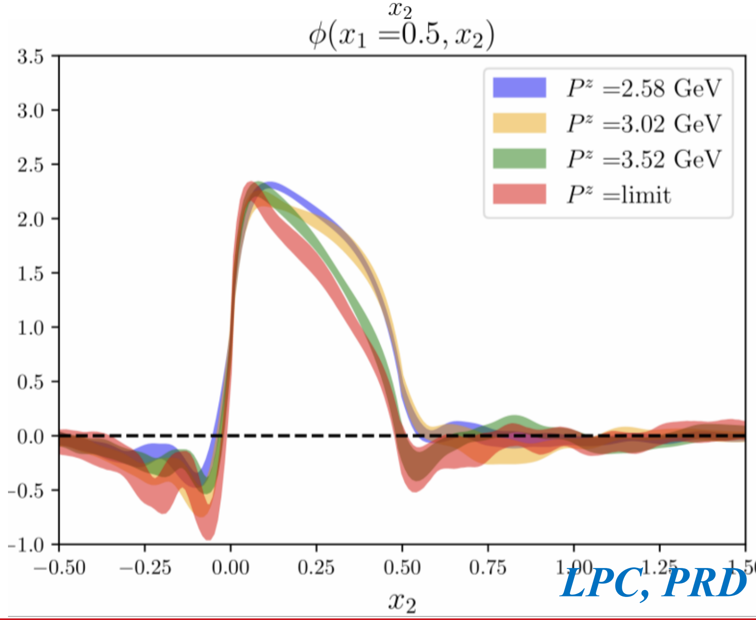
Renormalization & Matching



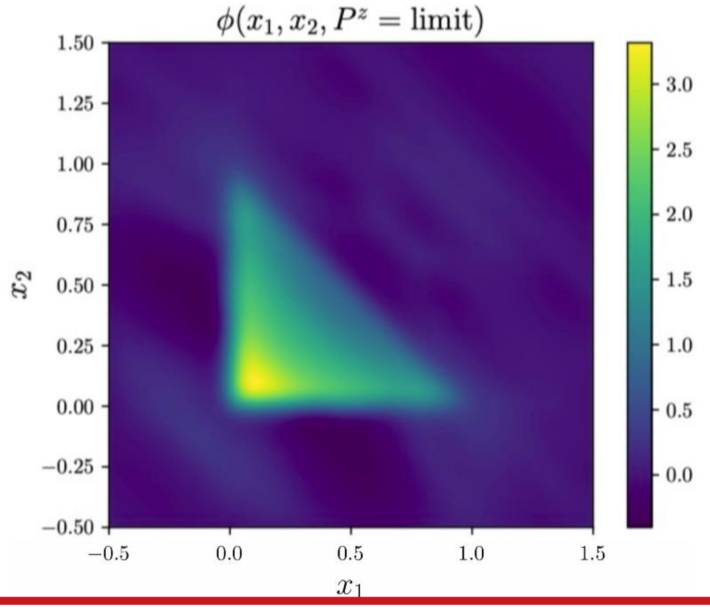
Matching



Large Pz limit



LCDA



LPC, PRD 111, 034510 (2025)

Summary & Outlook

- We have done:
  - ***LPC, PRD 111, 034510***, validated the concept of lattice calculation of baryon LCDA from LaMET
  - ***arXiv: 2508.08971***, successfully adopted the Hybrid renormalization on baryon matrix element

- We now proceeding:
  - Continuum & physical mass limit
  - Methods for limited FT, matching to light-cone
  - More potential strategies to enhance & optimize
    - **gradient flow, ……**

- Please stay tuned our future results for all leading twists LCDAs of Proton and Lambda !

Thanks for your attention!

Backup

x-dependent Baryon Light-cone DA from Lattice QCD

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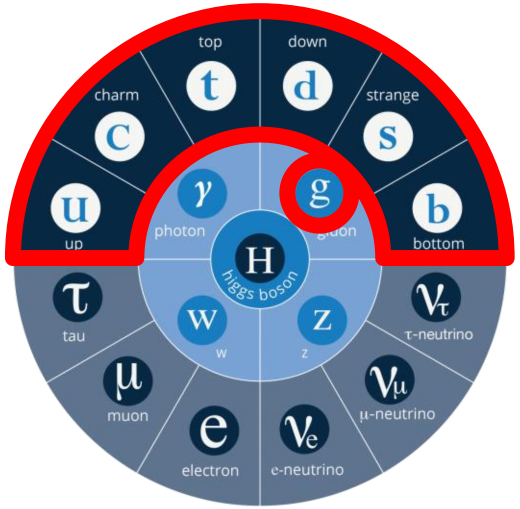
In collaboration with: 白颢阳(IHEP), 华俊(SCNU), 王伟(SJTU), 张家璐(SJTU)

(Lattice Parton Collaboration)

Nov. 11 @ BARYONS 2025

Why Baryons ?

Baryon — cornerstone particles

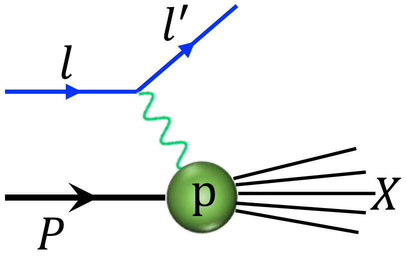


- Matter genesis in the Universe
- Tools to probe micro world
- Astronomy & Cosmology
- …………

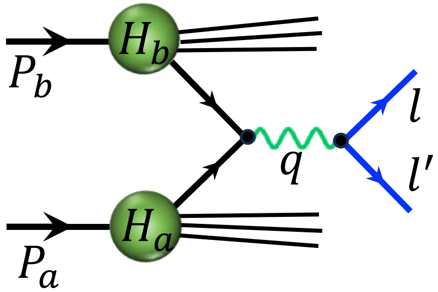
Why LCDAs ?

- Probe internal structure of nucleons in **Inclusive processes**
- **PDFs**

DIS

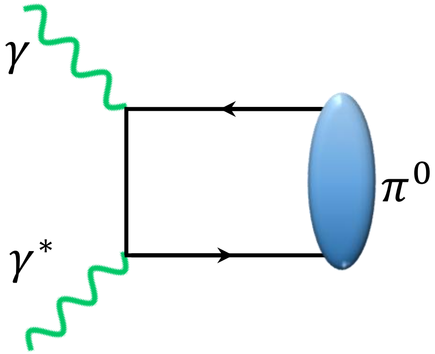


Drell-Yan

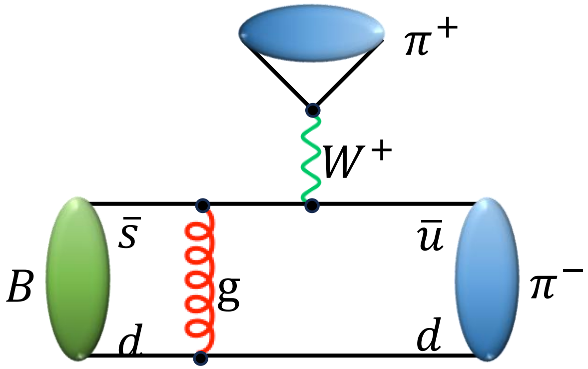


- Obtain richer QCD dynamical information in **Exclusive processes**
- **LCDAs**

$\gamma\gamma^* \to \pi$



$B \to \pi^+\pi^-$



Why Baryon LCDAs ?

In Exclusive Baryon Decays

Establish CPV in heavy baryon decays

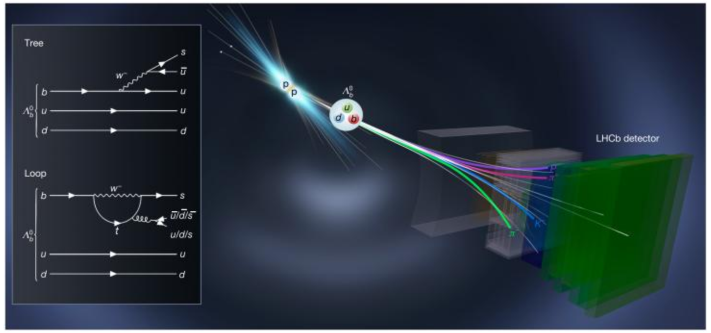
- 2025, first bservation of CPV in **<u>Baryon</u>**:

$\Lambda_b^0 \to p$

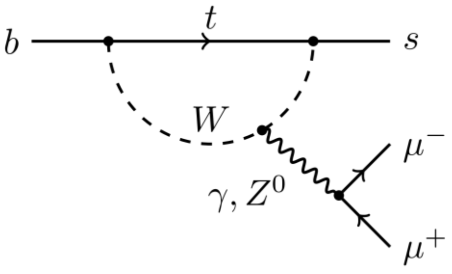
Observation of charge–parity symmetry breaking in baryon decays

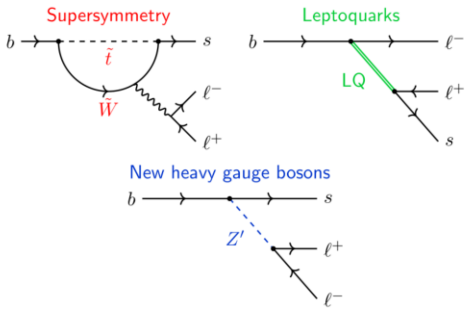
LHCb Collaboration

Nature **643**, 1223–1228 (2025) | Cite this article



Searching New Physics beyond SM





- b-baryon decays are highly sensitive to NP;
- Leading power FF can factorize out LCDAs

In Exclusive Scatterings

LHCb, Nature (2025);
J.J.Han, et.al. PRL 134, 221801 (2025)
W.Wang PLB 708 (2012)
Y.Jia, et al, PRL 135, 131903 (2025)

Lattice simulation

Nonlocal 2-point function related to baryon quasi-DA:

$$C_2(z_1,z_2;t,P^z)=\int \mathrm{d}^3x\ \mathrm{e}^{-\mathrm{i}\vec{x}\cdot\vec{P}}\langle 0|\underline{\mathcal{O}^{\gamma'}_{\text{Sink}}(\vec{x},t;z_1,z_2)}\,\underline{\overline{\mathcal{O}}^{\gamma}_{\text{Src}}(0,0;0,0)}\,\underline{T_{\gamma'\gamma}}|0\rangle$$

- **Extract & normalize quasi-DA from 2-point function**

$$C_2(z_1,z_2;P^z,t)=\sum_k\langle 0|\mathcal{O}^{\gamma'}_{\text{Sink}}(z_1,z_2;t)|B(P^z)_k\rangle\langle B(P^z)_k|\overline{\mathcal{O}}^{\gamma}_{\text{Src}}(0,0;t)|0\rangle T_{\gamma'\gamma}$$

$$\overset{t\to\infty}{\sim}\ \widetilde{\Phi}(z_1,z_2;P^z)\widetilde{\Phi}^*(0,0;P^z)|f_\Lambda|^2\left[u_\Lambda(P^z)u^\dagger_\Lambda(P^z)\right]^{\gamma'\gamma}T_{\gamma'\gamma}$$

$$\Longrightarrow\quad \frac{C_2(z_1,z_2;P^z,t)}{C_2(0,0;P^z,t)}=f(t)\frac{\widetilde{\Phi}(z_1,z_2;P^z)\widetilde{\Phi}^*(0,0;P^z)\cdots}{\widetilde{\Phi}(0,0;P^z)\widetilde{\Phi}^*(0,0;P^z)\cdots}\ \xrightarrow{t\to\infty}\ \frac{\widetilde{\Phi}(z_1,z_2;P^z)}{\widetilde{\Phi}(0,0;P^z)}$$

Lattice simulation

CLQCD Ensembles

Ensemble	size	Lattice spacing	π mass	measurements	Pz per unit	GeV
C24P29	24x72	0.10524 fm	292.3 MeV	864 *4 src *9 it0	0.490	1.47, 1.96, 2.45, 2.94
C32P23	32x64	0.10524 fm	227.9 MeV	954 *4 src *8 it0	0.368	1.47, 1.84, 2.21, 2.57, 2.94
C48P14	48x96	0.10524 fm	136.4 MeV	302 *4 src *16 it0	0.245	1.47, 1.96, 2.45, 2.94
F32P30	32x96	0.07753 fm	300.4 MeV	777 *4 src *8 it0	0.499	1.50, 2.00, 2.49, 2.99
F32P21	32x64	0.07753 fm	210.3 MeV	459 *4 src *8 it0	0.499	1.50, 2.00, 2.49, 2.99
G36P29	36x108	0.06887 fm	297.2 MeV	656 *6 src *8 it0	0.499	1.50, 2.00, 2.50, 3.00
H48P32	48x144	0.05199 fm	316.6 MeV	550 *6 src *9 it0	0.496	1.49, 1.98, 2.48, 2.98

Symmetries

Symmetries of quasi-DA

- isospin symmetry for "u, d" quarks (on a single configuration);

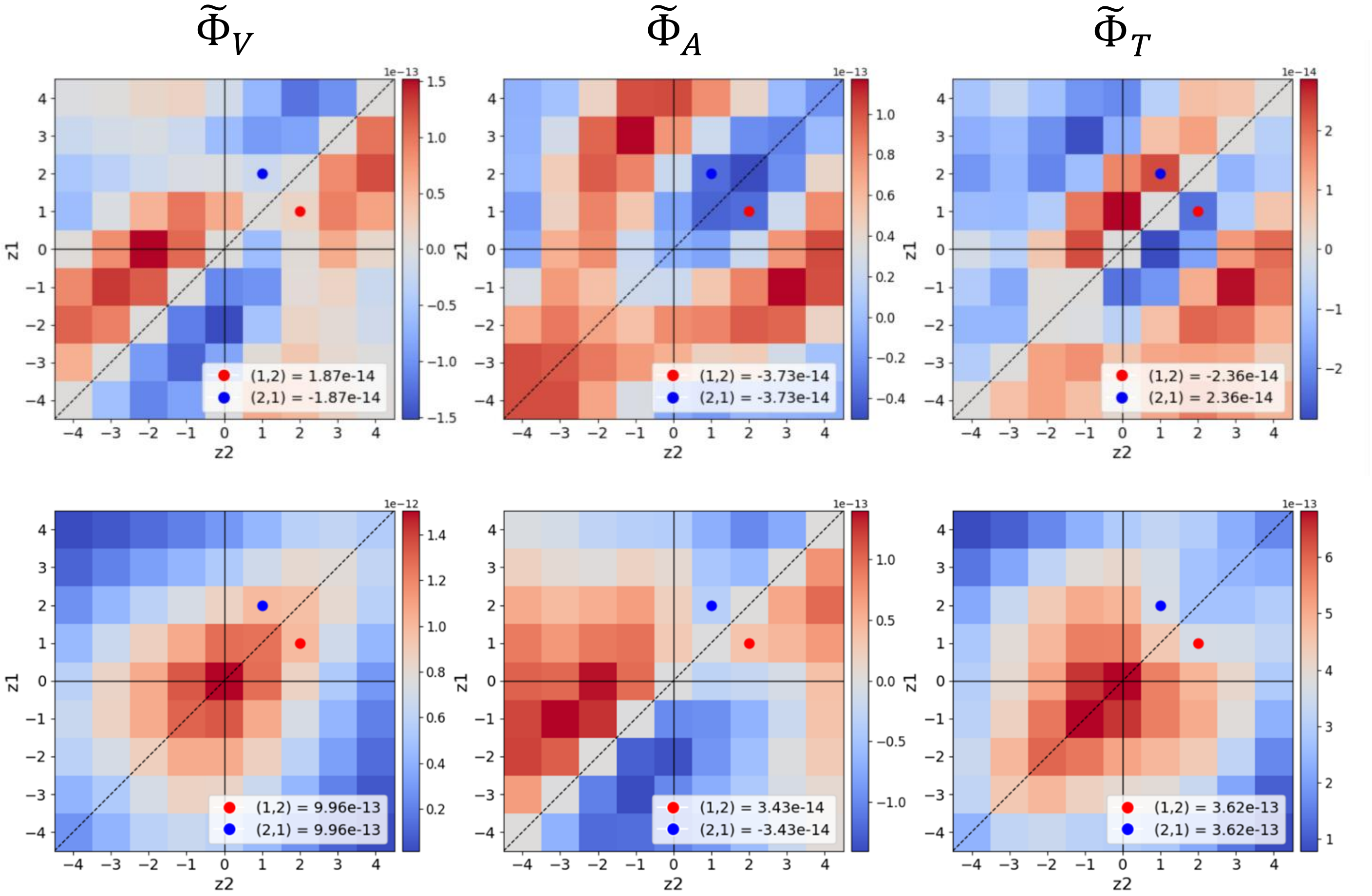
symmetry of z_1, z_2 exchanging:

$$
\begin{aligned}
V^{\Lambda}(x_2, x_1, x_3) &= -V^{\Lambda}(x_1, x_2, x_3)\,, \\
A^{\Lambda}(x_2, x_1, x_3) &= +A^{\Lambda}(x_1, x_2, x_3)\,, \\
T^{\Lambda}(x_2, x_1, x_3) &= -T^{\Lambda}(x_1, x_2, x_3)\,.
\end{aligned}
$$

Lambda

$$
\begin{aligned}
V^{B\neq\Lambda}(x_2, x_1, x_3) &= +V^{B}(x_1, x_2, x_3)\,, \\
A^{B\neq\Lambda}(x_2, x_1, x_3) &= -A^{B}(x_1, x_2, x_3)\,, \\
T^{B\neq\Lambda}(x_2, x_1, x_3) &= +T^{B}(x_1, x_2, x_3)\,,
\end{aligned}
$$

Proton



Symmetries

Symmetries of quasi-DA

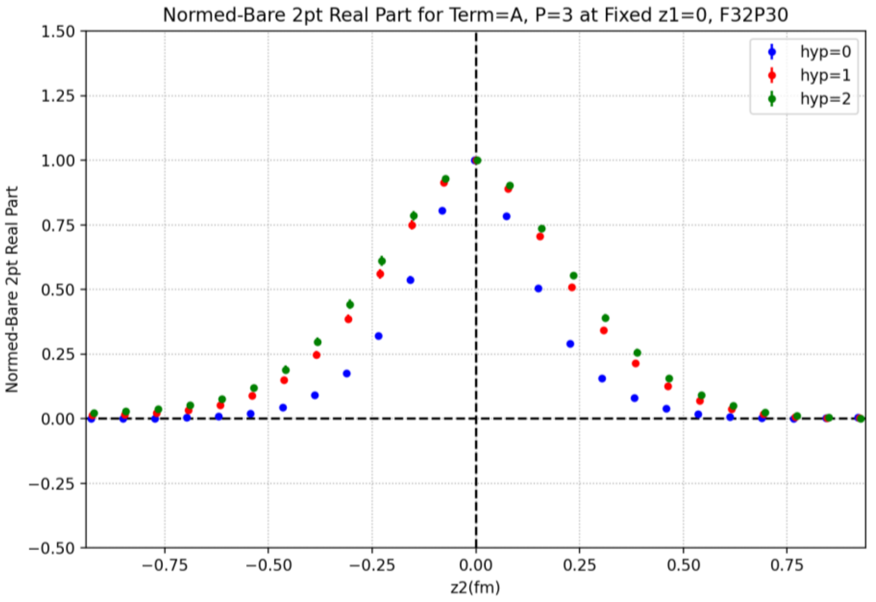
$$\tilde{\Phi}(z_1, z_2, \mu) = \int_0^1 \mathrm{d}x_1 \int_0^1 \mathrm{d}x_2\ \mathrm{e}^{\mathrm{i}(x_1 z_1 + x_2 z_2)P^z} \times \boxed{\tilde{\phi}(x_1, x_2, \mu)}$$

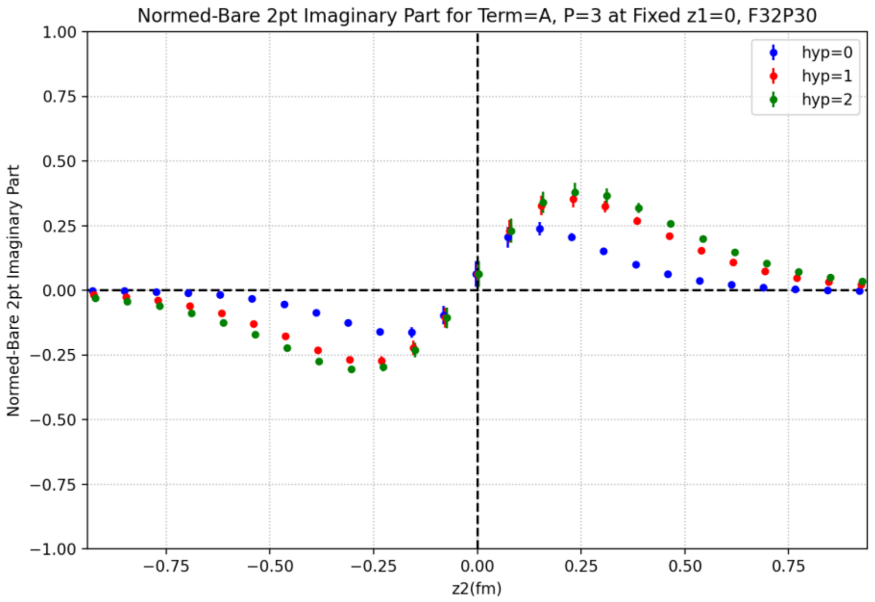
Pure real

- spatial parity symmetry (at statistical level);

the reality in momentum space $\tilde{\Phi}(x_1, x_2)$ leads to: $\tilde{\Phi}(z_1, z_2) = \tilde{\Phi}^*(-z_1, -z_2)$

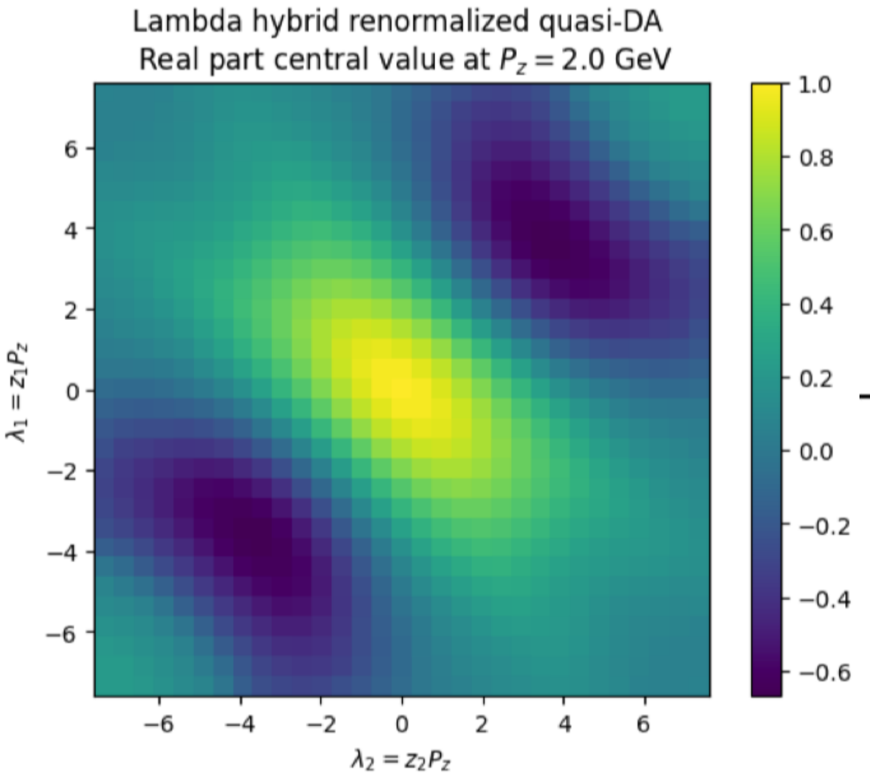
$$\mathrm{Re}\ \tilde{\Phi}(-z_1, -z_2) = \mathrm{Re}\ \tilde{\Phi}(z_1, z_2) \qquad \mathrm{Im}\ \tilde{\Phi}(-z_1, -z_2) = -\mathrm{Im}\ \tilde{\Phi}(z_1, z_2)$$

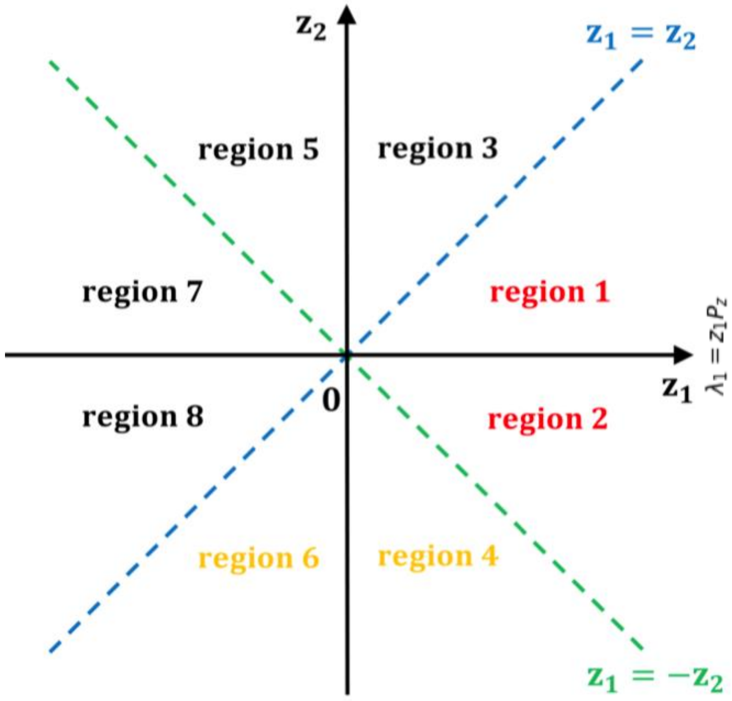


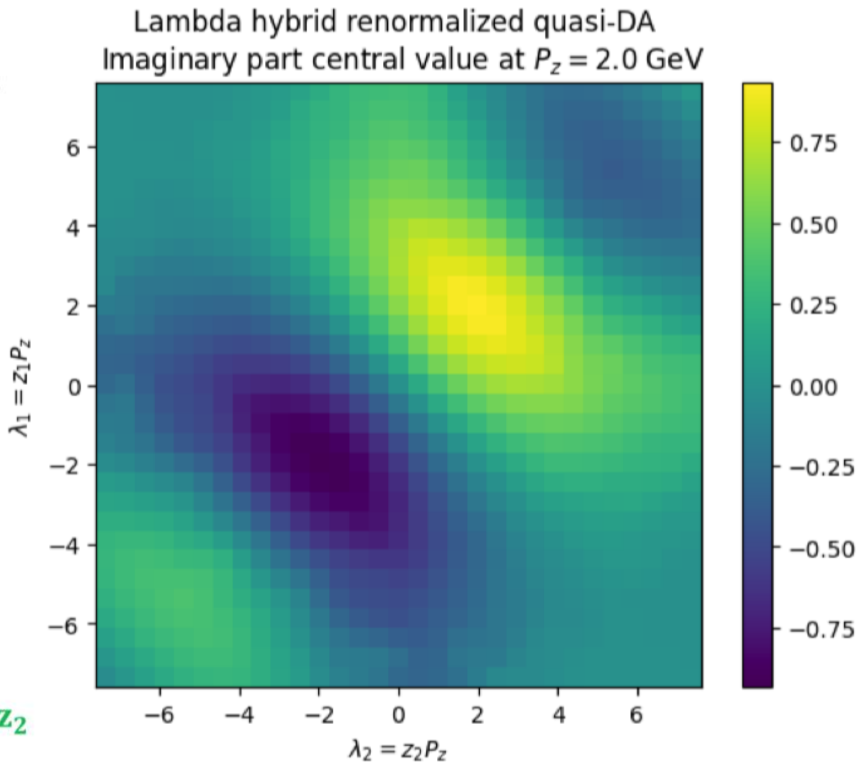


Symmetries

Symmetries of quasi-DA



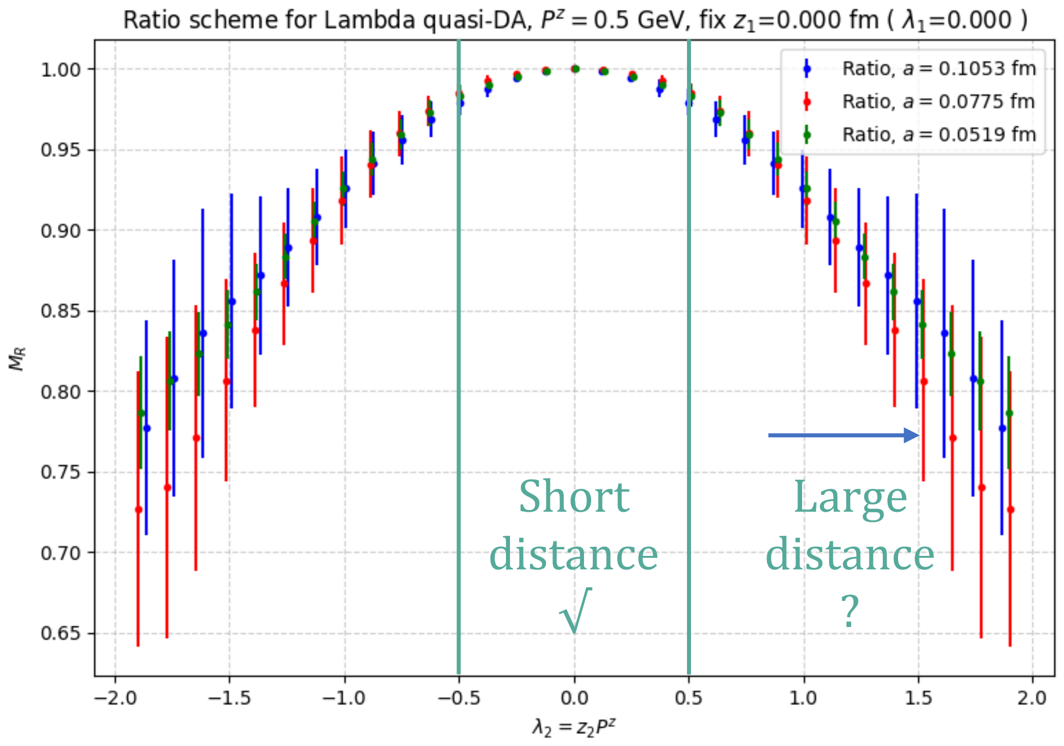




Ratio scheme

$$\tilde{\psi}^{\mathrm{ratio}}(z_1, z_2, P^z) = \frac{\tilde{\psi}^{\mathrm{bare}}(z_1, z_2, P^z)}{\tilde{\psi}^{\mathrm{bare}}(z_1, z_2, P^z = 0)}$$

Lambda A term $\quad P^z$=0.5 GeV



Uncontrollable IR structures on large z region !

Self-renormalization scheme

Step-1 (in all regions with linear div.)

b0

$$\ln M(z_1, z_2; P_z = 0; a) = \frac{k}{a \ln(a\Lambda_{\rm QCD})}\tilde{z} + g(z_1, z_2) + f(z_1, z_2)a^2 + \frac{\gamma_0}{b_0}\ln\frac{\ln(1/a\Lambda_{\rm QCD})}{\ln(\mu/\Lambda_{\overline{\rm MS}})} + \boxed{\ln\left[1 + \frac{d}{\ln(a\Lambda_{\rm QCD})}\right]}$$

Step-2 (in perturbative region)

$$g(z_1, z_2) - \ln Z_{\overline{\rm MS}}(z_1, z_2; \mu, \Lambda_{\overline{\rm MS}}) = m_0\tilde{z} + b_0$$

$$Z_{\overline{\rm MS}}(z_1, z_2; \mu, \Lambda_{\overline{\rm MS}}) = 1 + \frac{\alpha_s C_F}{2\pi}\left[\frac{7}{8}\ln\frac{z_1^2\mu^2 e^{2\gamma_E}}{4} + \frac{7}{8}\ln\frac{z_2^2\mu^2 e^{2\gamma_E}}{4} + \frac{3}{4}\ln\frac{(z_1 - z_2)^2\mu^2 e^{2\gamma_E}}{4} + 4\right]$$

Self-renormalization scheme

Renormalization factor

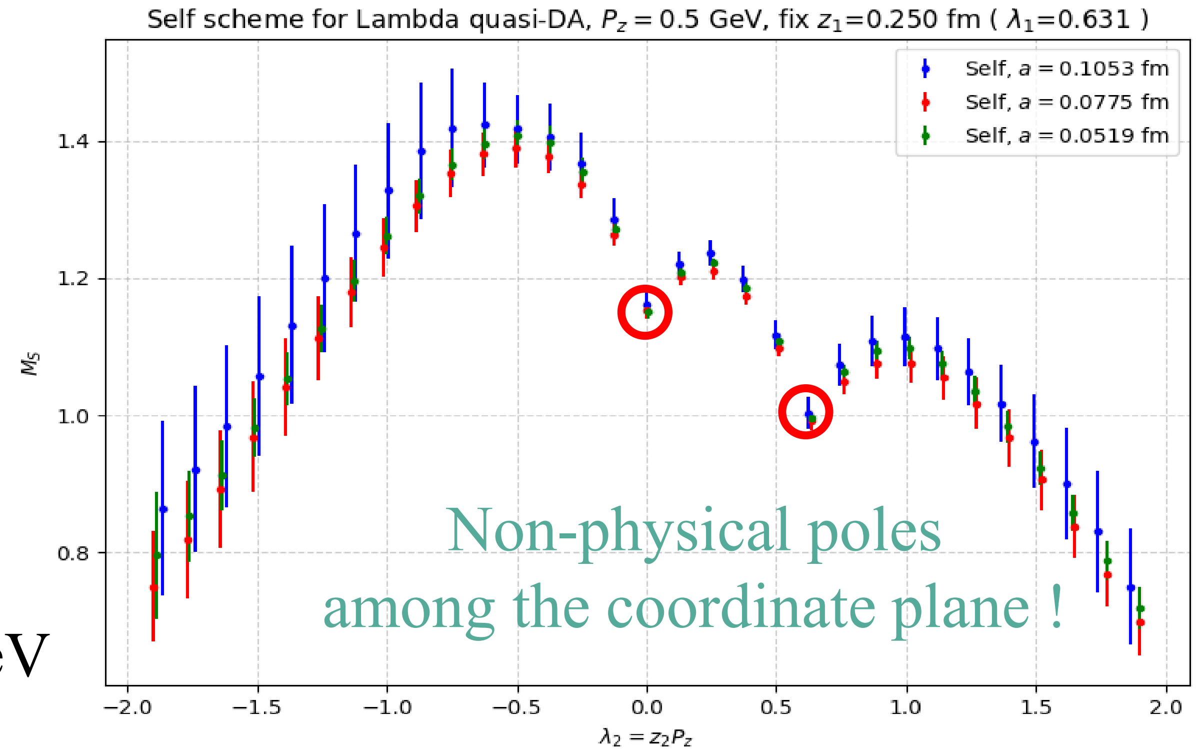
$$Z_R(z_1, z_2; a) = \exp\left[\frac{k}{a\ln(a\Lambda_{\mathrm{QCD}})}\tilde{z} + m_0\tilde{z} + f(z_1, z_2)a^2 + \frac{\gamma_0}{b_0}\ln\frac{\ln(1/a\Lambda_{\mathrm{QCD}})}{\ln(\mu/\Lambda_{\overline{\mathrm{MS}}})} + \ln\left[1 + \frac{d}{\ln(a\Lambda_{\mathrm{QCD}})}\right]\right]$$

Renormalized 0 & large-momentum matrix element

$$M_R(z_1, z_2; P_z = 0) = \exp\left(g(z_1, z_2) - m_0\tilde{z} - b_0\right)$$

$$M_R(z_1, z_2; P_z) = \frac{M(z_1, z_2; P_z; a)}{Z_R(z_1, z_2; a)}$$

P^z=0.5 GeV



Self-renormalization scheme

Bare Matrix Element

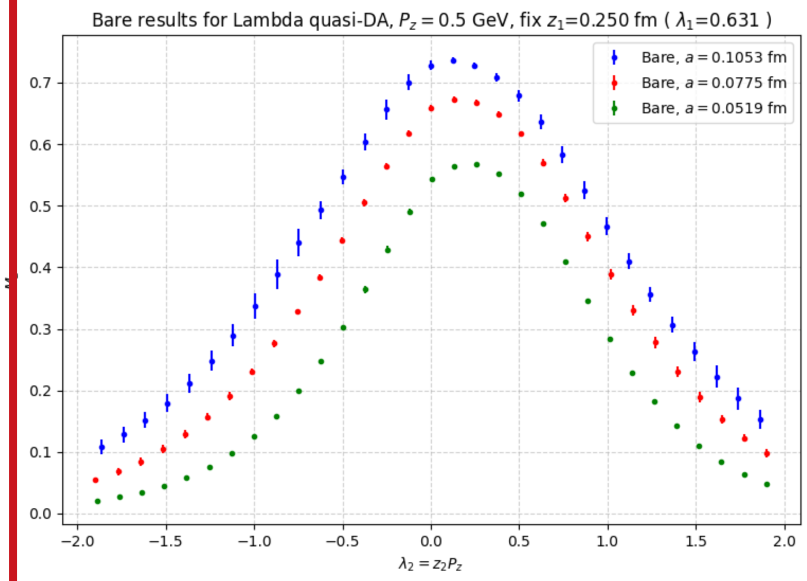
$$M(z_1, z_2; P_z; a)$$

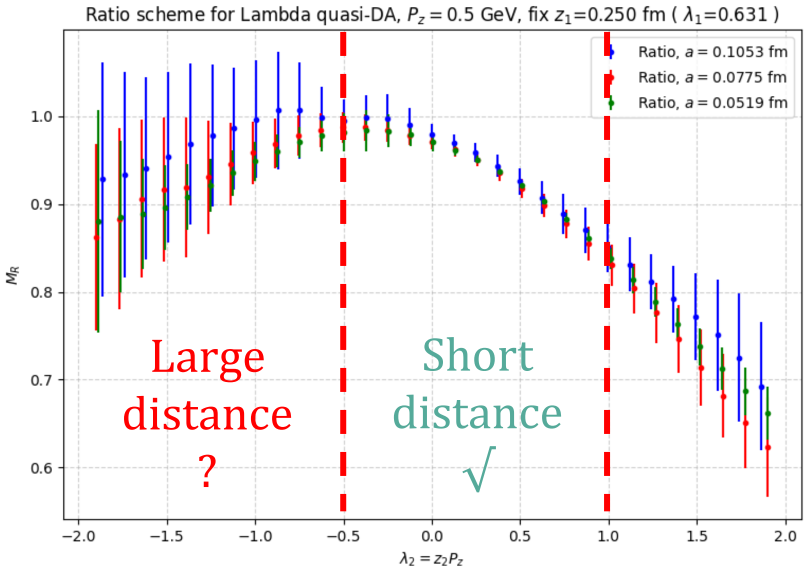
Ratio scheme

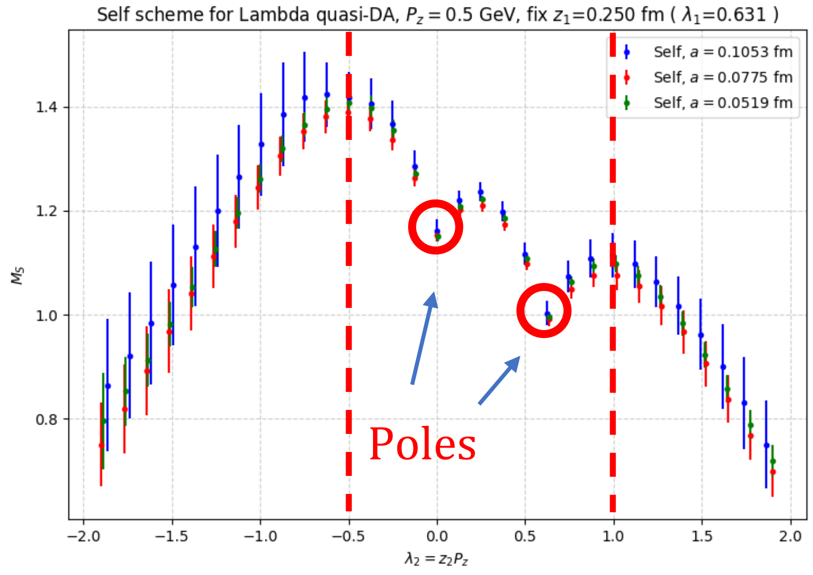
$$\frac{M(z_1, z_2; P_z; a)}{M(z_1, z_2; 0; a)}$$

Self scheme

$$\frac{M(z_1, z_2; P_z; a)}{Z_R(z_1, z_2; a)}$$







How to calculate LCDAs ?

2 Lattice methods for LCDA

Operator Product Expansion (OPE)

the local moments are consistent between light-cone coordinate and Euclidean space

- Focus on several lowest moments
- Problem from moments to full LCDA

Light baryon:

QCDSF, 2008, 2009;

RQCD, 2016, 2019, 2025 ……

Large momentum effective theory (LaMET)

the light-cone correlator relates to an equal-time correlator with a large momentum boost

- x-dependent LCDA directly
- Power corrections at both endpoints

Light baryon:

LPC, PRD 111, 034510 (2025)

LPC, arXiv: 2508.08971, ……