Eigenstates in coupled-channel scattering amplitude and their effects on spectrum

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T.Nisihibuchi and T.Hyodo, Phys. Rev. C **109**, no1, 015203 (2024),
arXiv:2506.21949 [hep-ph], arXiv:2507.18941 [hep-ph].

Recent results for $\Xi(1620)$

Recently new results of $\Xi(1620)$ are obtained

Belle experiment of $\Xi_c \to \pi\pi\Xi$ (2019)[1]

Ξ excited states are observed in $\pi^+\Xi^-$ spectrum.

The mass M_R and width Γ_R of $\Xi(1620)$

$$M_R = 1610.4 \pm 6.0(\text{stat.})^{+6.1}_{-4.2}(\text{syst.})\ \text{MeV}$$

$$\Gamma_R = 59.9 \pm 4.8(\text{stat.})^{+2.8}_{-7.1}(\text{syst.})\ \text{MeV}$$

ALICE experiment(2021)[2]

The scattering length f_0 of $K^-\Lambda$ was determined with femtoscopy in Pb-Pb collisions

[1]Belle collaboration, M.Sumihama et al., Phys. Rev. Lett. **122**, 072501 (2019).
[2]S. Acharya et al. (ALICE Collaboration)Phys. Rev. C **103**, 055201(2021).

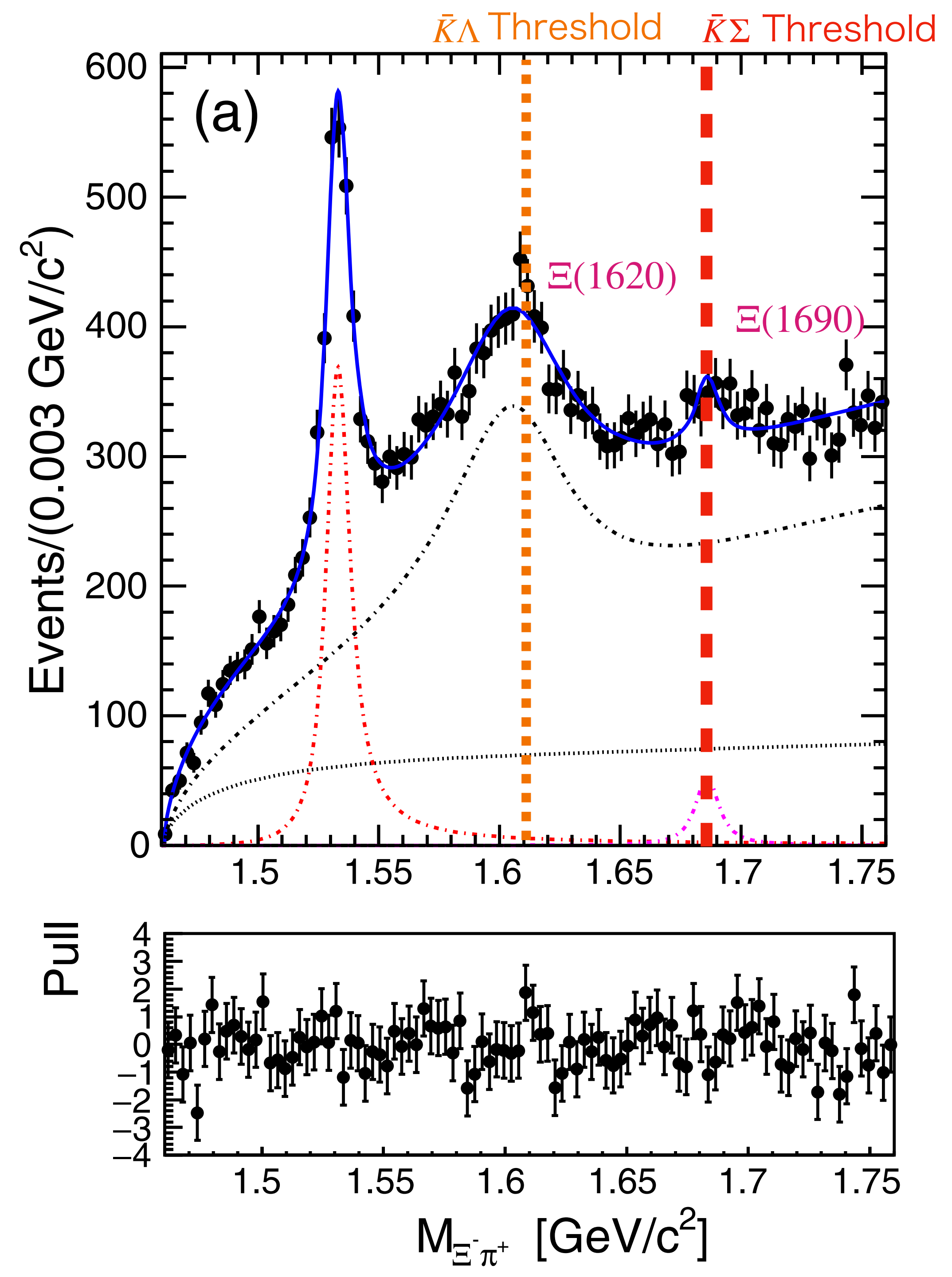


fig1.$\pi^+\Xi^-$ spectrum in $\Xi_c \to \pi\pi\Xi$ decay[1].

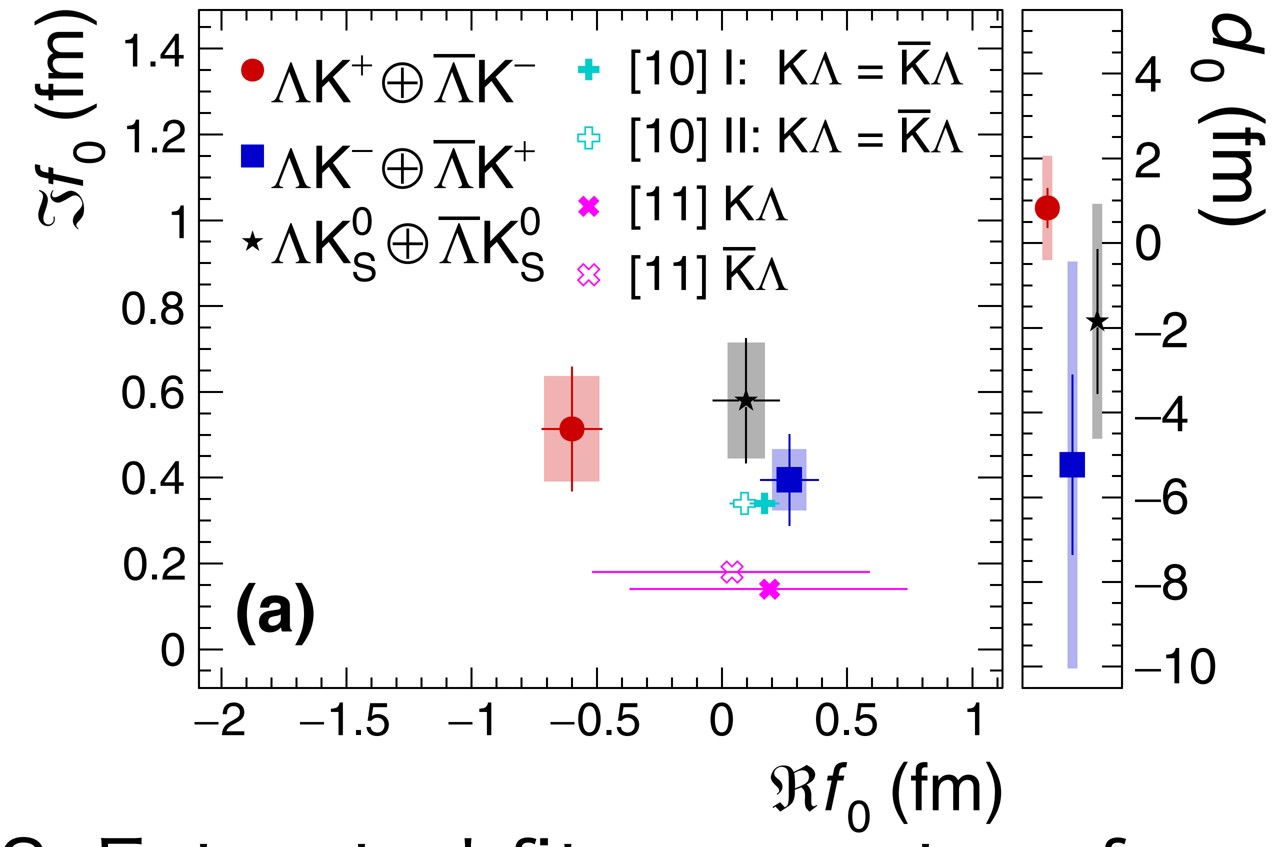
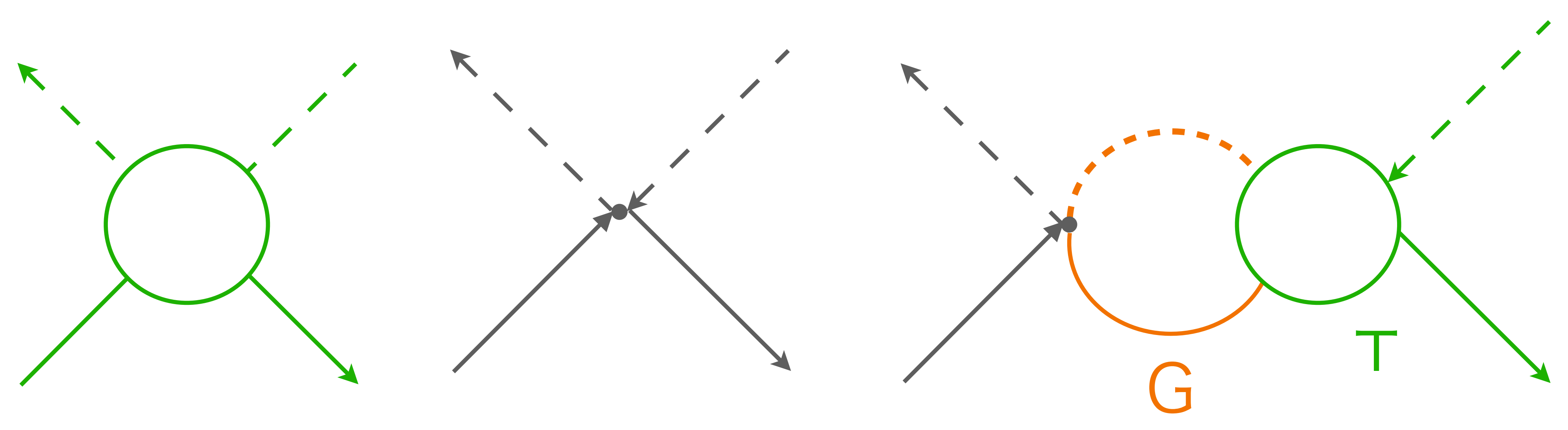


fig2. Extracted fit parameters for all of the $\bar{K}\Lambda$ system [2].

Formulation of the scattering model

The scattering amplitude $T_{ij}(W)$ **satisfies the scattering equation.**

$$T_{ij}(W) = V_{ij}(W) + V_{ik}(W)G_k(W, a_i)T_{kj}(W)$$



Interaction kernel $V_{ij}(W)$:Weinberg-Tomozawa interaction

$$V_{ij}(W) = -\frac{C_{ij}}{4f_i f_j} N_i N_j (2W - M_i - M_j)$$

f_i:Meson decay constant, N_i: Kinematical coefficient,

C_{ij}:Group theoretical coefficient, M_i:Baryon Mass

Loop function $G_i(W)$ (Removed divergence by dimensional regularization)

$$G_i(W) \to G_i(W, a_i)$$

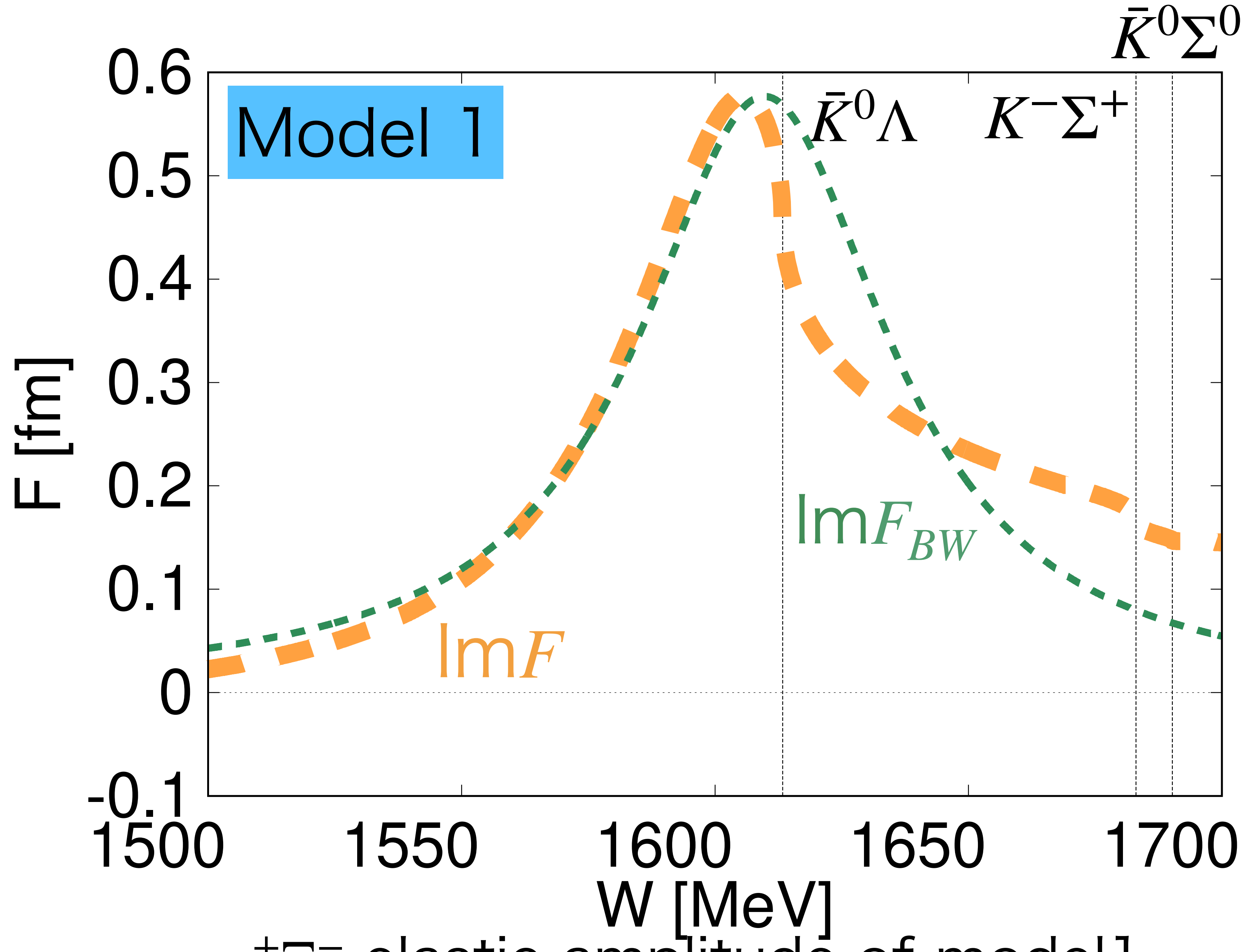
W:Total energy, a_i:subtraction constant

Construction of theoretical models

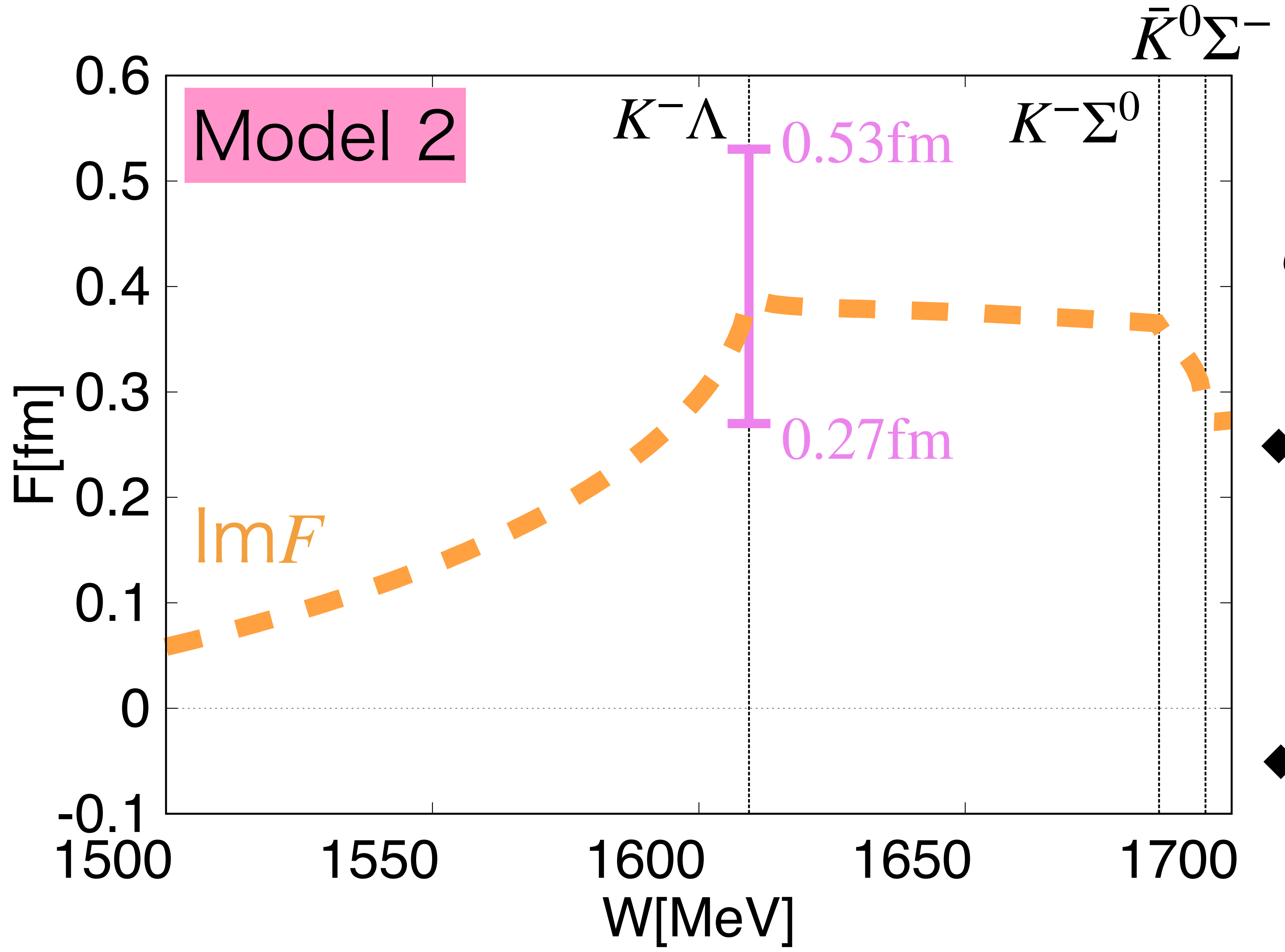
Construct the models which based on Belle and ALICE respectively

Model 1 Assume the pole position as $z_{\rm ex} = [1610 - 30i]$ MeV, and construct the model with the pole at $z_{\rm ex}$.

Model 2 Reproduce the $K^-\Lambda$ scattering length of ALICE.



$\pi^+\Xi^-$ elastic amplitude of model1 with BW amplitude



$K^-\Lambda$ elastic amplitude of Model2

Constructed by adjusting $a_{\pi\Xi}$ and $a_{\bar{K}\Lambda}$.

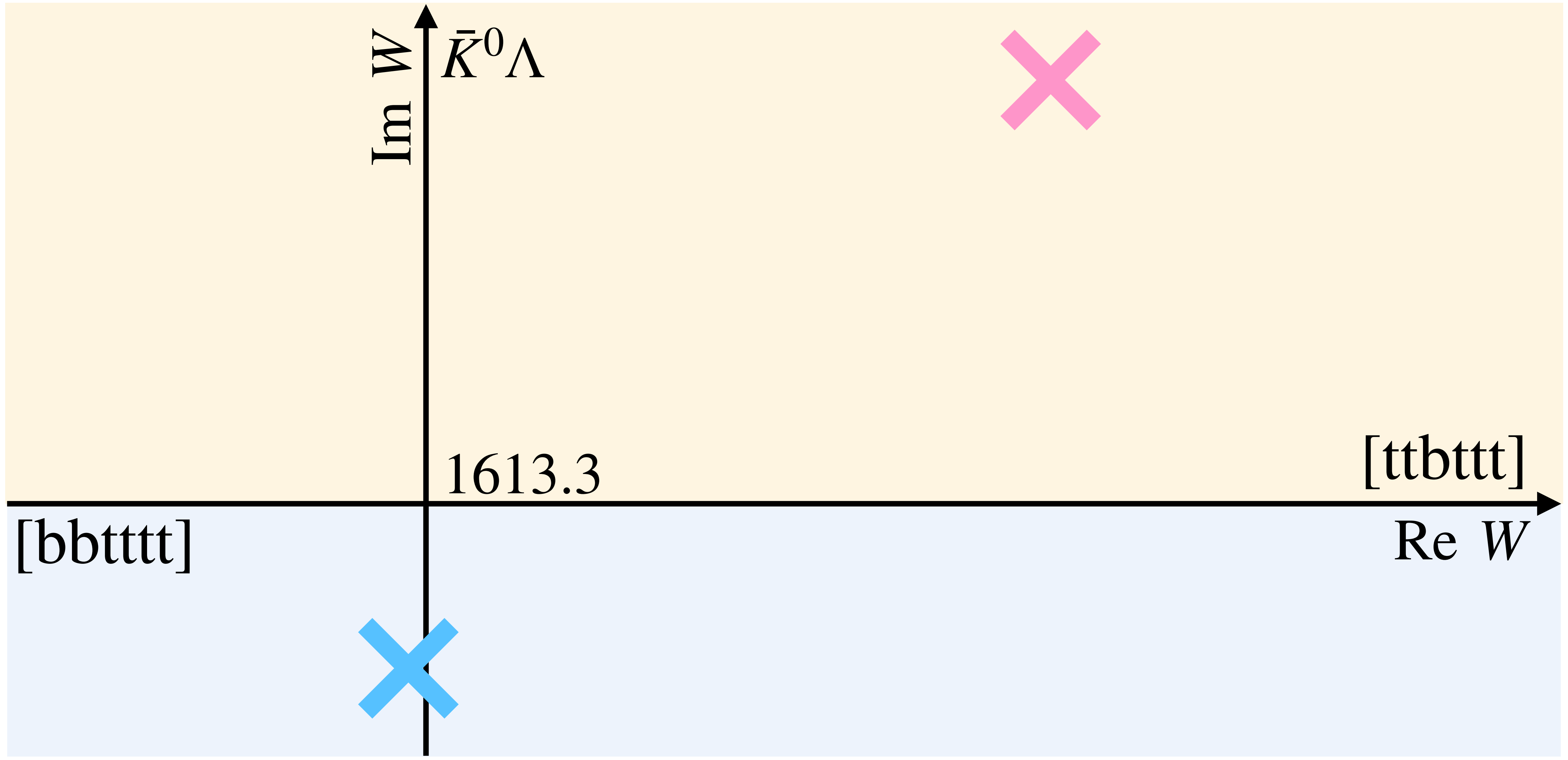
- The amplitude Model 1 has a peak near the $\bar{K}^0\Lambda$
- Model 2 amplitude has a cusp structure

[3]T.Nisihibuchi and T.Hyodo, Phys. Rev. C **109**, no1, 015203 (2024).

Poles of $\Xi(1620)$ in theoretical models

Pole position of each models as follows

	Pole of $\Xi(1620)$
Model 1	$z = 1610 - 30i$ MeV [bbtttt]
Model 2	$z = 1726 + 80i$ MeV [ttbttt]



pole position of Model1 and Model2

From pole position and Riemann sheet, we interpret these poles as follows.

Model 1 pole of $\Xi(1620)$: Quasi Bound(QB)

Model 2 pole of $\Xi(1620)$: Quasi Virtual(QV)

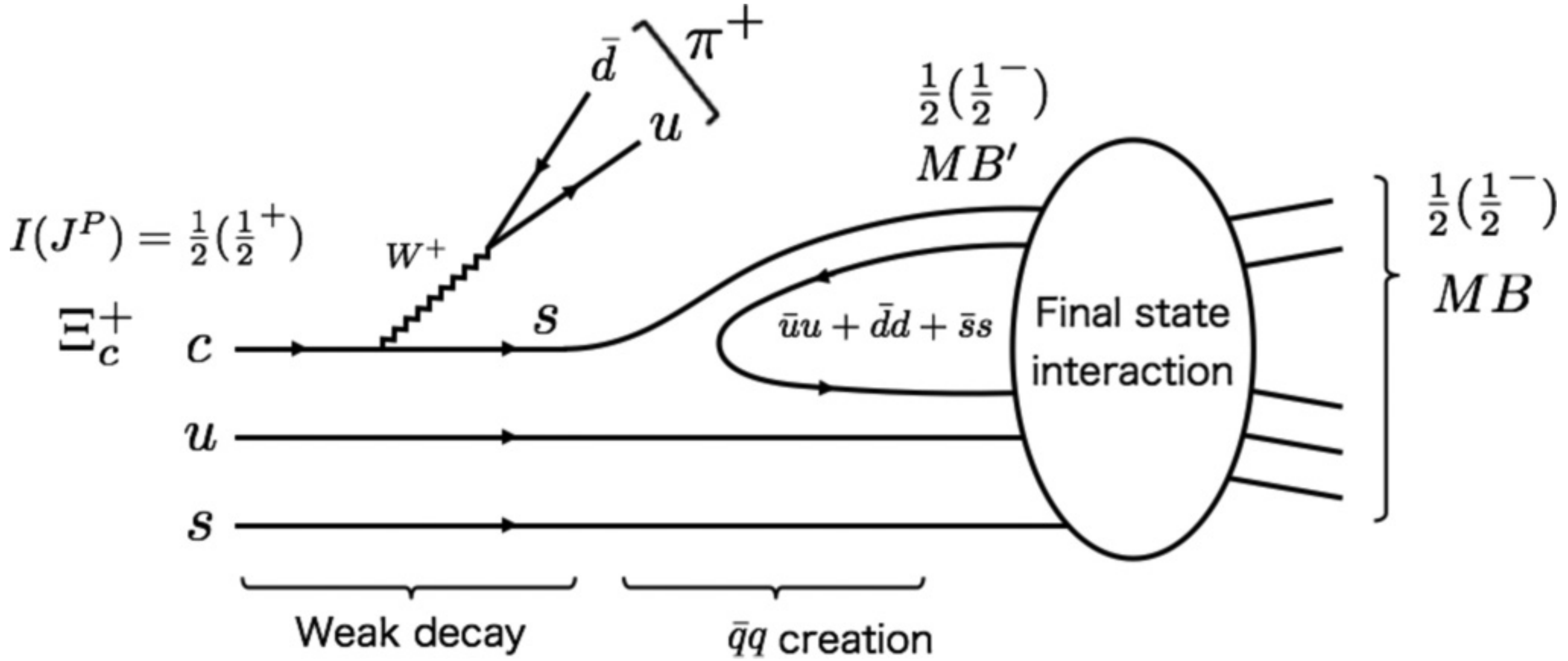
T.Nisihibuchi and T.Hyodo, Phys. Rev. C **109**, no1, 015203 (2024),

Formulation of $\mathscr{M}$ in 3-body decay

3-body decay($\Xi_c^+ \to \pi^+ MB$)

The decay amplitude to the final meson baryon state

$$\mathscr{M}_j = V_P \left(h_j + \sum_i h_i G_i(M_{\rm inv}) T_{ij}(M_{\rm inv}) \right)$$



The diagram for $\Xi_c^+ \to \pi^+ MB$ decay. [6]

V_P : the constant includes all dynamics before FSI.

h_i : the weight coefficient of intermediate state, $M_{\rm inv}$: Invariant Mass,

T_{ij} : Meson baryon scattering amplitude, G_i : Meson baryon loop function

[6]K.Miyahara, T.Hyodo, M.oka, J.Nieves and E.Oset Phys.Rev.C **95** (2017) 3, 035212

Spectrum shape in actual models

We calculated the invariant mass distributions for Model 1 and Model2.

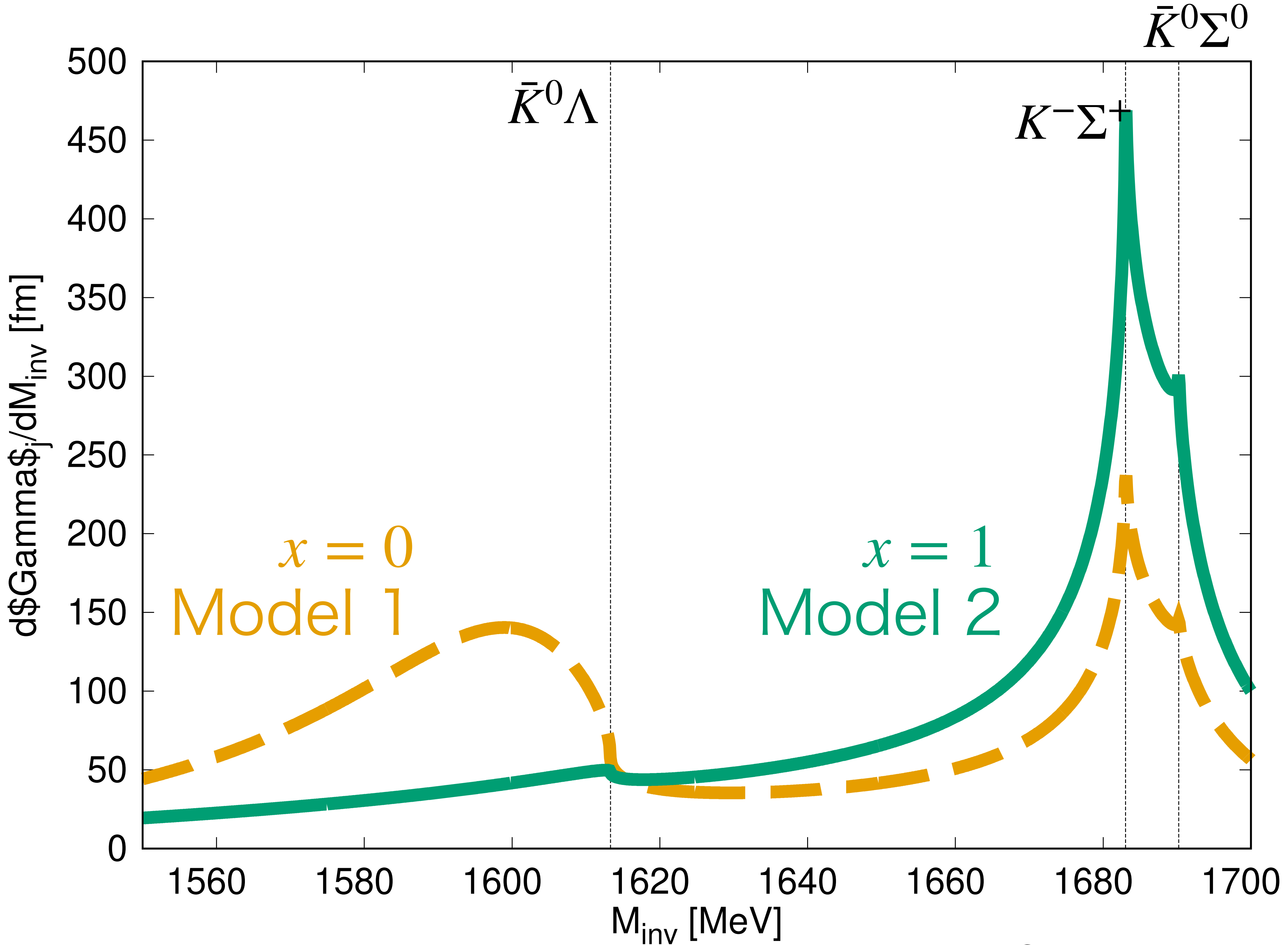
Confirm spectrum shape changing with intermediate states between models.

Model extrapolation by changing a_i

$$a_i(x) = xa_i'' + (1-x)a_i' \qquad (0 \le x \le 1)$$

a_i'・・・subtraction constant of Model 1

a_i''・・・subtraction constant of Model 2



$\pi^+\Xi^-$ invariant mass distribution of Models

In this talk, we calculated 4 intermediate states($x = 0.2, 0.4, \ldots, 1$)

T.Nisihibuchi and T.Hyodo, arXiv:2506.21949 [hep-ph].

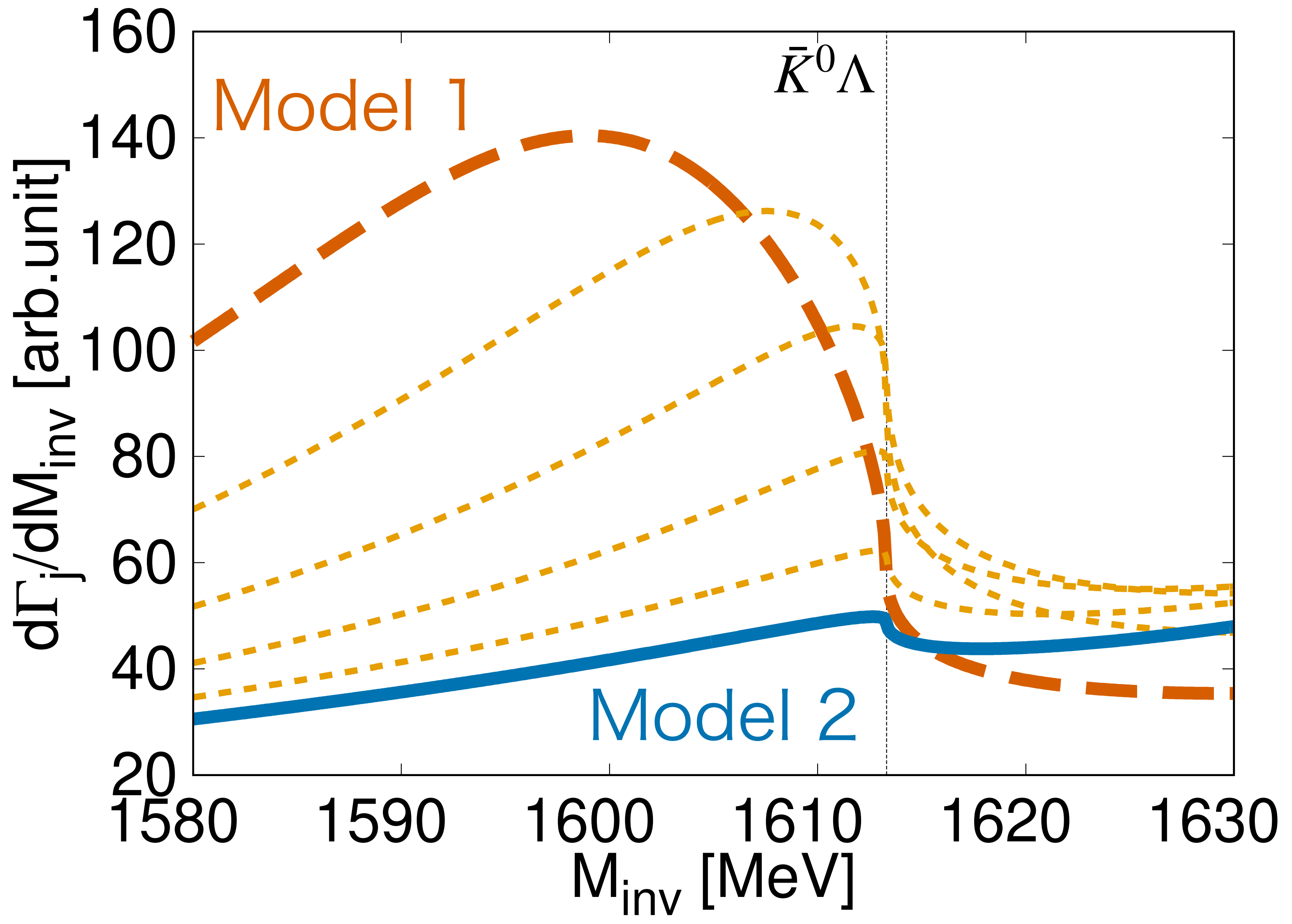
Spectra changing between models

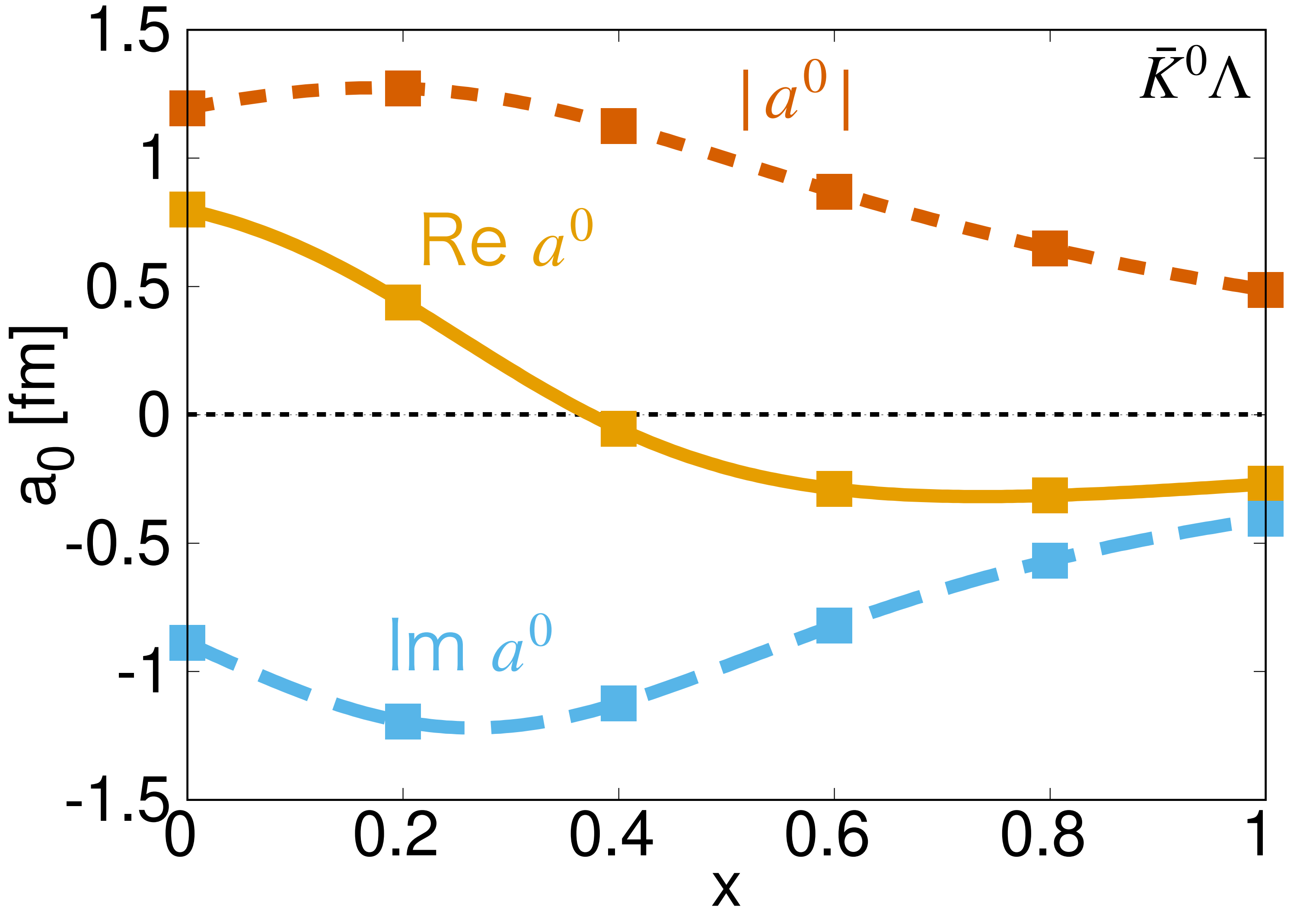
Focus on the near $\bar{K}^0\Lambda$ threshold

The peak near the $\bar{K}^0\Lambda$ threshold turns into a cusp and the height becomes lower (upper panel).

calculate the scattering length of $\bar{K}^0\Lambda$

- We found that spectra shape changing almost corresponds changing of scattering length.
  - When Re a^0 has positive··· **Spectrum has a peak**
  - When Re a^0 has negative···**Spectrum has a cusp**





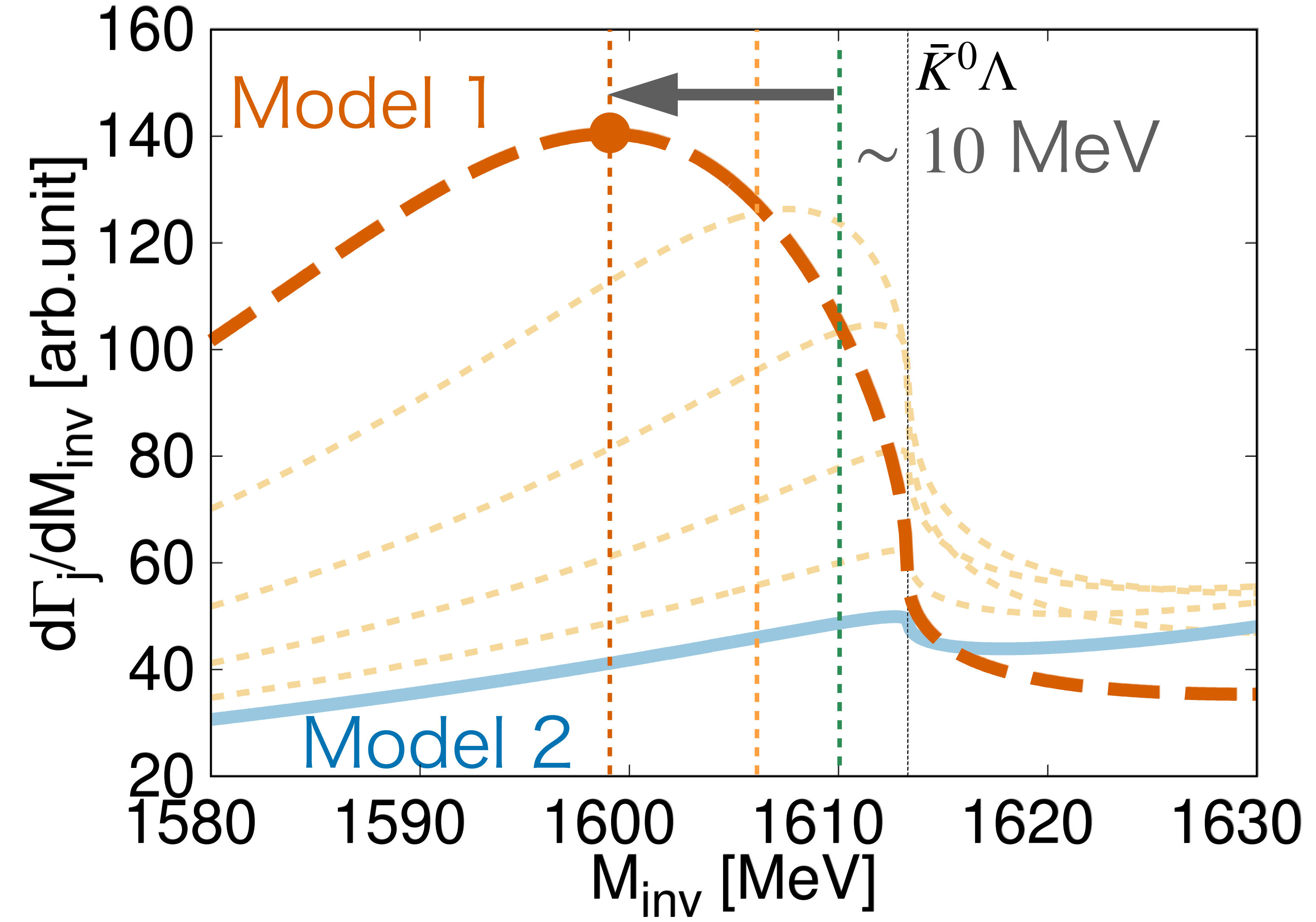
T.Nisihibuchi and T.Hyodo, arXiv:2507.18941 [hep-ph].

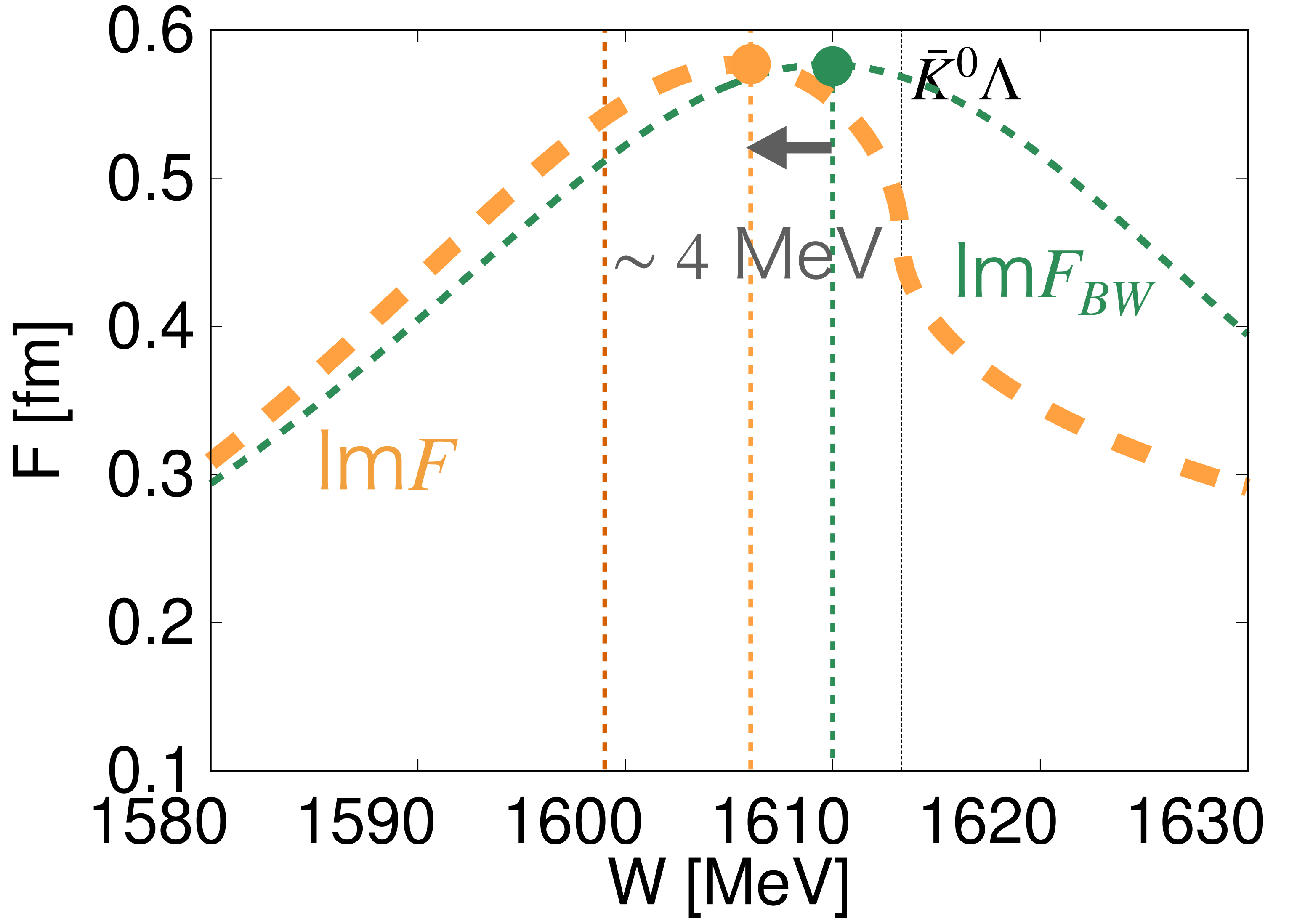
Effects for spectra near $\bar{K}\Lambda$ threshold

Threshold effect near the $\bar{K}\Lambda$ threshold

- Model 1 has a pole at $1610 - 30i$ MeV.
- The peak in the 3 body decay spectrum of Model1 locates on 1599 MeV (upper panel).
- In two body elastic scattering amplitude, the peak locate on 1606 MeV (lower panel).

Stronger threshold effect can be seen in the 3 body decay spectra.

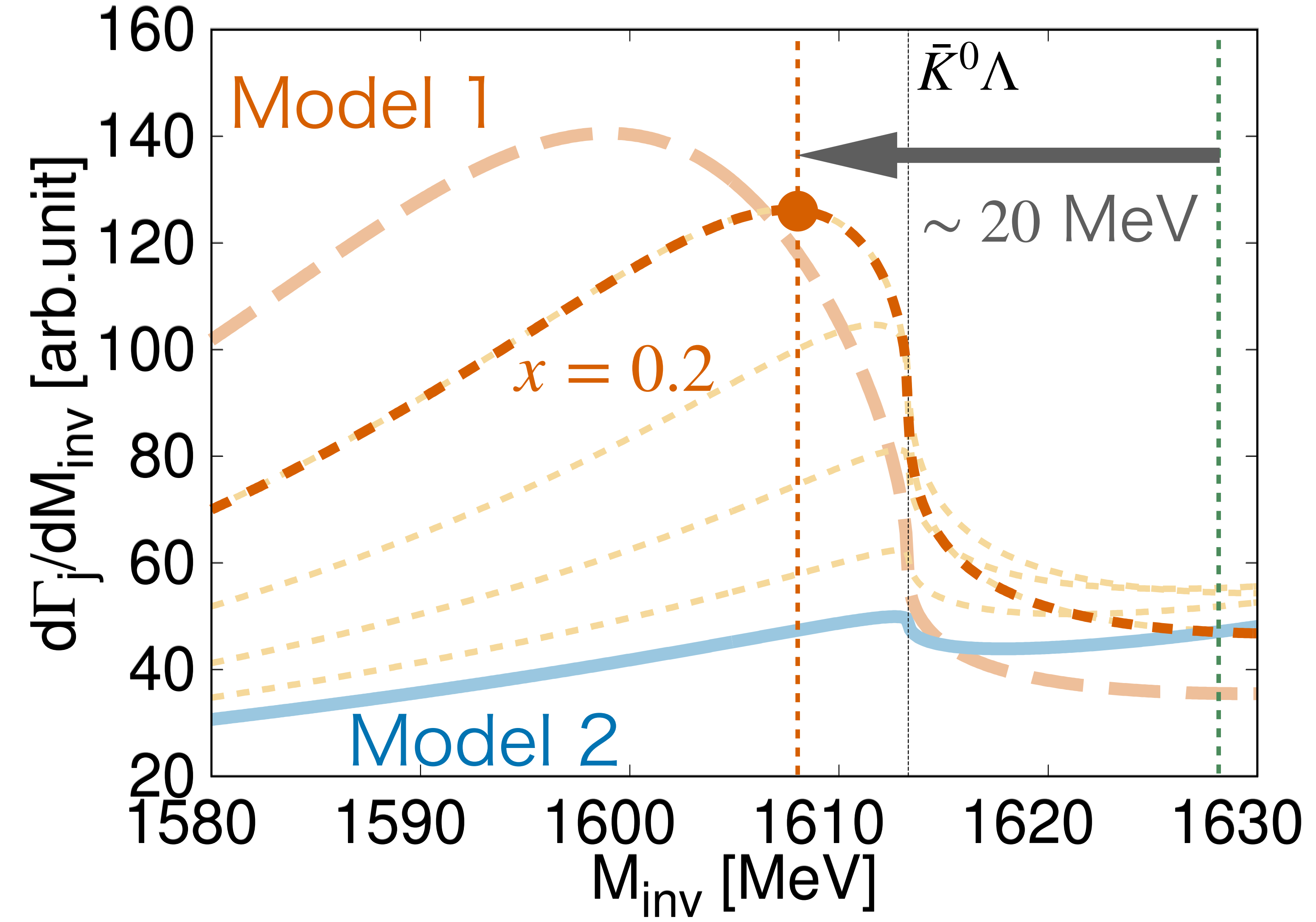




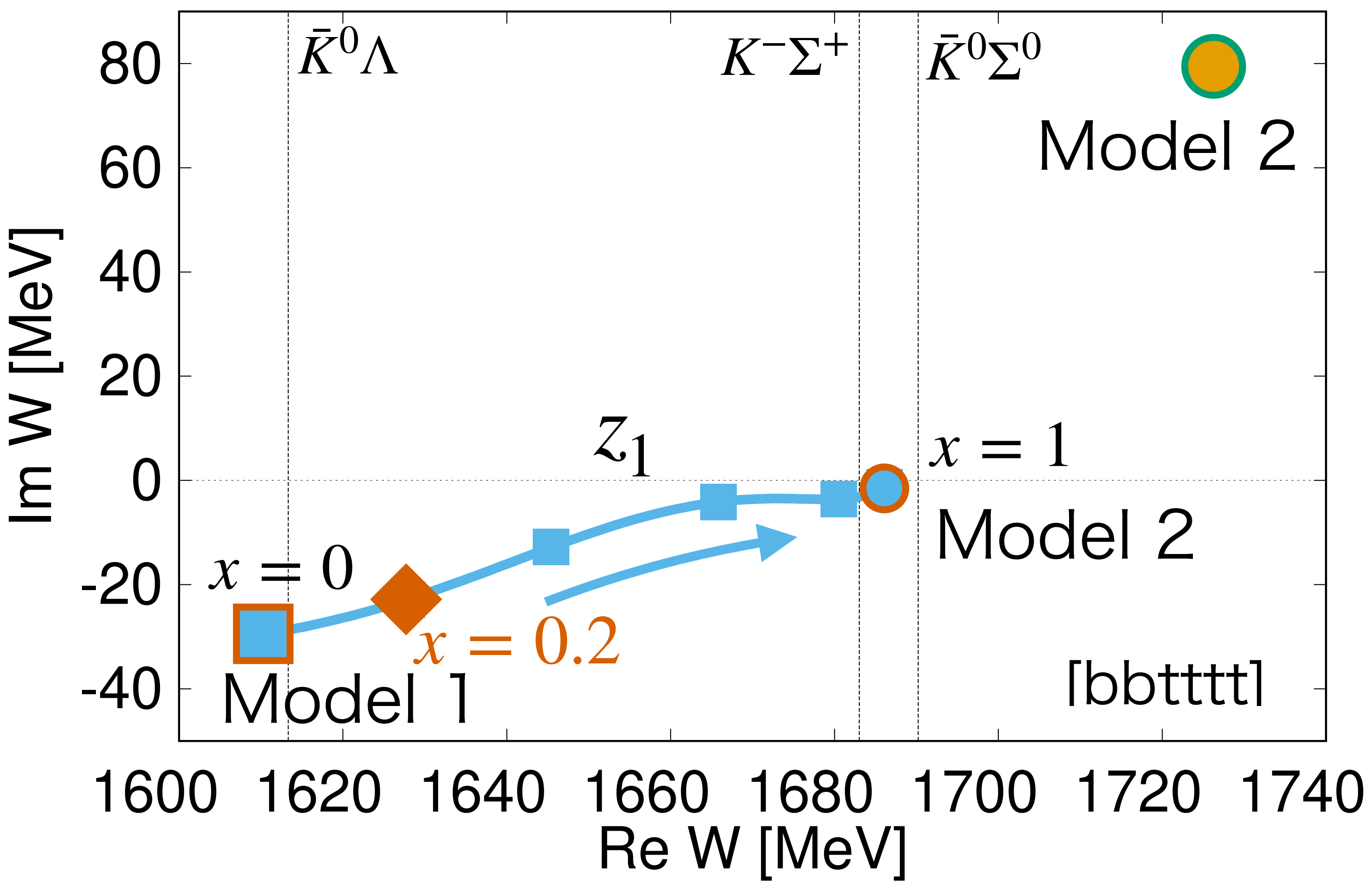
Effects for spectra near $\bar{K}\Lambda$ threshold

Compare with the pole trajectory

Focus on the intermediate state($x = 0.2$)



- The spectra of $x = 0.2$ has a clear peak structure at 1608 MeV near $\bar{K}^0\Lambda$ threshold (upper panel).



- But pole of $\Xi(1620)$ goes away from $\bar{K}^0\Lambda$ threshold.
  - It locates on $1628 - 23i$ MeV on [bbtttt] sheet, and the location is not physical relevant. (lower panel)

Due to a strong threshold effect, a peak structure appear emerged.

Summary

In recent years, experimental data about $\Xi(1620)$ have been reported, and theoretical studies have also been conducted actively.

- We construct the models based on Belle and ALICE(Model 1/Model 2)
- We calculate $\Xi_c \to \pi\pi\Xi$ decay for Model 1, Model 2.

Results

- The shape of spectra correlated with the sign of the Re a_0.
- Threshold effect appears more prominently than 2 body scattering.

Future work

- Investigate the threshold on 3 body decay amplitude in detail.

Back up

Formulation of $\mathscr{M}$ in 3 body decay

3 body decay($\Xi_c^+ \to \pi^+ \mathrm{MB}$)

The invariant mass distribution reduced to

$$\frac{d\Gamma_j}{dM_{\rm inv}} = \frac{1}{(2\pi)^3}\frac{p_{\pi^+}\tilde{p}_j M_j}{M_{\Xi_c^+}}|\mathscr{M}_j|^2$$

$$p_{\pi^+} = \frac{\lambda^{1/2}(M_{\Xi_c^+}^2, m_{\pi^+}^2, M_{\rm inv}^2)}{2M_{\Xi_c^+}}$$

$$\tilde{p}_j = \frac{\lambda^{1/2}(M_{\rm inv}^2, M_j^2, m_j^2)}{2M_{\rm inv}}$$

$$\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$$

$M_{\Xi_c^+}$: mass of Ξ_c^+, m_{π^+} : mass of π^+

p_{π^+} : three-momentum of the π^+ which emitted in weak decay (Ξ_c^+ rest flame)

$\tilde{p}_j$: three-momentum of meson baryon emitted in weak decay (MB rest flame)

Back up Detail of loop function

Loop function $G_i(W)$

$$G_i(W) = i\int \frac{d^4q}{(2\pi)^4}\frac{1}{q^2 - m_i^2 + i0^+}\frac{1}{(P-q)^2 - M_i^2 + i0^+}$$

Loop function $G_i(W, a_i)$ (Removed divergence by dimensional regularization)

$$G_i(W, a_i) = \frac{1}{16\pi^2}\left[a_i(\mu_{reg}) + \ln\frac{mM}{\mu_{reg}^2} + \frac{M^2 - m^2}{2W^2}\ln\frac{M^2}{m^2} + \frac{\lambda^{1/2}}{2W^2}\left\{\ln(W^2 - m^2 + M^2 + \lambda^{1/2})\right.\right.$$

$$\left.\left. + \ln(W^2 + m^2 - M^2 + \lambda^{1/2}) - \ln(-W^2 + m^2 - M^2 + \lambda^{1/2}) - \ln(-W^2 - m^2 + M^2 + \lambda^{1/2})\right\}\right]$$

$$\lambda^{1/2} = \sqrt{W^4 + m_k^4 + M_k^4 - 2W^2m_k^2 - 2m_k^2M_k^2 - 2M_k^2W^2}$$

Back up Definition of scattering length

- In this study, we define the scattering length f_0 as follows.
  (It is the value of scattering amplitude at threshold energy.)

$$f(k) = \frac{1}{\frac{1}{f_0} + \frac{d_0}{2}k^2 + \cdots - ik}$$

$f(k)$: Scattering amplitude

k: Complex momentum

r_0: effective range

- But in general, scattering length a_0 is defined as follow.
  (It is reverse sign of f_0.)

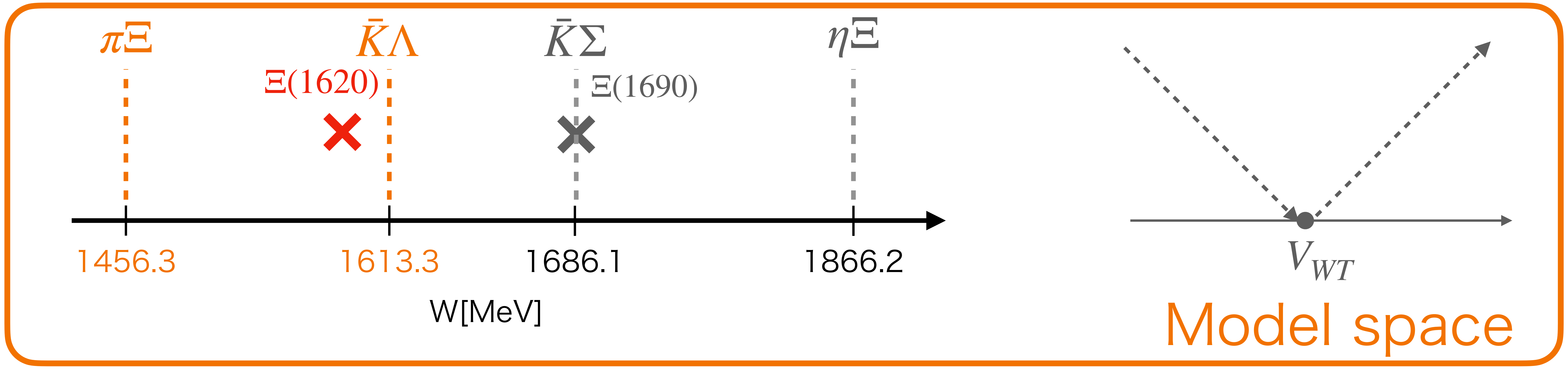
$$f(k) = \frac{1}{-\frac{1}{a_0} + \frac{r_0}{2}k^2 + \cdots - ik}$$

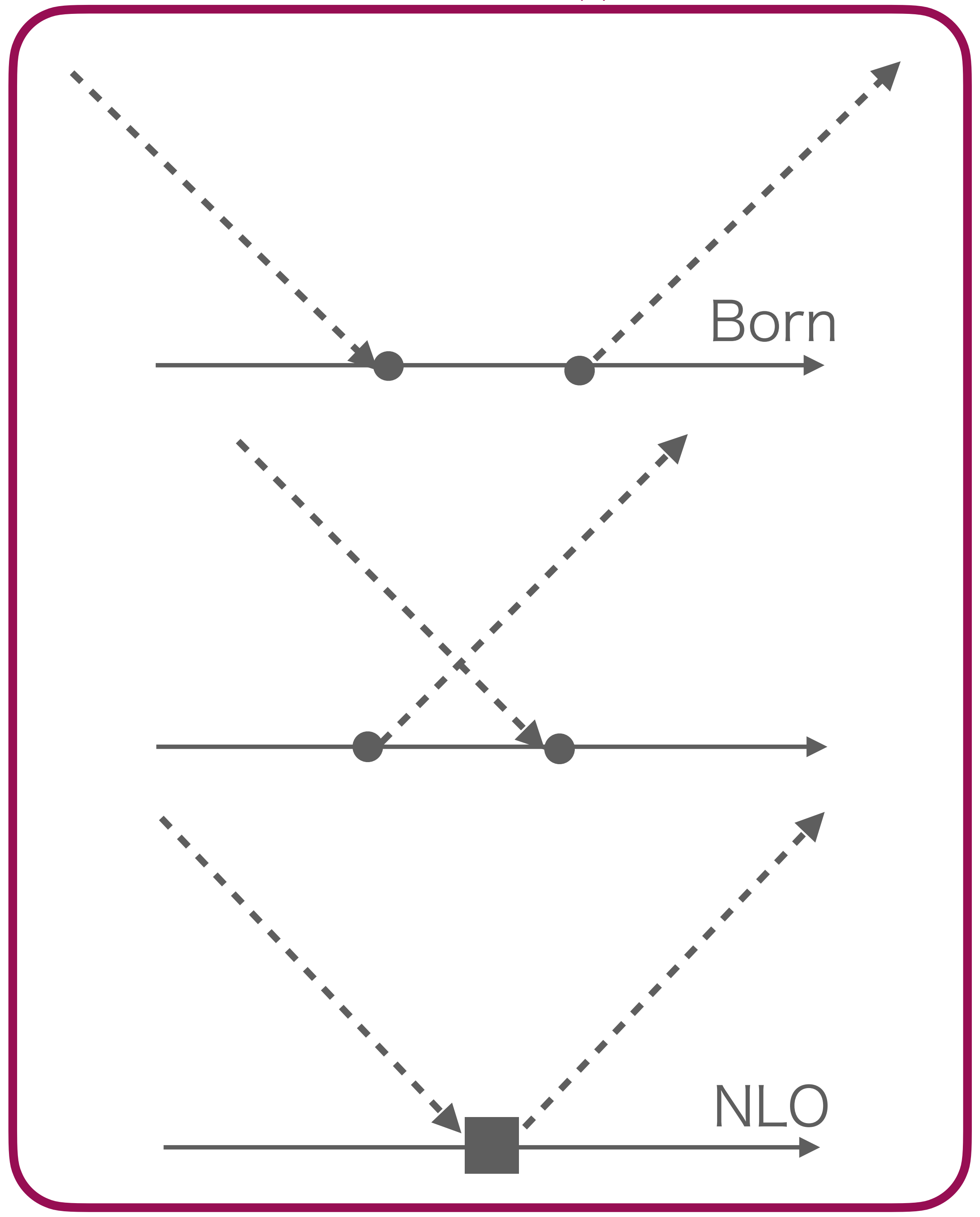
Back up The roles of subtraction constants

- By changing subtraction constants, the effects from outside of model space can be absorbed.

➡ When the subtraction constants closer to natural value, outside effect become smaller.

Effects from except V_{WT} .





Effects from other channels(Ξ^*_{uss},$\bar{K}^*\Lambda$,$\bar{K}^*\Sigma$,$\pi\bar{K}\Lambda$,$\cdots$).

[8] T.Hyodo, D.Jido and A.Hosaka Phys. Rev. C 78.025203 (2008)

Pole trajectory in actual models

Apply the model extrapolation to pole

Two poles that are supposed $\Xi(1620)$ are not continuously connected.

- z_1 : $\Xi(1620)$ pole of Model 1 [bbtttt]
- z_2 : $\Xi(1620)$ pole of Model 2 [ttbttt]

This result suggests that Model 1 and Model 2 poles have different physical origins.