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UNIVERSITY



Mauricio N. Ferreira

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Implications of the gluon mass gap in QCD observables

Dynamical mass generation in QCD

- At the level of the Lagrangian, QCD is a theory of (almost) massless fields only:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \bar{\psi}_f^i (i\gamma^\mu D_\mu - m_f)_{ij} \psi_f^j + \frac{1}{2\xi} (\partial^\mu A_\mu^a)^2 - \bar{c}^a \partial^\mu D_\mu^{ab} c^b$$

- Yet, hadrons are massive, and correspond to the vast majority of observable mass in the universe (see Craig's talk).

QCD generates its own mass, out of massless fields, though its nonperturbative dynamics.

M. N. F. and J. Papavassiliou, *Particles* **6**, no.1, 312-363 (2023).
M. Ding, C. D. Roberts and S. M. Schmidt, *Particles* **6**, 57-120 (2023).
D. Binosi, *Few Body Syst.* **63**, no.2, 42 (2022).

- In particular, the gluon self-interaction leads to a gluon mass gap, dynamically generated through the Schwinger mechanism.

J. M. Cornwall, *Phys. Rev. D* **26**, 1453 (1982).

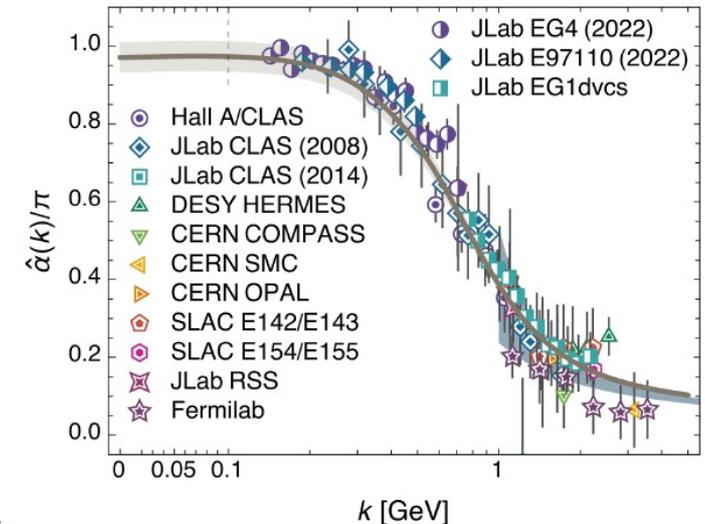
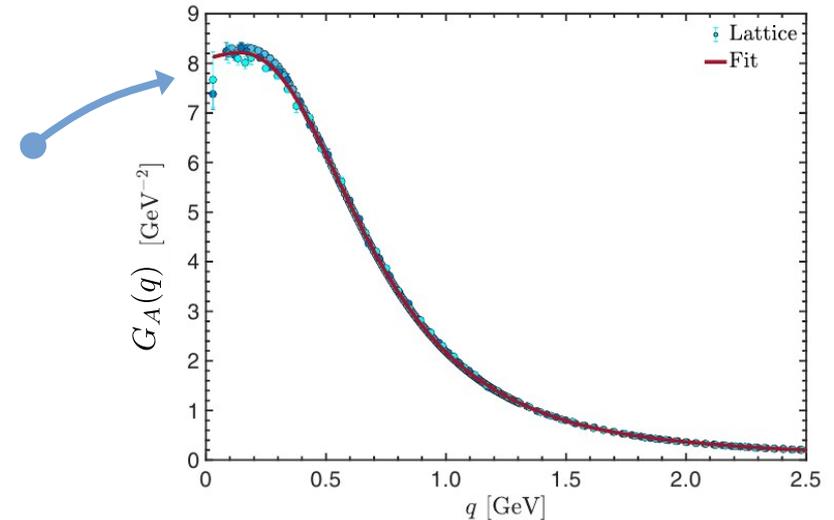
For a comprehensive review of the Schwinger mechanism, see M. N. F. and J. Papavassiliou, *Prog. Part. Nucl. Phys.* **144**, 104186 (2025).

Gluon propagator and its mass gap

The gluon mass scale is manifest in the infrared behavior of the gluon propagator, which saturates at the origin.

A. Cucchieri and T. Mendes, PoS **LATTICE2007**, 297 (2007).
I. L. Bogolubsky, et al, Phys. Lett. B **676**, 69-73 (2009).
A. C. Aguilar, et al, Phys. Rev. D **104**, no.5, 054028 (2021).

- **Unequivocal signal of gluon mass scale generation.**
- **Eliminates many infrared divergences** that are present at the perturbative level → QCD is well-defined in the infrared;
- Eliminates the Landau pole, leading to an **infrared finite and process independent effective charge.**
D. Binosi, et al, Phys. Rev. D **96**, no.5, 054026 (2017).
Z. F. Cui, et al, Chin. Phys. C **44**, no.8, 083102 (2020).
- One of the pillars of Emergent Hadron Mass (see Craig's talk).
M. Ding, C. D. Roberts and S. M. Schmidt, Particles **6**, 57-120 (2023).
C. D. Roberts, Symmetry **12**, no.9, 1468 (2020).
M. N. F. and J. Papavassiliou, Particles **6**, no.1, 312-363 (2023).



Defining the gluon mass gap

However, the **saturation value is gauge and renormalization point dependent**, and does not provide a measure of the gluon mass gap, m_{gap} .

One possibility to define the gluon mass gap, m_{gap} , is to identify it with the **“screening mass”** of the propagator:

m_{gap} is the real part of the location of the first (non-trivial) singularity of the propagator in the complex plane,

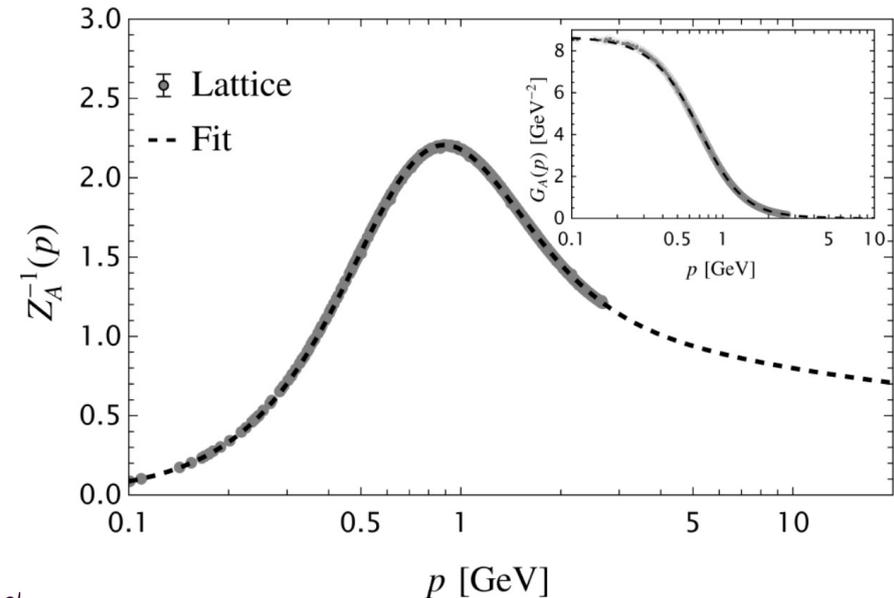
i.e.,

$$G_A(p) := \frac{1}{Z_A(p)} \frac{1}{p^2}, \quad m_{\text{gap}} = \omega_{\text{gap}}, \quad \omega_{\text{gap}} = (1 + i\gamma_{\text{gap}}) m_{\text{gap}},$$

$$\omega_{\text{gap}} = \min_{\omega_s \neq 0} \{ \text{Re } \omega_s \mid Z_A(-\omega_s^2) = 0 \}.$$

- This definition **generalizes** the notion of a **pole mass**, $1/(p^2 + m^2)$.
- Is **evidently RGI**.
- By Cauchy’s theorem, the screening mass is mapped to an exponential **suppression of long-range gluon modes** → decoupling of gluons at deep infrared.
- This suppression has **several physical connections**, some of which we will explore later.

First we have to ask, how can we extract the screening mass?



M. N. F., J. Papavassiliou, J. M. Pawłowski and N. Wink, 2508.20568, to appear in Eur. Phys. J. C.

Mauricio N. Ferreira ... mnferreira@nju.edu.cn ... 11/11/25 ... "Implications of the gluon mass gap in QCD observables"



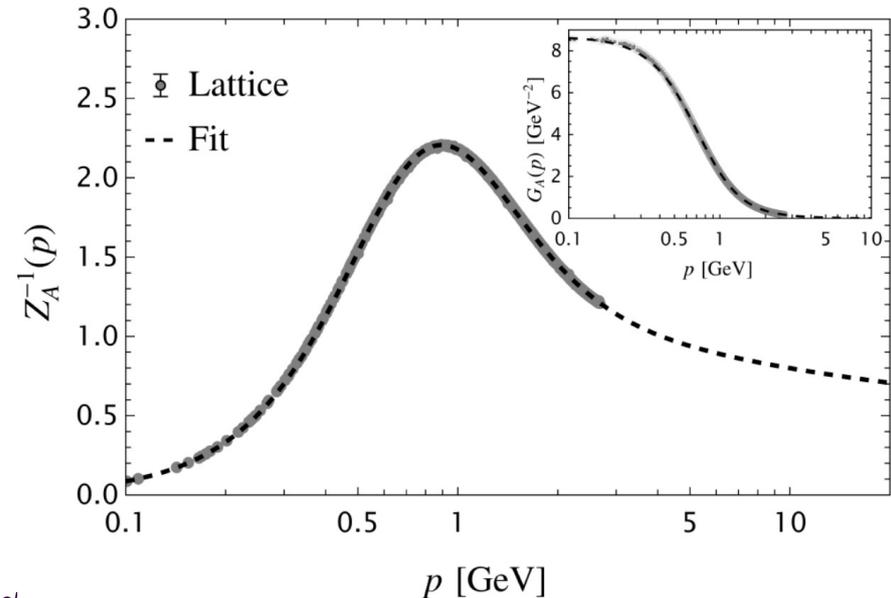
Extracting the screening mass

To extract m_{gap} , we fit Euclidean data with a functional form that incorporates the perturbative behavior in the UV, and the essentials of the Schwinger mechanism in the IR:

$$Z_{A,\text{fit}}(p) = Z_{A,\text{ir}}(x) + Z_{A,\text{uv}}(x), \quad x = p^2/m_{\text{gap}}^2,$$

$$Z_{A,\text{ir}}(x) = \frac{\left[z_{\text{sat}}/x - z_{\text{peak}} - z_{\text{gh}} \log\left(1 + \frac{1}{c_- x}\right) \right]}{(1 + c_+ x)^2},$$

$$Z_{A,\text{uv}}(x) = z_{\text{uv}} [1 + c_{\text{uv}} \log(1 + c_+ x)]^\gamma, \quad \gamma = \frac{13 - \frac{4}{3} N_f}{22 - \frac{4}{3} N_f}.$$



Extracting the screening mass

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The **singularity is obtained by directly complexifying the expression.**

- We find

$$m_{\text{gap}} = 686 \text{ MeV for } N_F = 0,$$

and

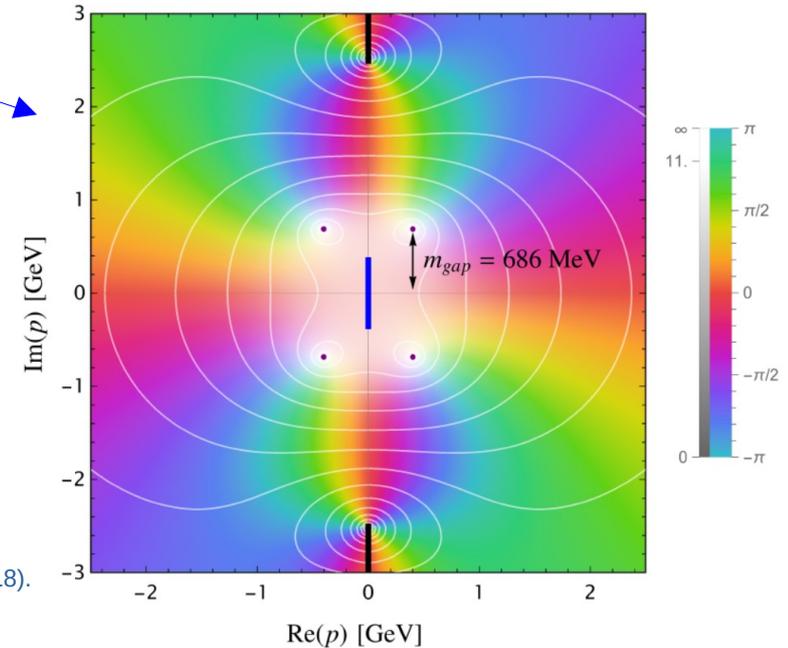
$$m_{\text{gap}} = 818 \text{ MeV for } N_F = 2 + 1,$$

M. N. F., J. Papavassiliou, J. M. Pawłowski and N. Wink, 2508.20568, to appear in Eur. Phys. J. C.

- Evidently, the detailed analytic structure depends heavily on the particular form of the fit, but the location of the first singularity is reliable.
- Other approaches, such as analytic cont. with the Schlessinger Point Method (SPM), and spectral reconstructions yield similar results ± 100 MeV.

D. Binosi and R. A. Tripolt, Phys. Lett. B **801**, 135171 (2020).

A. K. Cyrol, J. M. Pawłowski, A. Rothkopf and N. Wink, SciPost Phys. **5**, no.6, 065 (2018).



Confinement-deconfinement critical temperature and the gluon mass gap

The gluon mass gap is directly connected to the critical temperature, T_c , of the confinement-deconfinement phase transition.

In the continuum, the **Polyakov loop order parameter** can be related to the **Polyakov loop effective potential, directly sensitive to the gluon propagator**:

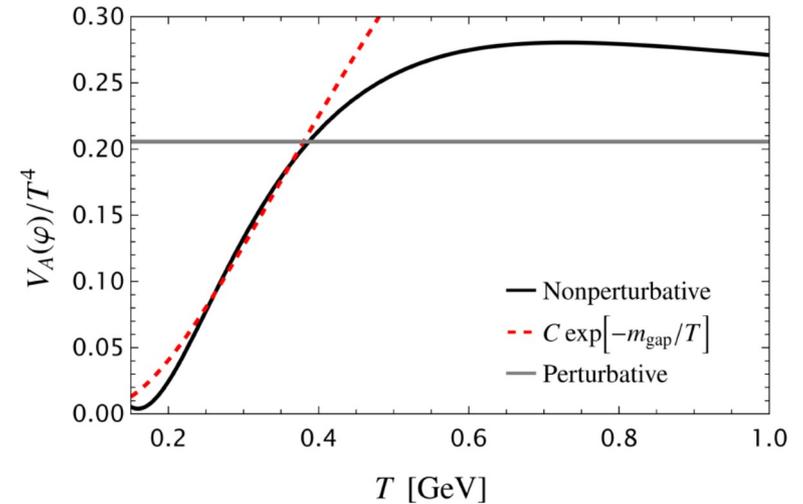
$$\frac{\delta(\Gamma - S)}{\delta A_0} = \frac{1}{2} \left[\text{Loop 1} - \text{Loop 2} \right] - \frac{1}{6} \left[\text{Loop 3} + \text{Loop 4} \right]$$

← For simplicity, we drop the 2-loop terms.

The key to relate T_c to m_{gap} is that:

- The **ghost contribution is confining**, whereas the **gluon is deconfining**.
- The gluon mass gap leads to an exponential suppression of the gluon contribution at low temperatures.
- The ghost remains massless and is not suppressed.
- At temperatures below a critical value, $T_c \propto m_{\text{gap}}$, the confining contribution of the ghost wins.
- We find $T_c = 275 \text{ MeV}$, which agrees with lattice for pure Yang-Mills SU(3).

J. Braun, H. Gies and J. M. Pawłowski, Phys. Lett. B **684**, 262-267 (2010).
L. Fister and J. M. Pawłowski, Phys. Rev. D **88**, 045010 (2013).



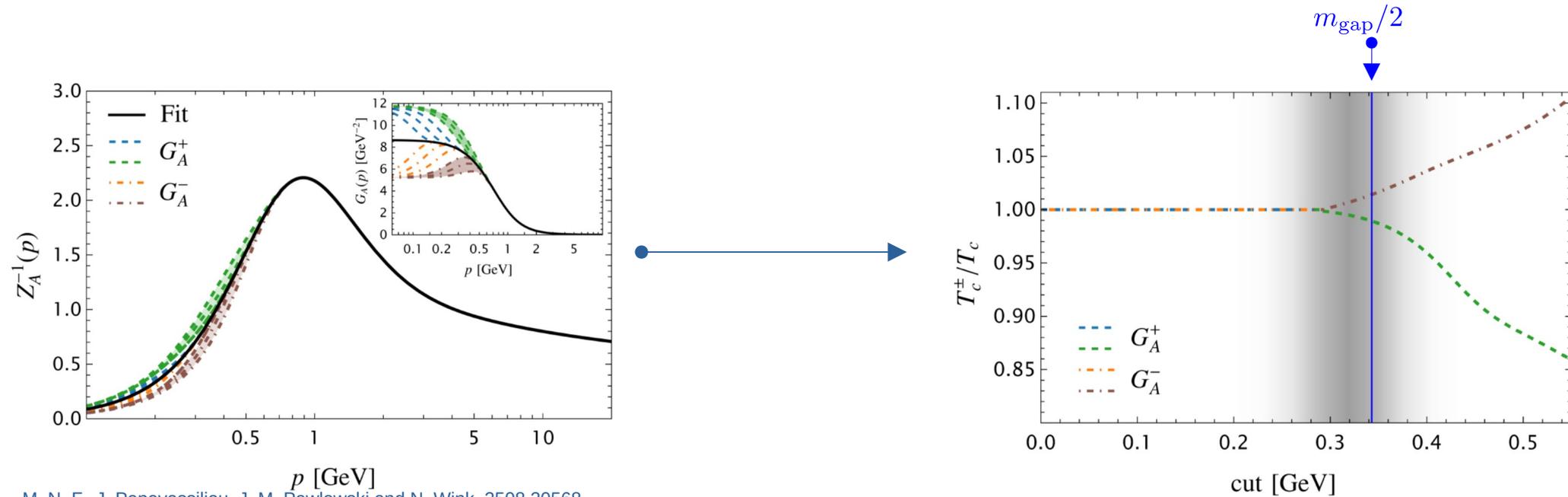
M. N. F., J. Papavassiliou, J. M. Pawłowski and N. Wink, 2508.20568, to appear in Eur. Phys. J. C.

Mauricio N. Ferreira ... mnferreira@nju.edu.cn ... 11/11/25 ... "Implications of the gluon mass gap in QCD observables"

Confinement-deconfinement critical temperature and the gluon mass gap

Moreover, the suppression of the gluon contribution implies that the details of the gluon propagator significantly below m_{gap} should not affect observables.

- We can test this notion by deforming the propagator by hand and seeing the effect on T_c .
- We find that T_c is insensitive to deformations of the propagator below $\approx m_{\text{gap}}/2$.



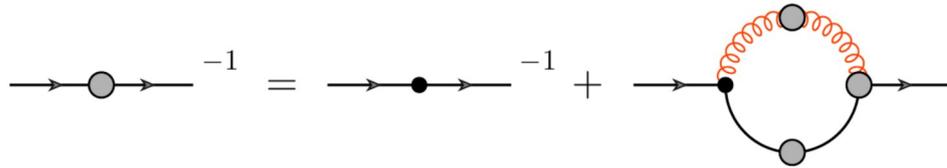
M. N. F., J. Papavassiliou, J. M. Pawłowski and N. Wink, 2508.20568, to appear in Eur. Phys. J. C.

Mauricio N. Ferreira ... mnferreira@nju.edu.cn ... 11/11/25 ... "Implications of the gluon mass gap in QCD observables"

Pion decay constant and the gluon mass gap

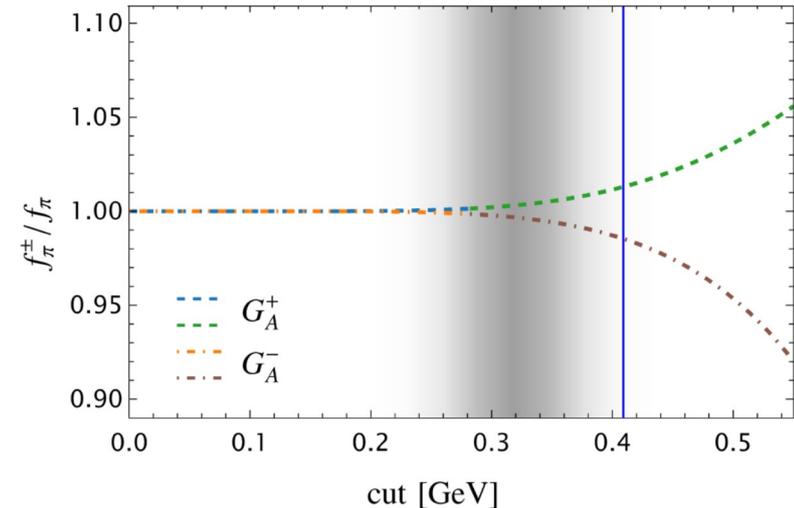
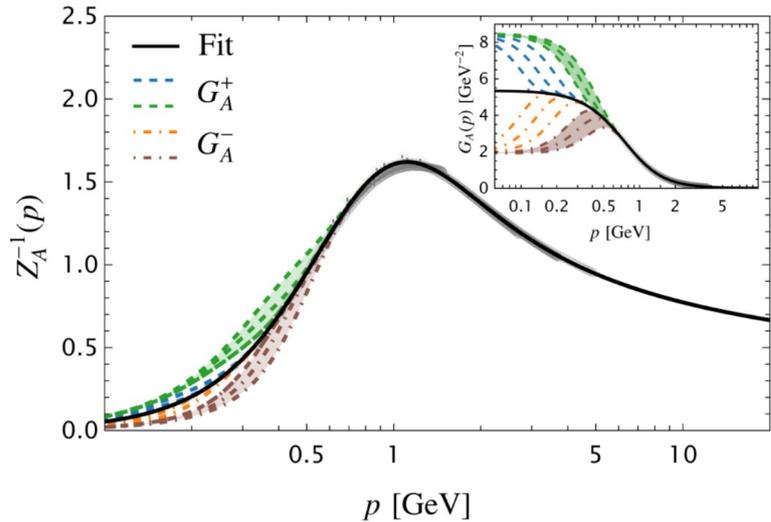
The same insensitivity to the infrared behavior is seen in the pion decay constant, f_π .

To see this, we solve the gap equation in the chiral limit, to compute the quark propagator, and compute f_π through the Pagels-Stokar formula.



$$f_\pi^2 = 4N_c \int \frac{d^4p}{(2\pi)^4} \frac{\bar{Z}_2}{Z_q(p)} \frac{M_q(p)}{[p^2 + M_q^2(p)]^2} \left[M_q(p) - \frac{p^2}{2} M_q'(p) \right].$$

Then we deform the gluon propagator in the same way done in the Yang-Mills case, and see that f_π is insensitive to deformations in the deep IR.



Conclusions

- **Gluon self-interactions** generate a **gluon mass gap through the Schwinger mechanism**, manifest by the **saturation of the gluon propagator at the origin**.
- A physically meaningful definition of the gluon mass gap is provided by the **screening mass**, defined from the first nontrivial singularity of the gluon propagator in the complex plane, which yields

$$m_{\text{gap}} = 686 \text{ MeV for } N_F = 0 ,$$

$$m_{\text{gap}} = 818 \text{ MeV for } N_F = 2 + 1 .$$

- Suppresses the gluon contribution to the Polyakov loop effective potential, **directly relating the gluon mass gap to confinement**.
- It also implies that observables should be insensitive to the deep infrared behavior of the gluon.
- We show this explicitly for two key QCD observables: the **confinement-deconfinement critical temperature** and the **pion decay constant are insensitive**.

Backup slides

Defining the gluon mass gap

However, the **saturation value is gauge and renormalization point dependent**, and does not provide a measure of the gluon mass gap, m_{gap} .

We seek a **Renormalization Group Invariant (RGI) definition of m_{gap}** .

The gluon dressing function, $Z_A(p)$,

$$G_A(p) := \frac{1}{Z_A(p)} \frac{1}{p^2},$$

offers three RGI points that serve as proxies for m_{gap} .

(i) An ultraviolet inflection point at $p_{\text{in}}^+ \approx 1.3 \text{ GeV}$, above which perturbation theory starts to take in;

(ii) A maximum at $p_{\text{peak}} = 900 \text{ MeV}$, at a scale known to be characteristic of the Schwinger mechanism;

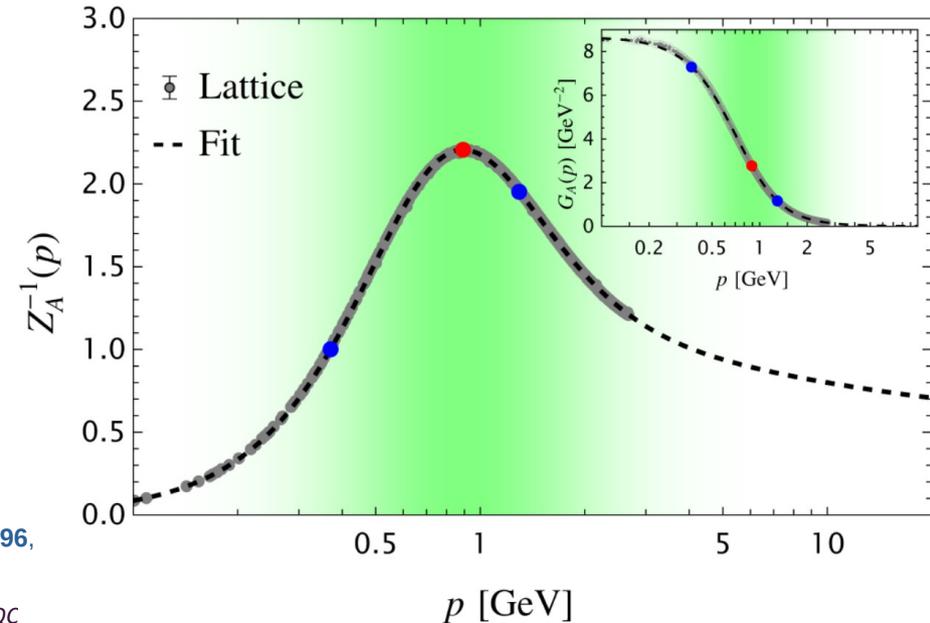
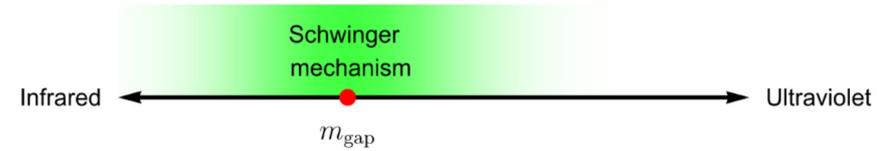
M. N. F. and J. Papavassiliou, Prog. Part. Nucl. Phys. **144**, 104186 (2025).

(iii) An infrared inflection point at $p_{\text{in}}^- \approx 370 \text{ MeV}$, which we will argue to be a decoupling scale.

These scales indicate: $0.4 \text{ GeV} \lesssim m_{\text{gap}} \lesssim 1.3 \text{ GeV}$.

- An inflection point of the Process Independent effective charge also leads to a gluon mass scale in this range.

D. Binosi, C. Mezrag, J. Papavassiliou, C. D. Roberts and J. Rodriguez-Quintero, Phys. Rev. D **96**, no.5, 054026 (2017).



Chiral symmetry breaking and the gluon mass gap

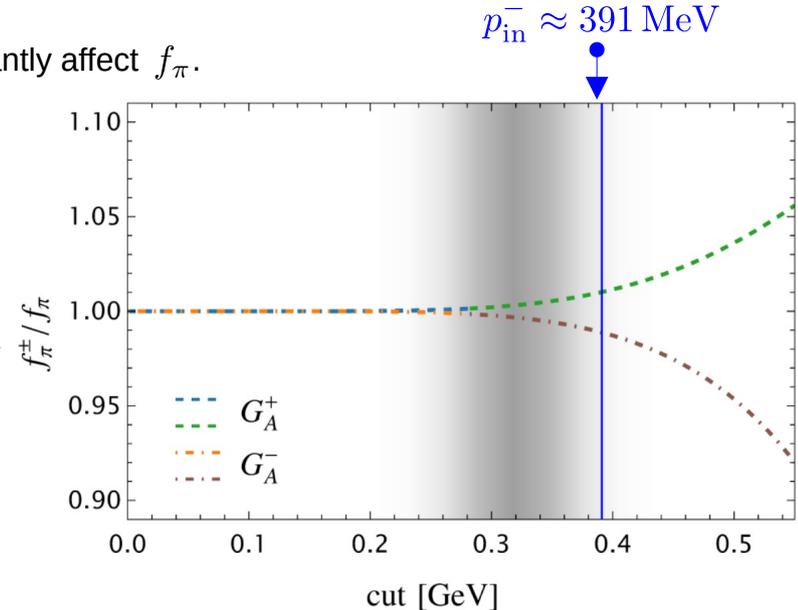
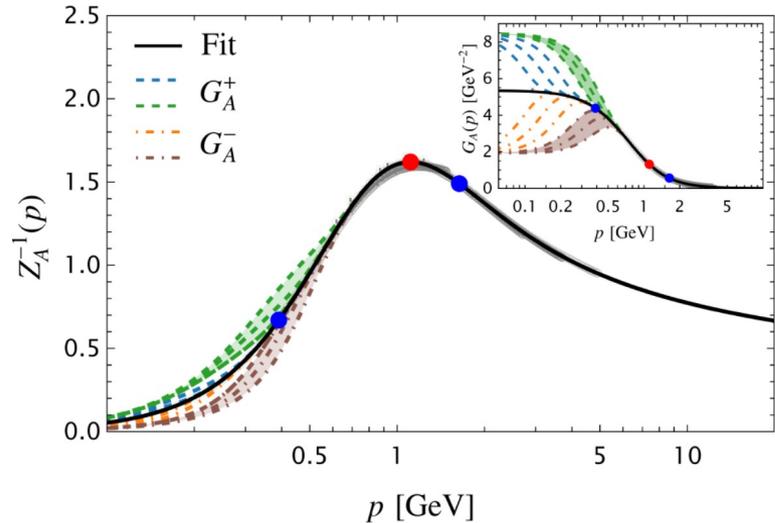
The second observable we consider is the pion decay constant, f_π .

The quark mass function, although RGI, is not an observable. Instead, we compute f_π . To this end we use the Pagels-Stokar formula:

$$f_\pi^2 = 4N_c \int \frac{d^4p}{(2\pi)^4} \frac{\bar{Z}_2}{Z_q(p)} \frac{M_q(p)}{[p^2 + M_q^2(p)]^2} \left[M_q(p) - \frac{p^2}{2} M_q'(p) \right].$$

For the central curve, we obtain $f_\pi = 93.2 \text{ MeV}$.

Again, deformations of the gluon propagator below the IR inflection point do not significantly affect f_π .



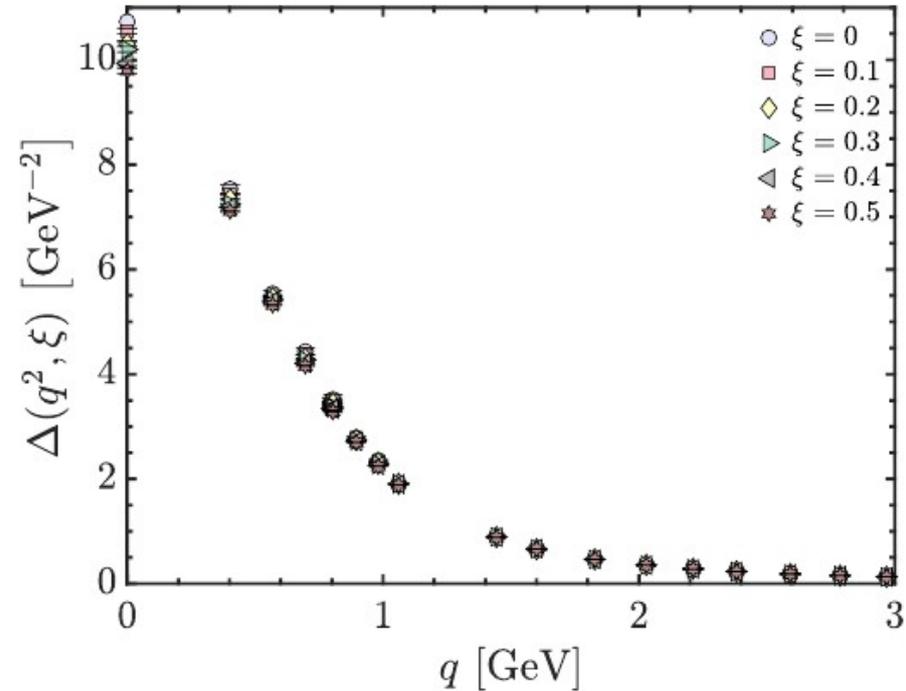
Gluon propagator and its mass gap

Gluon self-interaction can dynamically generate a mass gap.

J. M. Cornwall, Phys. Rev. D26, 1453 (1982).

The same is observed in linear covariant gauges

P. Bicudo, D. Binosi, N. Cardoso, O. Oliveira and P. J. Silva, Phys. Rev. D 92, no.11, 114514 (2015).



Gluon propagator and its mass gap

Gluon self-interaction can dynamically generate a mass gap.

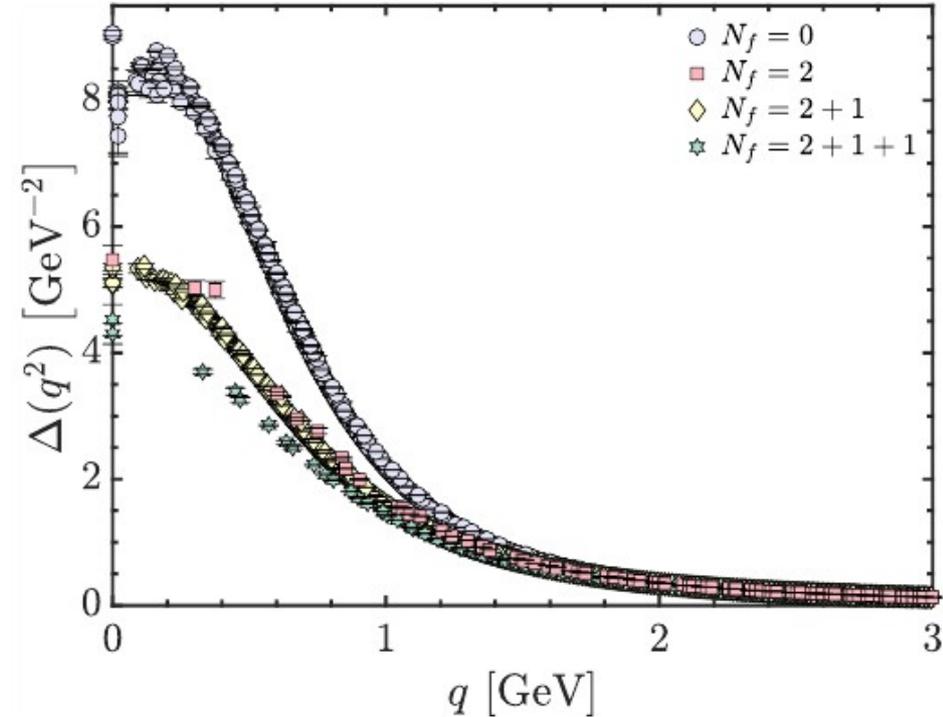
J. M. Cornwall, Phys. Rev. D26, 1453 (1982).

And with or without dynamical quarks

A. Ayala, A. Bashir, D. Binosi, M. Cristoforetti and J. Rodriguez-Quintero, Phys. Rev. D **86**, 074512 (2012).

D. Binosi, C. D. Roberts and J. Rodriguez-Quintero, Phys. Rev. D **95**, no.11, 114009 (2017).

A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, J. Rodríguez-Quintero and S. Zafeiropoulos, Eur. Phys. J. C **80**, no.2, 154 (2020).



Gluon mass generation mechanism must be driven by gauge sector dynamics: **truly, mass from nothing**

We can focus on pure Yang-Mills theory

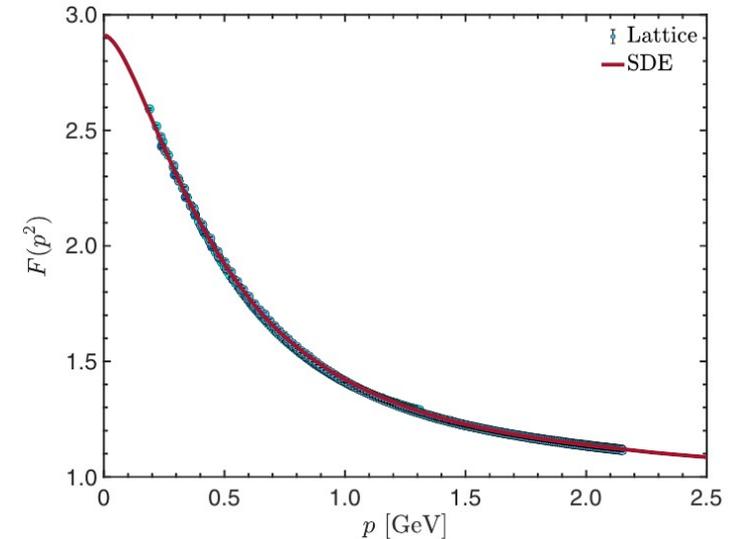
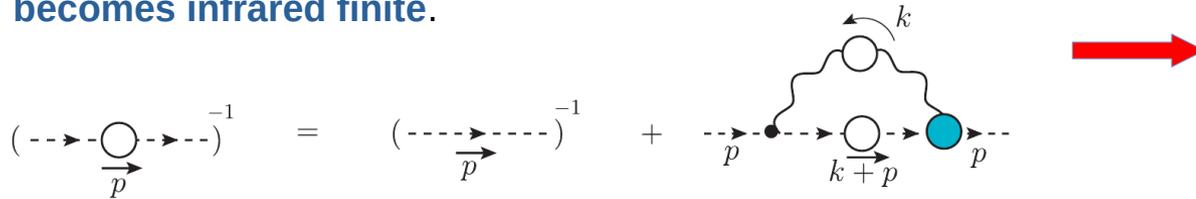
Implications

The generation of a gluon mass gap leaves distinctive imprints in other Schwinger functions. For example:

- The **ghost propagator**, $D(q^2)$, **remains massless**.
- But its **dressing function**, $F(q^2)$, given by

$$D(q^2) = \frac{iF(q^2)}{q^2}$$

becomes infrared finite.



A. Cucchieri and T. Mendes, PoS **LATTICE2007**, 297 (2007).

A. C. Aguilar, D. Binosi and J. Papavassiliou, Phys. Rev. D **78**, 025010 (2008).

P. Boucaud, J. P. Leroy, A. Le Yaouanc, J. Micheli, O. Pene and J. Rodriguez-Quintero, JHEP **06**, 099 (2008).

I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker and A. Sternbeck, Phys. Lett. B **676**, 69-73 (2009).

A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawłowski, N. Strodthoff, Phys. Rev. D **94**, 054005 (2016).

M. Q. Huber, Phys. Rept. **879**, 1-92 (2020).

A. C. Aguilar, C. O. Ambrosio, F. De Soto, M. N. F., B. M. Oliveira, J. Papavassiliou, J. Rodriguez-Quintero, Phys. Rev. D **104** no.5, 054028, (2021).

Implications

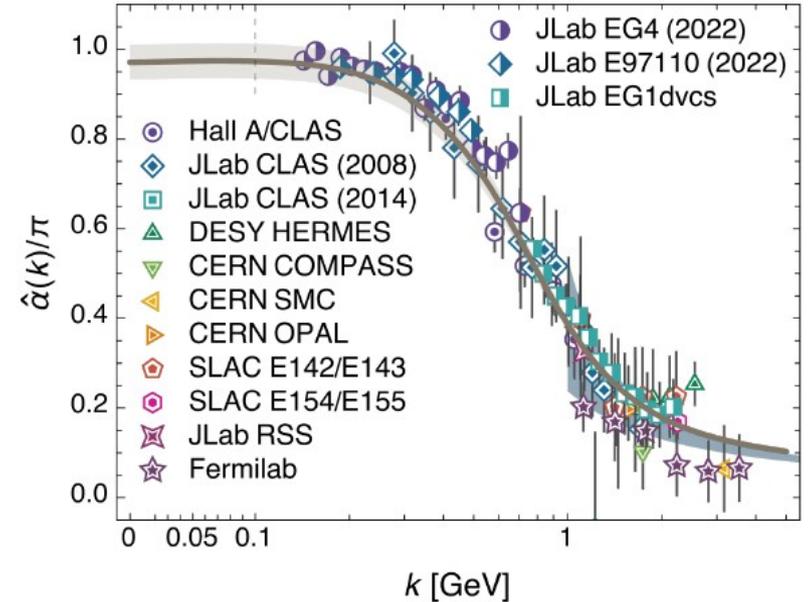
Together, the infrared finiteness of $\Delta(q^2)$ and $F(q^2)$ imply that:

- There is **no Landau pole in QCD**;
- QCD is a well-defined theory in the infrared.
- Leads to the construction of a **Renormalization Group Invariant** and **Process Independent effective charge**, $\hat{\alpha}(q^2)$, analogous to the Gell-Mann Low charge of QED.

D. Binosi, C. Mezrag, J. Papavassiliou, C. D. Roberts and J. Rodríguez-Quintero, Phys. Rev. D **96**, no.5, 054026 (2017).
Z. F. Cui, et al, Chin. Phys. C **44**, no.8, 083102 (2020).

- $\hat{\alpha}(q^2)$ is a key ingredient in modern studies of the hadron structure, as discussed in many talks in this conference.

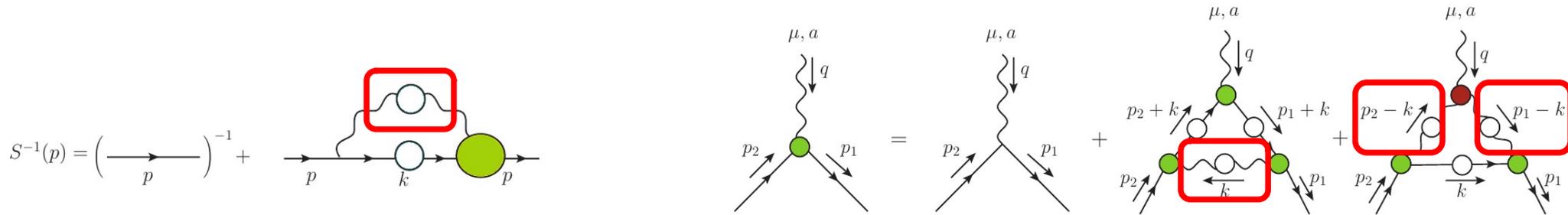
M. Ding, C. D. Roberts and S. M. Schmidt, Particles **6**, 57-120 (2023).



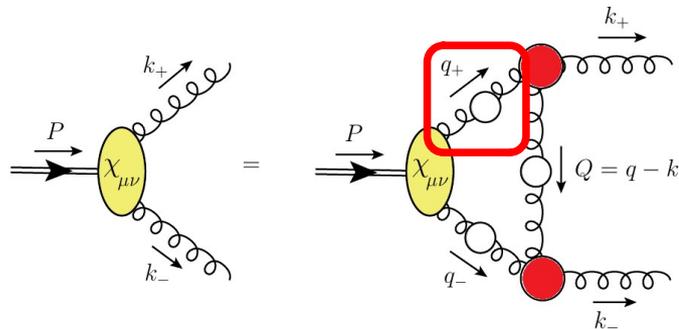
Implications

The gluon propagator appears everywhere in Dyson-Schwinger equations, Bethe-Salpeter equations, etc... Obvious examples:

1) The quark gap equation and the DSE for the quark-gluon vertex.



2) BSE equation for glueballs, the simplest of which is the pseudo-scalar



J. Meyers and E. S. Swanson, Phys. Rev. D **87**, no.3, 036009 (2013).
 S. S. Xu, Z. F. Cui, L. Chang, J. Papavassiliou, C. D. Roberts and H. S. Zong, Eur. Phys. J. A **55**, no.7, 113 (2019).
 E. V. Souza, M. N. F., A. C. Aguilar, J. Papavassiliou, C. D. Roberts, S.-S. Xu, Eur. Phys. J. A **56**, no.1, 25 (2020).
 M. Q. Huber, C. S. Fischer and H. Sanchis-Alepuz, Eur. Phys. J. C **80**, no.11, 1077 (2020).
 J. M. Pawłowski, C. S. Schneider, J. Turnwald, J. M. Urban and N. Wink, Phys. Rev. D **108**, no.7, 076018 (2023).

Origin of the gluon mass gap

Together, these results for the gauge sector Schwinger functions provide a solid framework for practical calculations. But this leaves us with a **question**:

How can the gluon acquire a mass gap?

- Gauge symmetry must be explicitly preserved;
- No associated mass term, $m^2 A^2$, in Lagrangian;
- No elementary scalar field for a Higgs mechanism.

Answer:

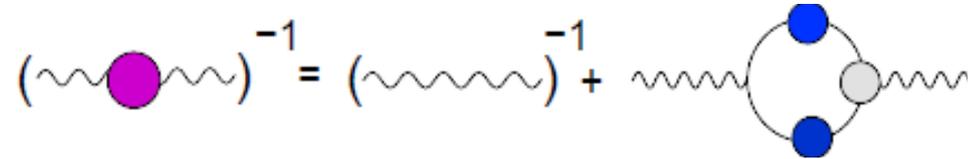
Through the Schwinger mechanism

Schwinger mechanism

A gauge boson may acquire mass, dynamically and without violating gauge symmetry if its vacuum polarization function develops a pole at zero momentum transfer.

J. S. Schwinger, Phys. Rev. **125**, 397 (1962); Phys. Rev. **128**, 2425 (1962).

Dyson-Schwinger equation for gauge boson propagator


$$\left(\text{wavy line with pink circle} \right)^{-1} = \left(\text{wavy line} \right)^{-1} + \text{wavy line with loop}$$

$$\Delta^{-1}(q^2) = q^2 [1 + \Pi(q^2)]$$

If, for some reason

$$\lim_{q \rightarrow 0} \Pi(q^2) = \frac{c}{q^2}, \quad c > 0$$

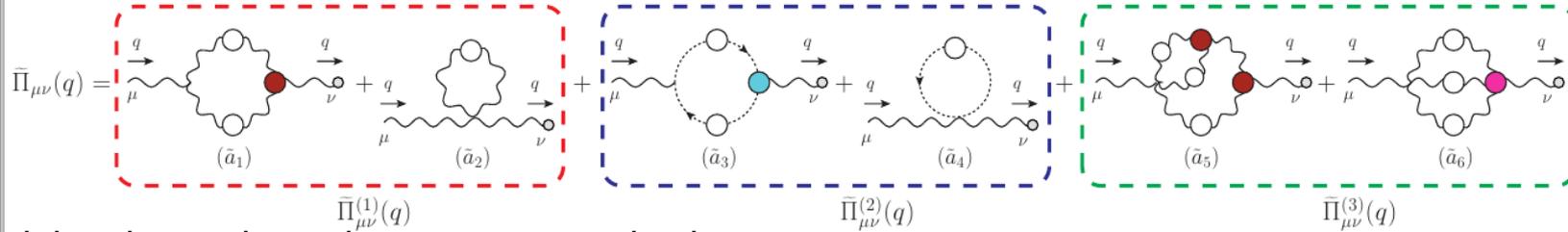
$$\Delta^{-1}(0) = c > 0$$

But how can the vacuum polarization acquire such a pole?

Vertex poles

From the gluon Schwinger-Dyson equation (in the PT-BFM scheme),

D. Binosi. and J. Papavassiliou, Phys. Rept. **479**, 1-152 (2009).



it has been shown in numerous works that:

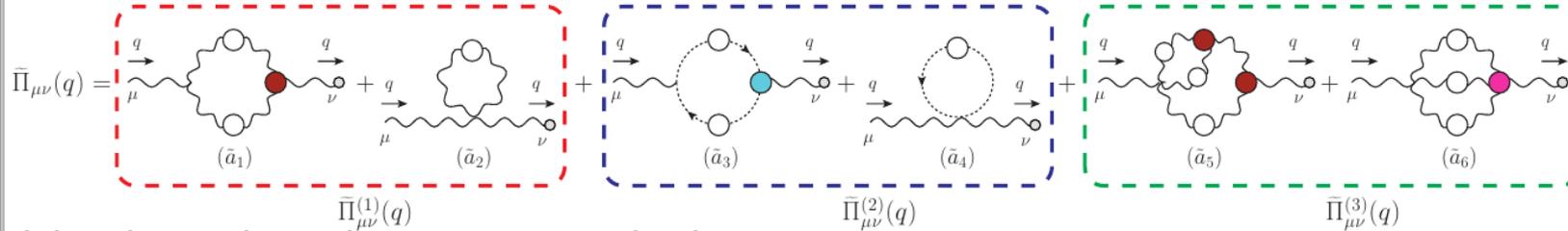
Pole in the vacuum polarization hinges on the existence of poles at zero momentum in the vertices = Schwinger poles

A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
 A. C. Aguilar and J. Papavassiliou, Phys. Rev. D **81**, 034003 (2010).
 A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
 A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawłowski, N. Strodthoff, Phys. Rev. D **94**, 054005 (2016).
 A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
 G. Eichmann, J. M. Pawłowski and J. M. Silva, Phys. Rev. D **104**, no.11, 114016 (2021).
 M. N. F. and J. Papavassiliou, arXiv:2501.01080.

Vertex poles

From the gluon Schwinger-Dyson equation, (in the PT-BFM scheme),

D. Binosi. and J. Papavassiliou, Phys. Rept. **479**, 1-152 (2009).



it has been shown in numerous works that:

Pole in the vacuum polarization hinges on the existence of poles at zero momentum in the vertices = Schwinger poles

- In the absence of such poles, the Ward identities of theory enforce diagrammatic cancellations culminating in

$$\int d^d k \frac{\partial \mathcal{F}_\mu(k)}{\partial k_\mu} = 0$$

Seagull identity

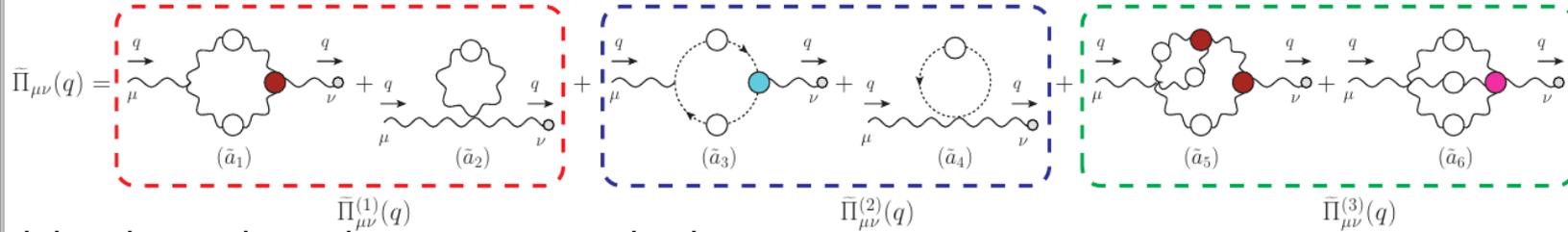
A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
 A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
 M. N. F. and J. Papavassiliou, arXiv:2501.01080.

- It's this seagull cancellation that enforces the masslessness of the photon.

Vertex poles

From the gluon Schwinger-Dyson equation (in the PT-BFM scheme),

D. Binosi. and J. Papavassiliou, Phys. Rept. **479**, 1-152 (2009).



it has been shown in numerous works that:

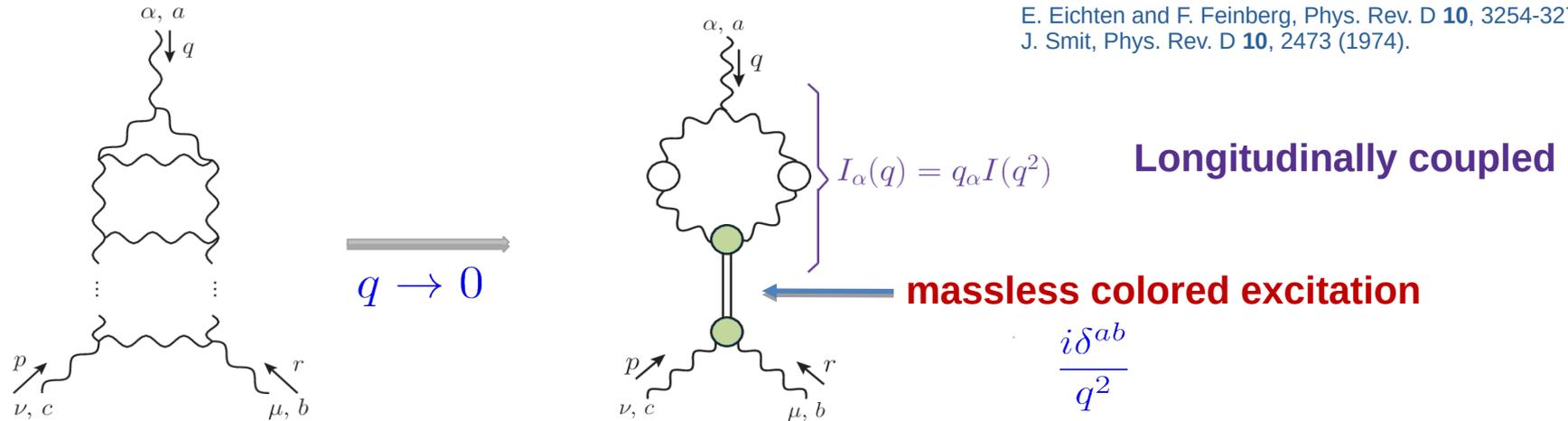
Pole in the vacuum polarization hinges on the existence of poles at zero momentum in the vertices = Schwinger poles

How can the vertices acquire Schwinger poles?

Massless bound state formalism

If the interaction is sufficiently strong \longrightarrow formation of **massless bound states**

E. Eichten and F. Feinberg, Phys. Rev. D **10**, 3254-3279 (1974).
J. Smit, Phys. Rev. D **10**, 2473 (1974).



Vertices of the theory acquire **longitudinally coupled poles at zero gluon momentum, e.g.:**

$$\Pi_{\alpha\mu\nu}(q, r, k) = \underbrace{\Gamma_{\alpha\mu\nu}(q, r, k)}_{\text{pole-free}} + \underbrace{\frac{q_\alpha}{q^2} g_{\mu\nu} 2(q \cdot r) \mathbb{C}(r^2)}_{\text{Schwinger pole}} + \dots$$

Residue function

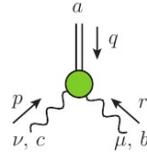
- A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
- D. Ibañez and J. Papavassiliou, Phys. Rev. D **87**, no.3, 034008 (2013).
- D. Binosi and J. Papavassiliou, Phys. Rev. D **97**, no.5, 054029 (2018).
- M. N. F. and J. Papavassiliou, Eur. Phys. J. C **84**, no.8, 835 (2024).

Mauricio N. Ferreira ... mnferreira@nju.edu.cn ... 11/11/25 ... "Implications of the gluon mass gap in QCD observables"

Bethe-Salpeter equation

The formation of a massless bound state is **dynamical and governed by a Bethe-Salpeter equation**.

At $q=0$, the scalar-gluon-gluon amplitude



is characterized by a single scalar form factor,

which is determined by the Bethe-Salpeter equation

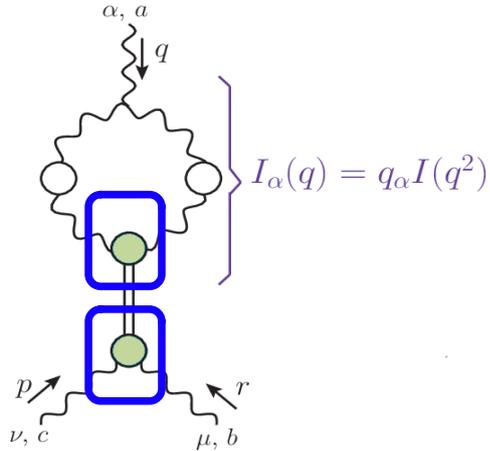
$$= t^{-1} \times \left[\text{Loop Diagram} \right]$$

where t is a constant that depends on $\mathbb{B}(r^2)$.

A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
 A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).
 M. N. F. and J. Papavassiliou, Eur. Phys. J. C **84**, no.8, 835 (2024).

Bethe-Salpeter equation

Then, the residue function $\mathbb{C}(r^2)$ is determined in terms of $\mathbb{B}(r^2)$



The residue function is proportional to the scalar-gluon-gluon vertex form factor

$$\mathbb{C}(r^2) = -I\mathbb{B}(r^2)$$

- The proportionality factor is the scalar-gluon transition amplitude, $I=I(0)$.
- Note that, since I is linear in $\mathbb{B}(r^2)$, then $\mathbb{C}(r^2)$ is quadratic

The sign of the residue function is fixed, turns out < 0

M. N. F. and J. Papavassiliou, Eur. Phys. J. C **84**, no.8, 835 (2024).

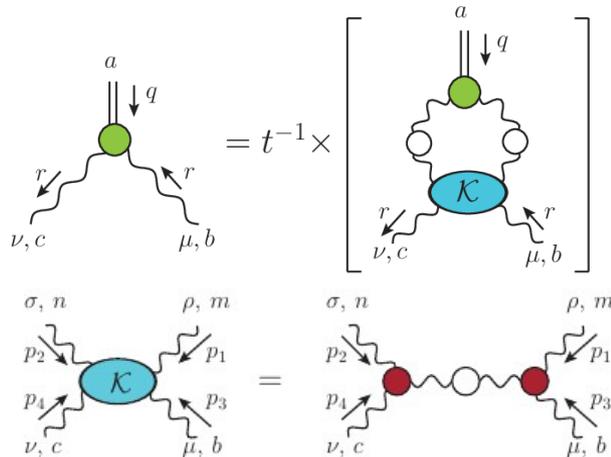
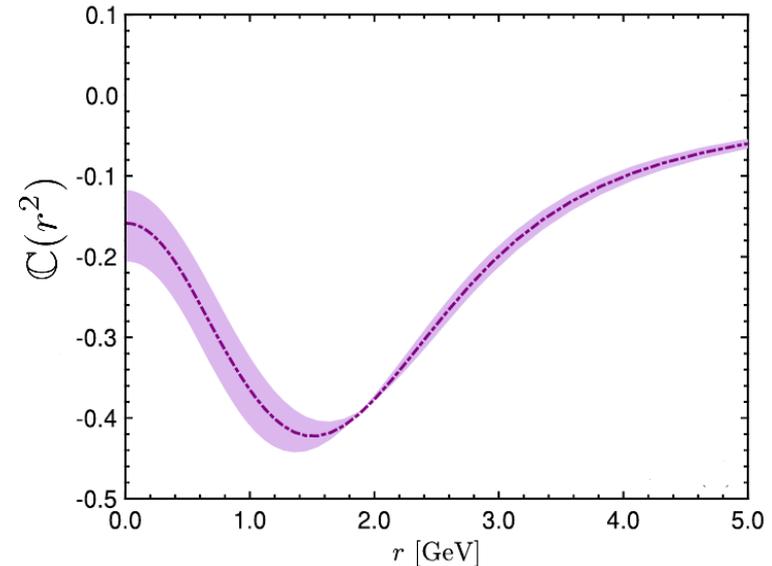
Bethe-Salpeter equation

With a "one-gluon exchange truncation", the BS equation admits **nontrivial solutions compatible with lattice ingredients** for the:

- Propagator;
- Vertex;
- and, value of the coupling $\alpha_s \approx 0.3$ @ $\mu = 4.3$ GeV

A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
 D. Binosi and J. Papavassiliou, Phys. Rev. D **97**, no.5, 054029 (2018).
 A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Eur. Phys. J. C **78**, no.3, 181 (2018).
 M. N. F. and J. Papavassiliou, Eur. Phys. J. C **84**, no.8, 835 (2024).

Residue function



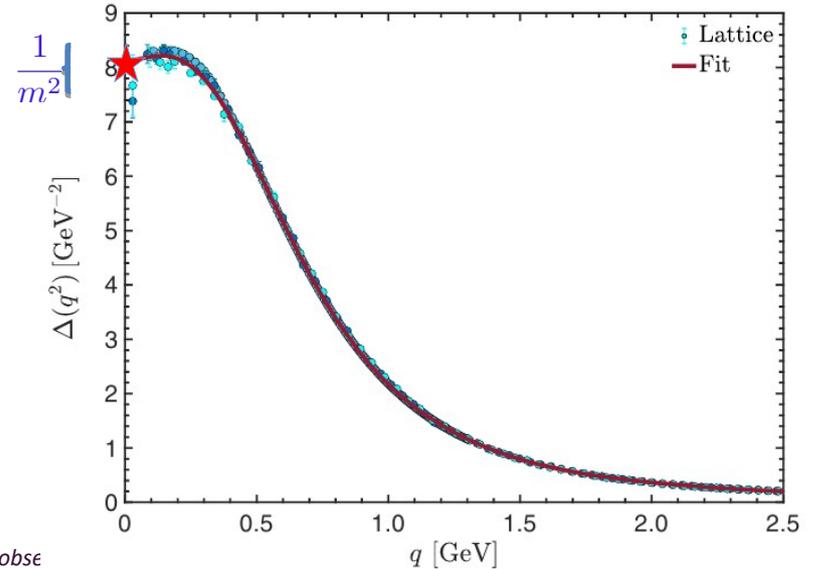
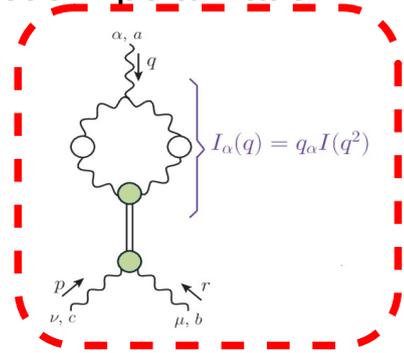
Gluon mass from Schwinger poles

The massless poles in the three-gluon vertex lead to a pole in the gluon vacuum polarization:

$$\Delta^{-1}(q^2) = q^2 + \mu, a \xrightarrow{q} \text{loop} \xrightarrow{q} \nu, b + \dots$$

$q \rightarrow 0$

$$m^2 = \underbrace{\mu, a \xrightarrow{q} \text{loop}}_{I_\mu(q)} \underbrace{\frac{i}{q^2}}_{\text{pole}} \underbrace{\text{loop} \xrightarrow{q} \nu, b}_{I_\nu(-q)}$$



- Quadratic expression guarantees $m^2 > 0$.

Renormalization

- Importantly, the **gluon mass generated by the Schwinger mechanism does not require a mass counterterm** for renormalization, which would break gauge symmetry.
- Instead, multiplicative renormalization is enforced by the standard renormalization constants, Z .

$$\Delta_R(q^2) = Z_A^{-1} \Delta(q^2),$$

$$\Gamma_R^\mu(q, p, r) = Z_1 \Gamma^\mu(q, p, r),$$

$$D_R(q^2) = Z_c^{-1} D(q^2),$$

$$\Gamma_R^{\alpha\mu\nu}(q, r, p) = Z_3 \Gamma^{\alpha\mu\nu}(q, r, p),$$

$$g_R = Z_g^{-1} g,$$

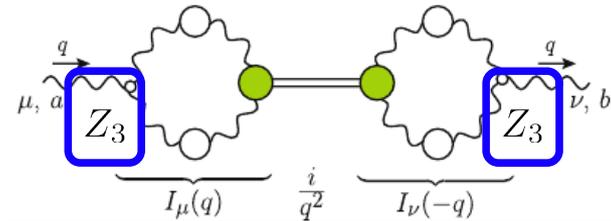
$$\Gamma_{R\alpha\beta\mu\nu}^{abcd}(q, r, p, t) = Z_4 \Gamma_{\alpha\beta\mu\nu}^{abcd}(q, r, p, t),$$

- Invoking the STIs for the renormalization constants, only that of the three-gluon vertex, Z_3 , is left.

$$Z_g^{-1} = Z_1^{-1} Z_A^{1/2} Z_c = Z_3^{-1} Z_A^{3/2} = Z_4^{-1/2} Z_A,$$



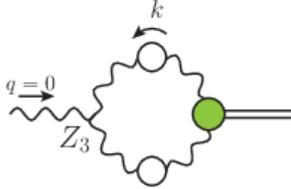
$$m^2 =$$



- It suffices to consider the renormalization of the scalar-gluon transition amplitude, I

Renormalization

Starting from the diagram for I , we "solve" for Z_3 to eliminate it as follows:

$$4gI_R = \text{diagram}$$


Renormalization

Starting from the diagram for I , we "solve" for Z_3 to eliminate it as follows:

$$4gI_R = \left[\text{Diagram with } Z_3 \text{ and } q=0 \right] = \left[\text{Diagram with } q=0 \text{ and } \mathcal{K} \right] \text{ Diagram with } k$$

DSE for regular part of the vertex

$$Z_3 = \text{Diagram with } \mathcal{K} \text{ and } q \text{ and } \alpha, a \text{ and } \nu, c \text{ and } \mu, b \text{ and } p \text{ and } r$$

Renormalization

Starting from the diagram for I , we "solve" for Z_3 to eliminate it as follows:

$$\begin{aligned}
 4gI_R &= \left[\text{Diagram with } Z_3 \text{ and } q=0 \right] \\
 &= \left[\text{Diagram with } q=0 \text{ and } \mathcal{K} \right] \\
 &= \left[\text{Diagram with } q=0 \text{ and } \mathcal{K} \right]
 \end{aligned}$$

DSE for regular part of the vertex

$$Z_3 = \left[\text{Diagram with } \alpha, a, q, p, r, \nu, c, \mu, b \right] - \left[\text{Diagram with } \mathcal{K} \text{ and } \alpha, a, q, p, r, \nu, c, \mu, b \right]$$

BS equation for $\mathbb{B}(r^2)$:

$$\left[\text{Diagram with } a, q, r, \nu, c, \mu, b \right] = t^{-1} \times \left[\text{Diagram with } \mathcal{K} \text{ and } a, q, r, \nu, c, \mu, b \right]$$

Renormalization

Starting from the diagram for I , we "solve" for Z_3 to eliminate it as follows:

$$\begin{aligned}
 4gI_R &= \text{Diagram with } Z_3 \text{ loop and tree-level vertex} \\
 &= \left[\text{Diagram with dressed vertex} - \text{Diagram with } \mathcal{K} \text{ loop and dressed vertex} \right] \text{Diagram with tree-level vertex} \\
 &= \text{Diagram with dressed vertex} \left[\text{Diagram with tree-level vertex} - \text{Diagram with } \mathcal{K} \text{ loop and tree-level vertex} \right] \\
 &= (1-t) \times \left[\text{Diagram with } Z_3 \text{ loop and tree-level vertex} \right]
 \end{aligned}$$

The Z_3 and the tree-level vertex are substituted by the dressed vertex and a **finite constant**.

Schwinger poles in lattice results?

Now, the **lattice can also compute the three-gluon vertex**. Can we see longitudinal poles in it?

Unfortunately, no!

The Schwinger **poles are longitudinally coupled**

$$\mathbb{\Gamma}_{\alpha\mu\nu}(q, r, k) = \underbrace{\Gamma_{\alpha\mu\nu}(q, r, k)}_{\text{pole-free}} + \underbrace{\frac{q_\alpha}{q^2} g_{\mu\nu} 2(q \cdot r) \mathbb{C}(r^2)}_{\text{Schwinger pole}} + \dots$$

But **lattice simulations only access transverse tensor structures**.



Lattice extracts the pole-free part of the vertex.

A smoking gun signal?

Question:

Is there a smoking-gun signal of the massless bound state poles, which can be tested with lattice inputs?

Answer:

Yes, the displacement of the Ward identities satisfied by the vertices.

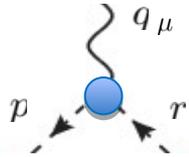
- The key observation is that the **Schwinger mechanism preserves the gauge symmetry**.
- If there is a massless bound state pole, the **propagators and pole-free parts of the vertices must change in shape to accommodate the pole contribution to the Ward identities**.

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

A toy example: scalar QED

Schwinger mechanism **off**

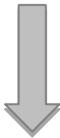


Ward-Takahashi identity

$$q^\mu \Gamma_\mu(q, r, p) = D^{-1}(p^2) - D^{-1}(r^2)$$

pole-free

$$q \rightarrow 0 \\ p \rightarrow -r$$



Taylor expansion

Textbook Ward identity

$$\Gamma_\mu(0, r, -r) = \frac{\partial D^{-1}(r^2)}{\partial r^\mu}$$

Schwinger mechanism **on**

$$\mathbb{\Gamma}_\mu(q, r, p) = \underbrace{\Gamma_\mu(q, r, p)}_{\text{pole-free}} + \frac{q^\mu}{q^2} C(q, r, p)$$

The Ward-Takahashi identity does **not** change

$$q^\mu \mathbb{\Gamma}_\mu(q, r, p) = q^\mu \Gamma_\mu(q, r, p) + C(q, r, p) \\ = D^{-1}(p^2) - D^{-1}(r^2)$$

$$q \rightarrow 0$$



Taylor expansion

Displaced Ward identity

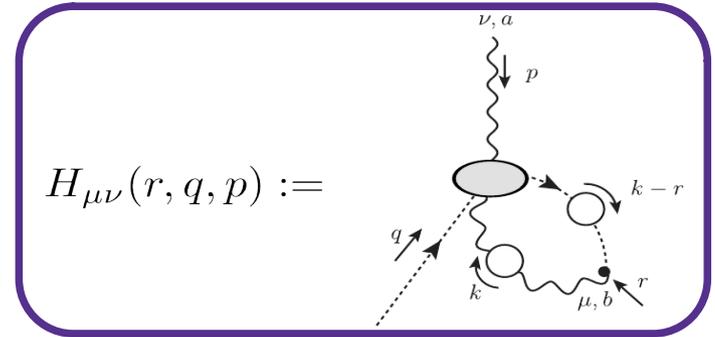
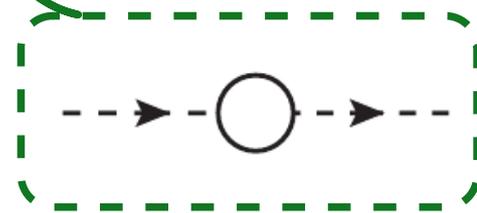
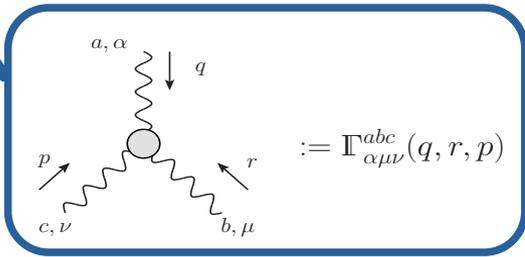
$$\underbrace{\Gamma_\mu(0, r, -r)}_{\text{pole-free}} = \frac{\partial D^{-1}(r^2)}{\partial r^\mu} - 2r_\mu \underbrace{\left[\frac{\partial C(q, r, p)}{\partial p^2} \right]_{q=0}}_{C(r^2)}$$

Displacement = Residue function

Ward identity displacement in QCD

The **same idea applies to QCD**, just more complicated due to **non-Abelian Slavnov-Taylor identities**:

$$q^\alpha \Pi_{\alpha\mu\nu}(q, r, p) = F(q^2) [\Delta^{-1}(p^2) P_\nu^\sigma(p) H_{\sigma\mu}(p, q, r) - \Delta^{-1}(r^2) P_\mu^\sigma(r) H_{\sigma\nu}(r, q, p)]$$



Then, assume the three-gluon vertex has a massless bound state pole:

$$\Pi_{\alpha\mu\nu}(q, r, k) = \Gamma_{\alpha\mu\nu}(q, r, k) + \frac{q_\alpha}{q^2} g_{\mu\nu} 2(q \cdot r) \mathbb{C}(r^2) + \dots$$

And expand around $q = 0$

Ward identity displacement in QCD

$$q^\alpha \Pi_{\alpha\mu\nu}(q, r, p) = F(q^2) [\Delta^{-1}(p^2) P_\nu^\sigma(p) H_{\sigma\mu}(p, q, r) - \Delta^{-1}(r^2) P_\mu^\sigma(r) H_{\sigma\nu}(r, q, p)]$$

$q \rightarrow 0$  Isolate classical tensor structure

Ward identity

$$L_{\text{sg}}(r^2) = F(0) \left[\frac{\mathcal{W}(r^2)}{r^2} \Delta^{-1}(r^2) + \frac{\partial \Delta^{-1}(r^2)}{\partial r^2} \right] + \mathbb{C}(r^2)$$

Displacement = Residue function

- ★ **Ingredients can (mostly) be computed with lattice simulations.**
- ★ **Combine ingredients and determine if there is a nontrivial displacement.**

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

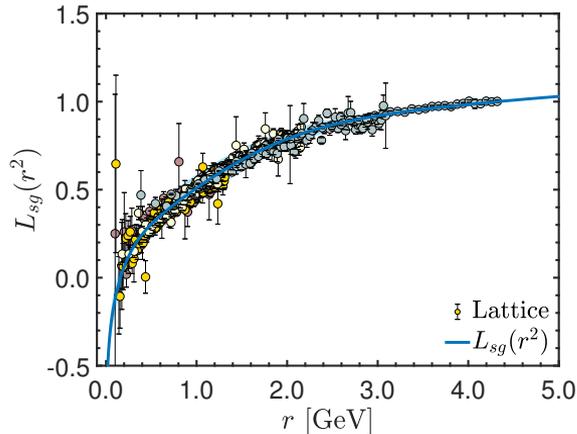
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

Ward identity displacement in QCD

$$q^\alpha \mathbb{\Gamma}_{\alpha\mu\nu}(q, r, p) = F(q^2) [\Delta^{-1}(p^2) P_\nu^\sigma(p) H_{\sigma\mu}(p, q, r) - \Delta^{-1}(r^2) P_\mu^\sigma(r) H_{\sigma\nu}(r, q, p)]$$

$q \rightarrow 0$  Isolate classical tensor structure
Ward identity

$$L_{sg}(r^2) = F(0) \left[\frac{\mathcal{W}(r^2)}{r^2} \Delta^{-1}(r^2) + \frac{\partial \Delta^{-1}(r^2)}{\partial r^2} \right] + \mathbb{C}(r^2)$$



Soft-gluon form factor of the three-gluon vertex

$$P_{\mu}^{\mu'}(r) P_{\nu}^{\nu'}(r) \mathbb{\Gamma}_{\alpha\mu'\nu'}(0, r, -r) = 2L_{sg}(r^2) r_{\alpha} P_{\mu\nu}(r)$$

$$P_{\mu\nu}(q) := g_{\mu\nu} - q_{\mu} q_{\nu} / q^2$$

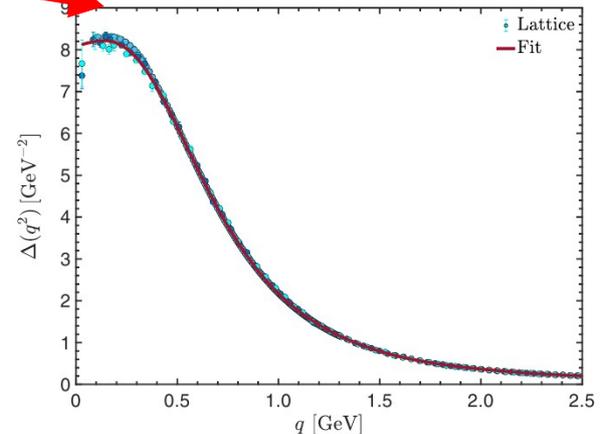
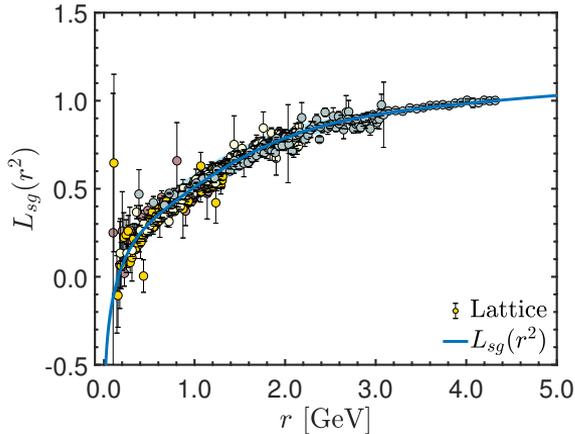
A. C. Aguilar, C. O. Ambrosio, F. De Soto, M.N. F., B. M. Oliveira, J. Papavassiliou and J. Rodriguez-Quintero, Phys. Rev. D 104 no.5, 054028, (2021).

Ward identity displacement in QCD

$$q^\alpha \Pi_{\alpha\mu\nu}(q, r, p) = F(q^2) [\Delta^{-1}(p^2) P_\nu^\sigma(p) H_{\sigma\mu}(p, q, r) - \Delta^{-1}(r^2) P_\mu^\sigma(r) H_{\sigma\nu}(r, q, p)]$$

$q \rightarrow 0$  Isolate classical tensor structure
Ward identity

$$L_{sg}(r^2) = F(0) \left[\frac{\mathcal{W}(r^2)}{r^2} \Delta^{-1}(r^2) + \frac{\partial \Delta^{-1}(r^2)}{\partial r^2} \right] + \mathbb{C}(r^2)$$

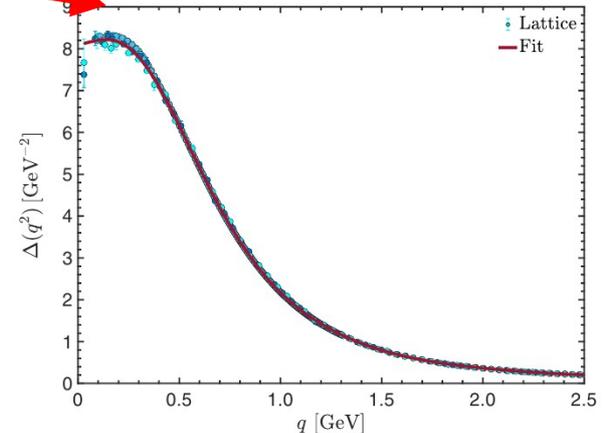
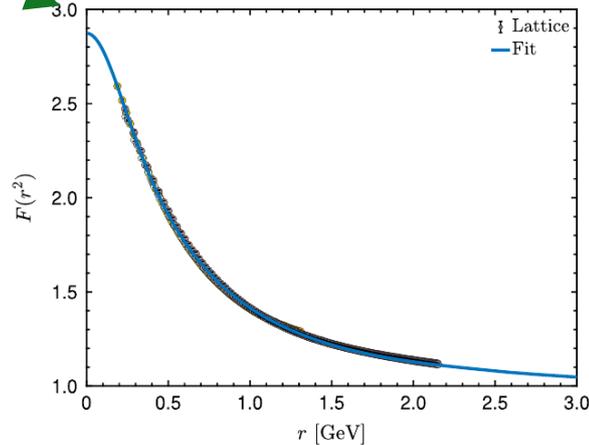
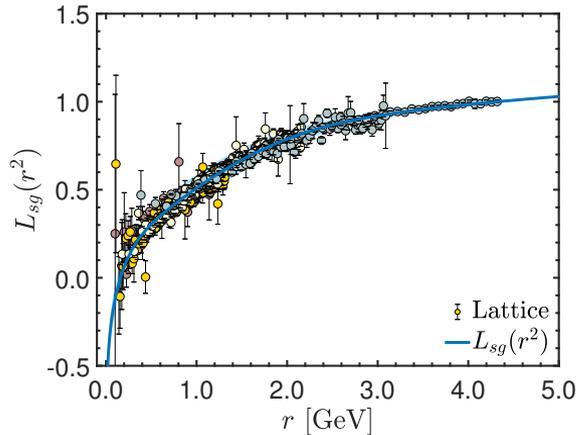


Ward identity displacement in QCD

$$q^\alpha \Pi_{\alpha\mu\nu}(q, r, p) = F(q^2) [\Delta^{-1}(p^2) P_\nu^\sigma(p) H_{\sigma\mu}(p, q, r) - \Delta^{-1}(r^2) P_\mu^\sigma(r) H_{\sigma\nu}(r, q, p)]$$

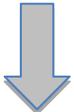
$q \rightarrow 0$  Isolate classical tensor structure
Ward identity

$$L_{sg}(r^2) = F(0) \left[\frac{\mathcal{W}(r^2)}{r^2} \Delta^{-1}(r^2) + \frac{\partial \Delta^{-1}(r^2)}{\partial r^2} \right] + \mathbb{C}(r^2)$$



Ward identity displacement in QCD

$$q^\alpha \Pi_{\alpha\mu\nu}(q, r, p) = F(q^2) [\Delta^{-1}(p^2) P_\nu^\sigma(p) H_{\sigma\mu}(p, q, r) - \Delta^{-1}(r^2) P_\mu^\sigma(r) H_{\sigma\nu}(r, q, p)]$$

$q \rightarrow 0$  Isolate classical tensor structure

Ward identity

$$L_{\text{sg}}(r^2) = F(0) \left[\frac{\mathcal{W}(r^2)}{r^2} \Delta^{-1}(r^2) + \frac{\partial \Delta^{-1}(r^2)}{\partial r^2} \right] + \mathbb{C}(r^2)$$

★ Only one ingredient not yet determined directly by lattice simulations.

Ward identity displacement in QCD

$$q^\alpha \mathbb{\Gamma}_{\alpha\mu\nu}(q, r, p) = F(q^2) [\Delta^{-1}(p^2) P_\nu^\sigma(p) H_{\sigma\mu}(p, q, r) - \Delta^{-1}(r^2) P_\mu^\sigma(r) H_{\sigma\nu}(r, q, p)]$$

$q \rightarrow 0$  Isolate classical tensor structure
Ward identity

$$L_{\text{sg}}(r^2) = F(0) \left[\frac{\mathcal{W}(r^2)}{r^2} \Delta^{-1}(r^2) + \frac{\partial \Delta^{-1}(r^2)}{\partial r^2} \right] + \mathbb{C}(r^2)$$

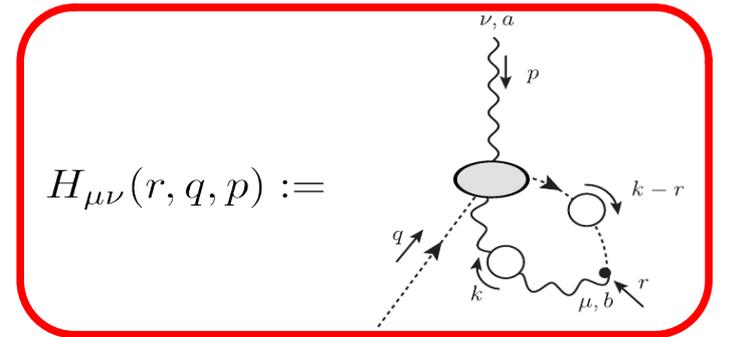
Partial derivative of the ghost-gluon kernel:

$$\mathcal{W}(r^2) = -\frac{1}{3} r^\alpha P^{\mu\nu}(r) \left[\frac{\partial H_{\nu\mu}(r, q, p)}{\partial q^\alpha} \right]_{q=0}$$

- **In principle, computable on the lattice, but not currently available.**

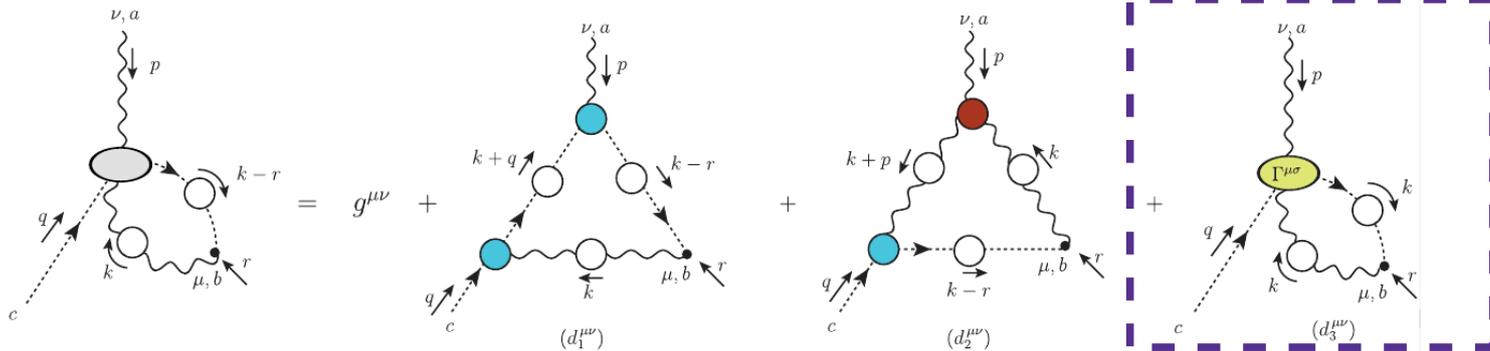
A. C. Aguilar, M. N. F. and J. Papavassiliou, Eur. Phys. J. C **81**, no.1, 54 (2021).

- **Resort to a lattice-driven SDE analysis.**



Lattice driven Schwinger-Dyson calculation

The $\mathcal{W}(r^2)$ can be obtained from the Schwinger-Dyson equation for the **ghost-gluon scattering kernel**

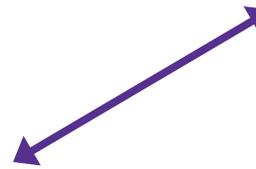


A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

(2% effect). M. Q. Huber, Eur. Phys. J. C **77**, 733 (2017).

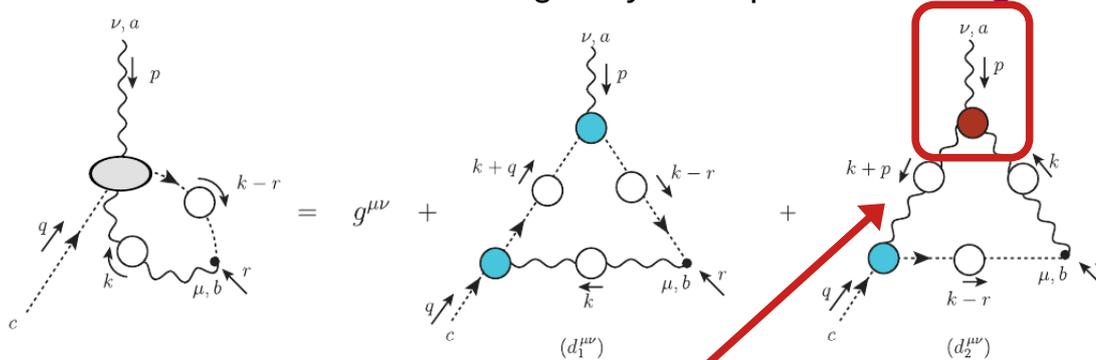
Depends on:

- 1) **Gluon and ghost propagators.**
- 2) **Four-point function probably subleading. Will be omitted.**



Lattice driven Schwinger-Dyson calculation

The $\mathcal{W}(r^2)$ can be obtained from the Schwinger-Dyson equation for the **ghost-gluon scattering kernel**



A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Depends on:

- 1) **Gluon and ghost propagators.**
- 2) **Four-point function probably subleading. Will be omitted.**
- 3) **General kinematics three-gluon vertex.**

By now well-determined by continuum and lattice studies (see Feliciano's talk).

G. Eichmann, R. Williams, R. Alkofer, M. Vujanovic, Phys. Rev. D **89**, 105014 (2014).

A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawłowski, N. Strodthoff, Phys. Rev. D **94**, 054005 (2016).

M. Q. Huber, Phys. Rev. D **101**, 114009 (2020).

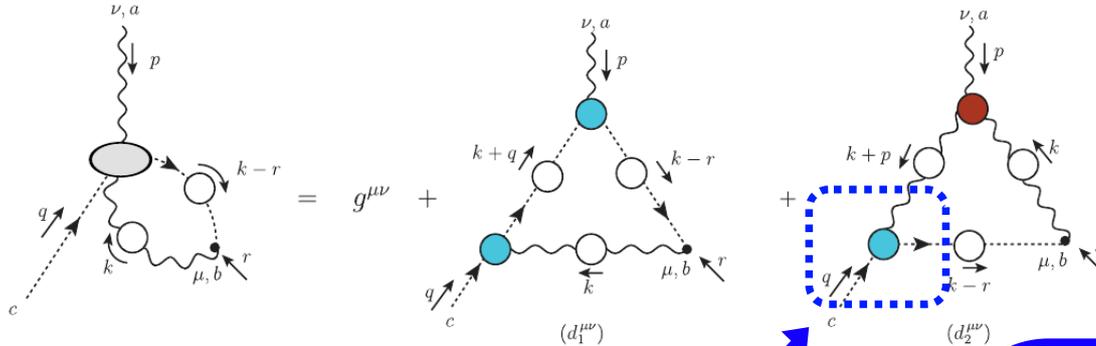
F. Pinto-Gómez, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B **838**, 137737 (2023).

A. C. Aguilar, M. N. F., J. Papavassiliou and L. R. Santos, Eur. Phys. J. C **83**, no.6, 549 (2023).

F. Pinto-Gómez, F. De Soto and J. Rodríguez-Quintero, Phys. Rev. D **110**, no.1, 014005 (2024).

Lattice driven Schwinger-Dyson calculation

The $\mathcal{W}(r^2)$ can be obtained from the Schwinger-Dyson equation for the **ghost-gluon scattering kernel**



A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Depends on:

- 1) **Gluon and ghost propagators.**
- 2) **Four-point function probably subleading. Will be omitted.**
- 3) **General kinematics three-gluon vertex.**
- 4) **General kinematics ghost-gluon vertex;**

Determined self-consistently through same SDE plus STI:

$$\Pi_\nu(r, q, p) = r^\mu H_{\mu\nu}(r, q, p)$$

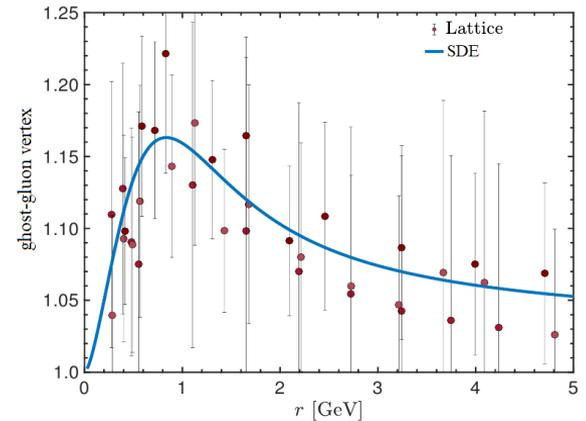
M. Q. Huber and L. von Smekal, JHEP 04, 149 (2013).

A. K. Cyrol, L. Fister, M. Mitter, et al. Phys. Rev. D 94, 054005 (2016).

A. C. Aguilar, C. O. Ambrósio, F. De Soto, M. N. F., et. al, Phys. Rev. D 104, no.5, 054028 (2021).

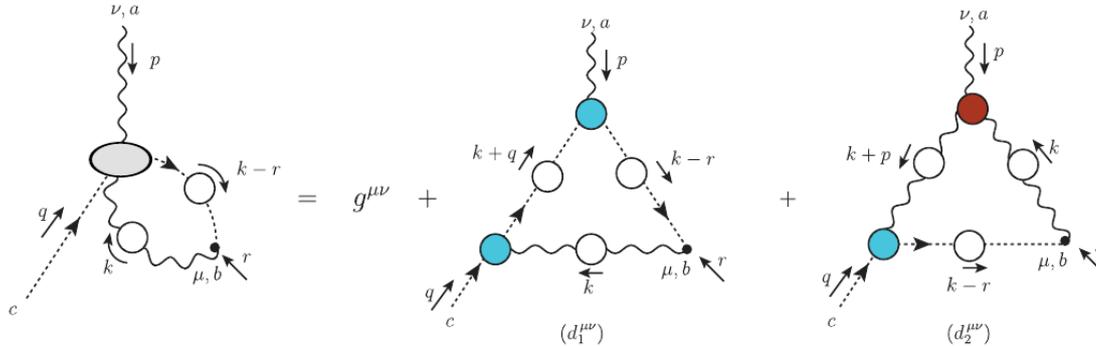
Reproduces available SU(3) lattice results:

E. -M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck, et al. Braz. J. Phys. 37, 193 (2007).



Lattice driven Schwinger-Dyson calculation

The $\mathcal{W}(r^2)$ can be obtained from the Schwinger-Dyson equation for the **ghost-gluon scattering kernel**



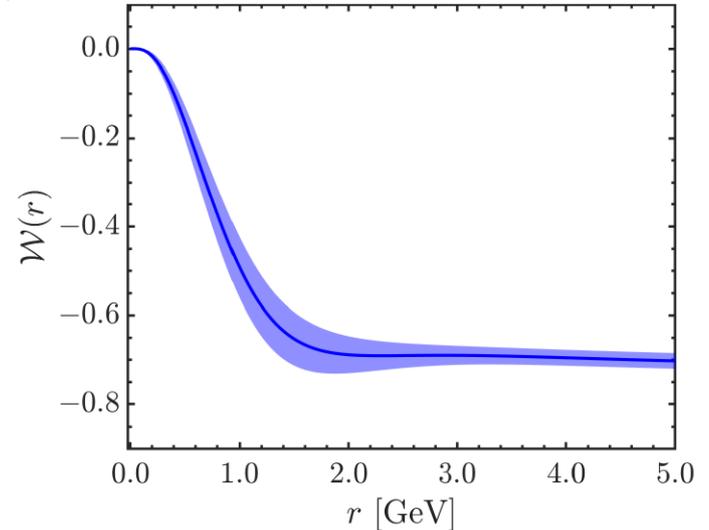
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Depends on:

- 1) **Gluon and ghost propagators.**
- 2) **Four-point function probably subleading. Will be omitted.**
- 3) **General kinematics three-gluon vertex.**
- 4) **General kinematics ghost-gluon vertex;**

With these ingredients at hand, we compute $\mathcal{W}(r^2)$

A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

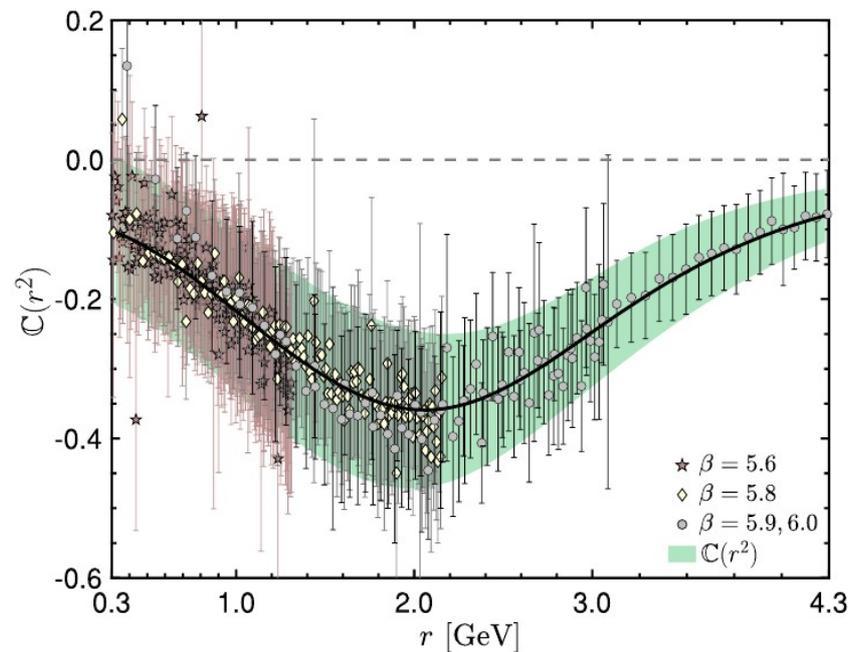
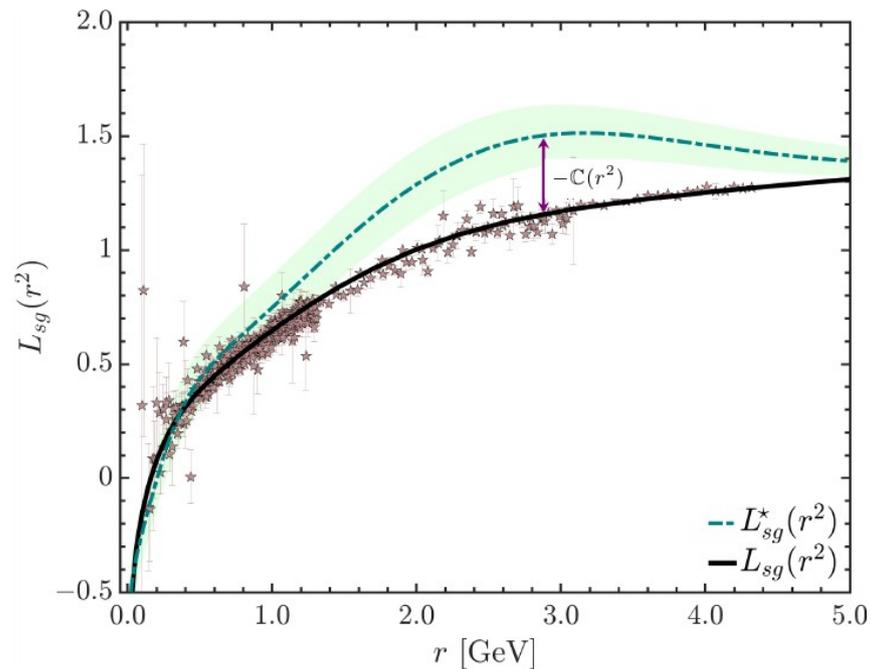


Results for $\mathbb{C}(r^2)$

With $\mathcal{W}(r^2)$ at hand, we can compute $L_{sg}^*(r^2)$ and determine $\mathbb{C}(r^2)$ as a **WI displacement**

$$L_{sg}^*(r^2) = F(0) \left[\frac{\mathcal{W}(r^2)}{r^2} \Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2} \right]$$

$$\mathbb{C}(r^2) = L_{sg}(r^2) - L_{sg}^*(r^2)$$



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Mauricio N. Ferreira ... mnferreira@nju.edu.cn ... 11/11/25 ... "Implications of the gluon mass gap in QCD observables"

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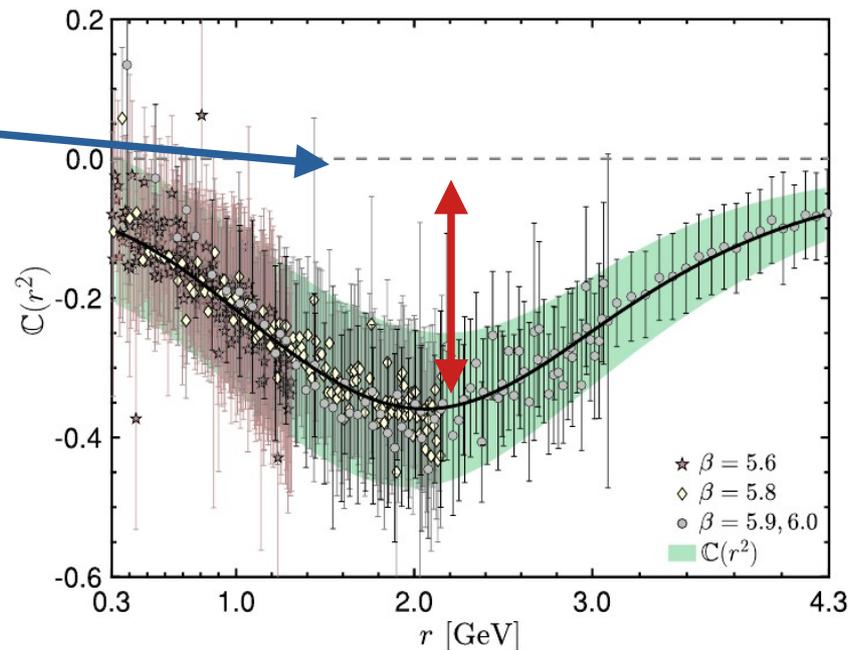
- $\mathbb{C}(r^2)$ obtained is clearly nonzero.
- Define the **null hypothesis**,

$$\mathbb{C}(r^2) = \mathbb{C}_0 := 0$$

p-value of null hypothesis is tiny:

$$P_{\mathbb{C}_0} = \int_{\chi^2=2630}^{\infty} \chi_{\text{PDF}}^2(515, x) dx = 7.3 \times 10^{-280}$$

- Even if the errors were doubled, the null hypothesis would still be discarded at the 5σ level.



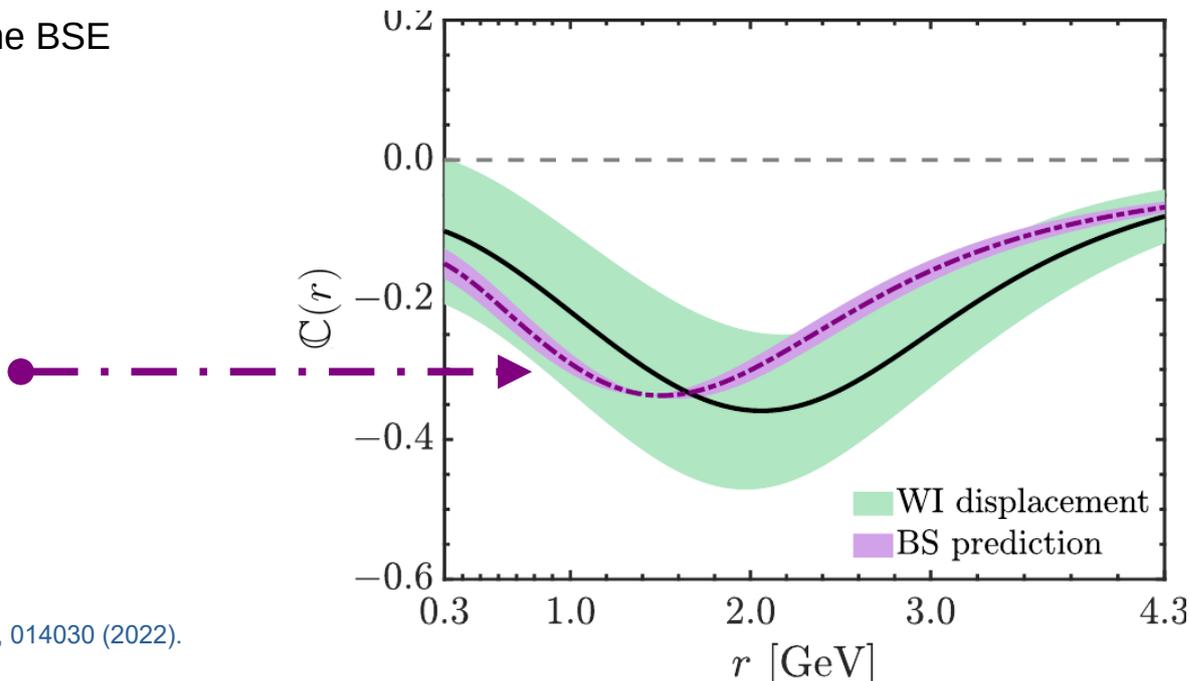
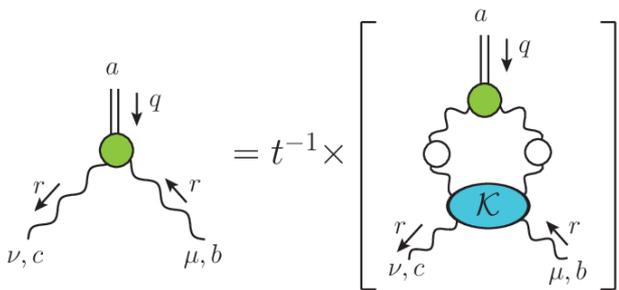
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- Moreover, we find good agreement with the BSE prediction.



A. C. Aguilar, M. N. F. and J. Papavassiliou, *Phys. Rev. D* **105**, no.1, 014030 (2022).

M. N. F. and J. Papavassiliou, *Particles* **6**, no.1, 312-363 (2023).

M. N. F. and J. Papavassiliou, *Eur. Phys. J. C* **84**, no.8, 835 (2024).