Understanding the Structure of Doubly-Heavy Tetraquarks based on the Diquark Model

Connecting quark-model spectra, GEM solutions and chiral EFT intuition

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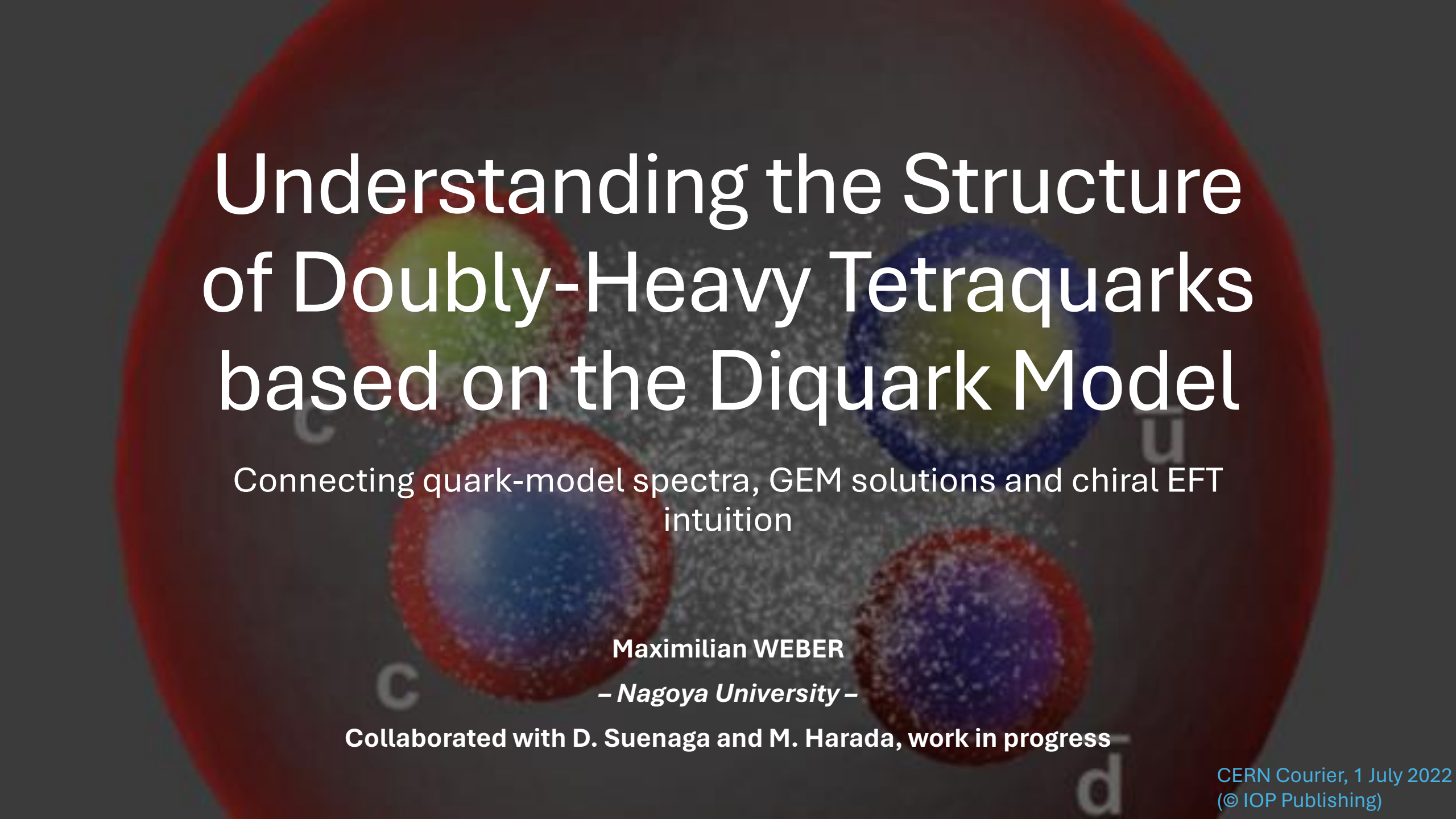
– Nagoya University –

Collaborated with D. Suenaga and M. Harada, work in progress



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Agenda

01	Motivation
02	Model & Method
03	Key Observation (Inverted Hierarchy)
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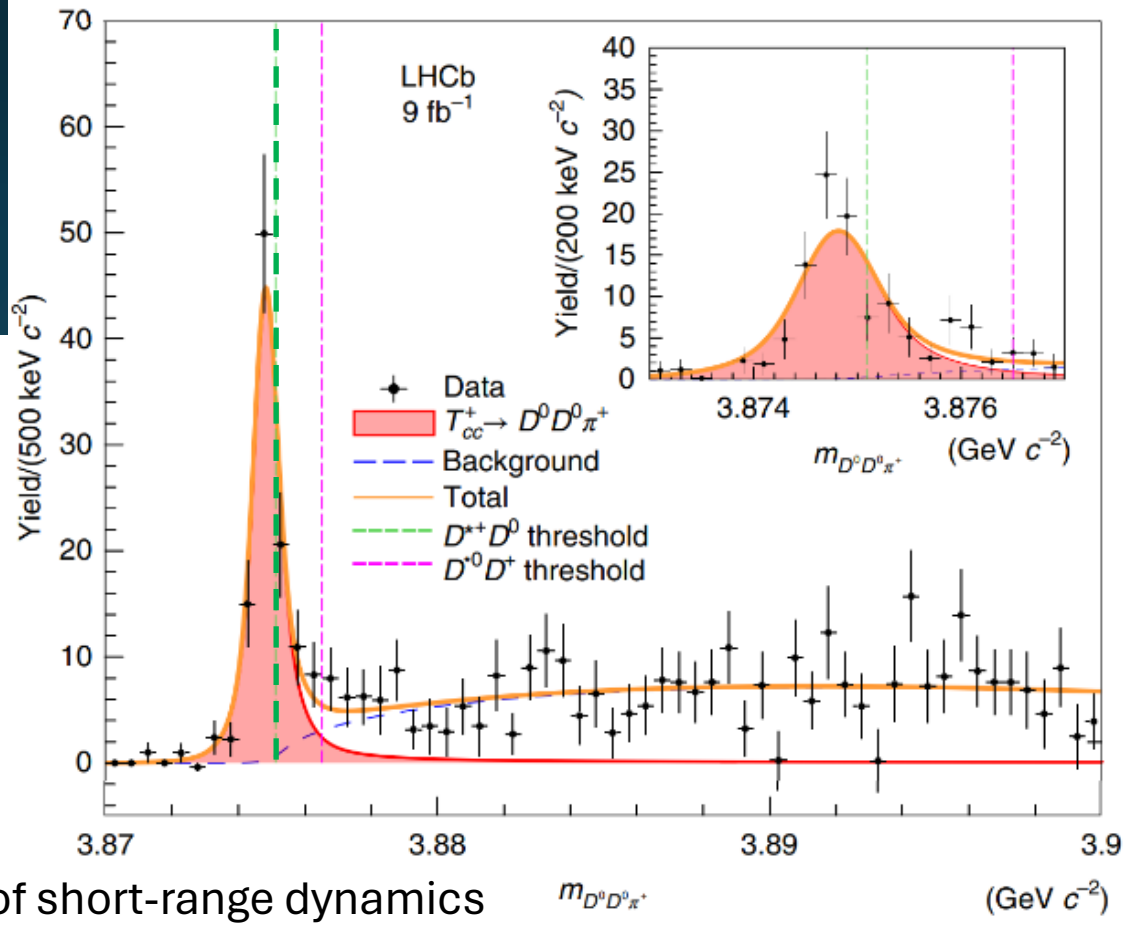
01 Motivation



- **Hadrons** = quarks bound by strong force (QCD)
  → baryons (qqq), mesons ($q\bar{q}$)
- Exotic hadrons (tetraquarks, pentaquarks) challenge standard quark model
- T_{cc}^{+} **discovered** by LHCb (2022): $cc\bar{u}\bar{d}$
  - Long lifetime and narrow width ≈ 410 keV
  - Near $D^{*+}D^0$ mass threshold → sensitive probe of short-range dynamics & diquark structure

  → Evidence for stable multiquark states beyond conventional hadrons
- **Remaining gap** in understanding the dynamics of doubly-heavy tetraquarks
  - Spatial arrangement & channel coupling between meson-meson and diquark-antidiquark configurations unknown

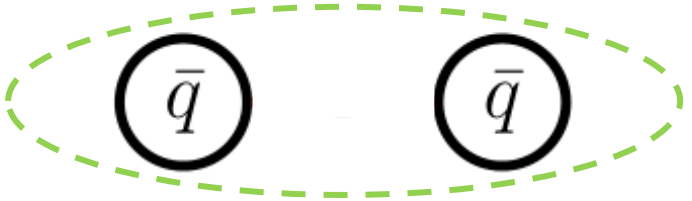
H.-X. Chen et al., Rept. Prog. Phys. 80, 076201 (2017);
LHCb Collaboration, Nat. Phys. 18, 751 (2022);
Y. Lyu et al., Phys. Rev. Lett. 131, 161901 (2023)

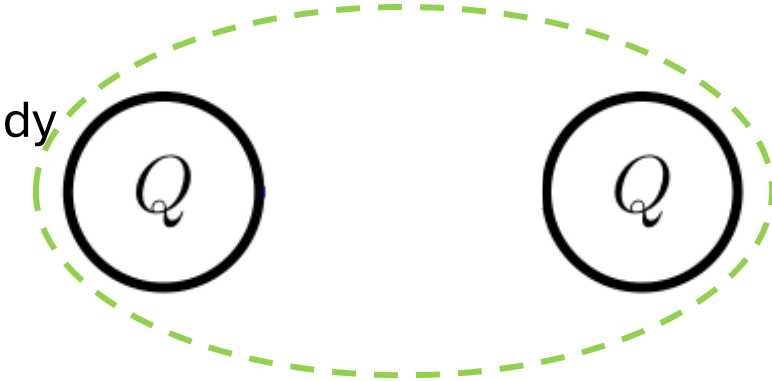
01 Motivation

- **Diquark Ansatz:**
  - Simplifies multiquark dynamics by treating two correlated quarks as a single effective degree of freedom (diquark)
  - Reduces a complex many-body QCD system to an effective two-body problem
  - Based on QCD color structure:
    - Two quarks can couple to color-sextet ($\mathbf{6}_c$) or color-antitriplet ($\bar{\mathbf{3}}_c$)
    - The $\bar{\mathbf{3}}_c$ channel is attractive, analogous to the color-singlet $q\bar{q}$ channel in mesons
  - **Scalar diquark ($\mathbf{0}^+$)**: antisymmetric in spin, flavor, and color → most tightly bound (e.g., light (ud) diquark)
  - **Axial-vector diquark ($\mathbf{1}^+$)**: symmetric in spin and flavor → required for identical heavy quarks (e.g., (cc)), stabilized by large charm mass

R. Jaffe and F. Wilczek, Phys. Rev. Lett. 91, 232003 (2003);
R. L. Jaffe, Phys. Rev. Lett. 38, 195 (1977);
E. J. Eichten and C. Quigg, Phys. Rev. Lett. 119, 202002 (2017)

01 Motivation





- Treat correlated quark pairs → reduces 4-body problem to 2-body
- Diquarks appear naturally in EFTs of dense matter
- EFT encodes short-range QCD into low-energy constants → predictive power
- T_{cc}^{+} **ideal testing ground** for diquark-based EFTs: near-threshold, assume sensitive to short-range correlations and diquark structure
- Chiral partners: states with same spin/flavor, opposite parity
- ***ρ*-mode:** excite relative motion **inside** diquark (parity flip/orbital)
- ***λ*-mode:** excite relative motion **between** heavy and light subsystem
- Chiral partners naturally line up with parity-flipped ρ-mode excitations of the light diquark

C. P. Burgess, Introduction to Effective Field Theory (2020);
Y. Lyu, et al., Phys. Rev. Lett. 131, 161901 (2023);
Y.-L. Ma and M. Harada, Physics Letters B 748, 463 (2015).

02 Model & Method

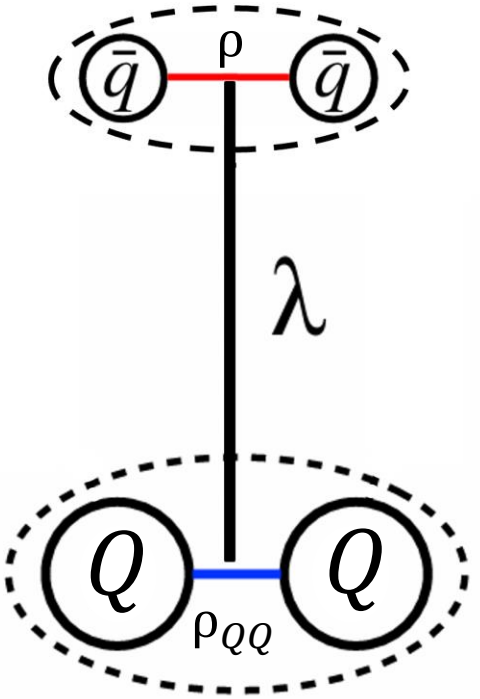
- Two-body (quark–diquark / quark–quark as effective two-body) ansatz
- Hamiltonian:

$$H = m_{QQ} + m_{\bar{q}\bar{q}} + \frac{p^2}{2\mu} + \sum_{a=1}^{8} \left(\lambda_{QQ}^{a} \cdot \lambda_{\bar{q}\bar{q}}^{a}\right) V_{SB}(r)$$

$$p = \frac{m_{QQ} p_{\bar{q}\bar{q}} - m_{\bar{q}\bar{q}} p_{QQ}}{m_{QQ} + m_{\bar{q}\bar{q}}} \qquad \mu = \frac{m_{QQ} m_{\bar{q}\bar{q}}}{m_{QQ} + m_{\bar{q}\bar{q}}}$$

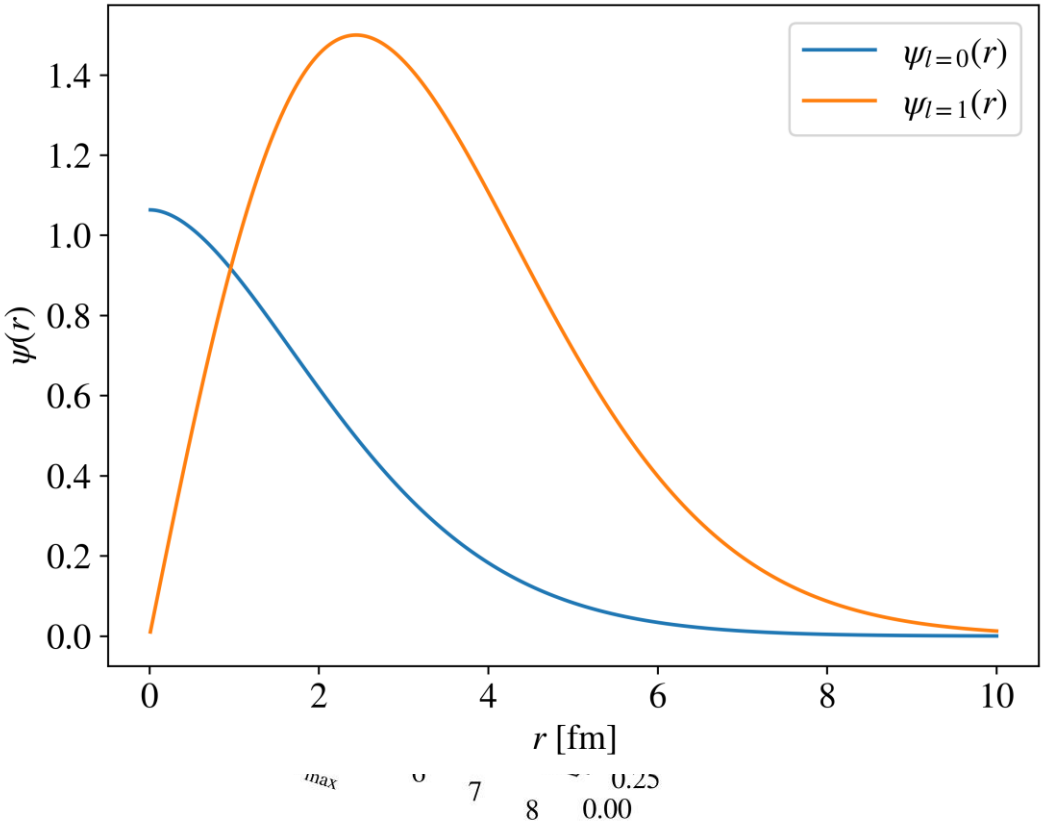
Silvestre-Brac potential: Silvestre-Brac, Few-Body Syst. 20 (1996)

$$V_{SB}(r) = -\frac{\kappa(1-\exp(-r/r_c))}{r} + \lambda r^p - \Lambda + \frac{2\pi}{3 m_{QQ} m_{\bar{q}\bar{q}}} \kappa'(1-\exp(-r/r_c)) \frac{\exp(-r^2/r_0^2)}{\pi^{3/2} r_0^3} \vec{\sigma}_{QQ} \vec{\sigma}_{\bar{q}\bar{q}}$$



$$r_0 = A \left(\frac{2 m_{QQ} m_{\bar{q}\bar{q}}}{m_{QQ} + m_{\bar{q}\bar{q}}} \right)^{-B}$$

02 Model & Method



- Solve Schrödinger equation numerically via Gauss

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2 + m_{QQ} + \sum_{n'=1}^{n_{\max}} [\quad] + \sum_{a=1}^{8} \quad - E\right]$$

$$\psi_{lm}(r) = \sum_{n=1}^{n_{\max}} c_{nl}\phi_{nlm}^{G}(r) \qquad \sum_{n=1}^{n_{\max}} c_{nl}\phi_{nl}^{G}(r)Y_{lm}(\hat{r})$$

- Numerical values: $n_{\max} = 20, r_1 = 0.1\ \text{fm}, r_{\max} = 6\ \text{fm}, p = 1, r_c = 0, m_u = m_d = 0.315\ \text{GeV}, m_s = 0.577\ \text{GeV}, m_c = 1.836\ \text{GeV}, m_b = 5.227\ \text{GeV}, \kappa = 0.5069, \kappa' = 1.836, \nu_n = 0.1653\ \text{GeV}^2, \Lambda = 0.8321\ \text{GeV}, B = 0.2204, \lambda = 1.6553\ \text{GeV}^{B-1}$

$$\phi_{nl}^{G}(r) = N_{nl} r^l e^{-\nu_n r^2} \qquad r_n = r_1 a^{n-1}$$

$$N_{nl} = \left(\frac{2^{l+2}(2\nu_n)^{l+3/2}}{\sqrt{\pi}(2l+1)!!}\right)^{1/2}$$

$$V_{SB}(r) = -\frac{\kappa}{r} + \lambda r - \Lambda + \frac{2\pi}{3m_{QQ}m_{\bar{q}\bar{q}}}\kappa' \frac{\exp(-r^2/r_0^2)}{\pi^{3/2}r_0^3}\vec{\sigma}_{QQ}\vec{\sigma}_{\bar{q}\bar{q}}$$

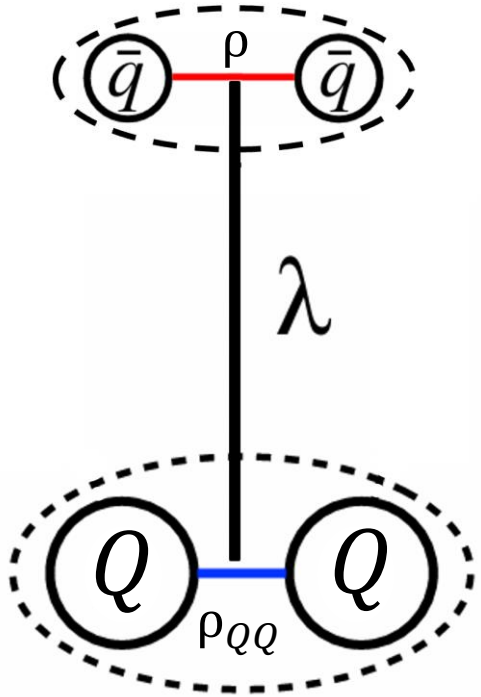
Silvestre-Brac, Few-Body Syst. 20 (1996)

Hiyama, Kino & Kamimura, Prog. Part. Nucl. Phys. 51 (2003)

03 Key Observation (Inverted Hierarchy)

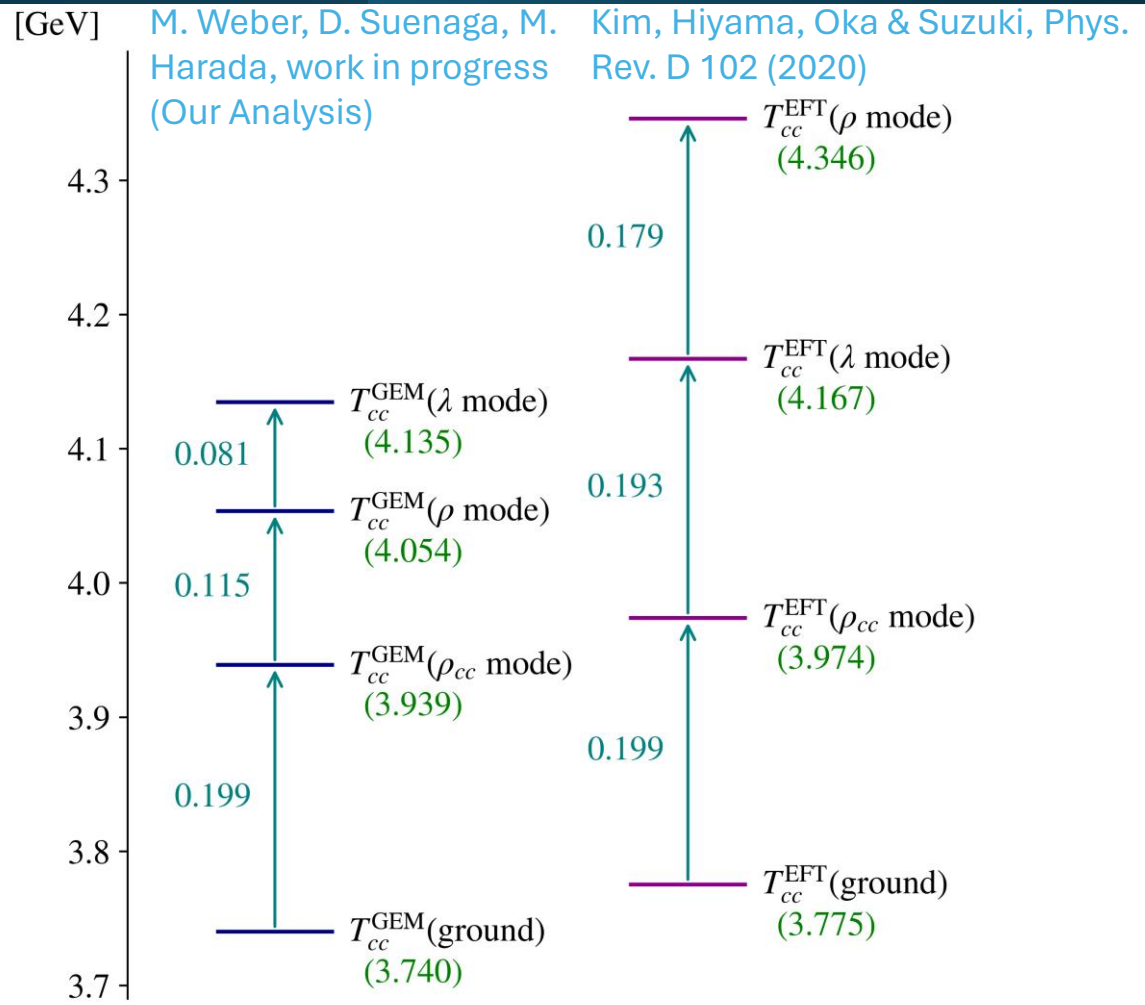
Mass predictions

(GeV)	Diquark model M. Weber, D. Suenaga, M. Harada, work in progress (Our Analysis)	EFT model Kim, Hiyama, Oka & Suzuki, Phys. Rev. D 102 (2020)
$m_{ud}(0^+)$	0.666	0.725
$m_{ud}(0^-,\lambda)$	1.121	1.406
$m_{cc}(1^+)$	3.500	/
$m_{cc}(1^-,\lambda)$	3.701	/
	0.455	0.681



Inverse mass hierarchy in λ/ρ-mode excitations

- With the diquark model masses we get $M(T_{cc}, \rho) < M(T_{cc}, \lambda)$ — an inverted ordering vs naive constituent mass expectations
- Using lattice data and EFT mass relation restores expected ordering — mass gap between $m_{ud}(0^+)$ and $m_{ud}(0^-)$ bigger



Naïve Expectations

- **Simple model**: baryons behave like particles in a harmonic oscillator (HO) potential

$$H = \sum_i \frac{\vec{p_i}^2}{2m_i} + \frac{1}{2}K\sum_{i<j} r_{ij}^2$$

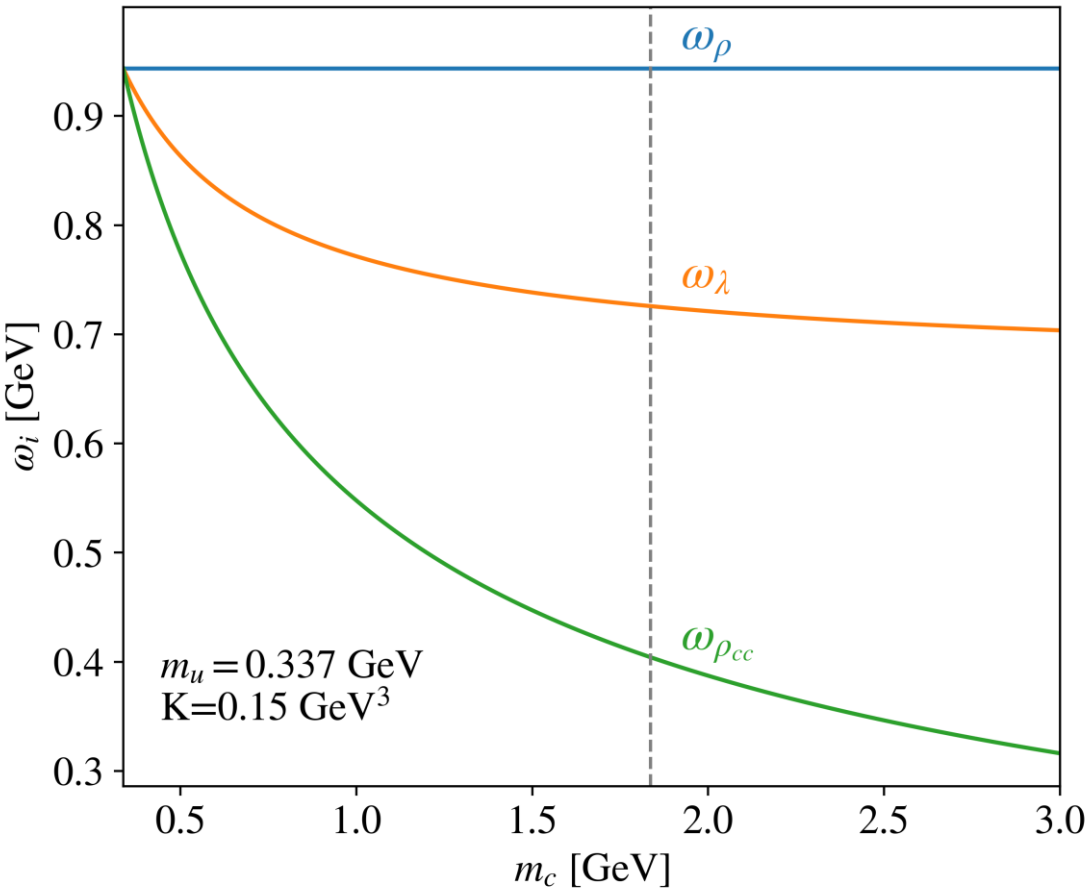
  - Predicts equidistant level spacing for excitations

$$E_n \propto \left(2\mathrm{n} + \mathrm{l} + \frac{3}{2}\right)\hbar\omega$$

  - Suggests a straightforward ordering of states (ground → ρ_{cc}-mode → λ-mode → ρ-mode)

$$\omega \sim \sqrt{K/\mu}$$

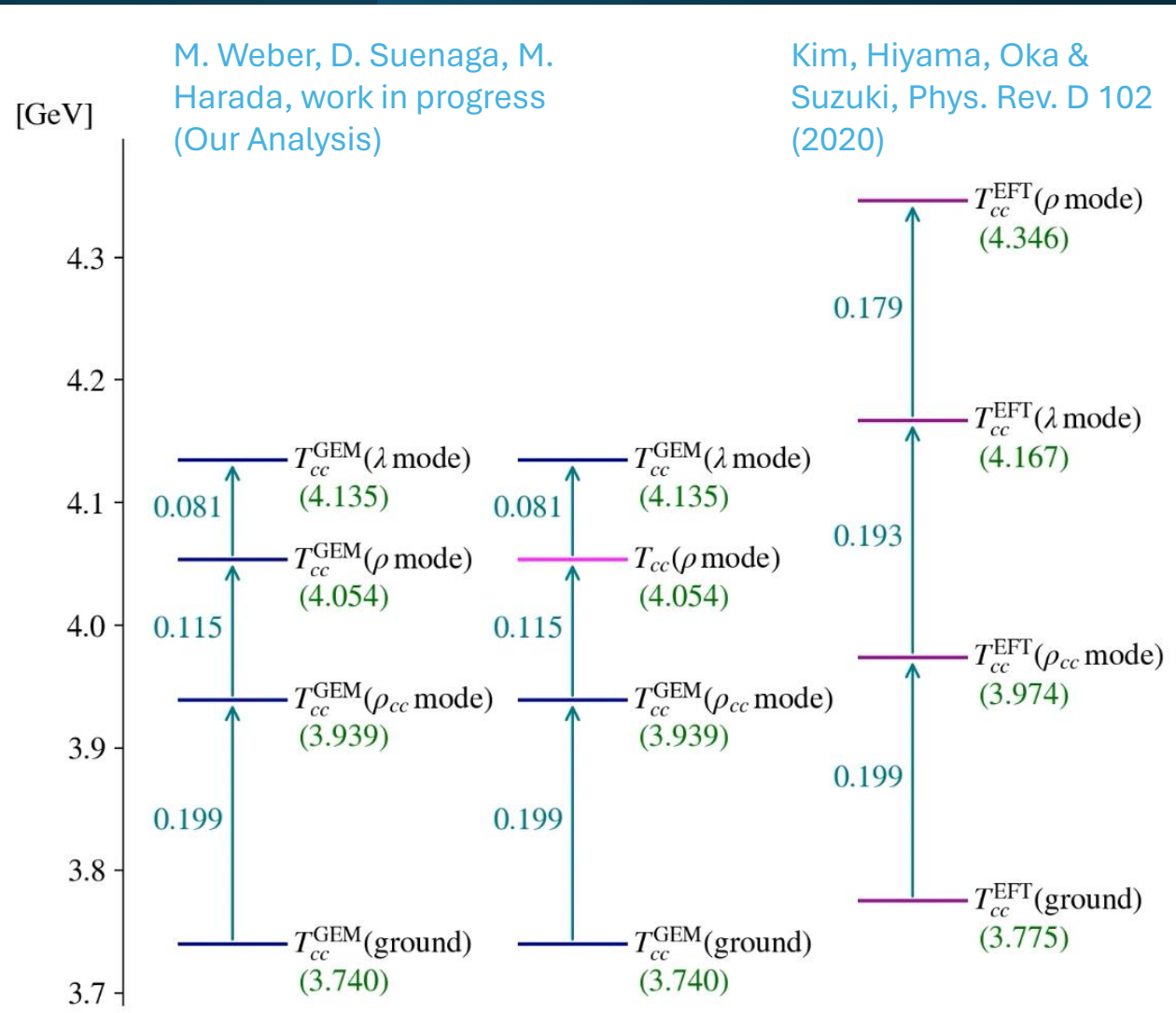
- **But**: real QCD spectra are more complex → need frameworks beyond HO



F. Buisseret and C. Semay, Phys. Rev. D 73, 114011 (2006);
L. Glozman and D. Riska, Physics Reports 268, 263 (1996)

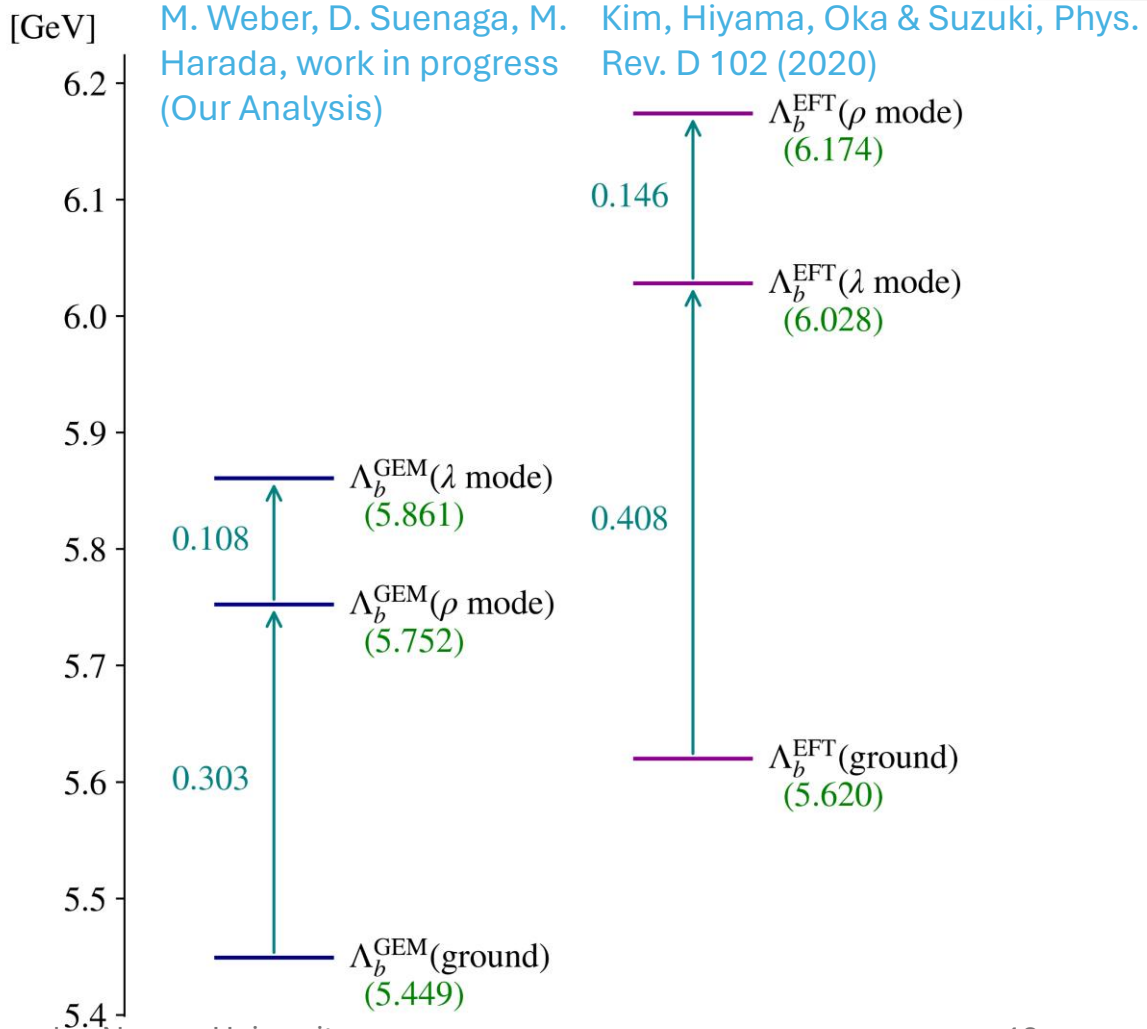
Restoring conventional ordering beyond a threshold

- This inversion changes for increasing the $m_{ud}(0^-) \in [1.121, 1.484]$ GeV
- As orbital angular momentum contributes via $l(l+1)/2\mu\langle r^2\rangle$, the heavier centrifugal energy can flip the ordering back
- There exists a threshold ($m_{ud}(0^-) = 1.225$ GeV) where centrifugal term dominates and pushes the ρ-mode energy up, restoring expected hierarchy



Cross-checks

- Verified across different systems $(T_{bb}, \Lambda_b, \Lambda_c)$
  - Sensitivity: diquark mass inputs from lattice vs EFT change quantitative numbers but keep the qualitative picture
  - Mass gap $m_{ud}(0^-) - m_{ud}(0^+)$ relevant
- Checked wavefunction radius-mean-square and expectation values



	$m_{ES} - m_{GS}$ (GeV)	$\sqrt{\langle r^2 \rangle}$ (fm)	μ (GeV)	$\Delta\langle T \rangle$ (GeV)	$\Delta\langle V_{\mathrm{mag}} \rangle$ (GeV)	$\Delta\langle V_{\mathrm{colomb}} \rangle$ (GeV)	$\Delta\langle V_{\mathrm{lin}} \rangle$ (GeV)
$m_{ud}(0^-)_{(\rho)}$	0.455	1.511	0.158	−0.041	0.182	0.047	0.267
$T_{cc}(1^+)$	0.000	0.495	0.559	0.000	0.000	0.000	0.000
$T_{cc}(0^-, \lambda)$	0.395	0.741	0.559	0.067	−0.020	0.137	0.210
$T_{cc}(1^+, \rho)$	0.313	0.404	0.849	0.003	0.000	−0.075	−0.069
$T_{cc}^{\mathrm{thresh}}(1^+, \rho)$	0.395	0.391	0.907	0.005	0.000	−0.089	−0.080
$T_{cc}^{\mathrm{EFT}}(1^+, \rho)$	0.541	0.371	1.003	0.009	0.000	−0.112	−0.095
$T_{cc}(1^+, \rho_{cc})$	0.199	0.493	0.564	0.000	0.000	−0.001	−0.002

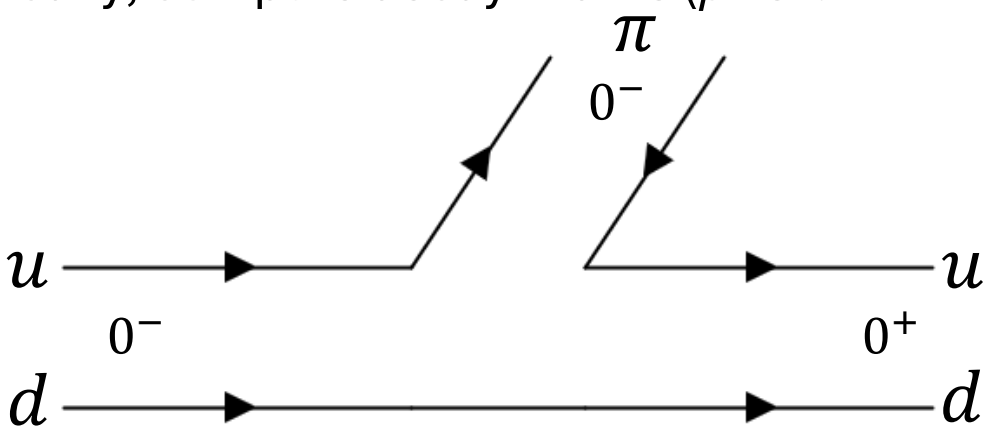
$$H = m_{QQ} + m_{\bar{q}\bar{q}} + T + V$$

04 Outlook & Next Steps

Yoshida, Hiyama, Hosaka, Oka & Sadato, Phys. Rev. D 92 (2015)

Barnes, Godfrey & Swanson, Phys. Rev. D 72 (2005)

- Use different potentials fitted to other sectors (Yoshida, Barnes)
  - Differentiation between different J → no degeneracy
  - Now: $m_{ud}(0^-, 1^-, 2^-)$, $T_{cc}(0^-, 1^-, 2^-)_\lambda$ and $T_{cc}(1^-; 0^-, 1^-, 2^-; 1^-, 2^-, 3^-)_\rho$
- Study in-medium modifications (density/temperature) of diquark masses — link to dense matter phases
- Compare to lattice excited spectra systematically; compute decay widths (ρ vs λ decay patterns)
  - Chiral partner couples strongly to state by emitting pion mass → ρ-mode



05 Summary

- Second excited state might already be chiral partner
- Centrifugal effects can restore conventional hierarchy beyond a threshold
- Results consistent across different heavy flavours — relevant for connecting diquark models with chiral EFT and dense QCD
- Consistency check via different potential
- Decay width calculations
- Experimental data needed for diquark mass

Thank you for your attention!