Internal structure of exotic hadrons with coupled channel potential in relation with scattering observables

I. Terashima and T. Hyodo, arXiv:2505.17657 [hep-ph].

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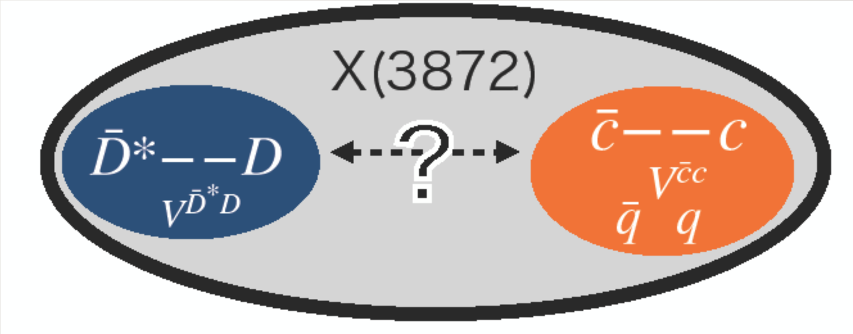
Introduction: Exotic hadron $X(3872)$

- There is no restriction by QCD which prohibits the mixing with each d.o.f
  - States with same quantum numbers mix by definition
- Structure of $X(3872)$ [A. Hosaka, T. Iijima, K. Miyabayashi, Y. Sakai, and S. Yasui, PTEP **2016** (2016)]
  - **Mixing with quark and hadron** degrees of freedom
  - Not enough experimental data and lattice QCD results

> - How about a channel coupling between **quark** and **hadron** degrees of freedom like $\mathbf{X(3872)}$?
> - Revealing the internal structure of exotic hadrons by compositeness



1 : Molecule ⟷ Compositeness ⟷ 0 : Elementary

Formulation: Channel coupling

✓ Formulation according to Feshbach method [H. Feshbach, Ann. Phys. 5, 357 (1958); ibid., 19, 287 (1962)]

■ Hamiltonian H with channel between quark potential V^q and hadron V^h

$$H = \begin{pmatrix} T^q & 0 \\ 0 & T^h + \Delta \end{pmatrix} + \begin{pmatrix} V^q & V^t \\ V^t & V^h \end{pmatrix}$$

T^q, T^h:Kinetic energy

Δ:Threshold energy

V^t:Transition potential

- Schrödinger equation with wave functions of quark and hadron channels $|q\rangle, |h\rangle$

$$H \begin{pmatrix} |q\rangle \\ |h\rangle \end{pmatrix} = E \begin{pmatrix} |q\rangle \\ |h\rangle \end{pmatrix}$$

$$\langle \boldsymbol{r}'_h \mid V^h_{\text{eff}}(E) \mid \boldsymbol{r}_h \rangle = \langle \boldsymbol{r}'_h \mid V^h \mid \boldsymbol{r}_h \rangle + \boxed{\sum_n \frac{\langle \boldsymbol{r}'_h \mid V^t \mid \phi_n \rangle \langle \phi_n \mid V^t \mid \boldsymbol{r}_h \rangle}{E - E_n}}$$

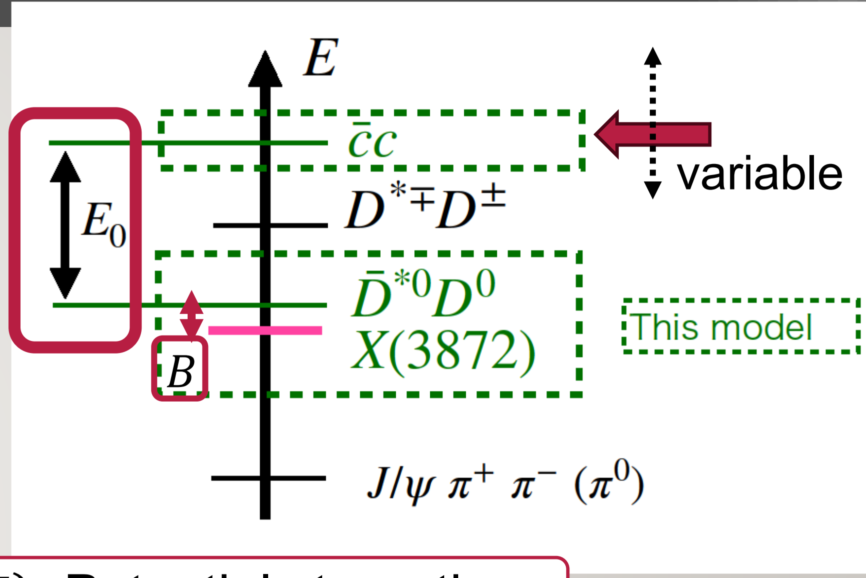
➢ Quark channel contribution. Sum of discrete eigenstates E_n

Formulation: $X(3872)$

◆Quark channel : $\bar{c}c$

$$H = \begin{pmatrix} T^q & 0 \\ 0 & T^h + \Delta \end{pmatrix} + \begin{pmatrix} V^q & V^t \\ V^t & V^h \end{pmatrix}$$

◆ Hadron channel : $D^0\,\bar{D}^{*0}$



$\langle \boldsymbol{r}'_h \mid V^t \mid \boldsymbol{r}_h \rangle$ ✓ Separable
✓ Yukawa

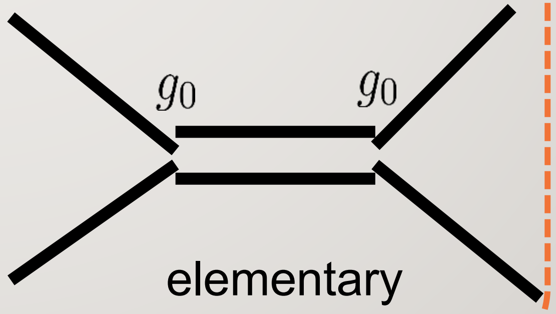
$$V_{\text{eff}}^{\bar{D}^* D}(\boldsymbol{r}, \boldsymbol{r}', E) = [\omega^q(E) + \omega^h] V(\boldsymbol{r}) V(\boldsymbol{r}')$$

$$= \omega(E) \frac{e^{-\mu r}}{r} \frac{e^{-\mu r'}}{r'}$$

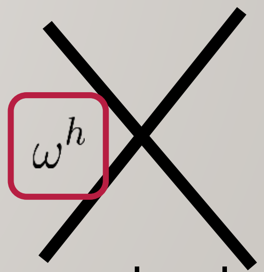
μ: cut-off

$\omega(E)$: Potential strength

$$\omega^q = \frac{g_0^2(B)}{E - E_0}$$



$$\omega^h \in \mathbb{R}$$



$g_0(B)$: coupling constant

➢ Determine to reproduce mass of $X(3872)$

$$g_0^2(B) = (B + E_0) \cdot (-1/G(E = -B) + \omega^h)$$

$G(E)$ is a loop function

Formulation: Compositeness

- Bound state wave function is normalized as:

[Kenta Miyahara and Tetsuo Hyodo. Phys. Rev. C, 93(1):015201, 2016.]

$$1 = \int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'\Psi_E^*(\boldsymbol{r}')\left(\delta(\boldsymbol{r}-\boldsymbol{r}') - \frac{\partial}{\partial E}V(\boldsymbol{r},\boldsymbol{r}',E)\right)\Psi_E(\boldsymbol{r})$$

- Definition of compositeness $1 = X + Z$

$$X = \int \mathrm{d}\boldsymbol{r}|\Psi_{E=-B}(\boldsymbol{r})|^2 \qquad Z = -\int \mathrm{d}\boldsymbol{r}\,\mathrm{d}\boldsymbol{r}'\Psi_E^*(\boldsymbol{r}')\frac{\partial}{\partial E}V(\boldsymbol{r},\boldsymbol{r}',E)\Psi_E(\boldsymbol{r})$$

$$\boxed{X = [1 + \frac{g_0^2\kappa\mu(\kappa+\mu)^3}{8\pi m^2(g_0^2 + (E-E_0)\omega^h)^2}]^{-1}}$$

- Phase shift $\delta(k)$

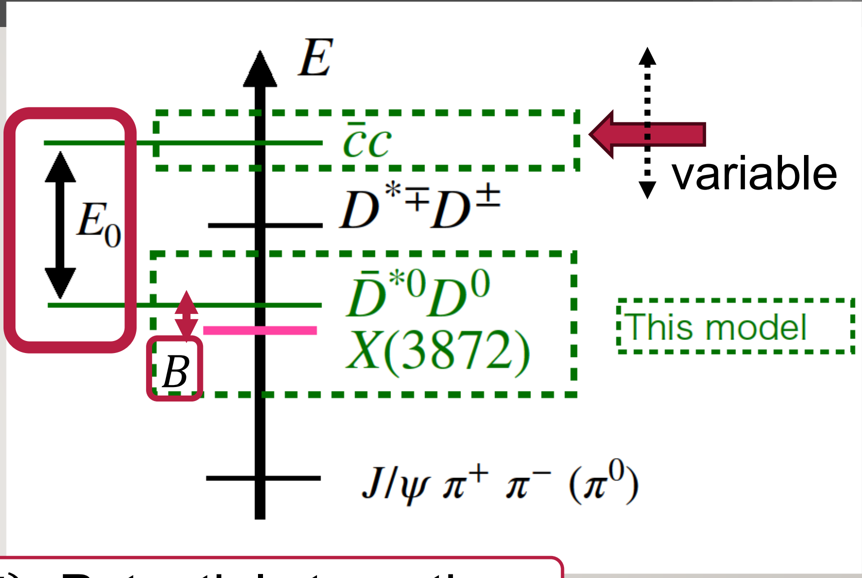
$$k\cot\delta(k) = -\frac{\mu[4\pi m\omega(E)+\mu^3]}{8\pi m\omega(E)} + \frac{1}{2\mu}\left[1 - \frac{2\mu^3}{4\pi m\omega(E)}\right]k^2 - \frac{1}{8\pi m\omega(E)}k^4$$

$$a_0 = \frac{d\delta(k)}{dk}|_{k\to 0}, \quad r_e = 2\frac{\mathrm{d}\ k\cot\delta(k)}{\mathrm{d}\ k^2}|_{k\to 0}$$

Parameters

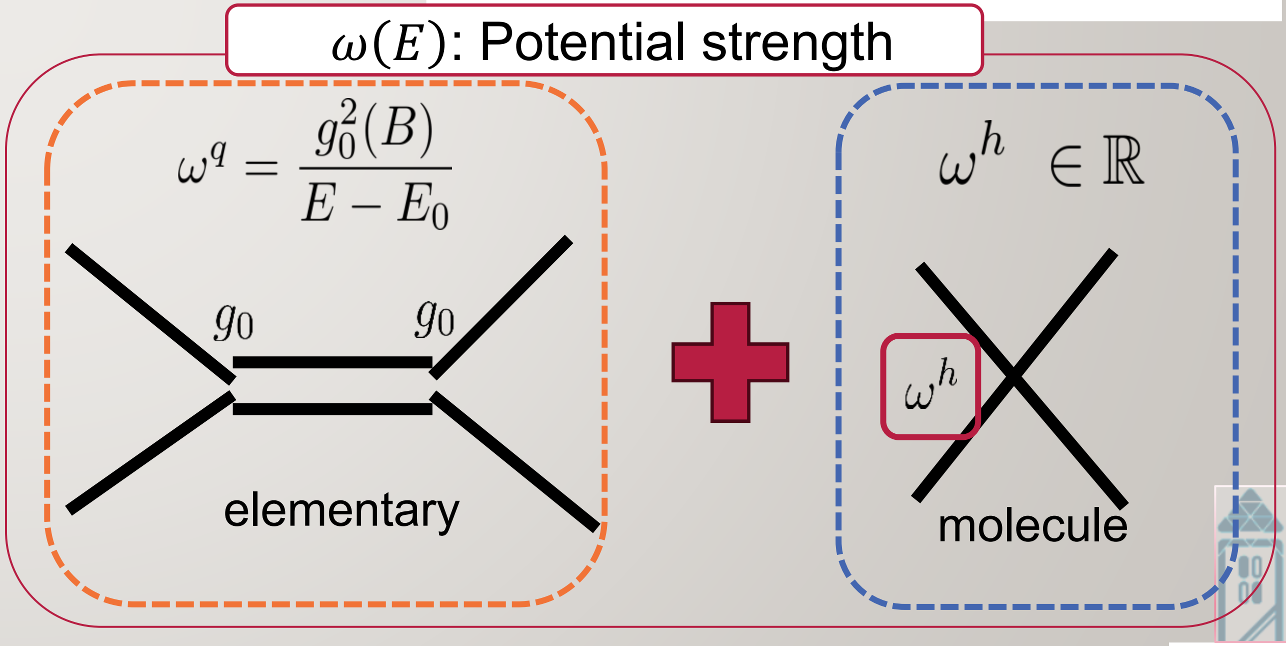
- Parameters in this model

Physical observable	Typical value
$\boldsymbol{E_0}$	78 [MeV] ($\chi_{C1}(2P)$)
B	0.04 [MeV]
μ	140 [MeV]
ω^h	0 [dim'less]



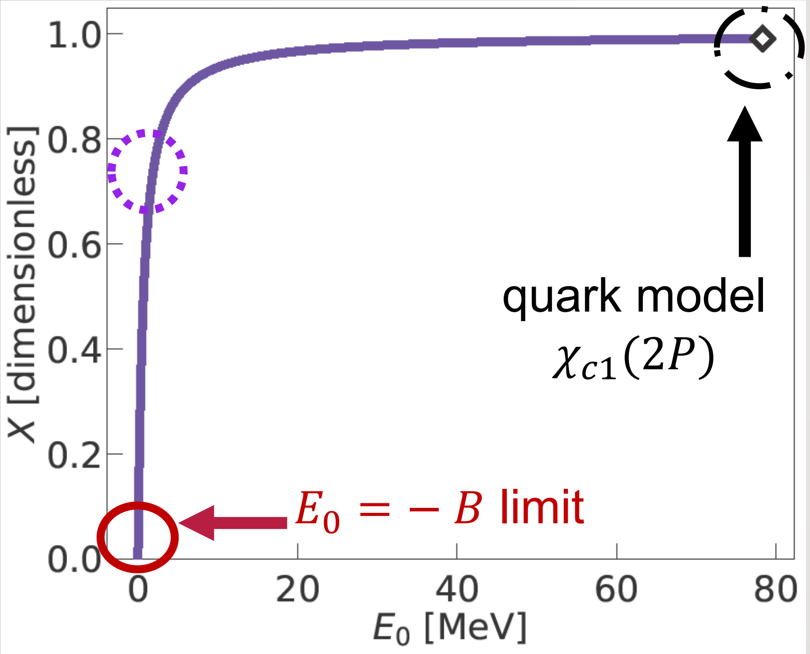
$$V_{\rm eff}^{\bar{D}^*D}(\boldsymbol{r},\boldsymbol{r}',E)=\omega(E)\frac{e^{-\boxed{\mu}r}}{r}\frac{e^{-\boxed{\mu}r'}}{r'}$$

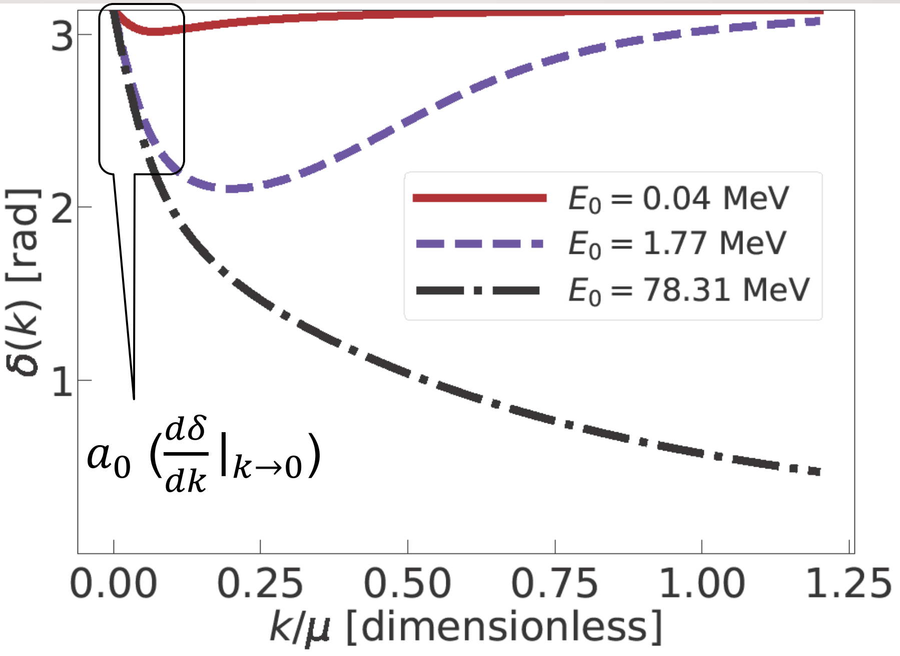
- Next…
  - ✓ Vary 1 and fix other 3 parameters
  - ✓ Compute X and δ, a_0, r_e
  - ✓ Apply to other exotics



Result : E_0 dependence

- Compare with X and scattering observables (phase shift δ, scattering length a_0, effective range r_e) with changing $c\bar{c}$ bare mass E_0 and fixing the others (B, μ, ω^h)





E_0 [MeV]	X [dim'less]	a_0 [fm]	r_e [fm]
0.04	0.1	5	-155
1.77	0.7	21	-3
78.31	1	24	3

- When E_0 is extremely small (enough to break the low-energy universality)
  - ✓ X changes significantly
  - ✓ neither the scatter length (slope of k=0) nor the effective range matches
- Scatter length is always positive but effective range changes from negative to positive

Formulation: Application

Parameters	
$\boldsymbol{E_0}$, B	← fix
μ, ω^h	← change

- Calculate compositeness X by changing μ and ω^h
- Set the limits of a_0 and r_e (given by 4 parameters), by the respective experimental values and errors
- Apply to other exotics (approximation: w/o charged and decay ch.*)

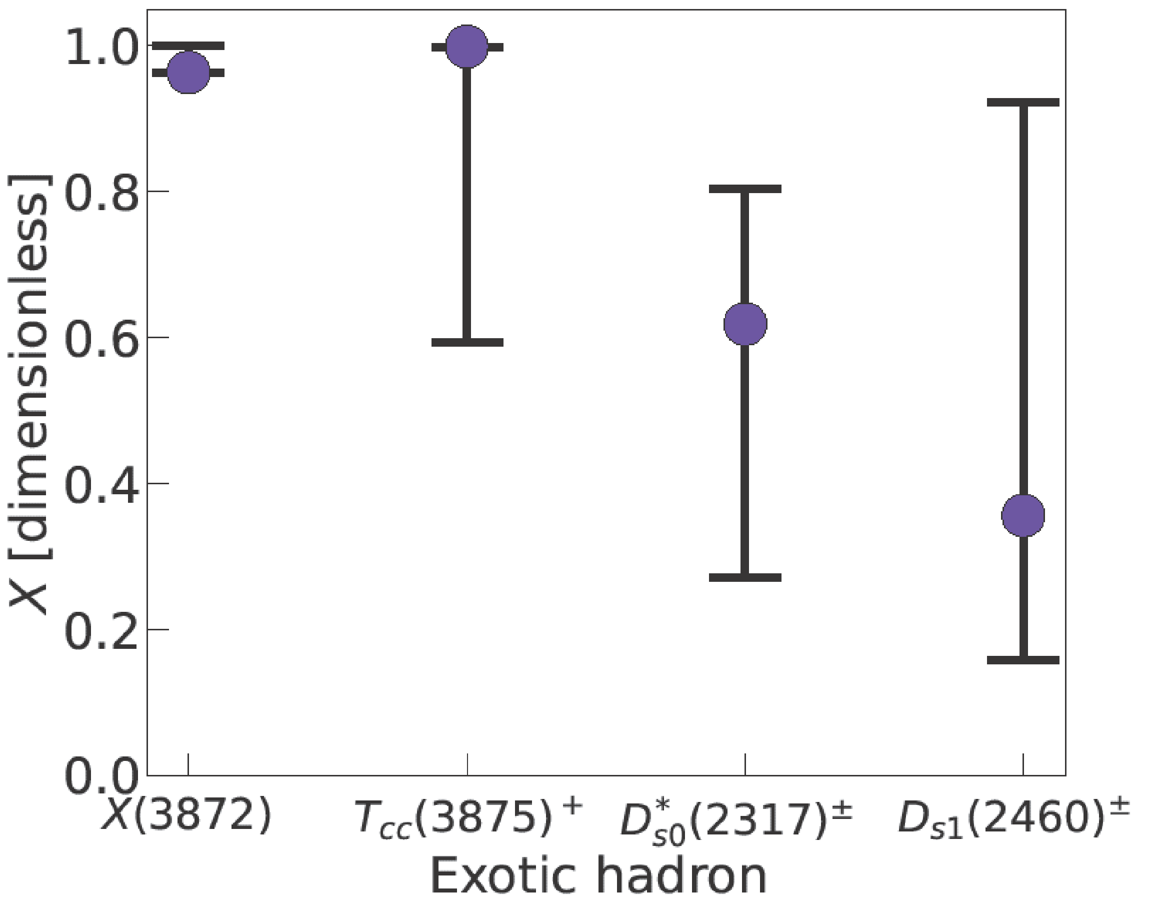
Exotic	System	B [MeV]	E_0 [MeV]	a_0 [fm]	r_e [fm]
$X(3872)$ [1, 2]	$D^0\ \bar{D}^{*0}$	0.040	78 [5]	28.6 ± 5.7	$-2.78 < r_e < 1$
$T_{cc}(3875)^+$ [3]	$D^0\ D^{*+}$	0.36	6.9 [6]	$1.3 \pm 0.5 \pm 0.1$	$-0.1 \pm 0.3 \pm 0.1$
$D_{s0}^*(2317)^\pm$ [4]	$D^\pm\ K^\pm$	45.0	117 [5]	$1.1 \pm 0.5 \pm 0.2$	$-0.2 \pm 0.3 \pm 0{,}1$
$D_{s1}(2460)^\pm$ [4]	$D^{*\pm}\ K^\pm$	44.7	25.8 [5]	7.16 ± 0.51	$0 < r_e < 11.9$

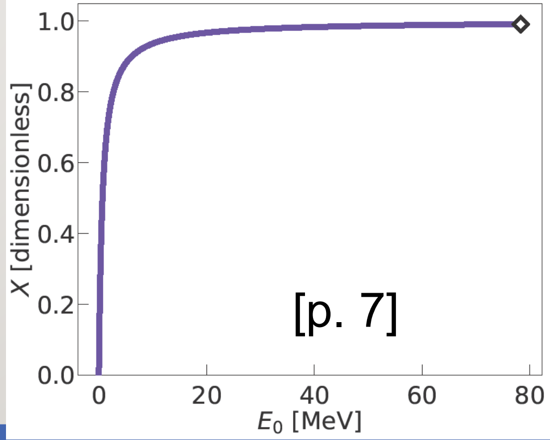
[1] V. Baru, X.-K. Dong, M.-L. Du, A. Filin, F.- K. Guo, C. Hanhart, A. Nefediev, J. Nieves, and Q. Wang, Phys. Lett. B 833, 137290 (2022).
[2] Jing Song, L. R. Dai, and E. Oset. Phys. Rev. D, 108(11):114017, 2023.
[3] Roel Aaij et al. Nature Commun., 13(1):3351, 2022.
[4] A. Mart´ınez Torres, E. Oset, S. Prelovsek, and A. Ramos, JHEP 05, 153.
[5] S. Godfrey and Nathan Isgur. Phys. Rev. D,32:189–231, 1985.
[6] Marek Karliner and Jonathan L. Rosner. Phys. Rev. Lett., 119(20):202001, 2017.

$^*\Gamma_{D_s} \simeq 3.5$ MeV

Result: Application

- Comparing compositeness X
  - $X(3872)$ and $T_{cc}(3875)^+$ → molecule
  - $D_{s0}^*(2317)^\pm$ → Prefer to molecule
  - $D_{s1}(2460)^\pm$ → Prefer to elementary
- $X(3872)$ v.s. $T_{cc}(3875)^+$
  - T_{cc} has smaller E_0 so that X can be smaller
- $D_{s0}^*(2317)^\pm$ v.s. $D_{s1}(2460)^\pm$
  - D_{s1} has smaller E_0 so that X can be smaller
  - $r_e^{D_{s1}}$ has larger error so that error of X can be larger





Summary

[I. Terashima and T. Hyodo, PhysRevC.108.035204 (2023)]
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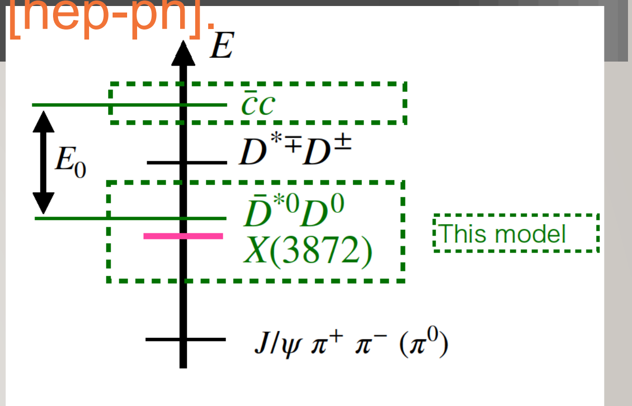
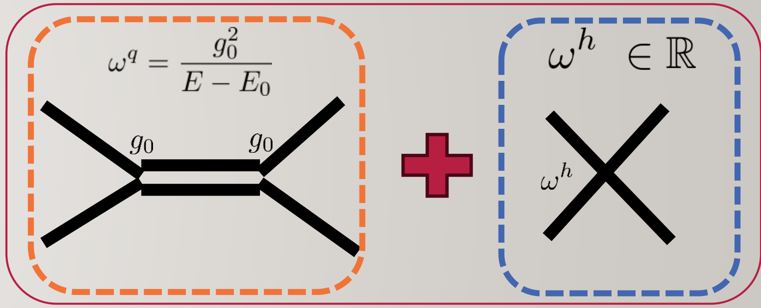


- Channel coupling between $c\bar{c}$ and $D\bar{D}^*$ in $X(3872)$

$$H = \begin{pmatrix} T^{c\bar{c}} & 0 \\ 0 & T^{\bar{D}^*D} + \Delta \end{pmatrix} + \begin{pmatrix} V^{c\bar{c}} & V^t \\ V^t & V^{\bar{D}^*D} \end{pmatrix}$$

- Effective potential with explicit V^q and V^h

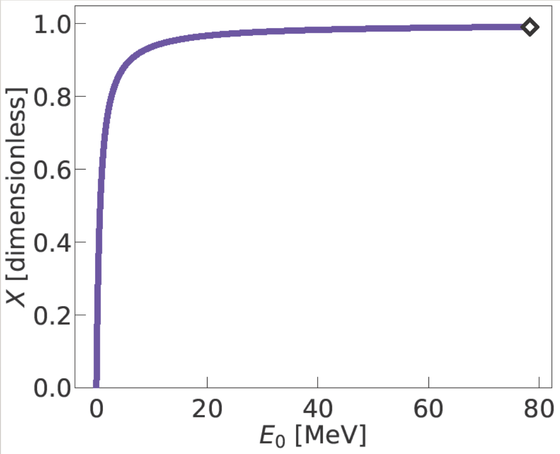
$$V_{\mathrm{eff}}^{\bar{D}^*D}(\boldsymbol{r}, \boldsymbol{r}', E) = [\omega^q(E) + \omega^h(E)]V(\boldsymbol{r})V(\boldsymbol{r}')$$



- Compositeness X in analytical form

$$X = [1 + \frac{g_0^2\kappa\mu(\kappa+\mu)^3}{8\pi m^2(g_0^2 + (E - E_0)\omega^h)^2}]^{-1}$$

- Parameter dependences for compositeness
  - Especially for E_0, changes from 0 (elementary) to 1 (molecule).



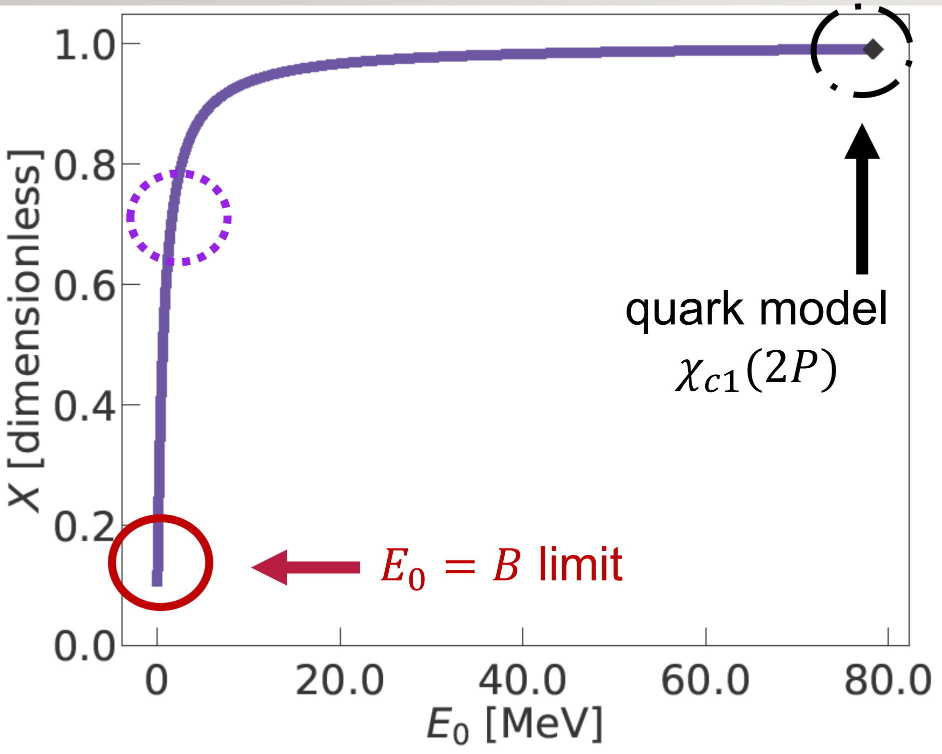
- Application to other exotics

	$X(3872)$	$T_{cc}(3875)^+$	$D_{s0}^*(2317)^{\pm}$	$D_{s1}(2460)^{\pm}$
X [dim'less]	$0.96^{+0.04}_{-0.01}$	$1.00^{+0.00}_{-0.40}$	$0.62^{+0.19}_{-0.35}$	$0.36^{+0.57}_{-0.20}$

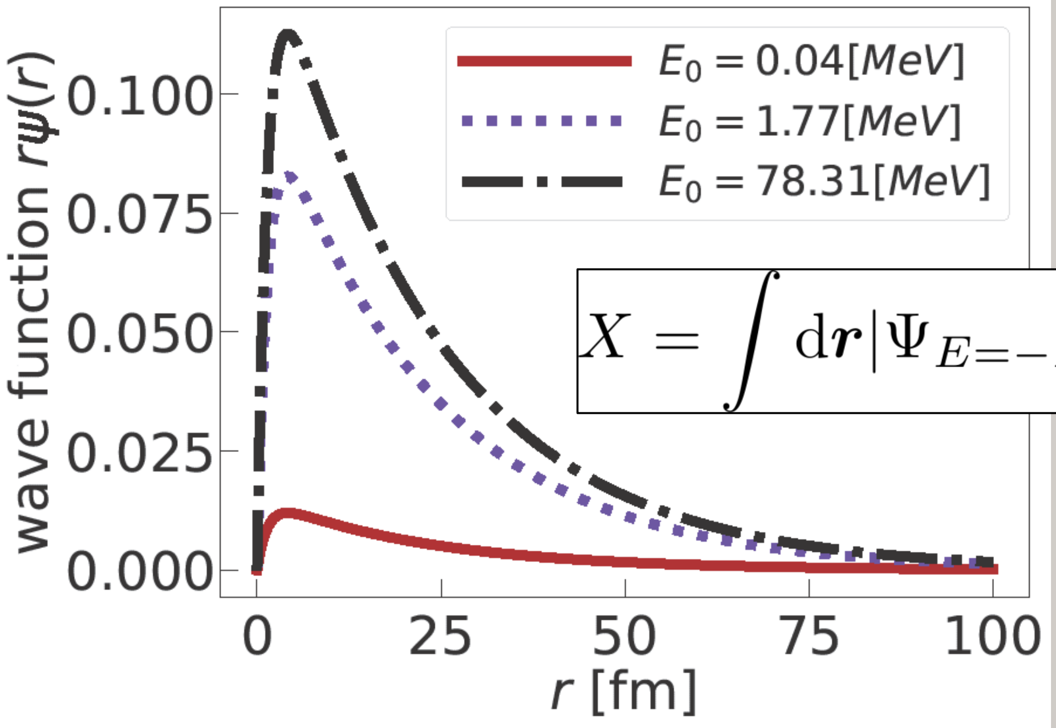
Result : E_0 dependence

Case: X changes

- Comparison of change in compositeness X and change in wave function



➢ Binding energy ↔ Quark model ($\chi_{c1}(2P)$)



$$X = \int \mathrm{d}\boldsymbol{r} |\Psi_{E=-B}(\boldsymbol{r})|^2$$

✓ When E_0 is extremely small (enough to break the low-energy universality), the compositeness changes significantly

✓ The shape of the wave function changes with compositeness X from

$$\Psi = \sqrt{X}\frac{\mu\kappa(\mu+\kappa)}{2\pi(\mu-\kappa)^2}\left(\frac{e^{-\mu r}}{r} - \frac{e^{-\kappa r}}{r}\right)$$

Result : E_0 dependence

E_0 [MeV]	X [dim'less]	a_0 [fm]	r_e [fm]
0.04	0.1	5	-155
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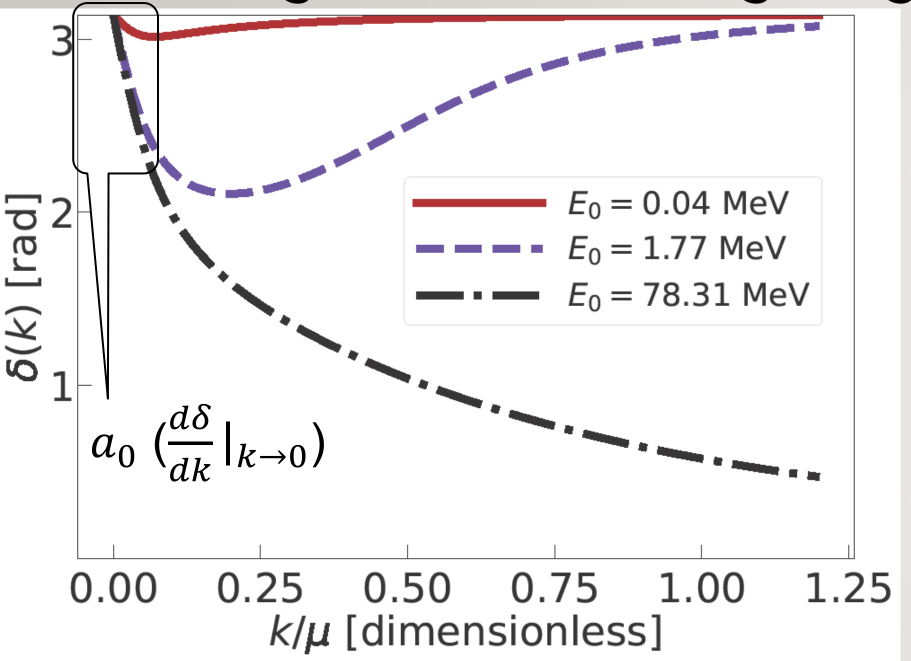
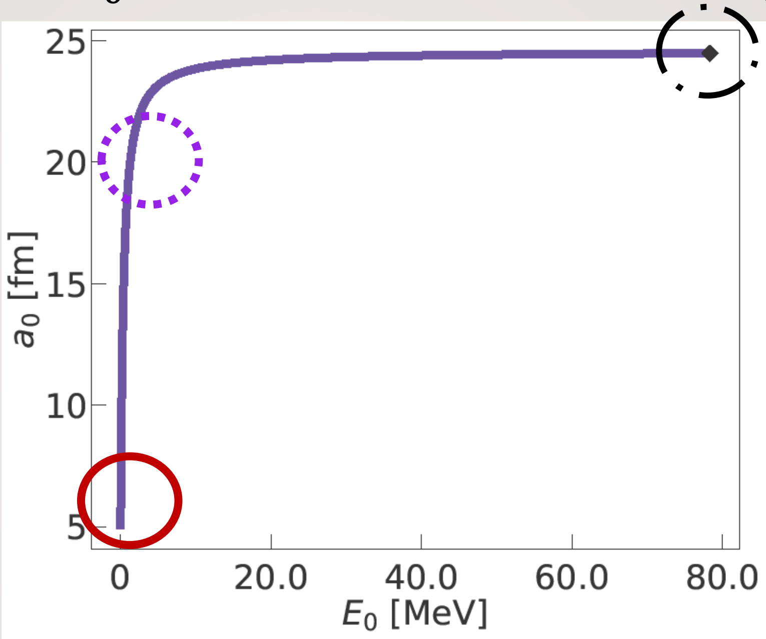
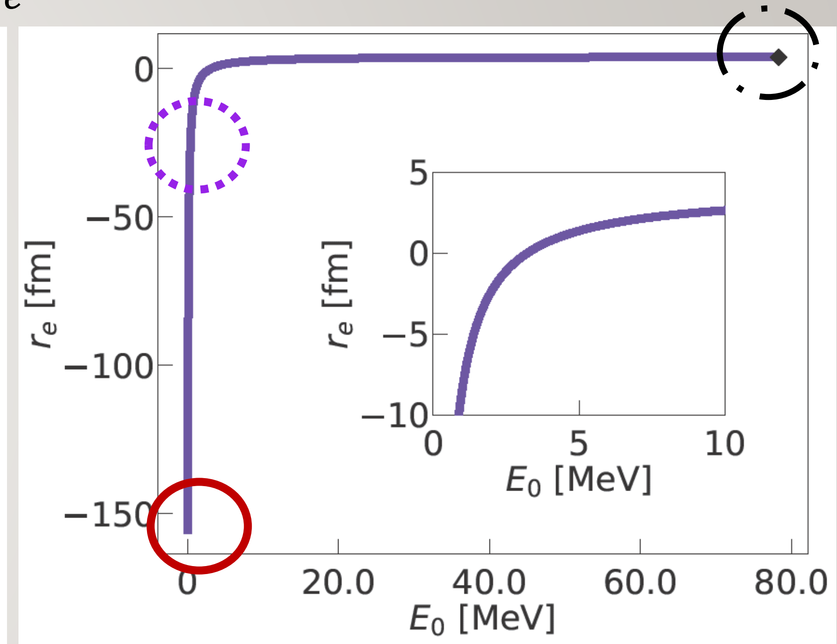
- Comparison of changes in phase shift $\delta(k)$ with changes in scattering length a_0 and effective range r_e







✓When E_0 is extremely small → neither the scatter length (slope of k=0) nor the effective range matches

✓Scatter length is always positive but effective range changes from negative to positiveV