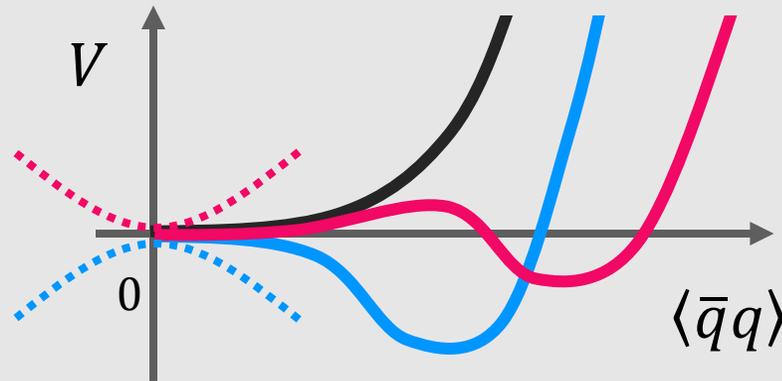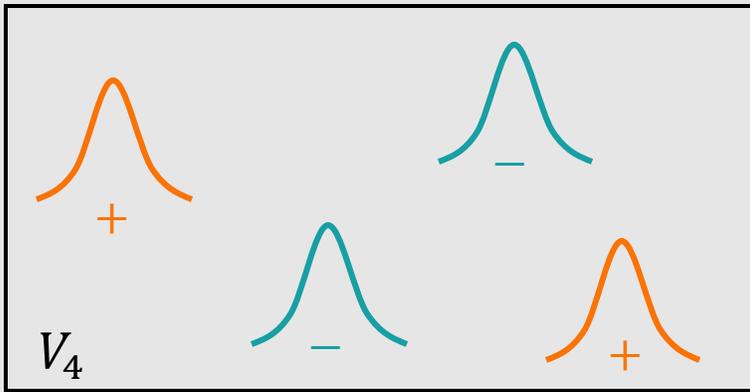# Different scenarios of dynamical chiral symmetry breaking in the interacting instanton liquid model via flavor symmetry breaking

Department of Physics, Institute of Science Tokyo
Yamato Suda and **Daisuke Jido**
BARYONS2025@ICC Jeju Korea 10-14 November 2025

Dynamical chiral symmetry breaking
- mechanism of DχSB
- instanton induced interaction

Model calculation
- quenched IILM & flavor SU(3)IILM
- flavor SU(2) & 2+1 flavor IILM

Summary & Outlook

Based on
YS, D. Jido, PRD **110**, 014037 (2024), YS, D. Jido, PRD **112**, 034011 (2025)

Dynamical chiral symmetry breaking

- **Dynamical chiral symmetry breaking (DχSB)**
  - Non-perturbative phenomenon
  - Nonzero quark condensate in the vacuum
  - Hadron properties in low energy
    - Hadron mass generation
    - (Pseudo) massless NG bosons
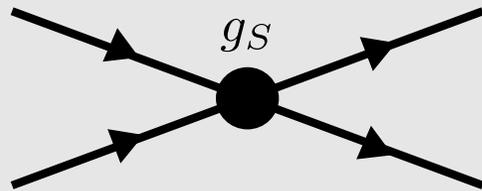
Dynamical chiral symmetry breaking

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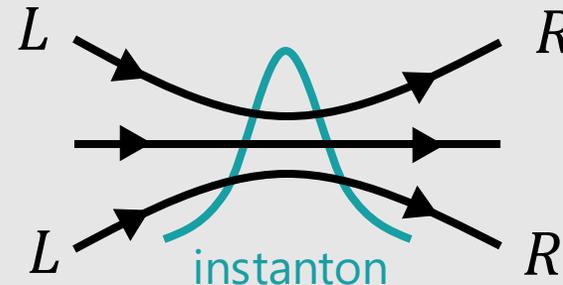   - (Pseudo) massless NG bosons

source of DχSB

- **4-fermion interaction**

g_S

$+$

- **Instanton induced interaction (III)**

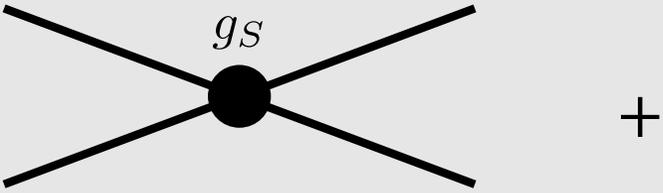L R

L instanton R

$+$ ...

[1] Y. Nambu, G. Jona-Lasinio (1961)

[2] G. 't Hooft, (1976); (1986)
[3] C. Callan, R. Dashen, D. Gross, (1978)

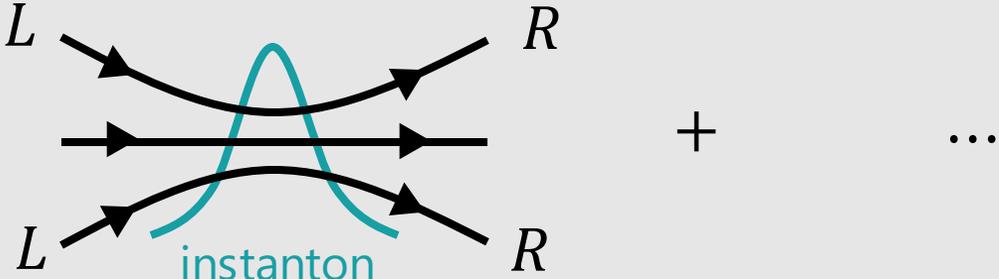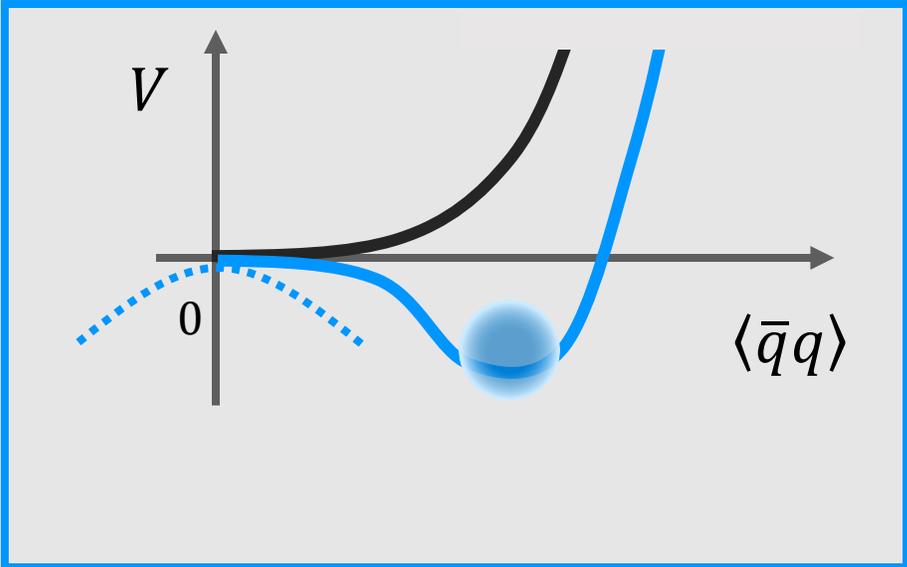# Which contribution is predominant in DχSB?

- **4-fermion interaction**

g_S

- **Instanton induced interaction (III)**

[4] S. Kono, *et al.,* PTEP (2021)
[5] YS, D. Jido, PRD (2024)

L R

L R

instanton

$+$

$+$

$\cdots$

large enough 4-fermi
finite (or zero) III

V

0

$\langle \bar{q}q \rangle$

V: effective potential
$\langle \bar{q}q \rangle$: quark condensate

Which contribution is predominant in DχSB?

- **4-fermion interaction**
- **Instanton induced interaction (III)**

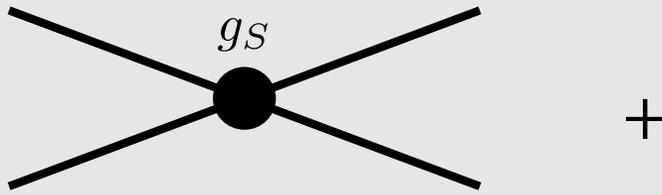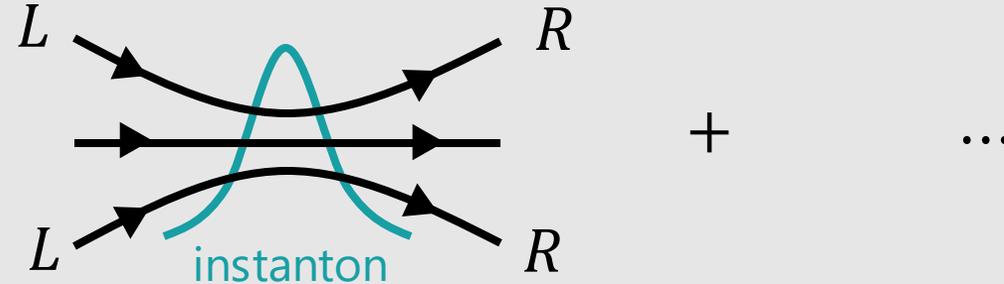[4] S. Kono, *et al.,* PTEP (2021)
[5] YS, D. Jido, PRD (2024)

g_S

L R

L R

instanton

$+$ $\cdots$

large enough 4-fermi
finite (or zero) III

not sufficient 4-fermi
sufficiently large III

V

0

$\langle \bar{q}q \rangle$

V

0

$\langle \bar{q}q \rangle$

V: effective potential
$\langle \bar{q}q \rangle$: quark condensate

5/17

Which contribution is predominant in DχSB?

- **4-fermion interaction**
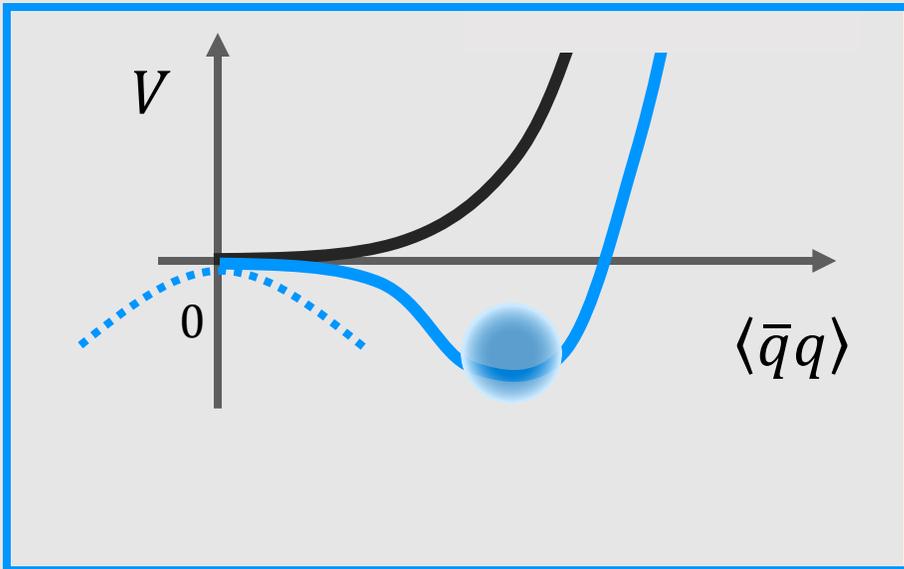- **Instanton induced interaction (III)**

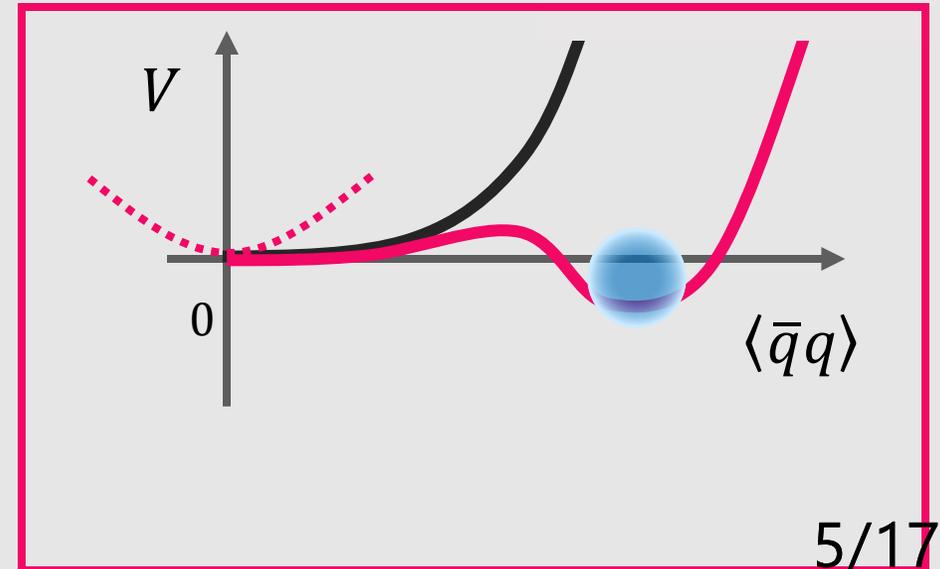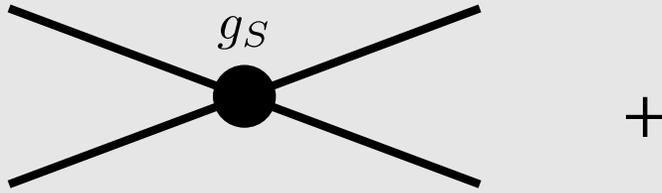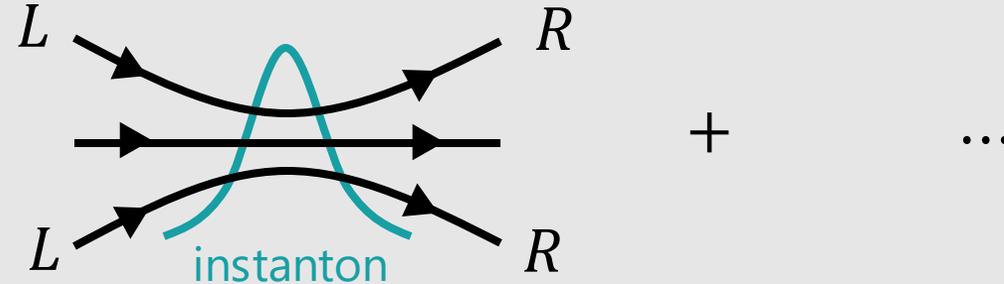[4] S. Kono, *et al.,* PTEP (2021)
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g_S

L R

L R

instanton

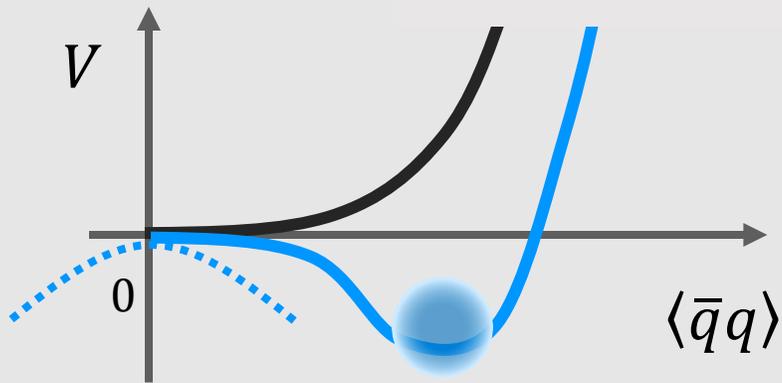$+$ $\cdots$

large enough 4-fermi
finite (or zero) III

not sufficient 4-fermi
sufficiently large III

- **definition**

$$C_2 = \left.\frac{\partial^2 V}{\partial \langle \bar{q}q \rangle^2}\right|_{\langle \bar{q}q \rangle = 0}$$

V

0

$\langle \bar{q}q \rangle$

V: effective potential
$\langle \bar{q}q \rangle$: quark condensate

V

0

$\langle \bar{q}q \rangle$

$C_2 < 0 \rightarrow$ Ordinary scenario

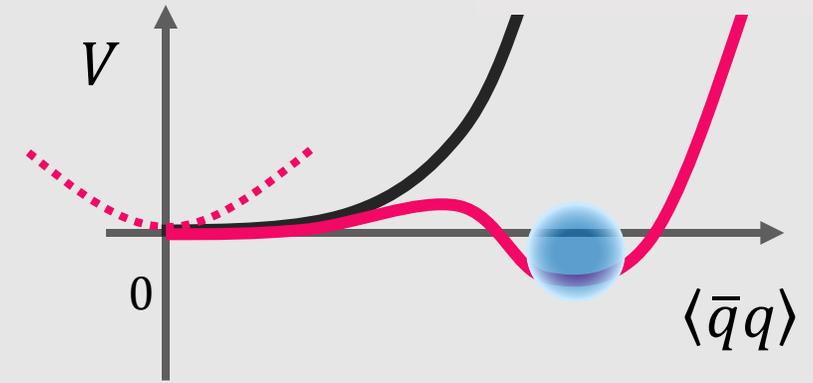$C_2 > 0 \rightarrow$ Anomaly-driven scenario

Connection to physical quantity

$$C_2 = \left.\frac{\partial^2 V}{\partial \langle \bar{q}q \rangle^2}\right|_{\langle \bar{q}q \rangle = 0}$$

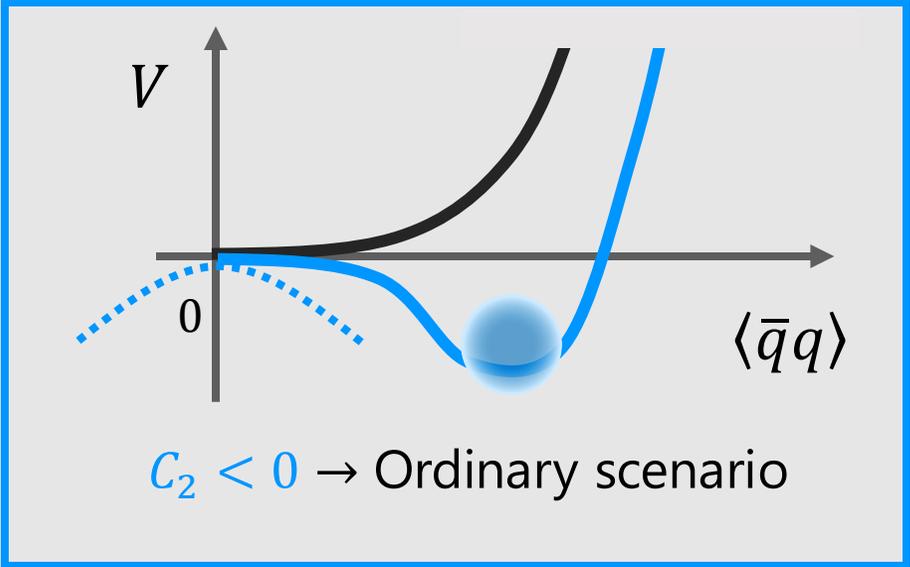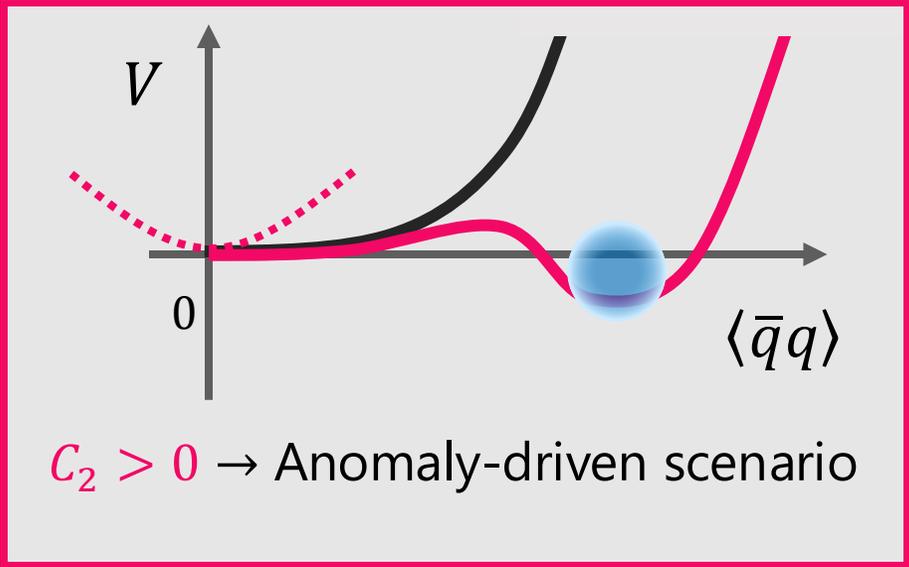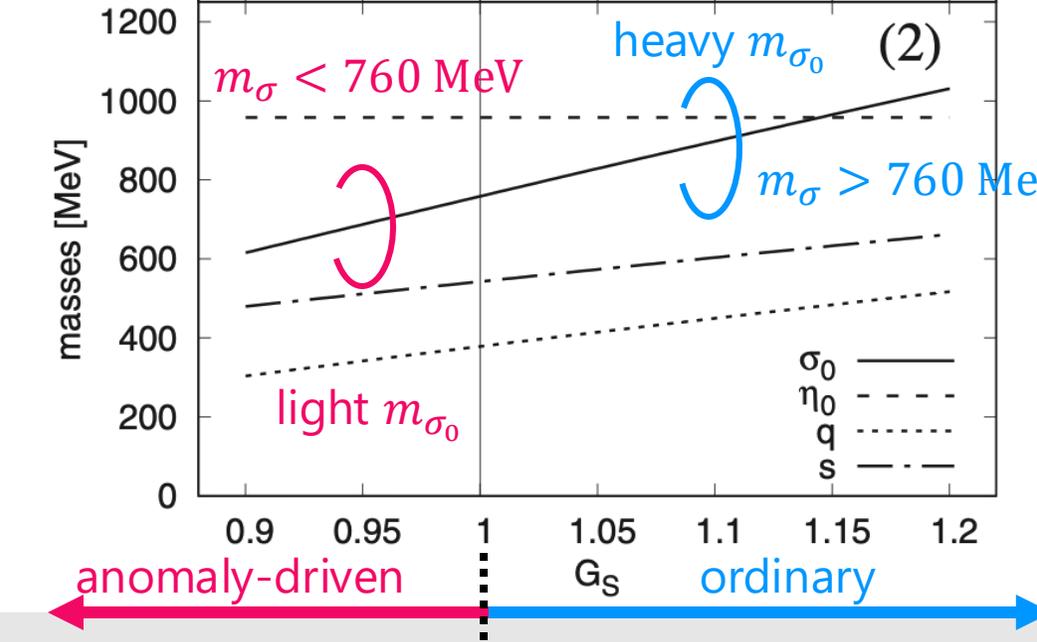V: effective potential
$\langle \bar{q}q \rangle$: quark condensate

$C_2 < 0 \rightarrow$ Ordinary scenario

$C_2 > 0 \rightarrow$ Anomaly-driven scenario

- NJL model calculation
  - mass of sigma $\sigma_0 = \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s)$
  - finite current quark masses m_q, m_s
    such that to reproduce m_π, m_K
  - anomaly term is determined to reproduce η'
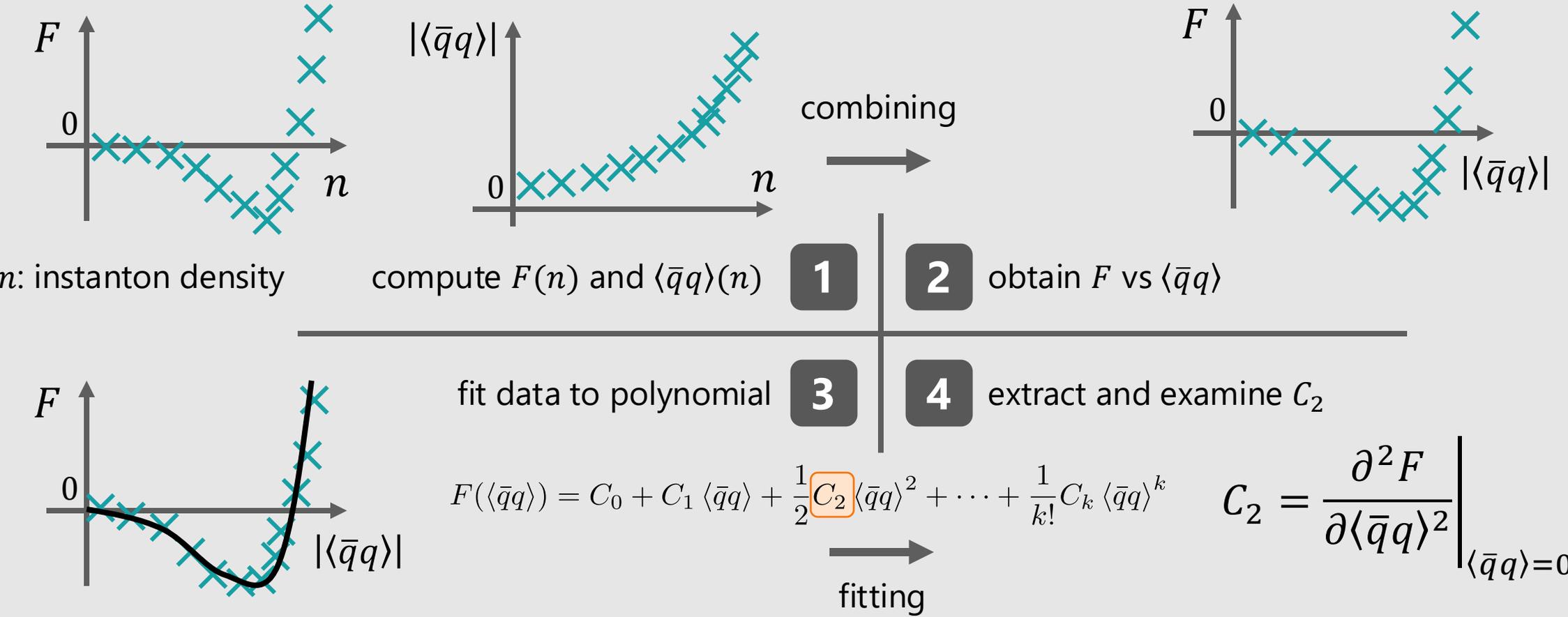

[4] S. Kono, *et al.,* PTEP (2021)

Outline of our study

[4] S. Kono, *et al.,* PTEP (2021)
[5] YS, D. Jido, PRD (2024)
[6] YS, D. Jido, PRD (2025)

- **Motivation**
  - different scenarios of DχSB are discussed in the chiral effective theories (LσM, NJL) [4]
  - how about in another model? Quenched & $SU(3)_f$ symmetric cases [5]
  - how about in more realistic case? $SU(2)_f$ symmetric & (2+1)-flavor cases [6]

- **Strategy**

n: instanton density   compute $F(n)$ and $\langle\bar{q}q\rangle(n)$ **1** **2** obtain F vs $\langle\bar{q}q\rangle$

fit data to polynomial **3** **4** extract and examine C_2

$$F(\langle\bar{q}q\rangle) = C_0 + C_1\langle\bar{q}q\rangle + \frac{1}{2}C_2\langle\bar{q}q\rangle^2 + \cdots + \frac{1}{k!}C_k\langle\bar{q}q\rangle^k$$

$$C_2 = \left.\frac{\partial^2 F}{\partial\langle\bar{q}q\rangle^2}\right|_{\langle\bar{q}q\rangle=0}$$

fitting

Interacting Instanton liquid model (IILM)

- **I**nteracting **I**nstanton **L**iquid **M**odel (**IILM**) [7] E. Shuryak (1982), E. Shuryak (1989), T. Schafer, E. Shuryak (1996); (1998), etc.
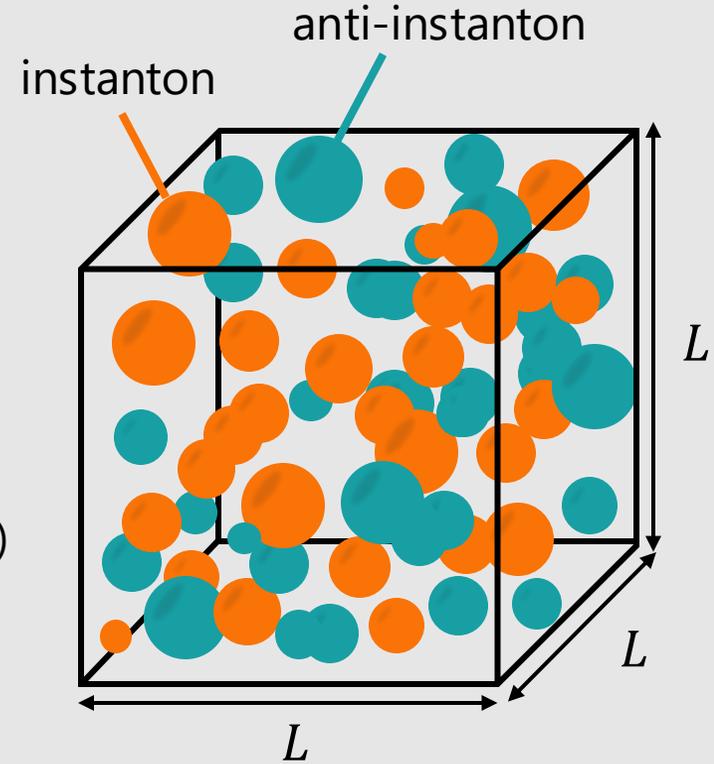 - QCD vacuum ≈ liquid of instantons (weakly interacting ensemble)

- **canonical partition function of the IILM**

instanton-instanton interaction

instanton-quark interaction

$$Z = \frac{1}{N_+! \, N_-!} \int \prod_{i=1}^{N_+ + N_-} d\Omega_i \, f(\rho_i) \, e^{-S_{\text{int}}} \prod_{f=1}^{N_f} \text{Det}(\widehat{D} + m_f)$$
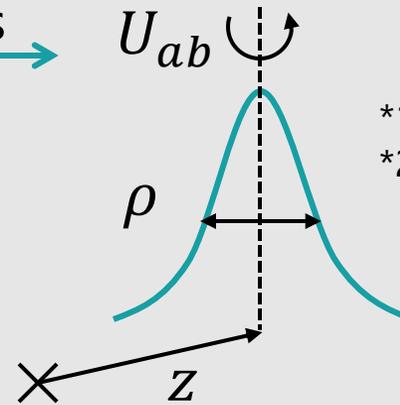
m_f : current quark mass (input)

semiclassical instanton amplitude

collective coordinates of instantons

U_{ab}

- **Physical quantities**
  - free energy density $F = -\frac{1}{V} \ln Z$ (logarithm of Z)
  - quark condensate $\langle \bar{q}q \rangle$ (2-pt corr. func.)

  obtained as a function of instanton density $n = \frac{N_+ + N_-}{V}$
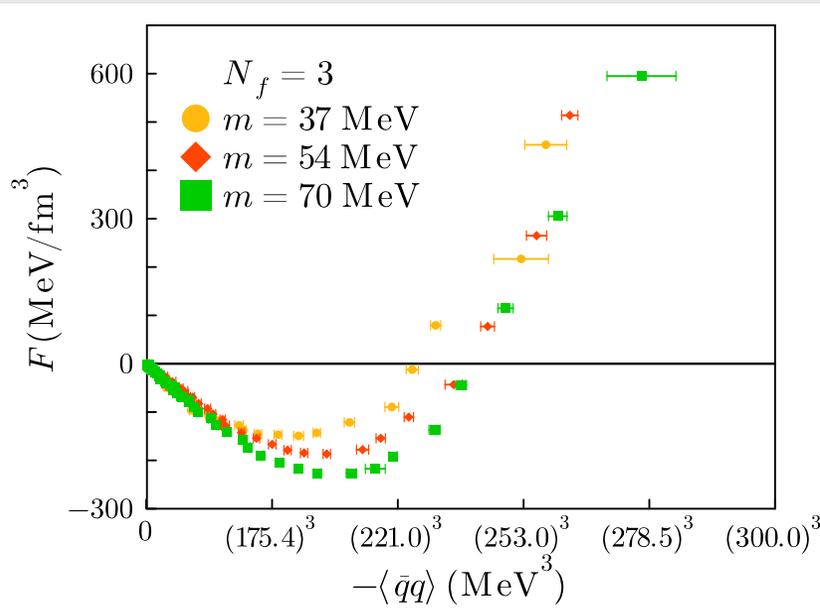
ρ

z

instanton   anti-instanton

L

L

L

*1 actual simulations are performed in 4d
*2 simulations cannot be performed
   at $m_f = 0$ due to computational cost

Estimates of C_2 in the IILM

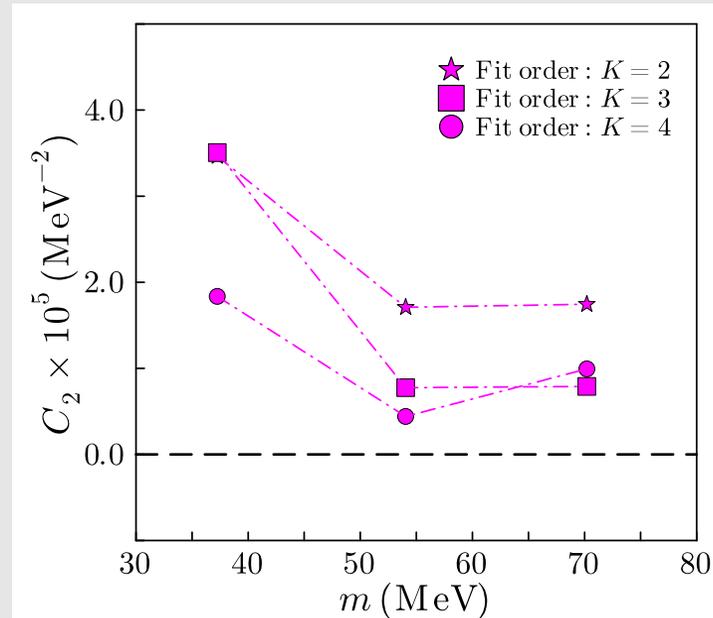[5] YS, D. Jido, PRD (2024)

▶ **flavor SU(3) symmetric case**

F vs $\langle \bar{q}q \rangle$ C_2 vs m Type of DχSB

fitting ▶

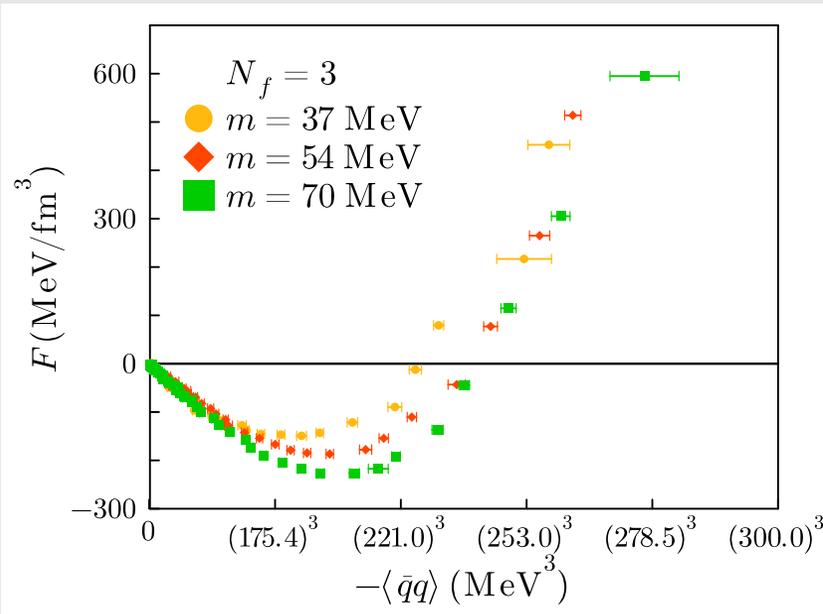$C_2 > 0$

↓

anomaly-driven scenario

↓

6-quark int. (III) predominant

Estimates of C_2 in the IILM

▶ **flavor SU(3) symmetric case**

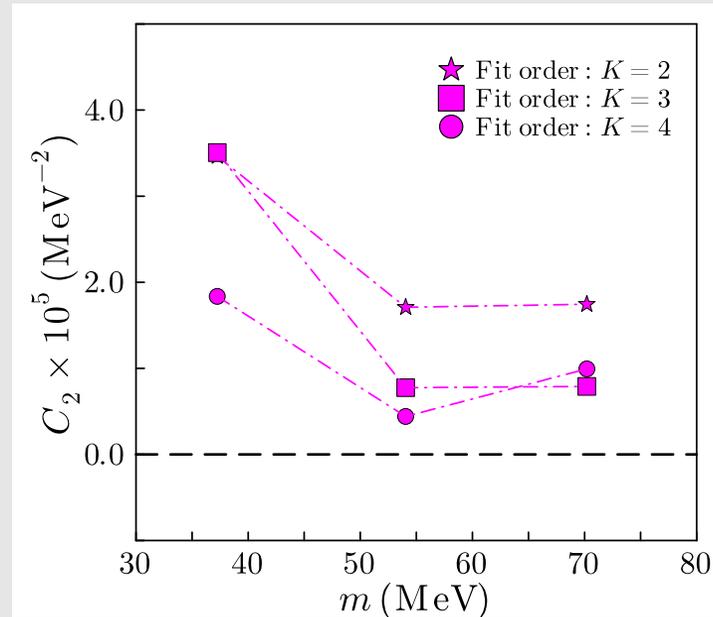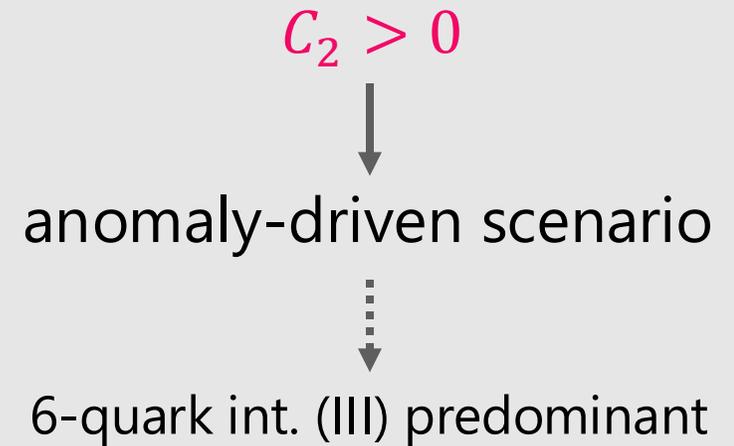| F vs $\langle \bar{q}q \rangle$	C_2 vs m	Type of DχSB

fitting ▶

$C_2 > 0$

↓

anomaly-driven scenario

⋮

6-quark int. (III) predominant

⇒ **Positive curvature implies predominant contribution to DχSB from the axial anomaly**

In the case of 2+1 flavor quarks

[5] YS, D. Jido, PRD (2024)
[6] YS, D. Jido, PRD (2025)

- **Current quark masses**
  - value in units of MeV can be obtained by multiplying Λ

Status	Quarks in IILM	m_q/Λ	m_s/Λ
[YS, Jido, (2024)]	SU(3)	$\{0.10, 0.15, 0.20, 0.30\}$	$\{0.10, 0.15, 0.20, 0.30\}$
[YS, Jido, (2025)]	SU(2)	$\{0.05, 0.08, 0.10, 0.15, 0.18, 0.20\}$	$-$
[YS, Jido, (2025)]	(2+1)-flavor	$\{0.08\}$ $\{0.10\}$ $\{0.15\}$ $\{0.20\}$ $\{0.25\}$ $\{0.30\}$	$\{0.10, 0.15, 0.30, 0.60, 0.90, 1.2\}$ $\{0.15, 0.30, 0.45, 0.60, 075, 0.90, 1.2\}$ $\{0.20, 0.30, 0.45, 0.60, 0.90, 1.2\}$ $\{0.25, 0.30, 0.45, 0.60, 0.90, 1.2\}$ $\{0.28, 0.30, 0.45, 0.60, 0.90, 1.2\}$ $\{0.45, 0.60, 075, 0.90, 1.0, 1.2\}$

- **How to handle the quark determinant in Z?**

$$Z \propto \int \prod_{i=1}^{N_+ + N_-} d\Omega_i \underbrace{f(\rho_i)}_{\downarrow} e^{-S_{\text{int}}} \prod_{f=1}^{N_f} \text{Det}(\widehat{D} + m_f)$$

includes $N_f = 3$ or 2

$$\begin{cases} \text{SU(3), SU(2): } N_f = 3 \text{ or } 2 \ \& \ \left[\text{Det}(\widehat{D} + m_q)\right]^{N_f} \\[2ex] \text{(2+1)-flavor: } N_f = 3 \ \& \ \left[\text{Det}(\widehat{D} + m_q)\right]^2 \text{Det}(\widehat{D} + m_s) \end{cases}$$

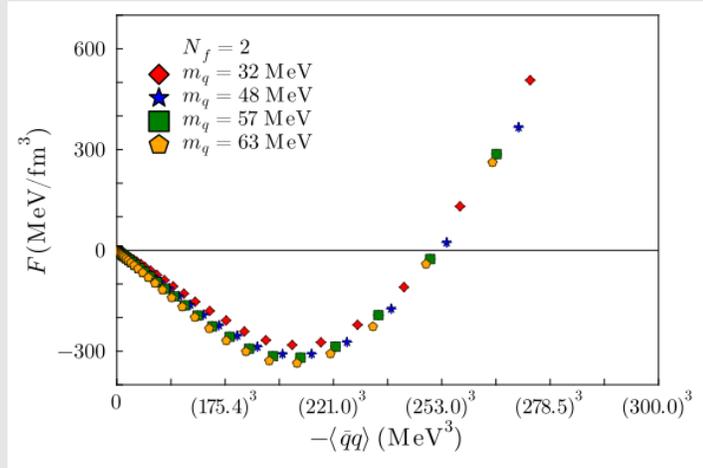# Estimates of C_2 in SU(2), (2+1)-flavor IILM

[6] YS, D. Jido, PRD (2025)
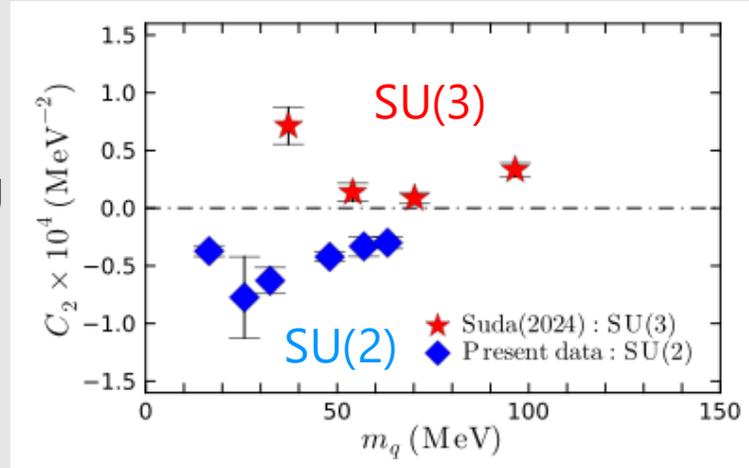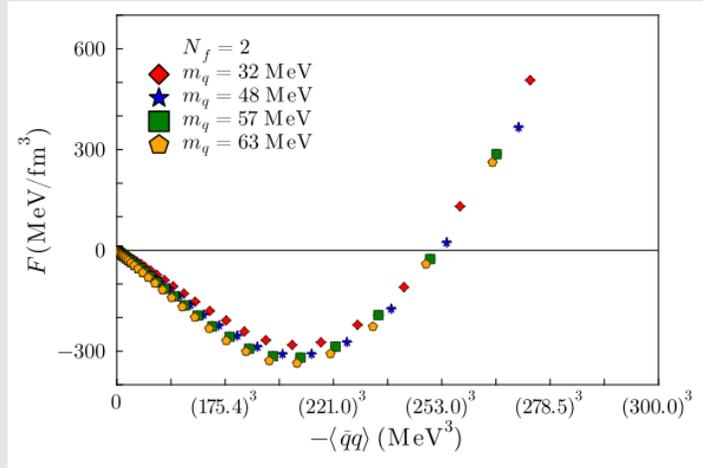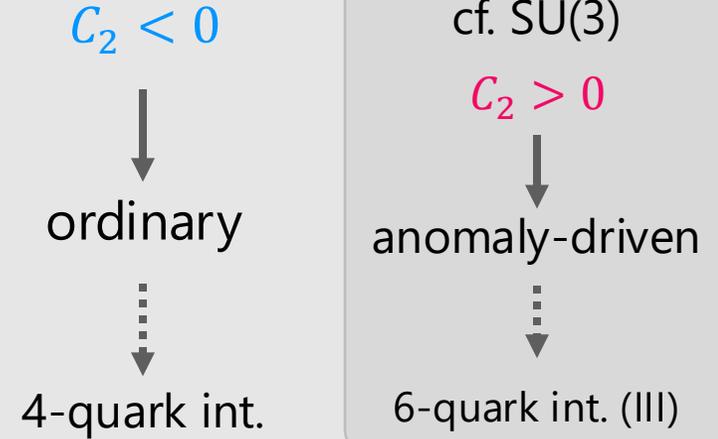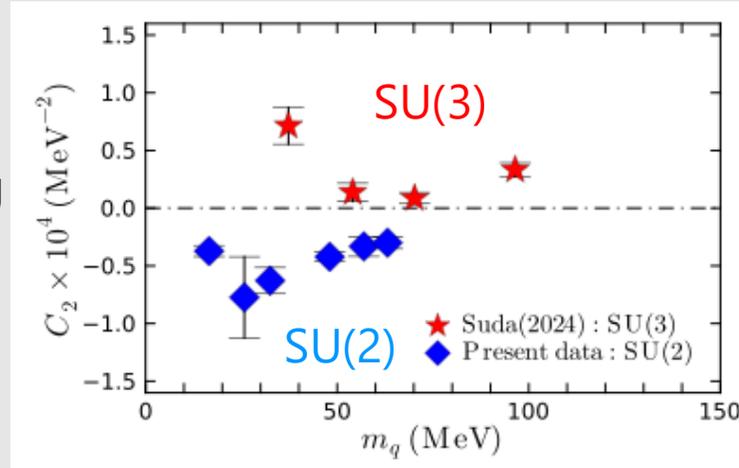
F vs $\langle \bar{q}q \rangle$

C_2 vs m

Type of DχSB

$N_f = 2$

fitting ▶

$C_2 < 0$

cf. SU(3)

$C_2 > 0$

↓

↓

ordinary

anomaly-driven

⋮

⋮

4-quark int.

6-quark int. (III)

Estimates of C_2 in SU(2), (2+1)-flavor IILM

[6] YS, D. Jido, PRD (2025)

F vs $\langle \bar{q}q \rangle$ C_2 vs m Type of DχSB

Ratio m_s/m_q-dependence of curvature

[6] YS, D. Jido, PRD (2025)

Ratio m_s/m_q-dependence of curvature

[6] YS, D. Jido, PRD (2025)

Summary & Outlook

[5] YS, D. Jido, PRD (2024)
[6] YS, D. Jido, PRD (2025)
[7] YS, D. Jido, in preparation

- types of dynamical chiral symmetry breaking (DχSB) are studied in the IILM [5,6]

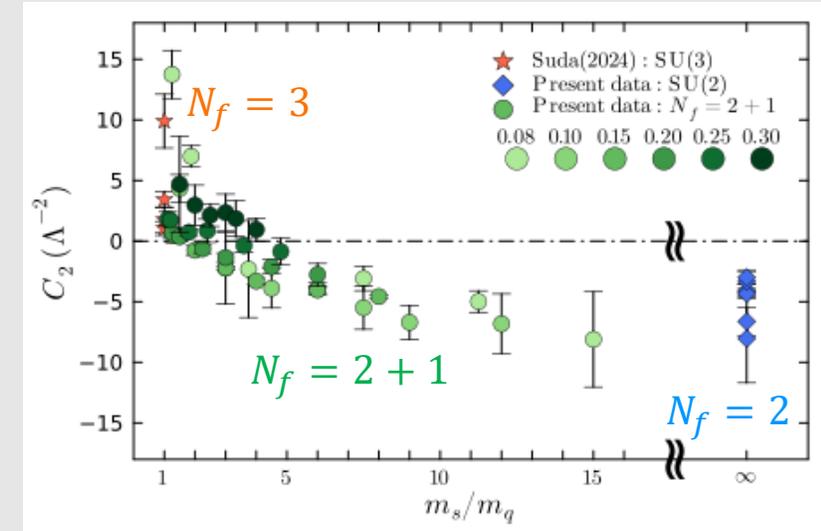|	C_2	type of DχSB	predominant source
$N_f = 3$	+	anomaly-driven	6-quark int. (instanton induced)
$N_f = 2 + 1$	+ $m_s \gg m_q$ $-$	anomaly-driven $m_s \gg m_q$ ordinary	6-quark int. (instanton induced) $m_s \gg m_q$ 4-quark int.
$N_f = 2$	$-$	ordinary	4-quark int.

$m_s = m_q$ ⟶ $m_s \gg m_q$

- suggestion
  SU(3) sym. system → anomaly-driven type of DχSB
  (2+1)-flavor system (e.g., real world) → ordinary type of DχSB
- How does such difference of types relate to vacuum quantity?
  → size of vacuum curvature and the scalar meson mass [7]

Back up slides

Interacting Instanton liquid model (IILM)

- **Details of IILM partition function** [7] E. Shuryak (1982), E. Shuryak (1989), T. Schafer, E. Shuryak (1996); (1998), etc.

$$Z = \frac{1}{N_+! \, N_-!} \int \prod_{i=1}^{N_++N_-} d\Omega_i \, f(\rho_i) \, e^{-S_{\text{int}}} \prod_{f=1}^{N_f} \text{Det}(\widehat{D} + m_f)$$

U_{ab}

ρ

z

collective coordinate

$d\Omega_i \equiv d\rho_i d^4 z_i dU_i$: collective coordinates, $f(\rho; \Lambda)$: semiclassical instanton amp.

- instanton interaction

$$S_{\text{int}} = \int d^4x \, G_{\text{top}}^{a\mu\nu} G_{\text{top}}^{a\mu\nu}(x) \approx \sum_{\text{all pairs } (i,j)} S_{\text{int}}^{(2)}(i,j)$$

$(\widehat{D} \equiv \gamma_\mu D_\mu)$

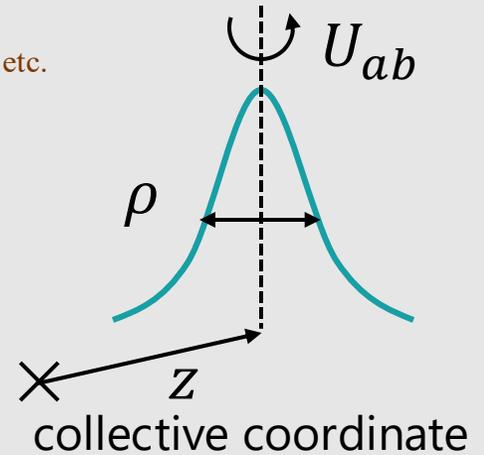Λ: scale parameter fixed such that $n = 1 \text{ fm}^{-4}$ in vacuum.

- instanton quark interaction

$$\text{Det}(\widehat{D} + m_f) \approx \left(\prod_{i=1}^{N_++N_-} 1.34 \, \rho_i \right) \text{Det}_{I,J}(-iT + m_f \mathbf{1})$$
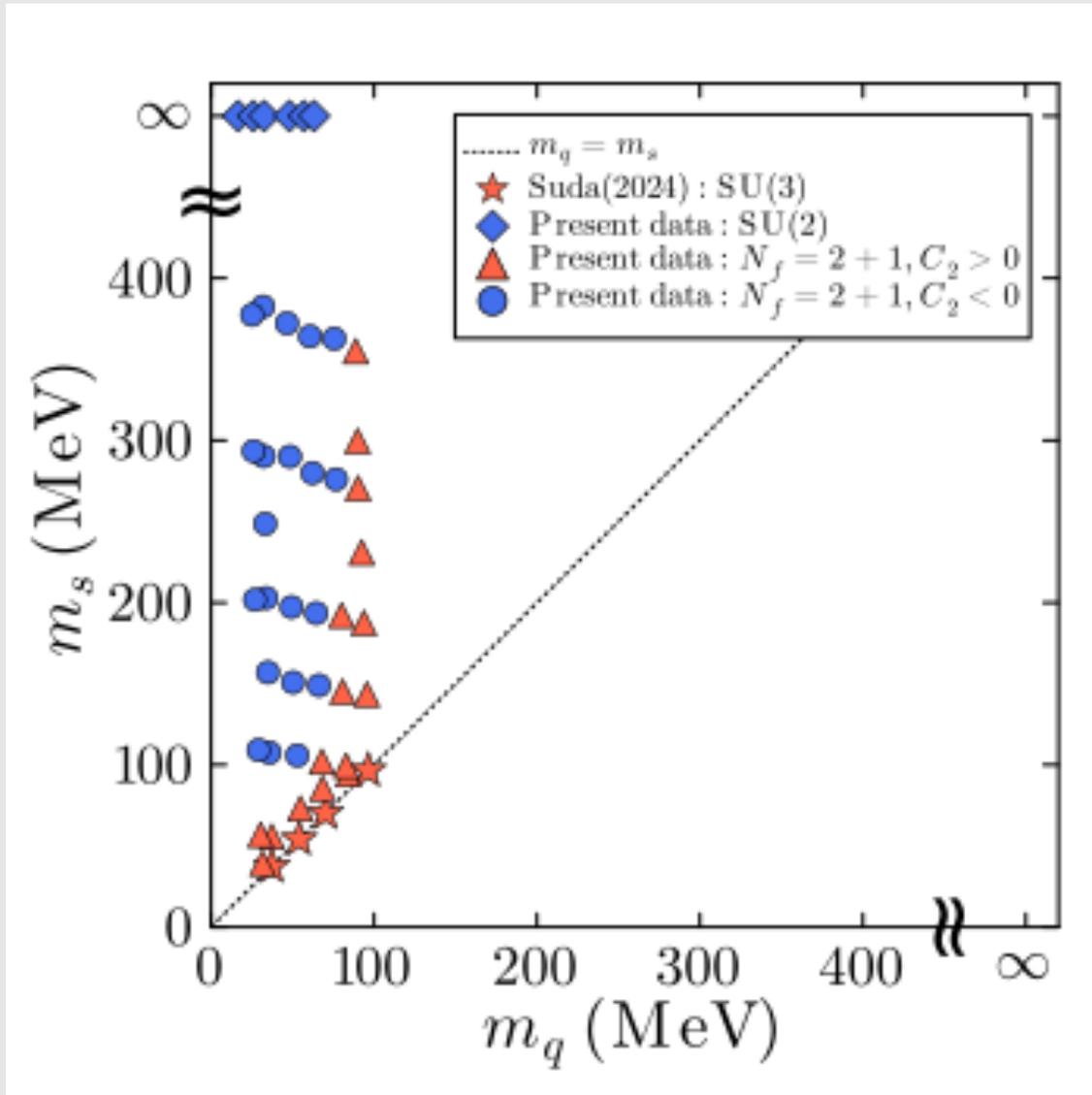
m_f : dynamical quark mass

$$(T)_{IJ} = \int d^4x \, \psi_{0,I}^*(x) i\gamma_\mu D_\mu \psi_{0,J}(x)$$

$\psi_{0,J}(x)$: quark zero-mode wave function in the instanton background

Curvature by IILM on m_q-m_s plane

- **Blue:** ordinary type DChSB
  ($C_2 < 0$ at the quark mass set)

- **Orange:** anomaly-driven type DChSB
  ($C_2 > 0$ at the quark mass set)

- s quark might decouple around
  $m_s = 100$ MeV from the system

- Note that at this stage,
  the physical point $(3.4, 92.2)$ MeV
  has not been accessed

[6] YS, D. Jido (2025)

Curvature in IILM with (2+1)-flavor quarks