The source of flavor SU(3) breaking of axial vector mesons as chiral partners of vector mesons

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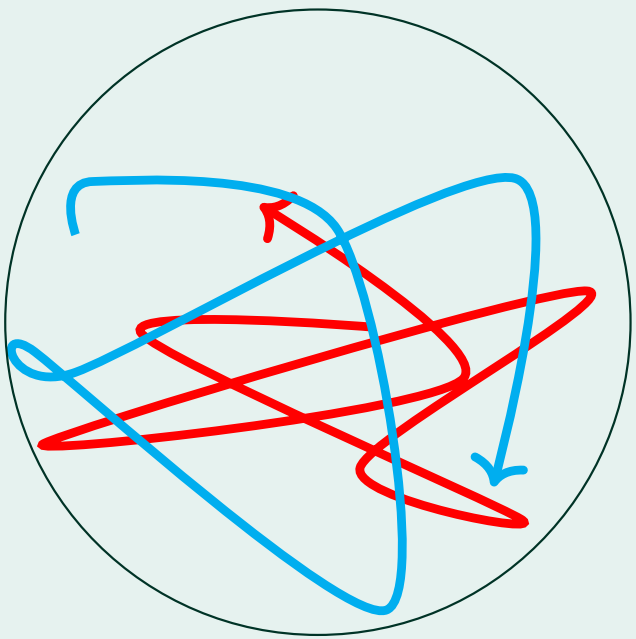
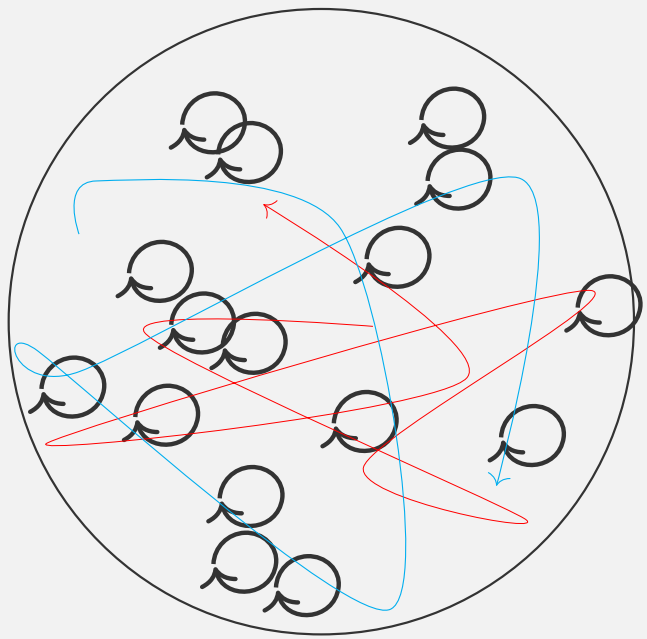
Hadron mass generation

How to interpret hadron mass generation

~~Current quark~~ Constituent quark



Vacuum structure



This picture needs to be modified in axial vector mesons

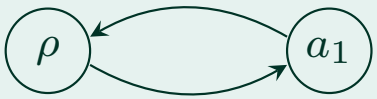
Chiral partner

Chiral partner should reflect the chiral symmetry breaking properties

Chiral partner

: A pair of fields which are transformed by chiral transformation each other

- **Degenerate between chiral partners if chiral symmetry is completely restored**

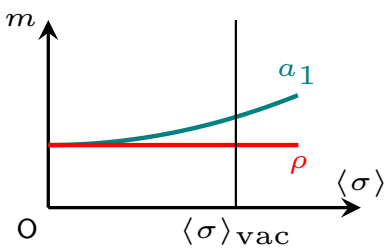


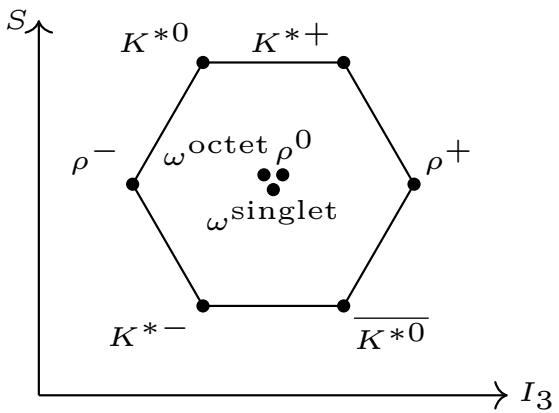
vector octet + axial vector octet

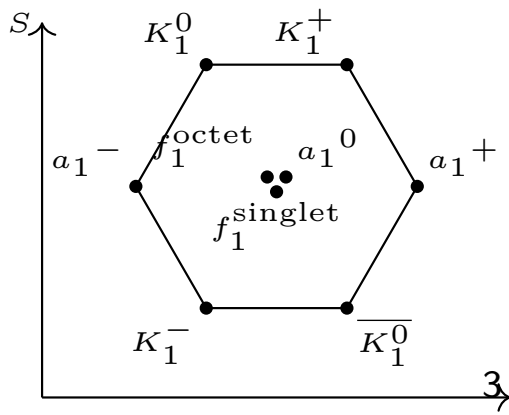
vector singlet

axial vector singlet

S. H. Lee., Subatomic Part. Cosmol., 1-2:100008, 2024.







Motivation

Explain axial vector mass spectrum without contradict to vector meson mass

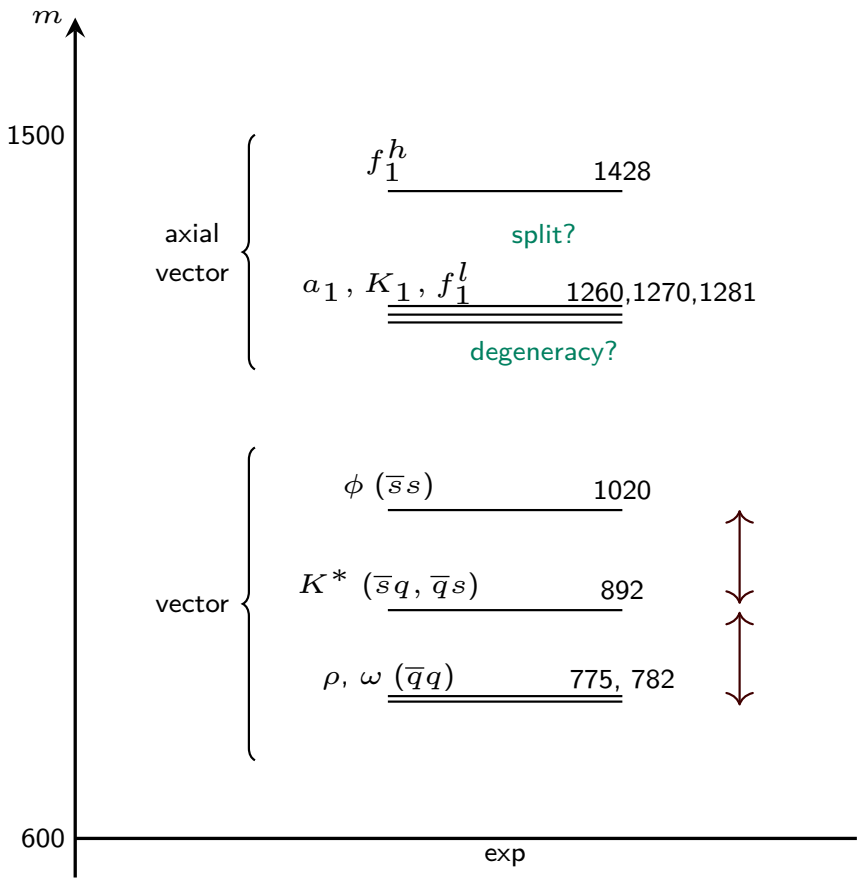
Constituent quark counting

- The mass difference have good agreement with the number of s quarks
- Effective mass
  $m^*_{q_N} \sim 300\ \mathrm{MeV}, m^*_{q_S} \sim 500\ \mathrm{MeV}$

This picture cannot explain axial vector mesons

The result for this study

- Try to make a scenario to explain axial vector meson



S. Navas et al. (Particle Data Group), Phys. Rev. D 110, 030001 (2024)

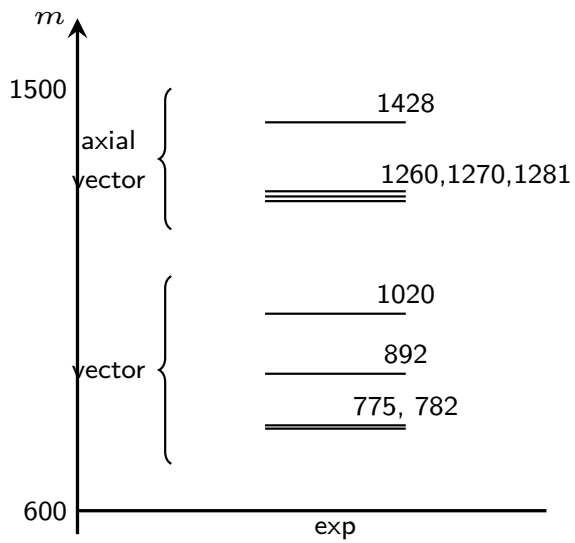
Idea

Singlet-octet splitting

- Only the $f_1(1420)$ breaks this degeneracy
- $f_1(1420)$ is from the group theoretical singlet
- This is equivalent not to be ideally mixed

SU(3) balance

- The $f_1(1285)$, $K_1(1270)$, and $a_1(1260)$ seems to be degenerate
- This degeneracy is for the cancellation of two sources of SU(3) symmetry breaking, $\langle\sigma\rangle$ and m_q



S. Navas et al. (Particle Data Group),
Phys. Rev. D 110, 030001 (2024)

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The model selections

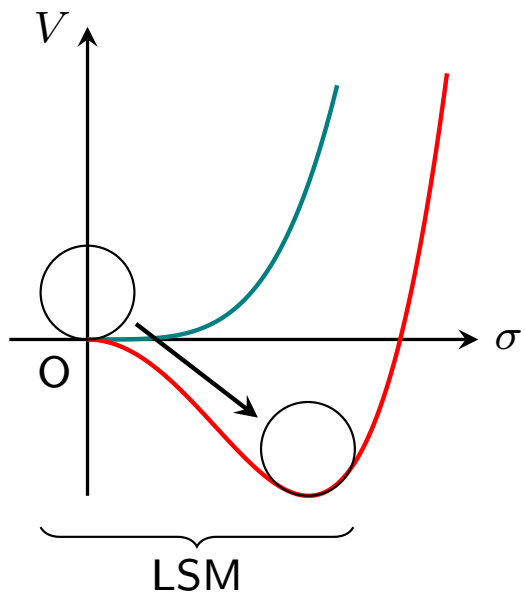
The chiral symmetry breaking mechanism should be contained in the model

Linear sigma model (LSM)

- Can describe both chiral symmetry is broken phase and completely restored phase
- Easy to analyze in-medium region
- Can see where the mass components come from

Extended linear sigma models

- Succeeded in explaining vector meson nonet



F. Giacosa et al., Prog.Part.Nucl.Phys. 143 (2025) 104176

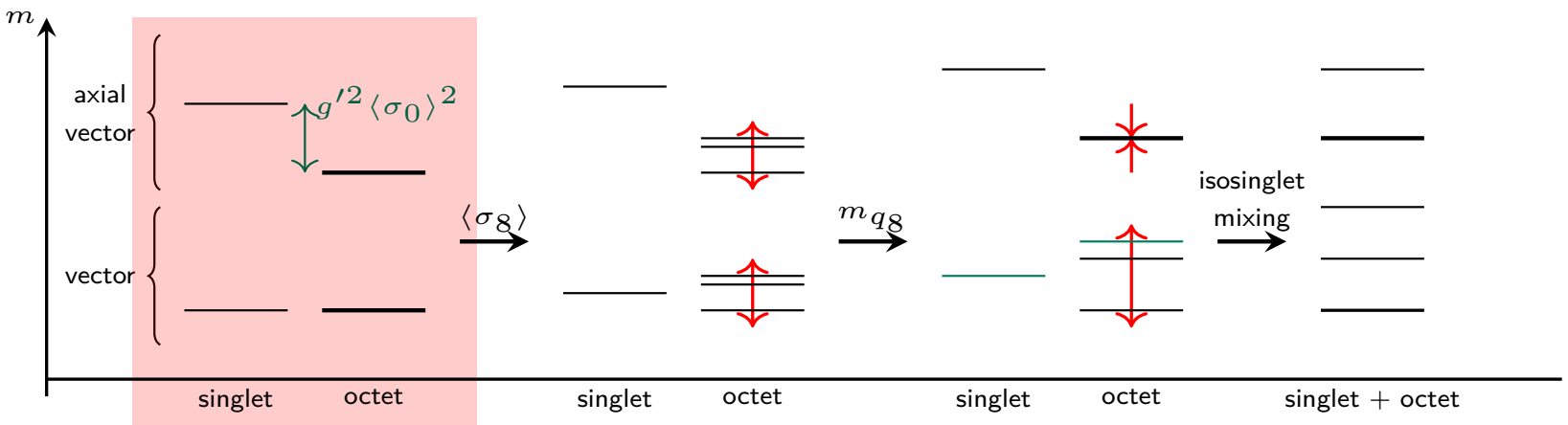
The model for singlet octet splitting

Singlet gauge coupling

- Assume that singlet and octet have different gauge coupling

$$D_\mu M = \partial_\mu M + igL_\mu M - igMR_\mu + ig' A_\mu^{\text{singlet}} M \tag{1}$$

- The singlet does not have chiral partner $\rightarrow$ do not have to choose same g value
- This g' term produces only axial vector singlet mass



The model for SU(3) balance

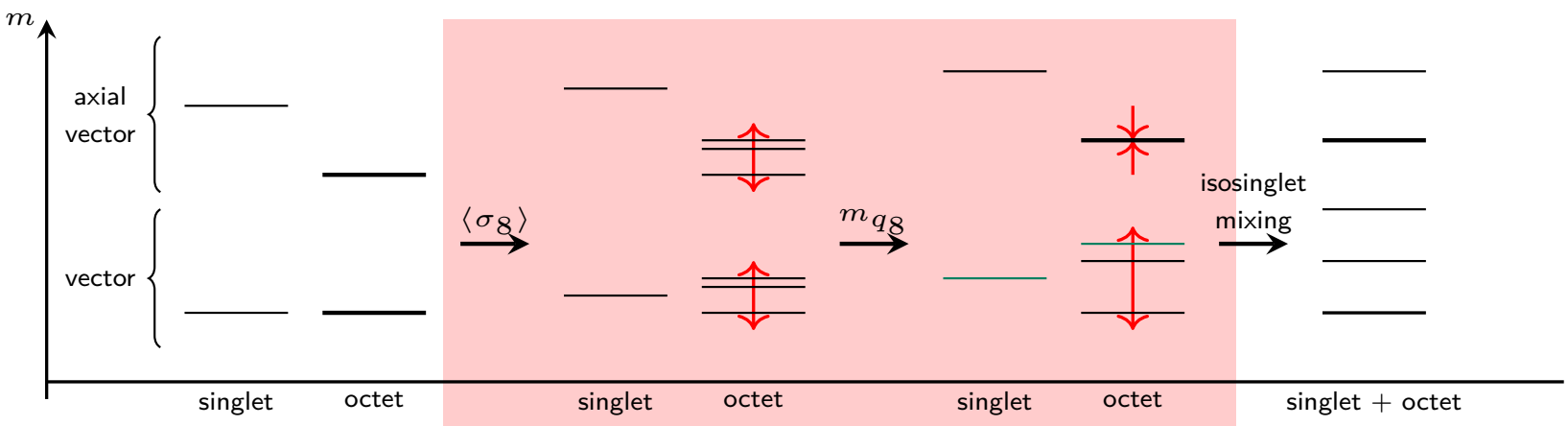
The source of SU(3) breaking

- Two different SU(3) breaking sources are needed
- Vacuum expectation value $\langle\sigma\rangle$ and current quark mass m_q are the sources

  $(m_{q_8} = \frac{m_{q_N} - 2m_{q_S}}{\sqrt{6}}, \quad m_{q_0} = \frac{m_{q_N} + m_{q_S}}{\sqrt{3}})$

$$\frac{\langle\sigma_S\rangle}{\langle\sigma_N\rangle} \simeq 0.76, \qquad \frac{m_{q_S}}{m_{q_N}} \simeq 27 \tag{2}$$

- Four point couplings



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Result

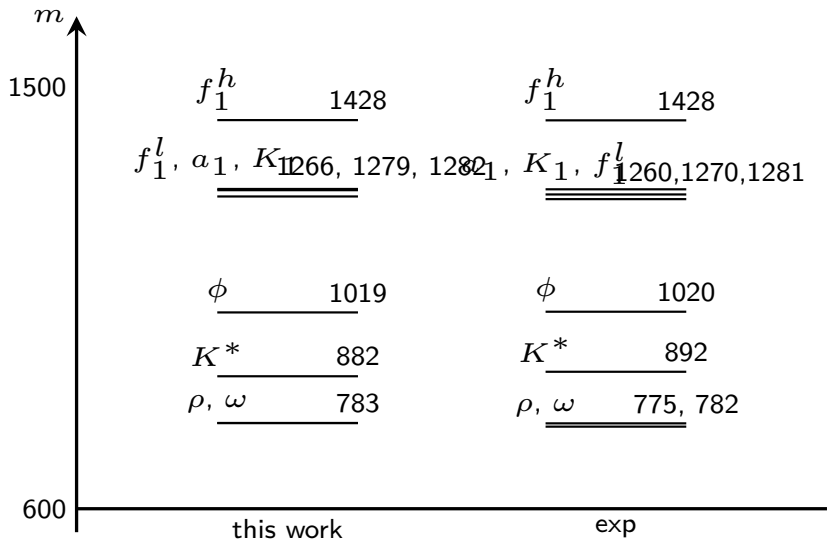
Vacuum mass

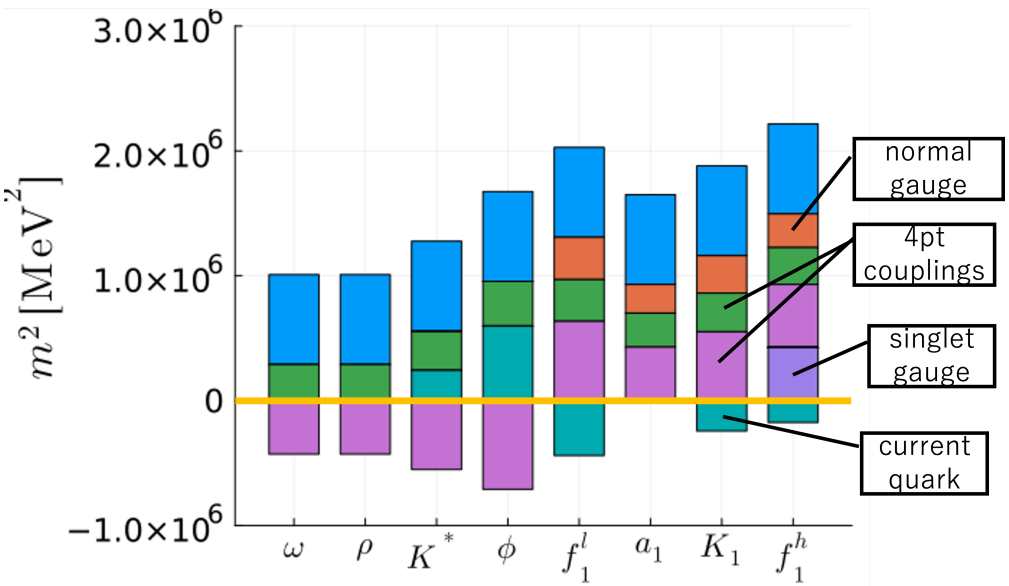
- Good fit result
  →this scenario is acceptable
- Octet-singlet splitting, SU(3) balance is achieved

Decay width

- Vector meson decay widths match naturally

S. Navas et al. (Particle Data Group), Phys. Rev. D 110, 030001 (2024)

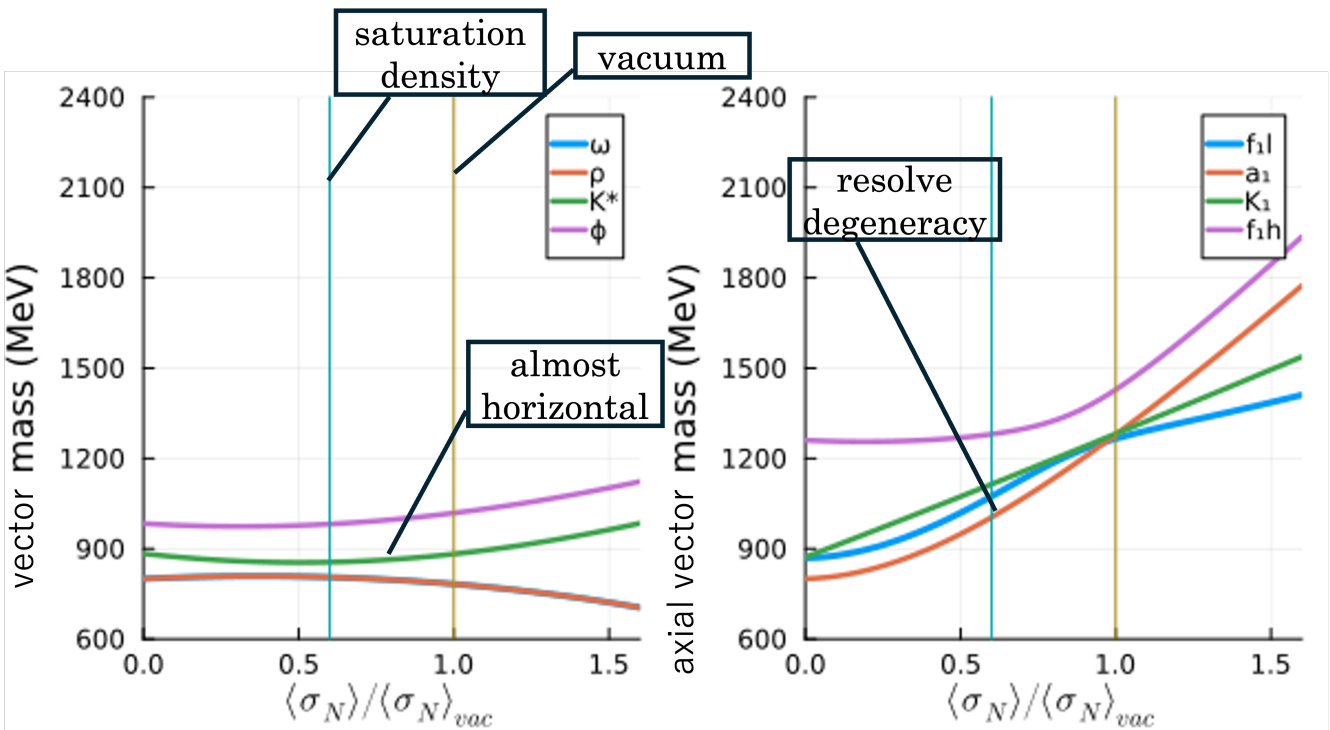




Result

The demonstration in fintie density

- Choose $\langle\sigma_S\rangle = \text{constant}$ and $\langle\sigma_N\rangle$ varies as a scenario
- Assume $\langle\sigma_N\rangle/\langle\sigma_N\rangle_{vac} \sim 0.6$
- Analize in-medium by moving $\langle\sigma\rangle$ around by hand
- Vector mesons are almost parallel to horizontal axis
- **Axial vector octet degeneracy is resolved**
- It could be observed by experiment in nuclear medium



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Summary and future work

Summary

Model construction

- Try to explain the axial vector mass without contradicting to vector spectrum
- Assume a combination of two interpretations, SU(3) balance and singlet octet splitting
- Construct a model with interpretations

Parameter fitting

- We confirm the scenario is an acceptable one
- SU(3) balance and signlet-octet splitting are achieved

Demonstration on finite density region

- Mass difference is smaller for vector mesons, and larger for axial vector mesons
- a_1, K_1 and $f_1(1285)$ mass differ about 50 MeV each in finite density

Future work

Finite density

- We can use relativistic mean-field prescription $\rightarrow$ more accurate calculations
- Comparison to the experimental result in finite density

Model Lagrangian

$$\mathcal{L} = \mathrm{Tr}[(D_\mu M)^\dagger (D^\mu M)] - \mu^2 \mathrm{Tr}[M^\dagger M] - \lambda_1 \left(\mathrm{Tr}[M^\dagger M]\right)^2 - \lambda_2 \mathrm{Tr}[(M^\dagger M)^2]$$
$$+ c_1 \left(\det M + \det M^\dagger\right) + \epsilon \mathrm{Tr}[\mathcal{M} M^\dagger + M \mathcal{M}] - \frac{1}{4}\mathrm{Tr}[L_{\mu\nu} L^{\mu\nu} + R_{\mu\nu} R^{\mu\nu}]$$
$$+ \frac{m_0^2}{2} \mathrm{Tr}\left[(L_\mu L^\mu + R_\mu R^\mu)\right] + h_2 \mathrm{Tr}(L_\mu M M^\dagger L^\mu + R_\mu M^\dagger M R^\mu) + 2 h_3 \mathrm{Tr}(L_\mu M R^\mu M^\dagger)$$
$$+ h_2' \mathrm{Tr}(L_\mu \mathcal{M} M^\dagger L^\mu + L_\mu M \mathcal{M} L^\mu + R_\mu \mathcal{M} M R^\mu + R_\mu M^\dagger \mathcal{M} R^\mu) + 2 h_3' \mathrm{Tr}(L_\mu \mathcal{M} R^\mu M^\dagger + L_\mu M R^\mu \mathcal{M}) \tag{3}$$

$$D_\mu M = \partial_\mu M + i g L_\mu M - i g M R_\mu^\dagger + i g' L_\mu^0 M - i g' M R_\mu^{0\dagger} \tag{4}$$

$$L_{\mu\nu} = \partial_\mu L_\nu - \partial_\nu L_\mu + i g [L_\mu, L_\nu], \quad R_{\mu\nu} = \partial_\mu R_\nu - \partial_\nu R_\mu + i g [R_\mu, R_\nu] \tag{5}$$

$$M = \Sigma + i\Pi, \qquad L_\mu = \frac{V_\mu + A_\mu}{2}, \qquad R_\mu = \frac{V_\mu - A_\mu}{2} \tag{6}$$

Model Lagrangian

$$\Sigma = \begin{pmatrix} \frac{1}{\sqrt{3}}\sigma + \frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}f_0 & a_0^+ & K_0^+ \\ a_0^- & \frac{1}{\sqrt{3}}\sigma - \frac{1}{\sqrt{2}}a_0^0 + \frac{1}{\sqrt{6}}f_0 & K_0^0 \\ K_0^- & \overline{K_0^0} & \frac{1}{\sqrt{3}}\sigma - \frac{2}{\sqrt{6}}f_0 \end{pmatrix}$$

$$\Pi = \begin{pmatrix} \frac{1}{\sqrt{3}}\eta_0 + \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & \pi^+ & K^+ \\ \pi^- & \frac{1}{\sqrt{3}}\eta_0 - \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 & K^0 \\ K^- & \overline{K^0} & \frac{1}{\sqrt{3}}\eta_0 - \frac{2}{\sqrt{6}}\eta_8 \end{pmatrix}$$

$$V_\mu = \begin{pmatrix} \frac{1}{\sqrt{3}}\omega_{0\mu} + \frac{1}{\sqrt{2}}\rho_\mu^0 + \frac{1}{\sqrt{6}}\omega_{8\mu} & \rho_\mu^+ & K_{0\mu}^{*+} \\ \rho_\mu^- & \frac{1}{\sqrt{3}}\omega_{0\mu} - \frac{1}{\sqrt{2}}\rho_\mu^0 + \frac{1}{\sqrt{6}}\omega_{8\mu} & K_{0\mu}^{*0} \\ K_{0\mu}^{*-} & \overline{K_{0\mu}^{*0}} & \frac{1}{\sqrt{3}}\omega_{0\mu} - \frac{2}{\sqrt{6}}\omega_{8\mu} \end{pmatrix}$$

$$A_\mu = \begin{pmatrix} \frac{1}{\sqrt{3}}f_{10\mu} + \frac{1}{\sqrt{2}}a_{1\mu}^0 + \frac{1}{\sqrt{6}}f_{18\mu} & a_{1\mu}^+ & K_{1\mu}^+ \\ a_{1\mu}^- & \frac{1}{\sqrt{3}}f_{10\mu} - \frac{1}{\sqrt{2}}a_{1\mu}^0 + \frac{1}{\sqrt{6}}f_{18\mu} & K_{1\mu}^0 \\ K_{1\mu}^- & \overline{K_{1\mu}^0} & \frac{1}{\sqrt{3}}f_{10\mu} - \frac{2}{\sqrt{6}}f_{18\mu} \end{pmatrix} \tag{7}$$

Chiral transformations

Chiral transformations

- Vector and axial vector singlet are not a chiral partner

$$v_\mu^a \xrightarrow{\text{axial}} v_\mu^a + f^{abc}\theta_A^b a_\mu^c, \qquad a_\mu^a \xrightarrow{\text{axial}} a_\mu^a + f^{abc}\theta_A^b v_\mu^c \tag{8}$$

- Antisymmetric structure constant for SU(3) f does not couple to singlet
- They are just a parity partner

S. H. Lee., Subatomic Part. Cosmol., 1-2:100008, 2024.

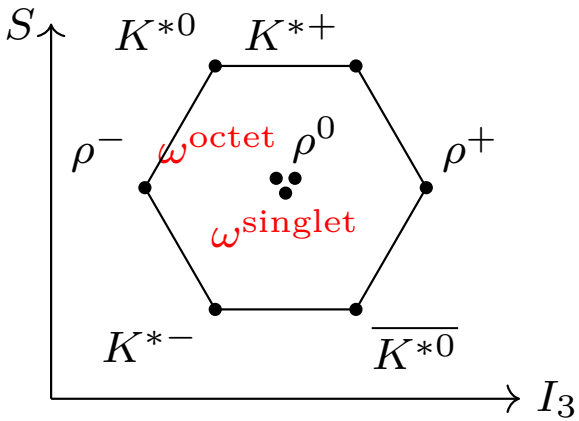
Isosinglet mixing

Are $f_1(1285)$ and $f_1(1420)$ ideally mixed?

Isosinglet mixing

- Singlet and octet isosinglet on same spins have same quantum number
- Mixing angle θ represents how singlet and octet isosinglet are mixed

$$\begin{pmatrix} |\phi\rangle \\ |\omega\rangle \end{pmatrix} = R(\theta) \begin{pmatrix} |\omega^{\text{singlet}}\rangle \\ |\omega^{\text{octet}}\rangle \end{pmatrix} \tag{9}$$



Ideal mixing

- The quark components u, d and s split due to a relation $m_s \gg m_u, m_d$.

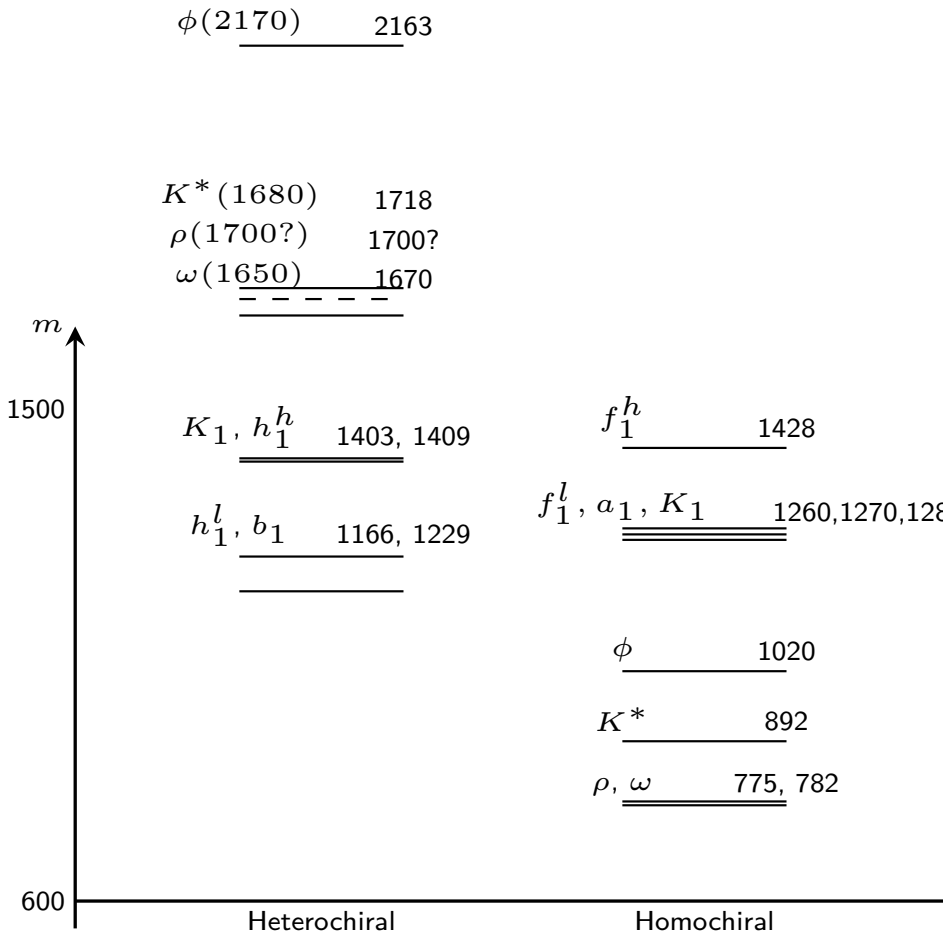
$$\omega^{\text{singlet}} = \frac{\overline{u}u + \overline{d}d + \overline{s}s}{\sqrt{3}}, \quad \omega^{\text{octet}} = \frac{\overline{u}u + \overline{d}d - 2\overline{s}s}{\sqrt{6}} \xrightarrow{\text{Ideal mixing}} \omega = \frac{\overline{u}u + \overline{d}d}{\sqrt{2}}, \quad \phi = \overline{s}s \tag{10}$$

- $\rho - \omega$ degeneracy is come from fact that both quark component are u and d.
- This picture is good in vector mesons (because the decays with OZI suppressed are rare)
- **There is no consensus for axial vector isosinglet $f_1(1285)$ and $f_1(1420)$**

Discussion: K_1 mixing

K_1 mixing

- Other explanation for axial vector spectrum: K_1 mixing angle $\sim 45^\circ$ and ideal mixing
- Another axial vector (hetero chiral) mesons ($h_1(1170)$, b_1, $K_1(1400)$, $h_1(1415)$)
- The $K_1(1270)$ mixes to $K_1(1400)$
- The K_1 mixing angle is not determined yet
- Our study suggest that the K_1 mixing angle need not be around 45°



S. Navas et al. (Particle Data Group), Phys. Rev. D 110, 030001 (2024)
Y.-J. Shi, J. Zeng, and Z.-F. Deng., Phys. Rev. D, 109(1):016027, 2024
H.-Y. Cheng, Phys. Lett. B, 707:116 – 120, 2012.

Result

$\langle\sigma\rangle$ dependence

- $\langle\sigma\rangle$ becomes small in finite density.
- There are several scenarios on $\langle\sigma_N\rangle$ and $\langle\sigma_S\rangle$ for finite density
- We choose the scenario that only the $\langle\sigma_N\rangle$ sensitive to ρ

$$\langle\sigma_N\rangle_{\rm med} \sim 0.6\langle\sigma_N\rangle_{\rm vac}$$
$$\langle\sigma_S\rangle_{\rm med} \sim \langle\sigma_S\rangle_{\rm vac} \qquad (11)$$

- We take cross sections for $\langle\sigma_S\rangle = \text{constant}$

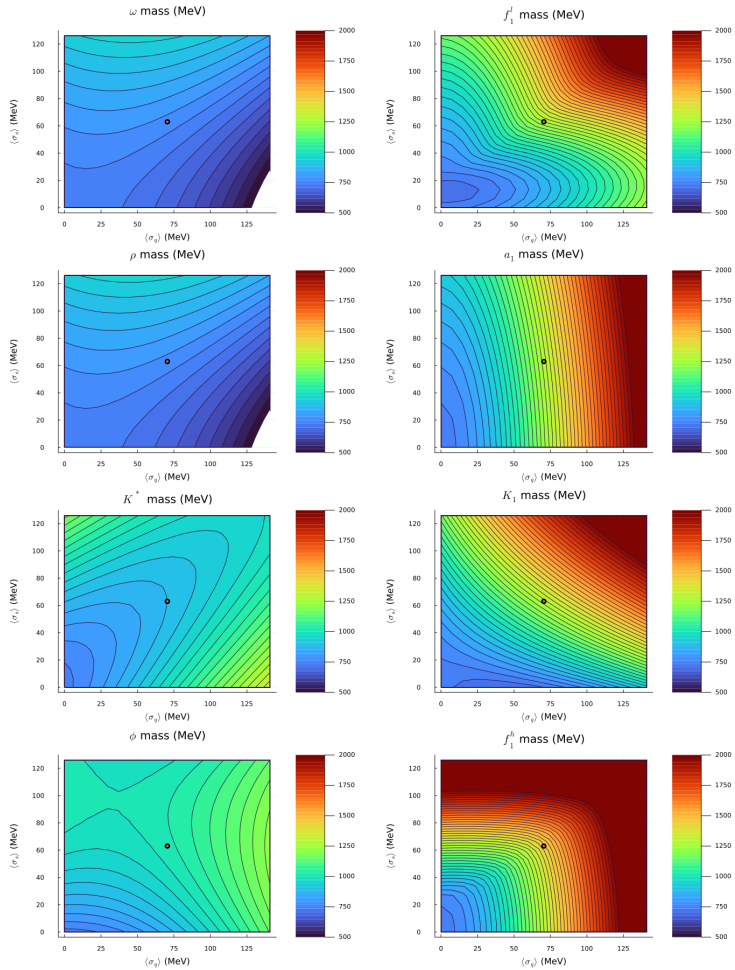


Fig.: x-axis $\langle\sigma_N\rangle$, y-axis $\langle\sigma_S\rangle$, color map displays each meson masses