Determination of Scattering Length from Lineshape of Cross Sections Depending on Incident Pion Momentum for $\pi^- p \to \phi n$ near Threshold

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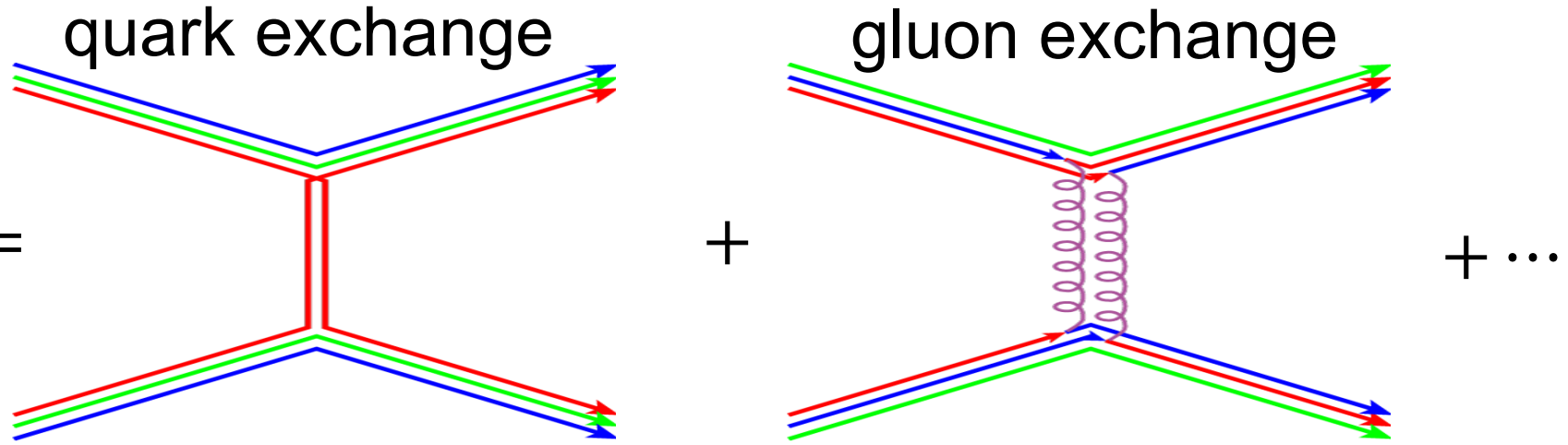
Contents

ϕN interaction

- Hadron interactions =



- Little is known about the gluon exchange.
- Gluon exchange is independent on flavors of the quark.

→ Studies of the gluon dynamics are fundamental and important

- ϕ meson consists of $s\bar{s}$.

→ **ϕN interaction is characterized by gluon exchange at leading order.**

Low energy ϕN scattering

- We have controversial scattering length a and effective range r.

$$p\cot\delta(p) \sim \frac{1}{a} + \frac{1}{2}rp^2$$

Photoproduction weak — I.I. Strakovsky *et al.*,PRC101, 045201 (2020)

$|a| = 0.063 \pm 0.010$ fm

Momentum correlation strongly attractive — ALICE collaboration, PRL 127, 172301 (2021)

$a = \left(0.85 \pm 0.34(\text{stat}) \pm 0.14(\text{syst})\right) + \text{i}\left(0.16 \pm 0.10(\text{stat}) \pm 0.09(\text{syst})\right)$ fm

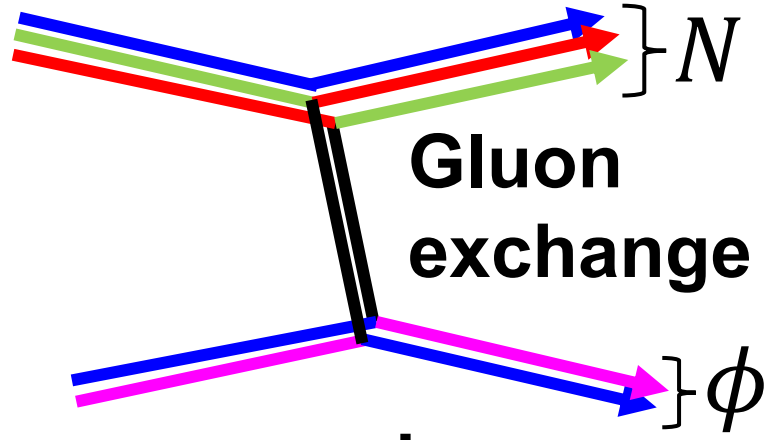
$r = 7.85 \pm 1.54(\text{stat}) \pm 0.26(\text{syst})$ fm

Lattice QCD strongly attractive

$a = 2.49 \pm 0.10^{+0.10}_{-0.48}$ fm

$r = 2.49$ fm

Y. Lyu *et al.*, PRD 106, 074507 (2022)



→ **We do not know whether ϕN interaction is strong or weak.**

$\pi^- p \to \phi n$ utilizing final state interactions

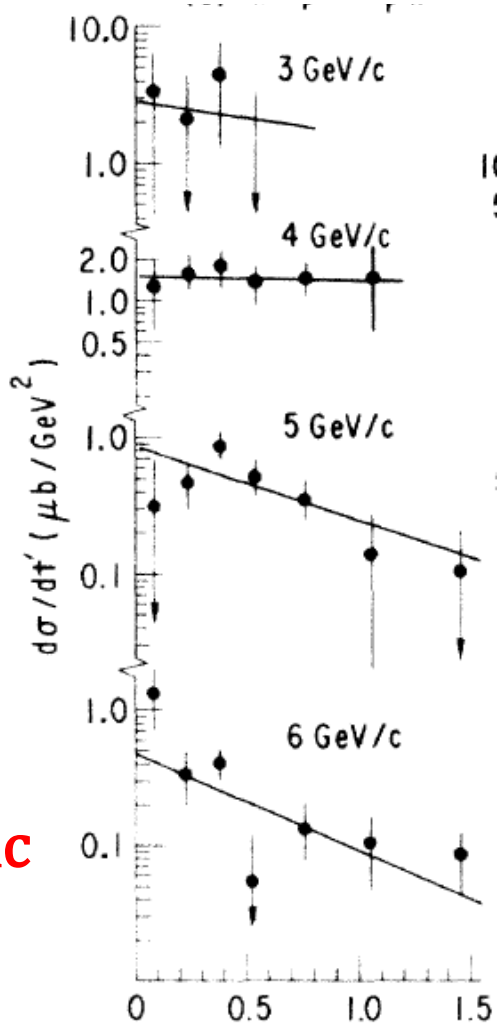
- We plan to evaluate the complex low energy ϕn scattering parameters, using the excitation function of $\pi^- p \to \phi n$ near threshold

- $\pi^- p \to \phi n$ seems to show the isotropic angular distribution.

$\to \pi^- p \to \phi n$ is adequate for the study of the ϕN interaction through final state interactions

- We have estimated it by solving scattering equations with final state interactions.

(low energy ϕN scattering)



$\pi^- p \to \phi n$
almost isotropic

D.S. Ayres et al.,
PRL32, 1463 (1974)

Scattering equation with final state interactions

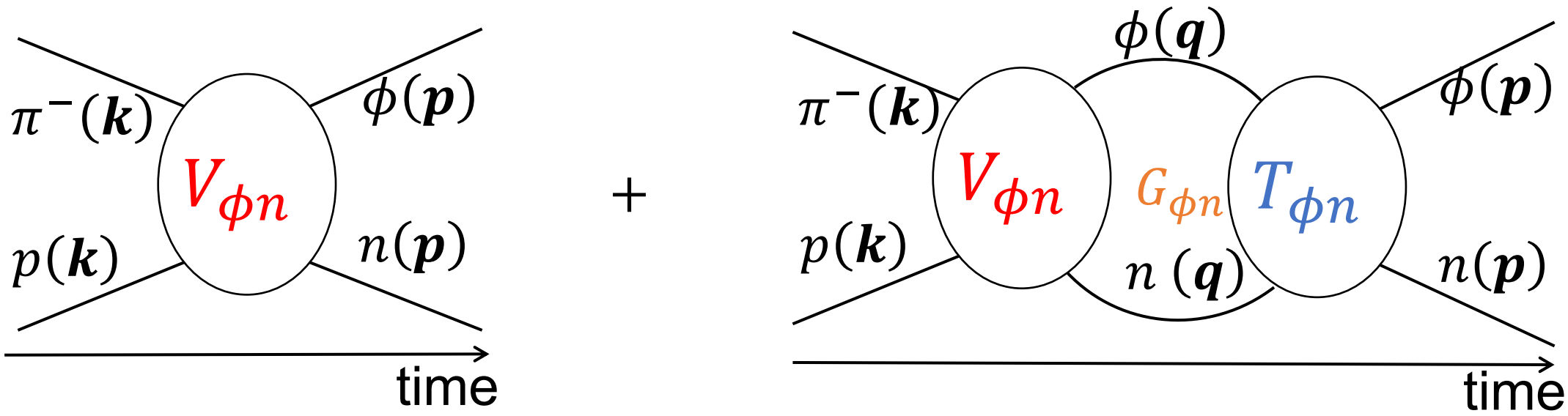
- We consider ϕn generation, followed by final state interactions.
  (low energy ϕn scattering)
- The transition amplitude T is given by

ϕn propagator

$$T = V_{\phi n} + T_{\phi n} G_{\phi n} V_{\phi n}$$

$V_{\phi n}$: ϕn production amplitude

$T_{\phi n}$: ϕn scattering amplitude



Total cross section and scattering parameters

- We approximate

on-shell approximation

$$\langle \phi n(\boldsymbol{p})|T_{\phi n}|\phi n(\boldsymbol{q})\rangle \sim \langle \phi n(\boldsymbol{p})|T_{\phi n}|\phi n(\boldsymbol{p})\rangle \sim -\frac{1}{(2\pi)^2\mu}\left[\frac{1}{a}+\frac{1}{2}rp^2-ip\right]^{-1}$$

ϕn scattering amplitude

Effective range expansion

It depends on ϕN scattering parameters.

$$\langle \phi n(\boldsymbol{q})|G_{\phi n}|\phi n(\boldsymbol{q})\rangle \sim 2\mu \int \frac{4\pi q^2 dq}{p^2-q^2-i\mu\Gamma_\phi}\exp\left[-\frac{q^2}{\Lambda^2}\right]$$

ϕn propagator

Gaussian form factor

$$\langle \phi n(\boldsymbol{q})|V_{\phi n}|\pi^- p(\boldsymbol{k})\rangle \sim \langle \phi n(\boldsymbol{p})|V_{\phi n}|\pi^- p(\boldsymbol{k})\rangle = \text{const.}$$

ϕn production amplitude

on-shell approximation

lineshape is meaningful.

- The total cross section $\sigma(W)$ of the $\pi^- p \to \phi n$ reaction is

$$\sigma(W) = \int_{2m_K}^{W-m_n} \frac{1}{16\pi W^2}\frac{p(W,m)}{k}\,|T(W,m)|^2 L(m)dm$$

Breit Wigner function.

Expected excitation functions

- We normalized total cross sections so that it becomes $25\ \mu\mathrm{b}$ at $p_\pi = 1.8\ \ \mathrm{GeV}/c$.

Photoproduction

$|a_{\phi N}| = 0.063 \pm 0.010$ fm, $r = 0$ fm

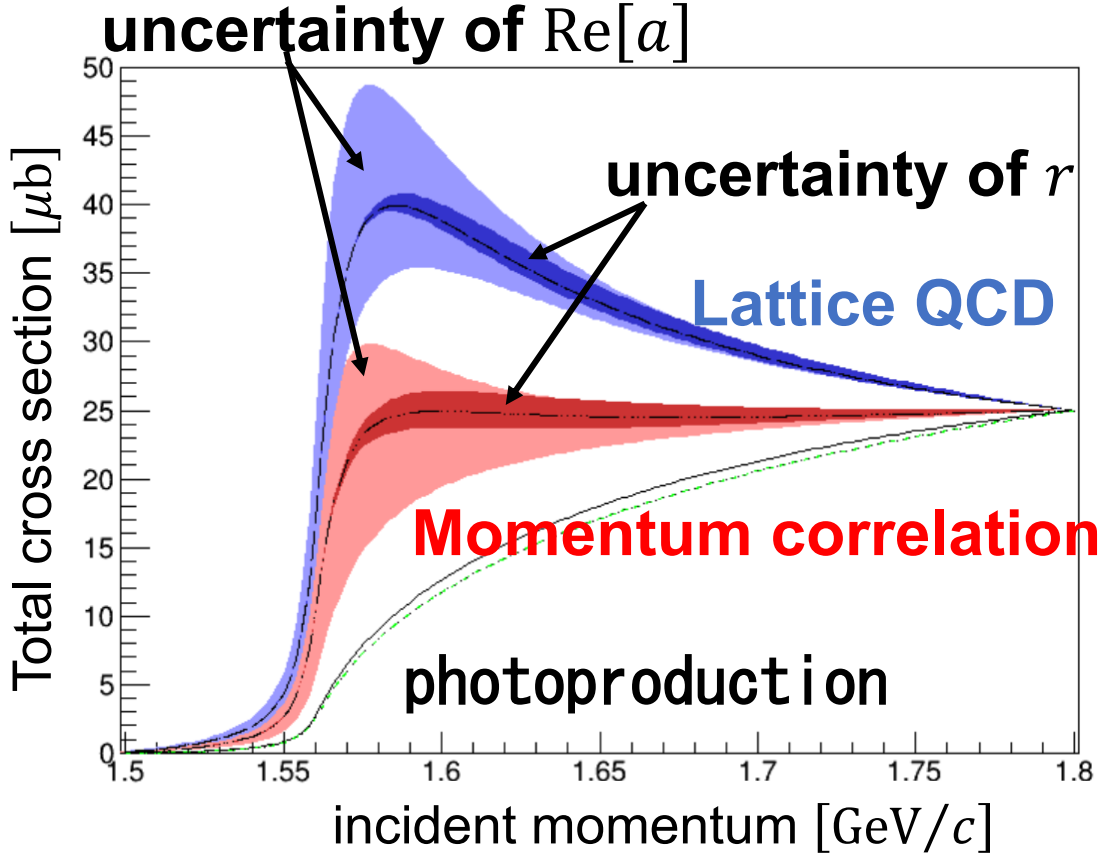
Momentum correlation

$a = (0.85 \pm 0.36) + \mathrm{i}0.16$fm

$r = 7.85 \pm 1.56$ fm

Lattice QCD

$a = 1.25^{+0.42}_{-0.24}$ fm, $r = 2.49^{+0.14}_{-0.49}$ fm



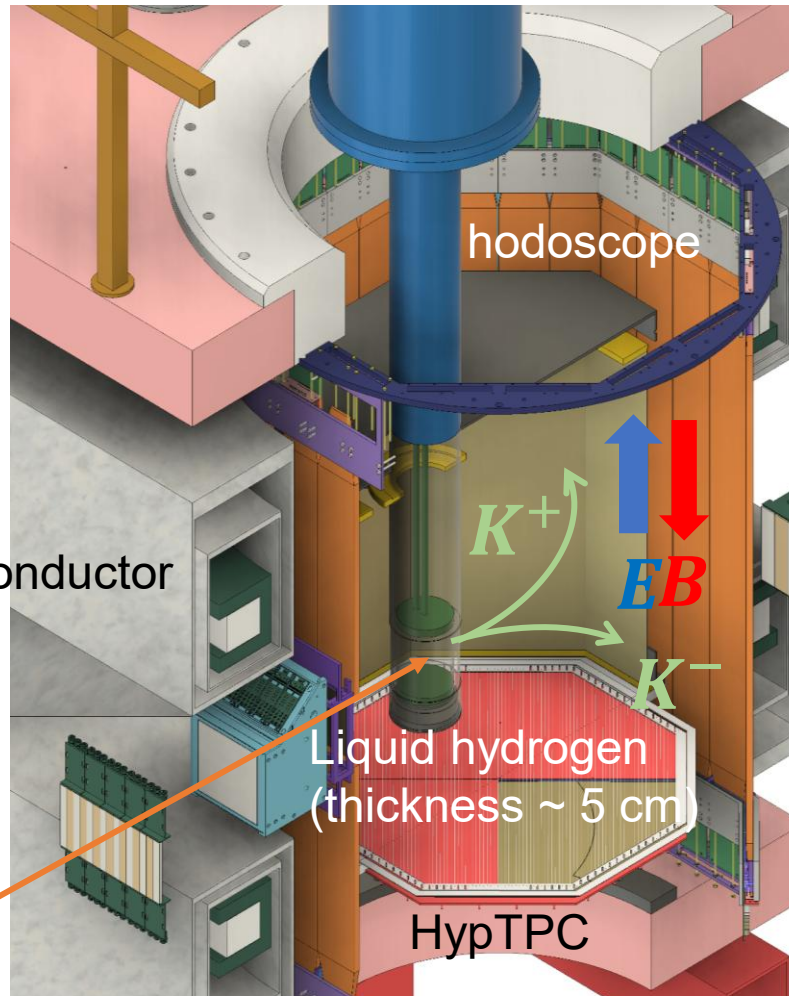
- The shape depends greatly on the values of scattering parameters.

→ **At least, we can distinguish three scattering parameters obtained previously.**

J-PARC E45 experiment

- The main purpose is a baryon spectroscopy using $(\pi, \pi\pi)$ reaction.
- Incident π^- momentum range is $1.5 \sim 1.8\ \mathrm{GeV}/c$
- We measure the total cross section for $\pi^- p \rightarrow \phi n$.
- ϕ is identified by K^+K^- pairs.
- K^+K^- is detected by a 3D trajectory detector, HypTPC.
- HypTPC momentum resolution is $\delta p/p \sim 1\sim4\%$.
- Neutron is identified by the missing mass.
- We assume total cross section $\sigma = 25\ \mu\mathrm{b}$ at $p_\pi = 1.8\ \mathrm{GeV}/c$.

O. I. Dahl *et al.*, PRL **163**, 1377 (1967)
H. Courant *et al.*,PRD **16**, 1 (1977)



Estimation of statistic uncertainty

- We created the simulation data $N_{\mathrm{measured}}(p)$ using Poisson distribution.
- We assume the same incident momentum as J-PARC E45 experiment.
  - momentum bite : $\delta p/p = \pm 2\%$
  - measurement time : 2.96 hours for each momentum set
  - acceptance and branching ratio: 100%

W[MeV]	p[MeV/c]
1940	1536.4
1970	1598.9
2000	1662.4
2025	1716.0
2075	1770.3

- We determined the fitting parameters by minimized χ^2.

$$\chi^2(a, r, \alpha) = \sum_i \frac{\left(N_{\mathrm{measured}}(p_i) - N_{\mathrm{expecte}}(p_i)\right)^2}{N_{\mathrm{measured}}(p_i)}$$

$$N_{\mathrm{expected}}(p) = \alpha \times f(a, r, p)$$ $f(a, r, p)$ is scattering equation

- The fitting parameters are α, $\mathrm{Re}[a]$, $\mathrm{Im}[a]$ and $\mathrm{Re}[r]$.
- $\mathrm{Im}[r]$ is fixed to 0 because we cannot determine $\mathrm{Im}[r]$.

Fitting result

- We assume incident pion momentum distribution is uniform.
- We used $a_0 = 0.85 + i\,0.16$ fm, $r_0 = 7.85$ fm.
- Result

  $\mathrm{Re}[a] = 0.856 \pm 0.015$ fm

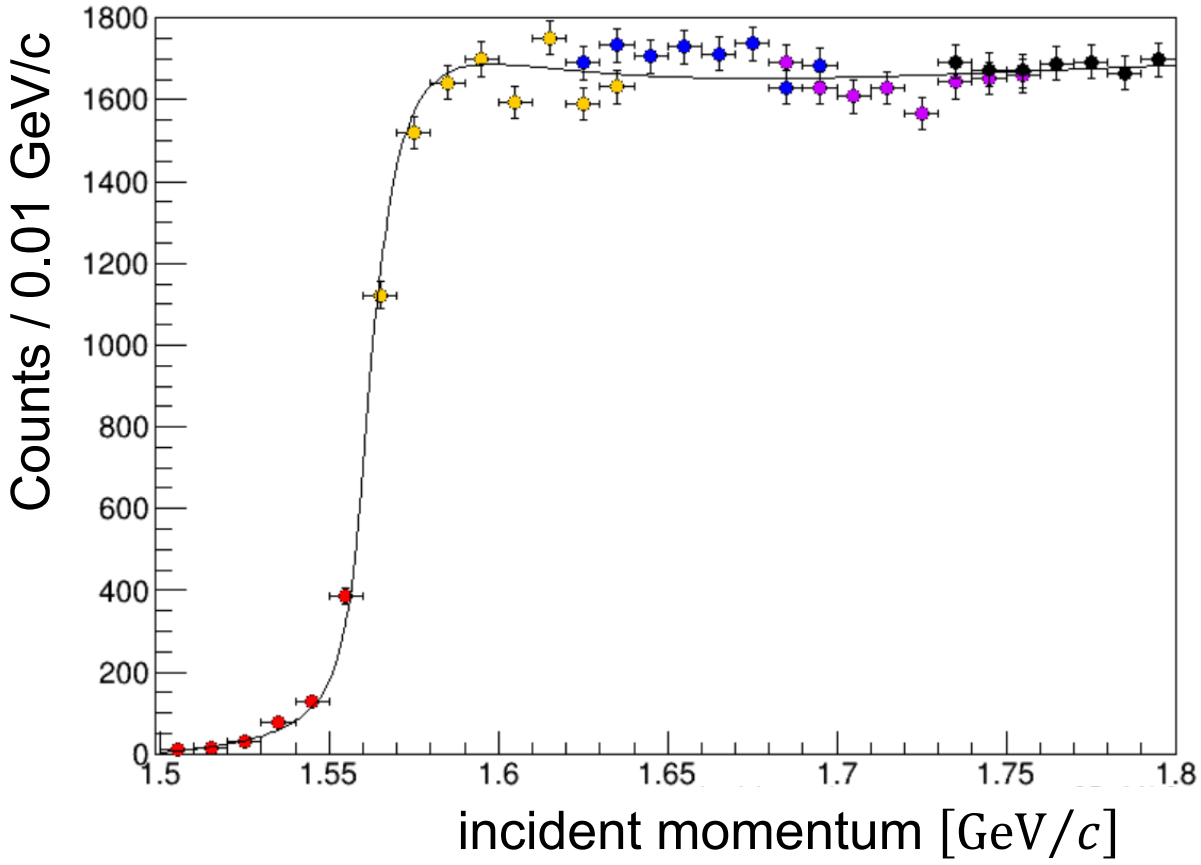
  $\mathrm{Im}[a] = 0.15 \pm 0.08$ fm

  $r = 7.84 \pm 0.24$ fm

  $\alpha = 316.0 \pm 1.4$

  χ^2_{min} / ndf $= 44.52 / 36$

- point : simulation data
- line : best fit



Summary

- We plan to measure the total cross section of $\pi^- p \to \phi n$ under the E45 physics program at J-PARC in order to determine the low energy ϕN scattering parameters.
- We estimate the excitation function of $\pi^- p \to \phi n$ using the scattering equation with a final-state interaction.
- We can determine $\mathrm{Re}\,[a]$ from measurement of total cross sections in about 4 days.
- Estimation of acceptance for identifying the reaction of interest is on-going

Back up

Vector meson dominance model in photoproduction

(I.I. Strakovsky L. Pentchev, A.I. Titov ,PRC101, 045201 (2020))

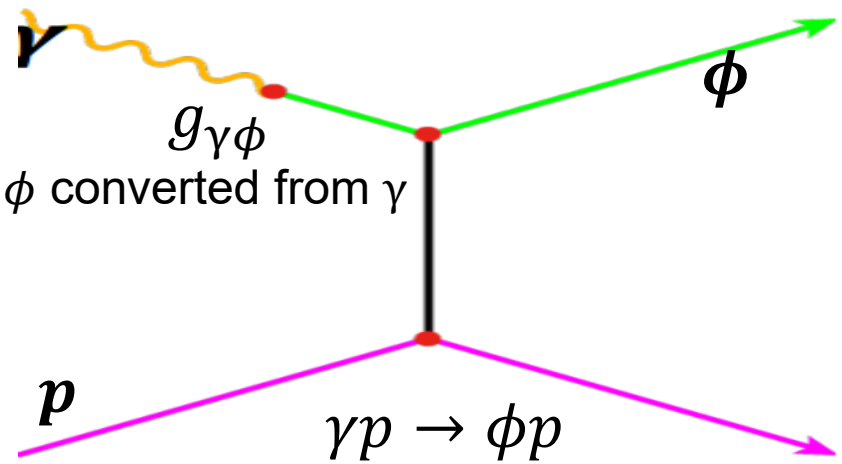
- using the $\gamma p \to \phi p$ reaction
- $|a_{\phi N}| = 0.063 \pm 0.010$ fm

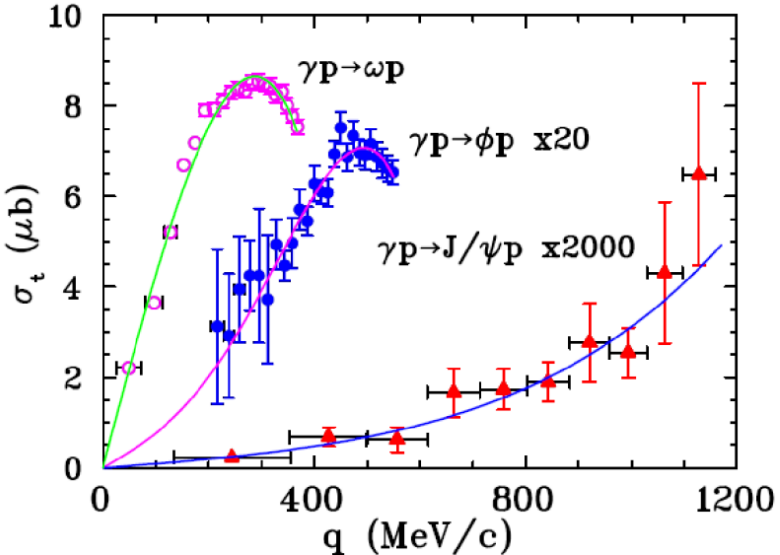
weakly interacting

- photon has a ϕ meson component in some position
- Near threshold, the cross section is expressed

$$\frac{d\sigma_{\gamma p \to \phi p}}{d\Omega} = \frac{q}{k}\frac{1}{64\pi}|T_{\phi p}|^2 = \frac{q}{k}\frac{\pi\alpha}{{g_{\gamma\phi}}^2}\frac{d\sigma_{\gamma p \to \phi p}}{d\Omega} = \frac{q}{k}\frac{\pi\alpha}{{g_{\gamma\phi}}^2}|a_{\phi p}|^2$$

- ϕ meson converted from γ may not be reached equilibrium.



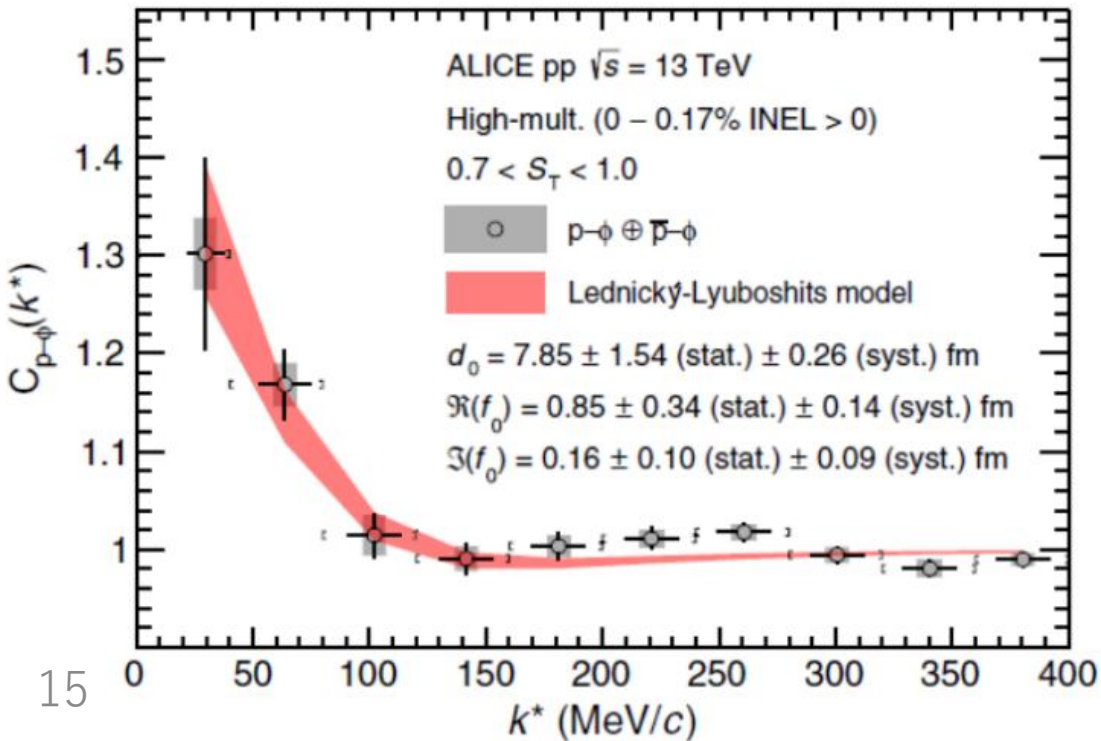


Momentum correlation function in high-energy pp scattering

(ALICE collaboration, PRL 127, 172301 (2021))

- analysis based on the Lednicky-Lyuboshits model
- There is a bump near the relative momentum $\boldsymbol{k^*}\sim\mathbf{0}$ MeV/*c*.
- $\mathrm{Re}[a_{\phi N}] = 0.85 \pm 0.34(\mathrm{stat}) \pm 0.14(\mathrm{syst})$ fm
- $\mathrm{Im}\,[a_{\phi N}] = 0.16 \pm 0.10(\mathrm{stat}) \pm 0.09(\mathrm{syst})$ fm
- $r_{\phi N} = 7.85 \pm 1.54(\mathrm{stat}) \pm 0.26(\mathrm{syst})$ fm

Strong attractive interaction

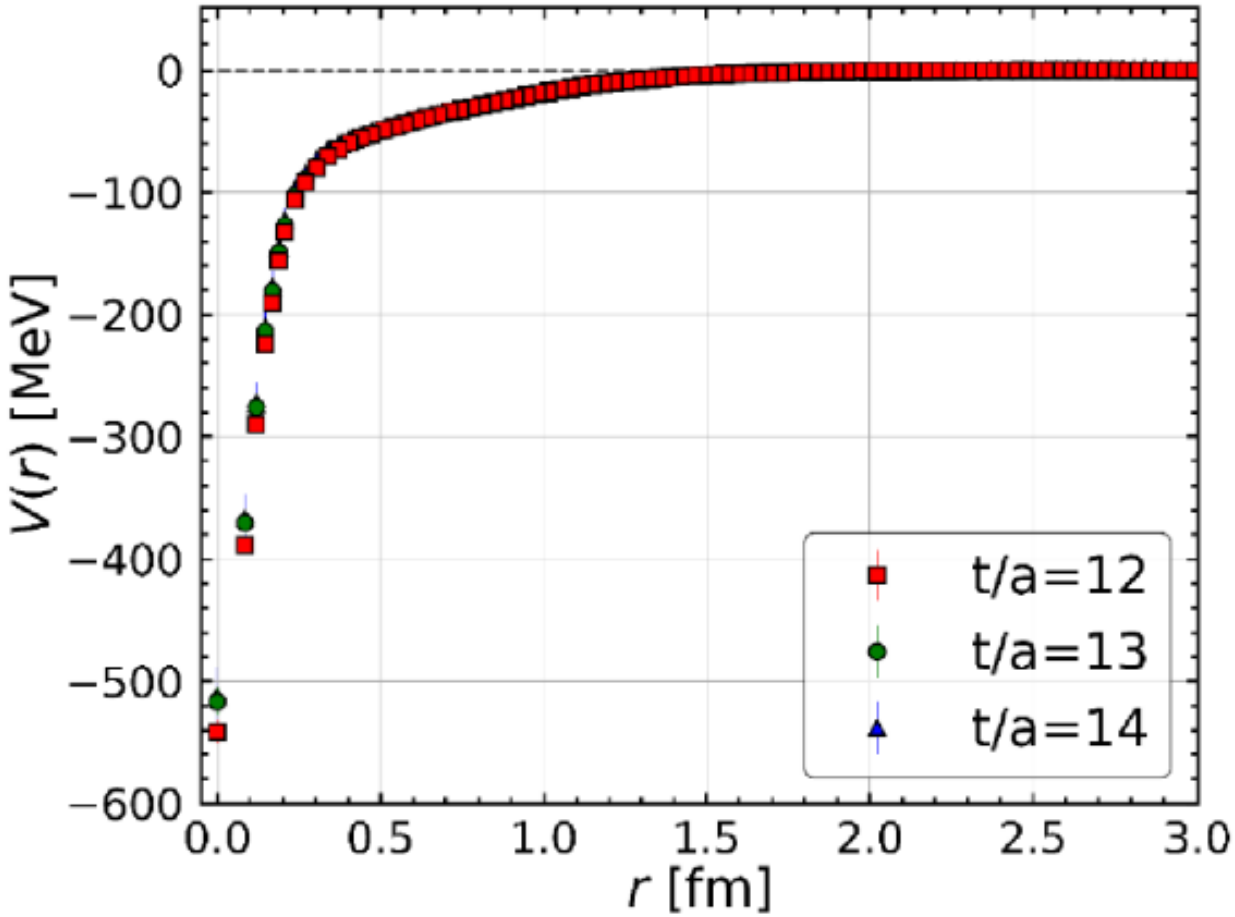


Lattice QCD calculation

(Y. Lyu *et al.*, PRD 106, 074507 (2022))

- HAL QCD method
- $r = 2.49 \pm 0.10^{+0.10}_{-0.48}$fm, $a = 1.25 \pm 0.23^{+0.36}_{-0.06}$ fm
- The most important result is potential $V(r)$.

Strong attractive interaction



$\gamma p \to \phi p$ vs $\pi^- p \to \phi n$

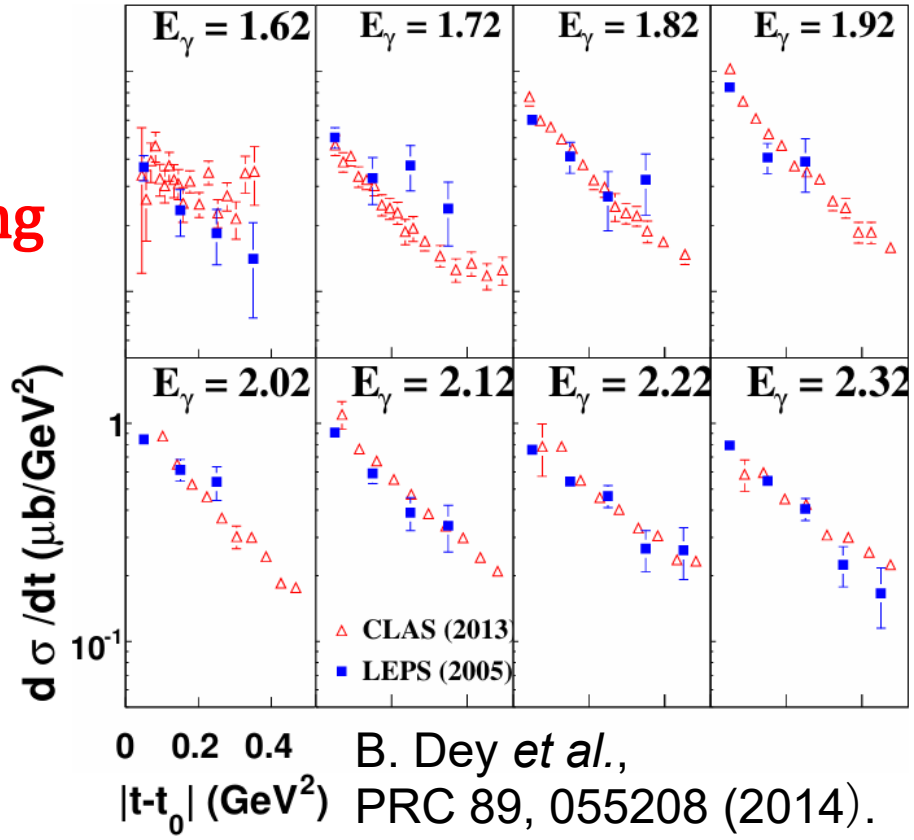
- $\gamma p \to \phi p$ shows the strongly forward peaking the threshold.

→ complicated for the low-energy scattering parameters to be obtained.

- $\pi^- p \to \phi n$ seems to show the isotropic angular distribution.

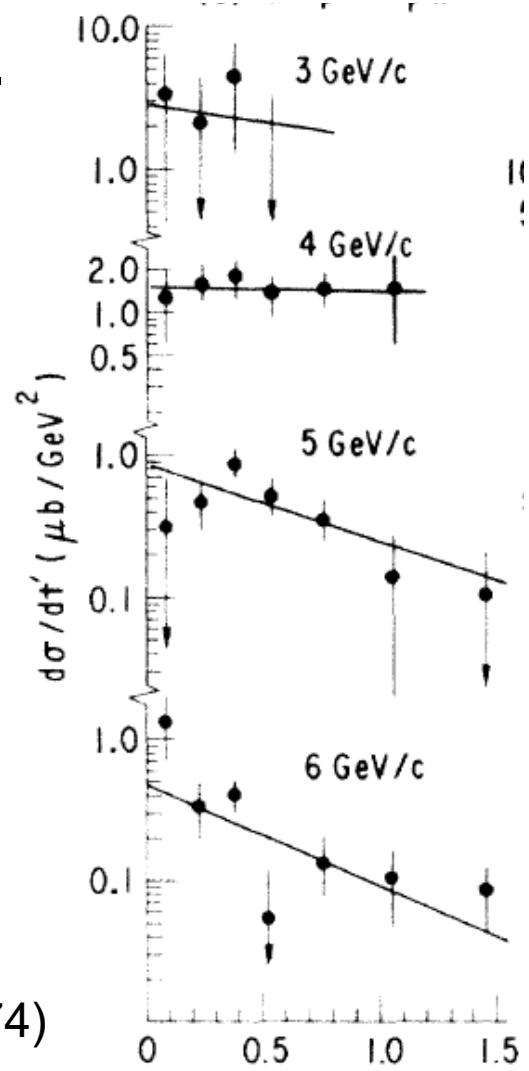
→ S-wave is expected to be dominant.

$\gamma p \to \phi p$
forward peaking



B. Dey *et al.*,
PRC 89, 055208 (2014).

$\pi^- p \to \phi n$
almost isotropic



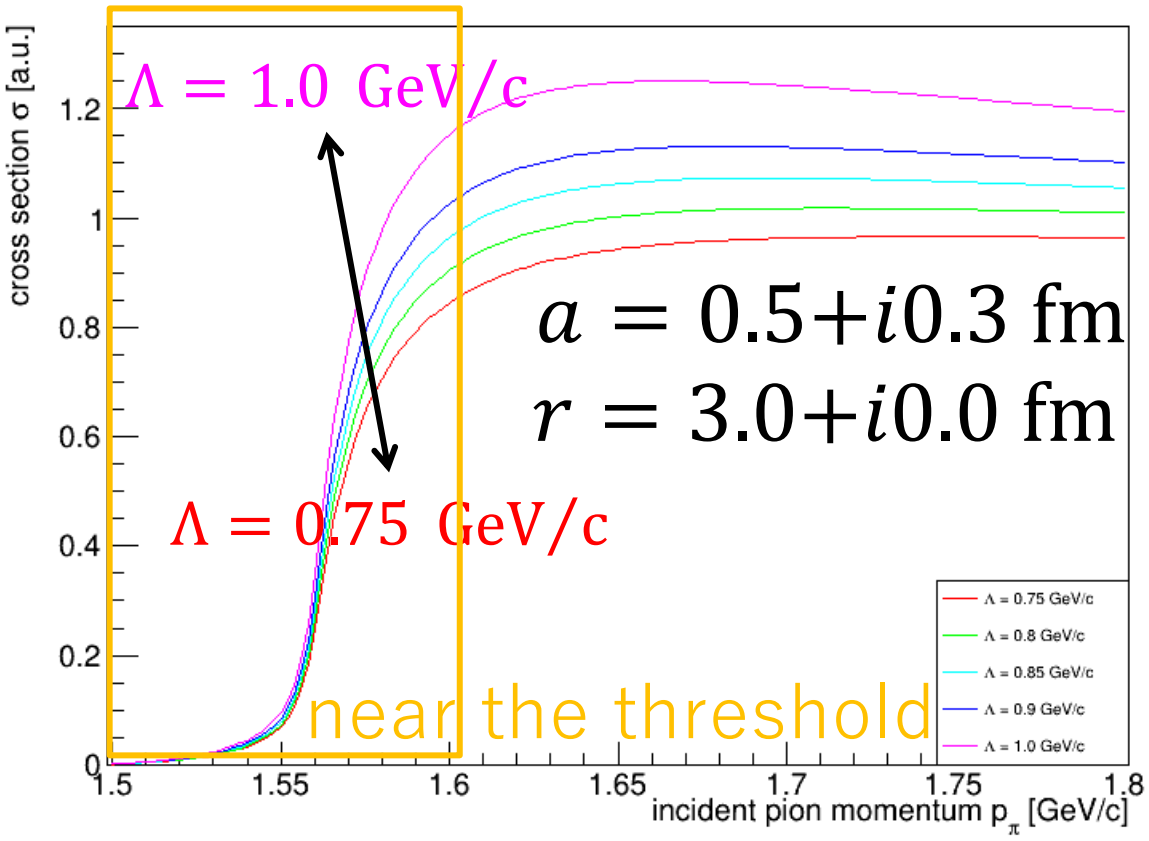
D.S. Ayres et al.,
PRL32, 1463 (1974)

Effect of different cutoff parameters

set to $\Lambda = 0.8$ GeV/c

(corresponds to hadron size ~ 0.6 fm)

We checked whether the excitation function strongly depends on Λ or not.



Different Re$[a]$ parameters

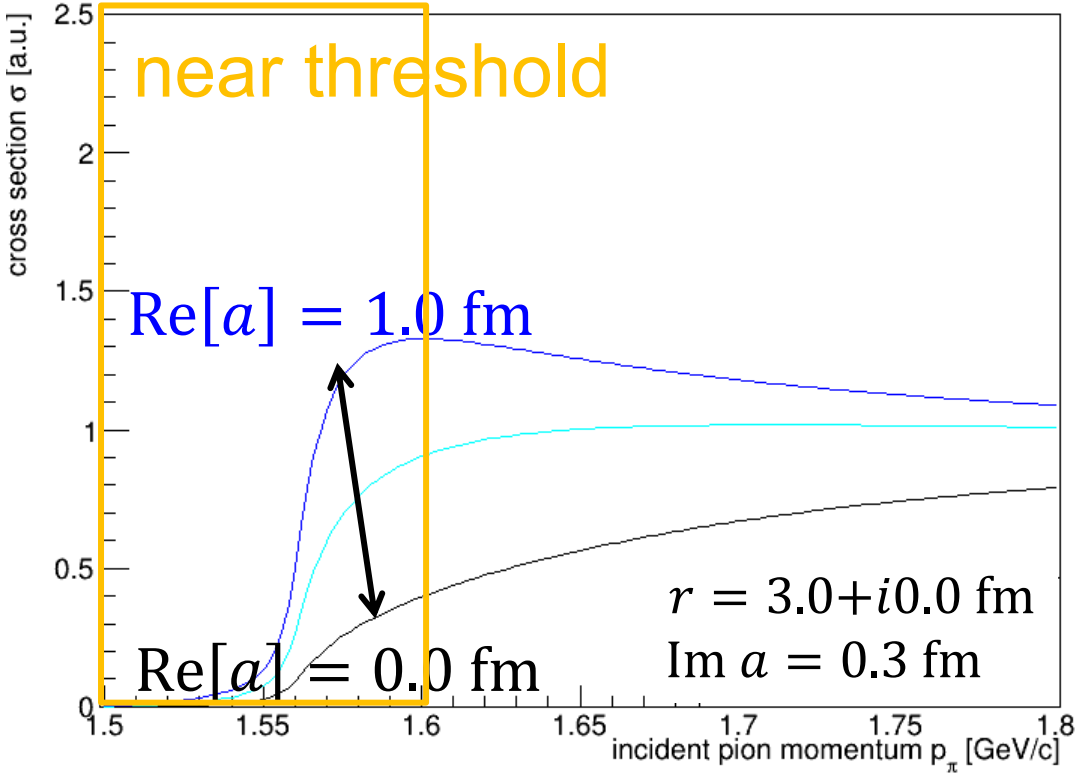
scattering length a = Re$[a]$+i Im$[a]$

running parameter

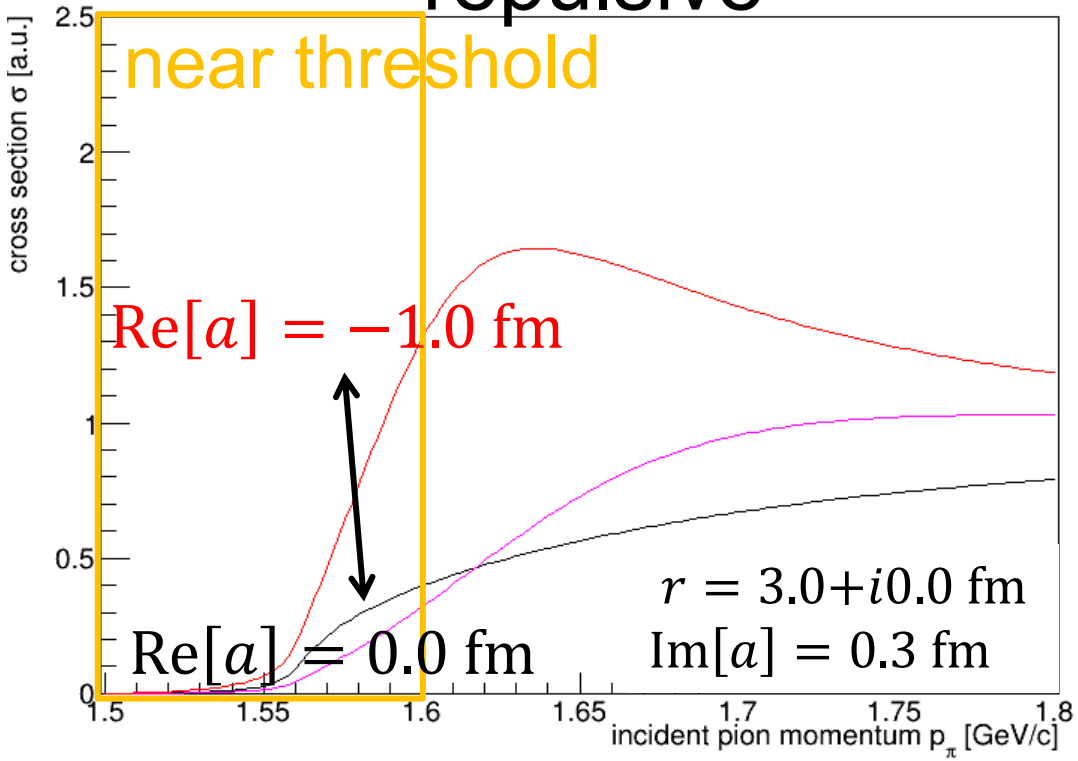
effective range r = Re$[r]$+i Im$[r]$

The line shapes are quite different depending on Re$[a]$.

attractive



repulsive



The other low energy scattering parameters

Even if we use some $\mathrm{Im}[a]$, $\mathrm{Re}[r]$ and $\mathrm{Im}[r]$, no significant change has been obtained.