Pion properties in isospin-asymmetric nuclear matter using in-medium chiral perturbation theory

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Introduction

- Dynamical chiral symmetry breaking is supposed to be the origin of the masses of hadrons in quantum chromodynamics (QCD).

- $\langle\Omega|[Q_5^a, P^a]|\Omega\rangle = \langle\bar{u}u + \bar{d}d\rangle = \langle\bar{q}q\rangle$: a sufficient condition for the spontaneous breaking of the chiral symmetry in the chiral limit.

- One expects the broken chiral symmetry to be partially restored in extreme conditions, such as at a high density or temperature.

- Once the partial restoration of chiral symmetry in a nuclear medium is confirmed, we can guess that the properties of the pions are influenced by in-medium effects.

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Introduction

- In this research, the in-medium mass, wave function renormalization, and decay constant of each pion in nuclear matter with finite isospin are explored

- The in-medium Gell-Mann-Oakes-Renner relation : $m_\pi^{*2} f_\pi^{*2} = -m_q \langle \bar{q}q \rangle^*$

- In this work, we assume that all in-vacuum calculations are done. Thus, we use experimental values for LECs.

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In-medium pion properties

- The normalization of the in-medium pion states

$$\langle \pi^{*\pm,0}(k_{\pi^{\pm,0}}) | \pi^{*\pm,0}(p_{\pi^{\pm,0}}) \rangle = (2\pi)^3 2\omega_{\pi^{\pm,0}}(\vec{p}) \delta^{(3)}(\vec{p} - \vec{k}).$$

$p_{\pi^{\pm,0}}$: an on-shell pion momentum of the pions

$\omega_{\pi^{\pm,0}}$: an in-medium pion energy

- The in-medium pion mass $\omega_{\pi^{\pm,0}}(\vec{0}) = m_\pi^*$.

- The in-medium pion mass is obtained by solving the mass equation

$$m_\pi^* = \sqrt{m_\pi^2 + \Sigma_\pi(m_\pi^*, \vec{0})}.$$

Σ_π : the in-medium pion self-energy

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In-medium pion properties

- The matrix elements of an axial-vector current $A_\mu^{\pm,0}$ [1] :

[1] Jido, Hatsuda, Kunihiro, Phys. Lett. B 670, 109 (2008)

$$\langle\Omega|A_\mu^{\pm,0}|\pi^{*\pm,0}(p_{\pi^{\pm,0}})\rangle = i\left[n_\mu N_{\pi^{\pm,0}}^*(p_{\pi^{\pm,0}}) + p_\mu^{\pi^{\pm,0}} F_{\pi^{\pm,0}}^*(p_{\pi^{\pm,0}})\right]e^{-ip_{\pi^{\pm,0}}\cdot x},$$

n_μ : a vector of the Lorentz frame of the nuclear matter

In the rest frame, $n_\mu = (1,0,0,0)$.

- The pure vacuum state : $|0\rangle$

- The asymptotic state of nuclear matter : $|\Omega\rangle$.

- The pion decay constants in nuclear matter :

$$f_\pi^{*(t)} = F_\pi^* + \frac{N_\pi^*}{m_\pi^*}, \qquad f_\pi^{*(s)} = F_\pi^*$$

$f_\pi^{*(t)}$: Temporal decay constant

$f_\pi^{*(s)}$: Spatial decay constant

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Correlation function approach

- The correlation function of the two axial-vector currents:

$$\Pi_{\mu\nu}^{\pm\mp,00}(p) \equiv \int d^4x \, e^{ip\cdot x}\langle\Omega|TA_\mu^{\pm,0}(x)A_\nu^{\mp,0}(0)|\Omega\rangle.$$

- This can be expanded as

$$\Pi_{\mu\nu}^{\pm\mp,00}(p) = \frac{i\left(n_\mu N_{\pi^{\pm,0}}^* + p_\mu^{\pi^{\pm,0}}(\vec{p})F_{\pi^{\pm,0}}^*\right)\left(n_\nu \bar{N}_{\pi^{\pm,0}}^* + p_\nu^{\pi^{\pm,0}}(\vec{p})\bar{F}_{\pi^{\pm,0}}^*\right)}{2\omega_{\pi^{\pm,0}}(\vec{p})(p^0 - \omega_\pi^{\pm,0}(\vec{p}) + i\epsilon)} + \cdots$$

$$= (i\hat{\Gamma}_\mu^{\pi^{\pm,0}}) \frac{i}{(p^2 - m_\pi^2 - \Sigma_\pi^{\pm,0}(p) + i\epsilon)}(-i\hat{\bar{\Gamma}}_\nu^{\pi^{\pm,0}}) + \cdots$$

$\hat{\Gamma}_\mu^{\pi^{\pm,0}}$: The vertex correction for the axial vector current

$$\langle\Omega|A_\mu^{\pm,0}|\pi^{*\pm,0}(p_{\pi^{\pm,0}})\rangle_{1PI} = i\hat{\Gamma}_\mu^{\pi^{\pm,0}}$$

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Correlation function approach

- The wave function renormalization:

$$Z_\pi = \left(1 - \frac{1}{2\omega_\pi(\vec{p})}\frac{\partial\Sigma_\pi(p)}{\partial p_0}\bigg|_{p_0=\omega_\pi(\vec{p})}\right)^{-1}$$

$$\Pi_{\mu\nu}^{\pm\mp,00}(p) = \hat{\Gamma}_\mu^{\pi^{\pm,0}}\frac{iZ_{\pi^{\pm,0}}}{2\omega_{\pi^{\pm,0}}(\vec{p})\left(p_0 - m^*_{\pi^{\pm,0}} - \mathcal{K}_{\pi^{\pm,0}}(\vec{p}) + i\epsilon\right)}\bar{\hat{\Gamma}}_\nu^{\pi^{\pm,0}} + \cdots,$$

$\mathcal{K}_{\pi^{\pm,0}}(\vec{p}) = \omega_{\pi^{\pm,0}}(\vec{p}) - m^*_{\pi^{\pm,0}}$: an in-medium kinetic energy.

- Effective vertex correction :

$$p_\mu \hat{f}_\pi \equiv \hat{\Gamma}_\mu^\pi.$$

- The calculation of the temporal decay constant:

$$f_\pi^{*(t)} = \sqrt{Z_\pi}\hat{f}_\pi.$$

Correlation function approach

- In a similar way, the two-point function of the pseudoscalar currents:

$$\Pi^{\pm,\mp,00}(p) \equiv \int d^4x \, e^{ip \cdot x} \langle \Omega | TP^{\pm,0}(x) P^{\mp,0}(0) | \Omega \rangle$$

$$= \frac{iG^*_{\pi^{\pm,0}} \bar{G}^*_{\pi^{\pm,0}}}{2\omega_{\pi^{\pm,0}}(\vec{p})(p^0 - \omega_\pi^{\pm,0}(\vec{p}) + i\epsilon)} + \cdots$$

$$= \left(i\hat{G}_{\pi^{\pm,0}} \right) \frac{i}{\left(p^2 - m_\pi^2 - \Sigma_\pi^{\pm,0}(p) + i\epsilon \right)} \left(-i\bar{\hat{G}}_{\pi^{\pm,0}} \right) + \cdots .$$

- The pseudo-scalar form factor G_π^* :

$$G_\pi^* = \sqrt{Z_\pi} \hat{G}_\pi,$$

where

$$\langle \Omega | P^{\pm,0}(x) | \pi^{\pm,0} \rangle_{1PI} = \hat{G}_{\pi^{\pm,0}} \, e^{-ip \cdot x} .$$

- The AP correlation function:

$$\Pi_{5\mu}^{\pm,\mp,00}(p) \equiv \int d^4x \, e^{ip \cdot x} \langle \Omega | TA_\mu^{\pm,0}(x) P^{\mp,0}(0) | \Omega \rangle .$$

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In-medium Gell-Mann-Oakes-Renner relation

- We start with the following correlation function:

$$\int d^4x \, e^{ip \cdot x} \partial^\mu \langle \Omega | T A_\mu^{\pm,0}(x) P^{\mp,0} | \Omega \rangle.$$

The operator relations: $\left[Q_5^{\pm,0}, P^{\mp,0} \right] = -i\bar{q}q, \; Q_5^{\pm,0} = \int d^3x A_0^{\pm,0}(x)$

The partially conserved axial vector current (PCAC) relation: $\partial^\mu A_\mu^{\pm,0} = m_q P^{\pm,0}$

- The Chiral Ward identity:

$$i \lim_{\vec{p} \to 0} \left[p^\mu \Pi_{5\mu}^{\pm\mp,00}(p) + m_q \Pi^{\pm\mp,00}(p) \right] = i \langle \bar{q}q \rangle^*,$$

$m_q = (m_u + m_d)/2$: The averaged quark mass

$\langle \Omega | \bar{q}q | \Omega \rangle \equiv \langle \bar{q}q \rangle^*$: In-medium quark condensate

In-medium Gell-Mann-Oakes-Renner relation

- The correlation functions:

$$\mathrm{i}\,\Pi_{5\mu}^{\pm\mp,00}(p) = -\mathrm{i}p^{\mu}\left(\sum_{n^{\pm,0}}\frac{\left(n_{\mu}N^{*}_{\pi_n^{\pm,0}} + p_{\mu}^{n^{\pm,0}}F^{*}_{\pi_n^{\pm,0}}\right)\bar{G}^{*}_{\pi_n^{\pm,0}}}{2\omega_{n^{\pm,0}}(\vec{p})(p^0 - \omega_{n^{\pm,0}}(\vec{p}) + i\epsilon)} + \sum_{n^{\mp,0}}\frac{\left(n_{\mu}\bar{N}^{*}_{\pi_n^{\mp,0}} + p_{\mu}^{n^{\mp,0}}\bar{F}^{*}_{\pi_n^{\pm,0}}\right)G^{*}_{\pi_n^{\mp,0}}}{2\omega_{n^{\mp,0}}(\vec{p})(p^0 - \omega_{n^{\mp,0}}(\vec{p}) - i\epsilon)}\right),$$

$$\mathrm{i}\,m_q\Pi^{\pm\mp,00}(p) = \mathrm{i}m_q\left(\sum_{n^{\pm,0}}\frac{G^{*}_{\pi_n^{\pm,0}}\bar{G}^{*}_{\pi_n^{\pm,0}}}{2\omega_{n^{\pm,0}}(\vec{p})(p^0 - \omega_{n^{\pm,0}}(p) + i\epsilon)} - \sum_{n^{\mp,0}}\frac{\bar{G}^{*}_{\pi_n^{\mp,0}}G^{*}_{\pi_n^{\mp,0}}}{2\omega_{n^{\mp,0}}(\vec{p})(q^0 - \omega_{n^{\mp,0}}(\vec{p}) - i\epsilon)}\right).$$

- The PCAC relation in terms of pion states:

$$\left(\omega_{n^{\pm,0}}(\vec{p})^2 - |\vec{p}|^2\right)F^{*}_{\pi_n^{\pm,0}} + \omega_{n^{\pm,0}}N^{*}_{\pi_n^{\pm,0}} = m_q G^{*}_{\pi_n^{\pm,0}}.$$

- The in-medium Gell-Mann-Oakes-Renner relation:

$$\frac{1}{2}\sum_{n^+}\left[\left|m^{*}_{\pi_n^+}F^{*}_{\pi_n^+} + N^{*}_{\pi_n^+}\right|^2\right] + \frac{1}{2}\sum_{n^-}\left[\left|m^{*}_{\pi_n^-}F^{*}_{\pi_n^-} + N^{*}_{\pi_n^-}\right|^2\right] = -m_q\langle\bar{q}q\rangle^{*},$$

$$\sum_{n^0}\left[\left|m^{*}_{\pi_n^0}F^{*}_{\pi_n^0} + N^{*}_{\pi_n^0}\right|^2\right] = -m_q\langle\bar{q}q\rangle^{*}.$$

In-medium Gell-Mann-Oakes-Renner relation

- The in-medium Gell-Mann-Oakes-Renner relation:

$$\frac{1}{2}\sum_{n^+}\left[\left|m^*_{\pi^+_n}F^*_{\pi^+_n} + N^*_{\pi^+_n}\right|^2\right] + \frac{1}{2}\sum_{n^-}\left[\left|m^*_{\pi^-_n}F^*_{\pi^-_n} + N^*_{\pi^-_n}\right|^2\right] = -m_q\langle\bar{q}q\rangle^*,$$

$$\sum_{n^0}\left[\left|m^*_{\pi^0_n}F^*_{\pi^0_n} + N^*_{\pi^0_n}\right|^2\right] = -m_q\langle\bar{q}q\rangle^*,$$

- Due to the power counting, we take only the lowest states in this work.

- The Gell-Mann-Oakes-Renner relation in terms of the temporal decay constants :

$$\frac{1}{2}\left(m^{*2}_{\pi^+}\left|f^{*(t)}_{\pi^+}\right|^2 + m^{*2}_{\pi^-}\left|f^{*(t)}_{\pi^-}\right|^2\right) = -m_q\langle\bar{q}q\rangle^*,$$

$$m^{*2}_{\pi^0}\left|f^{*(t)}_{\pi^0}\right|^2 = -m_q\langle\bar{q}q\rangle^*.$$

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In-medium chiral perturbation theory

- To derive the in-medium chiral perturbation theory, we develop the generating functional with the chiral Lagrangian between two asymptotic states of nuclear matter [2,3]:

$$Z[J] = \langle \Omega_{\text{out}} | \Omega_{in} \rangle$$

[2] Oller, José A., *PRC* 65,(2002): 025204.

[3] Meißner, Ulf-G., Jose A. Oller, and Andreas Wirzba., *Annals of Phys.* 297 (2002): 27-66.

- Asymptotic states:

$$|\Omega\rangle = \prod_{i}^{|p_i| \leq k_F^{p,n}} a^\dagger(p_i)|0\rangle,$$

i : the isospin and spin of the nucleon

Fermi momenta : $k_{\text{F}}^{\text{p,n}} = \left(3\pi^2 \rho_{p,n}\right)^{1/3}.$

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In-medium chiral perturbation theory

- After calculating the generating functional explicitly,
$$Z[J] = \int \mathcal{D}U\mathcal{D}\bar{N}\mathcal{D}N \langle \Omega_{\text{out}} | N_{t \to +\infty} \rangle \times e^{i\int d^4x(\mathcal{L}_\pi + \mathcal{L}_N + \mathcal{L}_{\pi N})} \langle N_{t \to -\infty} | \Omega_{in} \rangle,$$

  the Feynman rule for the in-medium nucleon propagator is obtained.

- The loop integral for the in-medium nucleon propagator is performed up to the Fermi momenta with the Heaviside step functions,
$$\Theta_{|\vec{p}|}^{p,n} \equiv \Theta\left(k_F^{p,n} - |\vec{p}|\right).$$

- The chiral Lagranigan:
$$\mathcal{L}_\pi^{(2)} = \frac{f^2}{4} Tr\{D_\mu U^\dagger D^\mu U + \chi^\dagger U + \chi U^\dagger\},$$

$$\mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} = \bar{N}(i\gamma^\mu \Gamma_\mu + ig_A \gamma^\mu \gamma^5 \Delta_\mu + c_1 Tr\{\chi_+\} + \frac{c_2}{2m_N^2} Tr\{u_\mu u_\nu\} D^\mu D^\nu$$

$$+ \frac{c_3}{2} Tr\{u_\mu u^\mu\} + \frac{c_4}{2} \gamma^\mu \gamma^\nu [u_\mu, u_\nu] + \cdots.$$

- We neglect the in-medium effects from the $\mathcal{O}(\rho^2)$ in the density expansion.

Self-energies

- Relevant Feynman diagrams :

Dashed line : pion propagators,
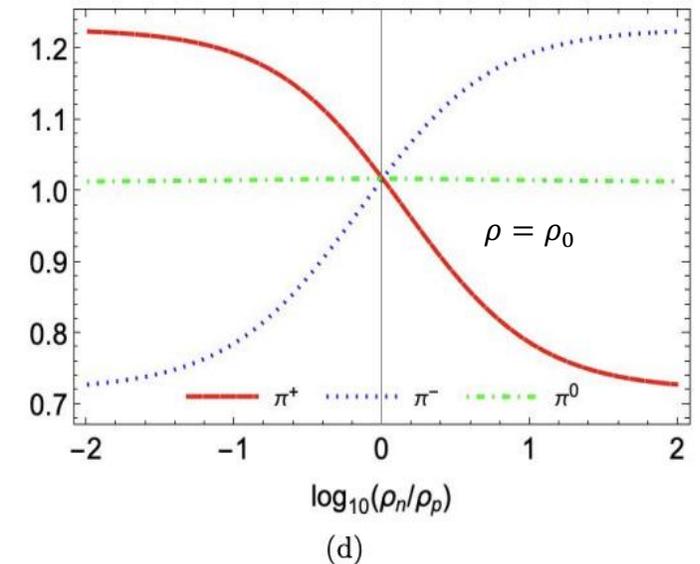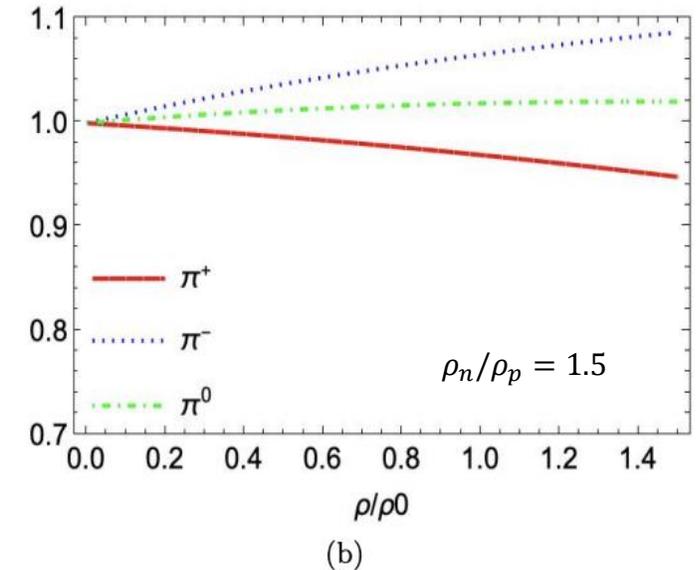
Thin line : the in-vacuum nucleon propagator

Thick line : the in-medium nucleon propagator.

- The in-medium pion masses are calculated with the following equations:

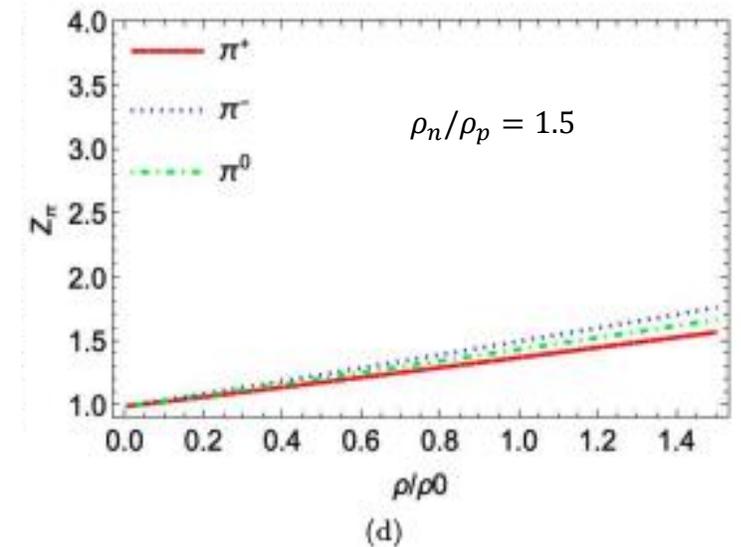$$m_\pi^* = \sqrt{m_\pi^2 + \Sigma_\pi(m_\pi^*, \vec{0})}.$$

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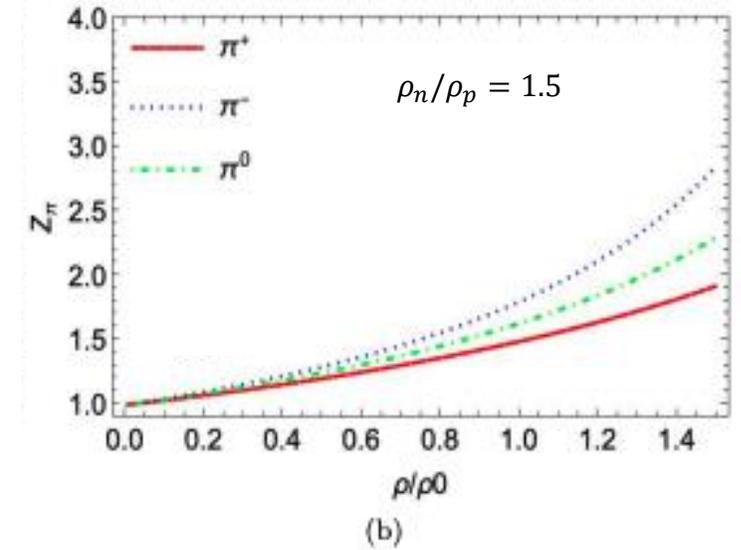SCIENCE TOKYO

Masses

- The first plot presents the results at $\rho_n/\rho_p = 1.5$, which corresponds to neutron-rich nuclear matter.

- The masses are affected not only by the density but also by the isospin of the medium.

- The overall behaviors are symmetric around the ratio $\rho_n/\rho_p = 1$.

- Under $r \leftrightarrow \frac{1}{r}$, the behaviors of the charged pions are flipped.
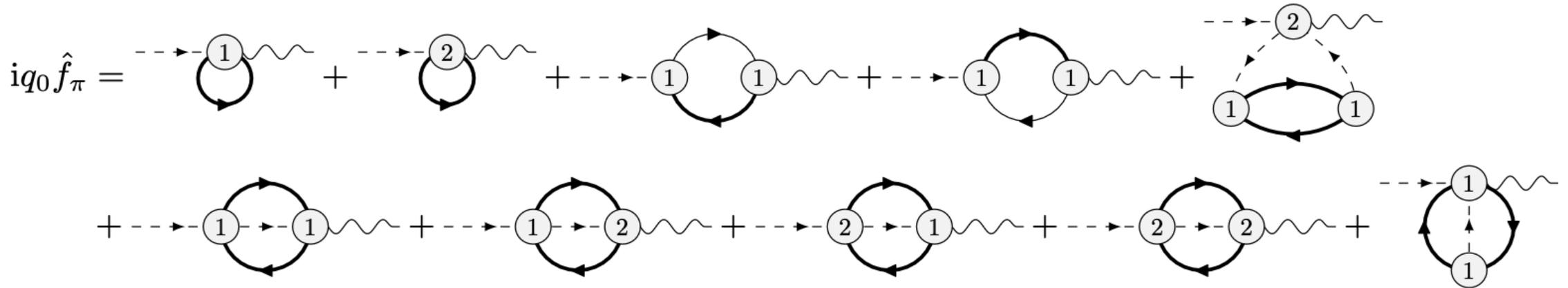
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(b)

(d)

Wave function renormalizations

- $Z_\pi = \left(1 - \dfrac{1}{2m_\pi^*}\dfrac{\partial \Sigma_\pi(p)}{\partial q_0}\bigg|_{q_0 = m_\pi^*}\right)^{-1}$

- The last result shows the small-density expansion.

(b)

(d)

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Pion decay constants
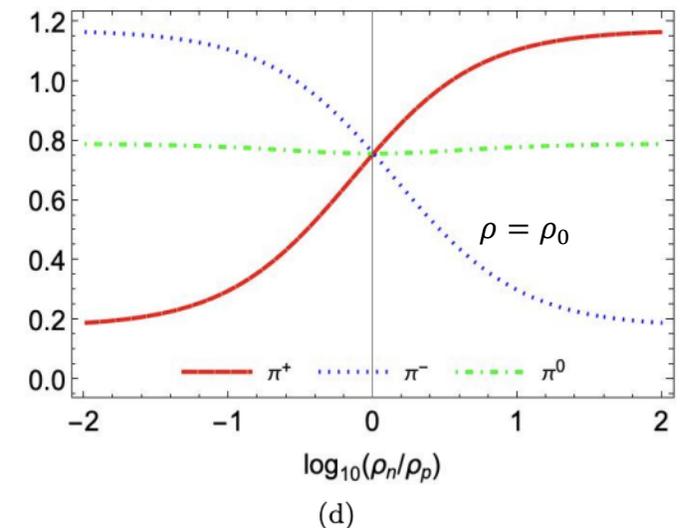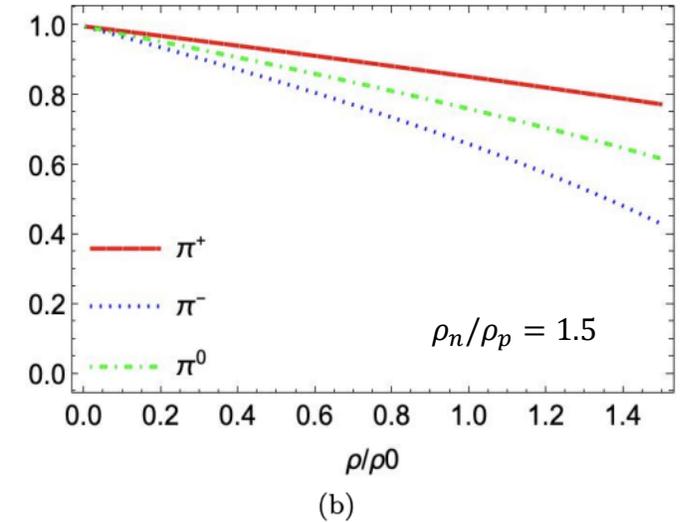
- The effective vertex corrections:

[4] Kwon, Kihong, Yamato Suda, Stephan Hübsch, and Daisuke Jido. *arXiv preprint arXiv:2507.01398* (2025).

- The wavy : axial vector currents.

- The detailed integral forms of the loop diagrams are shown in [4].

- With the wave function renormalization, the temporal pion decay constant is calculated as

$$f_\pi^{*(t)} = \sqrt{Z_\pi}\hat{f}_\pi.$$

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Pion decay constants

- The results are obtained in the pion rest frame, $q^\mu = (m_\pi^*, \vec{0})$.

- All decay constants decrease, and that of the negative pion reduces the fastest.

- The overall behaviors are symmetric around the ratio $\rho_n/\rho_p = 1$.

(b)

(d)

In-medium Gell-Mann-Oakes-Renner relation

- The in-medium Gell-Mann-Oakes-Renner relation:

$$\frac{1}{2}\left(m_{\pi^+}^{*2}\left|f_{\pi^+}^{*(t)}\right|^2 + m_{\pi^-}^{*2}\left|f_{\pi^-}^{*(t)}\right|^2\right) = -m_q\langle\bar{q}q\rangle^*,$$

$$m_{\pi^0}^{*2}\left|f_{\pi^0}^{*(t)}\right|^2 = -m_q\langle\bar{q}q\rangle^*.$$

- The relation is evaluated at $\rho = 0.6\,\rho_0$ and $\rho_n/\rho_p = 1.4$

- This gives about 23% reduction of the quark condensate in nuclear matter

- This result is in agreement with Ref. [5], which showed the 21-25% reduction.

[5] Nishi, et al., Nature Phys. 19, 788 (2023)

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Summary

- We have calculated the in-medium pion properties using in-medium chiral perturbation theory in the physical basis.

- The different densities of proton and neutron are compensated by the different Fermi momenta in actual calculations.

- Using the in-medium Gell-Mann-Oakes-Renner relations, we can estimate the partial restoration of the chiral symmetry in nuclear matter.

- The overall results are symmetric around the isospin symmetric case.

- The detailed calculations are shown in [4].

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References

[1] Jido, D., T. Hatsuda, and T. Kunihiro. "In-medium pion and partial restoration of chiral symmetry." *Physics Letters B* 670, no. 2 (2008): 109-113.

[2] Oller, José A. "Chiral Lagrangians at finite density." *Physical Review C* 65, no. 2 (2002): 025204.

[3] Meißner, Ulf-G., Jose A. Oller, and Andreas Wirzba. "In-medium chiral perturbation theory beyond the mean-field approximation." *Annals of Physics* 297, no. 1 (2002): 27-66.

[4] Kwon, Kihong, Yamato Suda, Stephan Hübsch, and Daisuke Jido. "Pion properties in isospin-asymmetric nuclear matter using in-medium chiral perturbation theory." *arXiv preprint arXiv:2507.01398* (2025)

[5] Nishi, Takahiro, Kenta Itahashi, DeukSoon Ahn, Georg PA Berg, Masanori Dozono, Daijiro Etoh, Hiroyuki Fujioka et al. "Chiral symmetry restoration at high matter density observed in pionic atoms." *Nature Physics* 19, no. 6 (2023): 788-793.

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