Effective gluon operators for heavy-light systems in the instanton vacuum

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Motivation

- Observables of heavy hadrons containing a heavy quark can be systematically expanded in powers of $1/m_Q$:

$$\langle O \rangle = \langle O \rangle^{(0)} + \frac{1}{2m_Q}\langle O \rangle^{(1)} + \frac{1}{(2m_Q)^2}\langle O \rangle^{(2)} + \cdots$$

where $\langle O \rangle^{(n)}$ represents the n-th order contribution in the inverse heavy-quark mass expansion.

- At leading order ($m_Q \to \infty$):

  - The heavy quark acts as a static color source

  - The dynamics are governed solely by the light quarks and gluons

- Starting from subleading order ($1/m_Q$):

  - Gluon operators begin to contribute, example: $\mathcal{O}_{\text{kin}} = \bar{h}_v(iD_\perp)^2 h_v, \ \mathcal{O}_{\text{mag}} = \frac{g}{2}\bar{h}_v\sigma_{\mu\nu}G^{\mu\nu}h_v$

  - These terms generate observable effects such as heavy meson mass splitting ($M_{D^*} - M_D$) and heavy quark spin-symmetry breaking

- In the instanton vacuum, the corresponding gluon fields acquire a microscopic origin from instanton-induced configurations, providing a nonperturbative basis for these operators.

Theoretical framework

- Gauge fields are represented as an ensemble of instantons and anti-instantons: $A_\mu = \sum_{\mathcal{Q}}^{N_+ + N_-} A_{\mathcal{Q}\mu}, \; A_{\mathcal{Q}\mu} = A_\mu(\xi_{\mathcal{Q}})$

where each (anti)instanton has its collective coordinates $\xi = (\rho, z, U)$: size ρ, position z and orientation U.

- The approximate action for light quarks:

$$\exp\left(-S[\psi^\dagger, \psi]\right) = \exp\left(\int d^4x \sum_f^{N_f} \psi_f^\dagger i\partial\!\!\!/ \psi_f\right) \prod_{\mathcal{Q}}^{N_+ + N_-} \prod_f^{N_f} \left(im_f - V_{\mathcal{Q}}[\psi_f^\dagger, \psi_f]\right)$$

where $V_\pm[\psi^\dagger, \psi]$ represents the fermion zero-mode contribution

- The action for heavy quarks LO on $1/m_Q$:

$$\exp\left(-S[h^\dagger, h]\right) = \exp\left[\int d^4x \, h^\dagger \left(i\partial_4 + \sum_{\mathcal{Q}}^{N_+ + N_-} A_{\mathcal{Q}4}\right) h\right]$$

- The heavy quark propagator with the account of the light quark determinant and QCD instanton vacuum properties

$$S_H \propto \int D\psi D\psi^\dagger \exp\left[\sum_f^{N_f} \int d^4x \left(\psi_f^\dagger i\partial\!\!\!/ \psi_f + \eta_f^\dagger \psi_f + \psi_f^\dagger \eta_f\right)\right] W_+^{N_+} W_-^{N_-} \bar{w}[\psi, \psi^\dagger]$$

$$\bar{w}[\psi, \psi^\dagger] = W_+^{-N_+} W_-^{-N_-} \int \prod_{\mathcal{Q}}^{N_+ + N_-} d\xi_{\mathcal{Q}} \prod_f^{N_f} \left(-V_{\mathcal{Q}}[\psi_f^\dagger, \psi_f]\right) \left(\partial_4 - i \sum_{\mathcal{Q}}^{N_+ + N_-} A_{\mathcal{Q}4}\right)^{-1}$$

$$W_\pm = \left\langle \prod_f^{N_f} (-V_\pm[\psi_f^\dagger, \psi_f]) \right\rangle$$ - one-instanton averages of zero-mode contributions

Effective heavy-light quark interaction

- Averaging over the ensemble of I's quand $\bar{I}$'s in low density instanton approximation we obtained effective N_f light flavor quarks and a heavy quark interaction
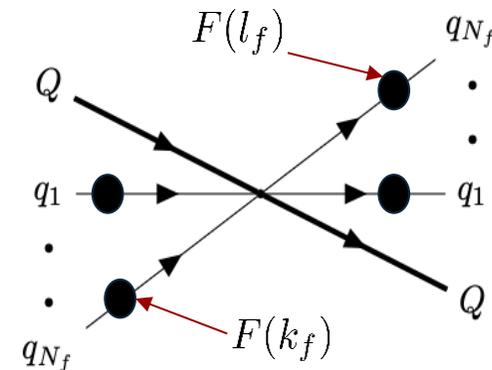
- Effective interaction at $N_f = 1$

$$S_{\text{int}} = \frac{M}{8}\frac{\Delta M_Q}{N/V}\int d^4x \left(h^\dagger \Gamma F(\partial)\psi\right)\left(F(\partial)\psi^\dagger \Gamma h\right), \qquad \Gamma = \{\mathbf{1}, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}/2\}$$

- Effective interaction at $N_f > 1$, LO on $1/N_c$

$$S_{\text{int}} = -i\left(\frac{2V}{N}\right)^{N_f-1}(iM)^{N_f}\int d^4z \det J_+(z) \int d^4x\, h^\dagger \frac{1}{N_c}\text{tr}_c\left(\partial_4\left(w_+ - \partial_4^{-1}\right)\partial_4\right)h + (+ \to -)$$

$$J_{\pm\,fg}(z) = \int \frac{d^4k_1 d^4k_2}{(2\pi)^8}e^{i(k_2-k_1)z}F(k_1)F(k_2)\psi^\dagger(k_1)\gamma_\pm\psi(k_2)$$

$$\frac{1}{N_c}\int d^3r\, e^{i\vec{p}\cdot\vec{r}}\,\text{tr}_c\langle\infty|(w_\pm - \partial_4^{-1})|-\infty\rangle \xrightarrow{\vec{p}\to 0} \frac{\Delta M_Q}{N/V}$$

Application (N_f=1)

- Averages of operators within effective theory in the $1/N_c$ limit

$$\langle \mathcal{O}[\psi, \psi^\dagger, h, h^\dagger] \rangle_{\text{eff}} = \frac{\int D\psi^\dagger D\psi Dh^\dagger Dh\, \mathcal{O}e^{-S_{\text{eff}}}}{\int D\psi^\dagger D\psi Dh^\dagger Dh\, e^{-S_{\text{eff}}}}$$

$$S_{\text{eff}}[\psi^\dagger, \psi, h^\dagger, h] = \int d^4x\, \psi^\dagger(i\slashed{\partial} + iMF^2(\partial))\psi + \int d^4x\, h^\dagger iv.\partial h + S_{\text{int}}[\psi^\dagger, \psi, h^\dagger, h]$$
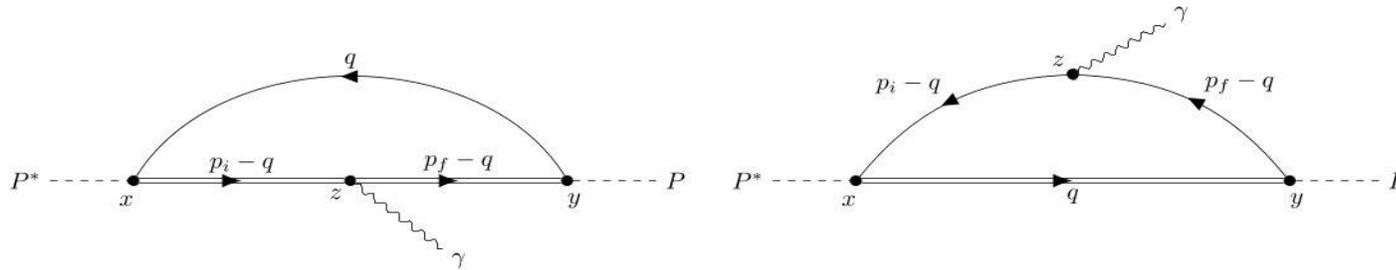
- After bosonization, one obtains an effective action that includes heavy mesons

- The weak decay constants: $\langle 0|J^A_\mu|P\rangle = -f_P\sqrt{m_P}v_\mu$, J^A_μ - renormalized axial-vector current

	[EFT($N_f = 1$)]	LQCD [FLAG,2021]	QCD SR [Pullin,2021]	PDG av.[PDG,2024]
f_D [MeV]	$204.6^{+13.6}_{-4.5}$	$209.0(2.4)$	$190(15)$	$203.8(4.7)(0.6)(1.4)$
f_B [MeV]	$191.2^{+30.0}_{-8.4}$	$192.0(4.3)$	192^{+20}_{-15}	$190.0(1.3)$

[K.Hong et al, Phys.Rev.D 110 (2024) 11, 114044]

Application II: radiative transitions

- Flavor conserving radiative transition: $\mathcal{M}\left(P_f^* \to P_f \gamma\right) = e\eta^\mu \langle P_f(v')| J_\mu^l + J_\mu^h |P_f^*(v,\epsilon)\rangle$
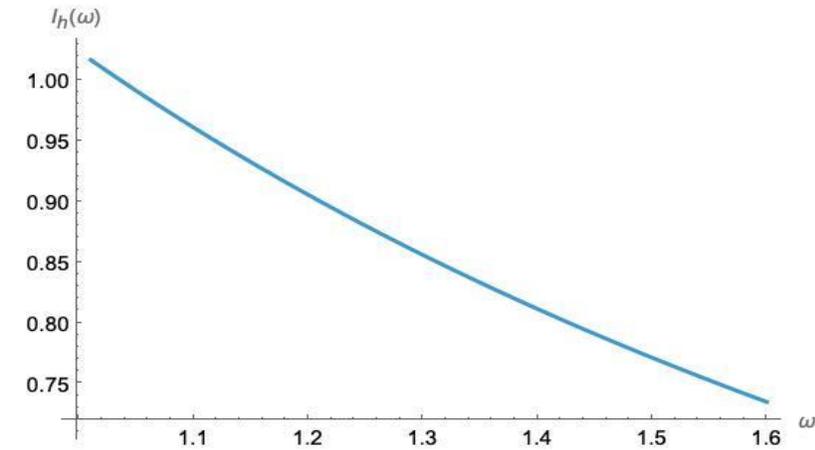

$$J_\mu^l = e_q \bar{\psi}\gamma_\mu \psi \qquad J_\mu^h = e_Q \bar{Q}\gamma_\mu Q$$

- Decay width at zero recoil $v = v'$, LO on $1/m_Q$

$$\Gamma(P^* \to P\gamma) = \frac{\alpha}{3}\frac{M_P}{M_{P^*}}|\vec{k}|^3 \left[\left(\frac{e_Q}{M_P} I_h(v.v' = 1)\right)^2 + (e_q I_l)^2\right]$$

Isgur-Wise function

Table 1: The decay widths of M1 radiative decays of the ground state heavy mesons. Preliminary theoretical calc.s (at zero recoil) vs PDG data.

Multiplet	Example Decay	in PDG Notations	Theory	Exp.
		$D^*(2007)^0 \to D^0 + \gamma$	2.6 keV	< 736 keV
$(0^-, 1^-)$	$1^- \to 0^-$	$D^*(2010)^+ \to D^+ + \gamma$	1.2 keV	1.33 keV
		$B^* \to B + \gamma$	0.02 keV	

Averages of operators containing gluons

- To account for gluon contributions at subleading order, we evaluate averages of gluonic operators in the instanton ensemble

$$\left\langle J_\Gamma(x_1) J_\Gamma^\dagger(x_2) \mathcal{F}[A] \right\rangle_{N_\pm} \propto \int D\psi^\dagger D\psi Dh^\dagger Dh \, J_\Gamma(x_1) J_\Gamma^\dagger(x_2) \exp \int d^4x \left[\psi_f^\dagger i\partial\!\!\!/ \psi_f + \eta_f^\dagger \psi_f + \psi_f^\dagger \eta_f \right]$$

$$\times \prod_{\mathcal{Q}}^{N_+ + N_-} \int d\xi_\mathcal{Q} \mathcal{F}[A] \prod_{\mathcal{Q}}^{N_+ + N_-} \prod_f^{N_f} \left(im_f - V_\mathcal{Q}[\psi_f^\dagger, \psi_f] \right) \exp \int d^4x \, h^\dagger \left(i\partial_4 + \sum_{\mathcal{Q}}^{N_+ + N_-} A_{\mathcal{Q},4} \right) h$$

J_Γ - fermion current, $\mathcal{F}[A]$ – function of gluon field

- $\mathcal{F}[A]$ can be decomposed into one-instanton and multi-instanton contributions:

$$\mathcal{F}[A(\xi)] = \sum_{\mathcal{Q}=\pm}^{N_\pm} \mathcal{F}[A_\mathcal{Q}(\xi_\mathcal{Q})] + [\text{multi} - \text{inst.}]$$

- The instanton medium is dilute ([ins. size]/[inter ins. distance])$^4 \ll 1$, multi-instanton contributions are numerically suppressed.

- Retain only one-instanton contributions and calculate averages of operators in the same N_c order as S_{eff}

Effective operators for gluons

- Represent the instanton ensemble averaged operator within the effective fermion theory

$$\left\langle J_\Gamma(x_1) J_\Gamma^\dagger(x_2) \mathcal{F}[A] \right\rangle_{N_\pm} = \left\langle J_\Gamma(x_1) J_\Gamma^\dagger(x_2) \text{``}\mathcal{F}\text{''}[\psi^\dagger, \psi, h^\dagger, h] \right\rangle_{\text{eff}}$$

$$= \frac{1}{Z_{\text{eff}}} \int D\psi D\psi^\dagger Dh Dh^\dagger \; J_\Gamma(x_1) J_\Gamma^\dagger(x_2) \text{``}\mathcal{F}\text{''}[\psi^\dagger, \psi, h^\dagger, h] \, e^{-S_{\text{eff}}}$$

- Effective operator:

$$\text{``}\mathcal{F}\text{''}[\psi, \psi^\dagger, h, h^\dagger] = N_+ \mathcal{Y}_{\mathcal{F}_+} \mathcal{R}_+ + N_- \mathcal{Y}_{\mathcal{F}_-} \mathcal{R}_-,$$

$$\mathcal{R}_\pm[\psi, \psi^\dagger, h, h^\dagger] = \frac{1}{N_+ N_-} \mathcal{Y}_\mp \exp\left[1 - \frac{Y_+}{N_+} + 1 - \frac{Y_-}{N_-}\right] \exp\left[\frac{1}{N_+} \int_x h^\dagger \mathcal{K}_+ h + \frac{1}{N_-} \int_x h^\dagger \mathcal{K}_- h\right]$$

- $Y_\pm$ - light quarks interaction vertex (t'Hooft vertex)

- $\mathcal{Y}_\pm$ - light quarks and a heavy quark interaction vertex

- $\mathcal{Y}_{\mathcal{F}_\pm}$ - light quarks , a heavy quark and instanton interaction vertex

Current status and ongoing work

Objective: study the contribution of effective gluon operators to the matrix elements of heavy–meson radiative transitions.

Current focus: the matrix elements receive two main types of $1/m_Q$ corrections:

- $\mathcal{O}(1/m_Q)$ contributions to the **QCD–matched HQET current**

$$\bar{Q}\gamma^\mu Q \rightarrow \bar{h}_{v'}\gamma^\mu h_v + \frac{1}{2m_Q}\bar{h}_{v'}\left(\gamma^\mu i\overrightarrow{D} - i\overleftarrow{D}\gamma^\mu\right)h_v$$

- $\mathcal{O}(1/m_Q)$ corrections arising from the **HQET subleading Lagrangian**

$$\langle M'|J(x)|M + \delta M\rangle \equiv \langle M'|J(x)|M\rangle + \frac{1}{2m_Q}\langle M'|i\int d^4x\, T\{J(x), \mathcal{L}_1(x)\}|M\rangle$$

$$\mathcal{L}_1 = \bar{h}_v(iD_\perp)^2 h_v + \frac{g}{2}\bar{h}_v\sigma_{\mu\nu}G^{\mu\nu}h_v$$

Expected Outcome:

- The resulting corrections are expected to be **numerically small**, consistent with HQET power counting.

- However, the framework allows for an **analytic test of Luke's theorem** within the effective theory.

At zero recoil $v.v' = 1$, the $1/m_Q$ corrections to hadronic matrix elements vanish

Discussion and Future Remarks

- Derived the effective heavy–light quark interaction within the QCD instanton vacuum framework

- Evaluated the weak decay constants and flavor-conserving radiative decays at zero recoil at leading order in 1/mQ; the results show good agreement with PDG averages

- Established a framework for constructing effective gluon operators in the QCD instanton vacuum for heavy–light quark system

- Demonstrated that instanton-induced configurations provide a microscopic, nonperturbative origin for gluonic corrections to heavy–hadron observables

- Extend calculations to higher-order corrections in $1/mQ$

- Explore further applications to heavy baryons and semileptonic transitions

- Provide a systematic comparison with other nonperturbative approaches such as lattice QCD and QCD sum rules.

Thank you for your attention!