Structure of the X(3915) within a quark model analysis

In progress for submission to arXiv

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Collaborated with

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BARYONS 2025

- X(3915) was first observed by Belle Collaboration in 2005.

  Phys. Rev. Lett. 94, 182002 (2005)

- X(3915) is a charmonium-like resonance with its quantum numbers $I^G(J^{PC}) = 0^+(0^{++})$.

- $X(3915) \rightarrow \omega J/\Psi$ is dominant → not a conventional $\bar{c}c$.

- Diquark model proposes that X(3915) is the lightest $\bar{c}c\bar{s}s$ state based on its decay pattern and the mass lying slightly below $D_s^+ D_s^-$ threshold.

  Phys. Rev. D 93, 094024 (2016)

- QCD Sum-Rule studies compute masses and decay constants showing X(3915) fits a $\bar{c}c\bar{s}s$ tetraquark.

  Eur. Phys. J. C 77, 78 (2017)   Phys. Rev. D 96, 114017 (2017)

- X(3915) provides insight into the internal structure of exotic hadrons, including other hidden charm exotics.

- We will discuss the internal structure of X(3915) as a $\bar{c}c\bar{s}s$ configuration from the perspective of $D_s^+ D_s^-$ threshold.

Quark Model Description

Hamiltonian

$$H = \sum_{i=1}^{n} \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) - \frac{3}{4} \sum_{i<j}^{n} \frac{\lambda_i^c}{2} \frac{\lambda_j^c}{2} \left(V_{ij}^C + V_{ij}^{CS} \right),$$

where n goes to 4 for tetraquarks.

- Quark potentials

$$V_{ij}^C = -\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0^2} - D,$$

$$V_{ij}^{CS} = -\frac{3}{16} \frac{\hbar^2 c^2 \kappa'}{m_i m_j c^4} \frac{e^{-r_{ij}/r_{0ij}}}{(r_{0ij}) r_{ij}} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

- Additional mass dependences

$$r_{0ij} = 1/ \left(\alpha + \beta \frac{m_i m_j}{m_i + m_j} \right),$$

$$\kappa' = \kappa_0 \left(1 + \gamma \frac{m_i m_j}{m_i + m_j} \right).$$

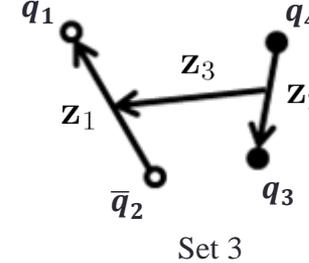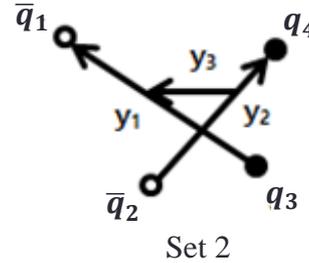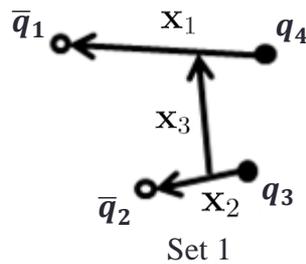- Our model Hamiltonian well describes hadron spectra

Quark Model Description

- Our Hamiltonian is based on two-body interactions $\boxed{\bar{q}_1 = \bar{c},\ \bar{q}_2 = \bar{s},\ q_3 = c,\ q_4 = s \text{ for } X(3915)}$

$$H = \sum_{i=1}^{n} \left(m_i + \frac{\mathbf{p}_i^2}{2m_i} \right) - \frac{3}{4} \sum_{i<j}^{n} \frac{\lambda_i^c}{2} \frac{\lambda_j^c}{2} \left(V_{ij}^C + V_{ij}^{CS} \right),$$

$$\boxed{I^G(J^{PC}) = 0^+(0^{++}).}$$

Set 1   Set 2   Set 3

● : quark
○ : antiquark

- Coordinate Set 1

$$\mathbf{x}_1 = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_4),\ \mathbf{x}_2 = \frac{1}{\sqrt{2}}(\mathbf{r}_2 - \mathbf{r}_3),\ \mathbf{x}_3 = \frac{1}{\mu}\left(\frac{m_1\mathbf{r}_1 + m_4\mathbf{r}_4}{m_1 + m_4} - \frac{m_2\mathbf{r}_2 + m_3\mathbf{r}_3}{m_2 + m_3} \right)$$

- Coordinate Set 2

$$\mathbf{y}_1 = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_3),\ \mathbf{y}_2 = \frac{1}{\sqrt{2}}(\mathbf{r}_4 - \mathbf{r}_2),\ \mathbf{y}_3 = \frac{1}{\mu}\left(\frac{m_1\mathbf{r}_1 + m_3\mathbf{r}_3}{m_1 + m_3} - \frac{m_2\mathbf{r}_2 + m_4\mathbf{r}_4}{m_2 + m_4} \right)$$

- Coordinate Set 3

$$\mathbf{z}_1 = \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2),\ \mathbf{z}_2 = \frac{1}{\sqrt{2}}(\mathbf{r}_3 - \mathbf{r}_4),\ \mathbf{z}_3 = \frac{1}{\mu}\left(\frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2} - \frac{m_3\mathbf{r}_3 + m_4\mathbf{r}_4}{m_3 + m_4} \right)$$

where

$$\mu = \left[\frac{m_1^2 + m_4^2}{(m_1 + m_4)^2} + \frac{m_2^2 + m_3^2}{(m_2 + m_3)^2} \right]^{1/2}$$

Under permutations (13)(24)
$$\mathbf{x}_1 \rightarrow -\mathbf{x}_2$$
$$\mathbf{x}_2 \rightarrow -\mathbf{x}_1$$

- Spatial Function

$$\psi^{Spatial}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = \left(\frac{2}{\pi} \right)^{\frac{9}{4}} a_1^{\frac{3}{4}} a_1^{\frac{3}{4}} a_3^{\frac{3}{4}} \exp\left[-a_1\mathbf{x}_1^2 - a_1\mathbf{x}_2^2 - a_3\mathbf{x}_3^2 \right]$$

Fitting Results

Table I. Measured and fitted masses of mesons.

Mesons	Experimental value (MeV)	Mass (MeV)	Variational parameter (fm^{-2})	Error (%)
D	1864.8	1860.4	$a = 4.4$	0.24
D^*	2007.0	2005.2	$a = 3.5$	0.08
η_c	2983.6	2995.4	$a = 14.3$	0.39
J/Ψ	3096.9	3117.5	$a = 11.0$	0.67
D_s	1968.3	1961.0	$a = 7.0$	0.37
D_s^*	2112.1	2097.6	$a = 5.4$	0.69
K	493.68	503.7	$a = 14.3$	1.23
K^*	891.66	875.0	$a = 2.6$	1.87
B	5279.3	5283.5	$a = 4.3$	0.07
B^*	5325.2	5339.2	$a = 3.9$	0.27
η_b	9398.0	9370.6	$a = 52.7$	0.30
Υ	9460.3	9485.7	$a = 40.8$	0.27
B_s	5366.8	5351.2	$a = 7.4$	0.29
B_s^*	5415.4	5410.3	$a = 6.6$	0.09
B_c	6275.6	6273.2	$a = 21.2$	0.02
B_c^*	$\cdots$	6357.7	$a = 17.4$	$\cdots$

Table II. Measured and fitted masses of baryons.

Baryons	Experimental value (MeV)	Mass (MeV)	Variational parameters (fm^{-2})	Error (%)
Λ	1115.7	1112.0	$a_1 = 2.6,\ a_2 = 2.6$	0.33
Λ_c	2286.5	2265.4	$a_1 = 2.7,\ a_2 = 3.5$	0.92
Ξ_{cc}	3621.4	3609.6	$a_1 = 7.3,\ a_2 = 3.0$	0.33
Λ_b	5619.4	5608.8	$a_1 = 2.7,\ a_2 = 3.9$	0.19
Σ_c	2452.9	2444.6	$a_1 = 2.0,\ a_2 = 3.5$	0.37
Σ_c^*	2517.5	2528.2	$a_1 = 1.8,\ a_2 = 3.1$	0.39
Σ_b	5811.3	5825.5	$a_1 = 1.9,\ a_2 = 3.8$	0.17
Σ_b^*	5832.1	5858.2	$a_1 = 1.9,\ a_2 = 3.6$	0.40
Σ	1192.6	1199.7	$a_1 = 2.0,\ a_2 = 2.9$	0.59
Σ^*	1383.7	1405.3	$a_1 = 1.7,\ a_2 = 2.2$	1.56
Ξ	1314.9	1316.6	$a_1 = 3.2,\ a_2 = 2.7$	0.13
Ξ^*	1531.8	1532.7	$a_1 = 2.8,\ a_2 = 2.1$	0.06
Ξ_c	2467.8	2459.5	$a_1 = 3.1,\ a_2 = 4.4$	0.44
Ξ_c^*	2645.9	2639.9	$a_1 = 2.3,\ a_2 = 4.0$	0.24
Ξ_b	5787.8	5790.3	$a_1 = 3.1,\ a_2 = 5.2$	0.03
Ξ_b^*	5945.5	5958.0	$a_1 = 2.4,\ a_2 = 4.9$	0.04
p	938.27	958.93	$a_1 = 2.2,\ a_2 = 2.2$	2.20
Δ	1232	1269.5	$a_1 = 1.7,\ a_2 = 1.7$	3.04

$$V_{ij}^{CS} = -\frac{3}{16}\frac{\hbar^2 c^2 \kappa'}{m_i m_j c^4}\frac{e^{-r_{ij}/r_{0ij}}}{(r_{0ij})r_{ij}}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \lambda_i^c \lambda_j^c$$

where

$$r_{0ij} = 1/\left(\alpha + \beta\frac{m_i m_j}{m_i + m_j}\right), \quad \kappa' = \kappa_0\left(1 + \gamma\frac{m_i m_j}{m_i + m_j}\right).$$

$\kappa = 107.7\,\text{MeV fm}, \quad a_0 = 0.0327338\,(\text{MeV}^{-1}\text{fm})^{1/2},$

$D = 950\,\text{MeV},$

$m_u = 320\,\text{MeV}, \qquad m_s = 612\,\text{MeV},$

$m_c = 1893\,\text{MeV}, \qquad m_b = 5285\,\text{MeV},$

$\alpha = 1.1349\,\text{fm}^{-1}, \ \beta = 0.0011554\,(\text{MeV fm})^{-1},$

$\gamma = 0.001370\,\text{MeV}^{-1}, \ \kappa_0 = 230.244\,\text{MeV}.$

For $\bar{c}c\bar{s}s$ with $I^G(J^{PC}) = 0^+(0^{++})$

- Color Space

 - Decomposition in terms of color SU(3)

$$[\mathbf{\bar{3}}]_{\bar{c}} \otimes [\mathbf{3}]_c \otimes [\mathbf{\bar{3}}]_{\bar{s}} \otimes [\mathbf{3}]_s = ([\mathbf{1}]_{\bar{c}c} \oplus [\mathbf{8}]_{\bar{c}c}) \otimes ([\mathbf{1}]_{\bar{s}s} \oplus [\mathbf{8}]_{\bar{s}s})$$
$$= \boxed{([\mathbf{1}]_{\bar{c}c} \otimes [\mathbf{1}]_{\bar{s}s})} \oplus ([\mathbf{1}]_{\bar{c}c} \otimes [\mathbf{8}]_{\bar{s}s}) \oplus ([\mathbf{8}]_{\bar{c}c} \otimes [\mathbf{1}]_{\bar{s}s}) \oplus \boxed{([\mathbf{8}]_{\bar{c}c} \otimes [\mathbf{8}]_{\bar{s}s})}$$
$$\boxed{(\bar{c}c)^{\mathbf{1}} \otimes (\bar{s}s)^{\mathbf{1}} \, , \quad (\bar{c}c)^{\mathbf{8}} \otimes (\bar{s}s)^{\mathbf{8}}}$$

- Spin Space

 - Decomposition in terms of spin SU(2)

$$[2] \otimes [2] \otimes [2] \otimes [2] = \underline{[5]} \oplus \underline{[3] \oplus [3] \oplus [3]} \oplus \underline{[1] \oplus [1]}.$$

| Spin 2 | Spin 1 | Spin 0 |

- For Spin 0, $\boxed{(\bar{c}c)_0 \otimes (\bar{s}s)_0 \, , \quad (\bar{c}c)_1 \otimes (\bar{s}s)_1}$ ← Satisfying charge conjugation (+)

 All possible 4 CS bases satisfy the symmetry of X(3915)

- For Spin 1, $(\bar{c}c)_0 \otimes (\bar{s}s)_1 \, , \quad (\bar{c}c)_1 \otimes (\bar{s}s)_0 \, , \quad \boxed{(\bar{c}c)_1 \otimes (\bar{s}s)_1}$ ← Satisfying charge conjugation (+)

 If $s \to q$, CS bases correspond to X(3872) with $I^G(J^{PC}) = 0^+(1^{++})$

 Only 2 of 6 CS bases satisfy the symmetry of X(3872)

Color-Spin States of X(3915)

- ## Color-Spin Bases of X(3915)

$$\left|CS_1'\right\rangle = \left|(\bar{c}c)_0^{\mathbf{1}}(\bar{s}s)_0^{\mathbf{1}}\right\rangle, \; \left|CS_2'\right\rangle = \left|(\bar{c}c)_0^{\mathbf{8}}(\bar{s}s)_0^{\mathbf{8}}\right\rangle$$
$$\left|CS_3'\right\rangle = \left|(\bar{c}c)_1^{\mathbf{1}}(\bar{s}s)_1^{\mathbf{1}}\right\rangle, \; \left|CS_4'\right\rangle = \left|(\bar{c}c)_1^{\mathbf{8}}(\bar{s}s)_1^{\mathbf{8}}\right\rangle$$

$$\left|CS_1\right\rangle = \left|(\bar{c}s)_1^{\mathbf{1}}(\bar{s}c)_1^{\mathbf{1}}\right\rangle, \; \left|CS_2\right\rangle = \left|(\bar{c}s)_0^{\mathbf{1}}(\bar{s}c)_0^{\mathbf{1}}\right\rangle$$
$$\left|CS_3\right\rangle = \left|(\bar{c}s)_1^{\mathbf{8}}(\bar{s}c)_1^{\mathbf{8}}\right\rangle, \; \left|CS_4\right\rangle = \left|(\bar{c}s)_0^{\mathbf{8}}(\bar{s}c)_0^{\mathbf{8}}\right\rangle$$

More appropriate for investigating the properties of $D_s^+ D_s^-$ threshold

- ## Transformation between CS Basis Sets

$$|CS_1'\rangle = -\frac{1}{2\sqrt{3}}|CS_1\rangle + \frac{1}{6}|CS_2\rangle - \sqrt{\frac{2}{3}}|CS_3\rangle + \frac{\sqrt{2}}{3}|CS_4\rangle$$

$$|CS_2'\rangle = -\sqrt{\frac{2}{3}}|CS_1\rangle + \frac{\sqrt{2}}{3}|CS_2\rangle + \frac{1}{2\sqrt{3}}|CS_3\rangle - \frac{1}{6}|CS_4\rangle$$

$$|CS_3'\rangle = -\frac{1}{6}|CS_1\rangle - \frac{1}{2\sqrt{3}}|CS_2\rangle - \frac{\sqrt{2}}{3}|CS_3\rangle - \sqrt{\frac{2}{3}}|CS_4\rangle$$

$$|CS_4'\rangle = -\frac{\sqrt{2}}{3}|CS_1\rangle - \sqrt{\frac{2}{3}}|CS_2\rangle + \frac{1}{6}|CS_3\rangle + \frac{1}{2\sqrt{3}}|CS_4\rangle$$

$$|CS_1\rangle = -\frac{1}{2\sqrt{3}}|CS_1'\rangle - \sqrt{\frac{2}{3}}|CS_2'\rangle - \frac{1}{6}|CS_3'\rangle - \frac{\sqrt{2}}{3}|CS_4'\rangle$$

$$|CS_2\rangle = \frac{1}{6}|CS_1'\rangle + \frac{\sqrt{2}}{3}|CS_2'\rangle - \frac{1}{2\sqrt{3}}|CS_3'\rangle - \sqrt{\frac{2}{3}}|CS_4'\rangle,$$

$$|CS_3\rangle = -\sqrt{\frac{2}{3}}|CS_1'\rangle + \frac{1}{2\sqrt{3}}|CS_2'\rangle - \frac{\sqrt{2}}{3}|CS_3'\rangle + \frac{1}{6}|CS_4'\rangle$$

$$|CS_4\rangle = \frac{\sqrt{2}}{3}|CS_1'\rangle - \frac{1}{6}|CS_2'\rangle - \sqrt{\frac{2}{3}}|CS_3'\rangle + \frac{1}{2\sqrt{3}}|CS_4'\rangle.$$

- ## Color-Spin Interaction Factor

$$V_{ij}^{CS} = -\frac{3}{16}\frac{\hbar^2 c^2 \kappa'}{m_i m_j c^4}\frac{e^{-r_{ij}/r_{0ij}}}{(r_{0ij})r_{ij}}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \lambda_i^c \lambda_j^c \longrightarrow K = -\sum_{i<j}^{n}\frac{1}{m_i m_j}\lambda_i^c \lambda_j^c \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$
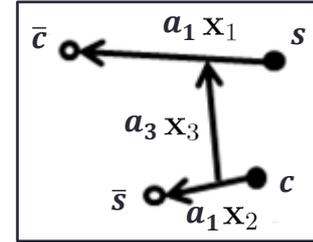
H. Yun, S. Noh, A. Park, S. Cho, S. H. Lee, et al, Phys. Rev. C 107, 014906 (2023)

Table III. Masses of X(3915) and the corresponding thresholds, obtained through variational method and our model Hamiltonian.

Configuration	J^{PC}	Threshold(MeV)	Measured mass(MeV)	Mass(MeV)	Variational parameters (fm^{-2})
$\bar{c}\bar{s}cs$	0^{++}	$D_s^+ D_s^-$ (3922.0)	3922.1	3922.1	$a_1 = 7.0, a_3 = 0.001$

$a_3 \to 0$

- ## Color-Spin Interaction Factor

$$K = -\sum_{i<j}^{n} \frac{1}{m_i m_j} \lambda_i^c \lambda_j^c \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

$K_{X(3915)} =$

$$
\begin{pmatrix}
\frac{16}{3}\left(\frac{1}{m_{\bar{c}}m_s}+\frac{1}{m_{\bar{s}}m_c}\right) & 0 & \frac{8\sqrt{2}(m_{\bar{c}}-m_s)(m_{\bar{s}}-m_c)}{3m_{\bar{c}}m_{\bar{s}}m_c m_s} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} \\
0 & -\frac{16}{m_{\bar{c}}m_s}-\frac{16}{m_{\bar{s}}m_c} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & 0 \\
\frac{8\sqrt{2}(m_{\bar{c}}-m_s)(m_{\bar{s}}-m_c)}{3m_{\bar{c}}m_{\bar{s}}m_c m_s} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & -\frac{8m_c m_s+(28m_c+2m_s+8m_{\bar{s}})m_{\bar{c}}+(2m_c+28m_s)m_{\bar{s}}}{3m_{\bar{c}}m_{\bar{s}}m_c m_s} & -\frac{4m_c m_s-14m_{\bar{c}}m_c-14m_{\bar{s}}m_s+4m_{\bar{c}}m_{\bar{s}}}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} \\
\frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & 0 & -\frac{4m_c m_s-14m_{\bar{c}}m_c-14m_{\bar{s}}m_s+4m_{\bar{c}}m_{\bar{s}}}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & \frac{2}{m_{\bar{c}}m_s}+\frac{2}{m_{\bar{s}}m_c}
\end{pmatrix}
$$

where $|CS_1\rangle = \left|(\bar{c}s)_1^{\mathbf{1}}(\bar{s}c)_1^{\mathbf{1}}\right\rangle$, $|CS_2\rangle = \left|(\bar{c}s)_0^{\mathbf{1}}(\bar{s}c)_0^{\mathbf{1}}\right\rangle$, $|CS_3\rangle = \left|(\bar{c}s)_1^{\mathbf{8}}(\bar{s}c)_1^{\mathbf{8}}\right\rangle$, $|CS_4\rangle = \left|(\bar{c}s)_0^{\mathbf{8}}(\bar{s}c)_0^{\mathbf{8}}\right\rangle$

$m_s = m_{\bar{s}} = 0.5 \text{ GeV}, \; m_c = m_{\bar{c}} = 1.5 \text{ GeV}$

$$
K_{X(3915)} =
\begin{pmatrix}
14.2 & 0 & -6.70 & 23.2 \\
0 & -42.7 & 23.2 & 0 \\
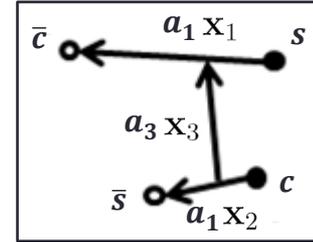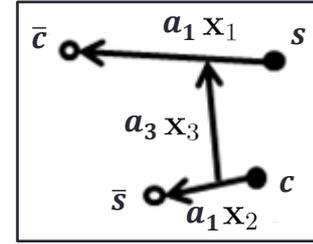-6.70 & 23.2 & -50.4 & 29.8 \\
23.2 & 0 & 29.8 & 5.33
\end{pmatrix}
$$

Color-Spin Structure of X(3915)

Table III. Masses of X(3915) and the corresponding thresholds, obtained through variational method and our model Hamiltonian.

Configuration	J^{PC}	Threshold(MeV)	Measured mass(MeV)	Mass(MeV)	Variational parameters (fm^{-2})
$\bar{c}\bar{s}cs$	0^{++}	$D_s^+ D_s^- (3922.0)$	3922.1	3922.1	$a_1 = 7.0, a_3 = 0.001$

$a_3 \to 0$

- **Color-Spin Interaction Factor**

$$K = -\sum_{i<j}^{n} \frac{1}{m_i m_j} \lambda_i^c \lambda_j^c \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

$K_{X(3915)} =$

$$
\begin{pmatrix}
\frac{16}{3}\left(\frac{1}{m_{\bar{c}}m_s} + \frac{1}{m_{\bar{s}}m_c}\right) & 0 & \frac{8\sqrt{2}(m_{\bar{c}}-m_s)(m_{\bar{s}}-m_c)}{3m_{\bar{c}}m_{\bar{s}}m_c m_s} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} \\
0 & -\frac{16}{m_{\bar{c}}m_s} - \frac{16}{m_{\bar{s}}m_c} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & 0 \\
\frac{8\sqrt{2}(m_{\bar{c}}-m_s)(m_{\bar{s}}-m_c)}{3m_{\bar{c}}m_{\bar{s}}m_c m_s} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & \frac{(2m_s+8m_{\bar{s}})m_{\bar{c}}+(2m_c+28m_s)m_{\bar{s}}}{3m_{\bar{c}}m_{\bar{s}}m_c m_s} & -\frac{4m_c m_s - 14m_{\bar{c}}m_c - 14m_{\bar{s}}m_s + 4m_{\bar{c}}m_{\bar{s}}}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} \\
\frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & 0 & -\frac{4m_c m_s - 14m_{\bar{c}}m_c - 14m_{\bar{s}}m_s + 4m_{\bar{c}}m_{\bar{s}}}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & \frac{2}{m_{\bar{c}}m_s} + \frac{2}{m_{\bar{s}}m_c}
\end{pmatrix}
$$

where $|CS_1\rangle = \left|(\bar{c}s)_1^{\mathbf{1}}(\bar{s}c)_1^{\mathbf{1}}\right\rangle$, $|CS_2\rangle = \left|(\bar{c}s)_0^{\mathbf{1}}(\bar{s}c)_0^{\mathbf{1}}\right\rangle$, $|CS_3\rangle = \left|(\bar{c}s)_1^{\mathbf{8}}(\bar{s}c)_1^{\mathbf{8}}\right\rangle$, $|CS_4\rangle = \left|(\bar{c}s)_0^{\mathbf{8}}(\bar{s}c)_0^{\mathbf{8}}\right\rangle$

- **Color-Spin Part of Hamiltonian** ($a_3 = 0.001$)

$$
\left\langle \sum_{i<j}^{4} V_{ij}^{CS} \right\rangle =
\begin{pmatrix}
74.99 & 0 & -0.41 & 0.36 \\
0 & -224.95 & 0.36 & 0 \\
-0.41 & 0.36 & -10.39 & 0.88 \\
0.36 & 0 & 0.88 & 28.12
\end{pmatrix}
$$

$$V_{ij}^{CS} = -\frac{3}{16}\frac{\hbar^2 c^2 \kappa'}{m_i m_j c^4}\frac{e^{-r_{ij}/r_{0ij}}}{(r_{0ij})r_{ij}}\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \lambda_i^c \lambda_j^c$$

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$$K = -\sum_{i<j}^{n} \frac{1}{m_i m_j} \lambda_i^c \lambda_j^c \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

$K_{X(3915)} =$

$$\begin{pmatrix}
\frac{16}{3}\left(\frac{1}{m_{\bar{c}}m_s} + \frac{1}{m_{\bar{s}}m_c}\right) & 0 & \frac{8\sqrt{2}(m_{\bar{c}}-m_s)(m_{\bar{s}}-m_c)}{3m_{\bar{c}}m_{\bar{s}}m_cm_s} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_cm_s} \\
0 & -\frac{16}{m_{\bar{c}}m_s} - \frac{16}{m_{\bar{s}}m_c} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_cm_s} & 0 \\
\frac{8\sqrt{2}(m_{\bar{c}}-m_s)(m_{\bar{s}}-m_c)}{3m_{\bar{c}}m_{\bar{s}}m_cm_s} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_cm_s} & \frac{(2m_s+8m_{\bar{s}})m_{\bar{c}}+(2m_c+28m_s)m_{\bar{s}}}{3m_{\bar{c}}m_{\bar{s}}m_cm_s} & -\frac{4m_cm_s-14m_{\bar{c}}m_c-14m_{\bar{s}}m_s+4m_{\bar{c}}m_{\bar{s}}}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_cm_s} \\
\frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_cm_s} & 0 & -\frac{4m_cm_s-14m_{\bar{c}}m_c-14m_{\bar{s}}m_s+4m_{\bar{c}}m_{\bar{s}}}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_cm_s} & \frac{2}{m_{\bar{c}}m_s} + \frac{2}{m_{\bar{s}}m_c}
\end{pmatrix}$$

D_s^- D_s^+

where $|CS_1\rangle = \left|(\bar{c}s)_1^{\mathbf{1}}(\bar{s}c)_1^{\mathbf{1}}\right\rangle$, $|CS_2\rangle = \left|(\bar{c}s)_0^{\mathbf{1}}(\bar{s}c)_0^{\mathbf{1}}\right\rangle$, $|CS_3\rangle = \left|(\bar{c}s)_1^{\mathbf{8}}(\bar{s}c)_1^{\mathbf{8}}\right\rangle$, $|CS_4\rangle = \left|(\bar{c}s)_0^{\mathbf{8}}(\bar{s}c)_0^{\mathbf{8}}\right\rangle$

- **Color-Spin Part of Hamiltonian** ($a_3 = 1.0 * 10^{-10}$)

$$\left\langle \sum_{i<j}^{4} V_{ij}^{CS} \right\rangle = \begin{pmatrix}
74.99 & 0.0 & -0.0002 & 0.0009 \\
0.0 & -224.95 & 0.0009 & 0.0 \\
-0.0002 & 0.0009 & -9.38 & 0.0011 \\
0.0009 & 0.0 & 0.0011 & 28.12
\end{pmatrix}$$

$$V_{ij}^{CS} = -\frac{3}{16} \frac{\hbar^2 c^2 \kappa'}{m_i m_j c^4} \frac{e^{-r_{ij}/r_{0ij}}}{(r_{0ij})r_{ij}} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \lambda_i^c \lambda_j^c$$

Table III. Masses of X(3915) and the corresponding thresholds, obtained through variational method and our model Hamiltonian.

Configuration	J^{PC}	Threshold(MeV)	Measured mass(MeV)	Mass(MeV)	Variational parameters (fm^{-2})
$\bar{c}\bar{s}cs$	0^{++}	$D_s^+ D_s^- (3922.0)$	3922.1	3922.1	$a_1 = 7.0, a_3 = 0.001$

$a_3 \to 0$

▪ Color-Spin Interaction Factor

$$K = -\sum_{i<j}^{n} \frac{1}{m_i m_j} \lambda_i^c \lambda_j^c \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$$

$K_{X(3915)} =$

$$\begin{pmatrix} \frac{16}{3}\left(\frac{1}{m_{\bar{c}}m_s}+\frac{1}{m_{\bar{s}}m_c}\right) & 0 & \frac{8\sqrt{2}(m_{\bar{c}}-m_s)(m_{\bar{s}}-m_c)}{3m_{\bar{c}}m_{\bar{s}}m_c m_s} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} \\ 0 & -\frac{16}{m_{\bar{c}}m_s}-\frac{16}{m_{\bar{s}}m_c} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & 0 \\ \frac{8\sqrt{2}(m_{\bar{c}}-m_s)(m_{\bar{s}}-m_c)}{3m_{\bar{c}}m_{\bar{s}}m_c m_s} & \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & -\frac{8m_c m_s+(28m_c+2m_s+8m_{\bar{s}})m_{\bar{c}}+(2m_c+28m_s)m_{\bar{s}}}{3m_{\bar{c}}m_{\bar{s}}m_c m_s} & -\frac{4m_c m_s-14m_{\bar{c}}m_c-14m_{\bar{s}}m_s+4m_{\bar{c}}m_{\bar{s}}}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} \\ \frac{4\sqrt{2}(m_{\bar{c}}+m_s)(m_{\bar{s}}+m_c)}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & 0 & -\frac{4m_c m_s-14m_{\bar{c}}m_c-14m_{\bar{s}}m_s+4m_{\bar{c}}m_{\bar{s}}}{\sqrt{3}m_{\bar{c}}m_{\bar{s}}m_c m_s} & \frac{2}{m_{\bar{c}}m_s}+\frac{2}{m_{\bar{s}}m_c} \end{pmatrix}$$

where $|CS_1\rangle = \left|(\bar{c}s)_1^{\mathbf{1}}(\bar{s}c)_1^{\mathbf{1}}\right\rangle$, $|CS_2\rangle = \left|(\bar{c}s)_0^{\mathbf{1}}(\bar{s}c)_0^{\mathbf{1}}\right\rangle$, $|CS_3\rangle = \left|(\bar{c}s)_1^{\mathbf{8}}(\bar{s}c)_1^{\mathbf{8}}\right\rangle$, $|CS_4\rangle = \left|(\bar{c}s)_0^{\mathbf{8}}(\bar{s}c)_0^{\mathbf{8}}\right\rangle$

▪ In threshold limit (ignoring $1/(m_{\bar{c}}m_c),\ 1/(m_{\bar{s}}m_s),\ 1/(m_{\bar{c}}m_{\bar{s}}),\ 1/(m_c m_s)$ terms)

$$K_{X(3915)} = \begin{pmatrix} \frac{32}{3m_c m_s} & 0 & 0 & 0 \\ 0 & -\frac{32}{m_c m_s} & 0 & 0 \\ 0 & 0 & -\frac{4}{3m_c m_s} & 0 \\ 0 & 0 & 0 & \frac{4}{m_c m_s} \end{pmatrix}$$

Table III. Masses of X(3915) and the corresponding thresholds, obtained through variational method and our model Hamiltonian.

Configuration	J^{PC}	Threshold(MeV)	Measured mass(MeV)	Mass(MeV)	Variational parameters (fm^{-2})
$\bar{c}\bar{s}cs$	0^{++}	$D_s^+ D_s^-$ (3922.0)	3922.1	3922.1	$a_1 = 7.0, a_3 = 0.001$

Table IV. Probability amplitudes for each CS basis in the ground state wave function of X(3915).

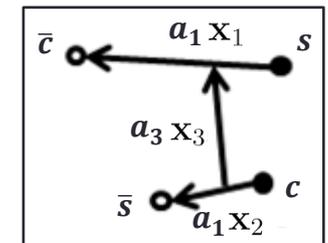| CS basis set	Amplitudes
$\{|CS_1\rangle, |CS_2\rangle, |CS_3\rangle, |CS_4\rangle\}$	$\{0.0, 1.0, 0.0, 0.0\}$
$\{|CS_1'\rangle, |CS_2'\rangle, |CS_3'\rangle, |CS_4'\rangle\}$	$\{0.03, 0.22, 0.08, 0.67\}$

where $|CS_1\rangle = \left|(\bar{c}s)_1^{\mathbf{1}}(\bar{s}c)_1^{\mathbf{1}}\right\rangle$, $|CS_2\rangle = \left|(\bar{c}s)_0^{\mathbf{1}}(\bar{s}c)_0^{\mathbf{1}}\right\rangle$, $|CS_3\rangle = \left|(\bar{c}s)_1^{\mathbf{8}}(\bar{s}c)_1^{\mathbf{8}}\right\rangle$, $|CS_4\rangle = \left|(\bar{c}s)_0^{\mathbf{8}}(\bar{s}c)_0^{\mathbf{8}}\right\rangle$

$|CS_1'\rangle = \left|(\bar{c}c)_0^{\mathbf{1}}(\bar{s}s)_0^{\mathbf{1}}\right\rangle$, $|CS_2'\rangle = \left|(\bar{c}c)_0^{\mathbf{8}}(\bar{s}s)_0^{\mathbf{8}}\right\rangle$, $|CS_3'\rangle = \left|(\bar{c}c)_1^{\mathbf{1}}(\bar{s}s)_1^{\mathbf{1}}\right\rangle$, $|CS_4'\rangle = \left|(\bar{c}c)_1^{\mathbf{8}}(\bar{s}s)_1^{\mathbf{8}}\right\rangle$

D_s^- D_s^+

▪ Well-separated $D_s^+ D_s^-$ configuration

$$\psi^{Spatial}(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3) = N \exp\left[-a_1\mathbf{x}_1^2 - a_1\mathbf{x}_2^2 - a_3\mathbf{x}_3^2\right]$$

$$\psi^{Spatial}(\mathbf{x}_1, \mathbf{x}_2) = N' \exp\left[-a_1\mathbf{x}_1^2 - a_1\mathbf{x}_2^2\right]$$

where N and N' are the normalization constants.

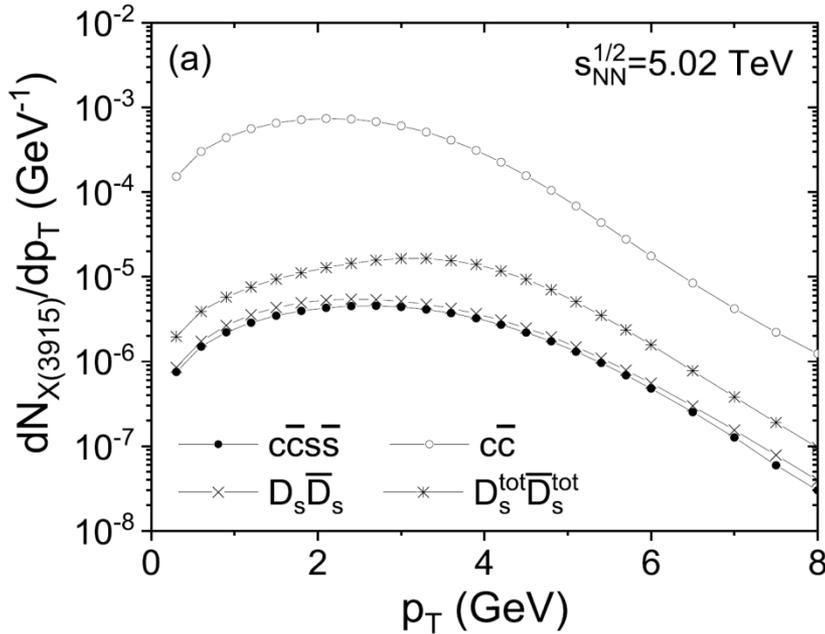Fig. 1(a) Transverse momentum distribution of X(3915) as $\bar{c}c, c\bar{c}s\bar{s}, D_s\bar{D}_s$ states.

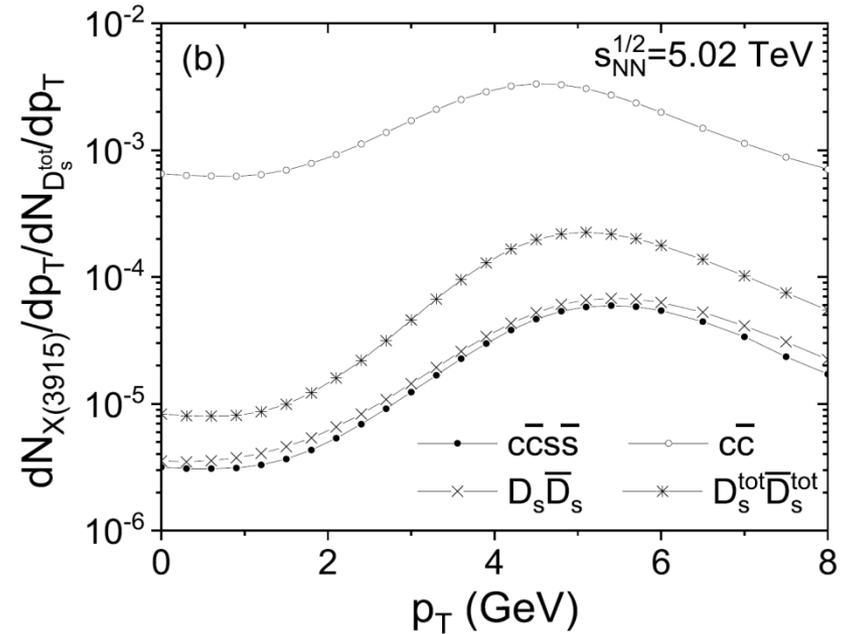Fig. 1(b) Transverse momentum distribution ratios between X(3915) and D_s meson.

Table IV. Yields of X(3915) for various possible states.

Yields	$c\bar{c}$	$c\bar{c}s\bar{s}$	$D_s\bar{D}_s$	$D_s^{tot}\bar{D}_s^{tot}$
Coal.	2.13×10^{-3}	1.62×10^{-5}	1.94×10^{-5}	5.67×10^{-5}

▪ The results would be helpful in identifying the internal structure of X(3915) from the measurement of its production in heavy ion collisions.

Summary

➤ X(3915) can provide a valuable insight for understanding other hidden-charm exotics.

➤ Our quark model analysis suggests that X(3915) is more consistent with a molecular structure, originating from the long-range interaction that extend beyond our quark model.

➤ p_T distribution and yields from our coalescence model are expected to provide valuable insight into the internal structure of X(3915).

➤ Continued theoretical and experimental studies are essential.

Back-up Slide

- Color octet-octet coupled singlet basis

$$|8 \otimes 8\rangle_1 = \sum_a (T^a)_{ij} (T^a)_{kl} |q_i \bar{q}_j q_k \bar{q}_l\rangle$$

  where T^a are the SU(3) Gell-Mann matrices.

- Using the standard Fierz identity for the generators

$$S_{ijkl} \equiv \sum_a (T^a)_{ij} (T^a)_{kl} = \frac{1}{2} \left(\delta_{il}\delta_{kj} - \frac{1}{3}\delta_{ij}\delta_{kl} \right)$$

- Applying the charge conjugation

$$\mathbf{C} \left(S_{ijkl} |q_i \bar{q}_j q_k \bar{q}_l\rangle \right) = S_{ijkl} |\bar{q}_i q_j \bar{q}_k q_l\rangle$$
$$= S_{jilk} |q_i \bar{q}_j q_k \bar{q}_l\rangle$$

Now the problem reduces to show that $S_{jilk} = S_{ijkl}$.

Since $\delta_{jk}\delta_{li} = \delta_{il}\delta_{kj}$ and $\delta_{ji}\delta_{lk} = \delta_{ij}\delta_{kl}$,

$$S_{jilk} = \frac{1}{2} \left(\delta_{jk}\delta_{li} - \frac{1}{3}\delta_{ji}\delta_{lk} \right) = \frac{1}{2} \left(\delta_{il}\delta_{kj} - \frac{1}{3}\delta_{ij}\delta_{kl} \right) = S_{ijkl}$$